Logic and Proof Spring 2021

Assignment 4 (Deadline: 6:00pm on 2 June 2021)

Prof Hongseok Yang

Submit your solutions in KLMS. (Reminder: We adopt a very strict policy for handling dishonest behaviours. If a student is found to copy answers from fellow students or other sources in his or her homework submission, she or he will get F.)

1. Consider the unification algorithm from the lecture notes.

   (a) Apply the algorithm to the set of literals

   $$\mathbf{L} = \{P(x, y), P(f(a), g(x)), P(f(z), g(f(z)))\}.$$

   (10 points)

   (b) Suppose we omit the *occur check* "does x occur in t" to improve efficiency. Exhibit literals L_1 and L_2 with no variable in common such that the unification algorithm fails to terminate on $\{L_1, L_2\}$.

   (10 points)

2. Express the following by formulas of first-order logic, using predicate $H(x)$ for "x is happy", $R(x)$ for "x is rich", $G(x)$ for "x is a graduate", and $C(x, y)$ for "y is a child of x".

   (a) Any person is happy if all their children are rich.

   (b) All graduates are rich.

   (c) Someone is a graduate if she or he is a child of a graduate.

   (d) All graduates are happy.

   Use first-order resolution (i.e. unification-based resolution) to show that (d) is entailed by (a), (b) and (c). Indicate the substitutions in each resolution step.

   (20 points)

3. Give an example of a finite set of clauses F in first-order logic such that $Res^*(F)$ is infinite.

   (20 points)

4. Professor Long claims that given closed formulas F and G in Skolem form, if every Herbrand model of F is a Herbrand model of G then $F \models G$. Is Professor Long correct? Justify your answer.

   (20 points)

5. A closed formula (i.e., formula without free variables) is in the class $\exists^*\forall^*$ if it has the form $\exists x_1 \dots \exists x_m \forall y_1 \dots \forall y_n F$, where F is quantifier-free and $m, n \geq 0$.

   (a) Prove that if a closed $\exists^*\forall^*$-formula over a signature with no function symbols has a model, then it has a finite model.

   (7 points)

(b) Suggest an algorithm for deciding whether a given closed $\exists^*\forall^*$-formula over a signature with no function symbols has a model.

(6 points)

(c) A closed formula is in the class $\exists^*$ if it has the form $\exists x_1 \ldots \exists x_m\, F$, where F is quantifier-free and $m \geq 0$. Prove that the validity problem for the class of closed $\exists^*$-formulas that may mention function symbols is undecidable.

(7 points)