

Lecture 6

The DPLL Algorithm

Introduction to Logic for Computer Science

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These slides are minor variants of those made by Prof Worrell and Dr Haase for their logic course at Oxford.

Davis–Putnam–Logemann–Loveland

DPLL algorithm:

- Combines search and deduction to decide satisfiability.
- Underlies most modern SAT solvers.
- Over 50 years old.



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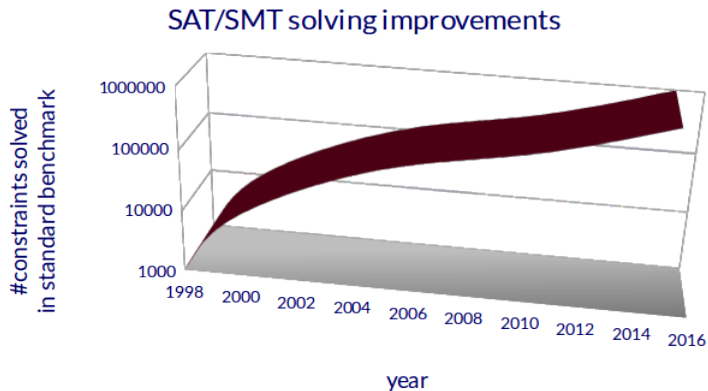


Dramatic progress of DPLL-based SAT solvers since 1990.

Emerged enhancement:

- **Clause learning.**
- **Non-chronological backtracking.**
- Branching heuristics.
- Lazy evaluation.

Performance increase of SAT solvers



DPLL: idea

Depth-first search.

At every unsuccessful leaf of search tree (called **conflict**), use resolution to compute a **conflict clause**.

Add the clause to the formula we're deciding about.



Think of conflict clauses as “caching” previous search results. So we “learn from previous mistakes”.

Conflict clauses also determine backtracking.

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- Sequence of assignments $\langle p_1 \mapsto b_1, \dots, p_r \mapsto b_k \rangle$ where p_1, \dots, p_k are distinct prop. variables and $b_1, \dots, b_k \in \{0, 1\}$.
- $p_i \mapsto b_i$ may be annotated with a clause and other info.

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3. Specialisation $F|_{\mathcal{A}}$.

- F simplified under \mathcal{A} .
- Delete any clause containing the true literal under \mathcal{A} , and remove from each remaining clause the false literal under \mathcal{A} .

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Example run with $\{\{\neg p_1\}, \{p_1, p_2, \neg p_3\}, \{p_3, p_4, p_5\}, \{p_4, \neg p_5\}\}$.

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Unit propagation – deduction step.

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Decision – search step.

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Clause learning – deduction step.

Terminology

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A **state** of algorithm is a pair of CNF formula F and valuation \mathcal{A} .

Successful state when $\mathcal{A} \models F$.

Conflict state when $\mathcal{A} \models \neg F$.

Note: Conflict state if $F|_{\mathcal{A}} \ni \square$. Successful state if $F|_{\mathcal{A}} = \emptyset$.

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Each assignment $p_i \mapsto b_i$ in \mathcal{A} classified as **decision assignment** or **implied assignment**.

$p_i \mapsto b_i$ by a decision strategy (step 5) is a **decision assignment**. p_i called **decision variable**.

$p_i \mapsto b_i$ by a unit propagation on a clause C (step 2) is an **implied assignment**. Denoted by $p_i \xrightarrow{C} b_i$.

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Example: start with set of clauses $F = \{C_1, \dots, C_5\}$, where

$$C_1 = \{\neg p_1, \neg p_4, p_5\},$$

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Let the current valuation is $\mathcal{A} = \langle p_1 \mapsto 1, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 1 \rangle$.

Notice that $F|_{\mathcal{A}}$ contains the unit clause $\{p_5\}$.

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Answer: Unit propagation generates $p_5 \xrightarrow{C_1} 1, p_6 \xrightarrow{C_2} 1, p_7 \xrightarrow{C_3} 1$. This leads to conflict, with C_4 being made false.

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Clause learning via conflict analysis.

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Ex: How can we find such C from a conflicted state (F, \mathcal{A}) ?

Clause learning algorithm

Suppose $\mathcal{A} = \langle p_1 \mapsto b_1, \dots, p_k \mapsto b_k \rangle$ leads to conflict.

Find associated clauses D_1, \dots, D_{k+1} by backward induction:

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Ex1: Run this algorithm on the example in the unit prop. slide.

Ex2: Prove the proposition.

Clause learning: example

In conflict of above example, learning generates clauses

$$D_8 := \{\neg p_1, \neg p_7, \neg p_5\} \quad (\text{clause } C_4)$$

$$D_7 := \{\neg p_1, \neg p_5, \neg p_6\} \quad (\text{resolve } D_8, C_3)$$

$$D_6 := \{\neg p_1, \neg p_5\} \quad (\text{resolve } D_7, C_2)$$

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The learned clause D_1 is a conflict clause with only decision variables, including the top-level one p_4 .

D_1 records that conflict arose from decision to make p_1, p_4 true.

Adding D_1 makes assignments validating p_1, p_4 unreachable.

Backtracking to the highest level where D_1 is a unit clause ($p_1 \mapsto 1$) and doing unit propagation lead to $p_4 \mapsto 0$.

Example: 4 queens

Problem: place 4 non-attacking queens on a 4x4 chess board.

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Variable p_{ij} models that there is a queen in the square (i, j) .

- ≥ 1 in each row: $\bigwedge_{i=1}^4 \bigvee_{j=1}^4 p_{ij}$.
- ≤ 1 in each row: $\bigwedge_{i=1}^4 \bigwedge_{j \neq j'=1}^4 (\neg p_{ij} \vee \neg p_{ij'})$.
- ≤ 1 in each column: $\bigwedge_{j=1}^4 \bigwedge_{i \neq i'=1}^4 (\neg p_{ij} \vee \neg p_{i'j})$.
- ≤ 1 on each diagonal: $\bigwedge_{i,j=1}^4 \bigwedge_k (\neg p_{i-k,i+k} \vee \neg p_{i+k,j+k})$.

Total number of clauses: $4 + 24 + 24 + 28 = 80$.

DPLL: 4 queens

Running the DPLL algorithm:

- Start with $p_{11} \mapsto 1$. Then,
 - 1) delete $\{p_{11}, p_{12}, p_{13}, p_{14}\}$;
 - 2) delete $\neg p_{11}$ and generate 9 new unit clauses;
 - 3) delete 65 clauses subsequently!

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- Next, set $p_{23} \mapsto 1$. Then,
 - 1) generate 4 new unit clauses: $\{\neg p_{24}\}, \{\neg p_{43}\}, \{\neg p_{32}\}, \{\neg p_{34}\}$;
 - 2) through the unit propagation of $\{\neg p_{34}\}$, reach UNSAT.

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- Fixing only two literals collapsed 80 clauses to 1 and ruled out 2^{14} of 2^{16} possible assignments!
- Backtrack and set $\langle p_{11} \mapsto 0, p_{12} \mapsto 1 \rangle$. Then,
 - 1) Delete $\{\neg p_{12}\}$ and generate 9 new unit clauses;
 - 2) Do unit propagation, which leaves only 1 clause $\{p_{43}\}$!

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 - 1) Delete $\{\neg p_{12}\}$ and generate 9 new unit clauses;
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- Answer: $p_{12}, p_{24}, p_{31}, p_{43} \mapsto 1$.

Summary

Resolution:

- Very simple sound and complete proof calculus.

Davis–Putnam algorithm:

- Uses resolution to decide SAT via variable elimination.
- But may generate huge intermediate formulas.

DPLL algorithm:

- Improves resolution with clause learning and backtracking.
- Efficient.
- Basis for modern SAT solvers.