

Laboratory 1 : Voltage and Current on a Lossless Transmission Line

In this laboratory, we will develop several simple functions to understand the behaviour of harmonic wave voltage $v(z,t)$ and current $i(z,t)$ along a lossless transmission line versus position z and time t .

1. Instantaneous forward and backward waves along a lossless transmission line

Write two MATLAB functions that give the instantaneous voltage and current for a forward wave and a backward wave on a lossless transmission line,

```
[ vf if ] = forward_wave(v0,omega,phi,Z0,vp,z,t)
[ vb ib ] = backward_wave(v0,omega,phi,Z0,vp,z,t)
```

using the following equations for instantaneous voltage and current on a lossless transmission line,

$$\begin{aligned}v_f(z,t) &= v_0 \cos\left(\omega\left(t - \frac{z}{v_p}\right) + \varphi\right) & i_f(z,t) &= \frac{v_0}{Z_0} \cos\left(\omega\left(t - \frac{z}{v_p}\right) + \varphi\right) \\v_b(z,t) &= v_0 \cos\left(\omega\left(t + \frac{z}{v_p}\right) + \varphi\right) & i_b(z,t) &= -\frac{v_0}{Z_0} \cos\left(\omega\left(t + \frac{z}{v_p}\right) + \varphi\right)\end{aligned}$$

where v_0 is the voltage amplitude, ω is the angular frequency, φ is the phase, Z_0 is the characteristic impedance of the lossless line, v_p is the phase velocity of the lossless line, z is the position on the line, and t is the time.

2. Movie of a forward wave and a backward wave

Consider the case of $v_0 = 1$ V, $\omega = 2\pi \times 10^9$ Hz, $\varphi = 0$ rad, $Z_0 = 50\ \Omega$ and $v_p = 2 \times 10^8$ m/s. Confine your numerical studies to a segment of transmission line $0 \leq z \leq 0.40$ m over a duration of time $0 \leq t \leq 5$ ns.

Using your functions above, write a MATLAB script that creates a movie for the forward going voltage wave and backward going voltage wave. To do this, create a vector z of distance values, a vector t of time values, a matrix vf of forward voltage wave values and a matrix vb of backward voltage wave values. Your variables should be defined such that $vf(m,n)$ corresponds to the forward voltage at position $z(m)$ and time $t(n)$. Be sure that you have sufficiently many points of distance and time to capture the wave propagation accurately in your movie. The script segment below can be used to create a movie from your values of z , t , vf and vb .

```
for k=1:length(t)
    plot(z, vf(k,:), 'b'); hold on;
    plot(z, vb(k,:), 'r'); hold off;
    xlabel('z [m]');
    ylabel('v(z,t) [V]');
    title('instantaneous voltage on a lossless line');
    legend('v_f(z,t)', 'v_b(z,t)');
    axis([0 4 -1 1]);
    M(k)=getframe;
end;
```

To replay the movie, execute `movie(M)` at the command line.

№ 1: Show your results to the teaching assistant.

An easy way to save a Graphics Interchange Format (GIF) file of your movie is the `movie2gif(M, 'file')` function, available for download here:

www.mathworks.com/matlabcentral/fileexchange/17463-movie-to-gif-converter

You may, optionally, wish to make a movie of the corresponding forward and backward current waves.

3. Movie of a standing wave

Consider again the case of $v_0 = 1$ V, $\omega = 2\pi \times 10^9$ Hz, $\varphi = 0$ rad, $Z_0 = 50$ Ω and $v_p = 2 \times 10^8$ m/s. Again, confine your numerical studies to a segment of transmission line $0 \leq z \leq 0.40$ m over a duration of time $0 \leq t \leq 5$ ns.

Using your functions above, write a MATLAB script that creates a movie of the voltage and current that result from the simultaneous presence of both the forward wave and backward wave, as defined by,

$$\begin{aligned}v_s(z, t) &= v_0 \cos\left(\omega\left(t - \frac{z}{v_p}\right) + \varphi\right) + v_0 \cos\left(\omega\left(t + \frac{z}{v_p}\right) + \varphi\right) \\i_s(z, t) &= \frac{v_0}{Z_0} \cos\left(\omega\left(t - \frac{z}{v_p}\right) + \varphi\right) - \frac{v_0}{Z_0} \cos\left(\omega\left(t + \frac{z}{v_p}\right) + \varphi\right)\end{aligned}$$

The resulting voltage $v_s(z, t)$ and current $i_s(z, t)$ are standing waves. Look carefully at your movie for voltage standing wave and your movie for current standing wave. Do the maxima in voltage and maxima in current occur at the same positions z ? Do the maxima in voltage and maxima in current occur at the same times t ?

№ 2: Show your results to the teaching assistant.

4. Phasor representation of waves

Write a MATLAB function that gives the instantaneous voltage $v(t)$ of a the complex phasor representation V ,

$$[v] = \text{ph2inst}(V, \omega, t)$$

using the following equation,

$$v(t) = \text{real}\{V \exp(j\omega t)\}$$

For efficiency, you may wish to write your function such that the input V is a vector of voltage phasor values $V(z)$ along the position z , the input ω is the radian frequency ω , the input t is a vector of time t values, and the output v is a matrix of corresponding instantaneous voltage values $v(z, t)$.

Confirm that your conversion function works by making a movie of the instantaneous voltage $v(z,t)$ versus time t corresponding to the phasor $V = j \exp(+j\beta z)$ with $\beta = (2\pi/0.20)$ rad/m. Again, confine your numerical studies to a segment of transmission line $0 \leq z \leq 0.40$ m over a duration of time $0 \leq t \leq 5$ ns. Does the wave travel in the direction you expect? Is the instantaneous voltage $v(z,t=0)$ what you expect? Can you create a wave travelling in the opposite direction?

№ 3: Show your results to the teaching assistant.