KIX 1002: ENGINEERING MATHEMATICS 2

Tutorial 3 – 4: 2nd Order Ordinary Differential Equations

1. Use the Wronskian to show whether the give set of functions is linearly dependent or linearly independent.
   (a) $y_1 = x$ $\quad y_2 = x + 1$
   (b) $y_1 = e^{\alpha x} \sin \beta x$ $\quad y_2 = e^{\alpha x} \cos\ \beta x$

2. Verify that each of the given functions is a solution of the differential equation, and use their Wronskian to show that these solutions are linearly independent.
   Verify the linear combination of the solutions is also a solution.
   (a) $y_1 = e^{x}$ $\quad y_2 = e^{-2x}$ $\quad y" + y' - 2y = 0$
   (b) $y_1 = x^2$ $\quad y_2 = x^{-1}$ $\quad x^2y" - 2y = 0$
   (c) $y_1 = e^{-x}$ $\quad y_2 = xe^{-x}$ $\quad y" + 2y' + y = 0$

3. Solve the following second order ODEs
   (a) $y" - 4y = 0$
   (b) $y" - y' - 6y = 0$
   (c) $6y" + y' - y = 0$
   (d) $y" + y = 0$
   (e) $y" + 4y' + 8y = 0$

4. Solve the following second order ODEs with initial-value conditions
   (a) $y" - 4y = 0$ $\quad y(0) = 4$ $\quad y'(0) = 12$
   (b) $y" - 6y + 9y = 0$ $\quad y(0) = -1.4$ $\quad y'(0) = 4.6$
   (c) $4y" - 8y + 3y = 0$ $\quad y(1) = \frac{4}{5}e^{\frac{1}{2}}$ $\quad y'(1) = \frac{2}{5}e^{\frac{1}{2}}$
   (d) $y" + y = 0$ $\quad y(0) = 3$ $\quad y'(0) = -0.5$
   (e) $y" + 2y' + 2y = 0$ $\quad y(0) = 1$ $\quad y'(0) = -1$

5. Obtain a general solution for nonhomogeneous problems using the method of undetermined coefficients.
   (a) $y" + 3y' + 2y = 2x^2$
   (b) $y" + 4y = 4x^2 + 6$
   (c) $y" - y = e^{x}$
   (d) $y" + 3y' - 4y = 10e^{x}$
   (e) $y" - 4y' + 3y = 2e^{3x}$
   (f) $y" + 4y' - 2y = 2x^2 - 3x + 6$
   (g) $y" + 3y' + 2y = 6$

6. Obtain a general solution for nonhomogeneous problems using the method of undetermined coefficients.
   (a) $y" - 2y' + 5y = e^x \cos 2x$
   (b) $y" + y = 2x \sin x$
   (c) $y" + 2y' + 2y = e^x \sin x$
   (d) $y" - 3y' + 2y = x^2 e^{3x}$

7. Solve the following nonhomogeneous with initial value using the method of undetermined coefficients.

(a) $5y" + y' = -6x$	$y(0) = 0$	$y'(0) = 0$
(b) $y" + 4y' + 5y = 35e^{-4x}$	$y(0) = -3$	$y'(0) = 1$
(c) $y" + 25y = 5x$	$y(0) = 5$	$y'(0) = -4.8$
(d) $y" - 2y' + y = 2x^2 - 8x + 4$	$y(0) = 0.3$	$y'(0) = 0.3$
(e) $y" - y' - 2y = 3e^{2x}$	$y(1) = e^{-1}$	$y'(1) = -e^{-1} + e^2$

8. Obtain a general solution using the method of **variation of parameter.**
   (a) $y" - 4y' + 4y = e^{2x}/x$
   (b) $y" + 2y' + y = x^{3/2}/e^x$
   (c) $y" - 2y' + y = e^x \sin x$
   (d) $y" + 2y' + 2y = e^{-x} \sec x$
   (e) $y" - 4y' + 4y = (x + 1)e^{2x}$
   (f) $y" - 5y' + 6y = 2e^x$
   (g) $y" + 2y' + y = 4e^{-x}$