CS423: Probabilistic Programming

Amortised Inference

Hongseok Yang
KAIST

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  (let [r (sample
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        a (observe (flip r) true)
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Importance sampling with proposal q(r).

1. Generate $(w_1,r_1), \ldots, (w_n,r_n)$ by running prog.

2. Estimate $\mathbb{E}_{p(r|a,b,c)}[f(r)] \approx \sum_i f(r_i)*(w_i/\sum_j w_j)$.

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$w_1 = 1 * p(.4)/q(.4)$

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$w_1 = .096 * p(.4)/q(.4)$

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$w_2 = .144 * p(.6)/q(.6)$
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How to find good q?

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How to find good q? Use amortised inference!

We often need to infer posterior of one model multiple times with different observations.

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Other examples:
Financial model,
captcha, brain, etc.

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Amortised inference. 1) Learn a proposal q(x; y) parameterized by obs. y via preprocessing.

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Amortised inference. 1) Learn a proposal $q(x; y)$ parameterized by obs. y via preprocessing. 2) Use $q(x;y_0)$ for any actual observation y_0 later.

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neural nets

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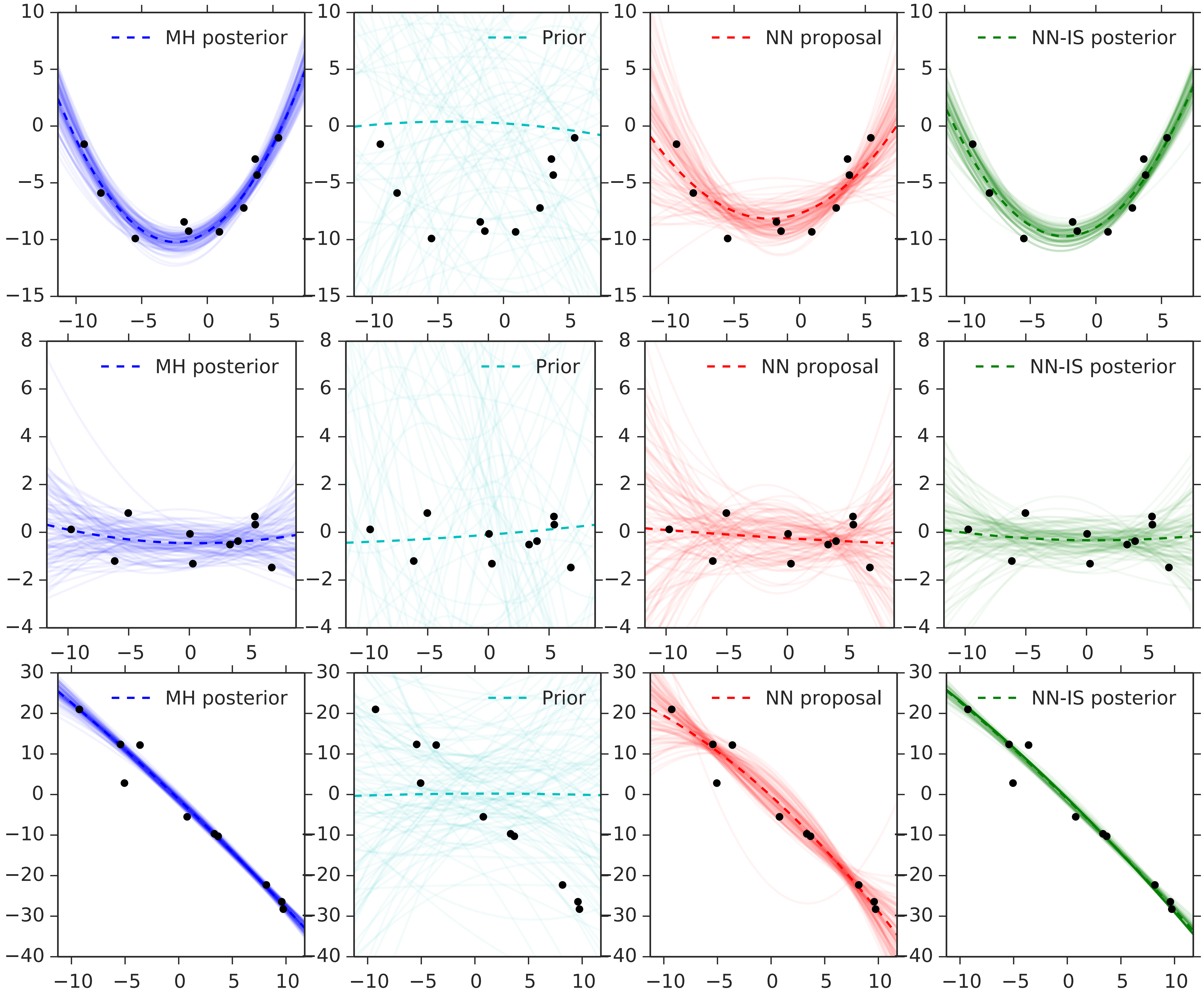
Other examples: Financial model, captcha, brain, etc.

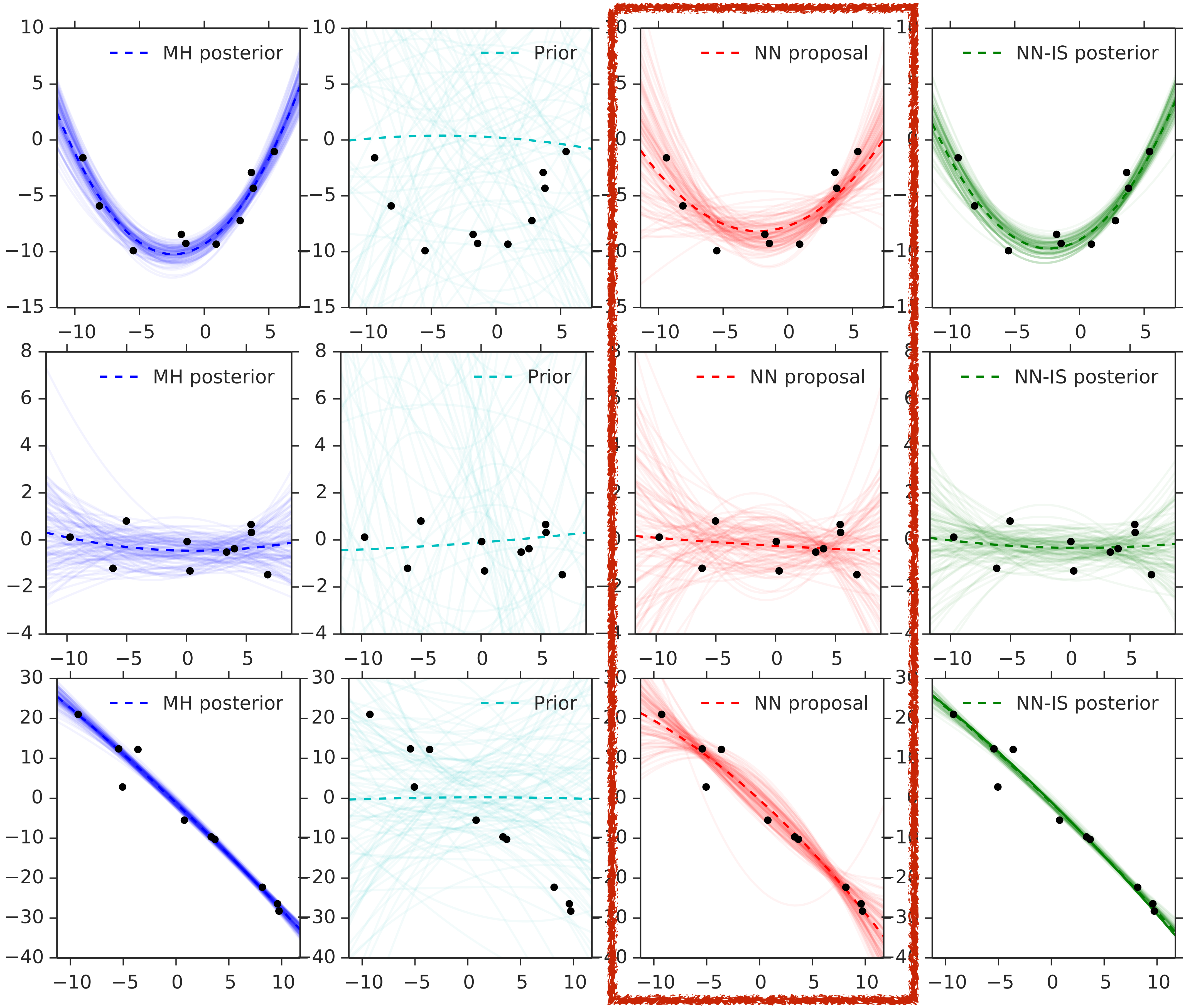
sample/object duality and reverse KL

Amortised inference. 1) Learn a proposal $q(x; y)$ parameterized by obs. y via preprocessing. 2) Use $q(x;y_0)$ for any actual observation y_0 later.

neural nets



Model for non-linear regression [Paige et al., ICML16]



Model for non-linear regression [Paige et al., ICML16]

Observed images

(W4kgvQ) (uV7FeWB) (MqhnpT)

more preprocessing

Samples

W4kgvQ	uV7EeWB	MqhnpT
WA4rjvQ	uV7FeWB	MypppT
Woxewd9	mTTEMMm	RIrpES
BKvu2Q	C9QDsoN	rS5FP2B

less preprocessing

Captcha solving [Le et al., AISTATS16]

Learning outcome

Can describe how amortised inference works for models written in math.

Can explain key ideas behind implementing amortised inference for probabilistic programs.

Proposal learning problem

Given:

1. joint dist. $p(x,y)$ for latent x and observed y,
2. IS proposal $q_\theta(x;y)$ parameterized by θ & y.

Find θ such that $q_\theta(x;y)$ is good for most y.

Proposal learning problem

Specified by $p(x)$ and $p(y|x)$.
Interested in $p(x|y)$.
But specific y not given yet.

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Differentiable wrt. θ for fixed x,y.
E.g. $q_\theta(x;y) = \text{normal}(x; f_\theta(y), g_\theta(y))$ for neural nets f,g.

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Proposal learning problem tackled by amortised inf.

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y sampled from p(y)

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Small KL divergence from p(x|y) to $q_\theta(x;y)$.

$\mathrm{KL}[\,p(x|y)\;||\;q_\theta(x;y)\,] = \mathbb{E}_{p(x|y)}[\,\log(p(x|y)/q_\theta(x;y))\,]$.

Proposal learning problem

$\text{argmin}_\theta\ \mathbb{E}_{p(y)}[\text{KL}[\ p(x|y)\ ||\ q_\theta(x;y)\ ]]$.

Solve this by stochastic gradient descent.

inf.

y sampled from p(y)

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Stochastic gradient descent for $\mathbb{E}_{p(y)}[\mathrm{KL}[p(x|y)||q_\theta(x;y)]]$

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Initialise θ

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

Initialise θ

$\theta \leftarrow \theta - 0.01 * \nabla_\theta \mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

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(until θ doesn't change much)

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

Learning rate

Initialise θ

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Can't compute, but can approximate.

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

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$\nabla_\theta \mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]] \approx -1/n * \sum_{i=1..n} \nabla_\theta \log q_\theta(x_i;y_i)$.

Stochastic gradient descent for $\mathbb{E}_{p(y)}[\mathrm{KL}[p(x|y)||q_\theta(x;y)]]$

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$\theta \leftarrow \theta - 0.0$

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Hard to sample x from p(x|y) for given y,
but easy to sample (x,y) from p(x,y).
Thus, no problem in sampling.

Can't compute, but can approximate.

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Exists since $q_\theta(x_i;y_i)$ is differentiable.

Can't compute, but can approximate.

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[Q] Prove that this is an unbiased estimator.

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Initialise θ

Repeat the following until θ doesn't change much:

1. Sample (x_1,y_1), …, (x_n,y_n) from $p(x,y)$
2. $G \leftarrow -1/n * \sum_{i=1..n} \nabla_\theta \log q_\theta(x_i;y_i)$
3. $\theta \leftarrow \theta - 0.01 * G$

Learning IS proposal $q_\theta(x;y)$ by amortised inference

Using stochastic gradient descent, solve:

$$\operatorname{argmin}_\theta \mathbb{E}_{p(y)}[\mathrm{KL}[p(x|y) \,||\, q_\theta(x;y)]].$$

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SVI: $\mathrm{argmin}_\theta \ \mathrm{KL}[q_\theta(x) \ || \ p(x|y_0)]$ for a given y_0.

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(b) Generated y vs given y_0.

Learning IS proposal $q_\theta(x;y)$

Lets us avoid sampling x from posterior $p(x|y_0)$ for given y_0. Just need to sample (x,y) from joint $p(x,y)$.

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What about probabilistic programs?

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

Initialise θ

Repeat the following until θ doesn't change:

1. Sample (x_1,y_1), …, (x_n,y_n) from $p(x,y)$
2. $G \leftarrow -1/n * \sum_{i=1..n} \nabla_\theta \log q_\theta(x_i;y_i)$
3. $\theta \leftarrow \theta - 0.01 * G$

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How to sample y?

Sample/observe duality

To sample observations, just replace sample by observe.

```
(defquery biased-coin []
  (let [r (sample
            (uniform-continuous 0 1))
        a (observe (flip r) true)
        b (observe (flip r) true)
        c (observe (flip r) true)]
    r))
```

Sample/observe duality

To sample observations, just replace sample by observe.

```
(defquery biased-coin-joint []
  (let [r (sample
            (uniform-continuous 0 1))
        a (sample (flip r))
        b (sample (flip r))
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    [r [a b c]]))
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Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

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Repeat the following until θ doesn't change:

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3. $\theta \leftarrow \theta - 0.01 * G$

Computed during execution.
Similar to the SVI case.
Just a new rule for sample.

Last remark

People also use "amortised inference" to mean parameter sharing via neural net in variational inf.

Assume not one but many observations $y_1, \ldots, y_n$.

1. Find separate $\theta_1, \ldots, \theta_n$ s.t. $q(x;\theta_i) \approx p(x|y_i)$.
2. Find one θ s.t. $q(x;f_\theta(y_i)) \approx p(x|y_i)$ where f_θ is a neural net.

Amortised inference means the second.

References

1. Inference networks for sequential Monte Carlo in graphical models. Paige et al. ICML'16.
2. Inference compilation and universal probabilistic programming. Le et al. AISTATS'17.