Differential Calculus, Reading

You probably did not see this one coming! But we did say "analytical methods" in the previous reading. No reason that should be limited to algebra...

It goes like this -- we determine the difference in the f(n) function for a given difference in n, and we use that to write a differential equation. Then we integrate -- it's just that easy!

Let's start easy -- O(1), constant complexity. The change in f(n) is zero for a change of 1 in n (or any change in n, for that matter).

$\Delta f = 0$ $\Delta n = 1$ $\frac{\Delta f}{\Delta n} = \frac{0}{1} = 0$ $\frac{df}{dn} = 0$ $f = \int 0 \cdot dn = constant \sim 1$

That's O(1).

O(n)

This applies to searching an array for a match. Adding one more value to the array adds one more operation, so (ignoring the constant multiplier):

$\Delta f = 1$ $\Delta n = 1$ $\frac{\Delta f}{\Delta n} = \frac{1}{1} = 1$ $\frac{df}{dn} = 1$ $f = 1 \cdot \int dn = n$

O(n^2)

This applies to nested for-loop sorting. In this case, adding one value to the array adds one more run of the inner loop with about n cycles, and adds one to each run of the inner loop. So (again ignoring the constant multiplier):

$\Delta f = n + n = 2n$ $\Delta n = 1$ $\frac{\Delta f}{\Delta n} = \frac{2n}{1} = 2n$ $\frac{df}{dn} = 2n$ $f = 2 \cdot \int n \cdot dn = 2 \cdot \frac{n^2}{2} = n^2$

So while differential calculus *sounds* scary, it actually makes the math easier.