

# CS492: Probabilistic Programming Introduction

Hongseok Yang  
KAIST

This Review ... discusses some of the state-of-the-art advances in the field, namely, **probabilistic programming**, Bayesian optimization, data compression and automatic model discovery.

Zoubin Ghahramani  
2015 Nature Review

Pyro

pyro.ai

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# PYRO

Deep Universal Probabilistic Programming

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What is probabilistic  
programming?

# (Bayesian) probabilistic modelling of data

- I. Develop a new probabilistic (generative) model.
2. Design an inference algorithm for the model.
3. Using the algo., fit the model to the data.

# (Bayesian) probabilistic modelling of data in a prob. prog. language

- I. Develop a new probabilistic (generative) model.
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# (Bayesian) probabilistic modelling of data in a prob. prog. language

as a program

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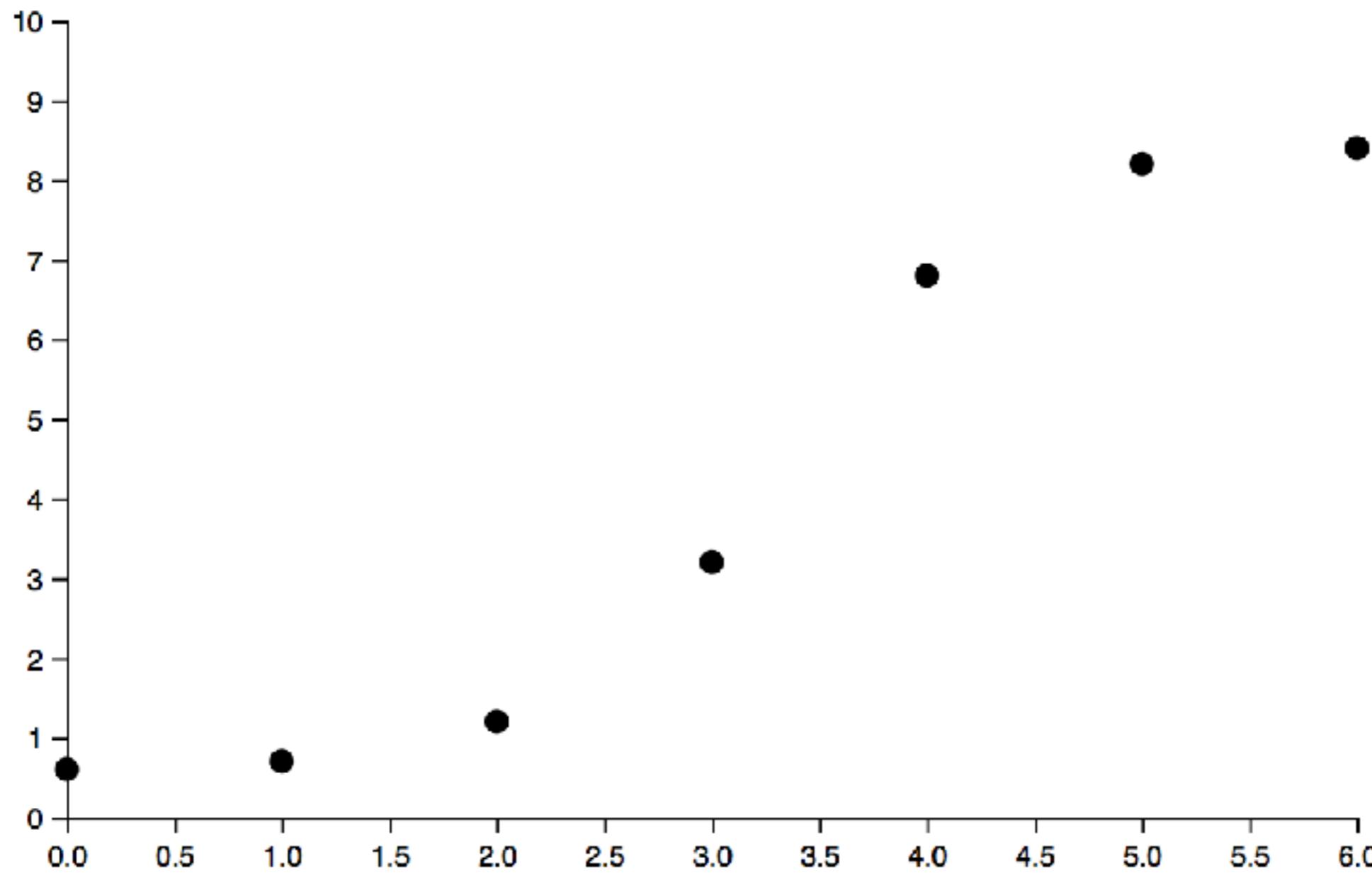
# (Bayesian) probabilistic modelling of data in a prob. prog. language

as a program

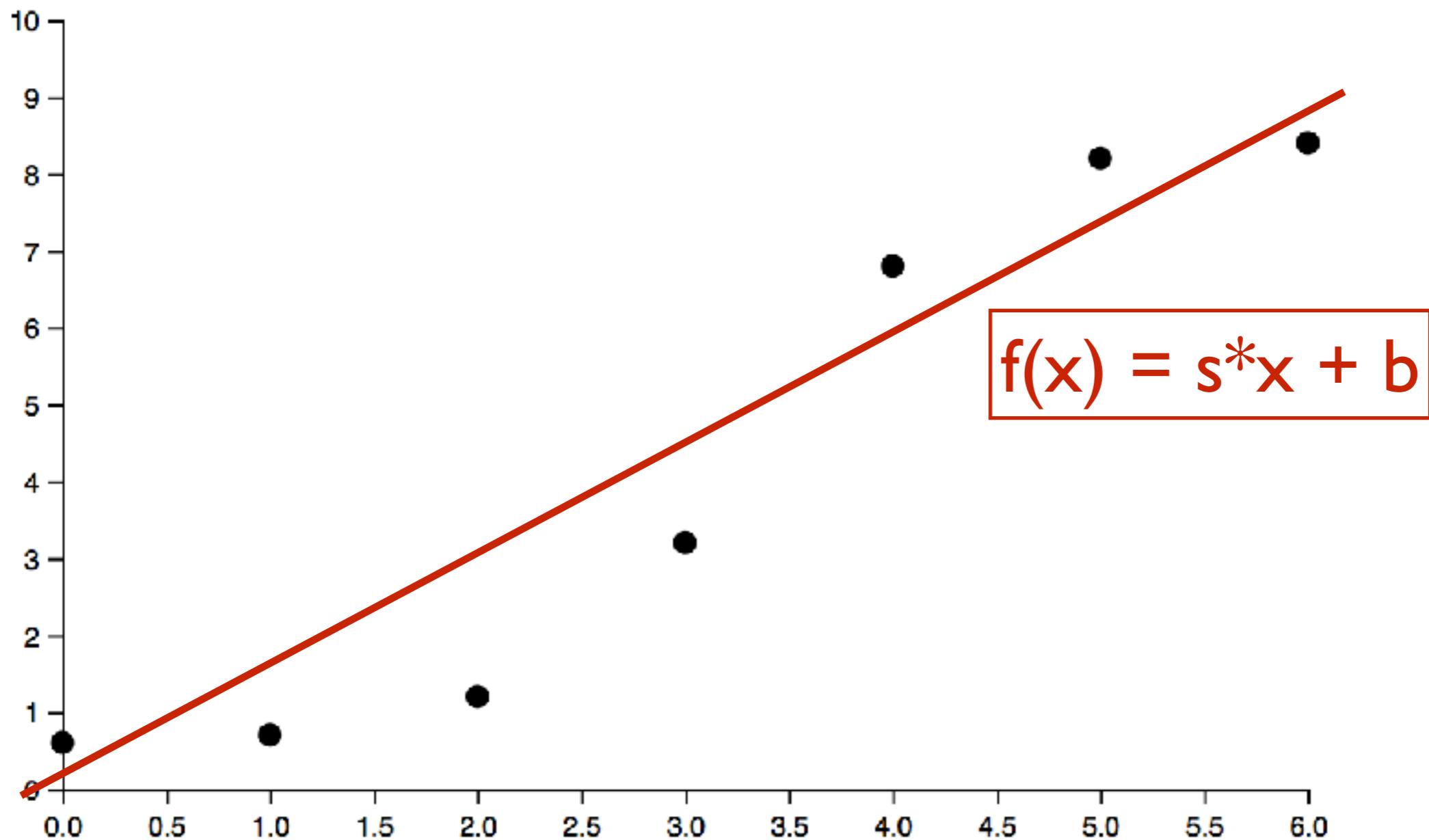
- I. Develop a new probabilistic (generative) model.
- ~~2. Design an inference algorithm for the model.~~
3. Using ~~the algo.~~, fit the model to the data.

a generic inference algo.  
of the language

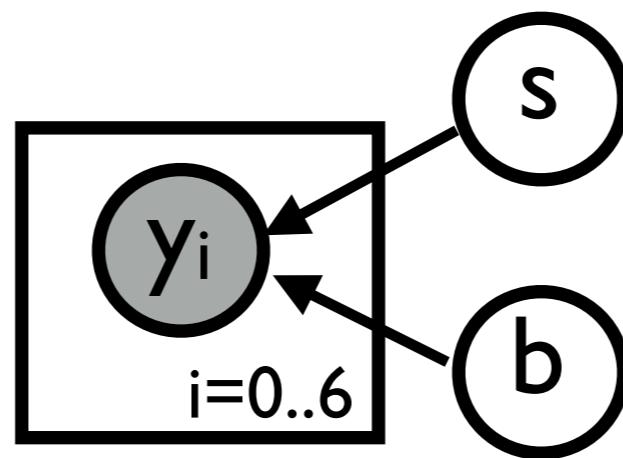
# Line fitting



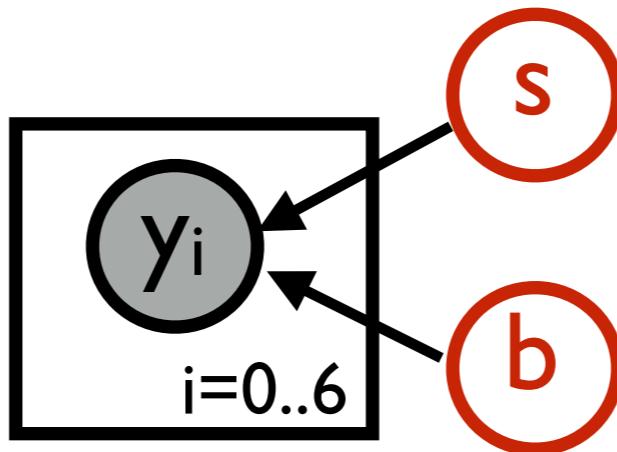
# Line fitting



# Bayesian generative model

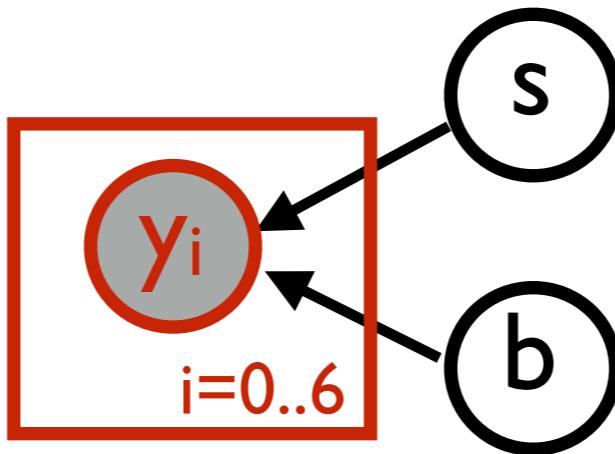


# Bayesian generative model



$s \sim \text{normal}(0, 2)$   
 $b \sim \text{normal}(0, 6)$

# Bayesian generative model



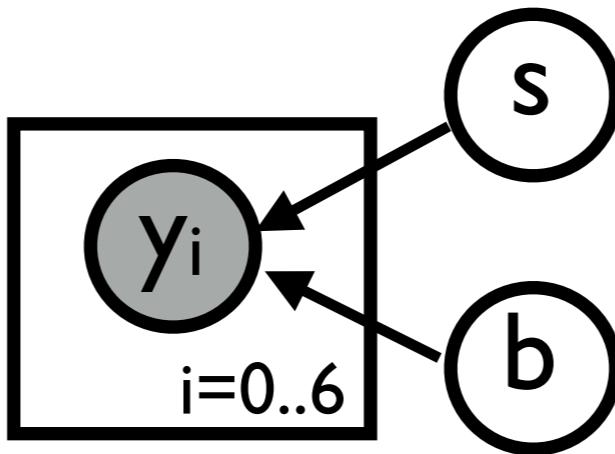
$s \sim \text{normal}(0, 2)$

$b \sim \text{normal}(0, 6)$

$f(x) = s*x + b$

$y_i \sim \text{normal}(f(i), 0.5)$   
where  $i = 0 .. 6$

# Bayesian generative model



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 $b \sim \text{normal}(0, 6)$   
 $f(x) = s*x + b$   
 $y_i \sim \text{normal}(f(i), 0.5)$   
where  $i = 0 .. 6$

Q: posterior of  $(s, b)$  given  $y_0=0.6, \dots, y_6=8.4$ ?

# Posterior of s and b given y<sub>i</sub>'s

$$p(s, b | y_0, \dots, y_6) = \frac{p(y_0, \dots, y_6 | s, b) \times p(s, b)}{p(y_0, \dots, y_6)}$$

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# Anglican program

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]
```

# Anglican program

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]

  (observe (normal (f 0) .5) .6)
  (observe (normal (f 1) .5) .7)
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  (observe (normal (f 5) .5) 8.2)
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```

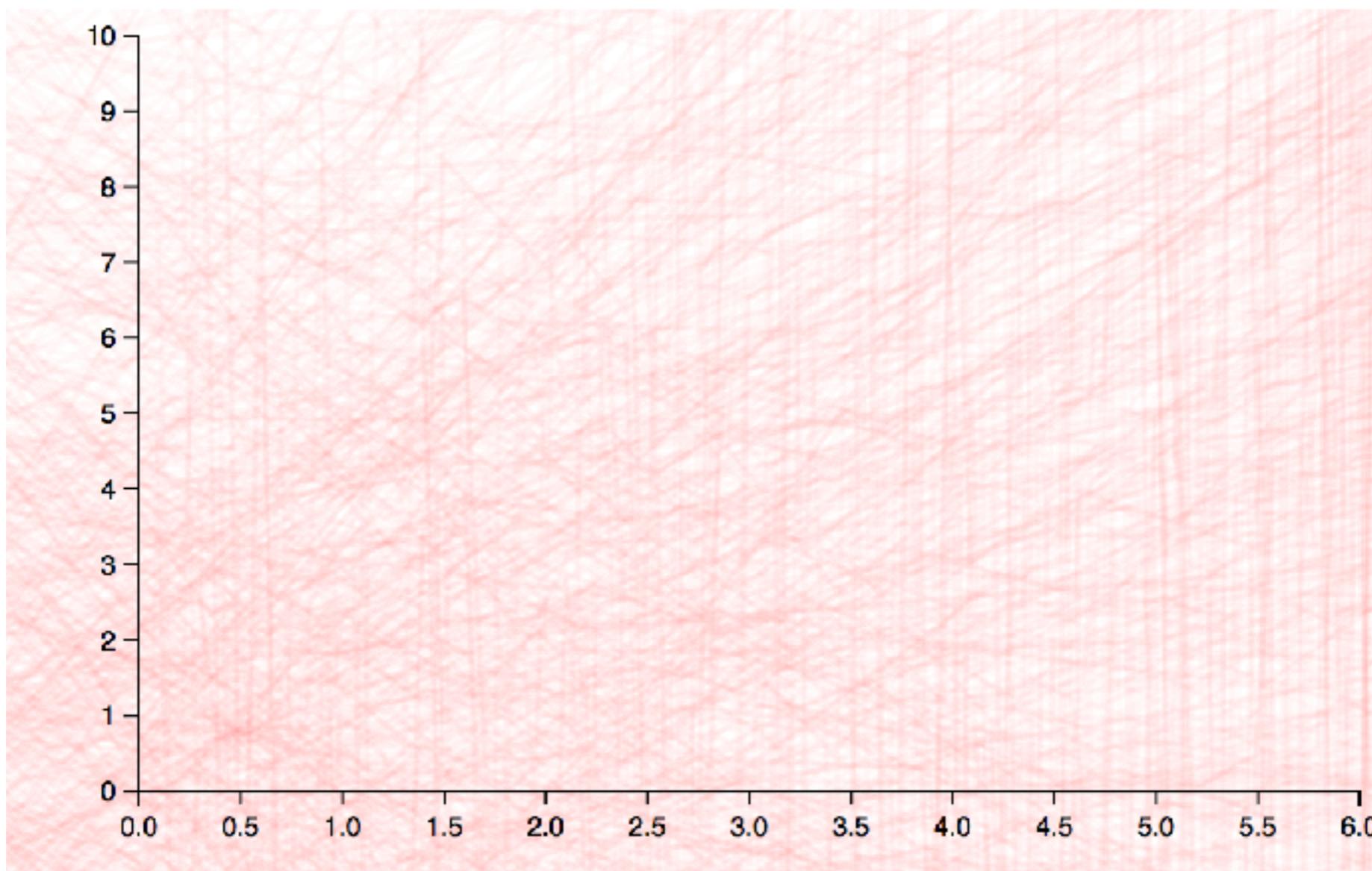
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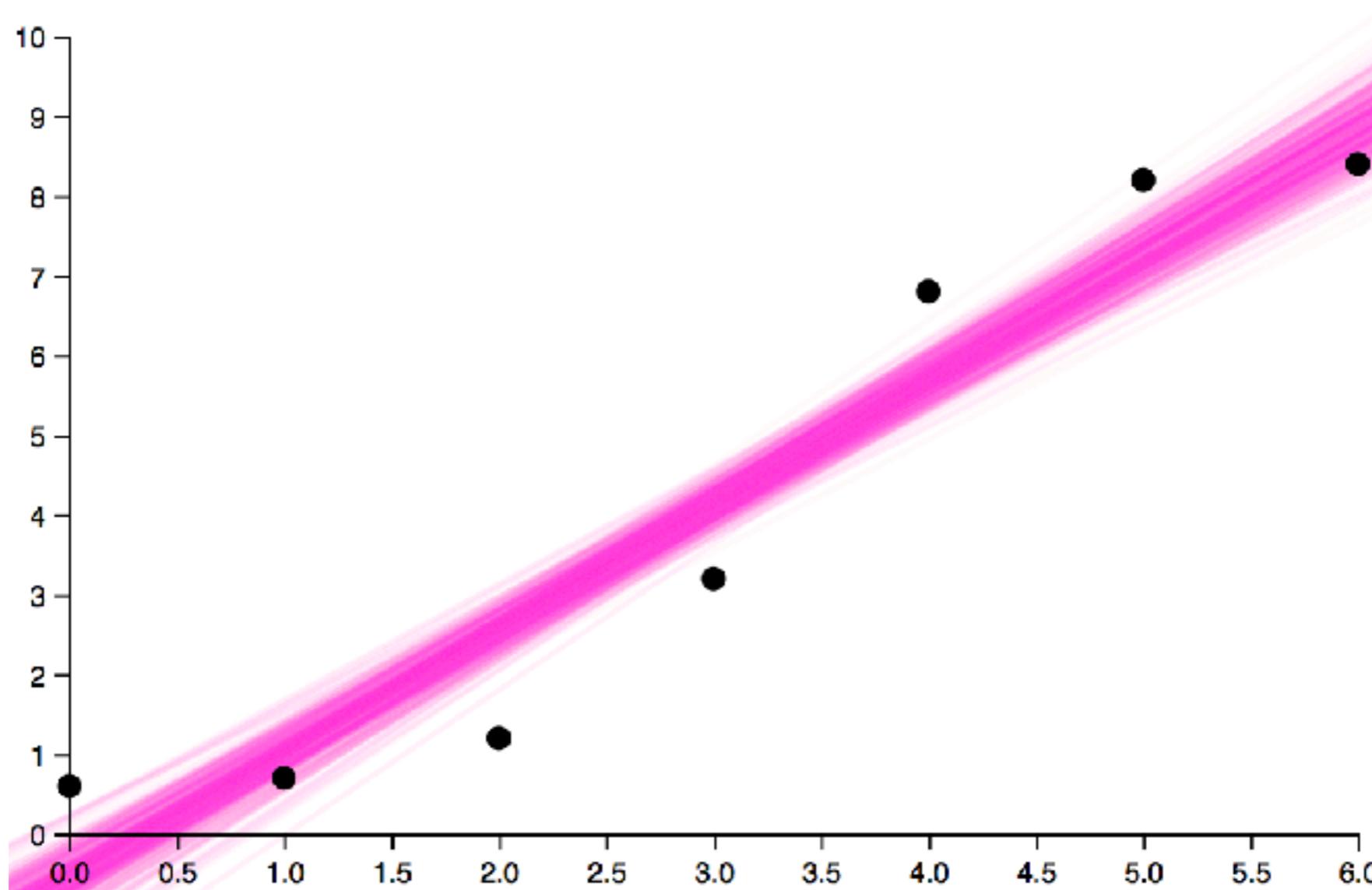
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  (observe (normal (f 6) .5) 8.4)

  [s b])
```

# Samples from prior



# Samples from posterior

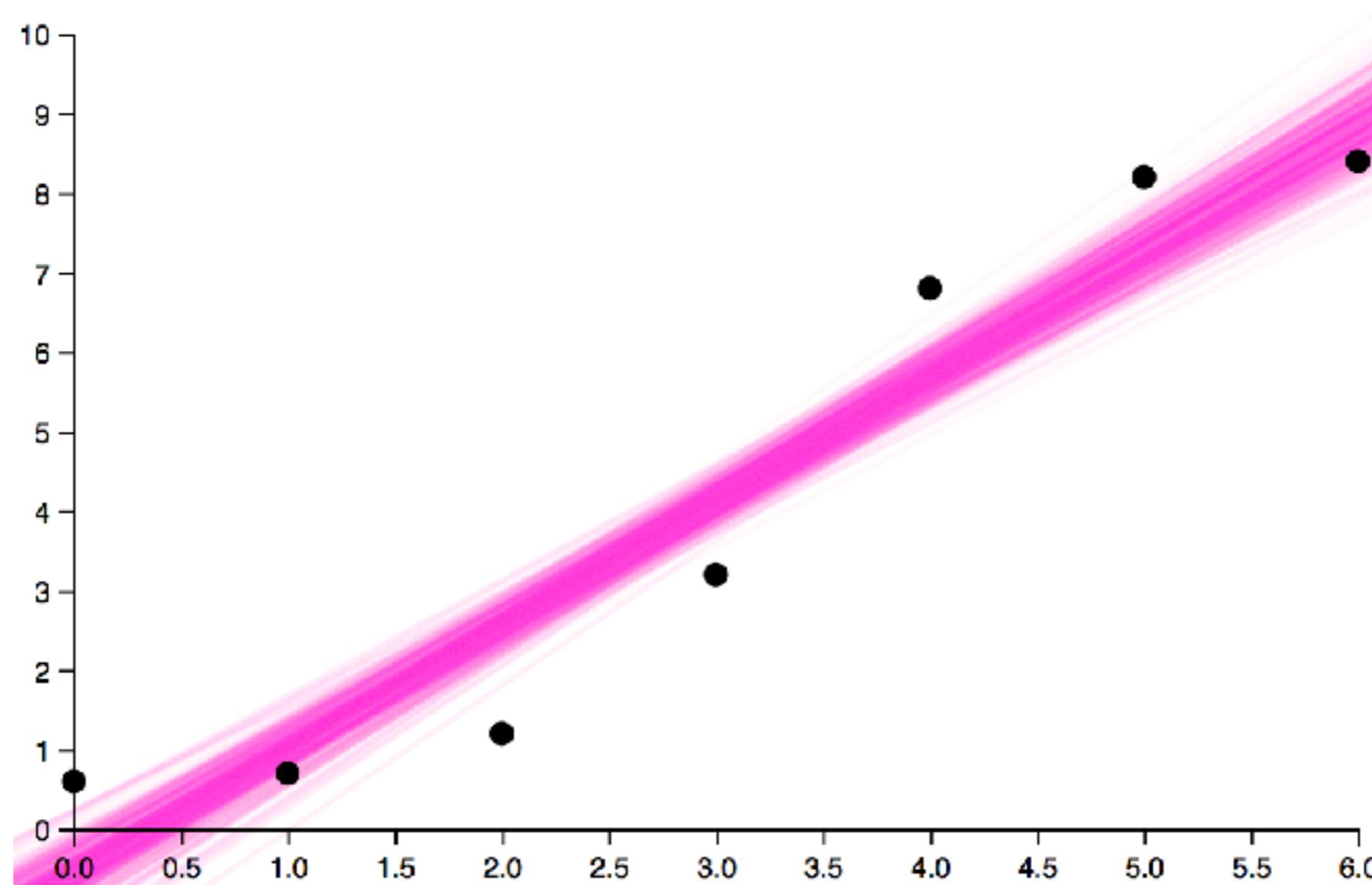


Why should one care  
about prob. programming?

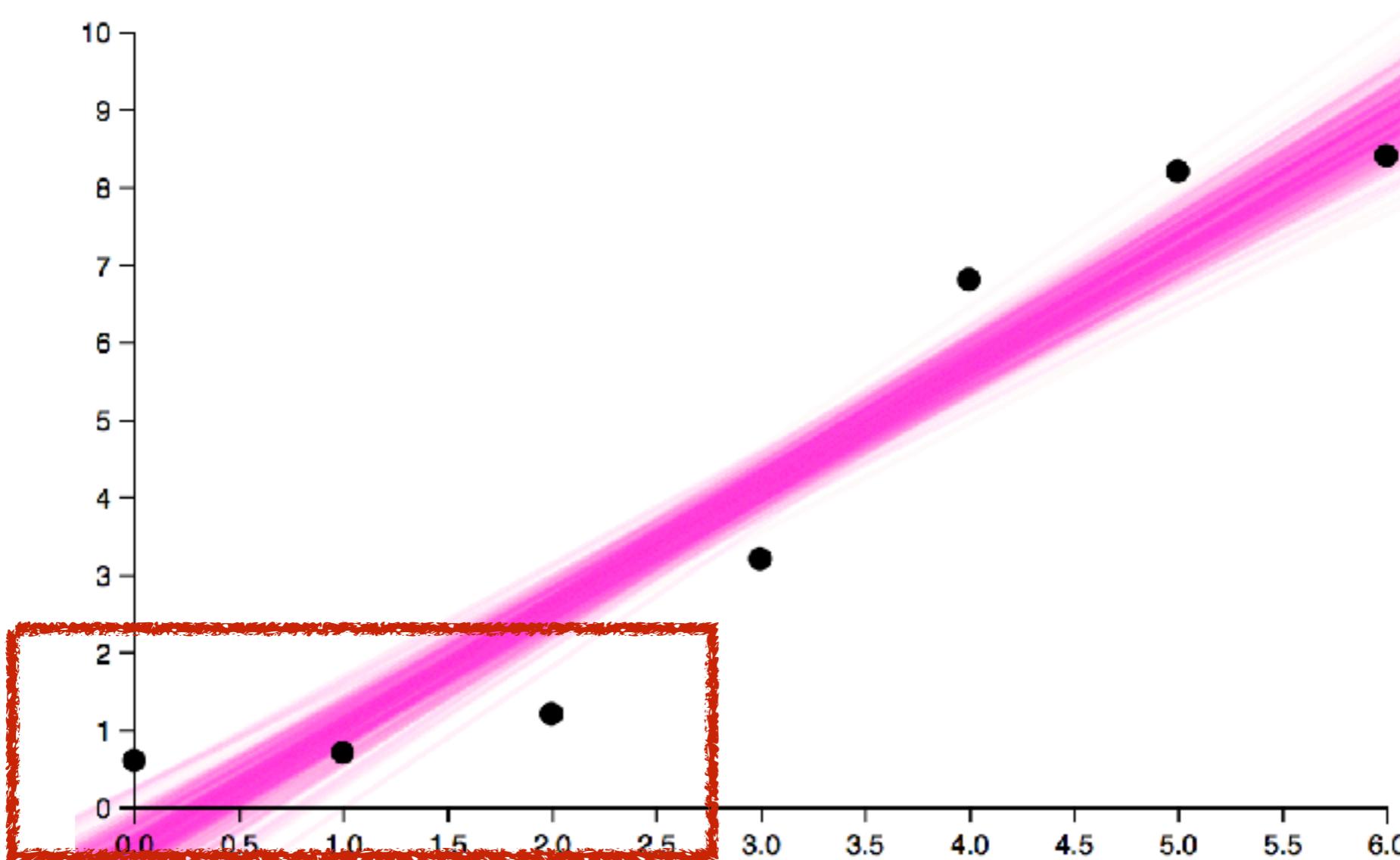
“Because probabilistic programming is a good way to build an AI.” (My ML colleague)

Prob. programming languages enable one to build and explore highly complex models.

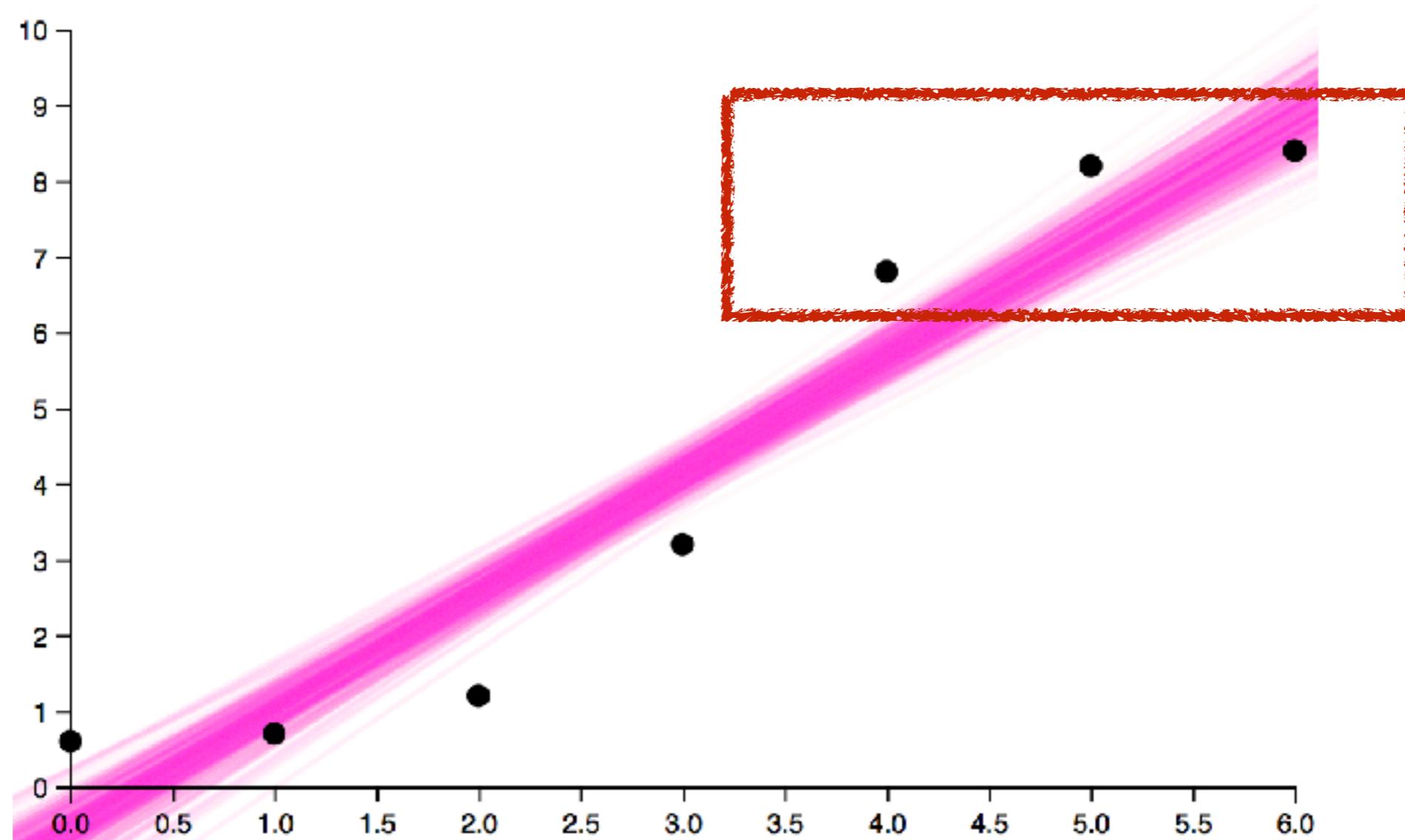
# Underfit?



# Underfit?



# Underfit?



```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]

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  [s b])
```

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```

[s b])

**Functions as first-class citizen.**

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]
```

```
(observe (normal (f 0) .5) .6)
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```

~~[s b]~~  
f)

Functions as first-class citizen.

```
(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))]
    (fn [x] (+ (* s x) b))))]
  f (F))
(observe (normal (f 0) .5) .6)
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(observe (normal (f 6) .5) 8.4)
```

~~Es ist~~  
f)

Functions as first-class citizen.

```
(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))])
    (fn [x] (+ (* s x) b))))  
f (add-change-points F 0 6)]  
(observe (normal (f 0) .5) .6)  
(observe (normal (f 1) .5) .7)  
(observe (normal (f 2) .5) 1.2)  
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```

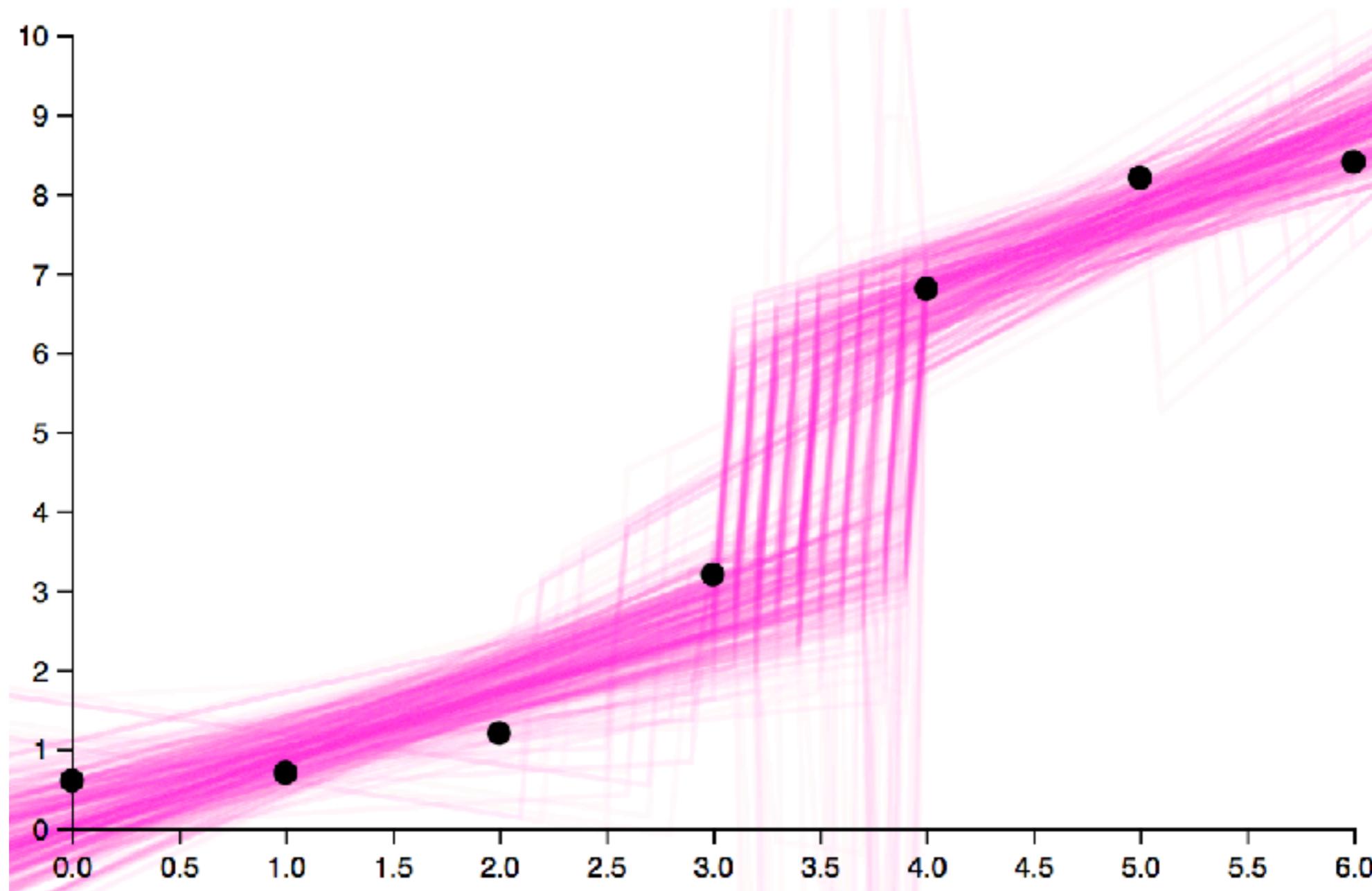
~~Exhibit~~  
f)

Functions as first-class citizen.

```
(defm add-change-points [F 1 u]
  (if (sample (flip 0.5)) (F)
    (if (sample (flip 0.5))
      (let [cp1 (sample
                  (uniform-continuous 1 u))
            f1 (F)
            f2 (F)]
        (fn [x] (if (< x cp1) (f1 x) (f2 x)))
      (let [cp1 (sample
                  (uniform-continuous 1 u))
            cp2 (sample
                  (uniform-continuous cp1 u))
            f1 (F)
            f2 (F)
            f3 (F)]
        (fn [x] (if (< x cp1) (f1 x)
                    (if (< x cp2) (f2 x)
                      (f3 x))))))))
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              cp2 (sample
                   (uniform-continuous cp1 u))
              f1 (F)
              f2 (F)
              f3 (F)]
          (fn [x] (if (< x cp1) (f1 x)
                      (if (< x cp2) (f2 x)
                        (f3 x))))))))
```

# Samples from posterior



```

(let [F (fn []
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    (fn [x] (+ (* s x) b))))]

  f (add-change-points F 0 6])
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```

~~Exercise~~  
f)

[Q] Change the model so that it finds constant functions with change points.

```

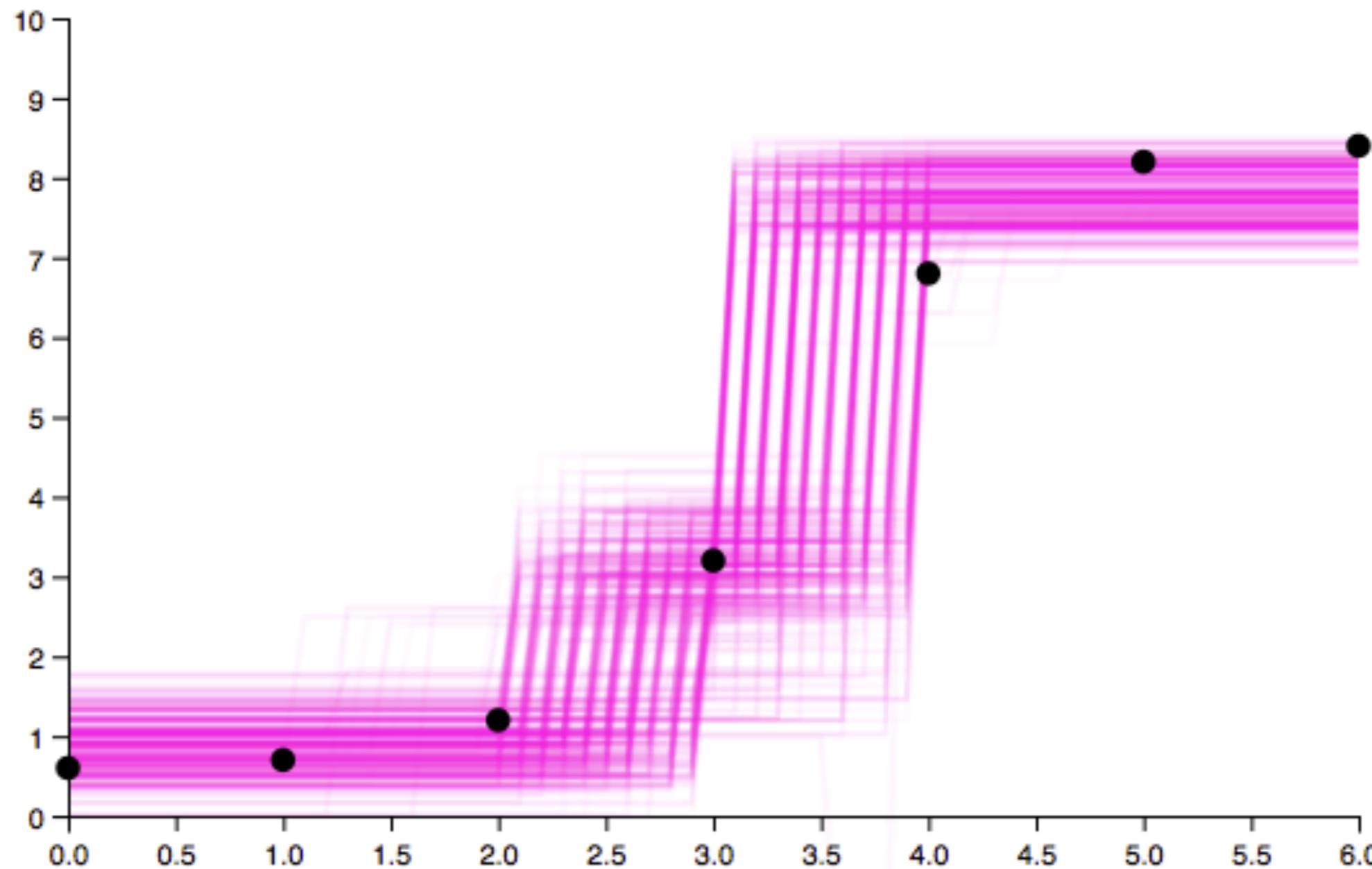
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~~Exercise~~  
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# Samples from posterior



# Three success stories:

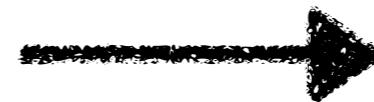
## I) captcha breaking



Le, Baydin, Wood [2016]

# Three success stories:

## I) captcha breaking



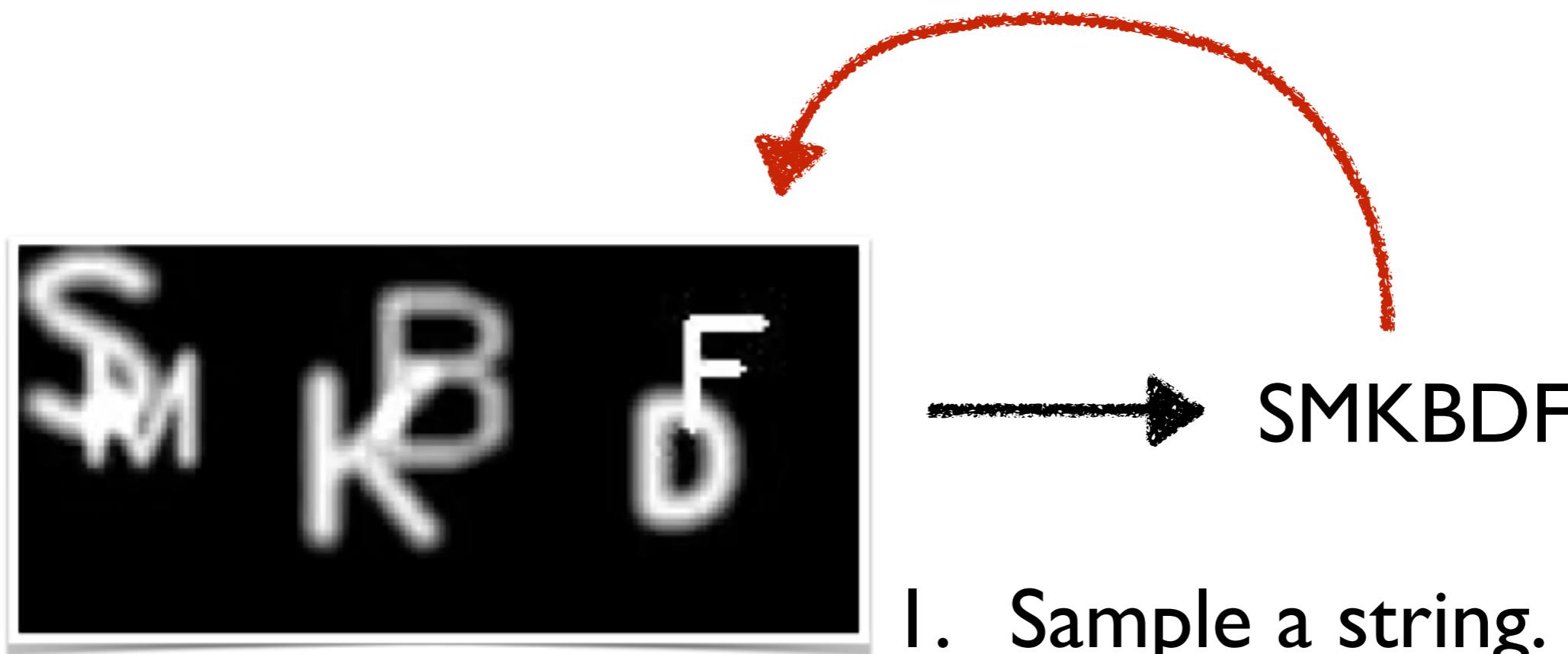
SMKBDF

I. Sample a string.

Le, Baydin, Wood [2016]

# Three success stories:

## I) captcha breaking

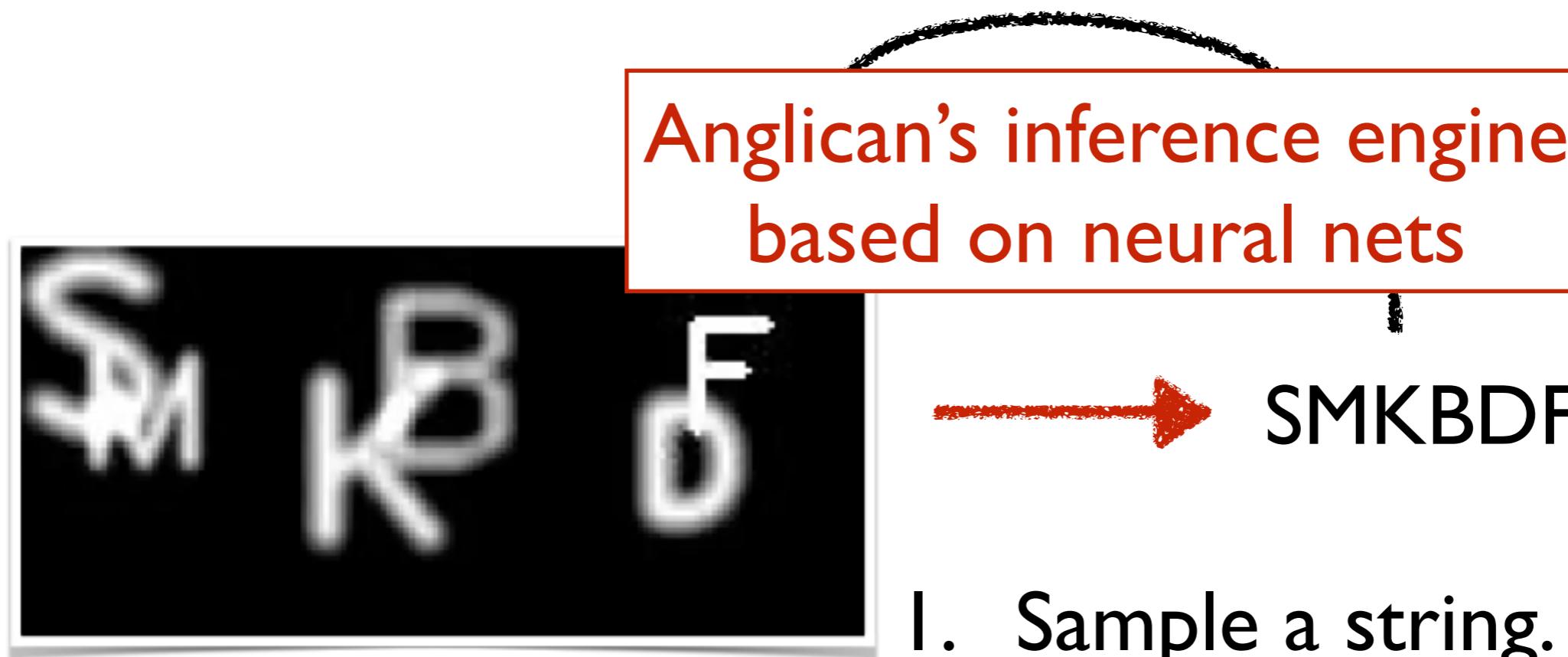


1. Sample a string.
2. Generate an image using complex JVM code.

Le, Baydin, Wood [2016]

# Three success stories:

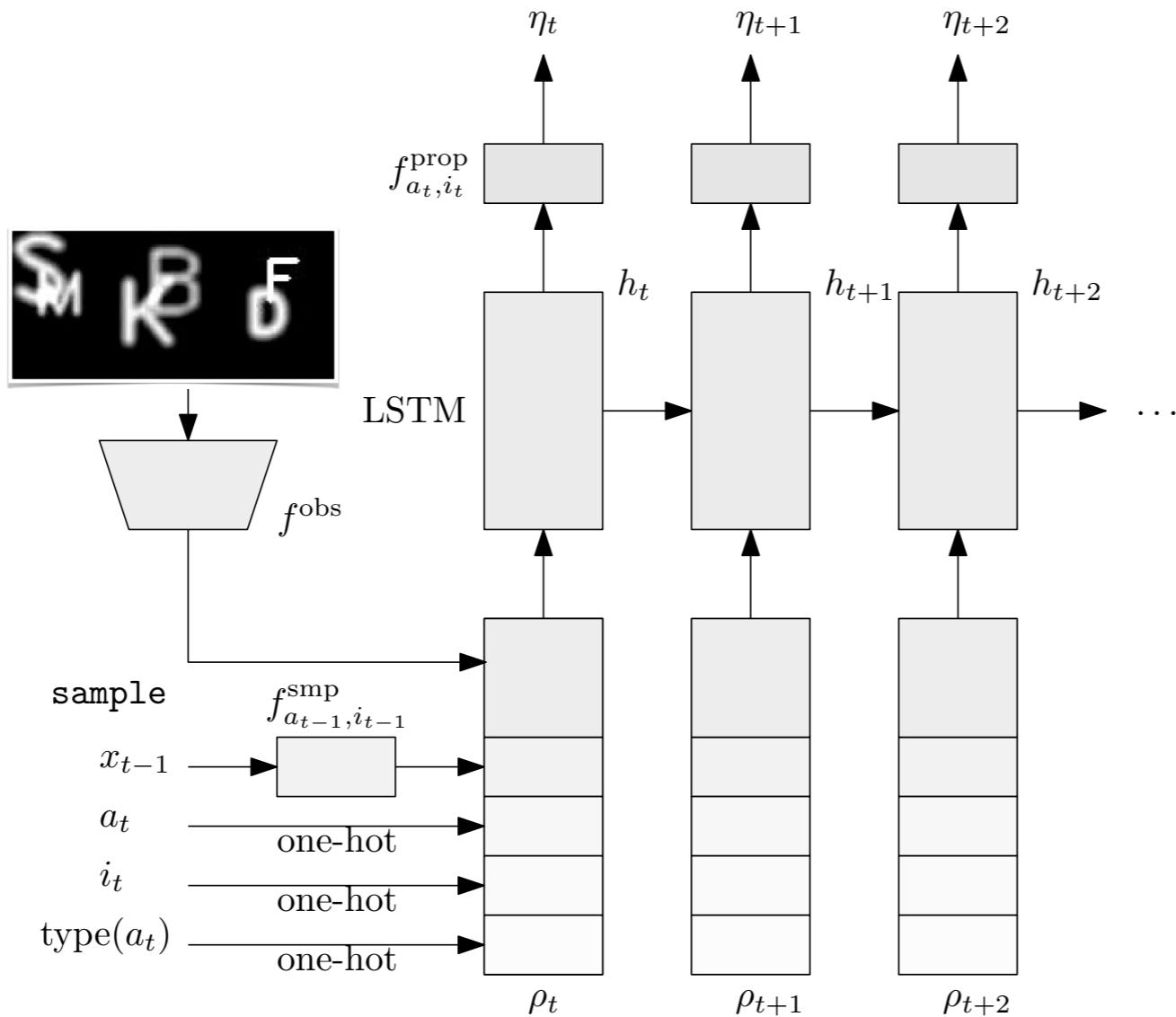
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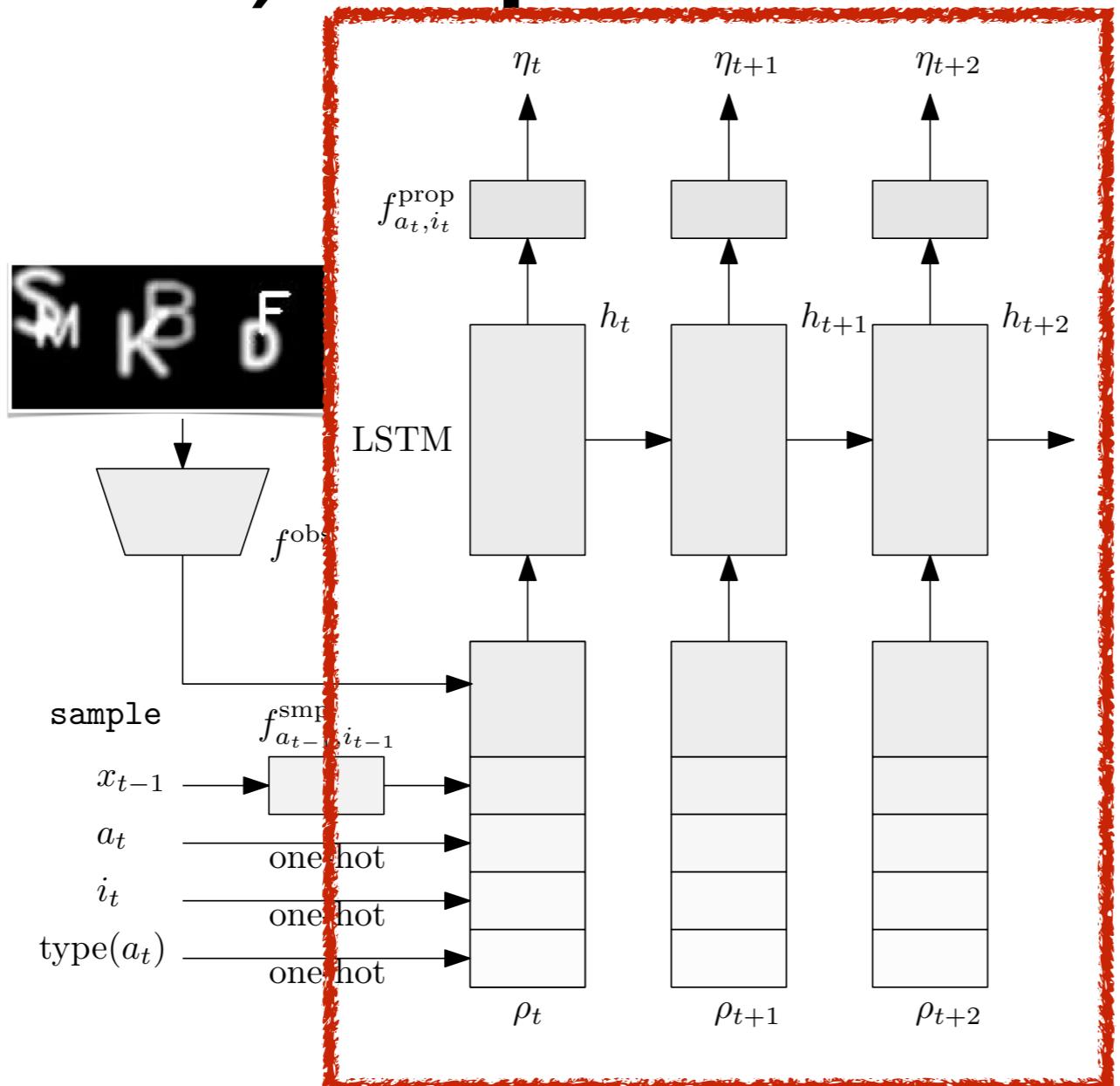
# Three success stories: I) captcha breaking



Le, Baydin, Wood [2016]

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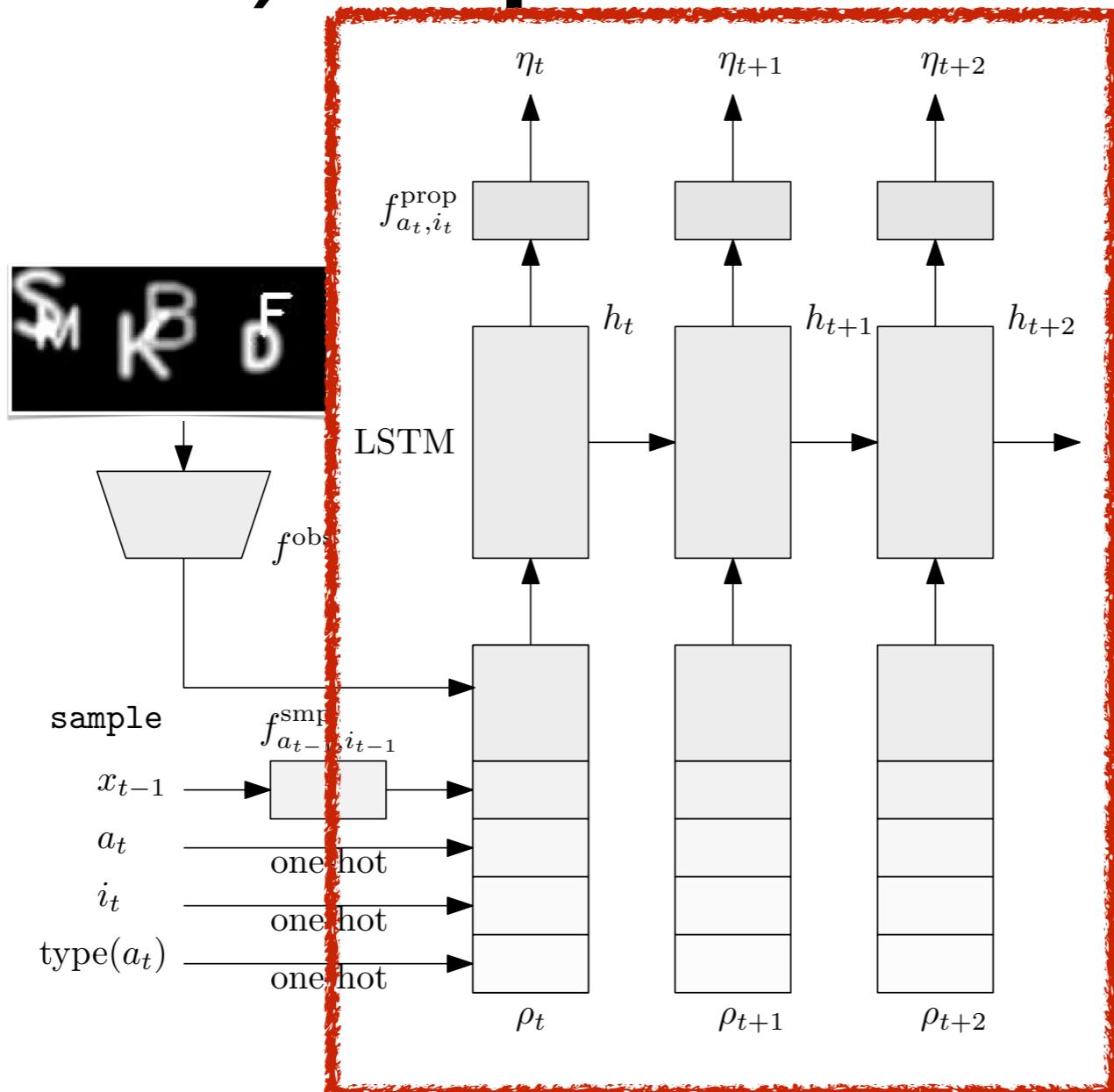


Neural net as a part  
of inference engine.

Le, Baydin, Wood [2016]

# Three success stories:

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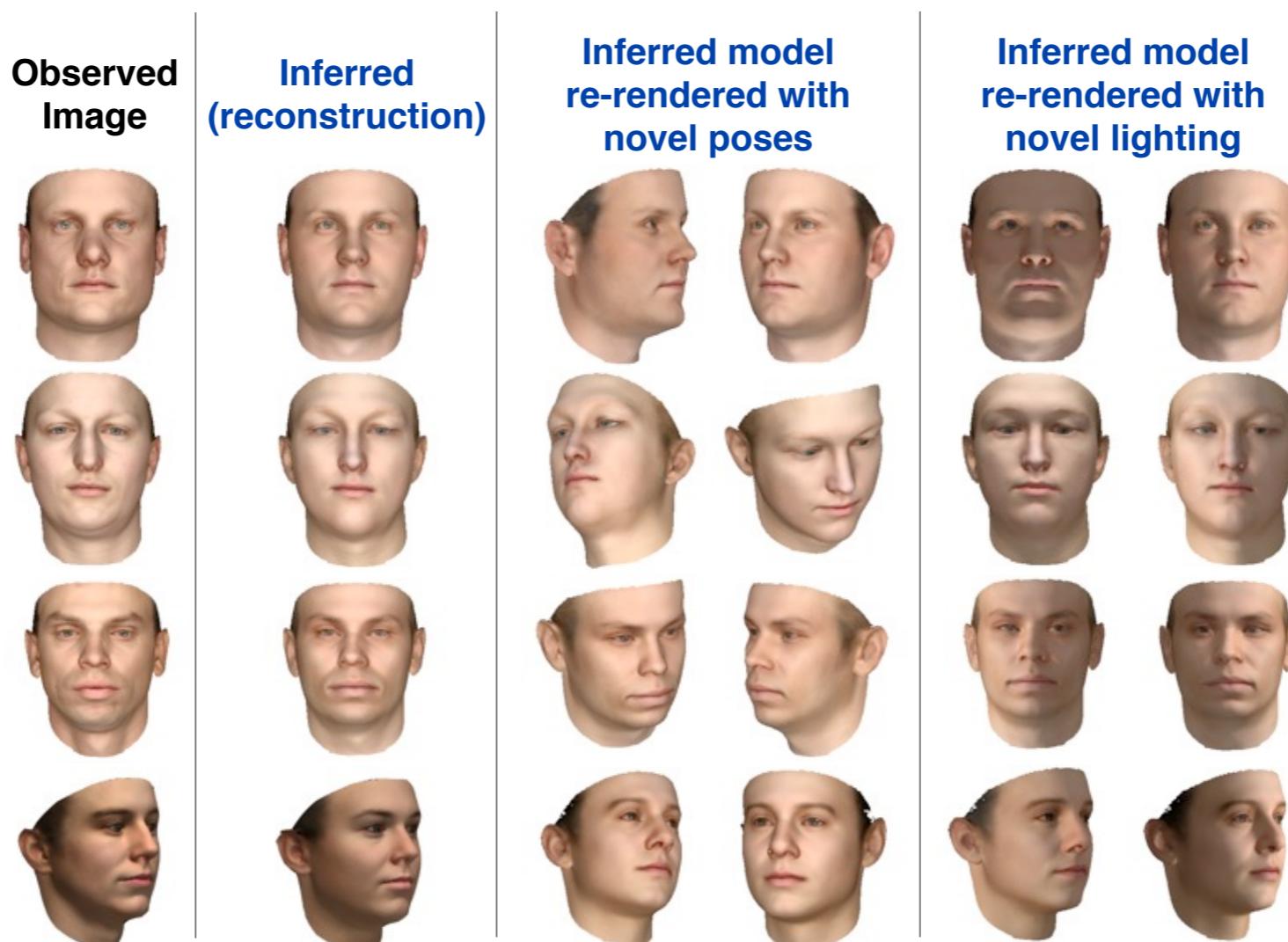


Neural net as a part  
of inference engine.

Approximates the  
inverse of the  
Captcha program.

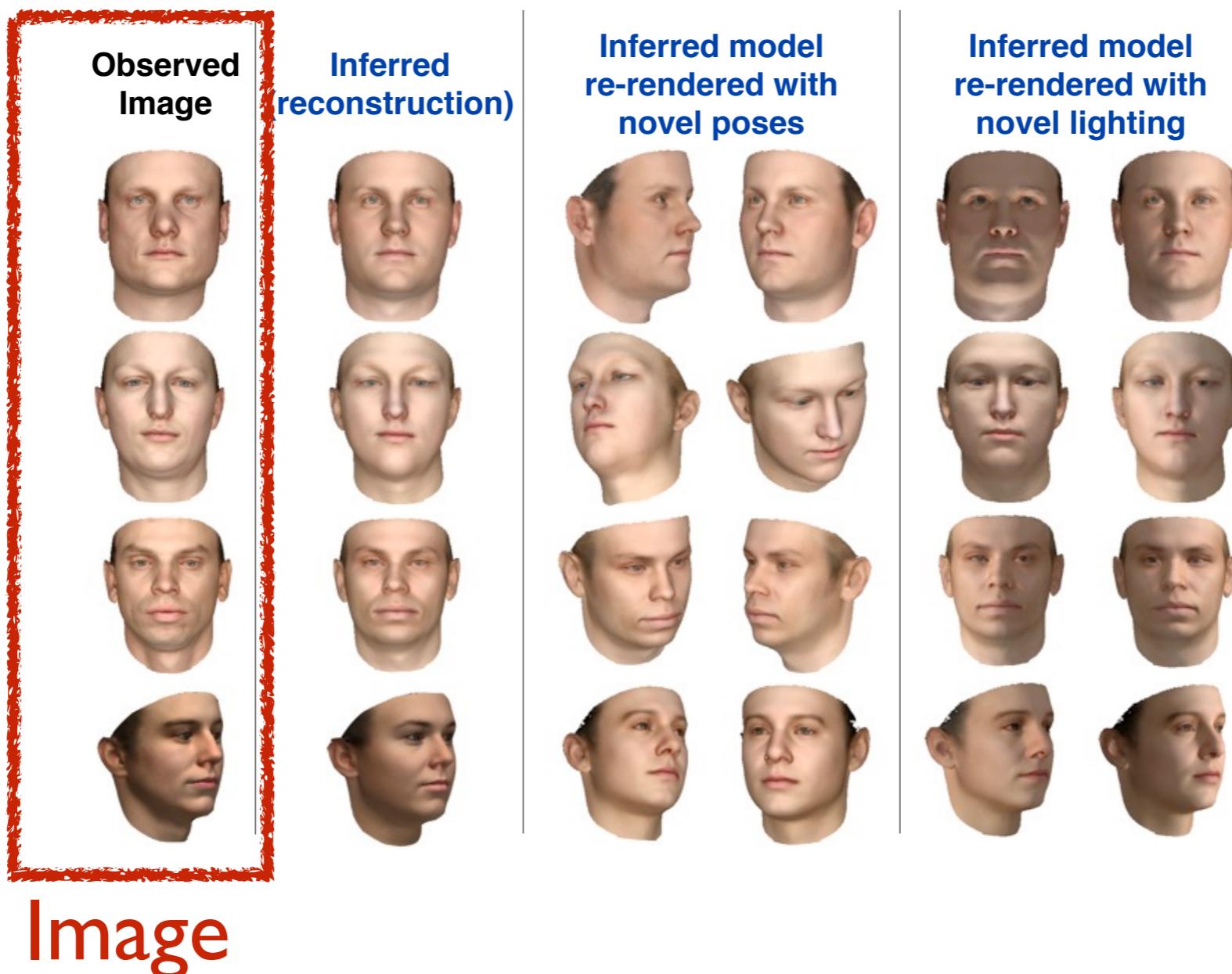
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# Three success stories: 2) inverse graphics



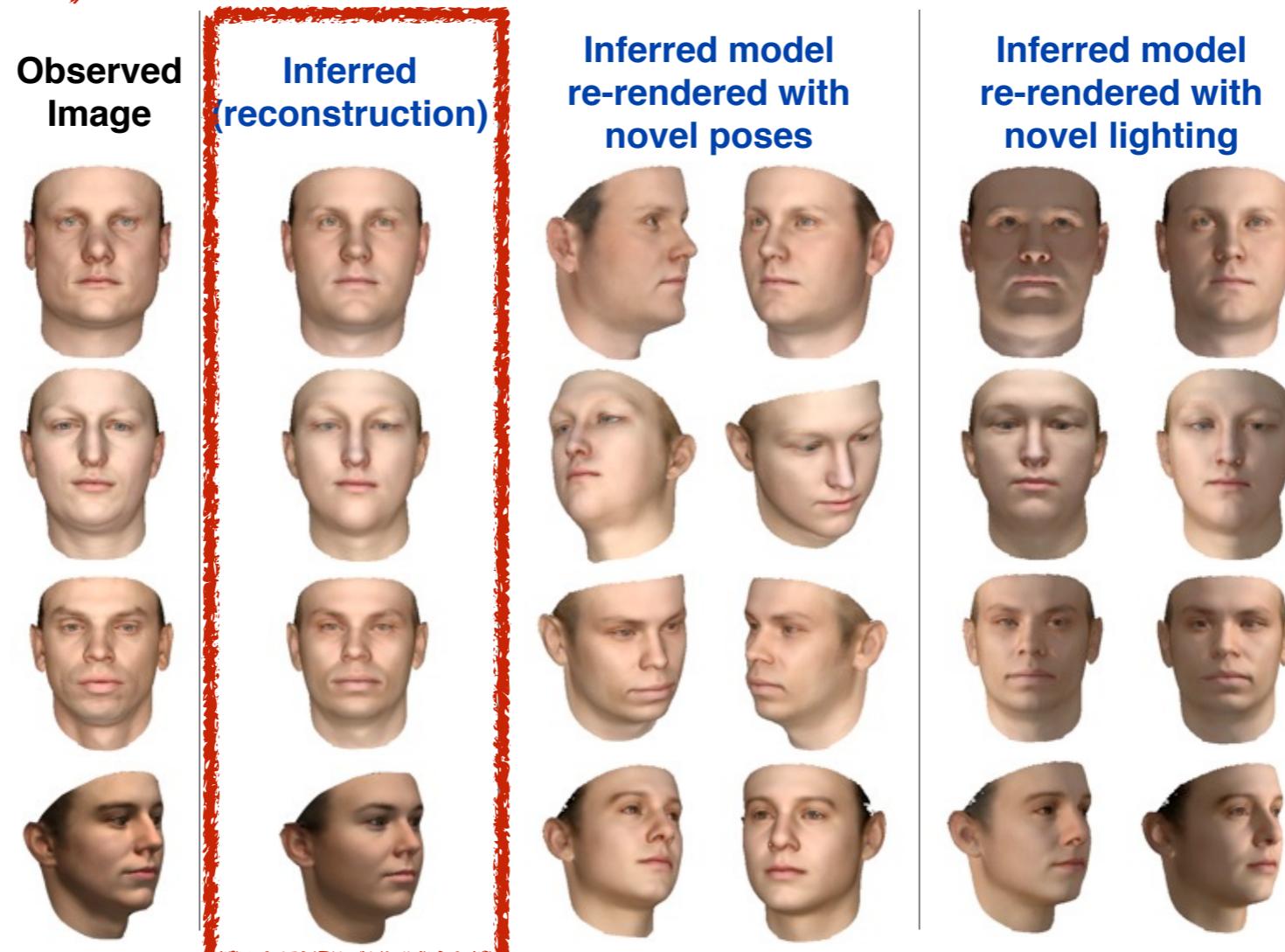
Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

# Three success stories: 2) inverse graphics



Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

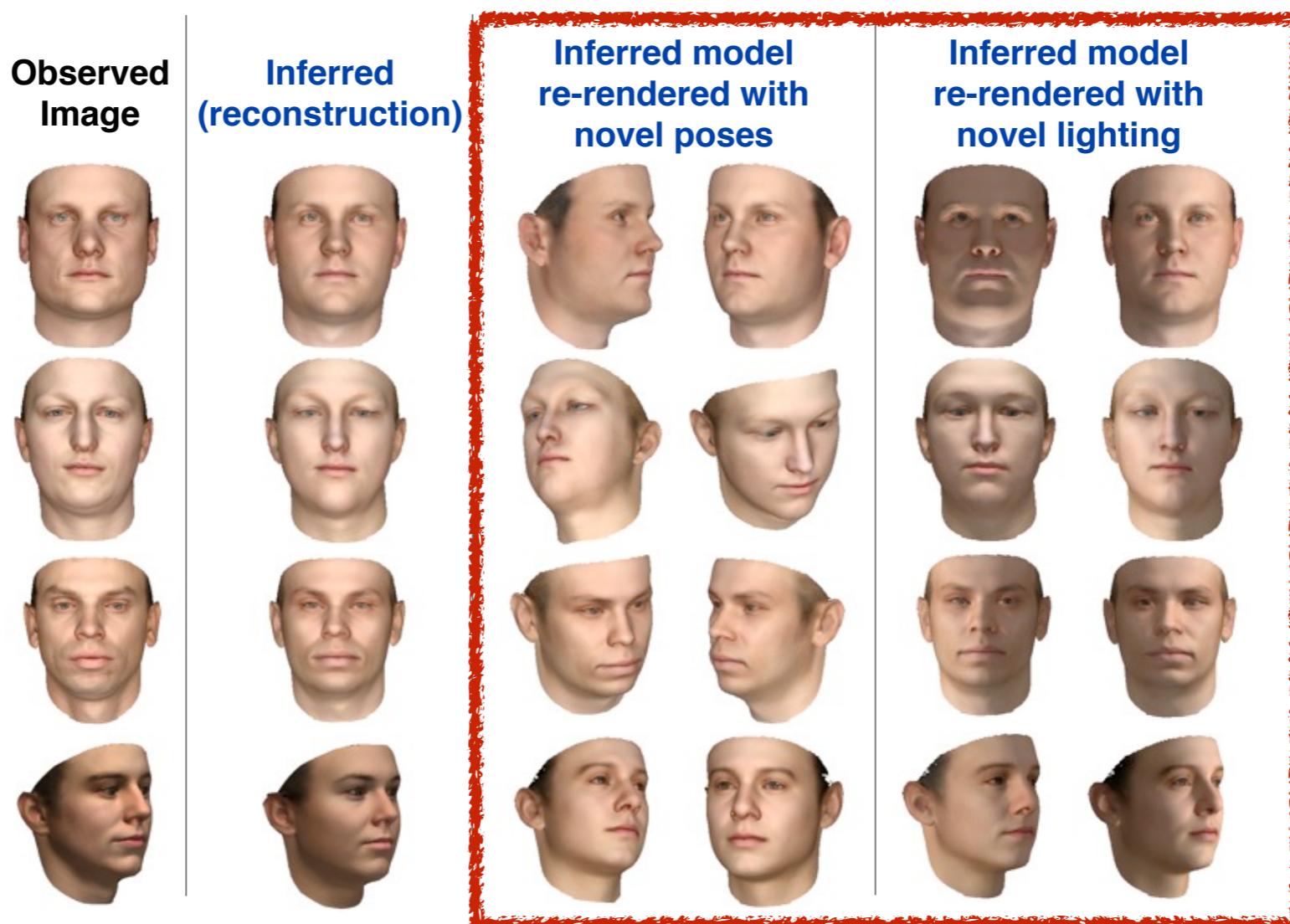
# Three success stories: 2) inverse graphics



3D model

Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

# Three success stories: 2) inverse graphics

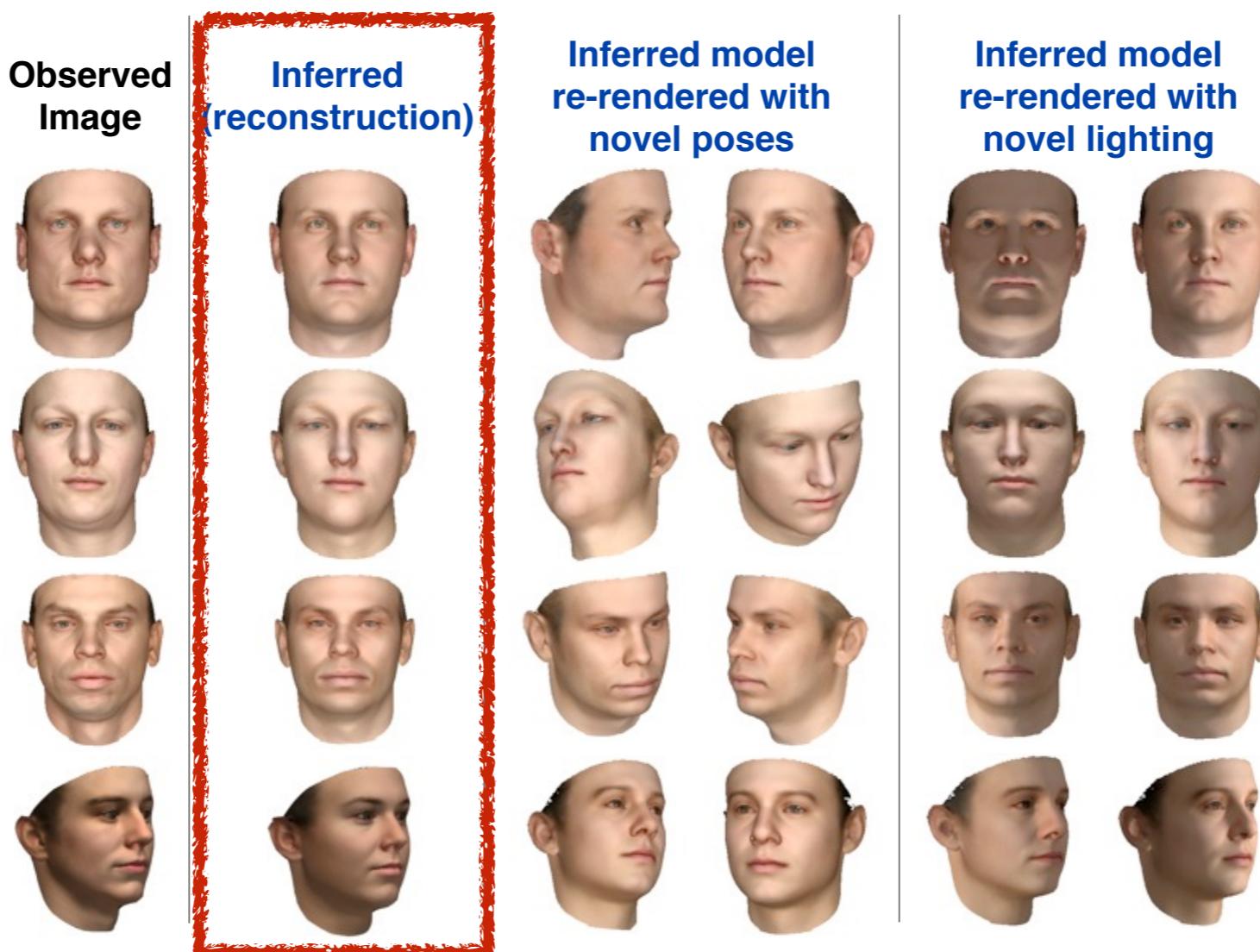


Transformed 3D models

Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

## I. Sample a 3D model.

# Success stories: Face graphics

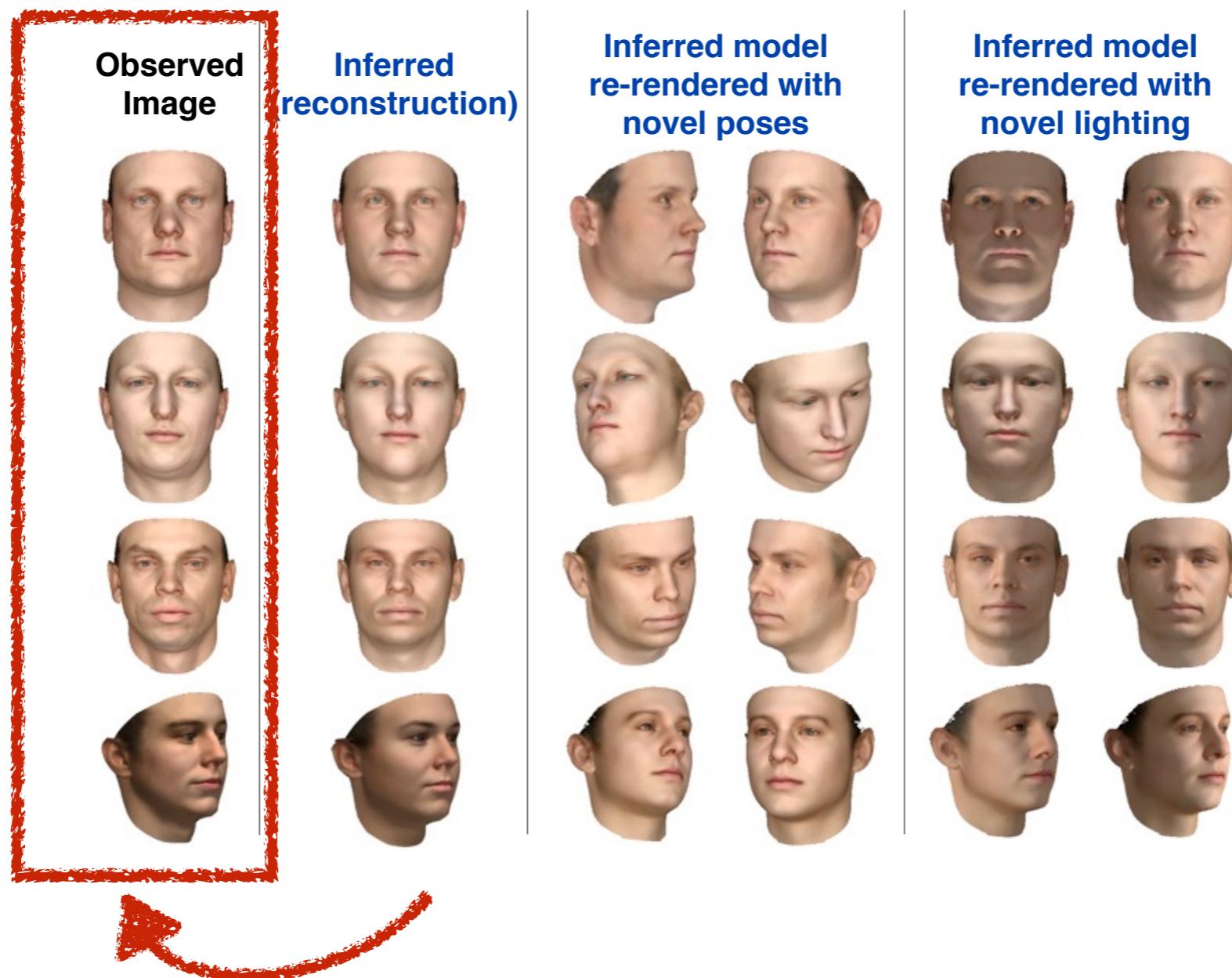


Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

TI

- I. Sample a 3D model.
2. Generate an image.

# Access stories: use graphics

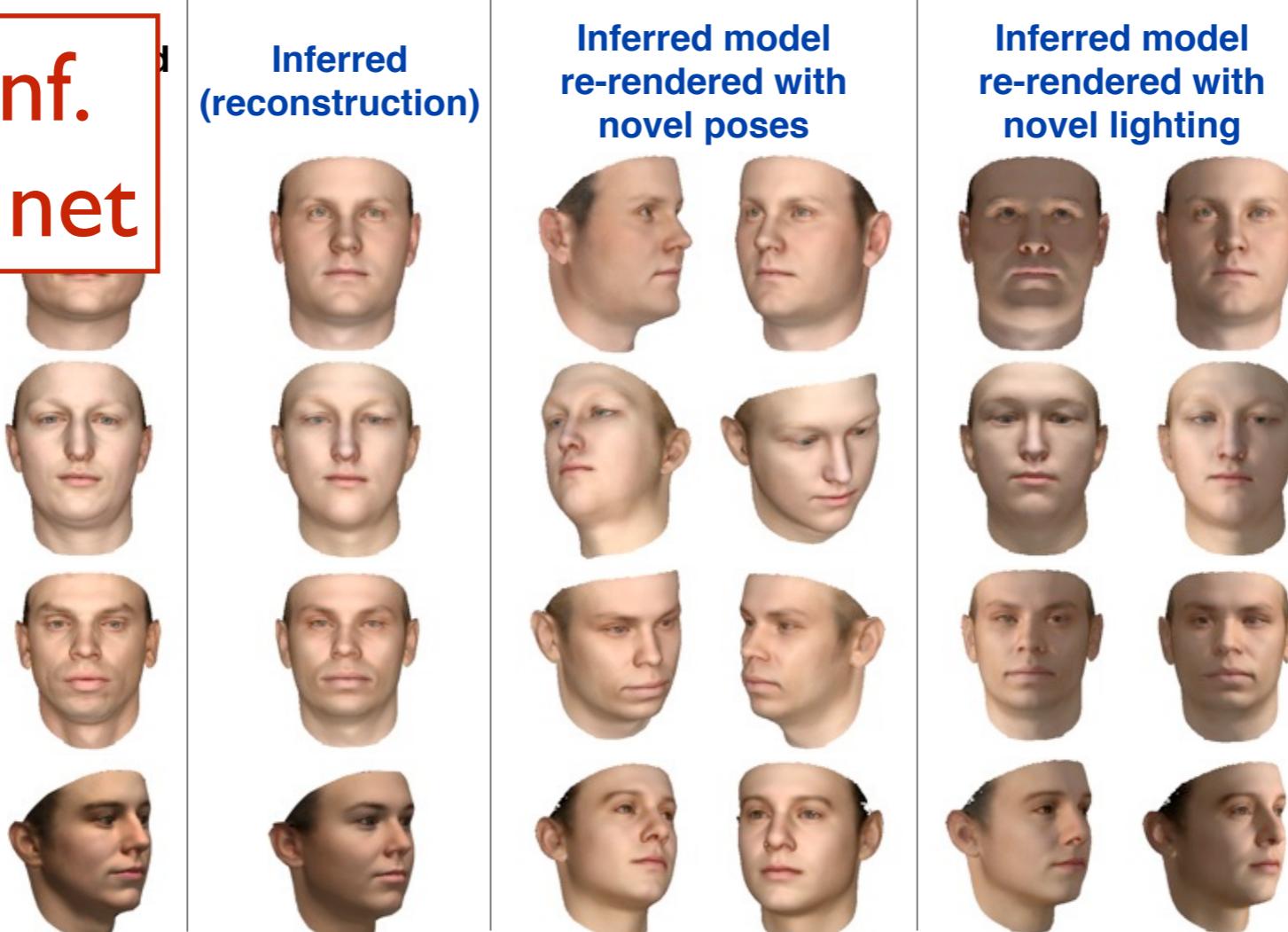


Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

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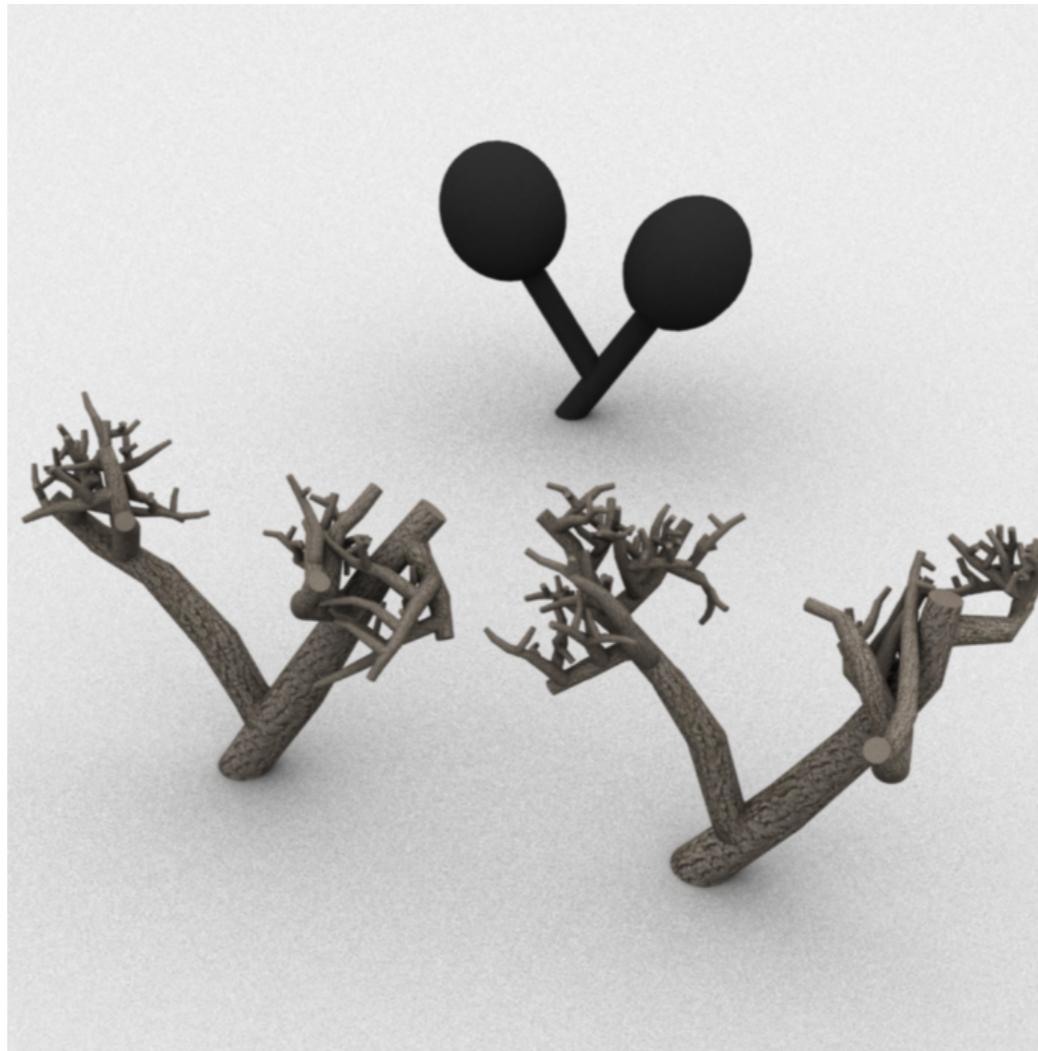
- I. Sample a 3D model.
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Posterior inf.  
with neural net



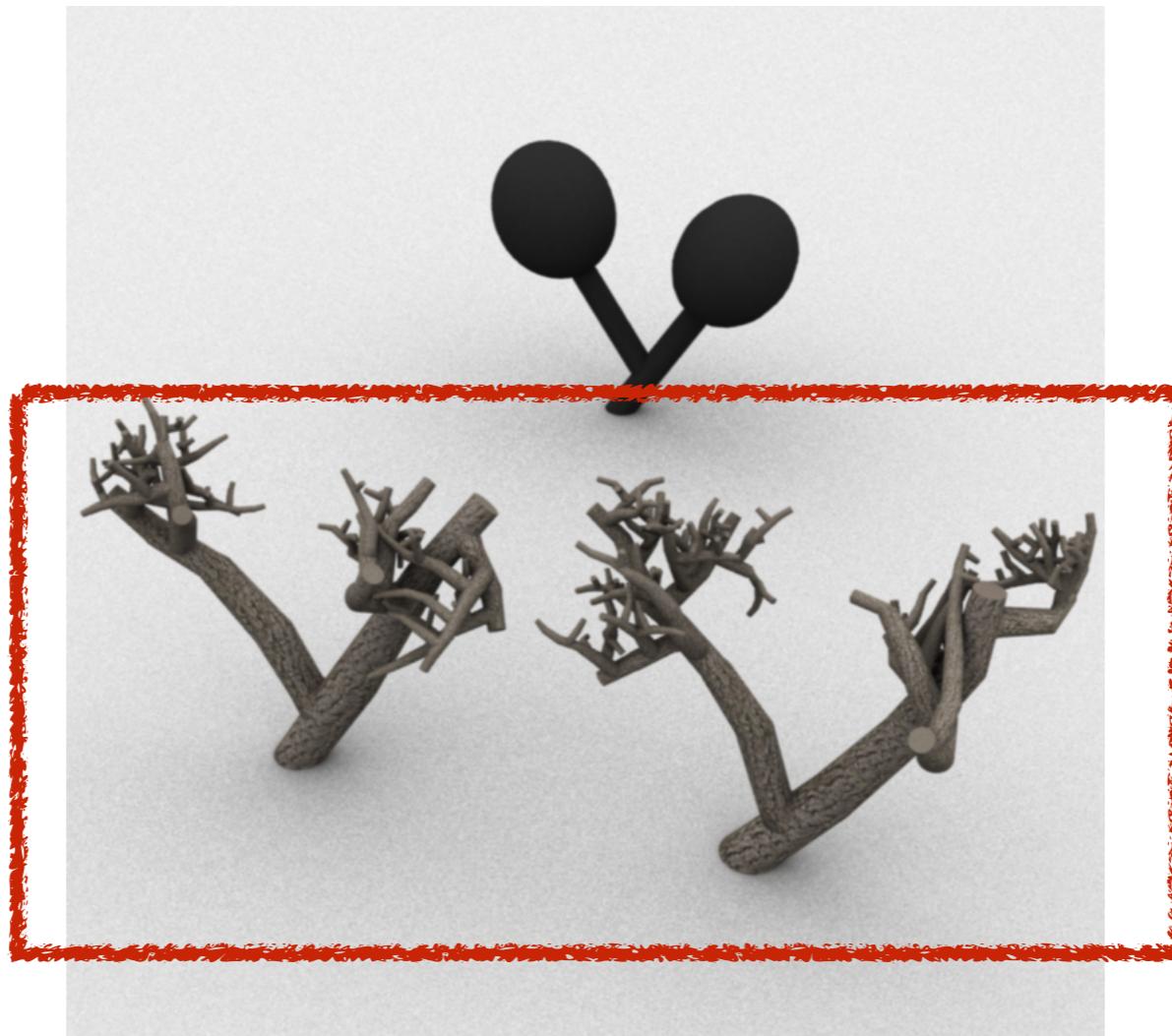
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# Three success stories: 3) procedural modelling



Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

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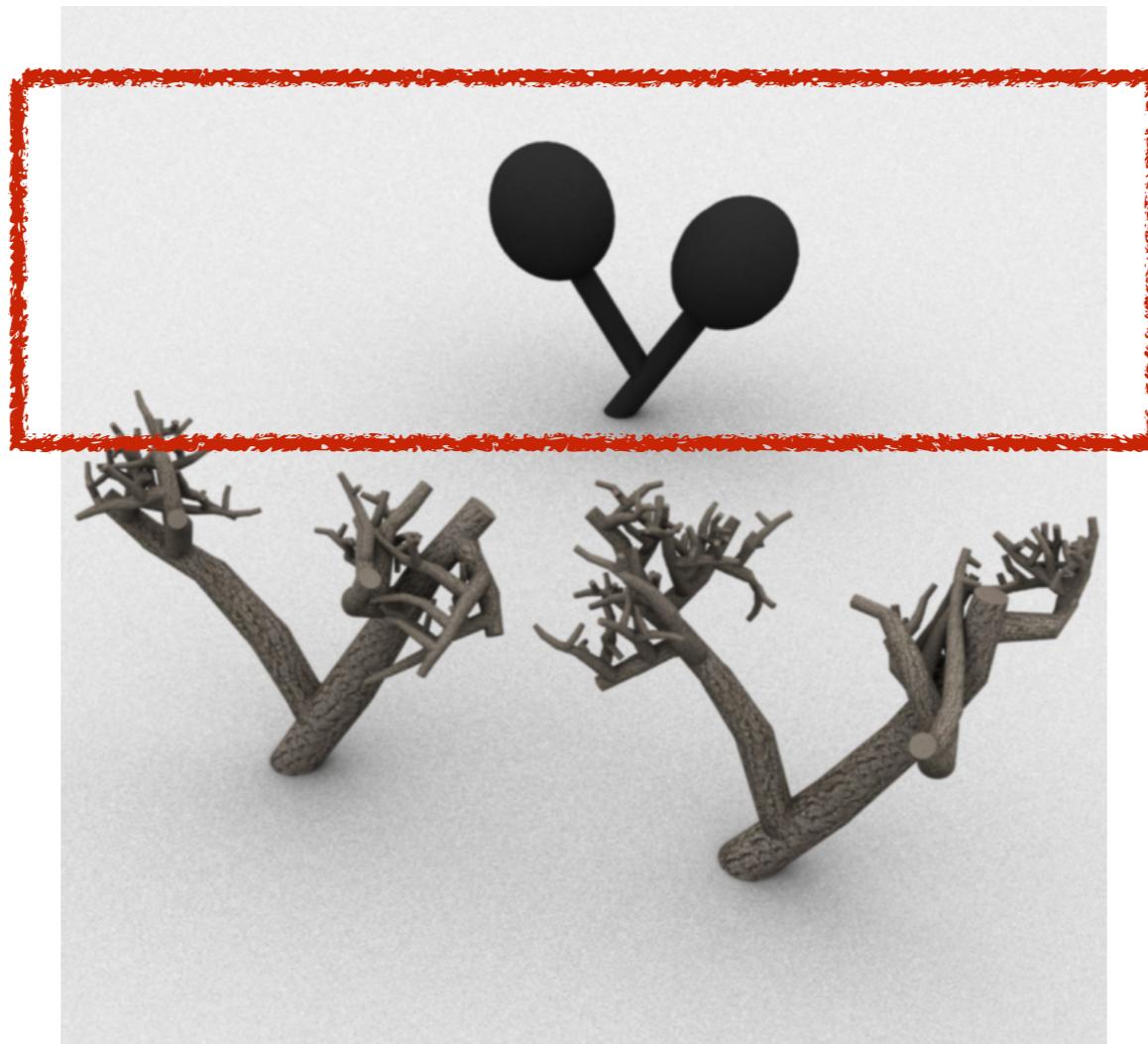


I. Sample a 3D object.

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

# Three success stories:

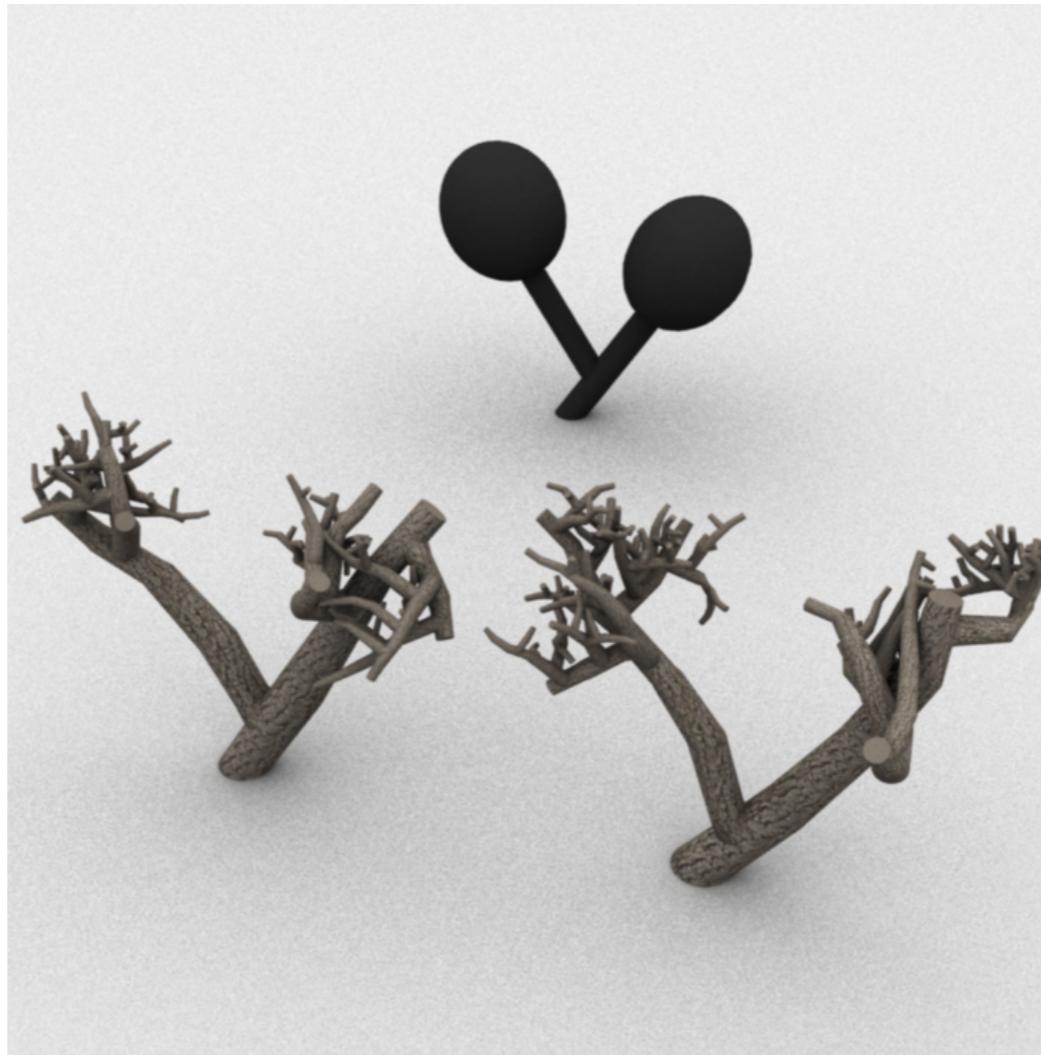
## 3) procedural modelling



1. Sample a 3D object.
2. Score the object.

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

# Three success stories: 3) procedural modelling



1. Sample a 3D object.
2. Score the object.  
**Used stochastic future.**

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

# Three succ

Asynchronous function  
call via future

```
future.create(function(i, frame, prev)
    if flip(T.branchProb(depth, i)) then
        -- Theta mean/variance based on avg weighted by
        local theta_mu, theta_sigma = T.estimateThetaD:
        local theta = gaussian(theta_mu, theta_sigma)
        local maxbranchradius = 0.5*(nextframe.center
        local branchradius = math.min(uniform(0.9, 1) *
        local bframe, prev = T.branchFrame(splitFrame,
        branch(bframe, depth+1, prev)
    end
```

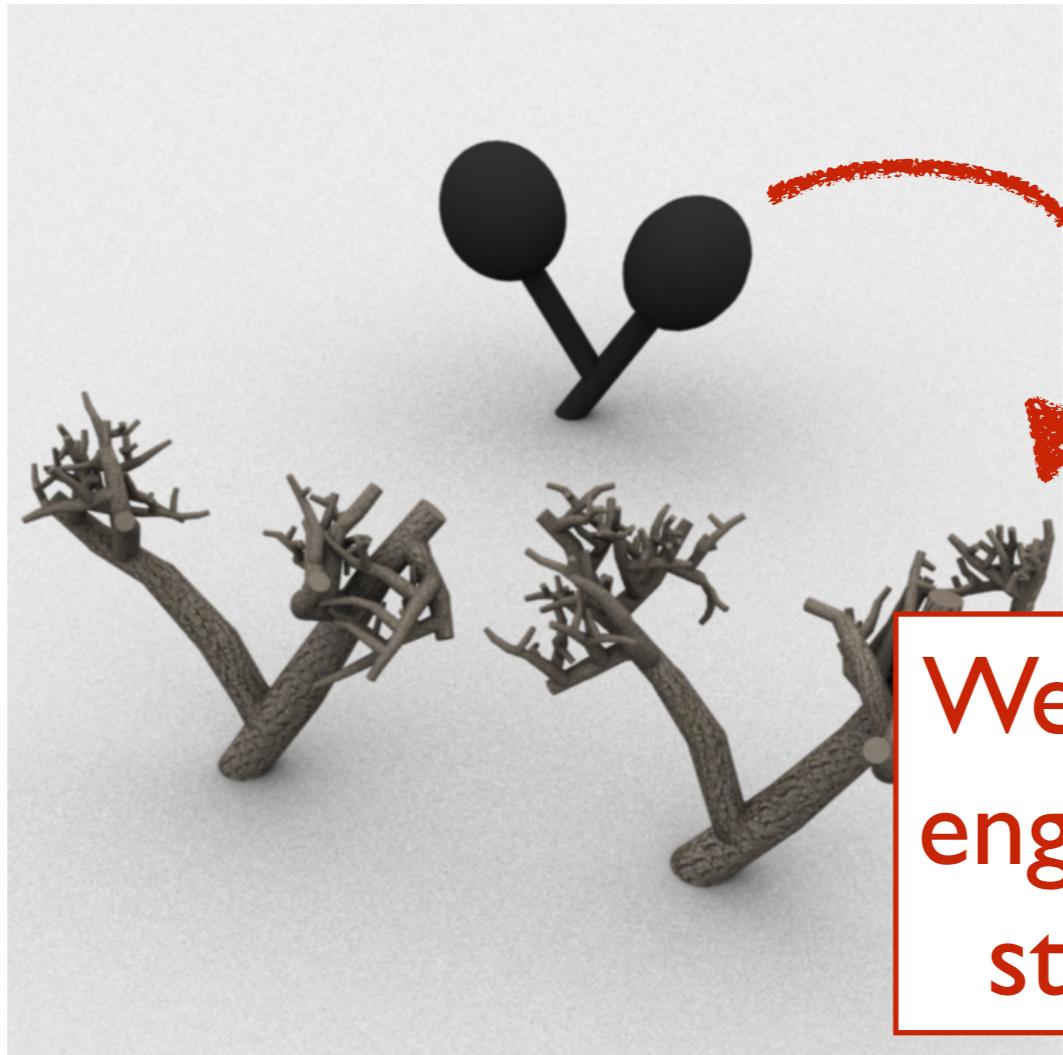
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Used stochastic future.

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

# Three success stories:

## 3) procedural modelling



WebPPL's inference  
engine that exploits  
stochastic future

1. Sample a 3D object.
2. Score the object.  
Used stochastic future.

Ritchie, Mildenhall, Goodman,  
Hanrahan [SIGGRAPH'15]

# Techniques used

- Changepoints: CPS transformation and new foundation of probability theory.
- Captcha and inverse graphics: neural nets and inference amortisation.
- Procedural modelling: sequential Monte Carlo algorithms and stochastic future.

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Prog. Languages,

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Prog. Languages, Machine Learning,

# Techniques used

- Changepoints: CPS transformation and new foundation of probability theory.
- Captcha and inverse graphics: neural nets and inference amortisation.
- Procedural modelling: sequential Monte Carlo algorithms and stochastic future.

Prog. Languages, Machine Learning, Probability Theory

# Overview of the course

# Objective

1. Learn how to write and reason about models in a prob. prog. language (PPL).
2. Learn results from ML/PL/prob. theory that are used for building effective PPLs.
3. Contribute to probabilistic programming.

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1. Learn how to write and reason about models in a prob. prog. language (PPL).
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3. Contribute to probabilistic programming.  
Group project by a group of 3-4 students.

# Webpage

<https://github.com/hongseok-yang/probprog18>

All the important announcements will be made  
in this webpage.

# Evaluation

- 3-5 homework exercises (20%).
- Group project (40%).
- Final exam (40%).

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- 3-5 homework exercises (20%).
- Group project (40%).
- Final exam (40%).

3-4 students form a group.

- Track A: New cool application of a PPL.
- Track B: Study one of two mathematical topics in depth, and teach fellow students.

Tasks: i) project. ii) 2 presentations (topic - 4 Apr, result - 10/15 May or 5/7 June). iii) report.

# Really important announcement

- I. Form a group and inform me and TA by the midnight of March 13 (Tuesday).
2. If your group wants to do Track B, contact me as early as possible. For logistic reasons, we will have only two groups on Track B.

# Somewhat important announcement

- Lecturer: Prof Hongseok Yang  
email: [hongseok00@gmail.com](mailto:hongseok00@gmail.com)
- TA: Kwonsoo Chae  
email: [kwonsoo.chae@gmail.com](mailto:kwonsoo.chae@gmail.com)
- Install Anglican. Try the changepoint example. Get hints from the webpage.

# Webpage

<https://github.com/hongseok-yang/probprog18>

All the important announcements will be made  
in this webpage.