LAB 4: SIMULATION OF ANGULAR VELOCITY CONTROL FOR A DC MOTOR

```
%--------------------------------------
% Fill in your student Name and ID.
Students.Name = 'Hongtao Xu';
Students.ID = '260773785';
%--------------------------------------
```

0. Objectives

In this lab, we will run different simulation experiments to track a desired reference signal with a simulated DC Motor. We will first run an open-loop experiment, then we will use state feedback to control the angular velocity of DC motor.

1. A State-Space Representation of DC Motor

A DC motor is a device that converts armature current into mechanical torque. A schematic of a DC motor is shown in the following

<<fig_dcmotorshematic.png>>

We can model the DC motor using the armature current i and the angular speed ω. The dynamics of the DC motor are given by the following two coupled differential equations:

$$J\frac{d\omega}{dt} + b\omega = K_t i$$

$$L\frac{di}{dt} + Ri = v - K_e\omega$$

where the parameters are defined as follow

```
Param.Kt = 0.034; % Torque proportionality cons
Param.Ke = 0.034; % Back electromotive proporti
Param.L = 0.018; % Electric inductance (H)
Param.R = 8.4; % Terminal resistance (Ohm)
```

```
Param.J = 2e-05; % Rotor moment of inertia (kg.m
Param.b = 1e-06; % Rotor viscous friction consta
```

From these differential equations we can represent the DC Motor model by the following state-space representation

$$\frac{d}{dt}x(t) = \begin{pmatrix} -\frac{b}{J} & \frac{Kt}{J} \\ -\frac{Ke}{L} & -\frac{R}{L} \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ \frac{1}{L} \end{pmatrix} v(t)$$

where $v(t)$ is the voltage and the state vector contains the angular velocity $\omega(t)$ and the current $i(t)$

$$x(t) = \begin{pmatrix} \omega(t) \\ i(t) \end{pmatrix}$$

Question 1 (1 mark)

Write the state-space representation of the DC Motor (Only A and B matrices).

```
A = [1e-06/2e-05 0.034/2e-05; -0.034/0.018 -8.4/
```

```
A = 2×2
10^3 ×
    0.0000    1.7000
   -0.0019   -0.4667
```

```
B = [0 ; 1/0.018]
```

```
B = 2×1
         0
   55.5556
```

2. Desired specifications

The objective of this lab is to track a reference signal ωref while satisfying the following requirements:

- Overshoot less than 10%.
- 5%-settling time less than 0.2 seconds.

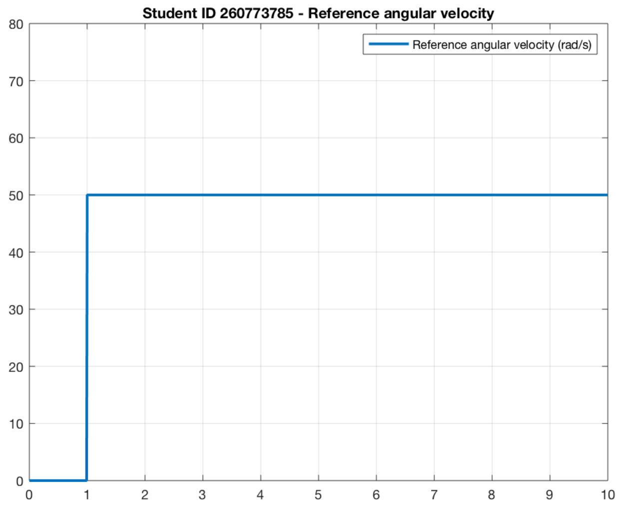
- Steady state gain of 1 $\pm$ 5%.

The Reference angular velocity is defined by the following:

$$\omega ref(t) = \begin{cases} 0, & 0 \leq t < 1 \\ 50, & t \geq 1 \end{cases}$$

In **MATLAB** we define ωref by:

```
T = 10; % Simulation duration
dt = 0.01; % Simulation step time
t = (0:dt:T)'; % Time array
TStart = 1; % Step start
wRef = zeros(size(t));
wRef(t >= TStart) = 50; % rad/s

figure;
clf;
plot(t, wRef, 'LineWidth', 2.0)
legend('Reference angular velocity (rad/s)');
xlim([0, T])
ylim([0, 80])
title(sprintf('Student ID %s - Reference angula
grid on
```



3. Open-loop Simulation Experiment

The state-space open-loop system is presented below.

<<fig_lab04_openloopdiagram.png>>

Note that if we think of the angular velocity $\omega(t)$ as the output of this system, then $C = (1\ 0)$ and

$$\omega(t) = Cx(t)$$

In this case, the transfer function from the voltage $v(t)$ to the angular velocity $\omega(t)$ is given by

$$G_O(s) = C(sI - A)^{-1}B$$

In **MATLAB**, we can find the transfer function of a state-space model using the function **ss** and **tf**, as shown below.

```
C = [1, 0];
D = 0;
A = [-Param.b/Param.J Param.Kt/Param.J; -Param.k
```

```
A = 2×2
10^3 ×
   -0.0000    1.7000
   -0.0019   -0.4667
```

```
B = [0; 1/Param.L]
```

```
B = 2×1
         0
   55.5556
```

```
Go = ss(A, B, C, D)
```

```
Go =


  A =
             x1       x2
   x1     -0.05     1700
   x2    -1.889   -466.7
```

```
tf(Go)
```

```
ans =


          9.444e04
```

```
--------------------
s^2 + 466.7 s + 3234
```

Question 2 (1 mark)

Recall that the DC gain can be obtained from a Laplace transfer function, by setting the laplace variable s to zero. From the state-space model representation (A,B,C,D), calculate the DC gain.

```
dcGainOL = C*inv(-A)*B
```

```
dcGainOL = 29.1996
```

Observe that if the open-loop system is a stable system. The DC Motor will be responded to a reference step signal without using a controller. However, the response will be determined by the eigenvalues of the open-loop A matrix (or equivalently, the poles of the open-loop transfer function).

Question 3 (1 mark)

Determine the poles of the open-loop transfer function from the state-space matrix A (Hint: **`help eig`**). Is the system stable? (Justify your answer.)

```
E = eigs(A)
```

```
E = 2×1
 -459.6804
   -7.0363
```

```
p1 = max(E)
```

```
p1 = -7.0363
```

```
p2 = min(E)
```

```
p2 = -459.6804
```

```
% Write your answer here.
```

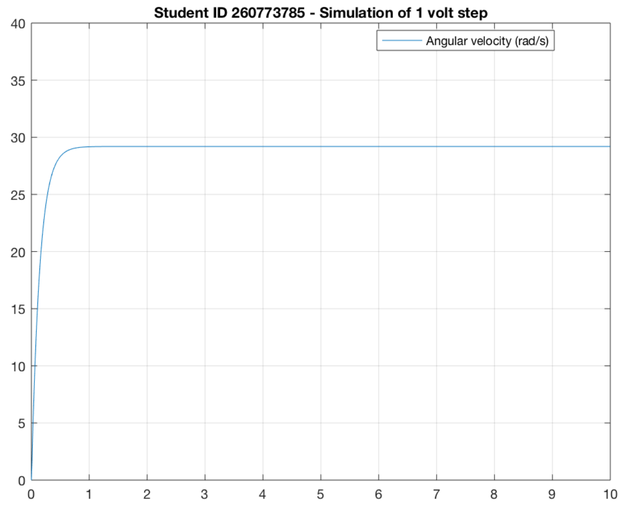
Following a fixed input $v0$ the angular velocity ω will attain a steady angular velocity $\omega\infty = DCgainv0$.

We can simulate the response of an LTI system to an arbitrary input using the **lsim** function.

Question 4 (0.5 mark)

Complete the following code to simulate the response of the system to a 1 volt input signal, and plot the results.

```
v = ones(size(t)); % create a signal with one vo
w = lsim(Go, v, t);

% Plot results
figure;
clf;
T = 10;
dt = 0.01;
t = (0:dt:T);
plot(t,w)
% [Answer here] plot the angular velocity w

legend('Angular velocity (rad/s)', ...
    'location', 'best');
xlim([0, T])
ylim([0, 40])
title(sprintf('Student ID %s - Simulation of 1 v
grid on
```



In open-loop we can generate the voltage v from ωref through the gain No.

$$v(t) = No\omega ref(t)$$

Question 5 (1 mark)

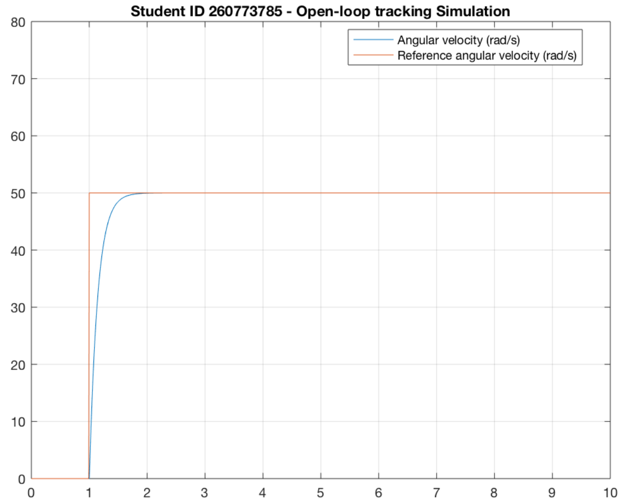
Find the value of No such the final value of the angular velocity output ω is the same as ωref (i.e. 50 rad/s)

```
% The voltage is generated from wRef and No
% [Answer here] Simulate the output angular velo
No = 1/dcGainOL
```

```
No = 0.0342
```

```
v = No * wRef;
w = lsim(Go, v, t);
% Plot results
figure;
clf;
plot(t,w)
hold on;
plot(t,wRef)
% [Answer here] Superpose the angular velocity w
legend('Angular velocity (rad/s)', ...
    'Reference angular velocity (rad/s)', ...
    'location', 'best');
xlim([0, T])
```

```
ylim([0, 80])
title(sprintf('Student ID %s - Open-loop trackir
grid on
```



Question 6 (0.5 marks)

Based on the plot above, determine if all the requirements are satisfied. Justify your answer.

Answer here:

From the diragram above, I can find out the angular velocity and wRef(angular velocity) have both the exact same initial velocity and final velocity. While there is no difference in the slope value from diagram 1 and 2. In this way, w Ref is the exact value as 50rad/s and it is a step funciton so that all the requirements are satisfied.

4. Closed-Loop Simulation Experiment

The closed-loop system is presented in the following figure.

<<fig_lab04_clsoedloopdiagram1.png>>

The full-state feedback control law is defined as follow

$$v(t) = N_c \omega_{ref} - Kx(t)$$

where gain K is a 1 by 2 matrix and the gain N_c is forward gain that will ensure achieving ω_{ref} at steady state.

Actually, this diagram is equivalent to an open-loop diagram but with a different state transition matrix $Acl = A - BK$

<<fig_lab04_clsoedloopdiagram2.png>>

Doing a state feedback with gain K is equivalent to changing the system dynamics from A to $A - BK$. Recall that the eigenvalues of a state transition matrix A are the poles of the system.

We want to choose K to specify the new poles of the closed-loop system $A - BK$ in a way that satisfies the desired specifications

- Overshoot less than 10%.
- 5%-settling time less than 0.2 seconds.

Then we will choose Nc similarly to the open-loop section to satisfy the steady state requirements:

- Steady state gain of 1 $\pm$ 5%.

Question 7 (2 marks)

Calculate the desired poles that satisfies the specifications (Use formula sheet for transient response of 2nd order system in myCourses if needed):

```
OT = 0.1
```

```
OT = 0.1000
```

```
Ts = 0.2
```

```
Ts = 0.2000
```

```
z=(-log(OT))/(sqrt((pi)^2+(log(OT))^2))
```

```
z = 0.5912
```

```
sigma1=3/Ts
```

```
sigma1 = 15
```

```
Wn=sigma1/z
```

```
Wn = 25.3741
```

```
Wd=Wn*(sqrt(1-(z)^2))
```

```
Wd = 20.4656
```

```
%those are continous time funciton
s=tf('s')
```

```
s =

  s

Continuous-time transfer function.
```

```
D1=s^2+2*z*s+(Wn)^2
```

```
D1 =

  s^2 + 1.182 s + 643.8

Continuous-time transfer function.
```

```
% here we can find out the poles
p1d=-sigma1+Wd*i
```

```
p1d = -15.0000 + 20.4656i
```

```
p2d=-sigma1-Wd*i
```

```
p2d = -15.0000 - 20.4656i
```

Instead of manual calculation of the value of the gain K that place the poles of $A - BK$ at [p1d, p2d], we will use the **`place`** command to calculate the gain K

```
K = place(A, B, [p1d, p2d]);
```

Question 8 (1 mark)

The closed-loop transfer function is

$$G_C(s) = C(sI - (A - BK))^{-1}B$$

1. What is the value of the DC gain of the closed-loop system.
2. What is the value Nc needed to compensate for the DC gain of the closed-loop system.

```
dcGainCL=C*(-inv(A-B*K))*B
```

```
dcGainCL = 146.6887
```

```
Nc=1/dcGainCL
```

```
Nc = 0.0068
```

Question 9 (1 mark)

Complete the following to simulate angular velocity control with state feedback that tracks the desired ωref .

[Answer here] Define the closed-loop system using ss

```
Gc = ss(A-B*K, B, C, D)
```

```
Gc =

  A =
              x1        x2
   x1      -0.05      1700
   x2    -0.3779    -29.95
```

```
stepinfo(Gc)
```

```
ans = struct with fields:
        RiseTime: 0.0723
    SettlingTime: 0.2335
     SettlingMin: 132.3905
     SettlingMax: 161.3576
       Overshoot: 10.0000
      Undershoot: 0
            Peak: 161.3576
        PeakTime: 0.1535
```

```
w = lsim(Gc, v, t)
```

```
w = 1001×1
     0
     0
     0
     0
     0
     0
```

```
% We also simulate the voltage for visualizatior
v = wRef * Nc
```

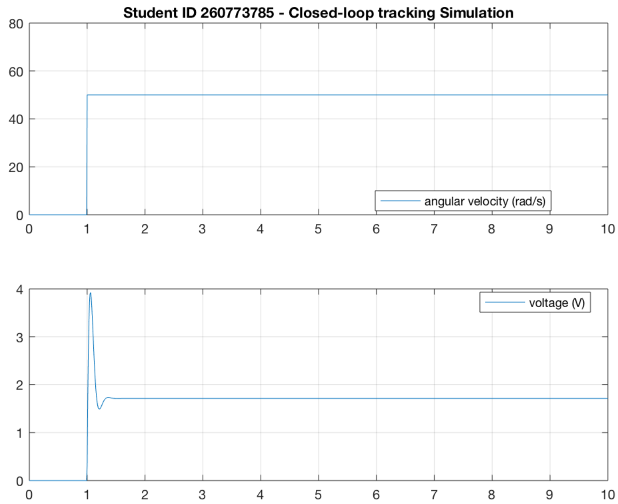
```
v = 1001×1
     0
     0
     0
     0
     0
     0
```

```
v=-lsim(ss(Gc.A,Gc.B,eye(2),Gc.D),Nc*wRef,t)*K'-

figure;
clf;

subplot(2, 1, 1)
plot(t, w)
plot(t, wRef)
% [Answer here] Superpose angular velocity w wit
% reference

legend('angular velocity (rad/s)', ...
    'Reference angular velocity (rad/s)', ...
    'location', 'best');
```

```
Warning: Ignoring extra legend entries.
```

```
xlim([0, T])
ylim([0, 80])
grid on
title(sprintf('Student ID %s - Closed-loop track
```

```
subplot(2, 1, 2)
plot(t,v)
legend('voltage (V)', 'location', 'best');
xlim([0, T])
grid on
```



Question 10 (1 marks)

Based on the plot above, determine if all the requirements are satisfied. Justify your answer.

```
% From the diagram above, I can see
% overshoot time for Gc is about 9.9%
% which is pretty close to 10%
% as required. Also, from that diagram
% Ts is about 0.2 s and angular
% frequency is about 50 rad/s,
% in this way, all the
% requirements are satisfied.
```