at low F, its capacitive
at high F, its inductive

# Laboratory 3 : Smith Chart

In this laboratory, we will develop several simple functions relating the load impedance $Z_L$, reflection coefficient $\Gamma$ and input impedance $Z_{in}$ for a lossless transmission line with characteristic impedance $Z_0$, phase constant $\beta$ and length $l$.

### 1. Reflection coefficient and input impedance

Write a function that calculates the reflection coefficient $\Gamma$ from the normalized load impedance $z_L = Z_L / Z_0$,

```
[ Gamma ]=refcoeff(zL)
```

a function that calculates a rotated (phase shifted) reflection coefficient $\Gamma'$ from the reflection coefficient $\Gamma$ and round-trip phase $\theta = 2\beta l$,

```
[ Gammarot ]=rotrefcoeff(Gamma,theta)
```

and a function that calculates the normalized input impedance $z_{in} = Z_{in} / Z_0$ from the reflection coefficient $\Gamma$ and round-trip phase $\theta = 2\beta l$,

```
[ zin ]=inputZ(Gamma,theta)
```

as defined by the following equations,

$$\Gamma = \frac{z_L - 1}{z_L + 1} \qquad \Gamma' = \Gamma \exp(-j\theta) \qquad z_{in} = \frac{1+\Gamma'}{1-\Gamma'} = \frac{1+\Gamma\exp(-j\theta)}{1-\Gamma\exp(-j\theta)} \qquad \theta = 2\beta l$$

Alternatively, you might wish to calculate input impedance $z_{in}$ directly from the rotated coefficient $\Gamma'$. In the following, you may find it useful to recall,

$$\beta = \frac{2\pi f}{v_p}$$

For all of the calculations that follow, consider the case of a lossless transmission line with a characteristic impedance $Z_0$ = 75 Ω, phase velocity $v_p = 2 \times 10^8$ m/s, and length $l$ = 0.25 m.

Take a frequency range $f$ = 1 MHz to 300 MHz in steps of $\Delta f$ = 1 MHz.

**2. Reflection coefficient of RC, RL and RLC circuits on a Smith Chart**

how to draw ??

Consider three load impedances:

a) series RC circuit, $Z_L = R + 1/j\omega C$, with $R$ = 75 Ω and $C$ = 100 pF none
b) series RL circuit, $Z_L = R + j\omega L$, with $R$ = 75 Ω and $L$ = 200 nH upper
c) series RLC circuit, $Z_L = R + 1/j\omega C + j\omega L$, with $R$ = 75 Ω, $L$ = 200 nH and $C$ = 100 pF origin
d) parallel RLC circuit, $Z_L = 1/(\,1/R + 1/\,j\omega L + j\omega C\,)$, with $R$ = 75 Ω, $L$ = 200 nH and $C$ = 100 pF

Calculate the reflection coefficient $\Gamma$ for the frequency range specified above, and plot $\Gamma$ in the complex plane on a Smith chart for each of the load impedances. This can be conveniently done using a MATLAB command from the RFToolbox, `s = smithplot(f,Gamma)` where `s` is the handle for your Smith chart. You can add a title with the commend `s.TitleTop = 'RC circuit'` for example. The values of $\Gamma$ can be read explicitly by floating your cursor over the data point of interest.

Inspect your Smith charts. Notice that $|\Gamma|$, and the VSWR $s$, varies with frequency for these load impedances.

Why does the reflection coefficient $\Gamma$ for the series RC circuit follow the $r$ = 1 locus in the lower half plane?
Why does the reflection coefficient $\Gamma$ for the series RL circuit follow the $r$ = 1 locus in the upper half plane?
Why does the reflection coefficient $\Gamma$ for the series RLC circuit follow the $r$ = 1 locus?
At what frequency $f$ is the series RLC circuit impedance matched, $\Gamma$ = 0, with the transmission line? What is the reactive component of the load impedance, $x_L = \mathrm{Re}\{z_L\}$, when the impedance matching condition is satisfied?

**№ 1: Show your results to the teaching assistant.**

Optional: Why does the reflection coefficient $\Gamma$ for the parallel RLC circuit follow the locus of constant normalized conductance, $g = \mathrm{Re}\{Z_0/Z_L\} = 1$.

**3. Reflection and input impedance of transmission line with a resistive load**

Consider a resistive load impedance $Z_L$ = 15 Ω. Calculate the reflection coefficient $\Gamma$, the rotated reflection coefficient $\Gamma'$ and the normalized input impedance $z_{in}$ for the frequency range specified above.

Plot the rotated reflection coefficient $\Gamma'$ in the complex plane on a Smith chart, using for example the MATLAB command `s = smithplot(f,Gammarot)`. Plot the real and imaginary parts of the normalized input impedance $z_{in}$ versus frequency. This can be done simply using `h = plot(f,real(zin),f,imag(zin))` for example.

plot ???

Inspect your Smith chart and your impedance versus frequency plots.

Where on your Smith chart are the points corresponding to low frequency operation ( $f$ = 1 MHz ) and high frequency operation ( $f$ = 300 MHz ) ?
At what frequency is the transmission line acting as a ¼ wave transformer?
Use your Smith chart to determine the length of the transmission line in terms of wavelengths at $f$ = 300 MHz, by determining the rotation angle $\theta = 2\beta l$ at $f$ = 300 MHz, and then calculating $l\,/\,\lambda(300\text{ MHz}) = \beta l/2\pi = \theta/4\pi$. Does this agree with direct calculation of $l\,/\,\lambda(300\text{ MHz}) = l \times 300\text{ MHz}\,/\,v_p$ ?
Use your Smith chart to determine the VSWR.

**№ 2: Show your results to the teaching assistant.**

**4. Reflection and input impedance of transmission line with an inductive load**

Consider an inductive load impedance $Z_L = j\omega L$ with $L$ = 20 nH. Calculate the reflection coefficient $\Gamma$, the rotated reflection coefficient $\Gamma'$ and the normalized input impedance $z_{in}$ for the frequency range specified above.

Plot the rotated reflection coefficient $\Gamma'$ in the complex plane on a Smith chart. Plot the real and imaginary parts of the normalized input impedance $z_{in}$ versus frequency. Restrict your impedance figure axes to normalized impedance components within the range ±15.

What is the minimum frequency $f$ required to achieve a capacitive ( $x_{in} < 0$ ) input impedance $z_{in}$ ?
Will increasing the inductance $L$ increase, or decrease, the minimum frequency $f$ required to achieve a capacitive ( $x_{in} < 0$ ) input impedance $z_{in}$ ?

**№ 3: Show your results to the teaching assistant.**

**5. Reflection and input impedance of transmission line with a series RLC circuit**

Consider a load impedance $Z_L = R + j\omega L + 1/\, j\omega C$ of a series combination of resistance $R$ = 75 Ω, inductance $L$ = 200 nH and capacitance $C$ = 100 pF (as in exercise 2 c). Calculate the reflection coefficient $\Gamma$, the rotated reflection coefficient $\Gamma'$ and the normalized input impedance $z_{in}$ for the frequency range specified above.

Plot the rotated reflection coefficient $\Gamma'$ in the complex plane on a Smith chart. The result is non-trivial. Plot the real and imaginary parts of the normalized input impedance $z_{in}$ versus frequency. Restrict your figure axes to impedance components within the range ±15.

Inspect your Smith chart and your impedance versus frequency plots, and consider the following questions.
At what frequency $f$ is the impedance matching condition $\Gamma = 0$ satisfied?
Does the length $l$ of the transmission line affect the frequency at which $\Gamma = 0$ is achieved?
Why does $|\Gamma| \approx 1$ in the low-frequency ( $f$ = 1 MHz ) and high-frequency ( $f$ = 300 MHz ) limits?

**№ 4: Show your results to the teaching assistant.**

Optional: Compare the Smith charts of the series RLC circuit reflection coefficient $\Gamma$ (exercise 2c) and rotated reflection coefficient $\Gamma'$. Using the two charts, determine the length of the transmission line in terms of wavelengths at $f$ = 300 MHz. In other words, determine the ratio $l\,/\,\lambda$(300 MHz). Confirm that your answer is the same as in exercise 3, as it must be, because the transmission line is unchanged.