A Hessenberg-Based Input Balanced Realization for All-Pass Systems

Tao Hong*, Chaogeng Huang†, Gang Li*, and Yong Ching Lim‡
* Zhejiang Key Lab. for Signal Processing, College of Information Engineering
Zhejiang University of Technology, 310023 Hangzhou, Zhejiang, P.R. China
Email: happyhongt@163.com
†Zhejiang Key Lab. for Signal Processing, College of Information
Zhejiang University of Finance & Economics, 310018 Hangzhou, Zhejiang, P.R. China
Email: cghuang@zufe.edu.cn
‡School of EEE Nanyang Technological University, Singapore 639798, Republic of Singapore
Email: eyclim@ntu.edu.sg

***Abstract*—In this paper, a novel Hessenberg-based input balanced realization for all-pass systems is derived. The expression of roundoff noise gain for this structure is obtained. An Nth order system implemented with a coupled all-pass sub-filters proposed in [18], in which each all-pass filter is realized using our proposed structure, yields a roundoff noise gain of $N + 1$, smaller than that of the classical optimal roundoff noise state-space realizations. A design example is presented to illustrate the behavior of the proposed structure and to compare it with a class of existing input balanced structures and the classical optimal roundoff noise realizations. It is shown that for the all-pass filters, the proposed structure outperforms the others in terms of minimizing roundoff noise as well as implementation efficiency.**

***Index Terms*—Digital filter structures, input balanced realizations, finite word length, all-pass filter.**

I. Introduction

It is well known that a well-designed Nth order digital filter of transfer function $H(z)$ has to be implemented using a digital device of limited capacity and finite precision. These lead to the so-called finite word length (FWL) effects which can degrade greatly the performance of the well-designed filter as these quantization errors are not taken into account at the design stage of the filter $H(z)$. In FWL implementation, different realizations may yield different performances, which also provides the freedom of degrees for minimizing the effects on the performance of the actually implemented digital filters [1], [2]. Traditionally, the optimal FWL structure design is to optimize a given criterion such as sensitivity measure [3] - [5] and roundoff noise gain [6] - [10]. As an effective technique for reducing the roundoff noise, error feedback technique has been applied into the state space realization of IIR filters [11] - [14]. Nevertheless, implementation complexity is a very key constraint condition for practical applications since that also means low consumption. Design of low complexity filters with high robustness against FWL errors has been an important topic.

Note that the optimal realizations are usually fully parameterized, such as the optimal roundoff noise state-space realization [6], [7]. The most straightforward approach is to design sparse structures. In [9], a sparse orthogonal structure was derived. Compared with the optimal state-space realization of an Nth order digital filter, this structure requires only $7N-3$ multiplications, instead of $(N+1)^2$, for computing one output sample, which can reduce the implementation complexity greatly. As a further study of this work, based on matrix decomposition an improved sparse structure was introduced, which only requires $4N-1$ multipliers [10]. Such a structure also yields lower roundoff noise gain. Other sparse optimal or quasi-optimal structures can be found in several publications [15] - [17].

All-pass digital filters having desirable properties that just shift the phase while keeping the amplitude constant, have found wide applications in the design of single-channel filters, transforms, equalizers, and so on. An all-pass filter yields complex pole-zero pairs that may lead to many nice properties such as low coefficient sensitivity and small roundoff noise. It is shown from [18] that the odd-order classical digital low-pass filters (Butterworth, Chebyshev, inverse Chebyshev, and elliptic designs) can be implemented as a sum of two all-pass sub-filters. In this case, the implementation of a digital filter as the sum of two all-pass sub-filters requires approximately half as many multipliers compared to a straightforward cascade form.

In [19], we developed a low complexity structure for IIR filter implementation. One of the objectives in this paper is to specify this structure for all-pass filters, the resultant structure is denoted as R_{new}, and to analyze the FWL performance of this obtained R_{new}. An expression for the roundoff noise gain of R_{new} is derived, which states that the roundoff noise gain of R_{new} for an Nth order all-pass filter is equal to $4N$. Applying this result to the structure proposed in [18] for an arbitrary Nth order IIR filter, the roundoff noise gain is $N+1$, smaller than that of the classical optimal realizations.

The outline of this paper is given as follows. Our new structure for all-pass systems is proposed in section II. In section III, the expression for the roundoff noise gain of our proposed structure is derived as well as the other three classical structures. A numerical example is presented in section IV to demonstrate the performance of this new structure. Section V offers some conclusions.

II. A Novel Structure for All-Pass Filters

Consider an Nth-order infinite impulse response (IIR) filter given by the following transfer function $H(z)$:

$$H(z) = \frac{b_0 + b_1 z^{-1} + ... + b_N z^{-N}}{1 + a_1 z^{-1} + ... + a_N z^{-N}} \triangleq \frac{N(z)}{D(z)} \tag{1}$$

Such a filter can be realized with its state-space equations:

$$\begin{cases} x(n+1) &= Ax(n) + Bu(n) \\ y(n) &= Cx(n) + du(n) \end{cases} \tag{2}$$

where $u(n), y(n)$ are the scalar input and output of the filter, respectively, $x(n) \in \mathcal{R}^{N\times 1}$ is the state vector, and $A \in \mathcal{R}^{N\times N}, B \in \mathcal{R}^{N\times 1}, C \in \mathcal{R}^{1\times N}, d \in \mathcal{R}$ are constant matrices, called a (state-space) realization, denoted as $R \triangleq (A, B, C, d)$, of $H(z)$, satisfying

$$H(z) = d + C(zI - A)^{-1}B \tag{3}$$

As well-known [2], the state-space representations that are equivalent to a given (initial) realization, say (A_0, B_0, C_0, d), can be characterized by $A = T^{-1}A_0T,\ B = T^{-1}B_0,\ C = C_0T$, where $T \in \mathcal{R}^{N\times N}$ is any non-singular coordinate transformation matrix.

The controllability and observability Gramians of the realization R of a stable $H(z)$ are defined as the solutions of the following Lyapounov equations, respectively

$$\begin{cases} W_c &= AW_cA^{\mathcal{T}} + BB^{\mathcal{T}} \\ W_o &= A^{\mathcal{T}}W_oA + C^{\mathcal{T}}C \end{cases} \tag{4}$$

where $\mathcal{T}$ denotes the transpose operator.

A realization is said *input* balanced, denoted as R_{IB}, if its W_c is equal to identity matrix: $W_c = I$, that is

$$I = AA^{\mathcal{T}} + BB^{\mathcal{T}} \tag{5}$$

The balanced realizations have very nice numerical properties such as low sensitivity and small roundoff noise gain [1], [2]. These realizations, like the optimal roundoff noise realizations R_{MRH}, are fully parametrized and hence slow down the processing.

As a special input balanced realization, the Hessenberg-based realization was proposed in [19] given as follows:

$$\begin{cases} A &= \begin{bmatrix} \cos\phi_1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} Q \triangleq Q_1 Q \\ B &= \begin{bmatrix} -\sin\phi_1 & 0 & \cdots & 0 \end{bmatrix}^{\mathcal{T}} \\ C &= \begin{bmatrix} \xi_1 & \xi_2 & \cdots & \xi_N \end{bmatrix},\ d = \xi_0 \end{cases} \tag{6}$$

where $|\phi_k| < \frac{\pi}{2}$ and $Q \triangleq \prod_{k=2}^{N} Q_k = Q_2Q_3\cdots Q_N$ with Q_k being the Givens rotation matrix of form[1]

$$Q_k \triangleq \begin{bmatrix} 1 &&&&&& \\ & \ddots &&&&& \\ && 1 &&&& \\ &&& \cos\phi_k & \sin\phi_k &&& \\ &&& -\sin\phi_k & \cos\phi_k &&& \\ &&&&& 1 && \\ &&&&&& \ddots & \\ &&&&&&& 1 \end{bmatrix} \tag{7}$$

for $k = 2, 3, \cdots, N$. For convenience, such a structure is referred to as R_{HB}, depicted in Fig. 1.

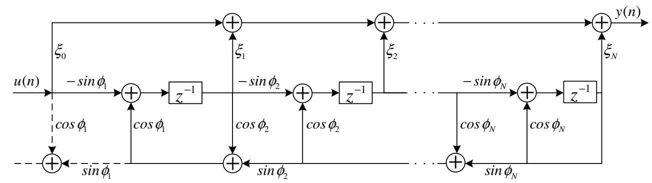

Fig. 1. Block diagram of the Hessenberg-based structure.

As is known, any Nth order all-pass system can be described as $H(z) = \frac{z^{-N}D(z^{-1})}{D(z)}$. One of the interesting properties of the all-pass system is that the product of its controllability and observability Gramians is equal to identity matrix. There exists some important results if the all-pass system is implemented using R_{HB} structure. Before giving our main result in this paper, we need the following lemma.

Lemma 1: Let $(\hat{A}, \hat{B}, \hat{C}, d)$ be an input balanced realization of an Nth order all-pass system $H(z)$. Furthermore, let $\hat{W}_c, \hat{W}_o$ be the controllability and observability Gramians of $(\hat{A}, \hat{B}, \hat{C}, d)$. Then

$$\hat{W}_c = \hat{W}_o = I \tag{8}$$

and

$$\begin{cases} d^2 + ||\hat{C}||_2^2 &= 1 \\ d\hat{B} + \hat{A}\hat{C}^{\mathcal{T}} &= 0 \end{cases} \tag{9}$$

where $||\cdot||_2$ is the l_2-norm.

Proof: It is well known that for any all-pass system, there exists $\hat{W}_c\hat{W}_o = I$. Since the input balanced realizations have an inherent property that $\hat{W}_c = I$, it is easy to show that $\hat{W}_o = I$.

Define

$$H(z) = d + \hat{C}(zI - \hat{A})^{-1}\hat{B} \triangleq d + \sum_{k=1}^{+\infty} q_k z^{-k} \tag{10}$$

It can be shown that $H(z)H(z^{-1}) = p_0 + \sum_{m=1}^{+\infty} p_m(z^{-m} + z^m)$, where $p_m = dq_m + \sum_{k=1}^{+\infty} q_{k+m}q_k,\ \forall m$. Since $q_k = \hat{C}\hat{A}^{k-1}\hat{B},\ \forall k > 0$, it turns out that $p_0 = d^2 +$

[1] The $(k-1, k)$th element of Q_k is $\sin\phi_k$.

$\hat{C}\sum_{k=1}^{+\infty}\hat{A}^{k-1}\hat{B}\hat{B}^{\mathcal{T}}(\hat{A}^{k-1})^{\mathcal{T}}C^{\mathcal{T}} = d^2+\hat{C}\hat{W}_cC^{\mathcal{T}} = d^2+||C||_2^2$ and $p_m = d\hat{C}\hat{A}^{m-1}\hat{B}+\hat{C}\hat{A}^m\hat{W}_cC^{\mathcal{T}} = \hat{C}\hat{A}^{m-1}(d\hat{B}+\hat{A}\hat{C}^{\mathcal{T}})$, $\forall m\neq 0$. It follows $H(z)$ being all-pass system that $H(z)H(z^{-1})=1$, which holds for (9). This completes the proof. ■

Now, we state our main result in this paper.

Theorem 1: Let (A,B,C,d) be defined as before for the Hessenberg-based realization R_{HB} of an Nth order all-pass system $H(z)$. Then,

$$C=\rho_d\sin\phi_1 v_1^{\mathcal{T}}Q,\ \ d=\rho_d\cos\phi_1 \tag{11}$$

where v_k denotes the elementary (column) vector of proper dimension, whose elements are all zero except the kth one which is equal to 1, and

$$\rho_d=\begin{cases}1, & if\ d>0\\ -1, & if\ d<0\end{cases} \tag{12}$$

Proof: It follows from Lemma 1 that $A^{\mathcal{T}}A+C^{\mathcal{T}}C=I$. Noting that $A=Q_1Q$, one gets

$$\begin{aligned} C^{\mathcal{T}}C=I-A^{\mathcal{T}}A &= I-Q^{\mathcal{T}}\begin{bmatrix}\cos^2\phi_1 & 0\\ 0 & I_{N-1}\end{bmatrix}Q\\ &= Q^{\mathcal{T}}\begin{bmatrix}\sin^2\phi_1 & 0\\ 0 & \mathbf{0}\end{bmatrix}Q \end{aligned} \tag{13}$$

which leads to $C=\pm\sin\phi_1 v_1^{\mathcal{T}}Q$, where I_{N-1} is an identity matrix with the dimension of $N-1$ and $\mathbf{0}$ is the matrix with all elements equal to zero. It follows from the first equation of (9) that $d=\rho_d\cos\phi_1$, where ρ_d is given by (12).

In fact, C can be uniquely determined. Based on obtained d and the second equation of (9), the first equation of (11) can be derived directly, which completes the proof. ■

Based on Theorem 1, the all-pass system $H(z)$ can be implemented by the following equivalent structure:

$$\begin{cases} x^{(1)}(n) &\triangleq x(n)\\ x^{(m)}(n) &= Q_{N-m+2}x^{(m-1)}(n),\ 2\le m\le N\\ w(n) &= x^{(N)}(n)\\ x(n+1) &= Q_1w(n)+Bu(n)\\ y(n) &= \rho_d\sin\phi_1 v_1^{\mathcal{T}}w(n)+du(n)\end{cases} \tag{14}$$

Such a structure is referred to as R_{new}, which is shown in Fig. 2

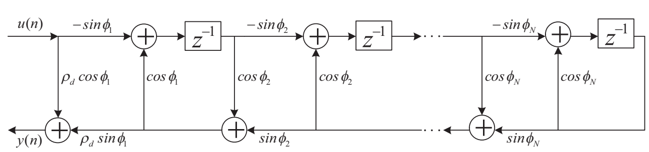

Fig. 2. An equivalent structure of the Hessenberg-based structure for all-pass system.

Remarks:

Simple calculation shows that the proposed structure needs $4N$ multiplications per filter output sample, while the R_{HB} needs $5N-1$ multiplications. Therefore, for an all-pass system, the proposed structure is more efficient than the R_{HB}. With less multiplications involved, it is also expected to have a smaller roundoff gain.

III. Roundoff Noise Analysis

One of the FWL related issues is the rounding operations due to multiplications between signals and non-trivial parameters[2]. In practice, less-than-double precision fixed-point arithmetic is usually utilized on account of its simpleness.

Let ξ_k be a structure parameter (multiplier) in a filter structure. Assume rounding occurs *after multiplication* (RAM). The product $\xi_k s_k(n)$ has to be rounded by a quantizer $q[\cdot]$. Denote $\epsilon_k(n)\triangleq q[\xi_k s_k(n)]-\xi_k s_k(n)$ as the roundoff noise and $E[\cdot]$ as the statistical average. As well known [6], [7], roundoff noises can be modelled as statistically independent white processes and $E[\epsilon_k^2(n)]=\sigma_0^2$ is constant, uniquely determined by the word length used for representing the states. Denote $\Delta y(n)$ as the corresponding output deviation of the filter due to $\epsilon_k(n)$. The roundoff noise gain for the parameter ξ_k is defined as

$$G_k\triangleq\frac{E[(\Delta y(n))^2]}{E[\epsilon_k^2(n)]}.$$

Let $H_k(z)=D_k+C_k(zI-A)^{-1}B_k$ be the transfer function between $\epsilon_k(n)$ and $\Delta y(n)$. According to the residue theory, it can be shown that

$$G_k=tr(D_k^{\mathcal{T}}D_k+B_k^{\mathcal{T}}W_o^kB_k)=tr(D_kD_k^{\mathcal{T}}+C_kW_c^kC_k^{\mathcal{T}}) \tag{15}$$

where W_c^k, W_o^k are the controllability and observability Gramians of the realization (A_k,B_k,C_k,D_k), respectively.

The total roundoff noise gain of the structure is defined as

$$G=\sum_k\delta(\xi_k)G_k$$

where $\delta(\xi_k)=0$ shows that ξ_k is a trivial parameter, otherwise it is equal to one.

A. *Roundoff Noise Gain for R_{MRH} and R_{IB}*

It was shown [6], [7] that for an Nth-order state-space realization satisfying (3), the roundoff noise gain of the optimal roundoff noise realization is given by

$$G_{MRH}=[1+\frac{1}{N}(\sum_{l=1}^{N}\sigma_l)^2](N+1) \tag{16}$$

where $\{\sigma_l\}$ are the Hankel singular values of the filter, with σ_l equal to the square root of the lth eigenvalue of W_cW_o.

Particularly, for an Nth order all-pass system, $W_cW_o=I$ is automatically satisfied and hence the corresponding roundoff noise gain can be simplified as

$$G_{MRH}^{AP}=(N+1)^2 \tag{17}$$

[2]Trivial parameters are defined as that are 0 and ±1. Other parameters are, therefore, referred to as nontrivial parameters.

It can be shown from [2] that the overall gain for the fully parametrized state-space realizations is given by

$$G = [tr(W_o) + 1](N + 1) \tag{18}$$

where $tr(\cdot)$ is the trace operator.

For the input balanced realizations R_{IB}, as $W_c = I$, we have $W_o = I$ and hence the roundoff noise gain is

$$G_{IB}^{AP} = (N + 1)^2 \tag{19}$$

B. Roundoff Noise Gain for the Proposed Structure

In the proposed structure, the structure parameters are $\{\sin\phi_k, \cos\phi_k\}$ in Q_k, B, C and d, respectively.

First of all, let us consider the quantization error caused by Q_1. In this case, (14) becomes

$$\left\{\begin{array}{rcl} \tilde{x}^{(1)}(n) & = & \tilde{x}(n) \\ \tilde{x}^{(m)}(n) & = & Q_{N-m+2}\tilde{x}^{(m-1)}(n),\ 2 \le m \le N \\ \tilde{w}(n) & = & \tilde{x}^{(N)}(n) \\ \tilde{x}(n+1) & = & q[Q_1\tilde{w}(n)] + Bu(n) \\ \tilde{y}(n) & = & \rho_d \sin\phi_1 v_1^{\mathcal{T}}\tilde{w}(n) + du(n) \end{array}\right.$$

Denote $e_1(n) \triangleq q[Q_1\tilde{w}(n)] - Q_1\tilde{w}(n)$. Therefore, $e_1(n) = v_1\epsilon_{Q_1}(n)$, where $\epsilon_{Q_1}(n)$ is the unit roundoff noise produced by Q_1. Denote $\Delta x(n) \triangleq \tilde{x}(n) - x(n)$, $\Delta x^{(m)}(n) \triangleq \tilde{x}^{(m)}(n) - x^{(m)}(n)$, $\Delta w(n) \triangleq \tilde{w}(n) - w(n)$ and $\Delta y(n) \triangleq \tilde{y}(n) - y(n)$. Then one has

$$\left\{\begin{array}{lcl} \Delta\tilde{x}^{(1)}(n) & = & \Delta\tilde{x}(n) \\ \Delta\tilde{x}^{(m)}(n) & = & Q_{N-m+2}\Delta\tilde{x}^{(m-1)}(n),\ 2 \le m \le N \\ \Delta\tilde{w}(n) & = & \Delta\tilde{x}^{(N)}(n) \\ \Delta\tilde{x}(n+1) & = & Q_1\Delta\tilde{w}(n) + v_1\epsilon_{Q_1}(n) \\ \Delta\tilde{y}(n) & = & \rho_d \sin\phi_1 v_1^{\mathcal{T}}\Delta\tilde{w}(n) \end{array}\right.$$

which is equivalent to the realization that is given by

$$\left\{\begin{array}{rcl} \Delta x(n+1) & = & A\Delta x(n) + v_1\epsilon_{Q_1}(n) \\ \Delta y(n) & = & C\Delta x(n) \end{array}\right. \tag{20}$$

Therefore, the corresponding roundoff noise gain for Q_1 is

$$G_{Q_1} = tr(v_1^{\mathcal{T}} W_o v_1) = W_o(1,1) = 1$$

where W_o is the observability Gramian of (A, B, C, d) for this all-pass system.

Similarly, denote $e_k(n) \triangleq q[Q_{N-k+2}x^{(k-1)}(n)] - Q_{N-k+2}x^{(k-1)}(n)$ for $k = 2, \cdots, N$. According to (7), $e_k(n)$ is of the form

$$e_k(n) = [\epsilon_{11}(n) + \epsilon_{12}(n)]v_{m-1} + [\epsilon_{21}(n) + \epsilon_{22}(n)]v_m$$

with the four $\epsilon_{ij}(n)$ being statically independent roundoff noises for $m = N - k + 2$.

With the same definition and analysis like $e_1(n)$, one has

$$\left\{\begin{array}{lcl} \Delta\tilde{x}^{(1)}(n) & = & \Delta\tilde{x}(n) \\ \Delta\tilde{x}^{(m)}(n) & = & Q_{N-m+2}\Delta\tilde{x}^{(m-1)}(n), \forall m \ne k \\ \Delta\tilde{x}^{(k)}(n) & = & Q_{N-k+2}\Delta\tilde{x}^{(k-1)}(n) + e_k(n) \\ \Delta\tilde{w}(n) & = & \Delta\tilde{x}^{(N)}(n) \\ \Delta\tilde{x}(n+1) & = & Q_1\Delta\tilde{w}(n) \\ \Delta\tilde{y}(n) & = & \rho_d \sin\phi_1 v_1^{\mathcal{T}}\Delta\tilde{w}(n) \end{array}\right.$$

which leads to

$$\left\{\begin{array}{rcl} \Delta x(n+1) & = & A\Delta x(n) + P_k e_k(n) \\ \Delta y(n) & = & C\Delta x(n) + R_k e_k(n) \end{array}\right.$$

where

$$P_k \triangleq \prod_{m=1}^{N-k+1} Q_m \tag{21}$$

and

$$R_k \triangleq \left\{\begin{array}{ll} \rho_d \sin\phi_1 v_1^{\mathcal{T}}, & k = N \\ \rho_d \sin\phi_1 v_1^{\mathcal{T}} \prod_{m=2}^{N-k+1} Q_m, & k \ne N \end{array}\right. \tag{22}$$

According to (15), the roundoff noise gain due to Q_k is

$$G_{Q_k} = tr(P_k^{\mathcal{T}} W_o P_k V_k + R_k^{\mathcal{T}} R_k V_k), \quad k = 2, \cdots, N \tag{23}$$

where

$$V_k \triangleq E[e_k(n)e_k^{\mathcal{T}}(n)] = 2(v_{m-1}v_{m-1}^{\mathcal{T}} + v_m v_m^{\mathcal{T}}), m = N-k+2$$

Substituting (21), (22) and $W_o = I$ into (23), it is interesting to note that $G_{Q_k} = 4$.

As to the first element of B and C, d presented in (11), it is easy to show that the corresponding roundoff noise gain is each equal to one.

The overall roundoff noise gain of our proposed structure is then given by

$$G_{new} = \sum_{k=2}^{N} G_{Q_k} + G_{Q_1} + 3 = 4N \tag{24}$$

IV. A Numerical Example

It is shown from [18] that a large range of Nth order IIR digital filter can be decomposed into the form of a parallel connection of two all-pass sub-filters, which can be expressed as $H(z) = \frac{1}{2}[H_1(z) + H_2(z)]$, where $H_k(z)$ is an N_kth order all-pass sub-filter for $k = 1, 2$ satisfying $N = N_1 + N_2$. Such a decomposition is denoted as S_{VMN}. In this case, the filter $H(z)$ yields the total roundoff noise gain $(\frac{1}{2})^2 G_1 + (\frac{1}{2})^2 G_2 + 1$, where G_k is the roundoff noise gain of H_k implemented with a structure.

In this section, we present a design example to illustrate the performance of our proposed structure R_{new}, and to compare it with other classical structures including the Hessenberg-form structure R_{HB}, the optimal roundoff noise realization R_{MRH}, the input balanced realization R_{IB}, For convenience, the structures implemented using two all-pass sub-filters in parallel are denoted as S_{VMN}^{MRH}, S_{VMN}^{IB}, S_{VMN}^{NL} and S_{VMN}^{HB}, respectively.

Example: This is a 7th order low-pass filter, generated using the MATLAB command $ellip(7, 0.25, 40, 0.2)$. The pass-band peak-to-peak ripple, the stop-band attenuation, and the normalized cut-off frequency are 0.25 dB, 40 dB, 0.1, respectively.

Based on the above approach mentioned, this filter is decomposed into two all-pass filters in parallel, i.e., $H(z) = \frac{1}{2}[H_1(z) + H_2(z)]$, where $H_1(z), H_2(z)$ are a 4th and a 3rd

TABLE II
STATISTICS FOR EACH OF THE FIVE STRUCTURES

$Structure$		R_{MRH}	R_{IB}	R_{HB}	R_{new}
G	$H(z)$	19.2149	26.0157	14.7041	–
	$H_1(z)+H_2(z)$	11.25	11.25	8.5734	8
N_{mul}	$H(z)$	$(N+1)^2$	$(N+1)^2$	$5N-1$	–
	$H_1(z)+H_2(z)$	41	41	33	28

TABLE I
THE COEFFICIENTS OF THE TWO ALL-PASS SUB-FILTERS FOR R_{new}.

	$H_1(z)$		$H_2(z)$	
m	$\sin\phi_k$	$\cos\phi_k$	$\sin\phi_k$	$\cos\phi_k$
1	0.7452	0.6669	0.7700	0.6380
2	0.4802	0.8772	0.3542	0.9352
3	0.1427	0.9898	0.4957	0.8685
4	-0.5729	0.8196	–	–

order all-pass sub-filters, respectively. TABLE I presents the coefficients of the two all-pass sub-filters for R_{new}. TABLE II shows the roundoff noise gain and the implementation complexity for the five structures, where G and N_{mul} are the roundoff noise gain and the number of multiplications using for computing each filter output sample for each structure, respectively.

Comments:

- It is observed from TABLE II that the proposed structure S_{VMN}^{new} yields the smallest roundoff noise, so is the implementation complexity.
- As can be seen from TABLE II, the system implemented using S_{VMN} based structure may reveal better performance than the corresponding structure implemented directly.

V. CONCLUSIONS

The main contribution of this paper is to derive a novel Hessenberg-based input balanced realization for all-pass system. Such a structure is sparse just having $4N$ multipliers. The expression of roundoff noise gain has been derived. A numerical example shows that our proposed structure outperform the others in terms of reducing roundoff noise and implementation complexity.

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