2018 MCM
Problem A: Multi-hop HF Radio Propagation

Background: On high frequencies (HF, defined to be 3 – 30 mHz), radio waves can travel long distances (from one point on the earth's surface to another distant point on the earth's surface) by multiple reflections off the ionosphere and off the earth. For frequencies below the *maximum usable frequency* (MUF), HF radio waves from a ground source reflect off the ionosphere back to the earth, where they may reflect again back to the ionosphere, where they may reflect again back to the earth, and so on, travelling further with each successive hop. Among other factors, the characteristics of the reflecting surface determine the strength of the reflected wave and how far the signal will ultimately travel while maintaining useful signal integrity. Also, the MUF varies with the season, time of day, and solar conditions. Frequencies above the MUF are not reflected/refracted, but pass through the ionosphere into space. In this problem, the focus is particularly on reflections off the ocean surface. It has been found empirically that reflections off a turbulent ocean are attenuated more than reflections off a calm ocean. Ocean turbulence will affect the electromagnetic gradient of seawater, altering the local permittivity and permeability of the ocean, and changing the height and angle of the reflection surface. A turbulent ocean is one in which wave heights, shapes, and frequencies change rapidly, and the direction of wave travel may also change.

Problem:

Part I: Develop a mathematical model for this signal reflection off the ocean. For a 100-watt HF constant-carrier signal, below the MUF, from a point source on land, determine the strength of the first reflection off a turbulent ocean and compare it with the strength of a first reflection off a calm ocean. (Note that this means that there has been one reflection of this signal off the ionosphere.) If additional reflections (2 through n) take place off calm oceans, what is the maximum number of hops the signal can take before its strength falls below a usable signal-to-noise ratio (SNR) threshold of 10 dB?

Part II: How do your findings from **Part I** compare with HF reflections off mountainous or rugged terrain versus smooth terrain?

Part III: A ship travelling across the ocean will use HF for communications and to receive weather and traffic reports. How does your model change to accommodate a shipboard receiver moving on a turbulent ocean? How long can the ship remain in communication using the same multi-hop path?

Part IV: Prepare a short (1 to 2 pages) synopsis of your results suitable for publication as a short note in *IEEE Communications Magazine*.

Your submission should consist of:

- One-page Summary Sheet,
- Two-page synopsis,
- Your solution of no more than 20 pages, for a maximum of 23 pages with your summary and synopsis.
- Note: Reference list and any appendices do not count toward the 23-page limit and should appear after your completed solution.