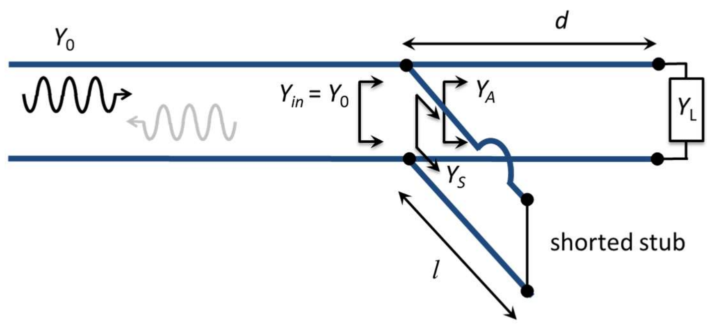
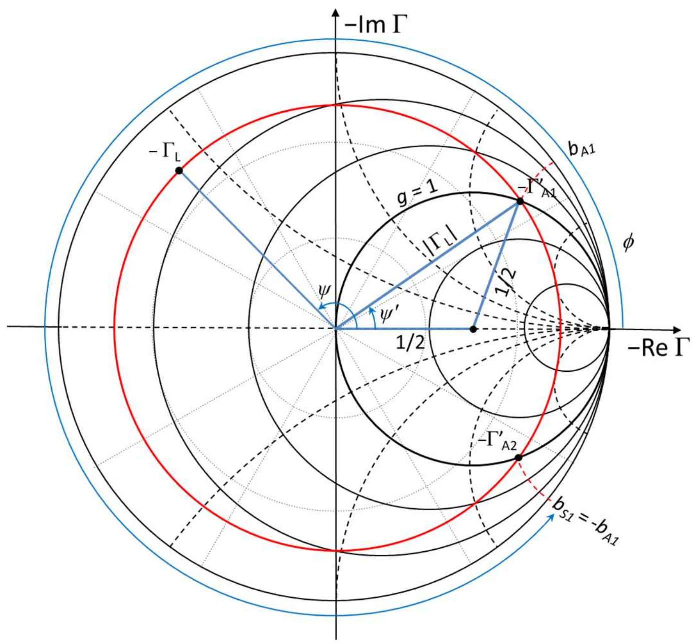
Laboratory 4: Impedance matching with a single stub circuit - design

In this laboratory, we will implement a code to design a single stub impedance matching circuit, and then analyze it's frequency dependent behaviour. The parameters of the single stub circuit are defined in the diagram below, $Y_0 = 1/Z_0$ the characteristic admittance of the lossless transmission line, $Y_S = 1/Z_S$ the input admittance of the shorted stub of length l, and $Y_A = 1/Z_A$ is the input admittance of the line with the load admittance $Y_L = 1/Z_L$.



The impedance matching condition is: $y_{\rm in} = 1 = y_{\rm A} + y_{\rm S} \quad \therefore \quad y_{\rm A} = 1 + jb_{\rm A} \qquad y_{\rm S} = jb_{\rm S} = \text{-}jb_{\rm A}$

For this problem it will be useful to use the Smith chart in admittance mode, shown below. Recall that the short circuit ($-\Gamma = 1$) is at the right hand side of this chart.



For our analysis it will be useful to make use of the following relations for admittance towards the load:

$$\text{-}\Gamma_{\rm L} = \frac{y_L - 1}{y_L + 1} = |\Gamma_{\rm L}|\exp(j\psi) \qquad \text{-}\Gamma'_{\rm A} = |\Gamma_{\rm L}|\exp(j\psi') \qquad y_{\rm A} = \frac{1 + (\text{-}\Gamma'_{\rm A})}{1 - (\text{-}\Gamma'_{\rm A})} = g_{\rm A} + jb_{\rm A}$$

Simple geometry of the triangle can be used to locate the intersection with $g_A = 1$: $\cos\psi' = |\Gamma_{\rm L}|$

The rotated reflection coefficient for the shorted stub is:

$$\text{-}\Gamma'_{\rm S} = \frac{(\text{-}jb_{\rm A}) - 1}{(\text{-}jb_{\rm A}) + 1} = \exp(j\phi)$$

Finally, the lengths can be inferred from the phase of the reflection coefficients

$$\frac{-\Gamma_L}{-\Gamma_A} = \exp[j(\psi - \psi')] = \exp(j2\beta d) \qquad \frac{1}{-\Gamma'_S} = \exp[j(2\pi - \phi)] = \exp(j2\beta l)$$

where β is the phase constant of the lossless transmission line sections.

1. Single shorted stub impedance matching circuit design

Write a function that calculates the lengths l and d for the single shorted stub (reporting the solution with shortest length d) given the normalized load impedance $z_L = Z_L / Z_0$ and the phase constant β,

```
[ l, d ]=shortedstubdesign(zL,beta)
```

You are free to design the flow your function however you wish. An example flow is as follows:

1. Calculate the normalized load admittance y_L.
2. Calculate the negative reflection coefficient for the load $-\Gamma_L$. It might be useful to work directly with the negative reflection coefficient, denoting it with a variable `nGammaL`.
3. Calculate the angle ψ of the reflection coefficient.
4. Calculate the angle ψ' of the rotated reflection coefficient. Choose the value of ψ' that minimizes $\psi - \psi'$.
5. Calculate the length d.
6. Calculate the rotated negative reflection coefficient $-\Gamma'_A$ corresponding to the selected ψ'.
7. Calculate the admittance y_A.
8. Calculate the shorted stub negative rotated reflection coefficient $-\Gamma'_S$.
9. Calculate the angle ϕ.
10. Calculate the length l.

You may find it useful to use the *modulo* function to fold phase angles into the interval [0,2π) using the MATLAB command `theta_new = mod(theta_old,2*pi).`

Test your function using a normalized load impedance $z_L = 0.70 - j0.95$ and phase constant $\beta = 2\pi$ rad/m. You should find $l = 0.111$ m and $d = 0.059$ m.

№ 1: Show your results to the teaching assistant.

Optional:

Can you develop a code for designing a matching circuit using a single *open-circuit* stub?

Laboratory 5: Impedance matching with a single stub circuit - analysis

1. Calculating the frequency response of a single shorted stub circuit

Write a function that calculates the reflection coefficient for a single stub circuit with lengths l and d, normalized load impedance z_L, and phase constant β,

```
[ Gamma ]=shortedstubresponse(l,d,zL,beta)
```

You are free to design the flow your function however you wish. An example flow is as follows:

1. Calculate the load reflection coefficient Γ_L.
2. Calculate the rotated load reflection coefficient Γ'_L.
3. Calculate the admittance y_A towards the load. write equations
4. Calculate the rotated reflection coefficient Γ'_S for the short circuited stub.
5. Calculate the admittance y_S towards the short.
6. Calculate the total admittance $y_{in} = y_A + y_S$.
7. Calculate the reflection coefficient Γ.

Test your function using the example of the previous laboratory, with normalized load impedance $z_L = 0.70 - j0.95$, phase constant $\beta = 2\pi$ rad/m and lengths $l = 0.111$ m and $d = 0.059$ m. You should find a reflection coefficient $\Gamma = 0$, corresponding to the matched condition (with possible deviation due to numerical rounding error).

№ 1: Show your results to the teaching assistant.

2. Analyzing the frequency response of a single shorted stub circuit

Design the lengths l and d of a single shorted stub circuit for a normalized load impedance $z_L = 0.4$, with a lossless transmission line of phase velocity $v_p = 2\text{x}10^8$ m/s and an operating frequency $f_0 = 1$ GHz (from which you can determine the phase constant β).

Calculate the reflection coefficient Γ versus frequency f for your single shorted stub circuit with the lengths l and d that you have designed. Take a frequency range $f = 1$ MHz to 2 GHz in steps of $\Delta f = 1$ MHz. Note that the phase "constant" β depends upon frequency.

Plot the reflection coefficient Γ on a Smith chart, using the MATLAB command `s = smithplot(f,Gamma)`. Inspect your Smith chart, which is presented in the impedance mode.

Plot the magnitude of the reflection coefficient $|\Gamma|$ versus frequency f.

Is impedance matching achieved at the design frequency $f = 1$ GHz ?
In the low frequency limit $f \rightarrow 0$, what is the input impedance? Does this agree with what you expect at dc?
The reflection coefficient $\Gamma = -1$, equivalent to that of a short circuit, at a frequency $f > 1$ GHz. Why?

№ 2: Show your results to the teaching assistant.

Repeat the exercise, and design a single shorted stub circuit for matching a load z_L = 0.04. Calculate the reflection coefficient Γ versus frequency *f*. Take a frequency range *f* = 1 MHz to 2 GHz in steps of Δ*f* = 1 MHz. Plot the reflection coefficient Γ on a Smith chart, and the magnitude of the reflection coefficient $|\Gamma|$ versus frequency *f*.

What do you notice about the frequency dependence of $|\Gamma|$ near the design frequency as compared to the single stub circuit with z_L = 0.4 ?

№ 3: Show your results to the teaching assistant.