LAB 8: FREQUENCY DOMAIN IDENTIFICATION OF QNET DC MOTOR

```
%----------------------------------------
% Fill in your student Name and ID.
Students.Name = 'Hongtao Xu';
Students.ID = '260773785';
%----------------------------------------
```

0. Objectives

In this lab, we will use frequency analysis to identify the transfer function of the Virtual Motor. To initiate a connection with the virtual motor, run the following:

```
Motor = VirtualMotor();
```

1. Frequency Response Analysis

When a cosine wave $x(t) = Mx\cos(\omega t)$ is injected into a linear system, the system will respond with the same frequency ω, a certain magnitude Mx and a certain phase angle relative to the input ϕ. The steady-state system output can be written as $y(t) = My\cos(\omega t + \phi)$.

If $G(j\omega)$ is the frequency response of this system, we define the magnitude and phase at ω by:

$$M(\omega) = \frac{My}{Mx} = ||G(j\omega)||$$

$$\phi(\omega) = \angle G(j\omega)$$

1.1 Bode plot

The Bode plot presents the magnitude and the phase shift between the input and output at each frequency. It is possible to draw a `point-by-point` Bode plot by injecting a cosine wave with a known frequency, once at a time, and measuring the magnitude and phase shift of the output after it reaches its steady-state.

In a bode plot the magnitude is represented in a logarithmic scale in decibels (db).

$$Mdb(\omega) = 20\log(M(\omega))$$

The phase at each frequency can be found by measuring the temporal delay between the output and the input when the input is a cosine wave. Another way to measure the phase shift is through the Lissajous method. A Lissajous curve (an ellipsoid) is found when we try to plot the output in function of the input (XY plot). In fact, we can show that at a frequency ω

$$\cos(\phi) = \frac{y*}{My}$$

Where $y*$ is the value of the output y when the input x is at the maximum Mx.

An illustration of using the Lissajous method (taken from Wikipedia) is shown below:

<<fig_lab08_lissajousphase.png>>

Question 1 (2.5 mark)

- Define the following transfer function in **MATLAB**

$$G(s) = \frac{s+4}{s2+s+1}$$

```
G = tf([1 4],[1 1 1])
```

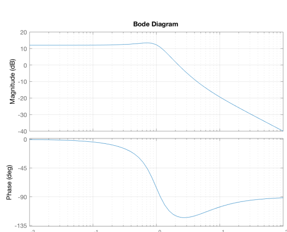
```
G =

      s + 4
  -----------
  s^2 + s + 1

Continuous-time transfer function.
```

- Plot the bode graph of this system. (Hint: help **bode**)

```
figure(1); clf;
% Plot Bode here
bode(G)
grid on
```



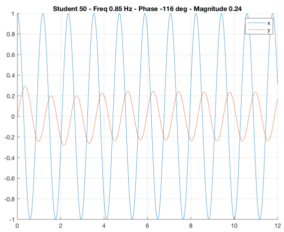
- Consider the following signal:

```
rng(datetime('now').Year*randi([1,20],1))
Freq = round(0.1+rand(1), 2);
t = 0:0.01:round(10/Freq);
```

```
x = cos(2*pi*Freq*t);
```

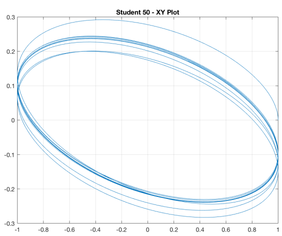
Using **lsim** simulate the output $y(t)$ representing the response of the system G to the signal $x(t)$.

```
y = lsim(G,x,t);
figure(2); clf;
title(sprintf('Student %d - Freq %3.2f Hz - Phase %+03d deg - Magnitude %4.2f',...
    Students.ID(1), Freq, round(180*angle(evalfr(G, 2*pi*Freq*1i))/pi), abs(evalfr(G, 2*pi*Freq*1i))));
hold on
plot(t, x)
plot(t, y)
grid on
legend('x', 'y')
```



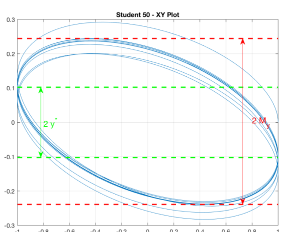
- Plot $y(t)$ in function of $x(t)$.

```
figure(3); clf;
% Plot here
plot(x,y)
title(sprintf('Student %d - XY Plot', Students.ID(1)));
grid on
```



- Find graphically the phase ϕ and magnitude M at the specified frequency.

```
showMyAndYStar(t, x, y, Freq);
```




```
y_star = (0.1068 + 0.1056)/2;
My = (0.2523 + 0.2472)/2;
% Write phase and magnitude here
% we calculate phi by the Lissajous method
phi = (acos(y_star/My) + pi) * 180/pi
```

```
phi = 244.8351
```

```
M = My
```

```
M = 0.2498
```

3. Identification of Qnet DC Motor Frequency Response

To avoid the non-linear domain of the DC Motor at low voltage we will stimulate the DC Motor with offsetted cosine waves. We choose the following voltage signal:

$$v(t) = 4 + 2\cos(2\pi f t)$$

To send a cosine wave of frequency $f = 0.5$ Hz to the Virtual Motor we do the following:

```
dt = 0.01; % sampling time
T = 15; % simulation duration
t = 0:dt:T; % define the time signal
vAmp = 2; % cosine amplitude
vOffset = 4; % voltage offset
Freq = 0.5; % frequency of cosine wave
```

```
Motor.reset();
Motor.input(vOffset + vAmp * cos(2*pi*Freq*t));
Motor.off();
Motor.save(Students, 'Lab08.Sinewave');
Motor.load('Lab08.Sinewave');
```

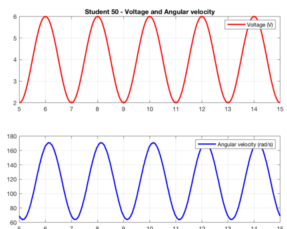
```
VirtualMotor::load::info: Recovering data collected by students: [260773785] .
```

For frequency analysis we are only interested at the steady-state response. We can retrieve the resulting angular velocity at steady state by:

```
delay = 5; % Ignore the first 5 seconds of experiment (transient response)
w = Motor.velocity(t >= delay);
v = Motor.voltage(t >= delay);
```

The following shows a plot of the temporal response.

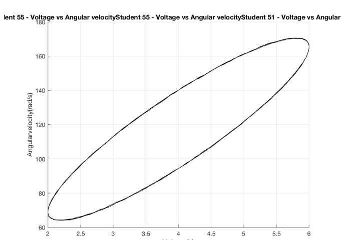
```
figure(4), clf;
subplot(2, 1, 1);
plot(t(t >= delay), v, 'r', 'linewidth', 2.0);
legend('Voltage (V)')
grid on
xlim([delay, T])
title(sprintf('Student %d - Voltage and Angular velocity', Students.ID(1)));
subplot(2, 1, 2);
plot(t(t >= delay), w, 'b', 'linewidth', 2.0);
legend('Angular velocity (rad/s)')
grid on
xlim([delay, T])
```



And the following shows the XY plot, which we can use to find $y*$, My and Mx.

```
w_filter = medfilt1(w, 10, 'truncate'); % we can filter the noise if needed

figure(5), clf; hold on;
title(sprintf('Student %d - Voltage vs Angular velocity', Students.ID));
plot(v, w_filter, 'k');
grid on
xlabel('Voltage (V)')
ylabel('Angularvelocity(rad/s)')
xlim([2 6])
```



Question 3 (3.0 marks)

Find the magnitude and phase of the transfer function of the Virtual Motor for the following frequencies.

```
FreqVect = [0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 2.0, 4.0, 6.0];
```

Measure graphically the phase and magnitude for each frequency and put it here. Note: The size of MagnitudeDB and PhaseDeg is the same as FreqVect.

```
% Answer here
Freq = 6.0
```

```
Freq = 6
```

```
%i get the frequency from freqvect

T = 2+5;
t = 0:dt:T;
% this is the duration time
% 5 is the delay time which we identify before

Motor.reset;
Motor.input(vOffset + vAmp * cos(2*pi*Freq*t));
Motor.off;
```

```
Motor.save(Students, ['Question3' num2str(Freq, '%3.2fHz')]);
% save the results
delay = 5;
v = Motor.voltage(t >= delay);
w = Motor.velocity(t >= delay);
% 5 is the delay as metioned in T
w_filter = medfilt1(w, 10, 'truncate');
figure(6); clf; set(gcf, 'Visible', 'on');
title(sprintf('Student %d - Voltage vs Angular velocity- Freq %3.2f Hz', Students.ID, Freq));
% name the title
plot(v,w)
grid on
xlabel('Voltage (V)')
ylabel('Angular velocity (rad/s)')
%plot the diagram
print(strrep(['XYPlot_Freq', num2str(Freq, '%03.2f'), 'Hz_Duration', num2str(T, '%02.1f'), 's'], '.', '_'), '-dp
%save the diagram
MagnitudeDB = [28.434 28.555 28.375 28.084 27.822 26.86 26.19 25.27 22.32 17.39 13.58]
```

```
MagnitudeDB = 1×11
   28.4340   28.5550   28.3750   28.0840   27.8220   26.8600   26.1900   25.2700   22.3200   17.3900   13.5800
```

```
PhaseDeg = [-11.00 -9.74 -14.223 -22.24 -37.4 -44.51 -53.66 -60.51 -54.97 -63.1 -33.647]
```

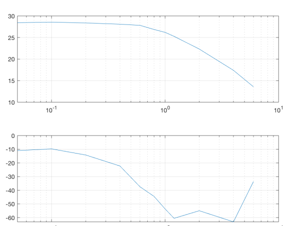
```
PhaseDeg = 1×11
  -11.0000   -9.7400  -14.2230  -22.2400  -37.4000  -44.5100  -53.6600  -60.5100  -54.9700  -63.1000  -33.6470
```

```
% i calculate the maganitude of DB from Freq, y and My;
% phasedegree =y/My
```

Question 4 (1.0 mark)

Draw the resulting Bode plot (Magnitude (dB) vs Frequency (Hz)) on a semilog plot and Phase (deg) vs Frequency (Hz) on a semilog plot). (Hint: help **`semilogx`**)

```
omega = 2 * pi * FreqVect;
figure(7);
clf;
subplot(2, 1, 1)
semilogx(FreqVect,MagnitudeDB)
grid on
subplot(2, 1, 2)
semilogx(FreqVect,PhaseDeg)
% i draw the both maganitude and phase here
grid on
```



Question 5 (1.0 mark)

From the Bode plot, measure the DC gain.

```
DC_gain = 28.43
```

```
DC_gain = 28.4300
```

Question 6 (1.0 mark)

From the Bode plot, measure the cut-off frequency (Frequency at which the magnitude drops by a factor of $-20\log 10\left(\frac{1}{\sqrt{2}}\right) = -3$ (dB) from the DC gain).

```
w_cutoff = 1.163*2*pi
```

```
w_cutoff = 7.3073
```

Question 7 (1.5 mark)

For a first order system the transfer function is expressed by

$$H(s) = \frac{DCgain}{1 + \frac{s}{\omega cutoff}}$$

- Write the transfer function of the identified DC Motor.

```
S = tf([DC_gain],[1/w_cutoff 1])
```

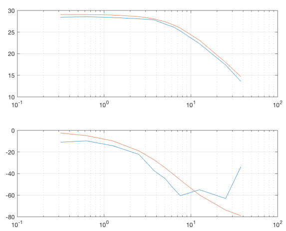
```
S =

     28.43
  ------------
  0.1368 s + 1

Continuous-time transfer function.
```

- Complete the code to superpose the experimental bode plot with the bode plot of the identified transfer function.

```
S_Freq(1,:) = (freqresp(S, omega, 'rad/s'));
S_MagnitudeDB = 20*log10(abs(H_Freq));
S_PhaseDeg = atan2(imag(S_Freq), real(S_Freq)) /pi * 180;
% define the magnitude in db and phase in degrees
figure(8)
subplot(211)
semilogx(omega, [MagnitudeDB; S_MagnitudeDB])
grid on
subplot(212)
semilogx(omega, [PhaseDeg; S_PhaseDeg])
grid on
```



- How accurate is the point-by-point bode plot for high frequencies? Comment on your results.

```
% from the magnitude plot, the point-by-point bode plot for high frequency is pretty accutare
% from the phase plot, it is not as accurate as the magnitude plot.
```

4. Useful Functions

4.1 Customize the XY plot

```
function showMyAndYStar(t, x, y, Freq)
max_y = max(y(t > round(5/Freq)));
max_x = max(x(t > round(5/Freq)));
min_y = min(y(t > round(5/Freq)));
min_x = min(x(t > round(5/Freq)));
star_y_1 = mean(y(t > round(5/Freq) & abs(x-max_x) < 0.05));
star_y_2 = mean(y(t > round(5/Freq) & abs(x-min_x) < 0.05));

gcf; hold on; axPos = get(gca, 'Position'); xMinMax = xlim; yMinMax = ylim;
plot([min_x, max_x], [max_y, max_y], '--r', 'linewidth', 2.0)
plot([min_x, max_x], [min_y, min_y], '--r', 'linewidth', 2.0)
annotation('doublearrow', [0.8, 0.8],...
    axPos(2)+(([max_y, min_y] - yMinMax(1)) / (yMinMax(2) - yMinMax(1)))*axPos(4), 'Color', 'red')
text(0.8, 0.0, '2 M_y', 'Color', 'red', 'FontSize', 14)
plot([min_x, max_x], [star_y_1, star_y_1], '--g', 'linewidth', 2.0)
plot([min_x, max_x], [star_y_2, star_y_2], '--g', 'linewidth', 2.0)
annotation('doublearrow', [0.2, 0.2],...
    axPos(2)+(([star_y_1, star_y_2] - yMinMax(1)) / (yMinMax(2) - yMinMax(1)))*axPos(4), 'Color', 'green')
text(-0.8, 0.0, '2 y^*', 'Color', 'green', 'FontSize', 14)
end
```