

CS423: Probabilistic Programming Introduction

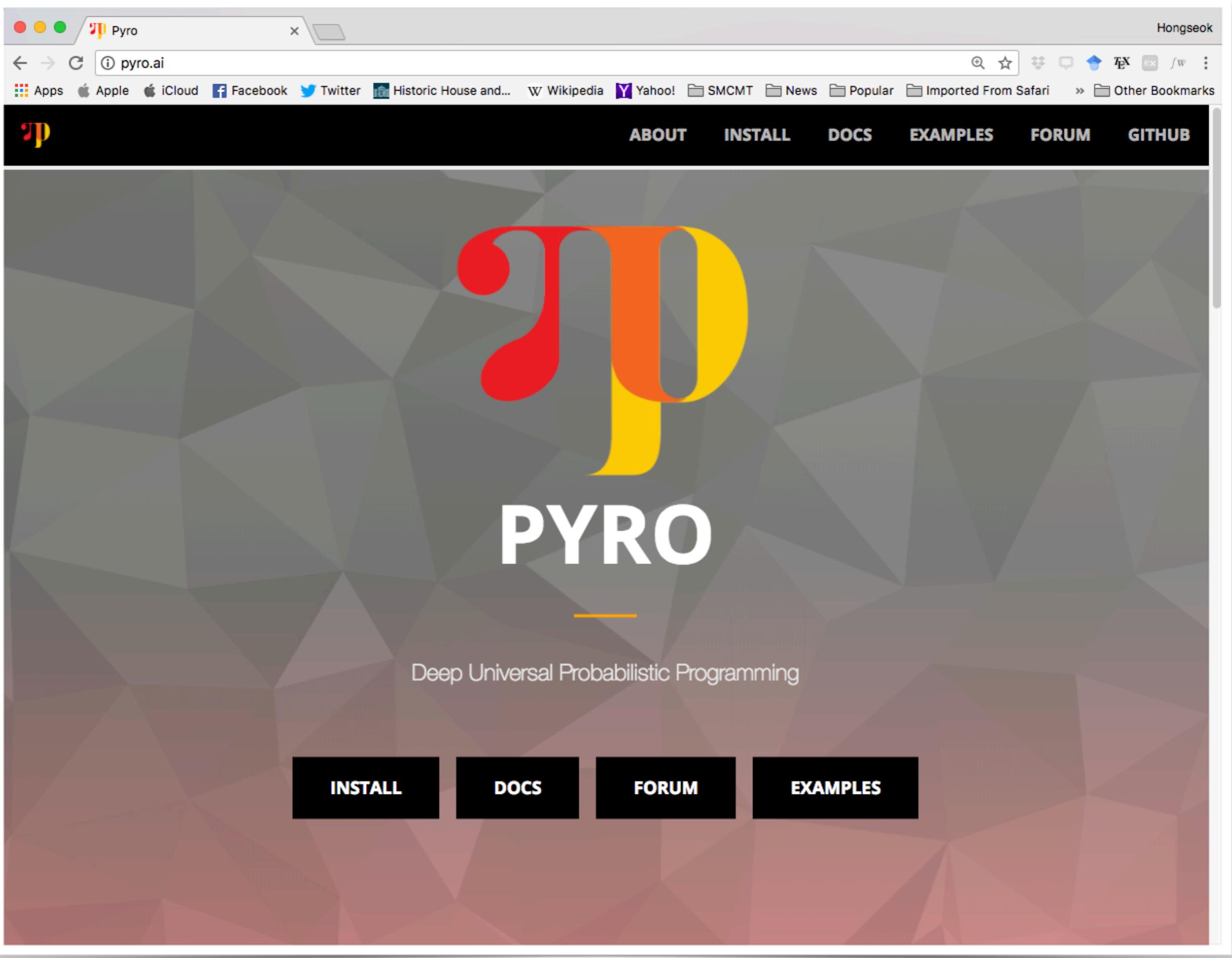
Hongseok Yang
KAIST

Can we capture the origins of human common sense ... in engineering terms? ... New tools from **probabilistic programming**, game engines and program learning ... are beginning to let us answer these questions.

Josh Tenenbaum
Invited talk at ICML'18 & IJCAI'18

This Review ... discusses some of the state-of-the-art advances in the field, namely, **probabilistic programming**, Bayesian optimization, data compression and automatic model discovery.

Zoubin Ghahramani
2015 Nature Review



What is probabilistic
programming?

(Bayesian) probabilistic modelling of data

- I. Develop a new probabilistic (generative) model.
2. Design an inference algorithm for the model.
3. Using the algo., fit the model to the data.

(Bayesian) probabilistic modelling of data in a prob. prog. language

- I. Develop a new probabilistic (generative) model.
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(Bayesian) probabilistic modelling of data in a prob. prog. language

as a program

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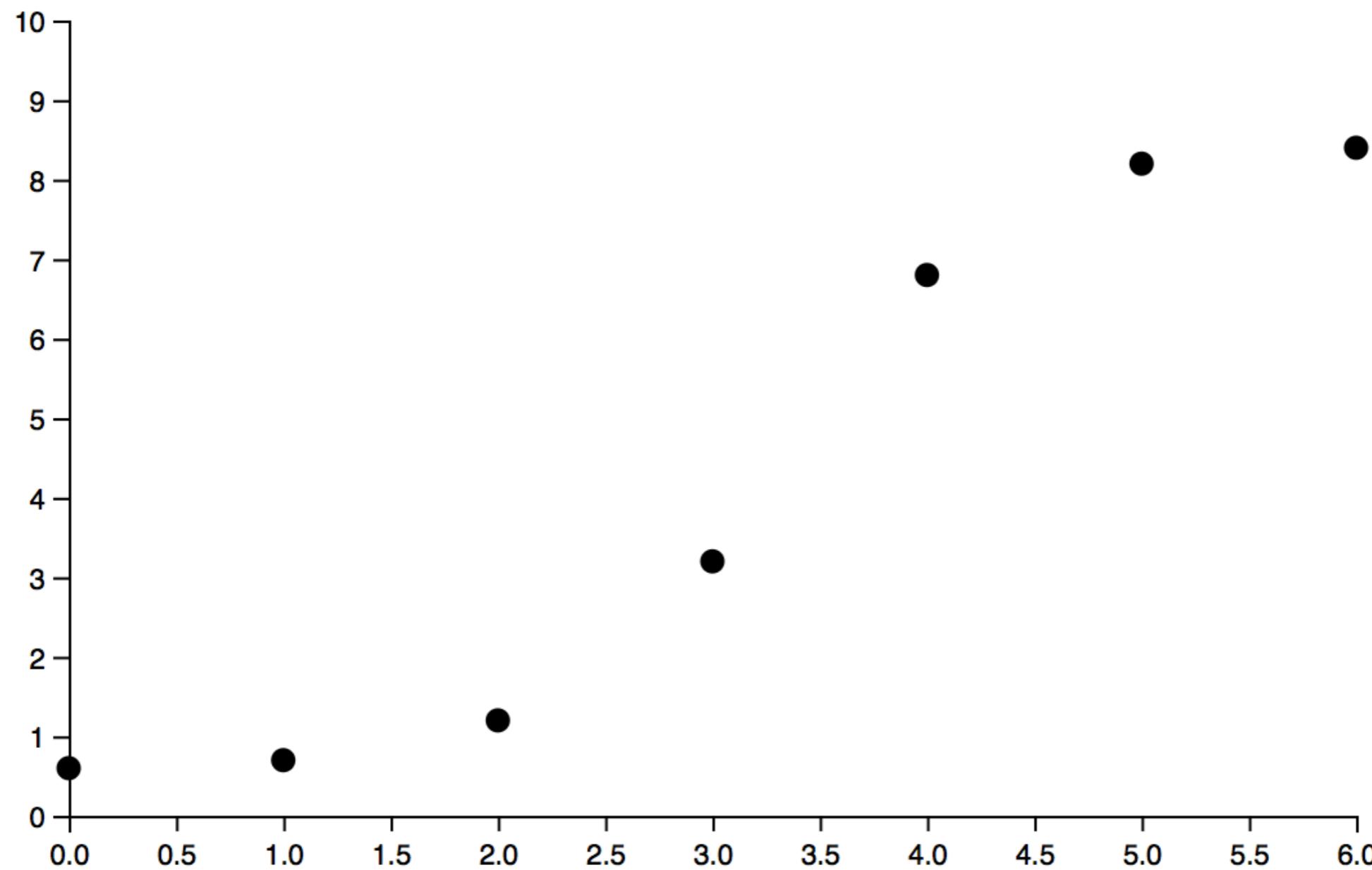
(Bayesian) probabilistic modelling of data in a prob. prog. language

as a program

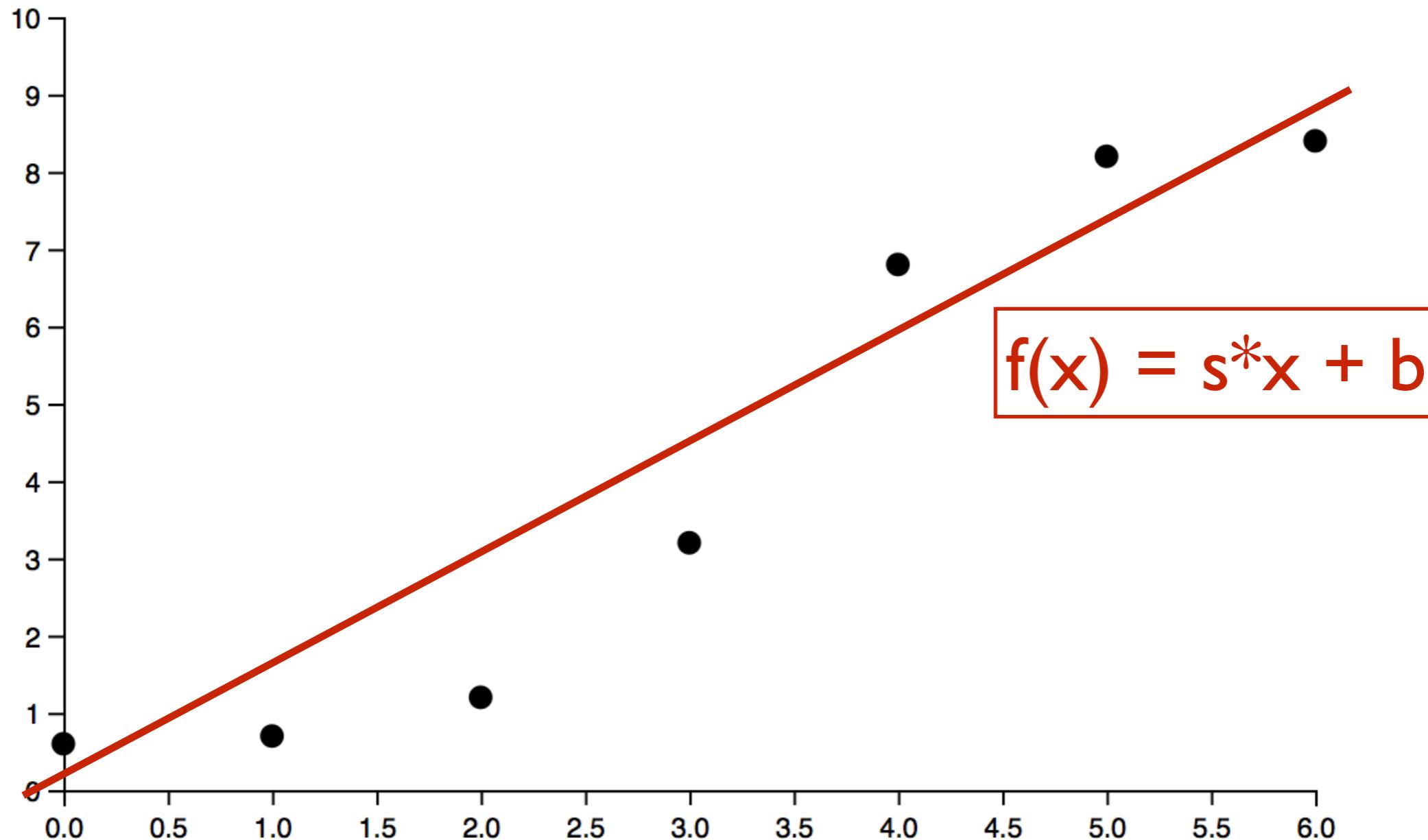
- I. Develop a new probabilistic (generative) model.
- ~~2. Design an inference algorithm for the model.~~
3. Using ~~the algo.~~, fit the model to the data.

a generic inference algo.
of the language

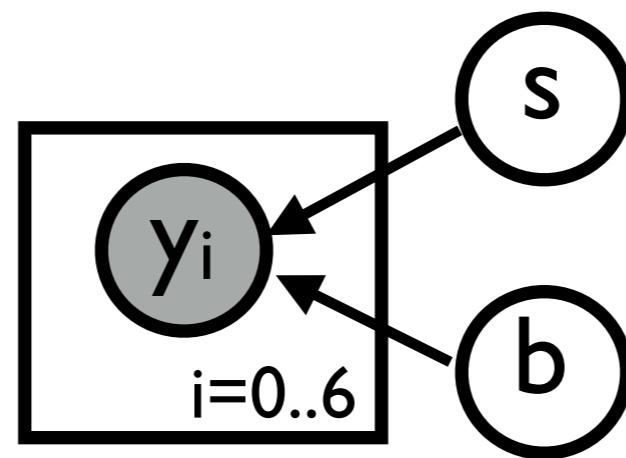
Line fitting



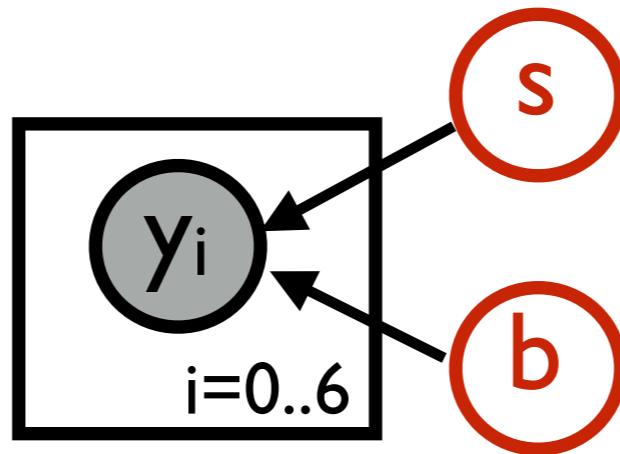
Line fitting



Bayesian generative model

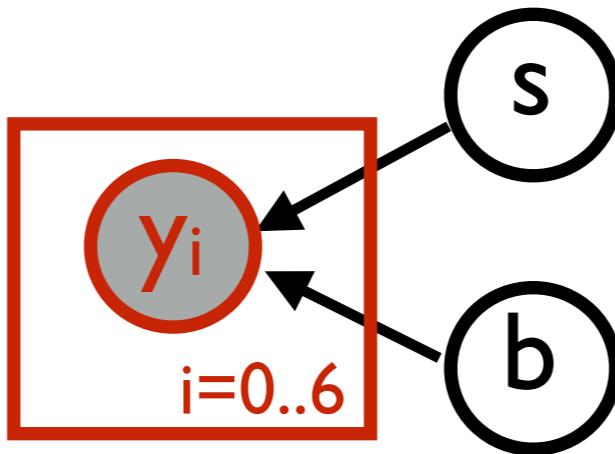


Bayesian generative model



$s \sim \text{normal}(0, 2)$
 $b \sim \text{normal}(0, 6)$

Bayesian generative model



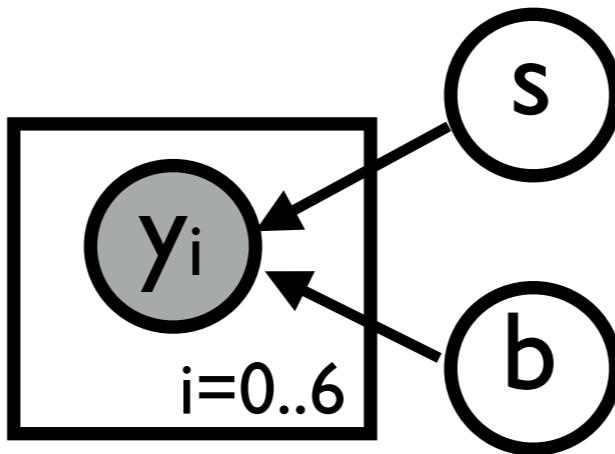
$s \sim \text{normal}(0, 2)$

$b \sim \text{normal}(0, 6)$

$f(x) = s*x + b$

$y_i \sim \text{normal}(f(i), 0.5)$
where $i = 0 .. 6$

Bayesian generative model



$s \sim \text{normal}(0, 2)$
 $b \sim \text{normal}(0, 6)$
 $f(x) = s*x + b$
 $y_i \sim \text{normal}(f(i), 0.5)$
where $i = 0 .. 6$

Q: posterior of (s, b) given $y_0=0.6, \dots, y_6=8.4$?

Posterior of s and b given y_i's

$$p(s, b | y_0, \dots, y_6) = \frac{p(y_0, \dots, y_6 | s, b) \times p(s, b)}{p(y_0, \dots, y_6)}$$

Posterior of s and b given y_i's

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Anglican program

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]
```

Anglican program

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]

  (observe (normal (f 0) 0.5) 0.6)
  (observe (normal (f 1) 0.5) 0.7)
  (observe (normal (f 2) 0.5) 1.2)
  (observe (normal (f 3) 0.5) 3.2)
  (observe (normal (f 4) 0.5) 6.8)
  (observe (normal (f 5) 0.5) 8.2)
  (observe (normal (f 6) 0.5) 8.4))
```

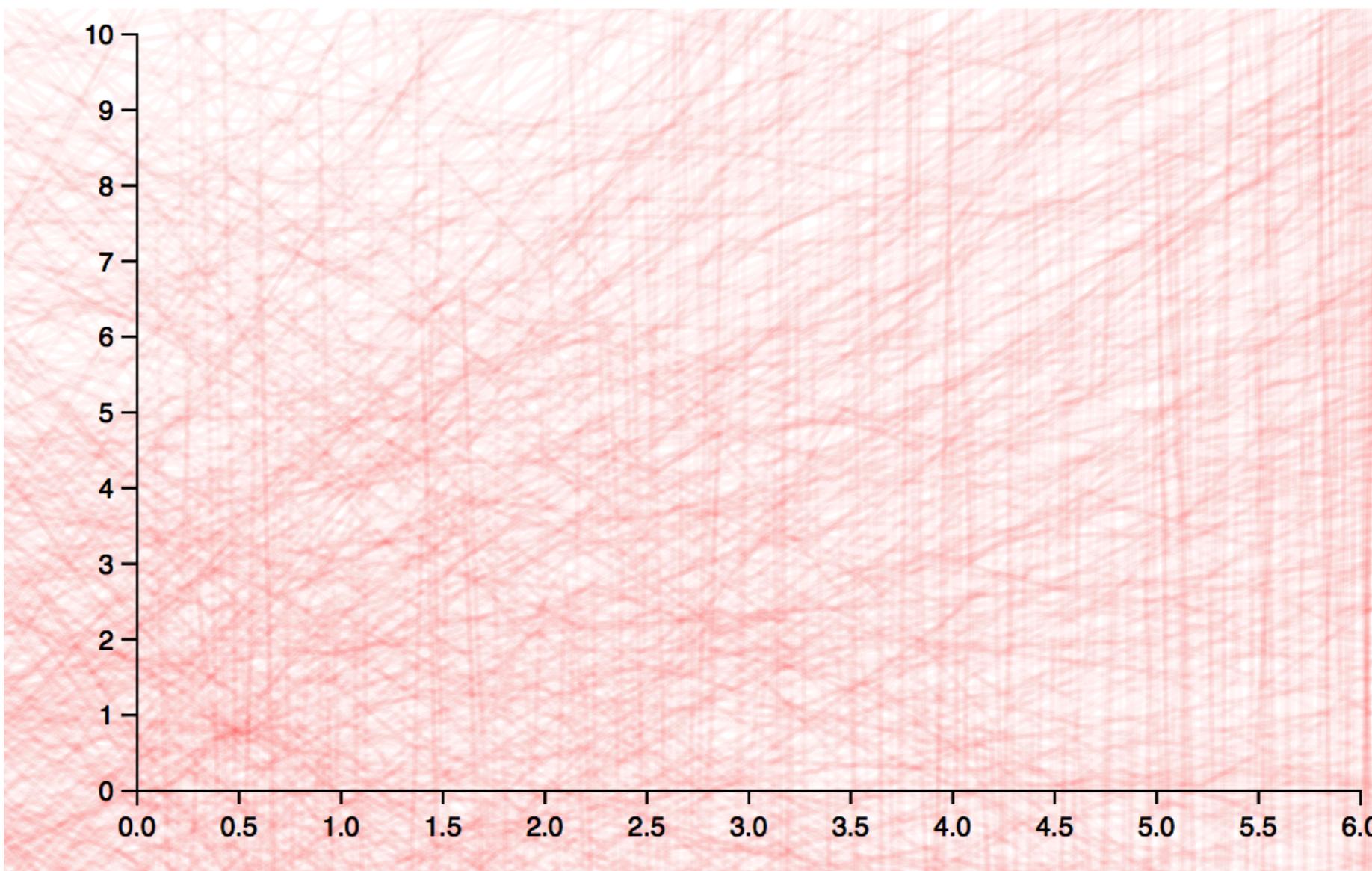
Anglican program

```
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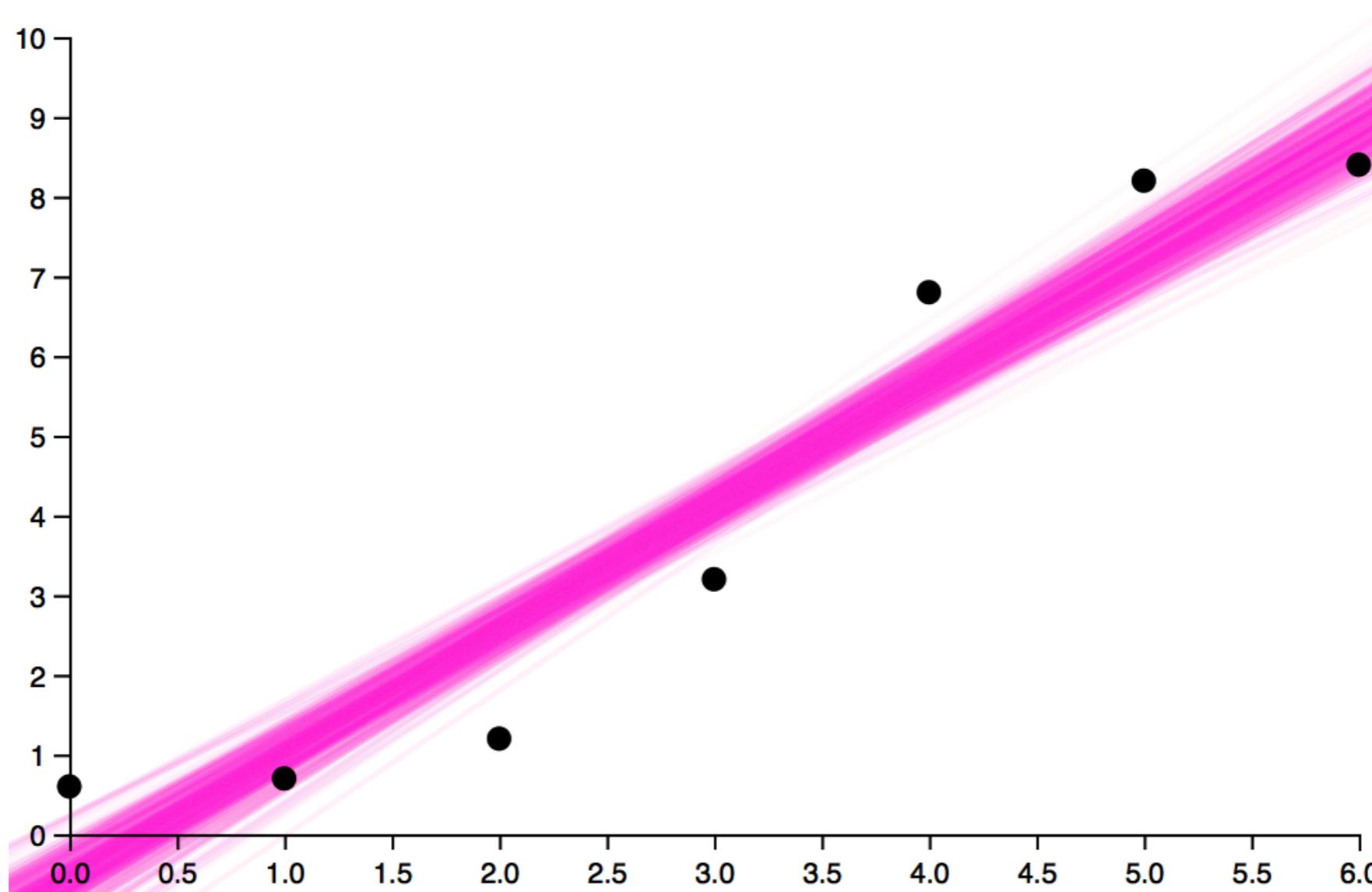
  (observe (normal (f 0) 0.5) 0.6)
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  [s b])
```

Samples from prior



Samples from posterior

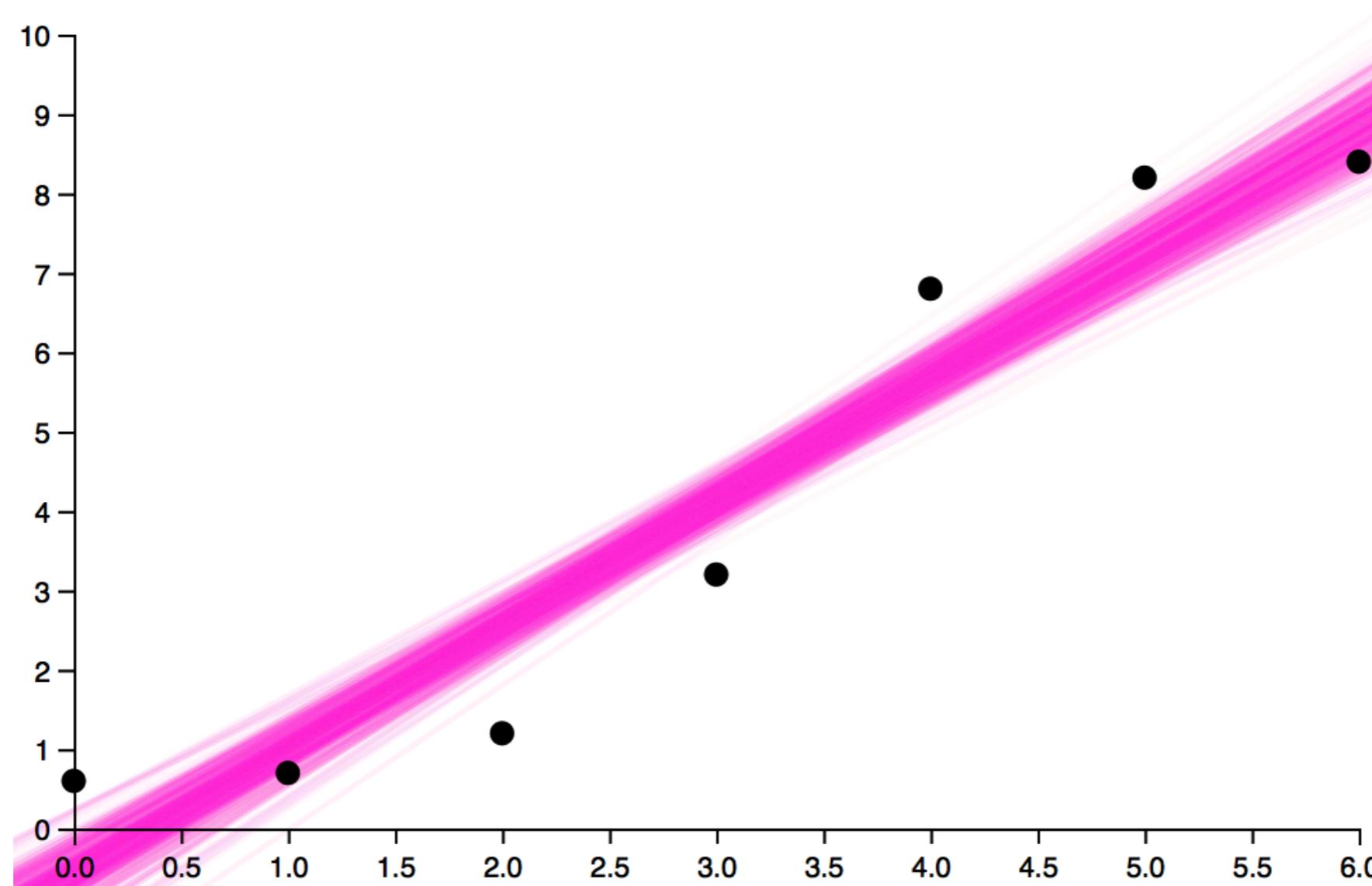


Why should one care
about prob. programming?

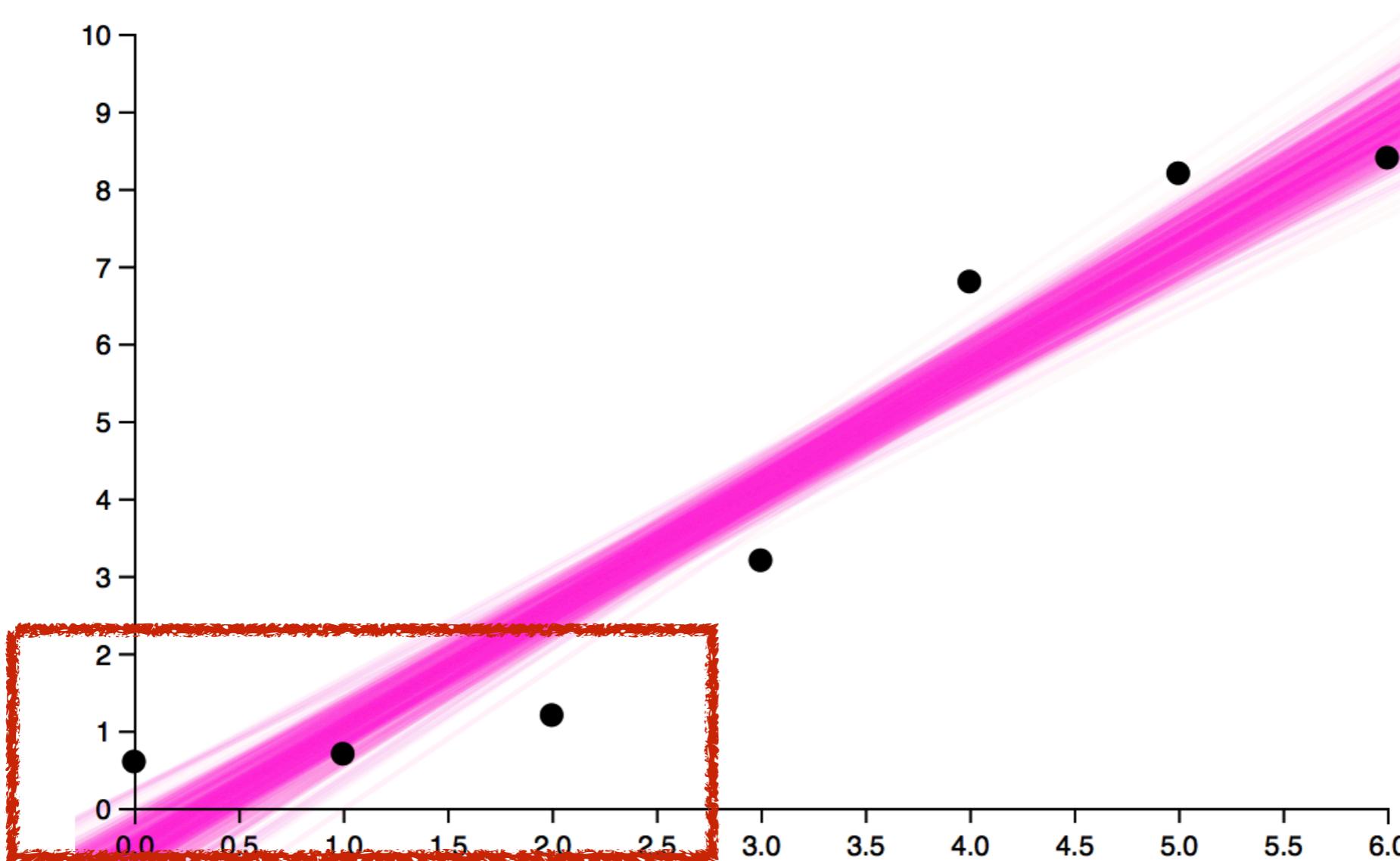
“Because probabilistic programming is a good way to build an AI.” (My ML colleague)

Prob. programming languages enable one to build and explore highly complex models.

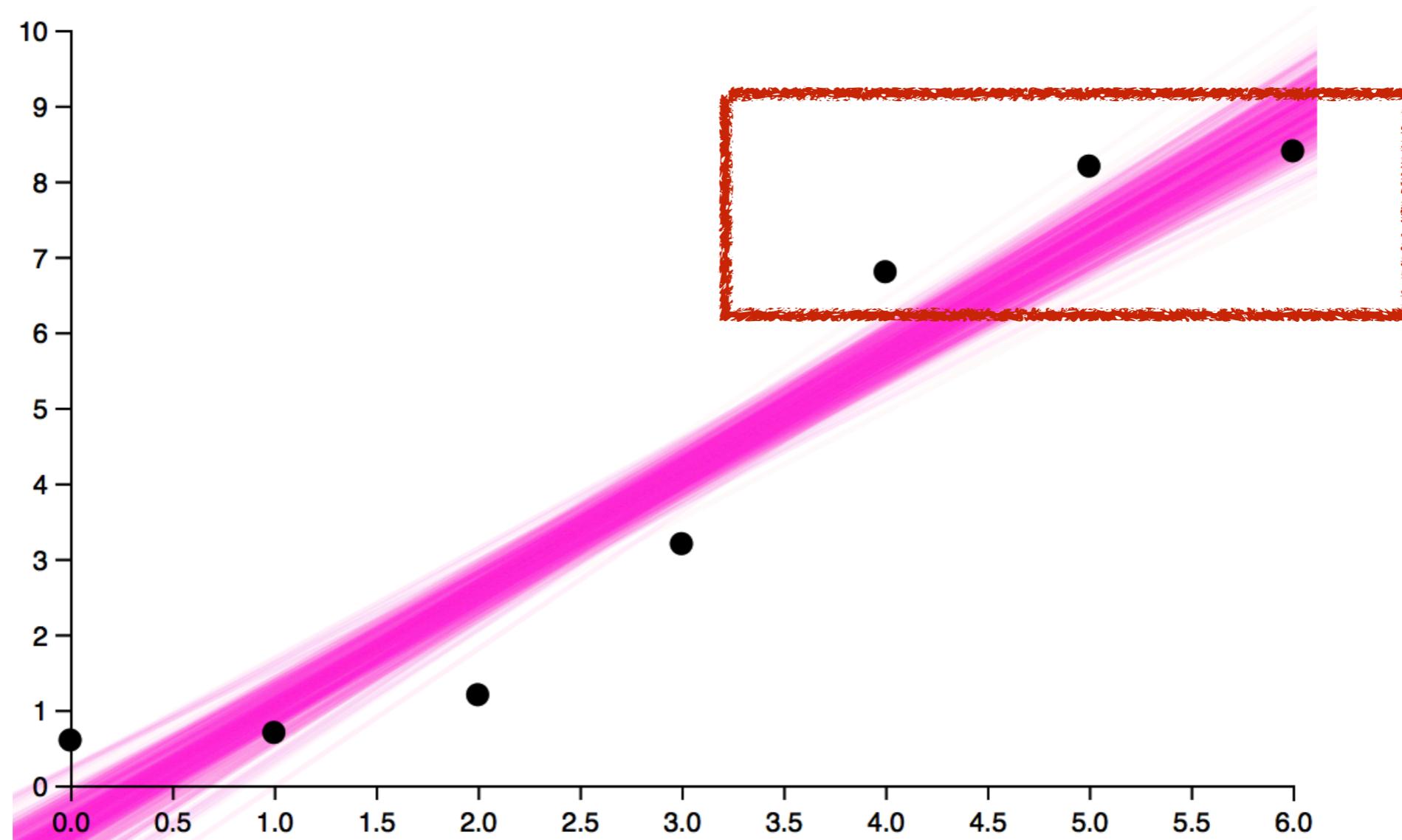
Underfit?



Underfit?



Underfit?



```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]

  (observe (normal (f 0) 0.5) 0.6)
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  [s b])
```

```
(let [s (sample (normal 0 2))
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```
(observe (normal (f 0) 0.5) 0.6)
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```

[s b])

Functions as first-class citizen.

```
(let [s (sample (normal 0 2))
      b (sample (normal 0 6))
      f (fn [x] (+ (* s x) b))]
```

```
(observe (normal (f 0) 0.5) 0.6)
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(observe (normal (f 6) 0.5) 8.4)
```

~~[s b]~~
f)

Functions as first-class citizen.

```
(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))]
    (fn [x] (+ (* s x) b))))]
  f (F))
(observe (normal (f 0) 0.5) 0.6)
(observe (normal (f 1) 0.5) 0.7)
(observe (normal (f 2) 0.5) 1.2)
(observe (normal (f 3) 0.5) 3.2)
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(observe (normal (f 6) 0.5) 8.4)
```

~~Es ist~~
f)

Functions as first-class citizen.

```

(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))]

    (fn [x] (+ (* s x) b)))))

f (add-change-points F 0 6)
(observe (normal (f 0) 0.5) 0.6)
(observe (normal (f 1) 0.5) 0.7)
(observe (normal (f 2) 0.5) 1.2)
(observe (normal (f 3) 0.5) 3.2)
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```

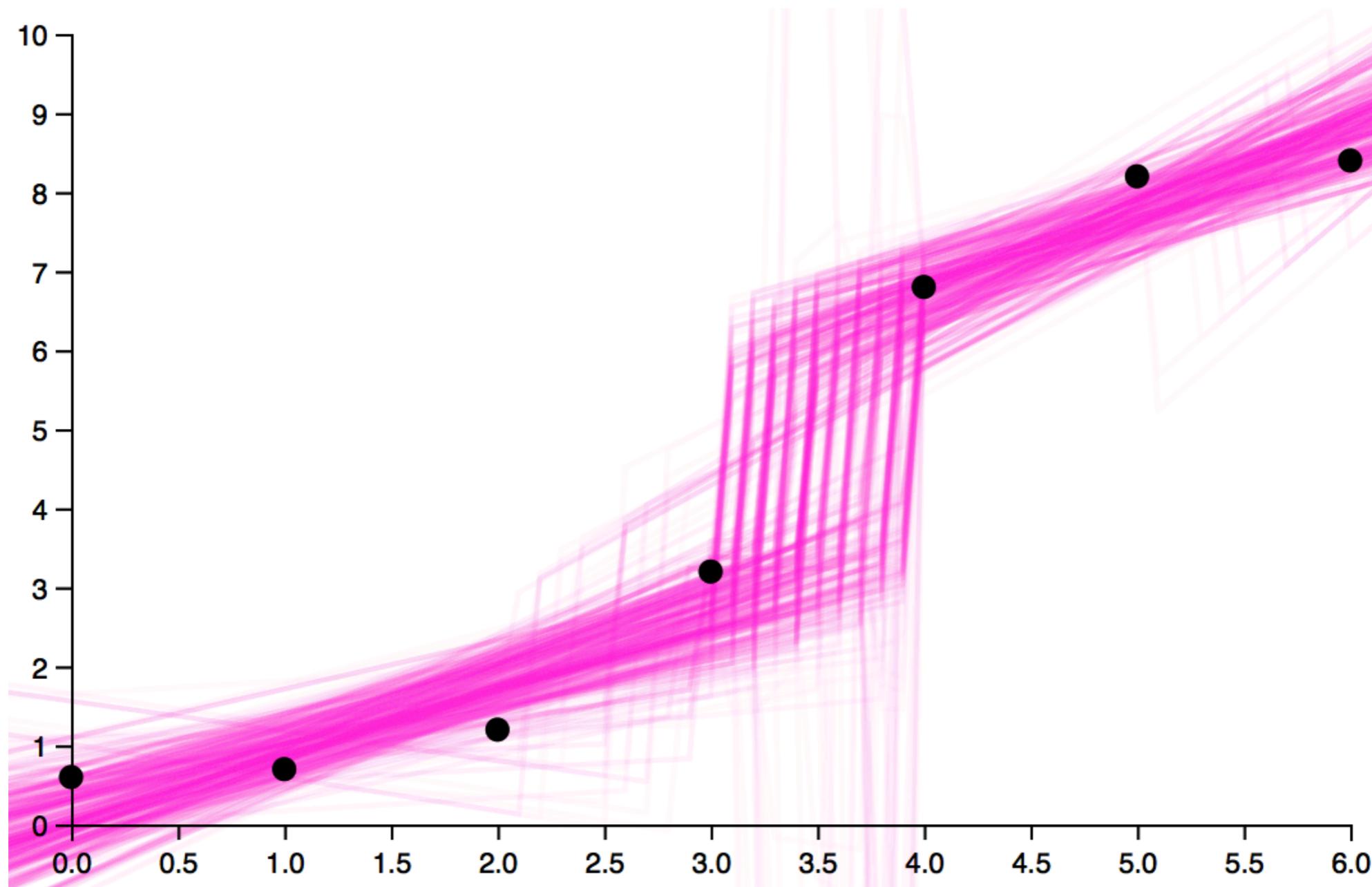
~~Exhibit~~
f)

Functions as first-class citizen.

```
(defm add-change-points [F 1 u]
  (if (sample (flip 0.5)) (F)
    (if (sample (flip 0.5))
      (let [cp1 (sample
                  (uniform-continuous 1 u))
            f1 (F)
            f2 (F)]
        (fn [x] (if (< x cp1) (f1 x) (f2 x))))
      (let [cp1 (sample
                  (uniform-continuous 1 u))
            cp2 (sample
                  (uniform-continuous cp1 u))
            f1 (F)
            f2 (F)
            f3 (F)]
        (fn [x] (if (< x cp1) (f1 x)
                    (if (< x cp2) (f2 x)
                      (f3 x)))))))
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(defm add-change-points [F 1 u]
  (if (sample (flip 0.5)) (F)
    (if (sample (flip 0.5))
        (let [cp1 (sample
                   (uniform-continuous 1 u))
              f1 (F)
              f2 (F)]
          (fn [x] (if (< x cp1) (f1 x) (f2 x)))
        (let [cp1 (sample
                   (uniform-continuous 1 u))
              cp2 (sample
                   (uniform-continuous cp1 u))
              f1 (F)
              f2 (F)
              f3 (F)]
          (fn [x] (if (< x cp1) (f1 x)
                      (if (< x cp2) (f2 x)
                        (f3 x))))))))
```

Samples from posterior



```

(let [F (fn []
  (let [s (sample (normal 0 2))
        b (sample (normal 0 6))])
    (fn [x] (+ (* s x) b))))
  f (add-change-points F 0 6)]
  (observe (normal (f 0) .5) .6)
  (observe (normal (f 1) .5) .7)
  (observe (normal (f 2) .5) 1.2)
  (observe (normal (f 3) .5) 3.2)
  (observe (normal (f 4) .5) 6.8)
  (observe (normal (f 5) .5) 8.2)
  (observe (normal (f 6) .5) 8.4))

```

~~Exercise~~
f)

[Q] Change the model so that it finds constant functions with change points.

```

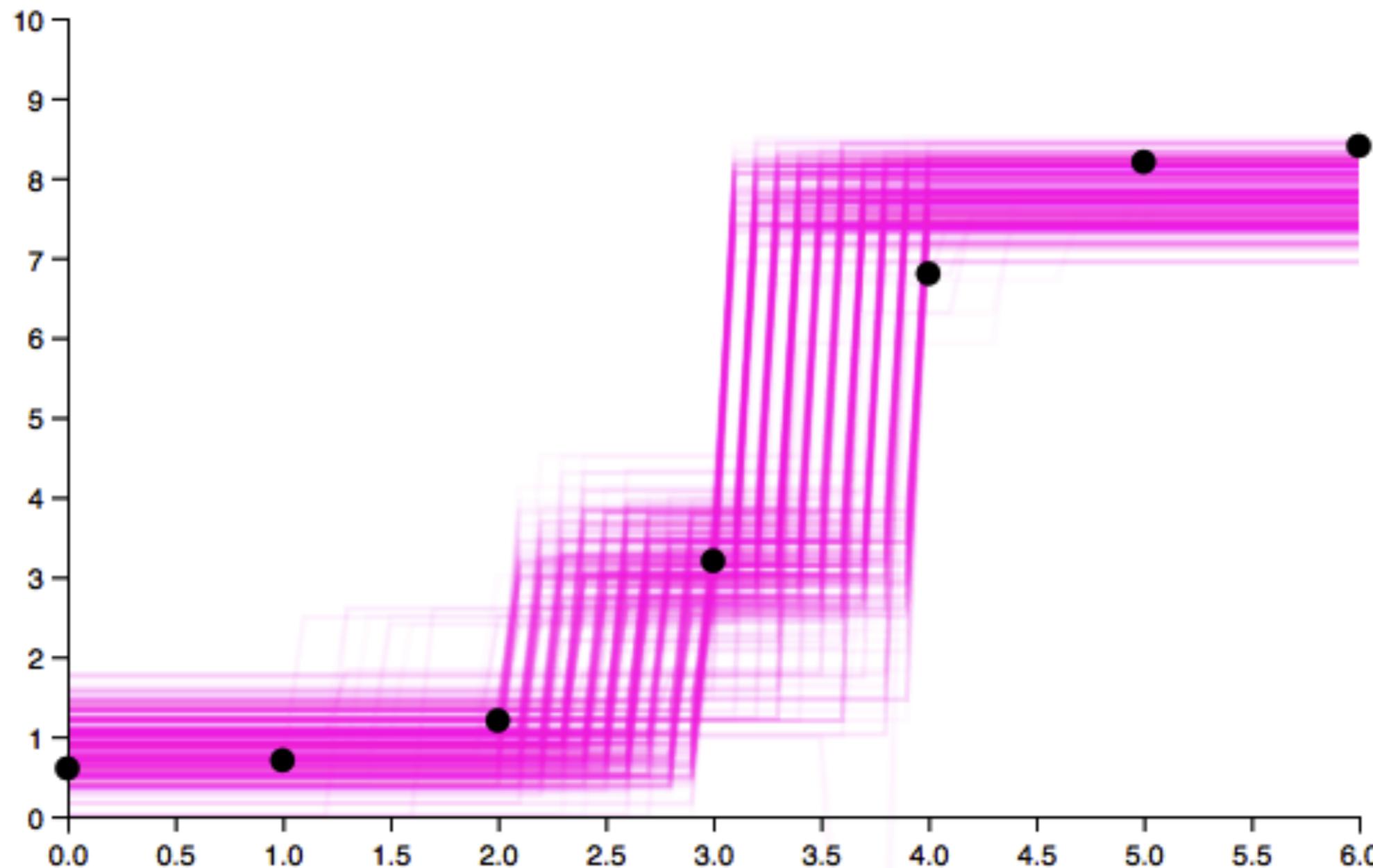
(let [F (fn []
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```

~~Exercise~~
f)

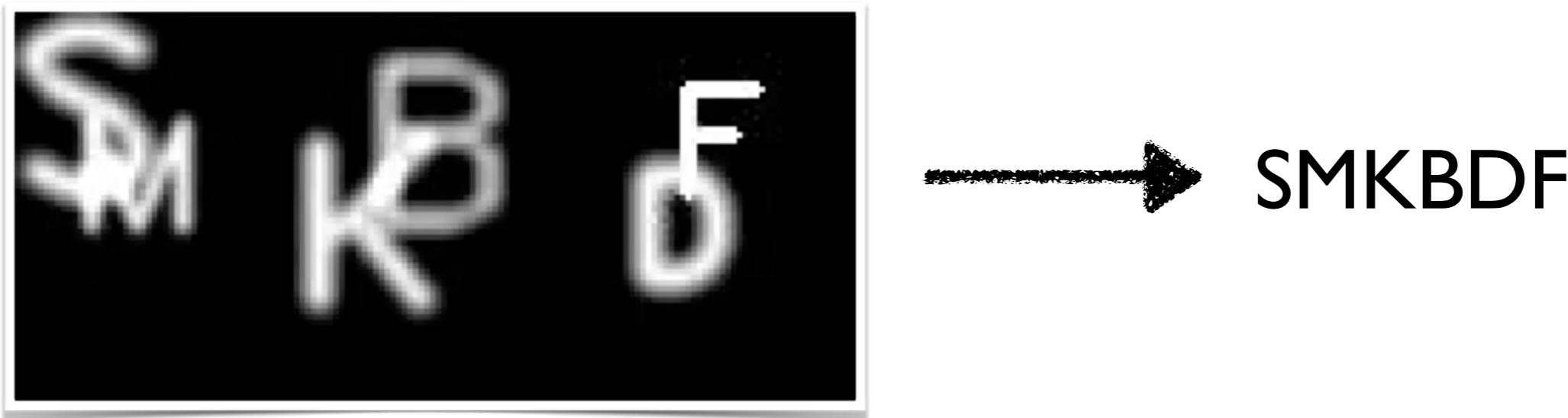
[Q] Change the model so that it finds constant functions with change points.

Samples from posterior



Three success stories:

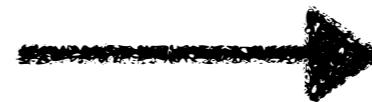
I) captcha breaking



Le, Baydin, Wood [2016]

Three success stories:

I) captcha breaking



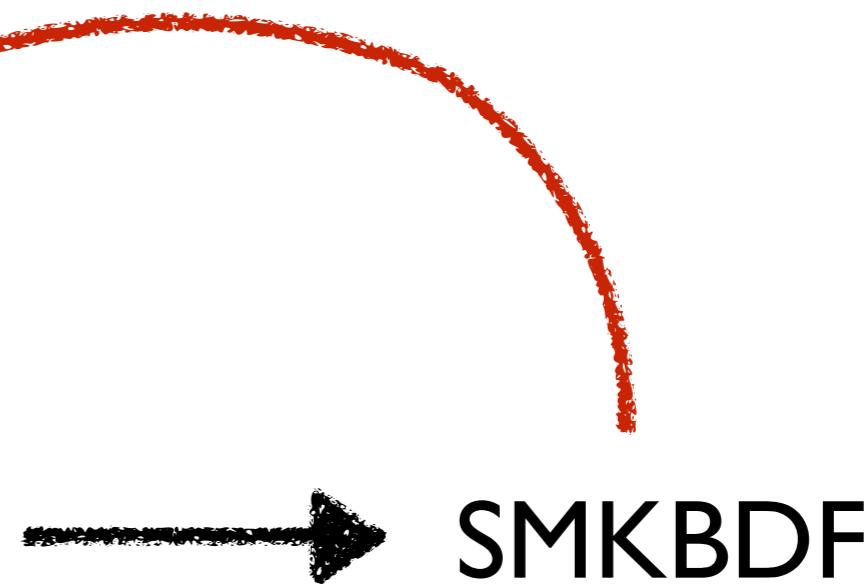
SMKBDF

I. Sample a string.

Le, Baydin, Wood [2016]

Three success stories:

I) captcha breaking

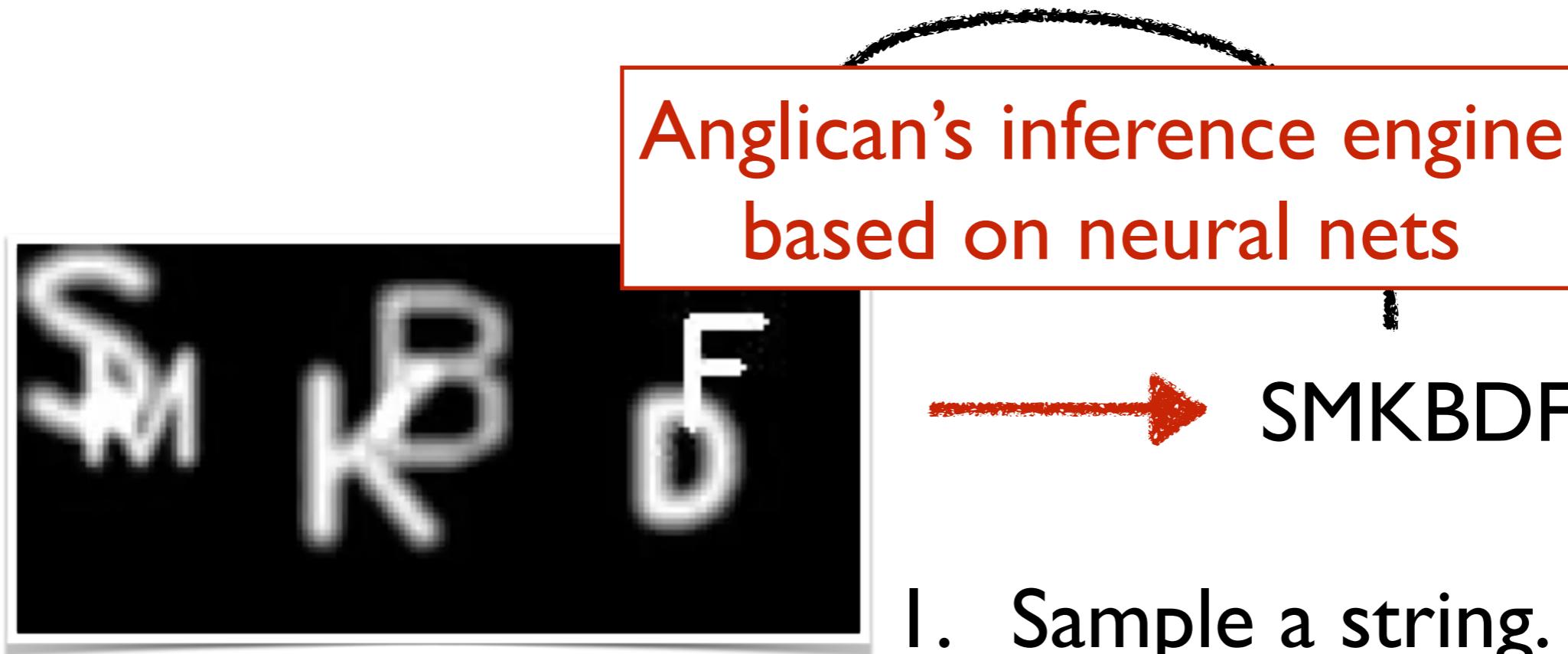


1. Sample a string.
2. Generate an image using complex JVM code.

Le, Baydin, Wood [2016]

Three success stories:

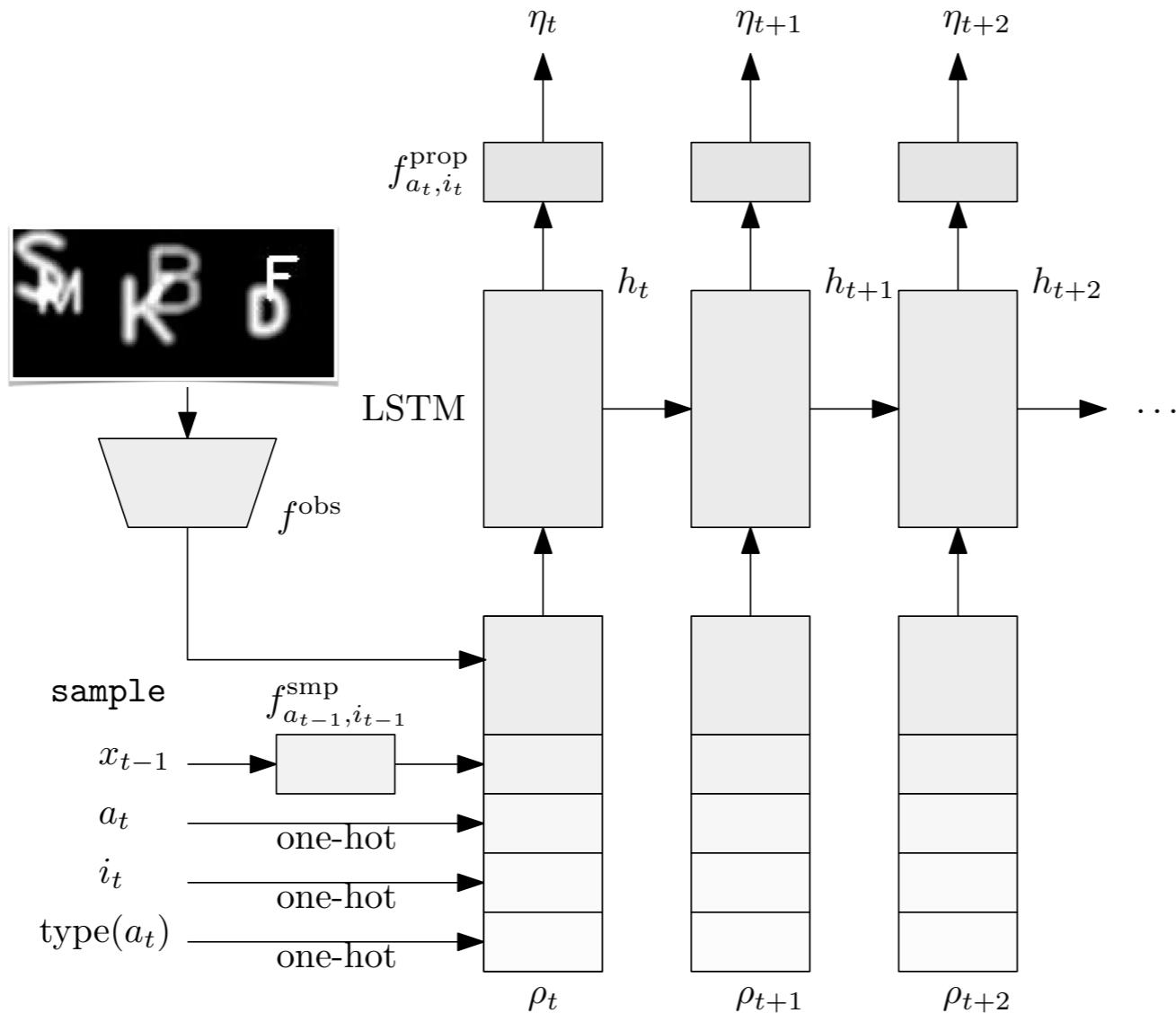
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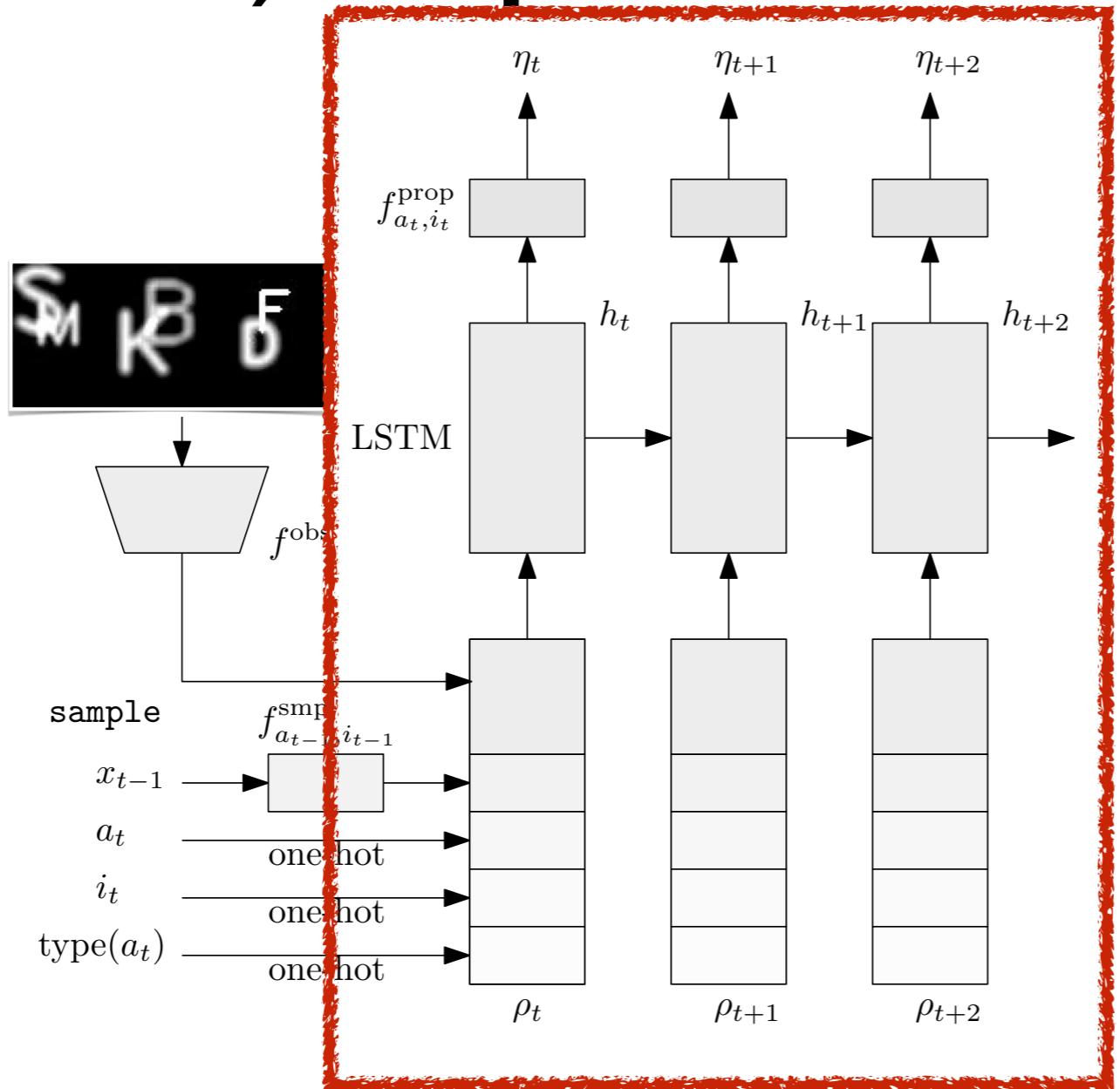
Three success stories: I) captcha breaking



Le, Baydin, Wood [2016]

Three success stories:

I) captcha breaking

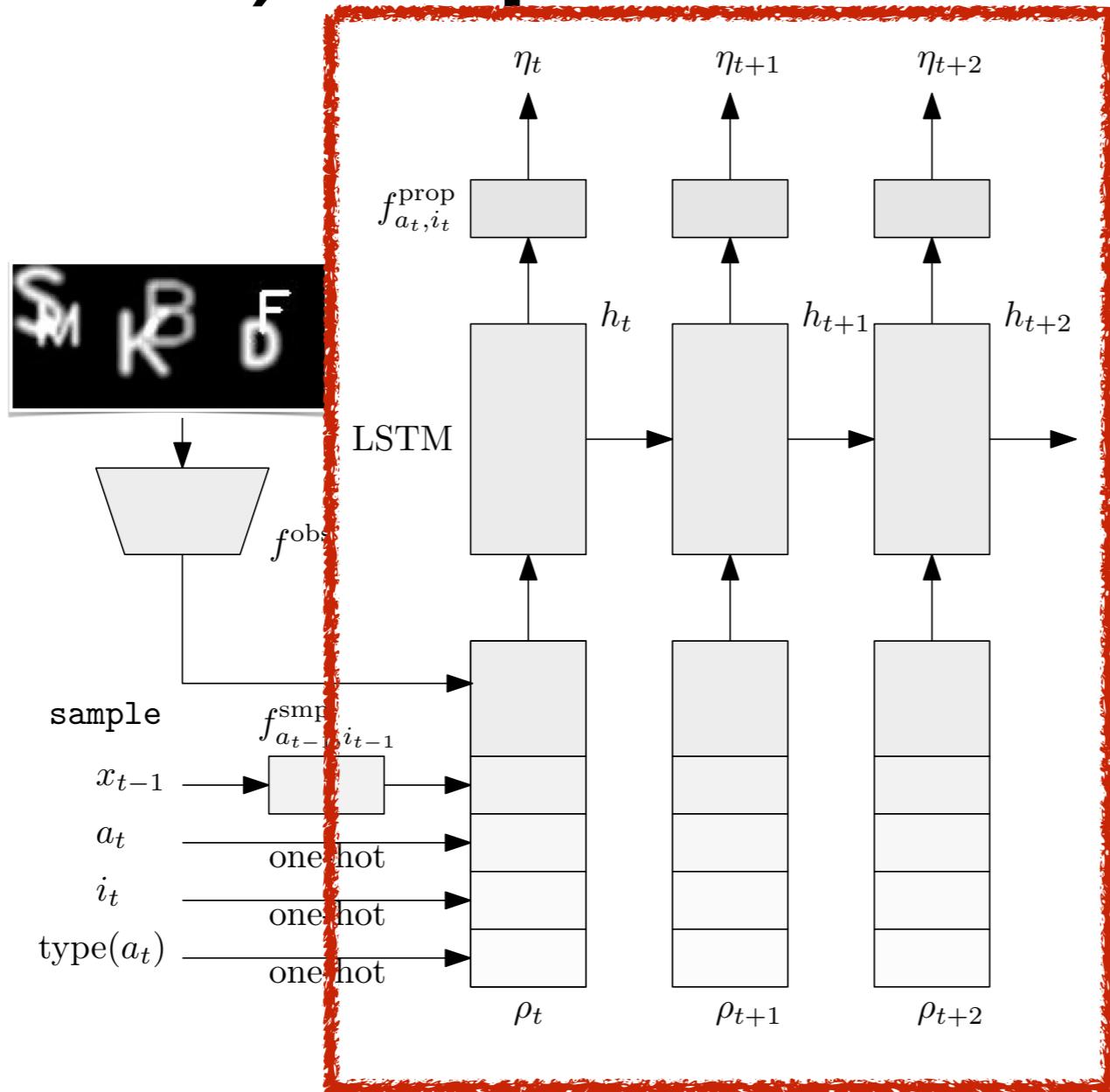


Neural net as a part
of inference engine.

Le, Baydin, Wood [2016]

Three success stories:

I) captcha breaking

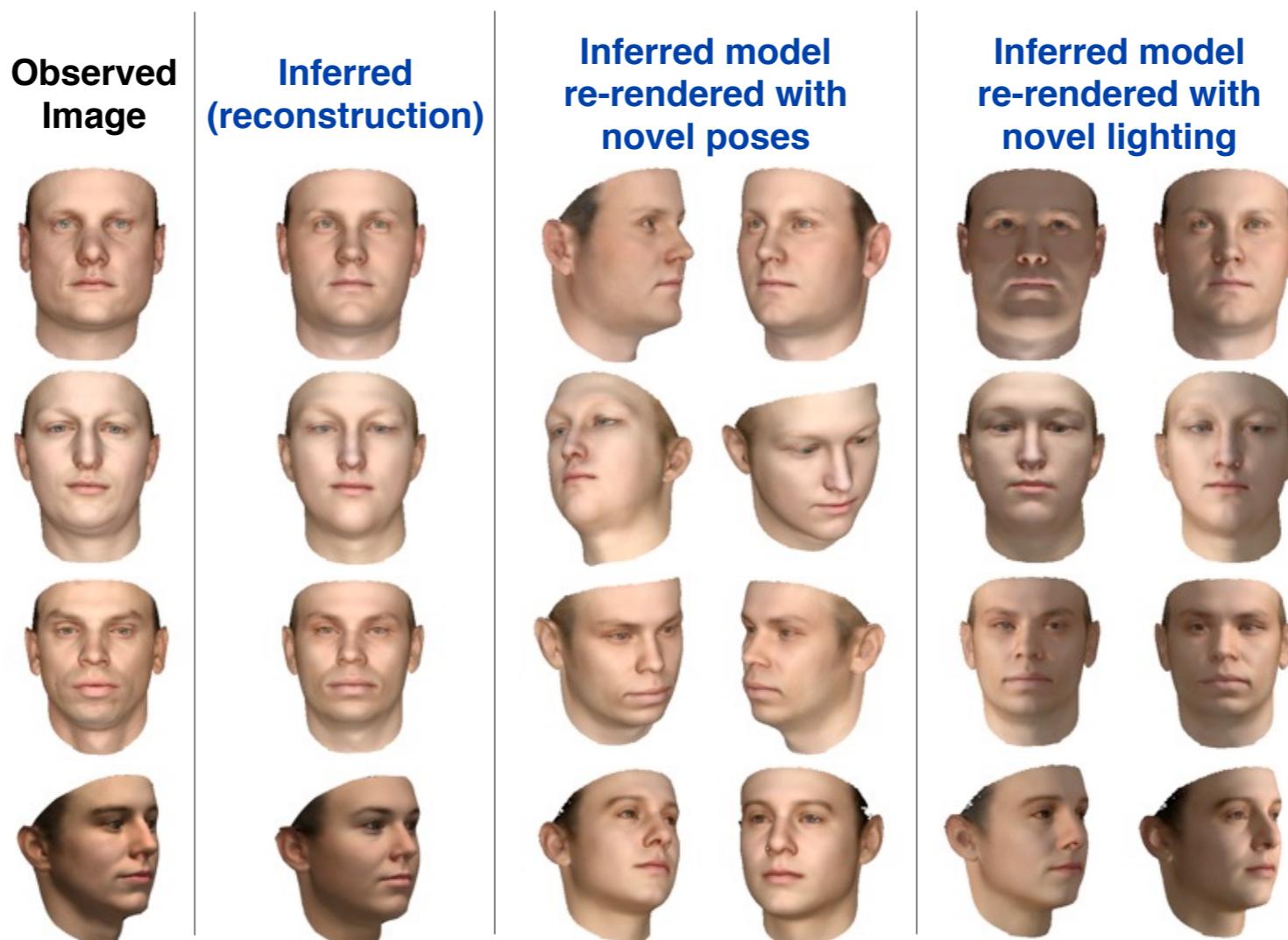


Neural net as a part
of inference engine.

Approximates the
inverse of the
Captcha program.

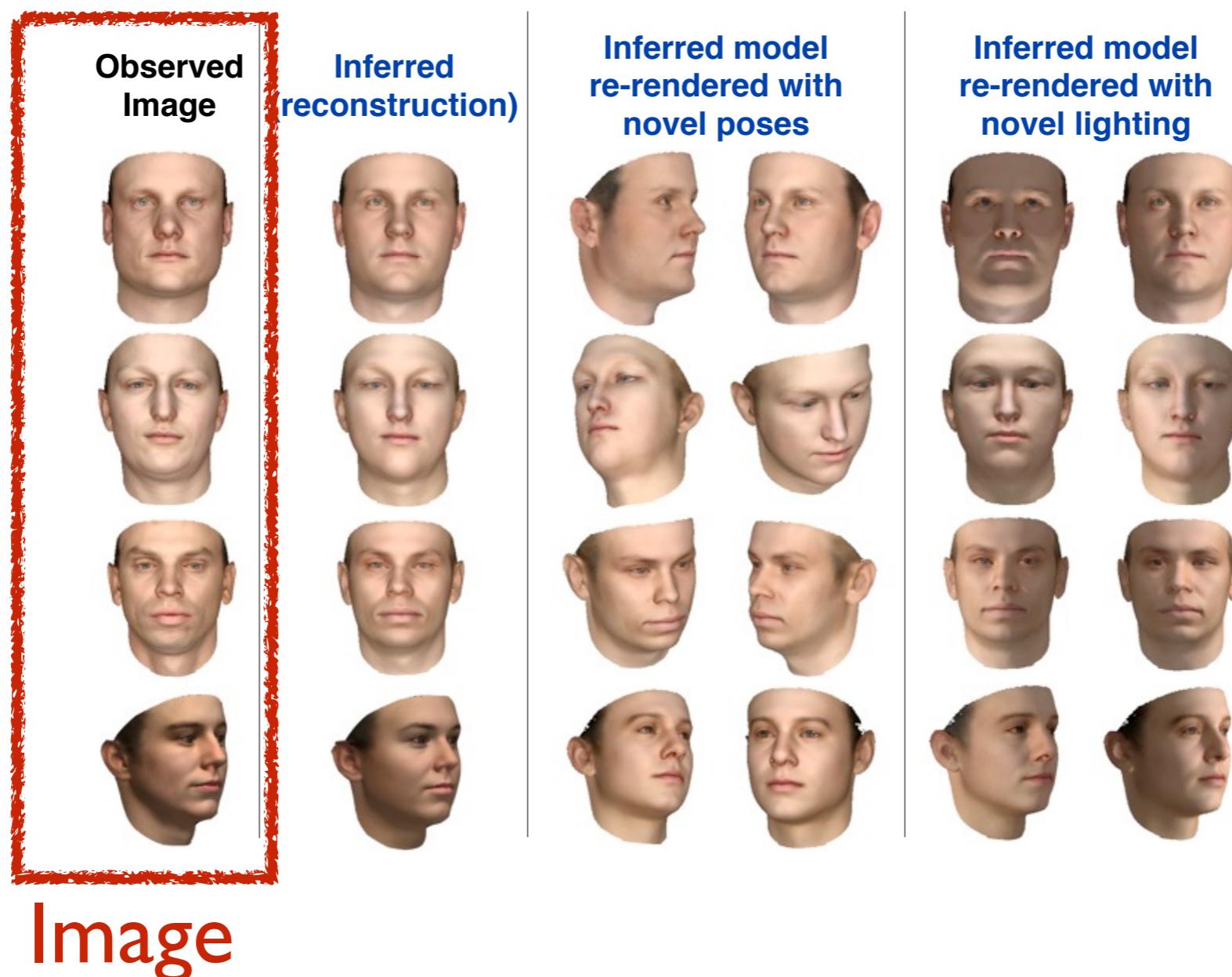
Le, Baydin, Wood [2016]

Three success stories: 2) inverse graphics



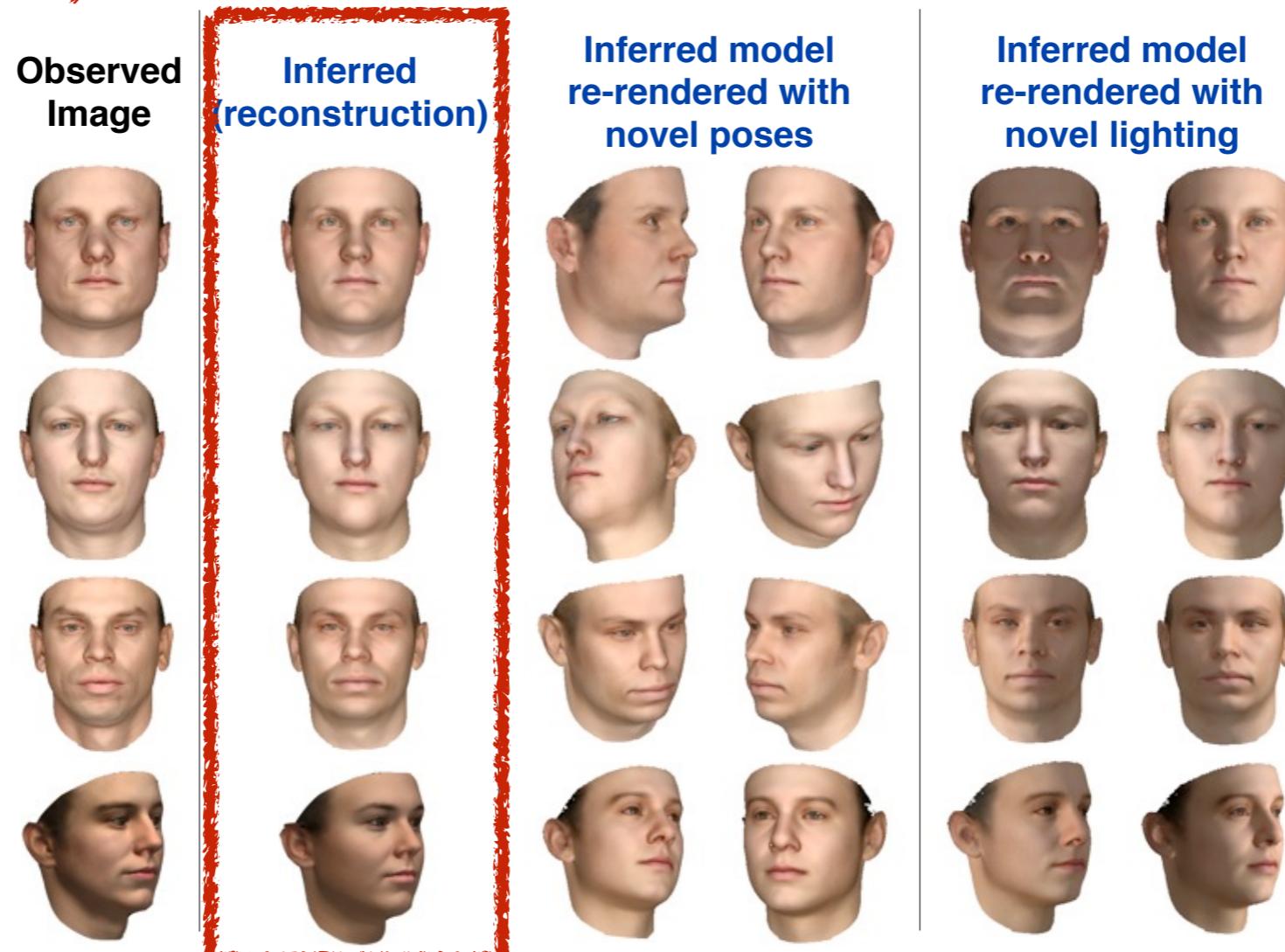
Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

Three success stories: 2) inverse graphics



Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

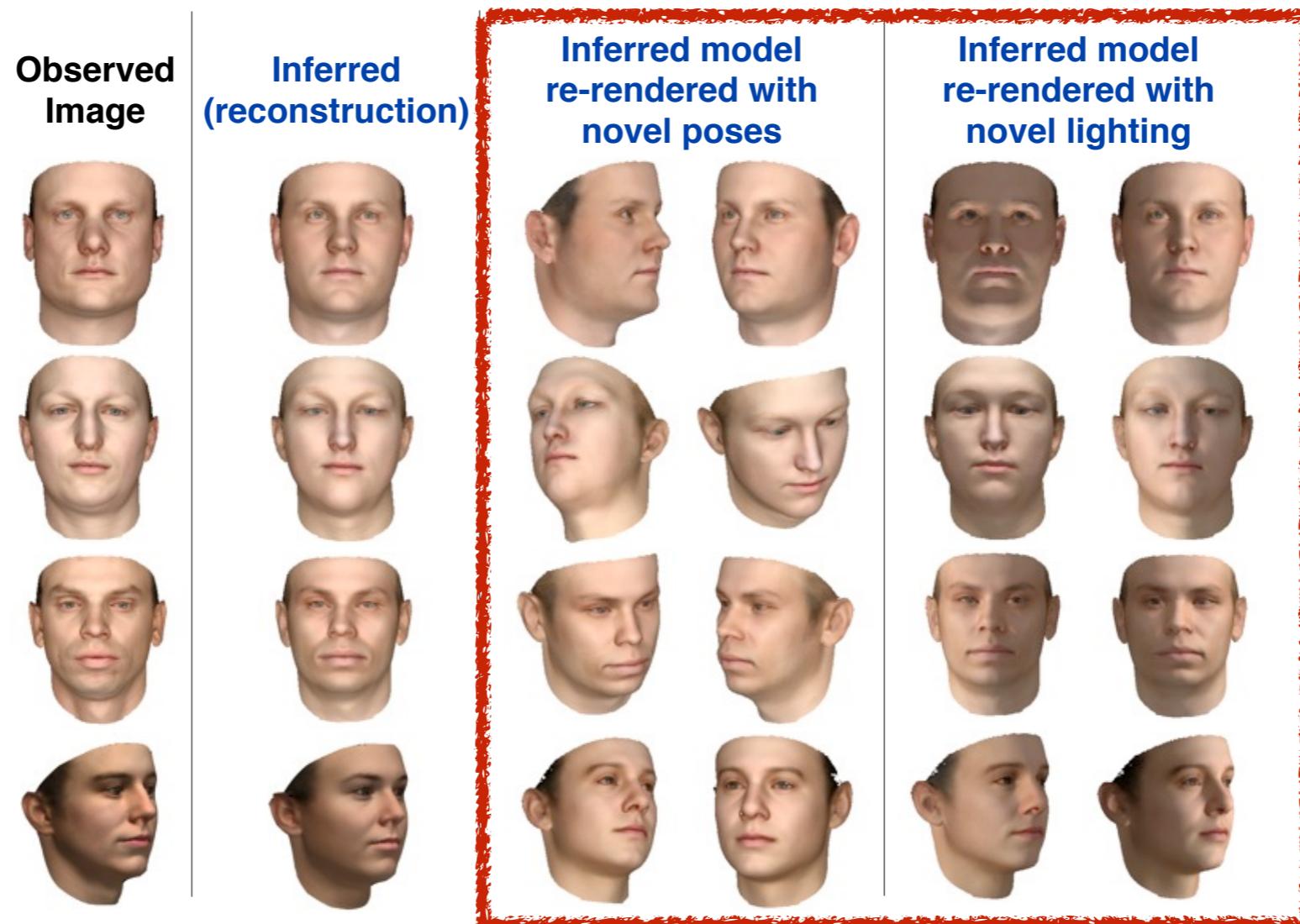
Three success stories: 2) inverse graphics



3D model

Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

Three success stories: 2) inverse graphics

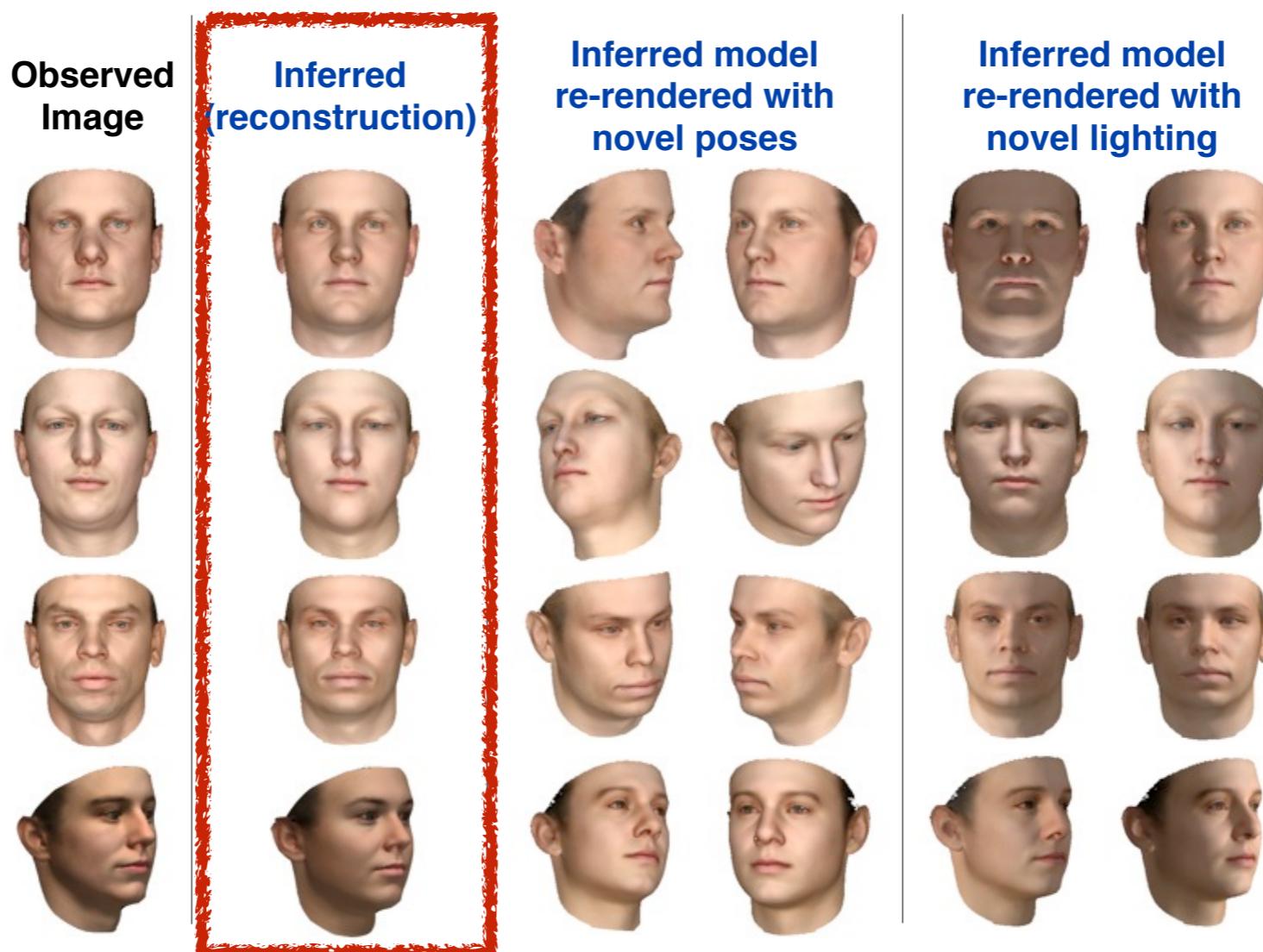


Transformed 3D models

Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

I. Sample a 3D model.

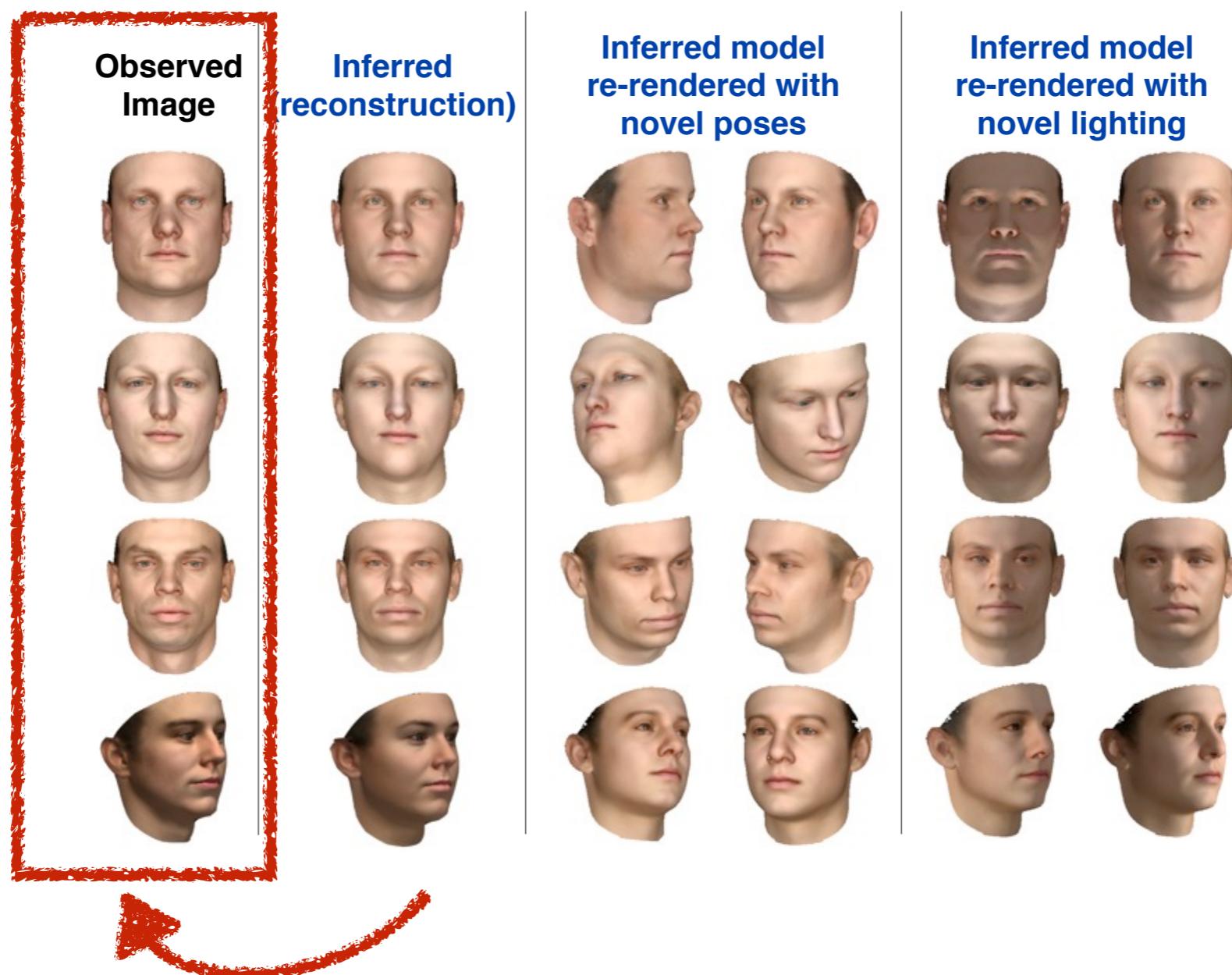
Access stories: -,-, diverse graphics



Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

TI I. Sample a 3D model. 2. Generate an image.

ccess stories: verse graphics



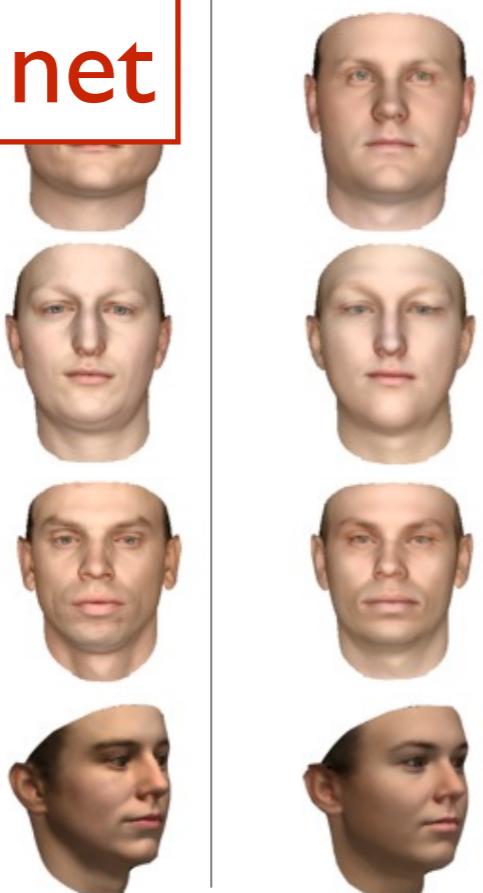
Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

Access stories: inverse graphics

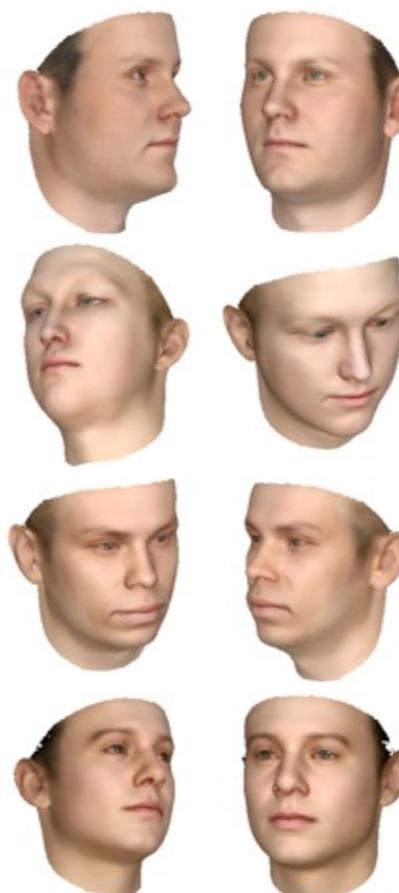
- I. Sample a 3D model.
- II. Generate an image.

Posterior inf.
with neural net

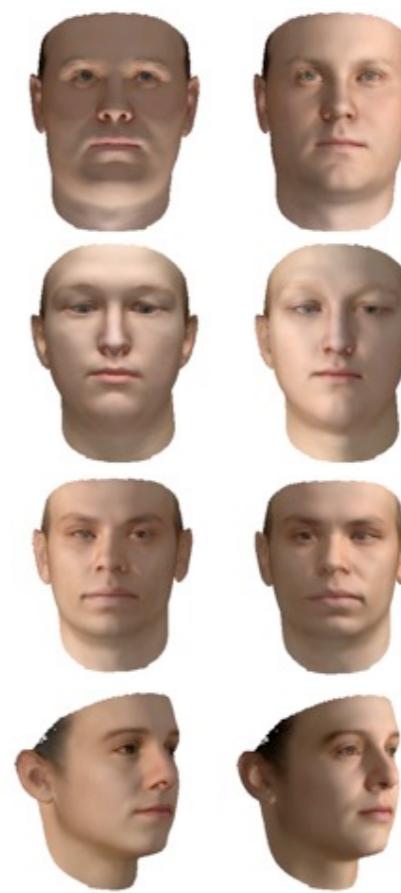
Inferred
(reconstruction)



Inferred model
re-rendered with
novel poses

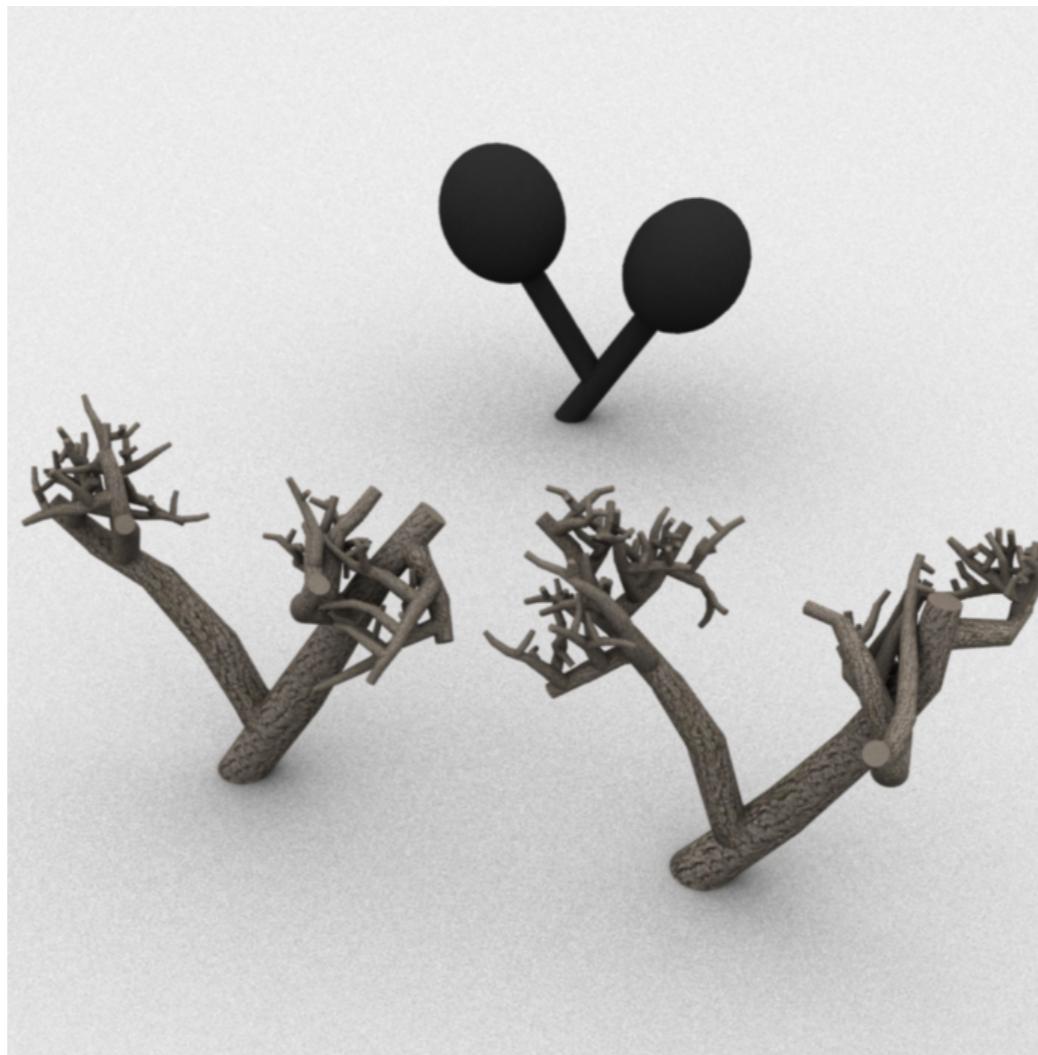


Inferred model
re-rendered with
novel lighting



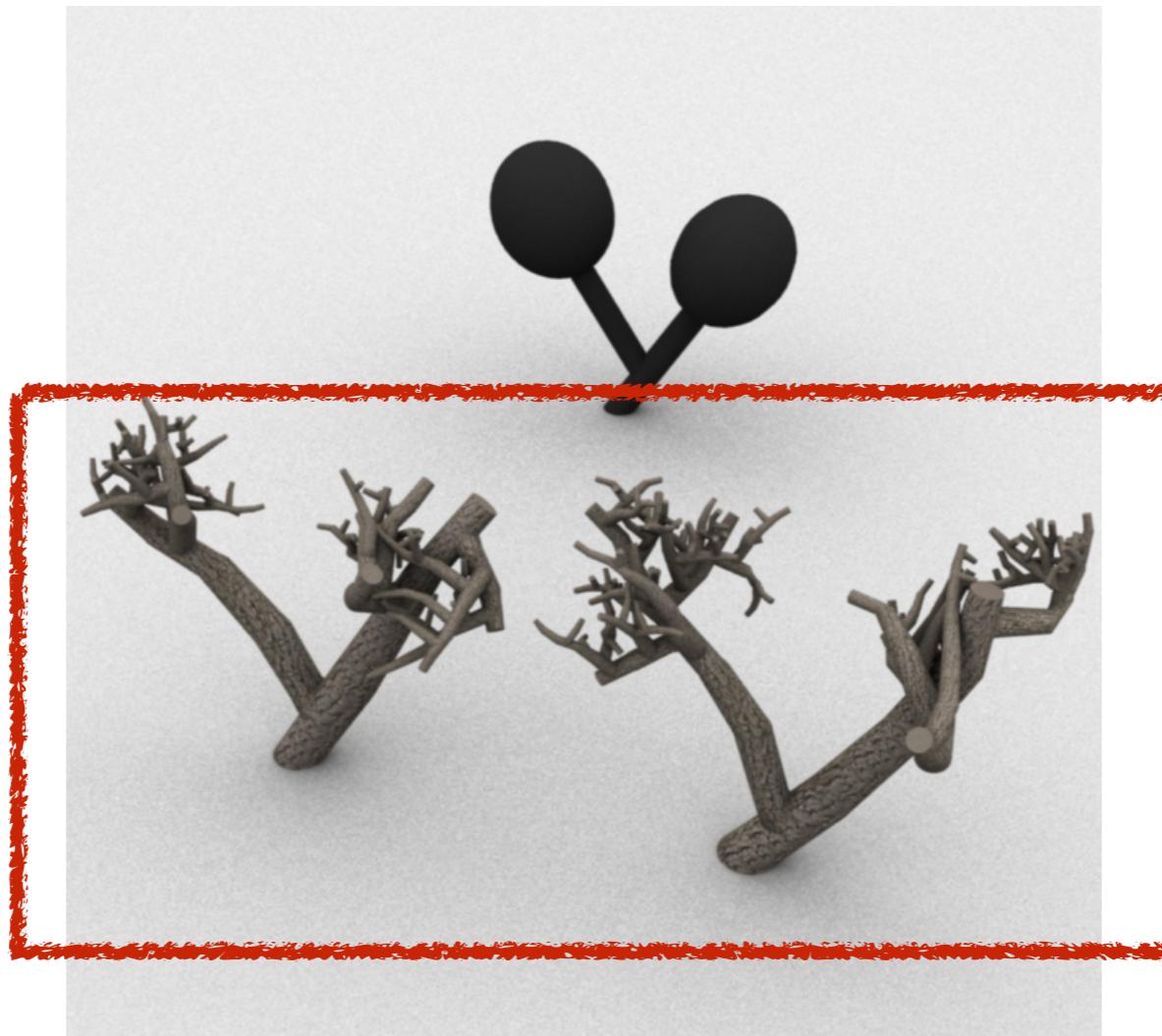
Kulkarni, Kohl, Tenenbaum, Mansinghka [CVPR'15]

Three success stories: 3) procedural modelling



Ritchie, Mildenhall, Goodman,
Hanrahan [SIGGRAPH'15]

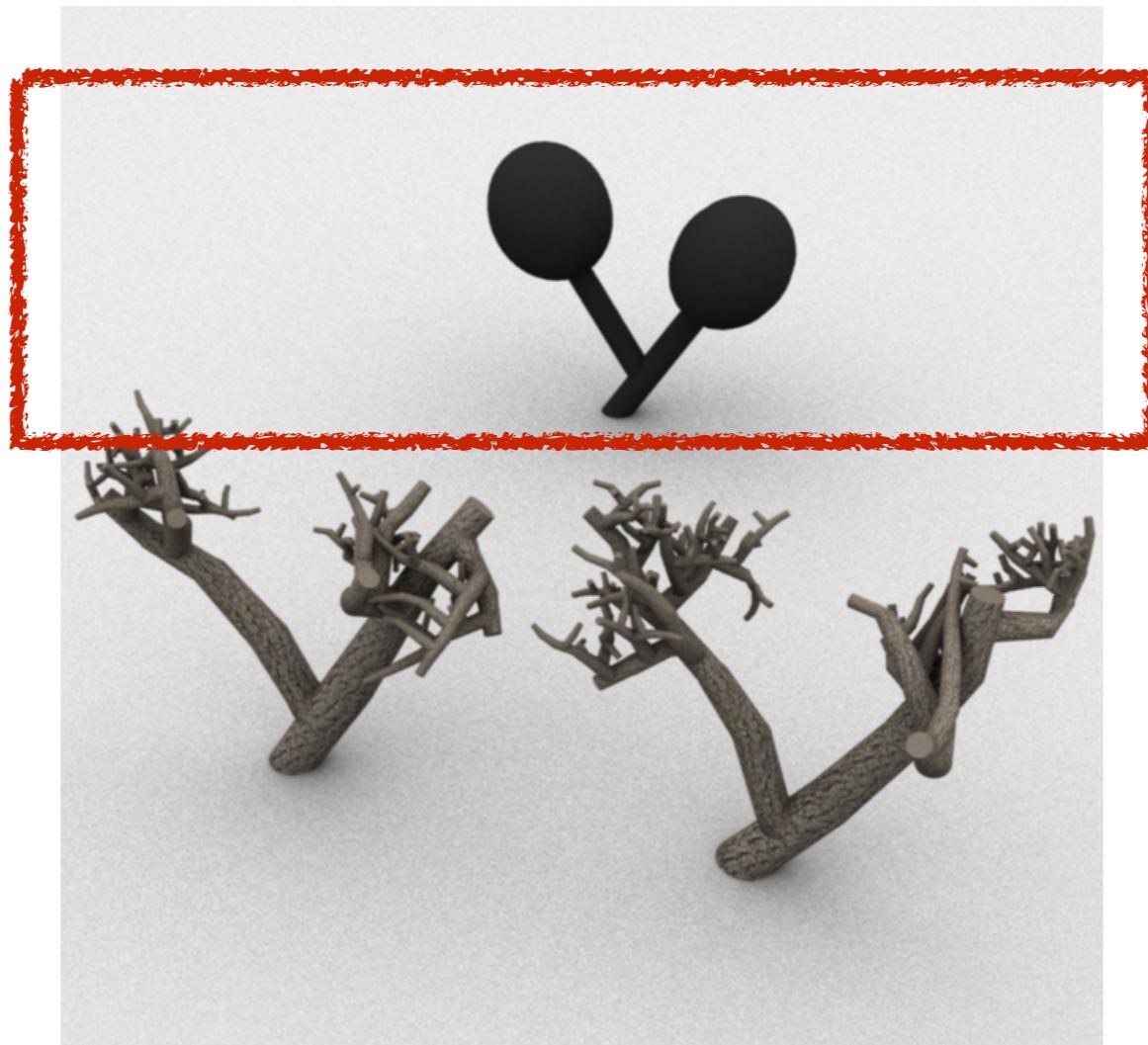
Three success stories: 3) procedural modelling



I. Sample a 3D object.

Ritchie, Mildenhall, Goodman,
Hanrahan [SIGGRAPH'15]

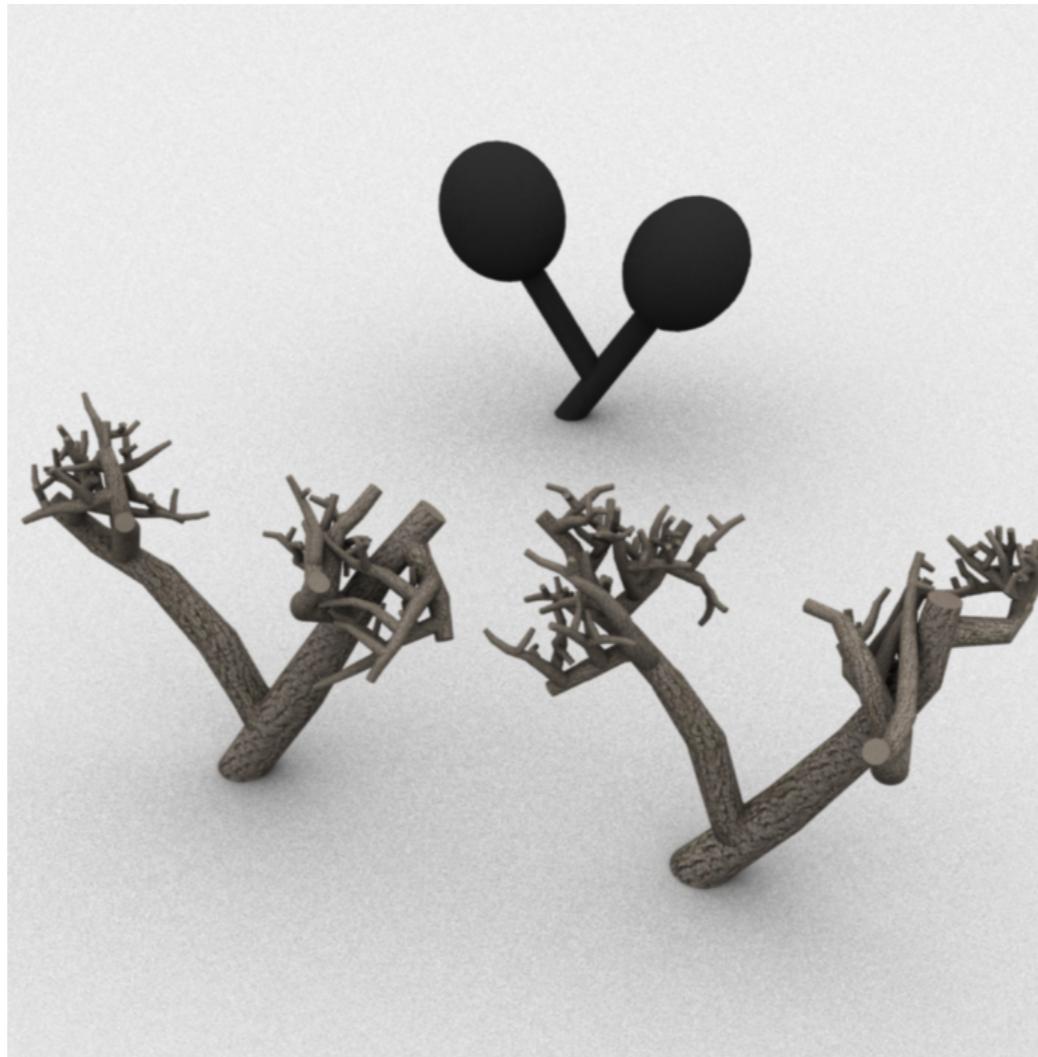
Three success stories: 3) procedural modelling



1. Sample a 3D object.
2. Score the object.

Ritchie, Mildenhall, Goodman,
Hanrahan [SIGGRAPH'15]

Three success stories: 3) procedural modelling



1. Sample a 3D object.
2. Score the object.
Used stochastic future.

Ritchie, Mildenhall, Goodman,
Hanrahan [SIGGRAPH'15]

Three succ

Asynchronous function
call via future

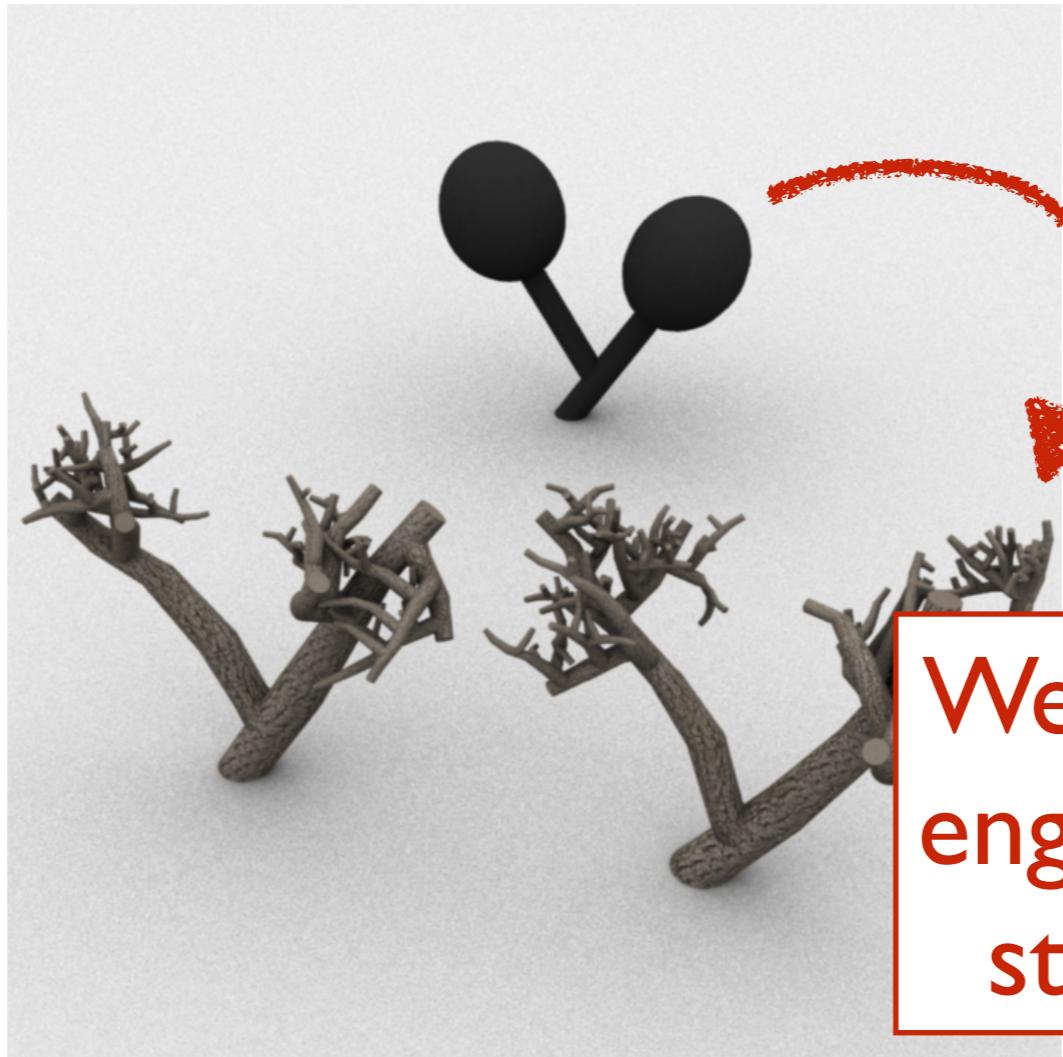
```
future.create(function(i, frame, prev)
    if flip(T.branchProb(depth, i)) then
        -- Theta mean/variance based on avg weighted by
        local theta_mu, theta_sigma = T.estimateThetaD:
        local theta = gaussian(theta_mu, theta_sigma)
        local maxbranchradius = 0.5*(nextframe.center
        local branchradius = math.min(uniform(0.9, 1) *
        local bframe, prev = T.branchFrame(splitFrame,
        branch(bframe, depth+1, prev)
    end
```

1. Sample a 3D object.
2. Score the object.

Used stochastic future.

Ritchie, Mildenhall, Goodman,
Hanrahan [SIGGRAPH'15]

Three success stories: 3) procedural modelling



WebPPL's inference
engine that exploits
stochastic future

1. Sample a 3D object.
2. Score the object.
Used stochastic future.

Ritchie, Mildenhall, Goodman,
Hanrahan [SIGGRAPH'15]

Techniques used

- Changepoints: CPS transformation and new foundation of probability theory.
- Captcha and inverse graphics: neural nets and inference amortisation.
- Procedural modelling: sequential Monte Carlo algorithms and stochastic future.

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Prog. Languages,

Techniques used

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Prog. Languages, Machine Learning,

Techniques used

- Changepoints: CPS transformation and new foundation of probability theory.
- Captcha and inverse graphics: neural nets and inference amortisation.
- Procedural modelling: sequential Monte Carlo algorithms and stochastic future.

Prog. Languages, Machine Learning, Probability Theory

Overview of the course

Objective

1. Learn how to write and reason about models in a prob. prog. language (PPL).
2. Learn results from ML/PL/prob. theory that are used for building effective PPLs.
3. Contribute to probabilistic programming.

Objective

1. Learn how to write and reason about models in a prob. prog. language (PPL).
2. Learn results from ML/PL/prob. theory that are used for building effective PPLs.
3. Contribute to probabilistic programming.
Group project by a group of 1-3 students.

Webpage

<https://github.com/hongseok-yang/probprog20>

All the important announcements will be made
in this webpage.

Evaluation

- 3-5 homework exercises (20%).
- Group project (40%).
- Final exam (40%).

Evaluation

- 3-5 homework exercises (20%).
- Group project (40%).
- Final exam (40%).

1-3 students form a group.

New cool application of a PPL.

Tasks:

- Project.
- 2 (likely online) presentations (topic - 27 Apr, result - 17/22/24 June).
- Report. 2 pages not including bibliography.

Honour code

- Strict policy for handling the violation of the standard honour code.
- If found to cheat in an exam or copy solutions or code, the student will get F.

Really important announcement

Form a group and inform me and TAs by the
midnight of 30 March (Monday).

Somewhat important announcement

- Lecturer: Prof Hongseok Yang
email: hongseok00@gmail.com
- TAs: Gwonsoo Che, Daeseok Lee
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- Install Anglican. Try the changepoint example. Get hints from the webpage.

Webpage

<https://github.com/hongseok-yang/probprog20>

All the important announcements will be made
in this webpage.