Laboratory 2 : Reflection on a Transmission Line

In this laboratory, we will develop several simple functions to understand the influence of the reflection coefficient Γ arising from a load impedance Z_L at $z = 0$ on the voltage and current along a transmission line.

1. Phasor voltage and current in the presence of reflection

Let us begin with a phasor description of the problem. Write a function that calculates the voltage phasor $V_s(z)$ and the current phasor $I_s(z)$ on a transmission line in the presence of an incident wave and a reflected wave using the same convention as in the lecture notes (load impedance Z_L located at $z = 0$),

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[ Vs Is ]=linephasor(V0,Gamma,gamma,Z0,z)
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as defined by the following equations,

$$V_s(z) = V_0 \exp(-\gamma z) + \overset{\text{Gamma}}{\Gamma} V_0 \exp(+\gamma z) \qquad\qquad I_s(z) = \frac{V_0}{Z_0}\exp(-\gamma z) - \Gamma\,\frac{V_0}{Z_0}\exp(+\gamma z)$$

where V_0 is the complex voltage phasor amplitude of the incident wave, γ is the phase constant of the line (where $\gamma = \alpha + j\,\beta$ in general), and Z_0 is the characteristic impedance of the transmission line.

2. Phasor magnitude on a lossless transmission line with resistive loads

Consider first a lossless transmission line with Z_0 = 100 Ω and $\gamma = j\ 2\pi$ rad/m, excited with an incident wave amplitude V_0 = 1 V. Restrict your attention to the length -2 m $\le z \le 0$ m. Plot the absolute value of the voltage and current phasors, $|V_s(z)|$ and $|I_s(z)|$, versus position z on the same figure in different colour so that you can compare both. You will find it useful to plot 100 x $|I_s(z)|$ such that the maxima in voltage and current are of comparable size in your figure. When reading your figures, take care that the units for $|V_s(z)|$ and $|I_s(z)|$ are different. Consider the following scenarios for the reflection coefficient, plotting a separate figure for each:

a) Γ = 0, corresponding to a matched load $Z_L = Z_0$ = 100 Ω.
b) Γ = +1/3, corresponding to the resistive load $Z_L = 2Z_0$ = 200 Ω.
c) Γ = +2/3, corresponding to the resistive load $Z_L = 5Z_0$ = 500 Ω.
d) Γ = +1, corresponding to the open circuit load $Z_L \to \infty$.

Do you observe the expected trend for the voltage standing wave ratio (VSWR) for a) to d) ? Recall that VSWR can be expressed as $s = \max|V_s(z)|/\min|V_s(z)|$.

Is there a need to define an independent quantity called the current standing wave ratio? Or would this quantity be identical to the VSWR?

Calculate the ratios $V_s(z{=}0)$ / $I_s(z{=}0)$ for a) to d). How do the ratios compare with the load impedances Z_L ?

№ 1: Show your results to the teaching assistant.

3. Phasor magnitude on a lossless transmission line with inductive loads

Use the same transmission line parameters and incident wave amplitude as above. Plot the absolute value of the voltage phasors $|V_s(z)|$ and $|I_s(z)|$ for the following four additional scenarios for the reflection coefficient:

e) $\Gamma = \exp(j\pi/4)$, corresponding to the inductive load $Z_L \approx 2.4j\, Z_0 = 240\, j\, \Omega$.
f) $\Gamma = \exp(j\pi/2) = +j$, corresponding to the inductive load $Z_L = j\, Z_0 = 100\, j\, \Omega$.
g) $\Gamma = \exp(j\, 3\pi/4)$, corresponding to the inductive load $Z_L \approx 0.41j\, Z_0 = 41\, j\, \Omega$.
h) $\Gamma = \exp(j\, 4\pi/4) = -1$, corresponding to the short circuit load $Z_L = 0\, \Omega$.

Comparing the sequence d) to h), what is changing in $|V_s(z)|$ and $|I_s(z)|$?

Calculate the ratios $V_s(z=0) / I_s(z=0)$ for e) to h). How do the ratios compare with the load impedances Z_L ?

№ 2: Show your results to the teaching assistant.

You may wish to continue the exercise, and investigate what happens for capacitive loads, and more general load impedances of the form $Z_L = R + j\,X$.

4. Phasor magnitude on a lossy transmission line with matched load

Consider now a transmission line with loss with a characteristic impedance $Z_0 = 100\, \Omega$ *, excited with an incident wave amplitude $V_0 = 1$ V, and terminated with a matched load $Z_L = Z_0$ such that $\Gamma = 0$. Restrict your attention to the length $-2 \text{ m} \le z \le 0$ m. Plot the absolute value of the voltage and current phasors $|V_s(z)|$ and $|I_s(z)|$ for the following scenarios of transmission line propagation constant:

a) α = 0.050 Np/m and $\beta = 2\pi$ rad/m.
b) α = 0.347 Np/m and $\beta = 2\pi$ rad/m.
c) α = 0.693 Np/m and $\beta = 2\pi$ rad/m.
d) α = 2 Np/m and $\beta = 2\pi$ rad/m.

How is the voltage $V(z=0)$ at the load affected by the increased attenuation in the sequence a) to d)?

Are the ratios $V_s(z=0) / I_s(z=0)$ equal to the load impedance $Z_L = Z_0$?

№ 3: Show your results to the teaching assistant.

* While Z_0 is in general a complex quantity for a transmission line with loss, it is possible to tune the parameters R, L, G, C of a transmission line with loss such that Z_0 is real. This is called a *dispersionless transmission line*. Can you derive the conditions for the dispersionless transmission line?

5. Phasor magnitude on a lossy transmission line with an open circuit load

Repeat exercise 4 above, now with an open circuit load such that Γ = +1. Plot the absolute value of the voltage and current phasors $|V_s(z)|$ and $|I_s(z)|$ for the same scenarios of transmission line propagation constant:

a) α = 0.050 Np/m and β = 2π rad/m.
b) α = 0.347 Np/m and β = 2π rad/m.
c) α = 0.693 Np/m and β = 2π rad/m.
d) α = 2 Np/m and β = 2π rad/m.

How is the standing wave component of the voltage on the line affected by the increased attenuation in the sequence a) to d)?

№ 4: Show your results to the teaching assistant.

6. Instantaneous voltage and current on a lossless transmission line with reflections

Let us now consider the behaviour of the instantaneous voltage $v_s(z,t)$ and current $i_s(z,t)$ on a lossless transmission line. This can be done by first calculating the phasor representation of the harmonic waves as you have done in the preceding exercises, and then converting the phasors to instantaneous time representations using the `ph2inst` function that you developed in laboratory 1, exercise 4 for precisely this purpose.

Assume an incident wave amplitude V_0 = 1 V, angular frequency $\omega = 2\pi \times 200 \times 10^6$ rad/s, characteristic impedance Z_0 = 100 Ω, and phase constant β = 2π rad/m. Restrict your attention to the length –2 m ≤ z ≤ 0 m and the time interval 0 s ≤ t ≤ 20 ns.

Make movies of the instantaneous voltage $v_s(z,t)$ and current $i_s(z,t)$ for the following three scenarios:

a) Γ = +2/3, corresponding to the resistive load $Z_L = 5Z_0$ = 500 Ω.
b) Γ = +1, corresponding to the open circuit load $Z_L \rightarrow \infty$.
c) $\Gamma = \exp(j\pi/2) = +j$, corresponding to the inductive load $Z_L = j Z_0 = 100 j$ Ω.

Carefully observe the simultaneous travelling wave components and standing wave components in the voltage and current of scenario a).

Carefully observe the standing waves in the instantaneous voltage and current of scenarios b) and c). What difference do you observe at the load (z = 0) between open circuit and inductive load ?

№ 5: Show your results to the teaching assistant.