Question 1

Find the limit of $\lim\limits_{x\to 5} \dfrac{x^2 - 25}{x^2 + x - 30}$.

$$\lim_{x\to 5} \frac{x^2-25}{x^2+x-30} = \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)}$$

Question 1

Find the limit of $\lim_{x \to 5} \dfrac{x^2 - 25}{x^2 + x - 30}$.

$$\begin{aligned}\lim_{x\to 5} \tfrac{x^2-25}{x^2+x-30} &= \lim_{x\to 5} \tfrac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)} \\ &= \lim_{x\to 5} \tfrac{x+5}{x+6}\end{aligned}$$

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\lim_{x\to 5} \tfrac{x^2-25}{x^2+x-30} &= \lim_{x\to 5} \tfrac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)} \\
&= \lim_{x\to 5} \tfrac{x+5}{x+6} \\
&= \tfrac{5+5}{5+6}
\end{aligned}$$

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Find the limit of $\lim_{x \to 5} \dfrac{x^2 - 25}{x^2 + x - 30}$.

$$\begin{aligned}
\lim_{x\to 5} \frac{x^2-25}{x^2+x-30} &= \lim_{x\to 5} \frac{\cancel{(x-5)}(x+5)}{\cancel{(x-5)}(x+6)} \\
&= \lim_{x\to 5} \frac{x+5}{x+6} \\
&= \frac{5+5}{5+6} \\
&= \mathbf{\frac{10}{11}}
\end{aligned}$$

Question 2

Find the limit of $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$.

$$\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x\to 9} \frac{\sqrt{x}-3}{x-9} \times \frac{\sqrt{x}+3}{\sqrt{x}+3}$$

Question 2

Find the limit of $\lim_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$.

$$\begin{aligned}\lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} \times \tfrac{\sqrt{x}+3}{\sqrt{x}+3} \\ &= \lim_{x\to 9} \tfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)}\end{aligned}$$

Question 2

Find the limit of $\lim\limits_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$.

$$
\begin{aligned}
\lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} \times \tfrac{\sqrt{x}+3}{\sqrt{x}+3} \\
&= \lim_{x\to 9} \tfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} \\
&= \lim_{x\to 9} \tfrac{1}{(\sqrt{x}+3)}
\end{aligned}
$$

Question 2

Find the limit of $\lim\limits_{x \to 9} \dfrac{\sqrt{x} - 3}{x - 9}$.

$$\begin{aligned}
\lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} \times \tfrac{\sqrt{x}+3}{\sqrt{x}+3} \\
&= \lim_{x\to 9} \tfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} \\
&= \lim_{x\to 9} \tfrac{1}{(\sqrt{x}+3)} \\
&= \tfrac{1}{\sqrt{9}+3}
\end{aligned}$$

Question 2

Find the limit of $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$.

$$
\begin{aligned}
\lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} &= \lim_{x\to 9} \tfrac{\sqrt{x}-3}{x-9} \times \tfrac{\sqrt{x}+3}{\sqrt{x}+3} \\
&= \lim_{x\to 9} \tfrac{\cancel{x-9}}{\cancel{(x-9)}(\sqrt{x}+3)} \\
&= \lim_{x\to 9} \tfrac{1}{(\sqrt{x}+3)} \\
&= \tfrac{1}{\sqrt{9}+3} \\
&= \boldsymbol{\tfrac{1}{6}}
\end{aligned}
$$

Question 3

Find the limit of $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$.

$$\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} = \lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \times \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}}$$

Question 3

Find the limit of $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$.

$$\begin{aligned}\lim_{x\to 0} \tfrac{\sqrt{2+x}-\sqrt{2}}{x} &= \lim_{x\to 0} \tfrac{\sqrt{2+x}-\sqrt{2}}{x} \times \tfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\ &= \lim_{x\to 0} \tfrac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x(\sqrt{2+x}+\sqrt{2})}\end{aligned}$$

Question 3

Find the limit of $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$.

$$
\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} &= \lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \times \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\
&= \lim_{x\to 0} \frac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})}
\end{aligned}
$$

Question 3

Find the limit of $\lim\limits_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$.

$$\begin{aligned}
\lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} &= \lim_{x\to 0} \frac{\sqrt{2+x}-\sqrt{2}}{x} \times \frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\
&= \lim_{x\to 0} \frac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})}
\end{aligned}$$

Question 3

Find the limit of $\lim_{x\to 0} \dfrac{\sqrt{2+x}-\sqrt{2}}{x}$.

$$\begin{aligned}
\lim_{x\to 0} \tfrac{\sqrt{2+x}-\sqrt{2}}{x} &= \lim_{x\to 0} \tfrac{\sqrt{2+x}-\sqrt{2}}{x} \times \tfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\
&= \lim_{x\to 0} \tfrac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \tfrac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \tfrac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} \\
&= \tfrac{1}{\sqrt{2}+\sqrt{2}}
\end{aligned}$$

Question 3

Find the limit of $\lim_{x \to 0} \dfrac{\sqrt{2+x} - \sqrt{2}}{x}$.

$$
\begin{aligned}
\lim_{x\to 0} \tfrac{\sqrt{2+x}-\sqrt{2}}{x} &= \lim_{x\to 0} \tfrac{\sqrt{2+x}-\sqrt{2}}{x} \times \tfrac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\
&= \lim_{x\to 0} \tfrac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \tfrac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0} \tfrac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} \\
&= \tfrac{1}{\sqrt{2}+\sqrt{2}} \\
&= \tfrac{1}{2\sqrt{2}} \times \tfrac{\sqrt{2}}{\sqrt{2}}
\end{aligned}
$$

Question 3

Find the limit of $\lim\limits_{x\to 0}\dfrac{\sqrt{2+x}-\sqrt{2}}{x}$.

$$
\begin{aligned}
\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x} &= \lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2}}{x}\times\frac{\sqrt{2+x}+\sqrt{2}}{\sqrt{2+x}+\sqrt{2}} \\
&= \lim_{x\to 0}\frac{(\sqrt{2+x})^2-(\sqrt{2})^2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0}\frac{2+x-2}{x(\sqrt{2+x}+\sqrt{2})} \\
&= \lim_{x\to 0}\frac{\cancel{x}}{\cancel{x}(\sqrt{2+x}+\sqrt{2})} \\
&= \frac{1}{\sqrt{2}+\sqrt{2}} \\
&= \frac{1}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}} \\
&= \mathbf{\frac{\sqrt{2}}{4}}
\end{aligned}
$$

Question 4

Find the limit of $\lim\limits_{\theta \to \frac{\pi}{2}} \dfrac{\tan \theta}{\sec \theta}$.

$$\lim_{\theta \to \frac{\pi}{2}} \frac{\tan \theta}{\sec \theta} = \lim_{\theta \to \frac{\pi}{2}} \frac{\left(\frac{\sin \theta}{\cancel{\cos \theta}}\right)}{\left(\frac{1}{\cancel{\cos \theta}}\right)}$$

Question 4

Find the limit of $\lim\limits_{\theta \to \frac{\pi}{2}} \dfrac{\tan\theta}{\sec\theta}$.

$$\begin{aligned} \lim_{\theta \to \frac{\pi}{2}} \frac{\tan\theta}{\sec\theta} &= \lim_{\theta \to \frac{\pi}{2}} \frac{(\frac{\sin\theta}{\cancel{\cos\theta}})}{(\frac{1}{\cancel{\cos\theta}})} \\ &= \lim_{\theta \to \frac{\pi}{2}} \sin\theta \end{aligned}$$

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Find the limit of $\lim\limits_{\theta \to \frac{\pi}{2}} \dfrac{\tan\theta}{\sec\theta}$.

$$\begin{aligned}\lim_{\theta \to \frac{\pi}{2}} \frac{\tan\theta}{\sec\theta} &= \lim_{\theta \to \frac{\pi}{2}} \frac{\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)}{\left(\frac{1}{\cancel{\cos\theta}}\right)} \\ &= \lim_{\theta \to \frac{\pi}{2}} \sin\theta \\ &= \sin\frac{\pi}{2}\end{aligned}$$

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$$\lim_{\theta \to \frac{\pi}{2}} \frac{\tan\theta}{\sec\theta} = \lim_{\theta \to \frac{\pi}{2}} \frac{\left(\frac{\sin\theta}{\cancel{\cos\theta}}\right)}{\left(\frac{1}{\cancel{\cos\theta}}\right)}$$
$$= \lim_{\theta \to \frac{\pi}{2}} \sin\theta$$
$$= \sin\frac{\pi}{2}$$
$$= \mathbf{1}$$

Question 5

Find the limit of $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta}$.

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} = \lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\theta}{\theta}$$

Question 5

Find the limit of $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$.

$$\begin{aligned}
\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} &= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\theta}{\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{\theta}{\sin\theta}
\end{aligned}$$

Question 5

Find the limit of $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta}$.

$$\begin{aligned}
\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} &= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\theta}{\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{\theta}{\sin\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \lim_{\theta \to 0} \frac{\theta}{\sin\theta}
\end{aligned}$$

Theorem

$\lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$

Question 5

Find the limit of $\lim\limits_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$.

$$
\begin{aligned}
\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} &= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\theta}{\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{\theta}{\sin\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \lim_{\theta \to 0} \frac{\theta}{\sin\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{1}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}}
\end{aligned}
$$

Theorem

$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

Question 5

Find the limit of $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$.

$$\begin{aligned}
\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} &= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\theta}{\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{\theta}{\sin\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \lim_{\theta \to 0} \frac{\theta}{\sin\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{1}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} \\
&= 0 \times 1
\end{aligned}$$

Question 5

Find the limit of $\lim_{\theta \to 0} \dfrac{\cos\theta - 1}{\sin\theta}$.

$$
\begin{aligned}
\lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} &= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\sin\theta} \times \frac{\theta}{\theta} \\
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&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \lim_{\theta \to 0} \frac{\theta}{\sin\theta} \\
&= \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} \times \frac{1}{\lim_{\theta \to 0} \frac{\sin\theta}{\theta}} \\
&= 0 \times 1 \\
&= \mathbf{0}
\end{aligned}
$$

Question 6

If $2x \leq g(x) \leq x^2 - x + 2$ for all x, evaluate $\lim\limits_{x \to 1} g(x)$.

$\lim_{x \to 1} 2x \leq \lim_{x \to 1} g(x) \leq \lim_{x \to 1}(x^2 - x + 2)$

Question 6

If $2x \leq g(x) \leq x^2 - x + 2$ for all x, evaluate $\lim_{x \to 1} g(x)$.

$\lim_{x\to 1} 2x \leq \lim_{x\to 1} g(x) \leq \lim_{x\to 1}(x^2 - x + 2)$

Therefore,

$$2 \leq \lim_{x \to 1} g(x) \leq 2$$

Question 6

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$\lim_{x \to 1} 2x \leq \lim_{x \to 1} g(x) \leq \lim_{x \to 1}(x^2 - x + 2)$

Therefore,

$$2 \leq \lim_{x \to 1} g(x) \leq 2$$

Hence,

$$\lim_{x \to 1} g(x) = \mathbf{2}$$

Question 7

Solve y' if $y = \sqrt{3x^2 - 2x + 3}$.

Let $u = 3x^2 - 2x + 3$, therefore $\frac{du}{dx} = 6x - 2$.

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Substitute $y = \sqrt{u} = u^{\frac{1}{2}}$, therefore $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$.

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Substitute $y = \sqrt{u} = u^{\frac{1}{2}}$, therefore $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$.

Putting back together,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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Putting back together,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{2}u^{-\frac{1}{2}} \cdot 6x - 2 \end{aligned}$$

Question 7

Solve y' if $y = \sqrt{3x^2 - 2x + 3}$.

Let $u = 3x^2 - 2x + 3$, therefore $\frac{du}{dx} = 6x - 2$.

Substitute $y = \sqrt{u} = u^{\frac{1}{2}}$, therefore $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$.

Putting back together,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{2}u^{-\frac{1}{2}} \cdot 6x - 2 \\ &= \frac{1}{2}(3x^2 - 2x + 3)^{-\frac{1}{2}} \cdot 6x - 2\end{aligned}$$

Question 7

Solve y' if $y = \sqrt{3x^2 - 2x + 3}$.

Let $u = 3x^2 - 2x + 3$, therefore $\frac{du}{dx} = 6x - 2$.

Substitute $y = \sqrt{u} = u^{\frac{1}{2}}$, therefore $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$.

Putting back together,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{2}u^{-\frac{1}{2}} \cdot 6x - 2 \\ &= \frac{1}{2}(3x^2 - 2x + 3)^{-\frac{1}{2}} \cdot 6x - 2 \\ &= \frac{6x - 2}{2\sqrt{3x^2 - 2x + 3}}\end{aligned}$$

Question 7

Solve y' if $y = \sqrt{3x^2 - 2x + 3}$.

Let $u = 3x^2 - 2x + 3$, therefore $\frac{du}{dx} = 6x - 2$.

Substitute $y = \sqrt{u} = u^{\frac{1}{2}}$, therefore $\frac{dy}{du} = \frac{1}{2}u^{-\frac{1}{2}}$.

Putting back together,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\
&= \frac{1}{2}u^{-\frac{1}{2}} \cdot 6x - 2 \\
&= \frac{1}{2}(3x^2 - 2x + 3)^{-\frac{1}{2}} \cdot 6x - 2 \\
&= \frac{6x - 2}{2\sqrt{3x^2 - 2x + 3}} = \mathbf{\frac{3x - 1}{\sqrt{3x^2 - 2x + 3}}}
\end{aligned}$$

Question 8

Solve y' if $y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$.

$y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$ can be rewrite as $y = 5(x^2 + x^{\frac{3}{2}})^{\frac{1}{3}}$.

Question 8

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$y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$ can be rewrite as $y = 5(x^2 + x^{\frac{3}{2}})^{\frac{1}{3}}$.

Let $u = x^2 + x^{\frac{3}{2}}$, therefore $\frac{du}{dx} = 2x + \frac{3}{2}x^{\frac{1}{2}}$.

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Let $u = x^2 + x^{\frac{3}{2}}$, therefore $\frac{du}{dx} = 2x + \frac{3}{2}x^{\frac{1}{2}}$.

Substitute $y = 5u^{\frac{1}{3}}$, therefore $\frac{dy}{du} = \frac{5}{3}u^{-\frac{2}{3}}$.

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Putting back together,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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Substitute $y = 5u^{\frac{1}{3}}$, therefore $\frac{dy}{du} = \frac{5}{3}u^{-\frac{2}{3}}$.

Putting back together,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{5}{3}u^{-\frac{2}{3}} \cdot \left(2x + \frac{3}{2}x^{\frac{1}{2}}\right)\end{aligned}$$

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Solve y' if $y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$.

$y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$ can be rewrite as $y = 5(x^2 + x^{\frac{3}{2}})^{\frac{1}{3}}$.

Let $u = x^2 + x^{\frac{3}{2}}$, therefore $\frac{du}{dx} = 2x + \frac{3}{2}x^{\frac{1}{2}}$.

Substitute $y = 5u^{\frac{1}{3}}$, therefore $\frac{dy}{du} = \frac{5}{3}u^{-\frac{2}{3}}$.

Putting back together,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{5}{3}u^{-\frac{2}{3}} \cdot \left(2x + \frac{3}{2}x^{\frac{1}{2}}\right) = \boldsymbol{\frac{5}{3}\left(x^2 + x^{\frac{3}{2}}\right)^{-\frac{2}{3}} \cdot \left(2x + \frac{3}{2}x^{\frac{1}{2}}\right)}\end{aligned}$$

Question 8

Solve y' if $y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$.

$y = 5\sqrt[3]{x^2 + \sqrt{x^3}}$ can be rewrite as $y = 5(x^2 + x^{\frac{3}{2}})^{\frac{1}{3}}$.

Let $u = x^2 + x^{\frac{3}{2}}$, therefore $\frac{du}{dx} = 2x + \frac{3}{2}x^{\frac{1}{2}}$.

Substitute $y = 5u^{\frac{1}{3}}$, therefore $\frac{dy}{du} = \frac{5}{3}u^{-\frac{2}{3}}$.

Putting back together,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{5}{3}u^{-\frac{2}{3}} \cdot \left(2x + \frac{3}{2}x^{\frac{1}{2}}\right) = \mathbf{\frac{5}{3}\left(x^2 + x^{\frac{3}{2}}\right)^{-\frac{2}{3}} \cdot \left(2x + \frac{3}{2}x^{\frac{1}{2}}\right)}\end{aligned}$$

Alternatively, $\frac{5\left(2x+\frac{3}{2}\sqrt{x}\right)}{3\left(x^2+\sqrt{x^3}\right)^{\frac{2}{3}}} = \frac{5\left(\frac{2x}{3}+\frac{1}{2}\sqrt{x}\right)}{\left(x^2+\sqrt{x^3}\right)^{\frac{2}{3}}} = \frac{5\left(\frac{2x}{3}+\frac{\sqrt{x^3}}{2x}\right)}{\left(x^2+\sqrt{x^3}\right)^{\frac{2}{3}}}$

Question 9

Solve y if $y = \ln \cos x^2$.

Let $u = \cos x^2$, therefore $\frac{du}{dx} = (-\sin x^2)(2x)$.

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Let $u = \cos x^2$, therefore $\frac{du}{dx} = (-\sin x^2)(2x)$.

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Putting back together,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

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Let $u = \cos x^2$, therefore $\frac{du}{dx} = (-\sin x^2)(2x)$.

Substitute $y = \ln u$, therefore $\frac{dy}{du} = \frac{1}{u}$.

Putting back together,

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{u} \cdot (-\sin x^2)(2x) \end{aligned}$$

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Solve y if $y = \ln \cos x^2$.

Let $u = \cos x^2$, therefore $\frac{du}{dx} = (-\sin x^2)(2x)$.

Substitute $y = \ln u$, therefore $\frac{dy}{du} = \frac{1}{u}$.

Putting back together,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\
&= \frac{1}{u} \cdot (-\sin x^2)(2x) \\
&= -\frac{2x \sin x^2}{\cos x^2}
\end{aligned}$$

Question 9

Solve y if $y = \ln \cos x^2$.

Let $u = \cos x^2$, therefore $\frac{du}{dx} = (-\sin x^2)(2x)$.

Substitute $y = \ln u$, therefore $\frac{dy}{du} = \frac{1}{u}$.

Putting back together,

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= \frac{1}{u} \cdot (-\sin x^2)(2x) \\ &= -\frac{2x \sin x^2}{\cos x^2} \\ &= \frac{6x - 2}{2\sqrt{3x^2 - 2x + 3}}\end{aligned}$$

Question 9

Solve y if $y = \ln \cos x^2$.

Let $u = \cos x^2$, therefore $\frac{du}{dx} = (-\sin x^2)(2x)$.

Substitute $y = \ln u$, therefore $\frac{dy}{du} = \frac{1}{u}$.

Putting back together,

$$\begin{aligned}
\frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\
&= \frac{1}{u} \cdot (-\sin x^2)(2x) \\
&= -\frac{2x \sin x^2}{\cos x^2} \\
&= \frac{6x - 2}{2\sqrt{3x^2 - 2x + 3}} = \mathbf{-2x \tan x^2}
\end{aligned}$$

Question 10

Differentiate $y = \log(4 + \cos x)$.

Note that $\frac{d}{dx}(\log_b(x)) = \frac{1}{\ln(b)\cdot x} = \frac{1}{x} \cdot \log e$.

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$$\frac{d}{dx}(\log 4 + \cos x) = \frac{1}{4 + \cos x} \cdot \log e \cdot \frac{d}{dx}(4 + \cos x)$$

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Differentiate $y = \log(4 + \cos x)$.

Note that $\frac{d}{dx}(\log_b(x)) = \frac{1}{\ln(b) \cdot x} = \frac{1}{x} \cdot \log e$.

$$
\begin{aligned}
\frac{d}{dx}(\log 4 + \cos x) &= \frac{1}{4 + \cos x} \cdot \log e \cdot \frac{d}{dx}(4 + \cos x) \\
&= \frac{1}{4 + \cos x} \cdot \log e \cdot (-\sin x)
\end{aligned}
$$

Question 10

Differentiate $y = \log(4 + \cos x)$.

Note that $\frac{d}{dx}(\log_b(x)) = \frac{1}{\ln(b) \cdot x} = \frac{1}{x} \cdot \log e$.

$$
\begin{aligned}
\frac{d}{dx}(\log 4 + \cos x) &= \frac{1}{4 + \cos x} \cdot \log e \cdot \frac{d}{dx}(4 + \cos x) \\
&= \frac{1}{4 + \cos x} \cdot \log e \cdot (-\sin x) \\
&= \boldsymbol{\frac{-(\log e)(\sin x)}{4 + \cos x}}
\end{aligned}
$$

Question 11

Find y' for $10e^{2xy} = e^{15y} + e^{13x}$.

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10e^{2xy}(2x \cdot y' + 2y) &= 15y'e^{15y} + 13e^{13x} \\
(20xe^{2xy} - 15e^{15y})y' &= 13e^{13x} - 20ye^{2xy}
\end{aligned}$$

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10e^{2xy}(2x \cdot y' + 2y) &= 15y'e^{15y} + 13e^{13x} \\
(20xe^{2xy} - 15e^{15y})y' &= 13e^{13x} - 20ye^{2xy} \\
\boldsymbol{y'} &= \boldsymbol{\frac{13e^{13x} - 20ye^{2xy}}{20xe^{2xy} - 15e^{15y}}}
\end{aligned}
$$

Question 12

Find $f'(x)$ if $f(x) = 2x(\arctan 5x)^2 + 6\tan(\cos 6x)$.

- $\frac{d}{dx}(\tan^{-1} 5x)^2 = 2\tan^{-1} 5x \cdot \frac{1}{1+(5x)^2} \cdot 5 = \frac{10\tan^{-1} 5x}{1+25x^2}$

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- $$\begin{aligned} \frac{d}{dx}2x(\tan^{-1} 5x)^2 &= 2x(\frac{10\tan^{-1} 5x}{1+25x^2}) + (\tan^{-1} 5x)^2 \cdot 2 \\ &= \frac{20x\tan^{-1} 5x}{1+25x^2} + 2(\tan^{-1} 5x)^2 \end{aligned}$$

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- $$\begin{aligned}\frac{d}{dx}6\tan(\cos 6x) &= 6\sec^2(\cos 6x)\frac{d}{dx}\cos 6x \\ &= 6\sec^2(\cos 6x) \cdot (-\sin 6x) \cdot 6 \\ &= -36(\sec^2(\cos 6x))\sin 6x\end{aligned}$$

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  $$= 6\sec^2(\cos 6x) \cdot (-\sin 6x) \cdot 6$$
  $$= -36(\sec^2(\cos 6x))\sin 6x$$

$$\therefore f'(x) = \boldsymbol{\frac{20x\tan^{-1} 5x}{1+25x^2} + 2(\tan^{-1} 5x)^2 - 36(\sec^2(\cos 6x))\sin 6x}$$

Question 13

Solve y' if $y = 4x\sinh^{-1}\left(\frac{x}{6}\right) + \tanh^{-1}\left(\frac{x}{6}\right) + \tanh^{-1}(\cos 10x)$.

- $$\frac{d}{dx}4x\sinh^{-1}(\frac{x}{6}) = 4x\left[\frac{1}{\sqrt{1+(\frac{x}{6})^2}}\cdot\frac{1}{6}\right] + \sinh^{-1}(\frac{x}{6})\cdot 4$$
$$= \frac{\frac{2x}{3}}{\sqrt{1+\frac{x^2}{36}}} + 4\sinh^{-1}(\frac{x}{6})$$

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$$= \frac{1}{1-\cos^2 10x}\cdot(-10\sin 10x)$$
$$= \frac{-10\sin 10x}{\sin^2 10x}$$
$$= -10\csc 10x$$

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$$= \frac{1}{1-\cos^2 10x}\cdot(-10\sin 10x)$$
$$= \frac{-10\sin 10x}{\sin^2 10x}$$
$$= -10\csc 10x$$

$$\therefore f'(x) = \frac{\frac{2x}{3}}{\sqrt{1+\frac{x^2}{36}}} + \mathbf{4\sinh^{-1}(\tfrac{x}{6}) - 10\csc 10x}$$

Question 14

Differentiate $y = \dfrac{1}{\sin^{-1} x}$.

Theorem

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

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Theorem

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$$y = (\sin^{-1} x)^{-1}$$

$$y' = -(\sin^{-1} x)^{-2}\frac{d}{dx}(\sin^{-1} x)$$

Question 14

Differentiate $y = \dfrac{1}{\sin^{-1} x}$.

Theorem

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$$\begin{aligned}
y &= (\sin^{-1} x)^{-1} \\
y' &= -(\sin^{-1} x)^{-2} \frac{d}{dx}(\sin^{-1} x) \\
&= -\boldsymbol{\frac{1}{(\sin^{-1} x^2)\sqrt{1-x^2}}}
\end{aligned}$$

Question 15

Differentiate $y = \left(x^3 - 1\right)^{100}$.

$$y' = 100(x^3 - 1)^{99} \frac{d}{dx}(x^3 - 1)$$

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$$
\begin{aligned}
y' &= 100(x^3 - 1)^{99} \frac{d}{dx}(x^3 - 1) \\
&= 100(x^3 - 1)^{99} \cdot 3x^2
\end{aligned}
$$

Question 15

Differentiate $y = \left(x^3 - 1\right)^{100}$.

$$\begin{aligned} y' &= 100(x^3 - 1)^{99} \frac{d}{dx}(x^3 - 1) \\ &= 100(x^3 - 1)^{99} \cdot 3x^2 \\ &= \mathbf{300x^2(x^3 - 1)^{99}} \end{aligned}$$