Tutorial 7: Engineering Applications of Matrices and Vectors

1. An electrical engineer supervises the production of three types of electrical components. Three kinds of materials are; metal, plastic and rubber – are required for production. The amounts needed to produce each component are

Component	Metal (g/component)	Plastic (g/component)	Rubber (g/component)
1	15	0.25	1.0
2	17	0.33	1.2
3	19	0.42	1.6

If totals of 2.12, 0.0434 and 0.164 kg of metal, plastic and rubber respectively are available each day, how many components can be produced per day?

$$\begin{bmatrix} 15 & 17 & 19 \\ 0.25 & 0.33 & 0.42 \\ 1 & 1.2 & 1.6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2120 \\ 43.4 \\ 164 \end{bmatrix}$$

$$D = \begin{vmatrix} 15 & 17 & 19 \\ 0.25 & 0.33 & 0.42 \\ 1 & 1.2 & 1.6 \end{vmatrix} = 0.13 \qquad D_{x_1} = \begin{vmatrix} 2120 & 17 & 19 \\ 43.4 & 0.33 & 0.42 \\ 164 & 1.2 & 1.6 \end{vmatrix} = 2.6$$

$$D_{x_2} = \begin{vmatrix} 15 & 2120 & 19 \\ 0.25 & 43.4 & 0.42 \\ 1 & 164 & 1.6 \end{vmatrix} = 5.2 \qquad D_{x_3} = \begin{vmatrix} 15 & 17 & 2120 \\ 0.25 & 0.33 & 43.4 \\ 1 & 1.2 & 164 \end{vmatrix} = 7.8$$

$$x_1 = \frac{D_{x_1}}{D} = \frac{2.6}{0.13} = \mathbf{20} \qquad x_2 = \frac{D_{x_2}}{D} = \frac{5.2}{0.13} = \mathbf{40} \qquad x_3 = \frac{D_{x_3}}{D} = \frac{7.8}{0.13} = \mathbf{60}$$

2. **A civil engineer involved in construction requires 4800, 5800 and 5700 m^3 of sand, fine gravel and coarse gravel respectively for building a project. There are three pits from which these materials can be obtained. The composition of these pits is**

	Sand %	Fine Gravel %	Coarse Gravel %
Pit 1	52	30	18
Pit 2	20	50	30
Pit 3	25	20	55

How many cubic meters must be hauled from each pit in order to meet the engineer's needs?

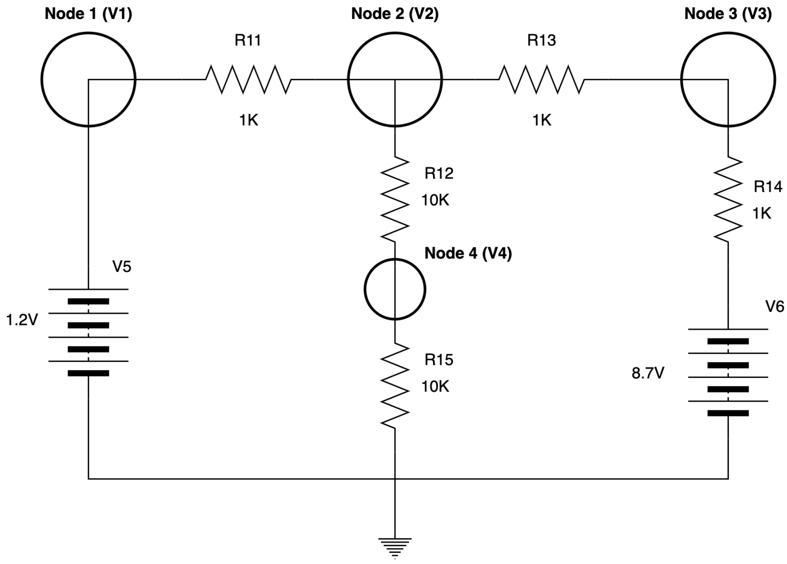
$$\begin{bmatrix} 0.52 & 0.30 & 0.18 \\ 0.20 & 0.50 & 0.30 \\ 0.25 & 0.20 & 0.55 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4800 \\ 5800 \\ 5700 \end{bmatrix}$$

$$D = \begin{vmatrix} 0.52 & 0.30 & 0.18 \\ 0.20 & 0.50 & 0.30 \\ 0.25 & 0.20 & 0.55 \end{vmatrix} = 0.086 \qquad D_{x_1} = \begin{vmatrix} 4800 & 0.30 & 0.18 \\ 5800 & 0.50 & 0.30 \\ 5700 & 0.20 & 0.55 \end{vmatrix} = 344.5$$

$$D_{x_2} = \begin{vmatrix} 0.52 & 4800 & 0.18 \\ 0.20 & 5800 & 0.30 \\ 0.25 & 5700 & 0.55 \end{vmatrix} = 613.3 \qquad D_{x_3} = \begin{vmatrix} 0.52 & 0.30 & 4800 \\ 0.20 & 0.50 & 5800 \\ 0.25 & 0.20 & 5700 \end{vmatrix} = 444$$

$$x_1 = \frac{D_{x_1}}{D} = \frac{344.5}{0.086} = \mathbf{4005.82} \qquad x_2 = \frac{D_{x_2}}{D} = \frac{613.3}{0.086} = \mathbf{7131.40} \qquad x_3 = \frac{D_{x_3}}{D} = \frac{444}{0.086} = \mathbf{5162.79}$$

3. By referring to the schematic below, find V1, V2, V3 and V4. Solve the problem using Cramer's rule.



Node equations

Node Equation #1: $V_1 = 1.2$ *[Since Node 1 directly connectted to source V_5]*

Node Equation #2: $\dfrac{V_2 - 1.2}{R_{11}} + \dfrac{V_2 - V_3}{R_{13}} + \dfrac{V_2 - V_4}{R_{12}} = 0$

Node Equation #3: $\dfrac{V_3 - V_2}{R_{13}} + \dfrac{V_3 - 8.7}{R_{14}} = 0$

Node Equation #4: $\dfrac{V_4 - V_2}{R_{12}} + \dfrac{V_4 - 0}{R_{15}} = 0$

Solving the node equations:

Substituting all values to Node Equation #2:

$$\begin{aligned} \frac{V_2 - 1.2}{1000} + \frac{V_2 - V_3}{1000} + \frac{V_2 - V_4}{10000} &= 0 \\ 10V_2 - 12 + 10V_2 - 10V_3 + V_2 - V_4 &= 0 \\ 21V_2 - 10V_3 - V_4 &= 12 \end{aligned} \tag{1}$$

Substituting all values to Node Equation #3:

$$\begin{aligned} \frac{V_3 - V_2}{1000} + \frac{V_3 - 8.7}{1000} &= 0 \\ V_3 - V_2 + V_3 - 8.7 &= 0 \\ -V_2 + 2V_3 &= 8.7 \end{aligned} \tag{2}$$

Substituting all values to Node Equation #4:

$$
\begin{aligned}
\frac{V_4 - V_2}{10000} + \frac{V_4 - 0}{10000} &= 0 \\
V_4 - V_2 + V_4 &= 0 \\
-V_2 + 2V_4 &= 0
\end{aligned} \tag{3}
$$

Putting Equation (1) – (3) into matrix and solve for V_2, V_3, V_4:

$$
\begin{bmatrix} 21 & -10 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 12 \\ 8.7 \\ 0 \end{bmatrix}
$$

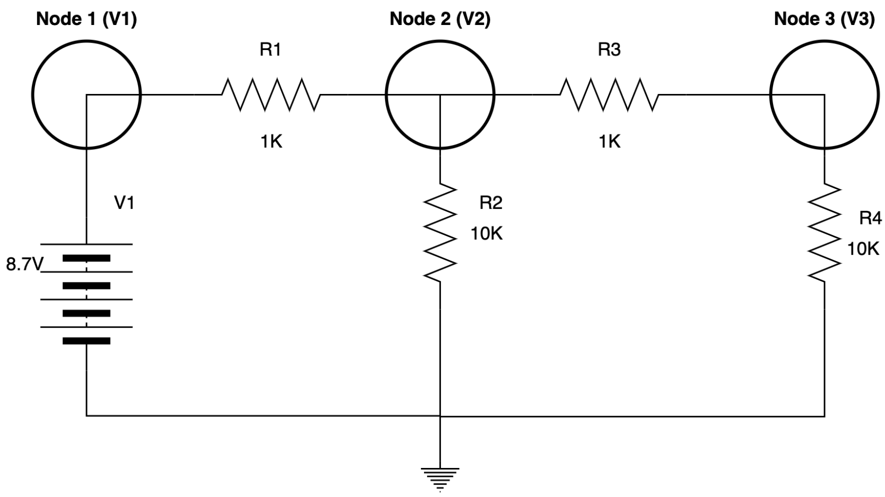
$$
D = \begin{vmatrix} 21 & -10 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = 62 \qquad D_{V_2} = \begin{vmatrix} 12 & -10 & -1 \\ 8.7 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = 222
$$

$$
D_{V_3} = \begin{vmatrix} 21 & 12 & -1 \\ -1 & 8.7 & 0 \\ -1 & 0 & 2 \end{vmatrix} = 380.7 \qquad D_{V_4} = \begin{vmatrix} 21 & -10 & 12 \\ -1 & 2 & 8.7 \\ -1 & 0 & 0 \end{vmatrix} = 111
$$

$$
\begin{aligned}
V_2 &= \frac{D_{V_2}}{D} \\
&= \frac{222}{62} \\
&= \mathbf{3.58\ V}
\end{aligned}
\qquad
\begin{aligned}
V_3 &= \frac{D_{V_3}}{D} \\
&= \frac{380.7}{62} \\
&= \mathbf{6.14\ V}
\end{aligned}
\qquad
\begin{aligned}
V_4 &= \frac{D_{V_4}}{D} \\
&= \frac{111}{62} \\
&= \mathbf{1.79\ V}
\end{aligned}
$$

$\therefore\ V_1 = \mathbf{1.2\ V};\ \ V_2 = \mathbf{3.58\ V};\ \ V_3 = \mathbf{6.14\ V};\ \ V_4 = \mathbf{1.79\ V}$

4. By referring to the schematic below, find V2, and V3. Solve the problem using Cramer's rule.



<u>**Node equations**</u>

Node Equation #1: $V_1 = 8.7$ *[Since Node 1 directly connectted to source V_1]*

Node Equation #2: $\dfrac{V_2 - V_1}{R_1} + \dfrac{V_2 - 0}{R_2} + \dfrac{V_2 - V_3}{R_3} = 0$

Node Equation #3: $\dfrac{V_3 - V_2}{R_3} + \dfrac{V_3 - 0}{R_4} = 0$

Solving the node equations:

Substituting all values to Node Equation #2:

$$\begin{aligned} \frac{V_2 - 8.7}{1000} + \frac{V_2 - 0}{10000} + \frac{V_2 - V_3}{1000} &= 0 \\ 10V_2 - 87 + V_2 + 10V_2 - 10V_3 &= 0 \\ 21V_2 - 10V_3 &= 87 \end{aligned} \tag{1}$$

Substituting all values to Node Equation #3:

$$\begin{aligned} \frac{V_3 - V_2}{1000} + \frac{V_3 - 0}{10000} &= 0 \\ 10V_3 - 10V_2 + V_3 &= 0 \\ -10V_2 + 11V_3 &= 0 \end{aligned} \tag{2}$$

Putting Equation (1) and (2) into matrix and solve for V_2, V_3:

$$\begin{bmatrix} 21 & -10 \\ -10 & 11 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 87 \\ 0 \end{bmatrix}$$

$$D = \begin{vmatrix} 21 & -10 \\ -10 & 11 \end{vmatrix} = 131 \quad D_{V_2} = \begin{vmatrix} 87 & -10 \\ 0 & 11 \end{vmatrix} = 957 \quad D_{V_3} = \begin{vmatrix} 21 & -87 \\ -10 & 0 \end{vmatrix} = 870$$

$$\begin{aligned} V_2 &= \frac{D_{V_2}}{D} \\ &= \frac{957}{131} \\ &= \mathbf{7.31\ V} \end{aligned} \qquad \begin{aligned} V_3 &= \frac{D_{V_3}}{D} \\ &= \frac{870}{131} \\ &= \mathbf{6.64\ V} \end{aligned}$$

$\therefore\ V_1 = \mathbf{8.7\ V};\ \ V_2 = \mathbf{7.31\ V};\ \ V_3 = \mathbf{6.64\ V}$