

CSCE 222 [503] Discrete Structures for Computing  
Spring 2015 – Philip C. Ritchey

**Problem Set 5**

**Due dates:** Electronic submission of the PDF file for this homework is due on **2/26/2015 (Thursday) before 11:59 p.m.** on <http://ecampus.tamu.edu>. A signed and stapled paper copy of the PDF is due on **2/27/2015 (Friday)** at the beginning of class.  
You must show your work. **No work → no credit.**

**Name:** (Han Hong)

**Resources.** Discrete Mathematics and Its Applications by Rosen, (additional people, books, articles, web pages, etc. that have been consulted when producing this homework)

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

**Signature:** \_\_\_\_\_

**Problem 1.** (11 points) Using a proof similar to the one in class for the geometric series, prove that

$$\sum_{i=0}^{n-1} i = \frac{n(n-1)}{2}$$

**Solution.**

$$\text{Let } S = \sum_{i=0}^{n-1} i$$

$$\Rightarrow 2S = \sum_{i=0}^n n + 1$$

$$\Rightarrow 2S = n(n+1)$$

$$S = \frac{n(n-1)}{2}$$

**Problem 2.** (11 points) Derive a general closed-form solution for the sum of an arithmetic progression  $\{a_n\}$ :

$$\sum_{i=m}^n a_i$$

**Solution.**

$$\sum_{i=m}^n a_i = \sum_{i=0}^n a_i - \sum_{i=0}^{m-1} a_i$$

$$\Rightarrow \frac{n(n+1)}{2} - \frac{m(m-1)}{2}$$

**Problem 3.** (12 points) Section 2.4, Exercise 17 d, f, h page 168  
 You must show your **work** to get **credit**.

**Solution.**

d.  $a_n = a_{n-1} + 2n + 3; a_0 = 4$

$$a_0 = 4$$

$$a_1 = 4 + 2(1) + 3 = 9$$

$$a_2 = 9 + 2(2) + 3 = 16$$

$$\Rightarrow a_n = (2 + n)^2$$

f.  $a_n = 3a_{n-1} + 1; a_0 = 4$

$$a_0 = 1$$

$$a_1 = 3(1) + 1 = 4$$

$$a_2 = 3(4) + 1 = 13$$

$$a_3 = 3(13) + 1 = 40$$

$$\Rightarrow \text{Geometric series: } \frac{3n(n+1)}{2} + 1h.a_n = 2na_{n-1}; a_0 = 1$$

$$a_1 = 2(1)(1) = 2$$

$$a_2 = 2(2)2 = 8$$

$$a_3 = 2(3)(8) = 48$$

$$\Rightarrow 2^n n!$$

**Problem 4.** (11 points) Section 2.4, Exercise 24, page 169

**Solution.**

a.  $B(k) = \frac{r}{12}B(k-1) - P$

b.  $P = \frac{r}{12}B(T-1)$

**Problem 5.** (11 points) Section 2.4, Exercise 34, page 169

**Solution.**

a.  $\Rightarrow \sum_{i=1}^3 ((i-1) + (i-2)) = ((1-1) + (1-2)) + ((2-1) + (2-2)) + ((3-1) + (3-2))$   
 $= 0 + 1 + 0 + 2 + 1 = 3.$

b.  $\Rightarrow \sum_{i=0}^3 ((3i+2*0) + (3i+2*1) + (3i+2*2))$   
 $= \sum_{i=0}^3 (9i + 6)$   
 $= 9*0 + 6 + 9*1 + 6 + 9*2 + 6 + 9*3 + 6$   
 $= 81$

c.  $\Rightarrow \sum_{i=1}^3 (0+1+2)$   
 $= \sum_{i=1}^3 3$   
 $= 3 + 3 + 3 = 9$

**Problem 6.** (11 points) Section 2.4, Exercise 36, page 169

**Solution.**

$$\sum_{k=1}^n \frac{1}{k(k+1)} = \sum_{k=1}^n \frac{1}{k} - \frac{1}{k+1} = (1 - \frac{1}{2}) + (\frac{1}{2} - \frac{1}{3}) \dots (\frac{1}{n} - \frac{1}{n+1})$$

ALL cancels out, leaves  $\frac{1}{n+1}$

**Problem 7.** (11 points) Section 2.4, Exercise 40, page 169

**Solution.**

Take sum from 1 to 200 minus sum from 1 to 99, will give result from 99 to 200  
 Answer: 379507500

**Problem 8.** (11 points) Section 2.4, Exercise 44, page 170

**Solution.**

$$\prod_{i=n}^1 i$$

**Problem 9.** (11 points) Section 2.4, Exercise 46, page 170

**Solution.**

$$\prod_{n=0}^4 j! = 0! * 1! * 2! * 3! * 4! = 1 * 1 * 2 * 6 * 24 = 288$$

**Checklist:**

- ☐ Did you add your **name**?
- ☐ Did you disclose all **resources** that you have used?  
(This includes all people, books, websites, etc. that you have consulted)
- ☐ Did you **sign** that you followed the Aggie honor code?
- ☐ Did you solve **every problem**?
- ☐ Did you submit the PDF file of your homework on **eCampus**?
- ☐ Did you submit a **signed and stapled** hardcopy of the PDF file **in class**?