Introduction to Logic for Computer Science Spring 2021

Assignment 2 (Deadline: 6:00pm on 16 April 2021)

Prof Hongseok Yang

Submit your solutions in KLMS. (Reminder: We adopt a very strict policy for handling dishonest behaviours. If a student is found to copy answers from fellow students or other sources in his or her homework submission, she or he will get F.

1. Show that for any CNF formula F one can compute in polynomial time an equisatifiable formula $G_1 \wedge G_2$, with G_1 a Horn formula and G_2 a 2-CNF formula. Justify your answer.

   (10 points)

2. A *renamable Horn formula* is a CNF formula that can be turned into a Horn formula by negating (all occurrences of) some of its variables. For example,

   $$(P_1 \vee \neg P_2 \vee \neg P_3) \wedge (P_2 \vee P_3) \wedge (\neg P_1)$$

   can be turned into a Horn formula by negating P_1 and P_2.

   Given a CNF-formula F, show how to derive a 2-CNF formula G such that G is satisfiable if and only if F is a renamable Horn formula. Show moreover that one can derive a renaming that turns F into a Horn formula from a satisfying assignment for G. **(10 points)**

3. (a) Show that a Horn-renamable CNF formula in which no unit clauses occur has a satisfying assignment which makes in every clause all literals true except at most one. By unit clause, we mean a clause with only one literal.

   (15 points)

   (b) Show that when run on a Horn-renamable CNF formula F in which no unit clauses occur the Walk-SAT algorithm moves in every iteration with probability at least 1/2 towards a satisfying assignment of F.

   (15 points)

4. Using resolution, show that $A \wedge B \wedge C$ is a consequence of

   $$F = (\neg A \vee B) \wedge (\neg B \vee C) \wedge (A \vee \neg C) \wedge (A \vee B \vee C)\,.$$

   (10 points)

5. Using resolution, or otherwise, show that there is a polynomial-time algorithm to decide satisfiability of those CNF formulas F in which each propositional variable occurs at most twice. Explain why your answer is correct and briefly explain why it meets the required time bound. **(10 points)**

6. *Positive resolution* is a restriction of ordinary resolution, which is defined as follows: derive a resolvent from C_1 and C_2 only if C_1 is a positive clause, i.e., it consists only of positive literals. By adapting the completeness proof from lectures, show that if a propositional formula F is an unsatisfiable CNF formula then one can derive the empty clause from F using only positive resolution. **(30 points)**