Causality

Jihoon Ko, Seungwoo Lee, Junho Han and Yongsu Baek

Lecture Objectives

1. To understand **what** is causality and **why** causality matters

2. To explore major **categories** of causality and their details

3. To learn how to deal with causality in **mathematical languages**

4. To find out how the intuitive causal reasoning can conflict with the logic of probability and statistics in **Paradoxes**

How can human get success on the Earth?

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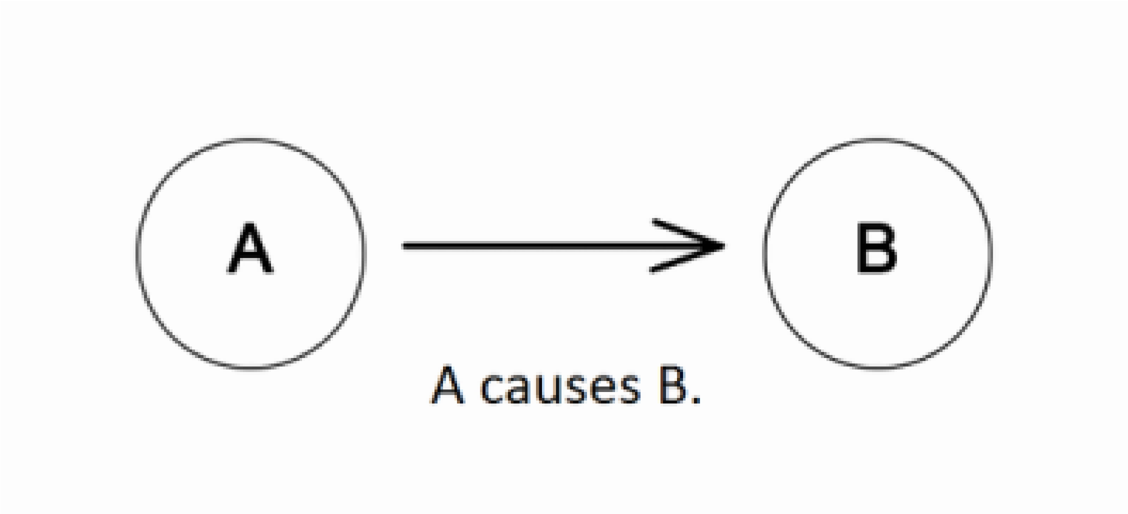
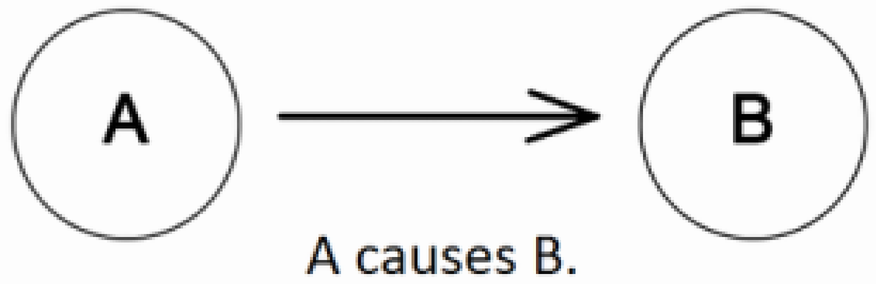


How can human get success on the Earth?

Causality



How can human get success on the Earth?

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How can human get success on the Earth?

new plans, imaginations, predictions
or causal thinking

We always think with causality (Causal thinking)

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1. The most advanced tool for managing causality.

2. Our brains store and construct an incredible amount of **causal knowledge** supplemented by data.

3. We can use this to answer most pressing questions of our time, but other species and (current) robots can't.

4. What if we unlock **the logic behind our causal thinking**?

Some causality questions:

1. Did the new tax law *cause* our sales to go up, or *was it* our advertising campaign?
2. *How effective* is a given treatment in *preventing* a disease?
3. I'm about to quit my job. *Should I*?

“The society and our daily life constantly demand answers to causality questions.”

Yet science gave us no useful methodologies even to articulate them **in mathematical languages** until very recently!
It’s called **do-calculus**. We will discuss it later.

Statistics vs Causality as scientific tools

Statistics vs Causality as scientific tools

Statistics is a quite useful tool,
but it explains only parts of the whole nature.

Statistics vs Causality as scientific tools

Observation

X	Y	Z
1	2	4
2	3	6
3	4	8
7	8	16
9	10	20
12	13	26

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Statistics can say that X+1=Y=Z/2, which is **correlation** of X, Y and Z.

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Causality says that there can be two different **models** behind data.

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Model 1.

Statistics vs Causality as scientific tools

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Cause → Effect

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Model 1.

Cause → Effect

Model 2.

Effect ← Cause

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True model behind data?

S … **correlation** of X, Y and Z.

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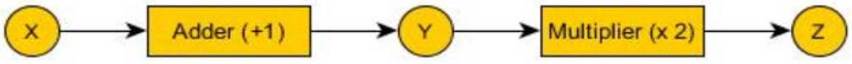
Model 1. 

Cause → Effect

Model 2. 

Effect ← Cause

Statistics vs Causality as scientific tools

Observation

X	Y	Z
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S **correlation** of X, Y and Z.

True model behind data?

Causality says that there can be two different **models** behind data.

It cannot explain cause and effect relationship.
So statistics is a model-blind method.

Model 2. 

Effect ← Cause

Statistics vs Causality as scientific tools

X=3 Y=4 Z=8 ← Observation

Model 1.

Cause → Effect

Model 2.

Effect ← Cause

Statistics vs Causality as scientific tools

What will happen to X if we **change** Y?

X=? Y=7 Artificially changed environment

X=3 Y=4 Z=8 Observation

Model 1.

Cause Effect

Model 2.

Effect Cause

Statistics vs Causality as scientific tools

What will happen to X if we **change** Y?

X=? Y=7 ← Artificially changed environment

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Model 1.

Cause → Effect

Model 2.

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Statistics vs Causality as scientific tools

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Model 1.

Cause → Effect

Model 2.

Effect ← Cause

Statistics vs Causality as scientific tools

What will happen to X if we **change** Y?

X still remains 3 !! X=? Y=7 ← Artificially changed environment

X=3 Y=4 Z=8 ← Observation

Model 1.

Cause → Effect

Model 2.

Effect ← Cause

Statistics vs Causality as scientific tools

What will happen to X if we **change** Y?

Statistics cannot answer to this question because it is a model-blind method.

Model 1.

Cause → Effect

Model 2.

Effect ← Cause

Statistics vs Causality as scientific tools

What will happen to X if we **change** Y?

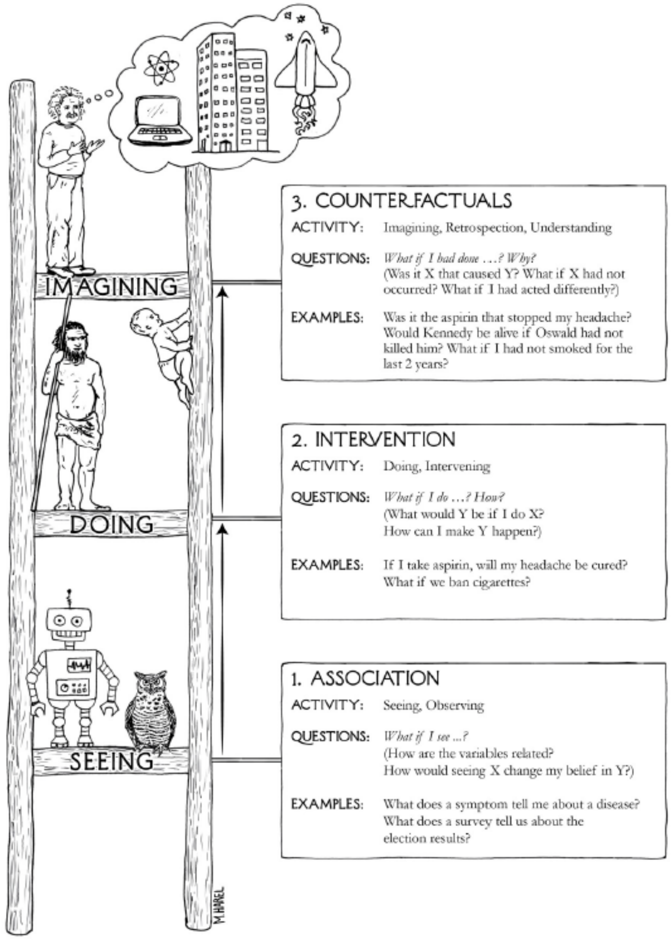
1. This kind of questions cannot be resolved by statistics.

2. Causality considers model unlike statistics, so it can handle this question.

3. However, figuring out the model behind data is a difficult research area, *which is not a goal of our lecture*. (Finding exact model may be impossible.)

4. Big data companies such as *Facebook* and *Youtube* know this, so they not only collect a lot of observations but also **constantly perform experiment** by changing environment.

3-steps Ladder of Causality



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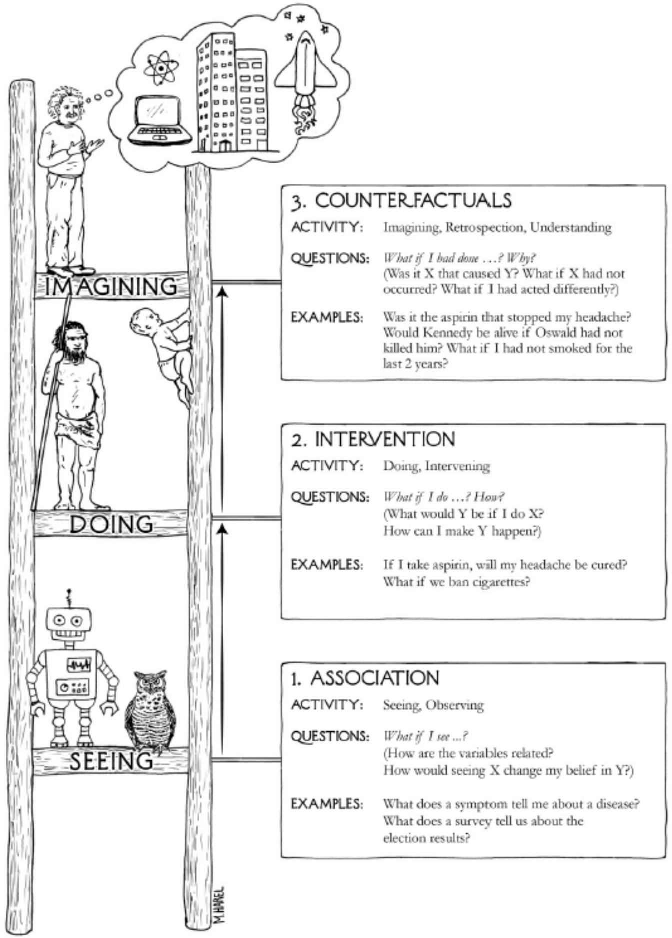

1. Association(=statistics)
   - looking for regularities in **observations**, which is exactly same as what statistics does.

3-steps Ladder of Causality

2. Intervention (=change environment/not just observe)
   - imagining what will happen if we **intervene, do or fix** some factors which doesn't occur yet.

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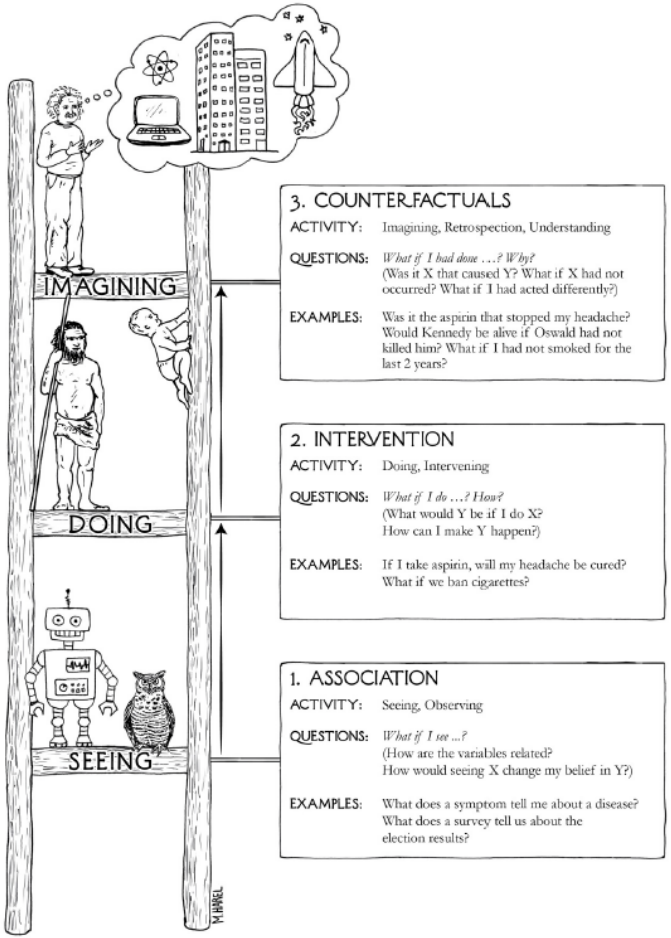

3-steps Ladder of Causality

3. Counterfactuals (=counter to facts)
   - imagining what will happen if we **negate** the past observed factors which already occurred.

2. Intervention (=change environment/not just observe)
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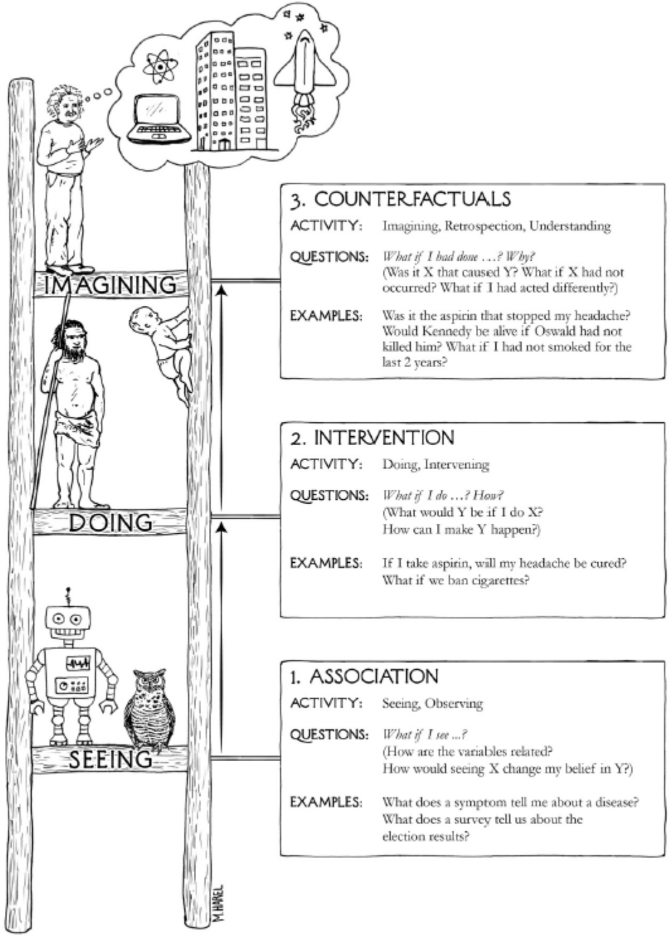

3-steps Ladder of Causality (examples)

Level (Symbol)	Typical Activity	Typical Questions	Examples
1. Association $P(y\|x)$	Seeing	What is? How would seeing X change my belief in$Y$?	What does a symptom tell me about a disease? What does a survey tell us about the election results?
2. Intervention $P(y\|do(x), z)$	Doing Intervening	What if? What if I do X?	What if I take aspirin, will my headache be cured? What if we ban cigarettes?
3. Counterfactuals $P(y_x\|x', y')$	Imagining, Retrospection	Why? Was it X that caused Y? What if I had acted differently?	Was it the aspirin that stopped my headache? Would Kennedy be alive had Oswald not shot him? What if I had not been smoking the past 2 years?

Fig. 1. The Causal Hierarchy. Questions at level i can only be answered if information from level i or higher is available.

The Seven Tools of Causal Inference with Reflections on Machine Learning, Judea Pearl

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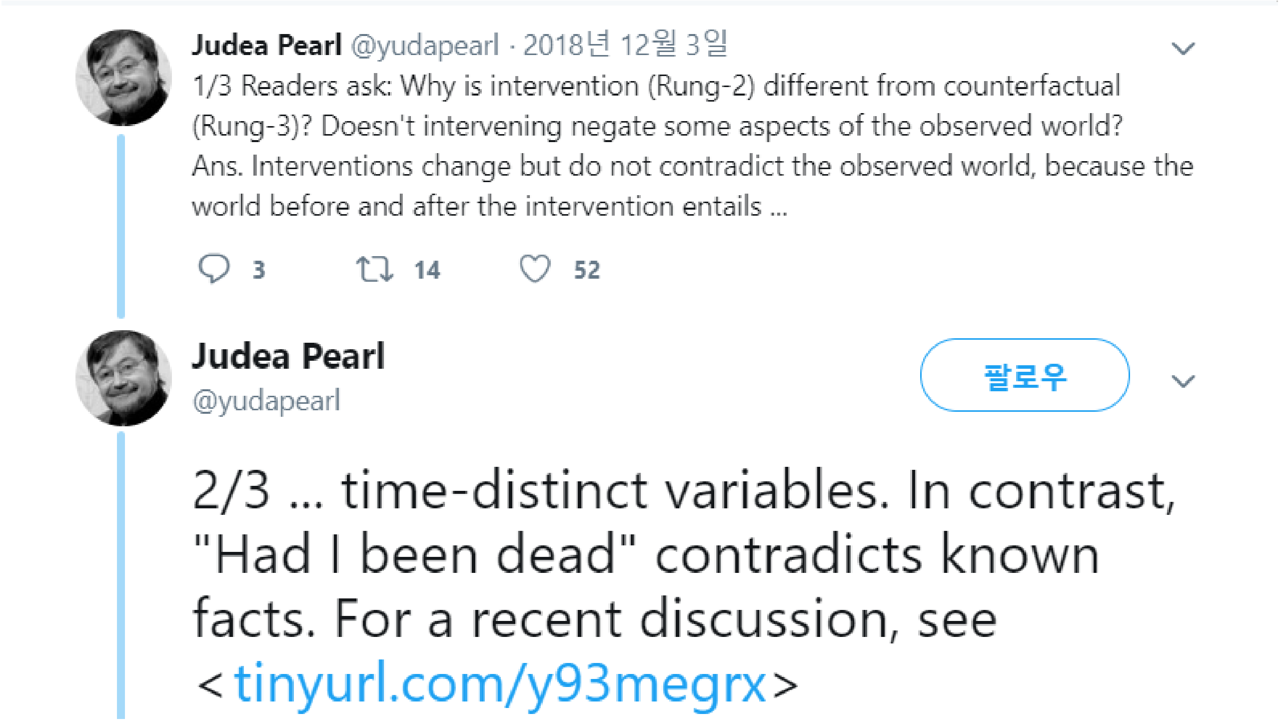
difference: intervention and counterfactuals



Judea Pearl @yudapearl · 2018년 12월 3일

1/3 Readers ask: Why is intervention (Rung-2) different from counterfactual (Rung-3)? Doesn't intervening negate some aspects of the observed world? Ans. Interventions change but do not contradict the observed world, because the world before and after the intervention entails ...

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팔로우

2/3 ... time-distinct variables. In contrast, "Had I been dead" contradicts known facts. For a recent discussion, see <tinyurl.com/y93megrx>

difference: intervention and counterfactuals

Question: *Given that Hilary Clinton* ***did not win the 2016 presidential election****, and given that she* ***did not visit Michigan 3 days before the election****, and given* ***everything else we know about the circumstances of the election****, what can we say about the probability of Hilary Clinton winning the election, had she visited Michigan 3 days before the election?*

difference: intervention and counterfactuals

Answer: *probability that she* ***hypothetically*** *wins the election*

- she lost the election
- she did not visit Michigan
- any other relevant an observable facts
- she ***hypothetically*** visits Michigan

difference: intervention and counterfactuals

$$Intervention : p(y|do(x))$$

$$Counterfactual : p(y'|do(x'))?$$

difference: intervention and counterfactuals

$$Intervention : p(y|do(x))$$

difference: intervention and counterfactuals

$$Intervention : p(y|do(x))$$

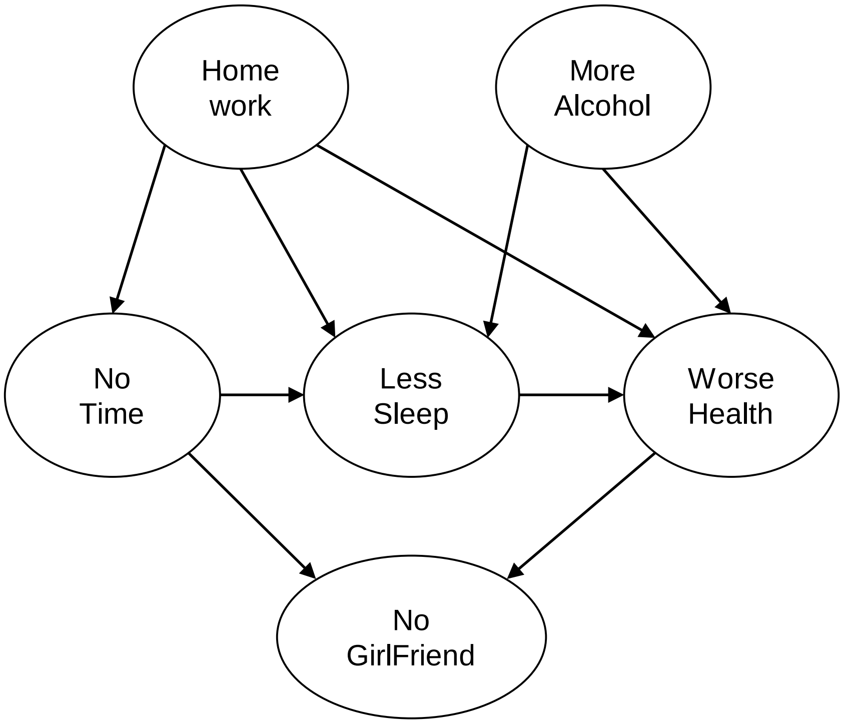
$$Counterfactual : p(y'|x, y, do(x')))$$

Theoretical Approach to Causality

- Causal diagram
- Effect of Observation & Intervention
- D-separation
- Do-calculus
- Examples

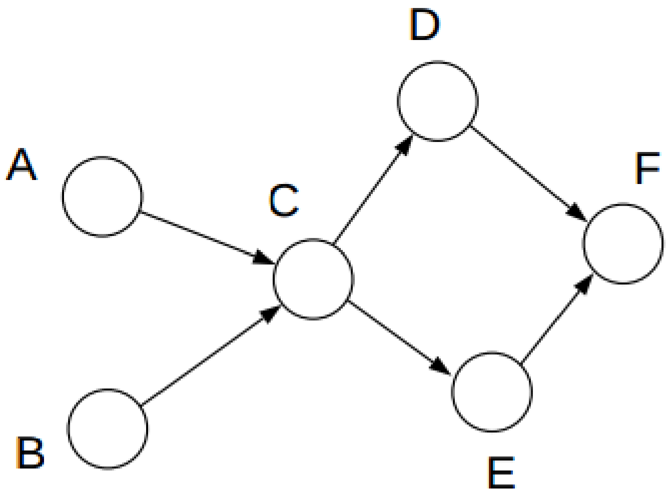
Causal Diagram

- It is Directed Acyclic Graph(DAG)
- Vertex represents each feature(factors)
- Edge represents Cause-Effect relation among factors
- How to formulate mathematically?



Probability Graphical Model

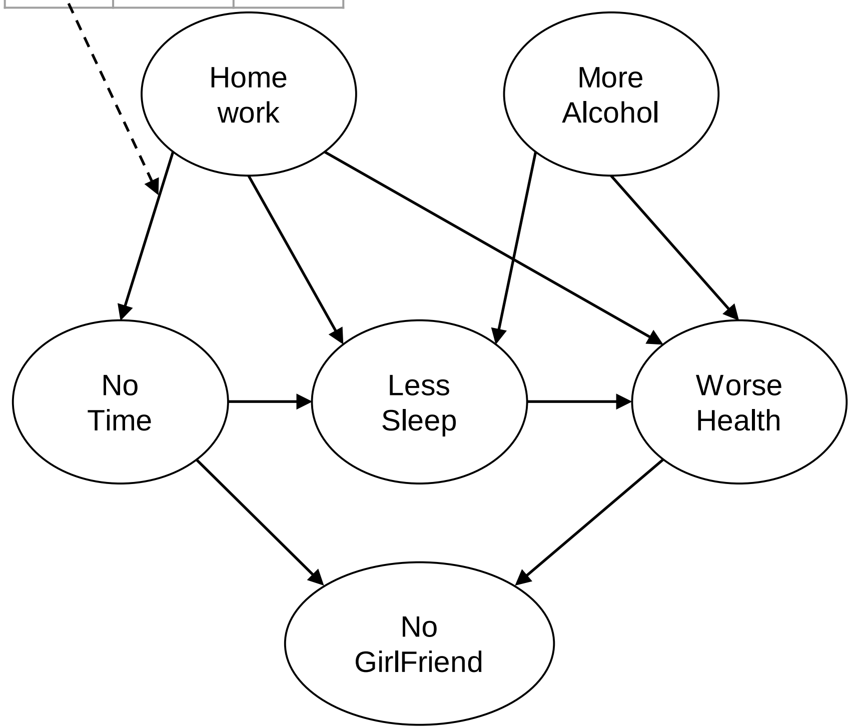
- Each Vertex represents random variable.
- Edge represents ***conditional dependency*** between two random variable
- If there is no edge between two random variable, then they are ***conditionally independent***(p(A|B) = p(A))



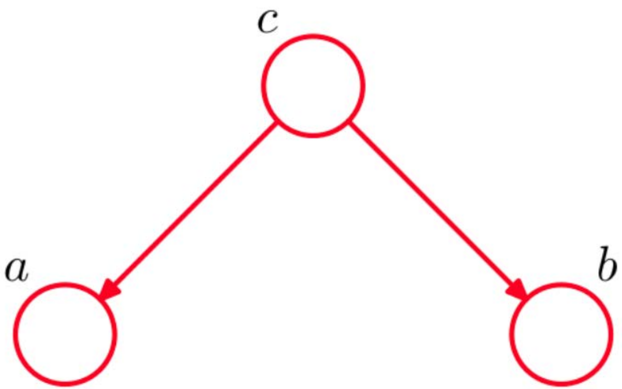
Causal Diagram +PGM

- Combining PGM and Causal Diagram can formulate causality problem.
- However, ***observation*** might affect Independence relations

	No HW	HW
No time	0.1	0.99
time	0.9	0.1



Observation change dependence - (1)



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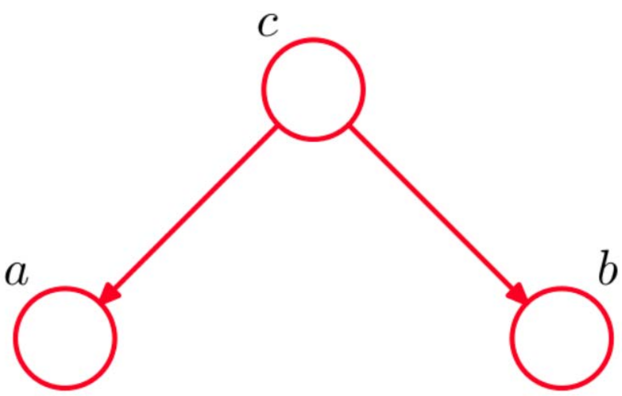

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A. No

$$
\begin{aligned}
p(a,b) &= \sum_c p(a,b,c) = \sum_c p(a|b,c)p(b,c) \\
&= \sum_c p(a|b,c)p(b|c)p(c) = \sum_c p(a|c)p(b|c)p(c))
\end{aligned}
$$

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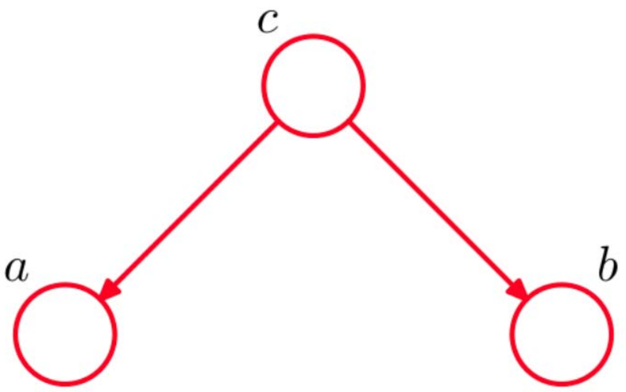

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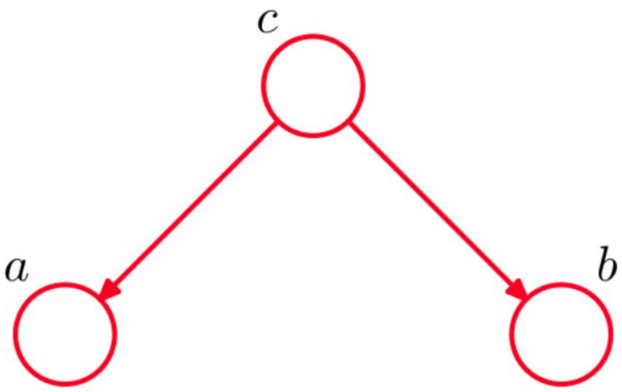

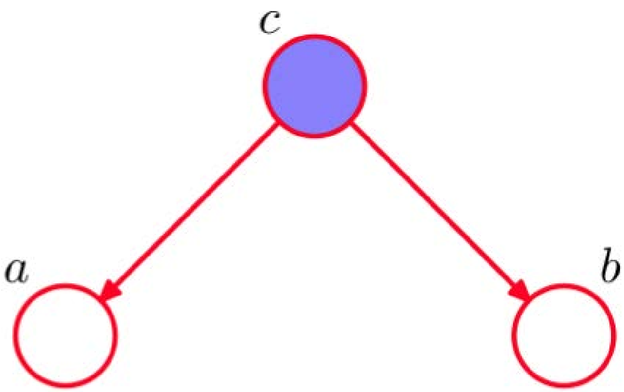

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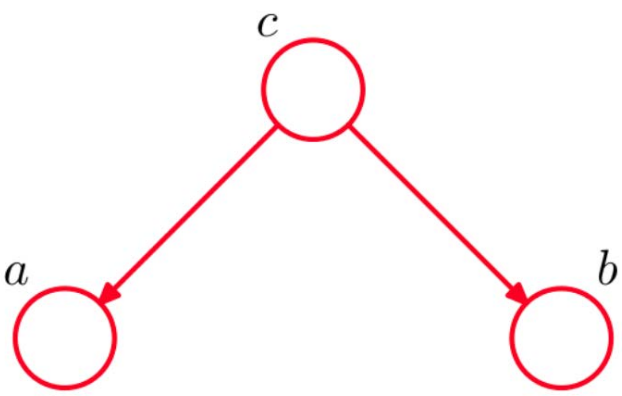

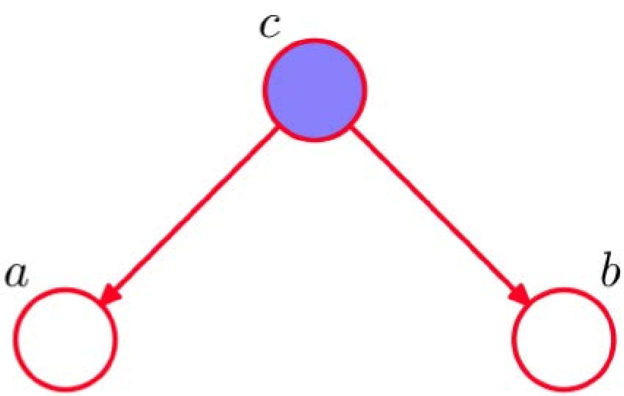

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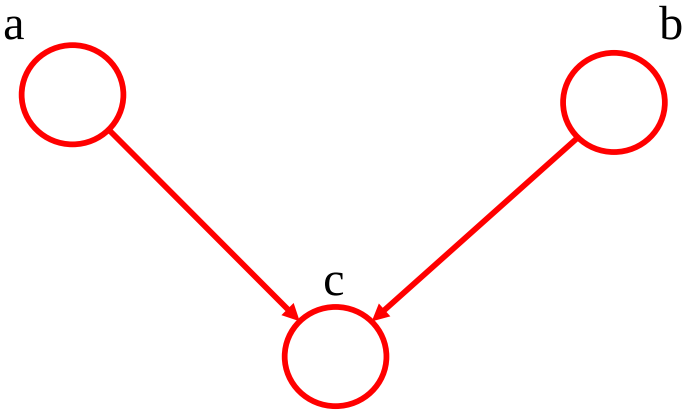
A. Yes

$$\begin{aligned} p(a,b|c) &= \frac{p(a,b,c)}{p(c)} \\ &= p(a|c)p(b|c) \end{aligned}$$

Observation change dependence - (2)



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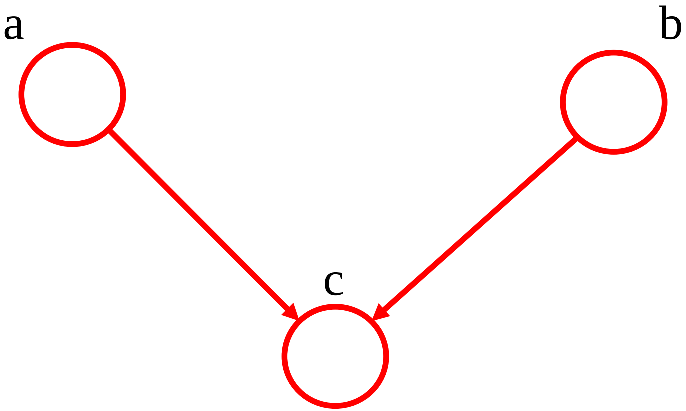

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A. Yes $p(a,b) = p(a)p(b)$

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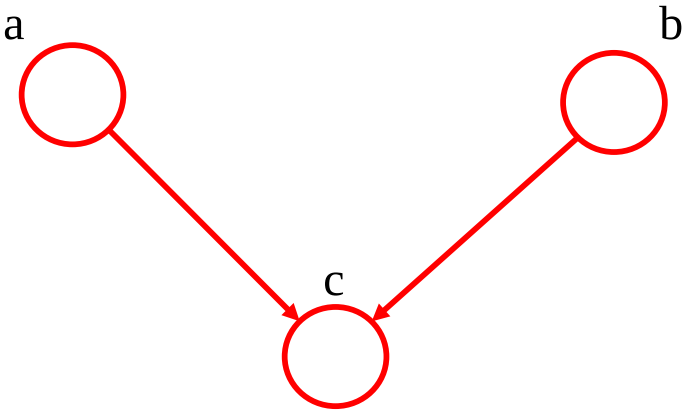

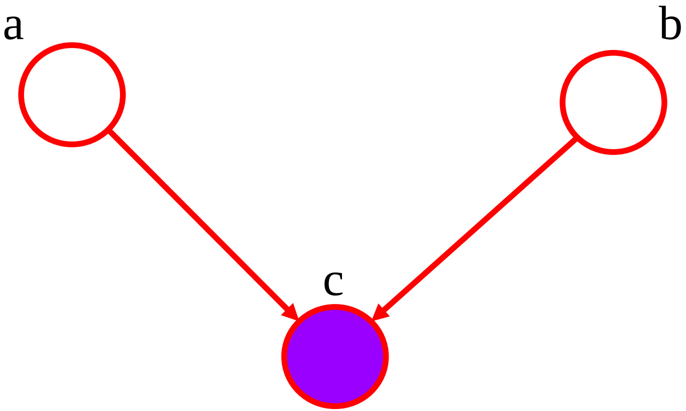

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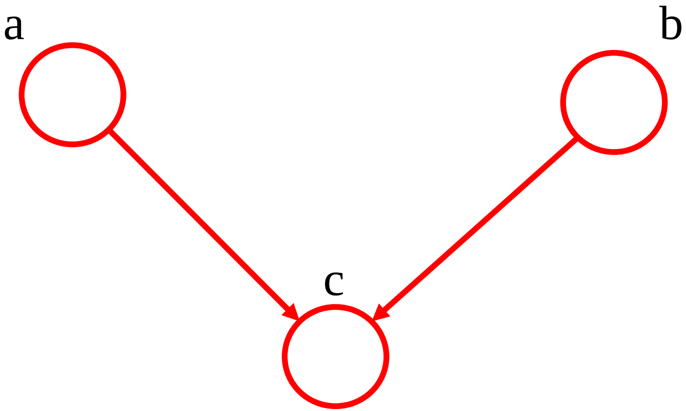

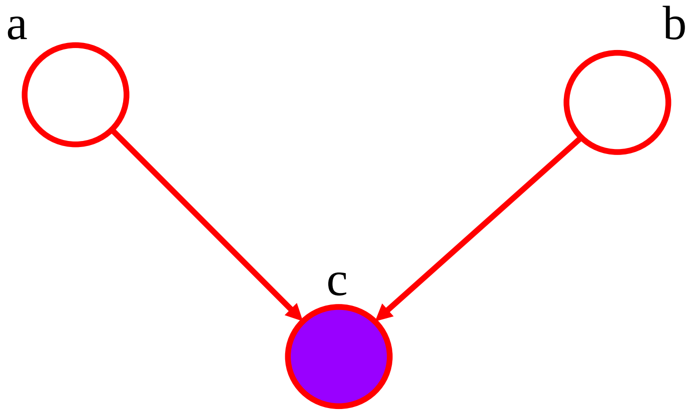

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A. No

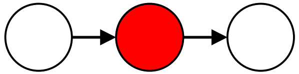
$$p(a,b,c) = p(c|a,b)p(a)p(b)$$

$$p(a,b|c) = \frac{p(a,b,c))}{p(c))} = \frac{p(c|a,b)p(a)p(b)}{p(c))}$$

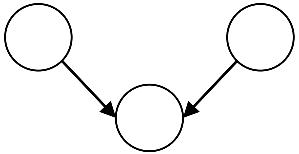
D-separation

‘***d-separation***’ is a criterion for deciding, from a causal diagram, whether a set A of variables is independent of another set B given a third set C, notated as $A \perp\!\!\!\perp B | C$

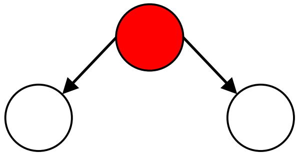
Examples :



Causal/Evidential ‘Chain’



‘Collider’

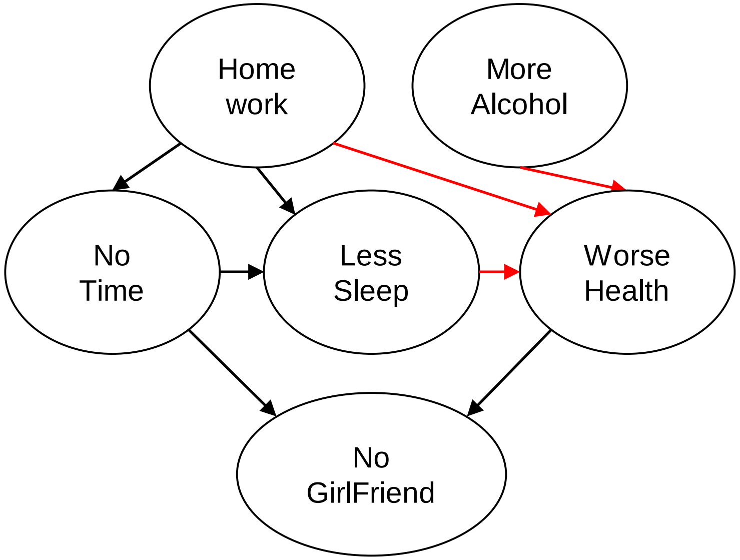


‘Confounder’

Using d-separation, we can predict the effect of ***observation***

Q. How about ***intervention***?

Intervention in causal diagram



Hypothesis :

If I'm *Healthier*, then Can I make Girlfriend?

Problem :

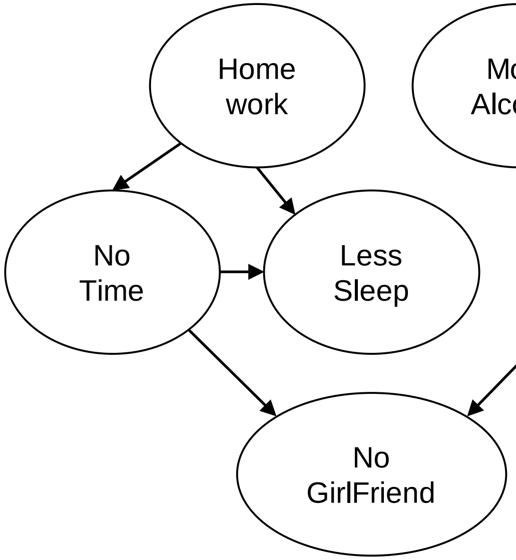
We ***cannot*** just fix a value in causal diagram.

Effect variable affects the cause variable.

ex) : Better health means there is fewer HW, more sleep, and Low alcohol consumption.

Q. How do we ***remove*** the effect on ancestor variables?

Intervention in causal diagram

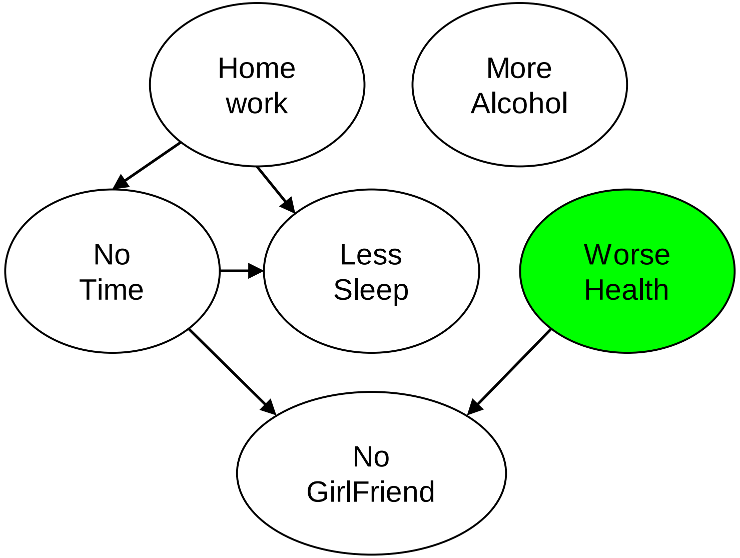


Solution :

By removing the in-edge of intervention node, we can make variables ***independent*** to intervention value

ex) Now Sleep, Homework, Alcohol are independent to health. So, we can think it as observation

Intervention in causal diagram



Solution :
By removing the in-edge of intervention node, we can make variables ***independent*** to intervention value

ex) Now Sleep, Homework, Alcohol are independent to health. So, we can think it as observation

Result :
Making Girlfriend still depends on *Homework*

Do-Calculus

- Calculus to discuss causality in a formal language by Judea Pearl
- A new operator, do(), marks an action or an ***intervention*** in the model.
- Example: $p(y|do(x))$ instead of $p(y|x)$

Main goal: *to generate probabilistic formulas for the effect of interventions in terms of the observed probabilities.*

Notation for do-calculus

- $G_{\overline{X}}$ denotes the perturbed graph in which all edges pointing to X have been deleted
- $G_{\underline{X}}$ denotes the perturbed graph in which all edges pointing from X have been deleted.
- Z(W) denote the set of nodes in Z which are not ancestors of W

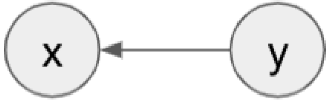
Pearl's 3 rules

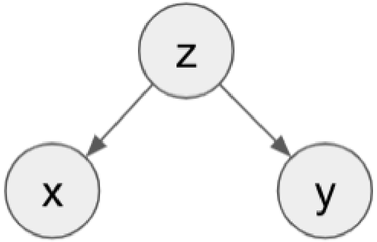
1. Ignoring Observations
$$p(y|do(x), z, w) = p(y|do(x), w) \quad if (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X}}}$$

1. Action/Observation Exchange (the back-door criterion)
$$p(y|do(x), do(z), w) = p(y|do(x), z, w) \quad if (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X},\underline{Z}}}$$

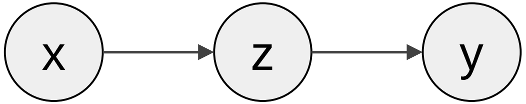
1. Ignoring Actions/Interventions
$$p(y|do(x), do(z), w) = p(y|do(x), w) \quad if (Y \perp\!\!\!\perp Z|X, W)_{G_{\overline{X},\overline{Z(W)}}}$$

Do-Calculus: Simple Example



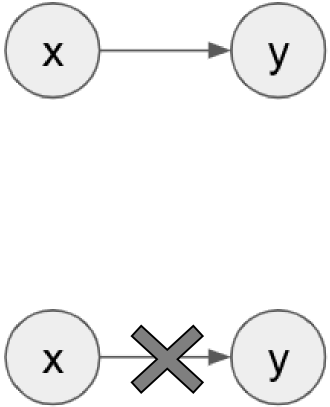


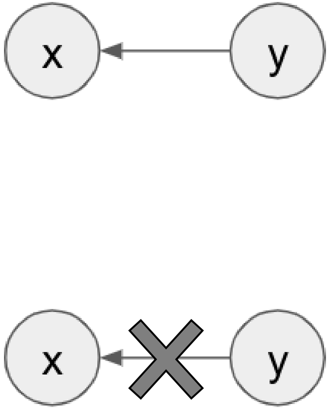
$$p(y|do(x)) = ?$$

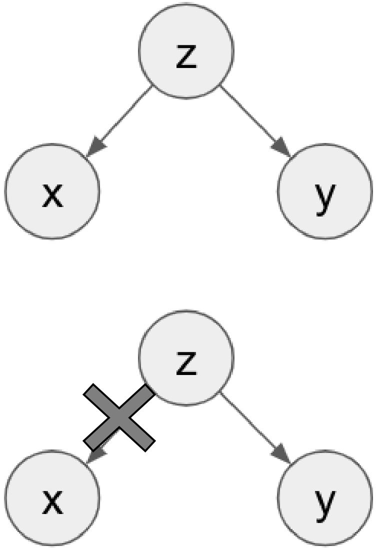


$$p(y|do(z), x) = ?$$

Do-Calculus: Simple Example



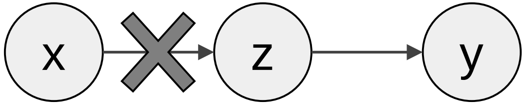




$$p(y|do(x)) = p(y|x)$$

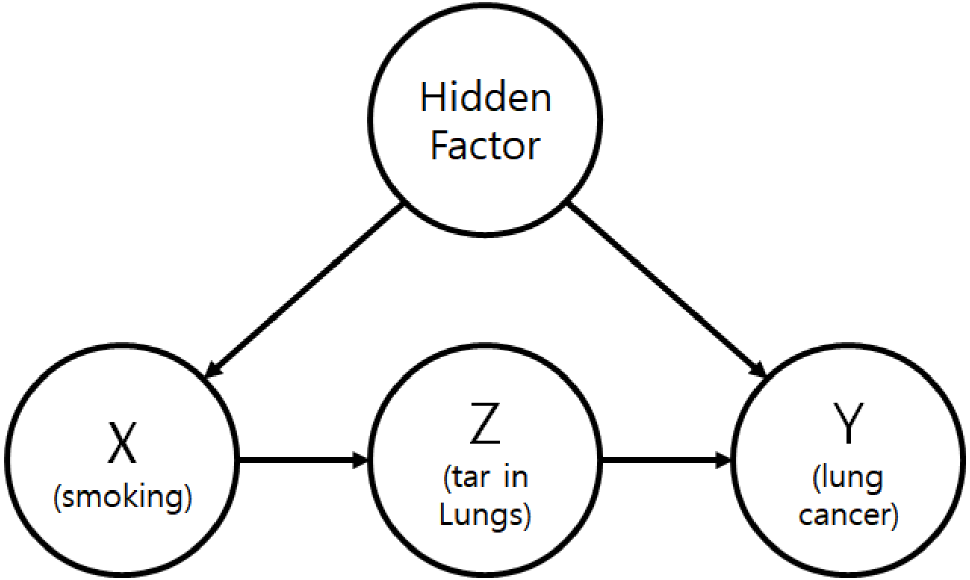
$$p(y|do(x)) = p(y)$$

Do-Calculus: Simple Example

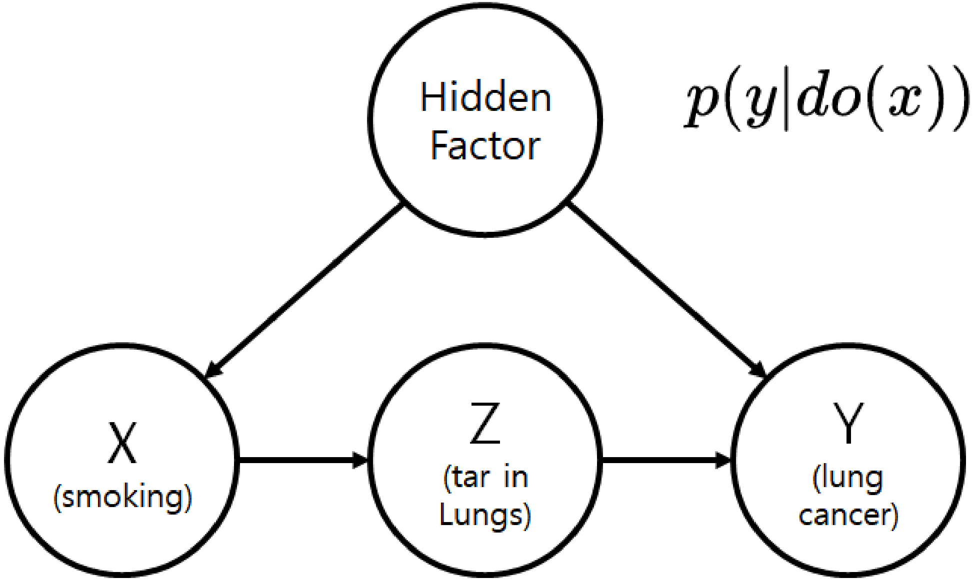


$$p(y|do(z),x) = p(y|do(z))$$
$$= p(y|z)$$

Do-Calculus: Example

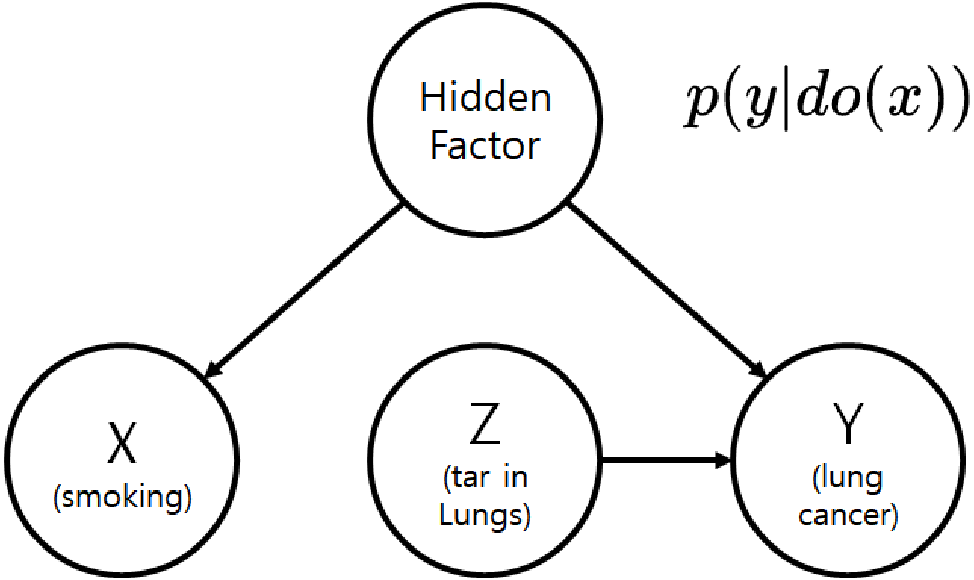


$$p(y|do(x)) =?$$



$$p(y|do(x)) = \sum_z p(y|z, do(x))p(z|do(x))$$

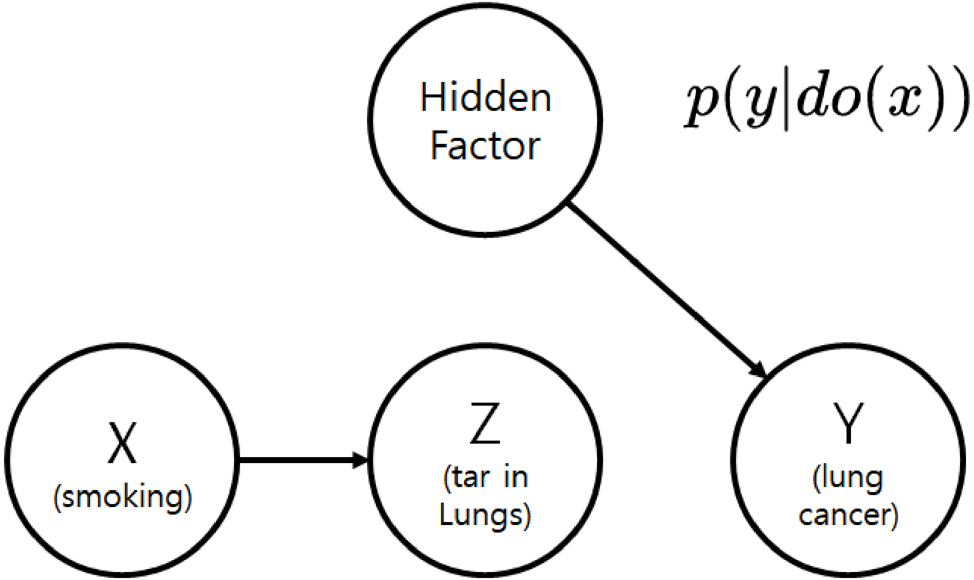
G



$G_{\underline{X}}$

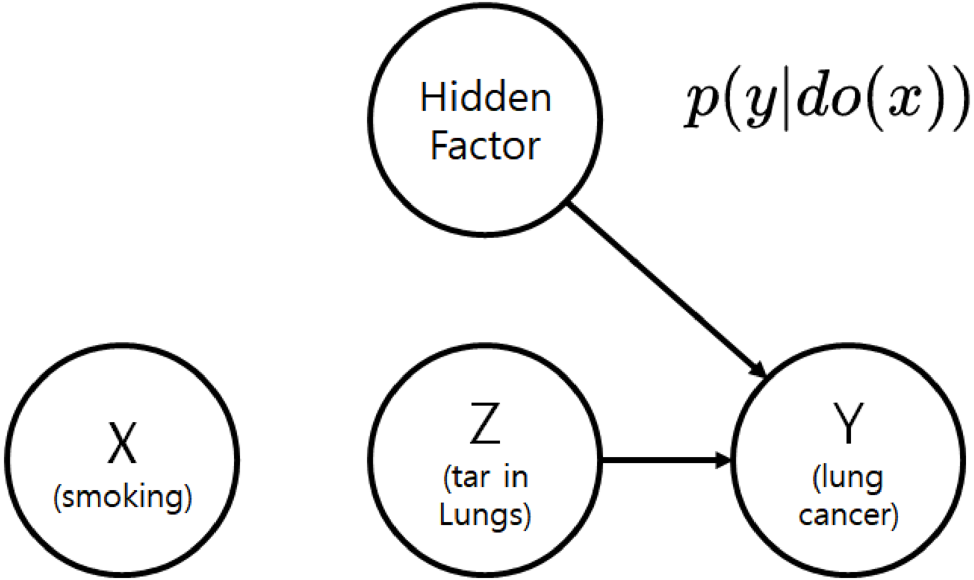
$$p(y|do(x)) = \sum_z p(y|z, do(x))p(z|do(x))$$
$$= \sum_z p(y|z, do(x))p(z|x)$$

$(\because (Z \perp\!\!\!\perp X)_{G_{\underline{X}}})$ (Use rule 2)



$G_{\overline{X},\underline{Z}}$

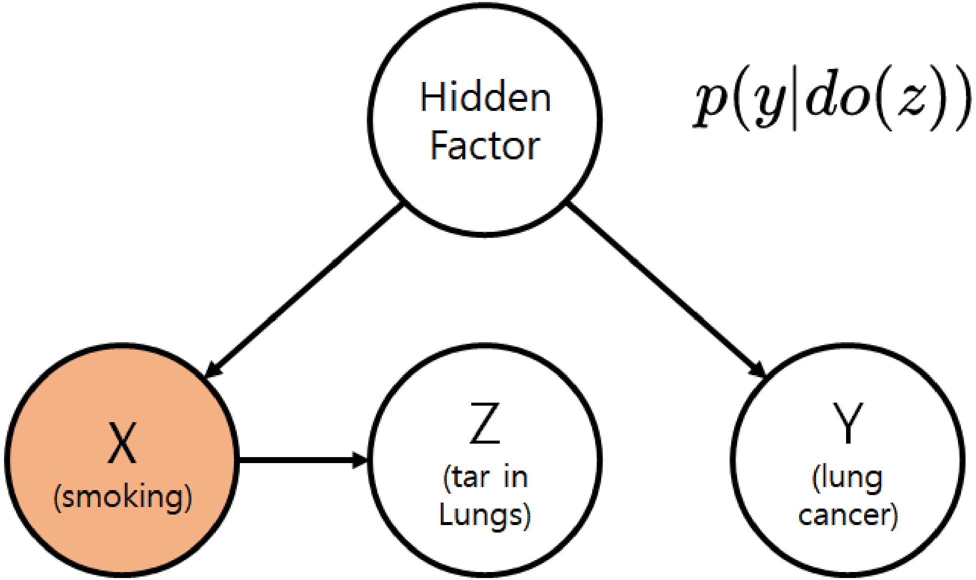
$$p(y|do(x)) = \sum_z p(y|z, do(x))p(z|do(x))$$
$$= \sum_z p(y|z, do(x))p(z|x)$$
$$= \sum_z p(y|do(z), do(x))p(z|x)$$
$$(\because (Y \perp\!\!\!\perp Z|X)_{G_{\overline{X},\underline{Z}}})$$ (Use rule 2)



$G_{\overline{Z},\overline{X}}$

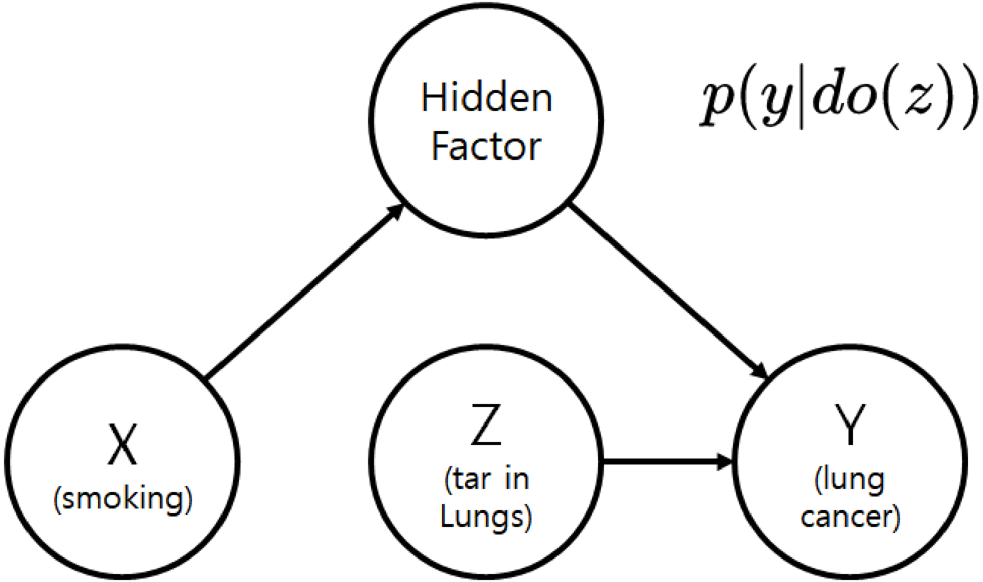
$$
\begin{aligned}
p(y|do(x)) &= \textstyle\sum_z p(y|z, do(x))p(z|do(x)) \\
&= \textstyle\sum_z p(y|z, do(x))p(z|x) \\
&= \textstyle\sum_z p(y|do(z), do(x))p(z|x) \\
&= \textstyle\sum_z p(y|do(z))p(z|x)
\end{aligned}
$$

$(\because (Y \perp\!\!\!\perp X|Z)_{G_{\overline{Z},\overline{X}}})$ (Use rule 3)



$G_{\underline{Z}}$

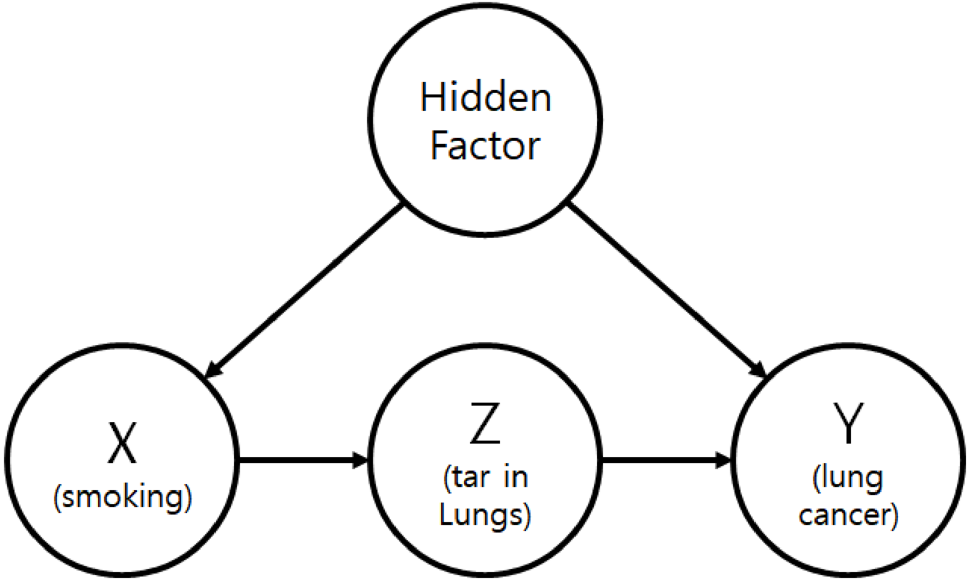
$$p(y|do(z)) = \sum_{x'} p(y|x', do(z))p(x'|do(z))$$
$$= \sum_{x'} p(y|x', z)p(x'|do(z))$$
$$(\because (Y \perp\!\!\!\perp Z|X)_{G_{\underline{Z}}})$$ (Use rule 2)



$G_{\overline{Z}}$

$$\begin{aligned} p(y|do(z)) &= \textstyle\sum_{x'} p(y|x', do(z))p(x'|do(z)) \\ &= \textstyle\sum_{x'} p(y|x', z)p(x'|do(z)) \\ &= \textstyle\sum_{x'} p(y|x', z)p(x') \end{aligned}$$

$(\because (Z \perp\!\!\!\perp X)_{G_{\overline{Z}}})$ (Use rule 3)



$$p(y|do(x)) = \sum_{z,x'} p(y|z,x')p(z|x)p(x')$$

Paradoxes - Objective

- To explain why people find the paradox surprising or unbelievable

- To identify the class of scenarios in which the paradox can/cannot occur

- When we have to make a choice between two plausible yet contradictory statements, to tell us which statement is correct

Causality and Paradoxes?

- reveal the way the brain works, the shortcuts it takes, and things it finds conflicting.

- shine a spotlight onto patterns of **intuitive causal** reasoning that clash with the **logic of probability and statistics**.

Monty Hall Dilemma - *Review*

Let's Make a Deal

Resolving Monty Hall's Dilemma

Switch or Stay?

Door 1	Door 2	Door 3	Outcome If You Switch	Outcome If You Stay
Auto	Goat	Goat	Lose	Win
Goat	Auto	Goat	Win	Lose
Goat	Goat	Auto	Win	Lose

$$P(car = 2 | goat = 3) = \frac{2}{3}$$

Resolving Monty Hall's Dilemma - Bayesian Reasoning

- prior: $P(car = 1) = P(car = 2) = P(car = 3) = \frac{1}{3}$
- Observe: The door 3 is opened and revealed the goat. (goat=3)
- posterior: $P(car = 1|goat = 3) = \frac{P(goat=3|car=1)P(car=1)}{P(goat=3|car=1)P(car=1)+P(goat=3|car=2)P(car=2)+P(goat=3|car=3)P(car=3)}$

$$= \frac{1/2}{1/2+1+0} = \frac{1}{3}$$

$$P(car = 2|goat = 3) = \frac{2}{3}$$

$$P(car = 3|goat = 3) = 0$$

Resolving Monty Hall's Dilemma - Causal Reasoning

- When we condition on a **collider**, we create a **spurious dependency**

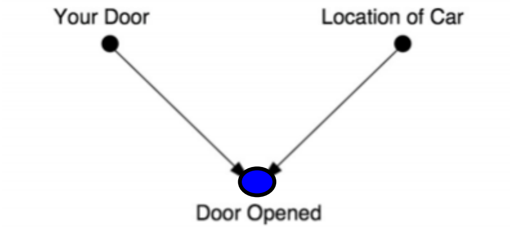


FIGURE 6.1. Causal diagram for *Let's Make a Deal*.

Let's Fake a Deal

- Monty Hall open the door ***randomly***, which is not the door you open.

Door You Chose	Door with Auto	Door Opened by Host	Outcome If You Switch	Outcome If You Stay
1	1	2 (goat)	Lose	Win
1	1	3 (goat)	Lose	Win
1	2	2 (auto)	Lose	Lose
1	2	3 (goat)	Win	Lose
1	3	2 (goat)	Win	Lose
1	3	3 (auto)	Lose	Lose

Let's Fake a Deal

- Monty Hall open the door ***randomly***, which is not the door you open.

Door You Chose	Door with Auto	Door Opened by Host	Outcome If You Switch	Outcome If You Stay
1	1	2 (goat)	Lose	Win
1	1	3 (goat)	Lose	Win
1	2	2 (auto)	Lose	Lose
1	2	3 (goat)	Win	Lose
1	3	2 (goat)	Win	Lose
1	3	3 (auto)	Lose	Lose

$$P(car = 2 | goat = 3) = \frac{1}{2}$$

Let's Fake a Deal

Your choice of a door and the producer's choice of where to put the car are independent

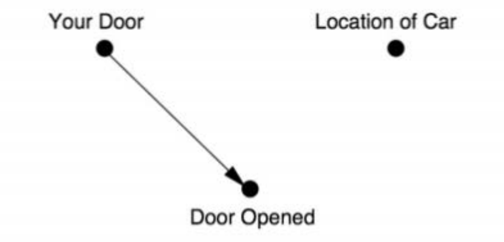
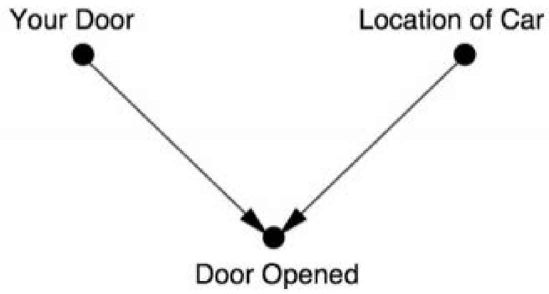


FIGURE 6.2. Causal diagram for *Let's Fake a Deal*.

What we learned

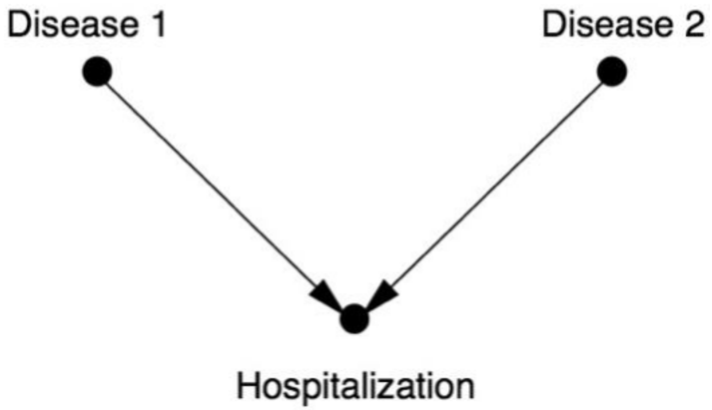
- Data generating process is also important

- Why we see it as a paradox in the first place.
  - Brain - Not to do *probability problems*, but to do *causal problems*
  - **Causeless correlation**



Collider Bias: Berkson's Bias

- Disease 1 and 2 have no relation to each other
- Neither Disease 1 nor 2 is ordinarily severe enough to cause hospitalization, but the combination is.
  - we would expect Disease 1 to be highly correlated with Disease 2 in the hospitalized population
  - Conditioning on a collider creates a **spurious association**



Berkson's Paradox

- About 7.5 % of people in general population have a bone disease
- “admission rate bias” or “Berkson bias”

	General Population			Hospitalized in Last Six Months		
Respiratory disease? ↓	*Bone disease?* ↓			*Bone disease?* ↓		
	Yes	No	% Yes	Yes	No	% Yes
Yes	17	207	7.6	5	15	25.0
No (control)	184	2,376	7.2	18	219	7.6

Lesson

- Collider bias can occur by the process by which observations are selected

- We should be careful to this bias

Simpson's Paradox

- BBG?
  - Bad for men
  - Bad for women
  - Good for people
- Take the Drug or not?

	Control Group (No Drug)		Treatment Group (Took Drug)	
	Heart attack	*No heart attack*	*Heart attack*	*No heart attack*
Female	1 5%	19	3	37
Male	12 < 30%	28	7.5% 8	12
Total	13 < 22%	47	40 % 11	49

> 18 %

Why does it happen? - Simpson's reversal

We have used an overly simple word "better" to describe a complex averaging process over uneven seasons

The denominators are not distributed evenly year to year

	Hits/At Bats			
	1995	*1996*	*1997*	*All Three Years*
David Justice	104/411 = .253	45/140 = .321	163/495 = .329	312/1,046 = .298
Derek Jeter	12/48 = .250	183/582 = .314	190/654 = .291	385/1,284 = .300

Return to BBG: Should I take it?

C: taking drug, E: heart attack occurred, F: female

We know that

$$P(E|do(C), F) < P(E|do(\neg C), F)$$

$$P(E|do(C), \neg F) < P(E|do(\neg C), \neg F)$$

$P(E|do(C)) > P(E|do(\neg C))$?

Return to BBG: Should I take it?

"Sure-Thing Principle"

"An action C that increases the probability of an event E in each ***subpopulation*** *must also increase the probability of E in the* ***population*** *as a whole, provided that the action does not change the distribution of the subpopulations."*

Our causal intuitive: the drug does not change the sex

$$P(F|do(C)) = P(F|do(\neg C)) = P(F)$$

Sure-Thing Principle

$$
\begin{aligned}
P(E|do(C)) &= P(E|do(C),F)P(F|do(C)) + P(E|do(C),\neg F), P(\neg F|do(C)) \\
&= P(E|do(C),F)P(F) + P(E|do(C),\neg F)P(\neg F) \\
&< P(E|do(\neg C),F)P(F) + P(E|do(\neg C),\neg F)P(\neg F) \\
&= P(E|do(\neg C))
\end{aligned}
$$

No BBG, but BBB!!

Lesson

- “Causal relationships are governed by the laws of probability calculus” -> **No!** Causality is governed by its own logic and this logic requires a major extension of probability calculus: *do-calculus*

- Our decision is driven by **causal** and not by statistical consideration

Q&A