KIX1001: Engineering Mathematics I (2018/19)

TUTORIAL 14: Frobenius Method

Part I: Using method of reduction of order, find y_2 such that y_1 , y_2 form a basis.

1.
$$2t^2y'' + ty' - 3y = 0$$
,

$$y_1(t) = t^{-1}, \qquad t \neq 0$$

2.
$$t^2y'' - (t^2 + 2t)y' + (t+2)y = 0,$$
 $y_1(t) = t$

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3.
$$y'' + 6y' + 9y = 0$$
,

$$y_1(t) = e^{-3t}$$

4.
$$(x-1)y'' - xy' + y = 0$$
, $y_1(x) = e^x$, $x > 1$

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5.
$$xy'' - y' + 4x^3y = 0,$$

$$y_1(x) = \sin x^2, \quad x > 0$$

Part II: Discuss whether two Frobenius series solutions exist or do not exist for the following equations.

1.
$$2x^2y'' + x(x+1)y' - (\cos x)y = 0$$

2.
$$x^4y'' - (x^2\sin x)y' + 2(1 - \cos x)y = 0$$

Part III: Apply Frobenius Method to find the basis of solutions of the following differential equations.

1.
$$2xy'' + y' + y = 0$$

2.
$$xy'' + 2y' + xy = 0$$

3.
$$xy'' + (1-2x)y' + (x-1)y = 0$$

4.
$$2ty'' + (1+t)y' + y = 0$$

5.
$$x(1-x)y'' - 3xy' - y = 0$$