Laboratory 6: Wave propagation in lossy media

In this laboratory, we will explore wave propagation in lossy media, including liquid water and silicon.

For a medium with complex permittivity $\epsilon_e = \epsilon' - j\epsilon''$ and permeability $\mu = \mu_0$, the propagation constant γ and the intrinsic impedance η are given by:

$$\gamma = \alpha + j\beta = j\omega\sqrt{\epsilon_e \mu} \qquad \eta = \sqrt{\frac{\mu}{\epsilon_\mathrm{e}}}$$

As you will soon learn, the loss tangent $\tan(\delta_D)$ is a useful parameter for describing the degree of loss per unit wavelength of propagation distance, and is given by,

$$tan(\delta_D) = \frac{\epsilon''}{\epsilon'}$$

Recall further that the wavelength in the medium is given by $\lambda = 2\pi/\beta$ and the attenuation per unit length in [dB/m] is given by $A/\ell = 8.69\ \alpha$.

1. Wave propagation in pure water

The normalized complex permittivity $\epsilon(f)/\epsilon_0$ of pure water is very well modelled by the function,

$$\frac{\epsilon(f)}{\epsilon_0} = \epsilon(\infty) + \frac{\epsilon(0) - \epsilon(\infty)}{1 + j2\pi f\tau}$$

over the frequency range 0 Hz < f < 300 GHz where the parameters $\epsilon(\infty) = 5.2$, $\epsilon(0) = 78.3$ and $\tau = 8.3\ ps$ at T = 25 °C. The origin of this behaviour is the orientational response of water molecules to electric fields. The permeability $\mu = \mu_0$.

Write a function that calculates the normalized complex permittivity $\epsilon(f)/\epsilon_0$ of pure water for a frequency f:

```
[ eps_norm_H20 ]=purewatereps(f)
```

Calculate the complex permittivity over a frequency range 1 kHz < f < 300 GHz with frequency distributed on a logarithmic scale, as setup for example by `f = 10.^[3:0.05:log10(300e9)]` . Plot the normalized real component, ϵ'/ϵ_0, and the normalized negative imaginary component, ϵ''/ϵ_0, of the normalized permittivity using a logarithmic frequency scale, using `semilogx(f,real(eps_norm_H20),f,-imag(eps_norm_H20))` for example.

Calculate and plot (on logarithmic frequency scale) the loss tangent $\tan(\delta_D)$.

Is the “dielectric constant” really a constant?
At what frequency is ϵ''/ϵ_0 a maximum?

№ 1: Show your results to the teaching assistant.

Calculate the propagation constant $\gamma \ = \alpha + j\beta$ versus frequency *f*.

Calculate the attenuation per unit length A/ℓ in [dB/m] versus frequency *f* and plot on a log-log scale. This can be done using the command line `loglog(f,norm_lambda)` for example.

Calculate the wavelength ratio λ/λ_0 versus frequency *f*, where λ is the wavelength in pure water and λ_0 is the wavelength in vacuum. Plot λ/λ_0 versus the logarithmic frequency scale.

Review your plots carefully.

How should one choose frequency *f* if one wants to transmit an electromagnetic wave with minimum attenuation through pure water?
At what frequency is the attenuation per unit length A/ℓ = 3 dB/m ?
Is the wavelength λ in pure water larger or smaller than the wavelength λ_0 in vacuum at *f* = 1 GHz ? In other words, does pure water shorten or lengthen the wavelength of an EM wave in the microwave frequency range?

№ 2: Show your results to the teaching assistant.

2. Wave propagation in sea water

Outside of a strictly controlled laboratory environment, water contains ions from dissolved salts of varying concentration. The normalized complex permittivity $\epsilon(f)/\epsilon_0$ of sea water is very well modelled by the function,

$$\frac{\epsilon(f)}{\epsilon_0} = \epsilon(\infty) + \frac{\epsilon(0) - \epsilon(\infty)}{1 + j2\pi f\tau} + \boxed{\frac{\sigma/\epsilon_0}{j2\pi f}}$$

over the frequency range 0 Hz < *f* < 300 GHz where the parameters $\epsilon(\infty) = 5.2$, $\epsilon(0) = 78.3$ and $\tau = 8.3\,ps$ at the temperature *T* = 25 °C, as before, and the conductivity $\sigma = 4$ S/m for typical ion concentrations in salt water. Note that sea water differs from pure water by the addition of ionic conductivity, seen in the last term. The permeability $\mu = \mu_0$.

Write a function that calculates the normalized complex permittivity $\epsilon(f)/\epsilon_0$ of sea water for a frequency *f*:

```
[ eps_norm_salt_H20 ]=seawatereps(f)
```

Calculate the complex permittivity over a frequency range 1 kHz < *f* < 300 GHz with frequency distributed on a logarithmic scale, as in the previous exercise.

Plot the real component ϵ'/ϵ_0 and the negative imaginary component ϵ''/ϵ_0 of the normalized permittivity versus frequency *f*. You may want to plot the permittivity components on a log-log scale due to the divergence in ϵ''/ϵ_0 at low frequencies.

Calculate and plot (on logarithmic frequency scale) the loss tangent $\tan(\delta_D)$.

Why is $\epsilon''/\epsilon_0 \propto 1/f$ at “low frequencies” *f* ?
What is the approximate range of frequencies $\epsilon''/\epsilon_0 \propto 1/f$?

№ 3: Show your results to the teaching assistant.

Calculate the propagation constant $\gamma = \alpha + j\beta$ versus frequency *f*.

Calculate the attenuation per unit length A/ℓ in [dB/m] versus frequency *f* and plot on a log-log scale.

Calculate the wavelength ratio λ/λ_0 versus frequency *f*, where λ is the wavelength in pure water and λ_0 is the wavelength in vacuum. Plot λ/λ_0 versus frequency. A log-log scale will be useful here.

Review your plots carefully.

How should one choose frequency *f* if one wants to transmit an electromagnetic wave with minimum attenuation through sea water?
At what frequency is the attenuation per unit length A/ℓ = 3 dB/m ?
Which medium, pure water or sea water, has a greater attenuation per unit length A/ℓ ? Does your answer depend on frequency?
What is the wavelength λ in sea water at *f* = 1 kHz ? How does this compare to the wavelength λ_0 in vacuum at *f* = 1 kHz ?

№ 4: Show your results to the teaching assistant.

Optional: Fresh water suitable for aquatic life has a typical conductivity $\sigma = 0.02$ S/m. How does the attenuation versus frequency of fresh water compare to sea water?

Consider the implications of your results for the interaction of electromagnetic waves with saline water, in the context of communications, radar, microwave heating and biomedical imaging.