Zhang

Zhang

May 19, 2021

1 Equivalent resistence

Suppose there are totally n buses where $1, \cdots, n-1$ is load bus and n is generator bus. Then we could get that

$$\begin{pmatrix} Y_{11} & \cdots & Y_{1n} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn} \end{pmatrix} \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} I_1 \\ \vdots \\ I_n \end{pmatrix} \tag{1}$$

Note that $\sum_i I_i = 0$ and the equation could hold if V_i plus any constant voltage V_C. Thus We suppose $V_n = 0$ and the equation could be rewritten as

$$\begin{pmatrix} Y_{11} & \cdots & Y_{1n-1} \\ \vdots & \ddots & \vdots \\ Y_{n1} & \cdots & Y_{nn-1} \end{pmatrix} \begin{pmatrix} V_1 \\ \vdots \\ V_{n-1} \end{pmatrix} = \begin{pmatrix} I_1 \\ \vdots \\ I_n \end{pmatrix} \tag{2}$$

Let Y^+ denotes the pseudoinverse of matrix Y. And

$$I_i^* = (I_1, \cdots, I_n)^T = \left(0, \cdots, \underset{i^{th}}{1}, \cdots, 0, 1\right)^T. \tag{3}$$

Then the equivalent resistence $R_i = R_{ni}$ between generator n and load i could be represented as

$$R_i = R_{ni} = \left(Y^+ I_i^*\right)_i. \tag{4}$$

And the equivalent electrical conductance $Y_i = Y_{ni}$ between generator n and load i could be represented as

$$Y_i = Y_{ni} = 1/R_i = 1/\left(Y^+ I_i^*\right)_i. \tag{5}$$

2 $\sum_i Y_i$

Similar to the part(1), Let

$$I_{all} = \left(I_1^*, \cdots, I_{n-1}^*\right) = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \\ -1 & \cdots & -1 \end{pmatrix}. \tag{6}$$

Then

$$diag\left\{R_1,\cdots,R_{n-1}\right\} = diag\left\{Y^{+}I_{all}\right\}. \tag{7}$$

where *diag* on the left hand side indicates choose the main diagnose element of matrix to be a new matrix and *diag* on the right hand side indicates select the element to be the diagnose of the new matrix.

And

$$\sum_i Y_i = \sum_i diag\left\{Y_1,\cdots,Y_{n-1}\right\} = \sum_i diag\left\{1/R_1,\cdots,1/R_{n-1}\right\} = \sum_i diag\left\{Y^{+}I_{all}\right\}^{-1}. \tag{8}$$

3 $\sum_i \left(Y_i - Y_i^{'}\right)$

Suppose there are to admittance matrix Y and Y'. In order to calculate their difference $\sum_i \left(Y_i - Y_i^{'}\right)$.

$$\sum_i \left(Y_i - Y_i'\right) = \sum_i diag\left\{Y_1,\cdots,Y_{n-1}\right\} - \sum_i diag\left\{Y_1{}',\cdots,Y_{n-1}{}'\right\} \tag{9}$$

$$= \sum_i diag\left\{\frac{1}{R_1} - \frac{1}{R_1{}'},\cdots,\frac{1}{R_{n-1}} - \frac{1}{R_{n-1}{}'}\right\} \tag{10}$$

$$= \sum_i diag\left\{\frac{R_1{}' - R_1}{R_1 R_1{}'},\cdots,\frac{R_{n-1}{}' - R_{n-1}}{R_{n-1}R_{n-1}{}'}\right\} \tag{11}$$

$$= \sum_i diag\left\{\frac{R_1{}' - R_1}{R_1 R_1{}'},\cdots,\frac{R_{n-1}{}' - R_{n-1}}{R_{n-1}R_{n-1}{}'}\right\} \tag{12}$$

$$= \sum_i diag\left\{Y_i\right\} diag\left\{R_i{}' - R_i\right\} diag\left\{Y_i{}'\right\} \tag{13}$$

$$= \sum_i diag\left\{Y^{+}I_{all}\right\}^{-1} diag\left\{Y'^{+}I_{all} - Y^{+}I_{all}\right\} diag\left\{Y'^{+}I_{all}\right\}^{-1} \tag{14}$$

$$= \sum_i diag\left\{Y^{+}I_{all}\right\}^{-1} diag\left\{\left(Y^{+}YY'^{+} - Y^{+}Y^{'}Y'^{+}\right)I_{all}\right\} diag\left\{Y'^{+}I_{all}\right\}^{-1} \tag{15}$$

$$= \sum_i diag\left\{Y^{+}I_{all}\right\}^{-1} diag\left\{Y^{+}\left(Y - Y'\right)Y'^{+}I_{all}\right\} diag\left\{Y'^{+}I_{all}\right\}^{-1} \tag{16}$$

More detailed of code can be viewed at https://github.com/hongshen-zhang/Voltage_Collapse_AC_model/tree/main/calculate_Y