# Computer Graphics 2019

## 11. Complex 3D

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2019-12-04

# General spline curves

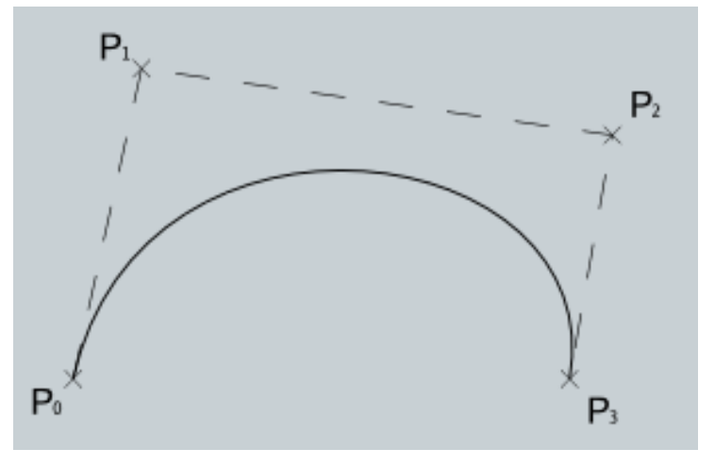
parametric curve

basis functions

$$\mathbf{P}(t) = \sum_i \mathbf{P}_i \, B_i(t)$$

$$t \in [t_0, t_1)$$

control pints



# Bézier curve

## Bézier curve

$$\boldsymbol{C}(t)=\sum_{i=0}^{n}\boldsymbol{P}_i B_{i,n}(t),\quad t\in[0,1]$$

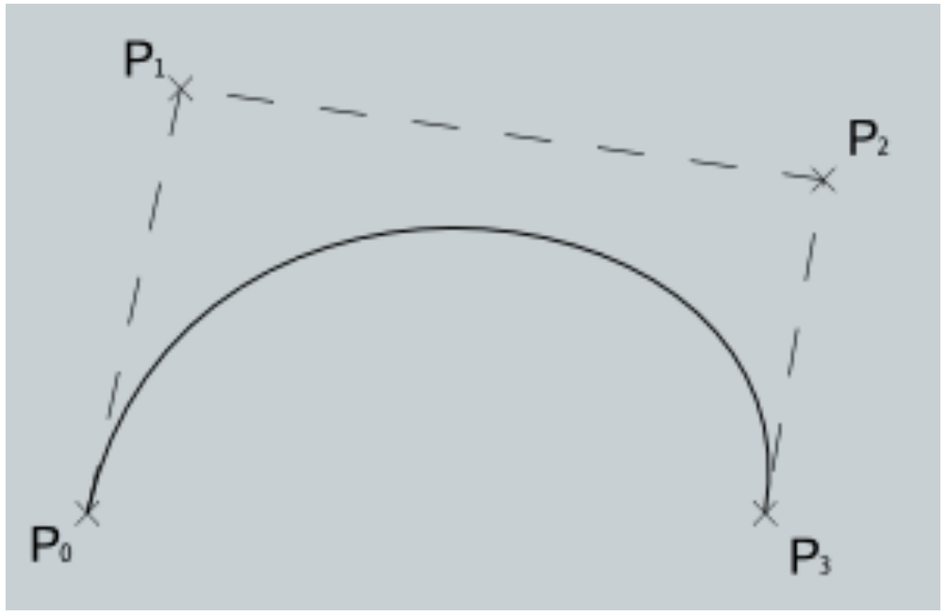
**where, $P_i$ ($i$=0,1,…,n) are control points.**

$$B_{i,n}(t)=C_n^i t^i(1-t)^{n-i},\ t\in[0,1]$$

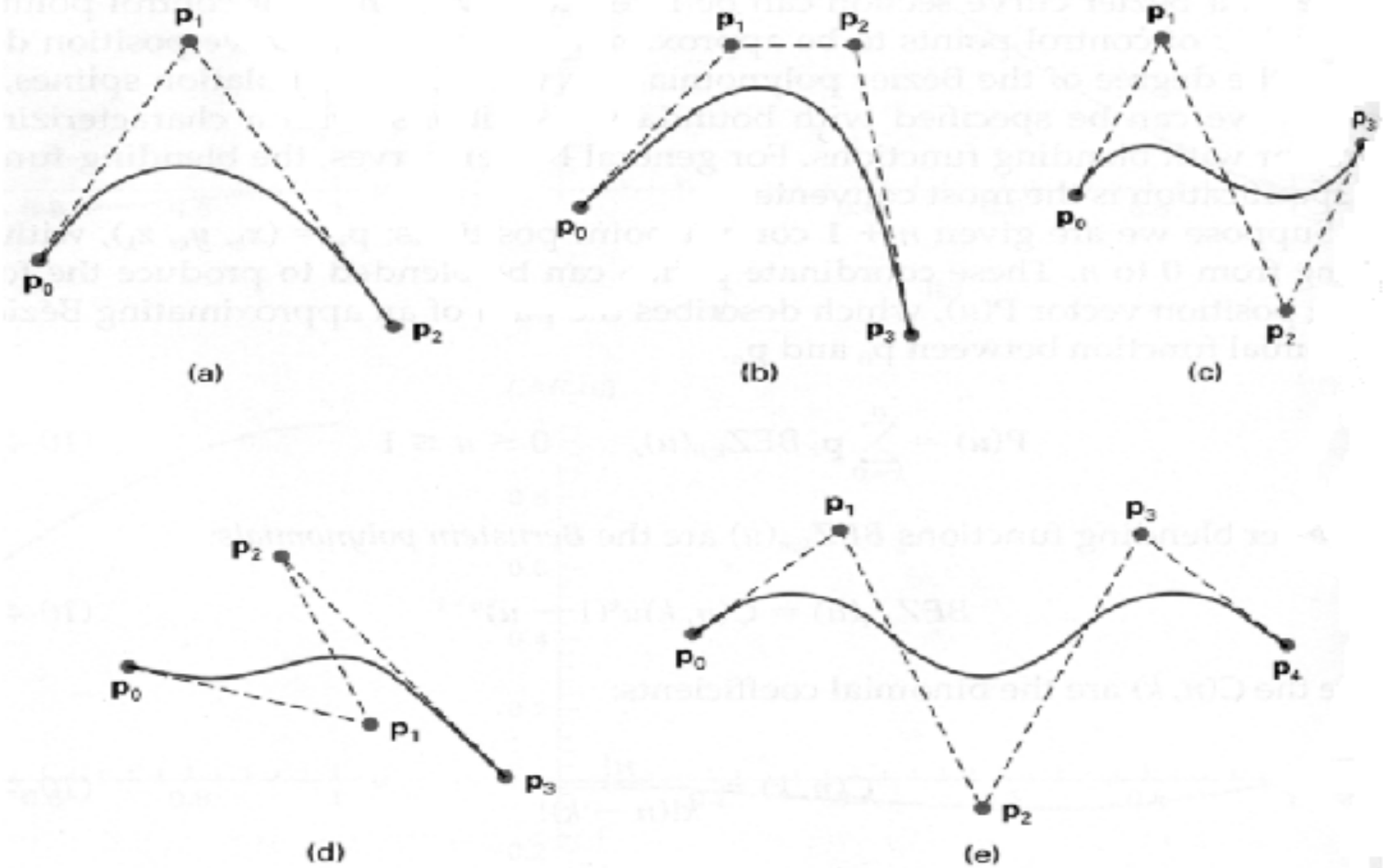
**Bernstein basis**

$$\begin{cases}\mathrm{X}(t)=\sum_{i=0}^{n}x_i B_{i,t}(t)\\ \mathrm{Y}(t)=\sum_{i=0}^{n}y_i B_{i,t}(t)\end{cases}$$

$$C(t)=\begin{pmatrix}\mathrm{X}(t)\\ \mathrm{Y}(t)\end{pmatrix},\quad P_i=\begin{pmatrix}x_i\\ y_i\end{pmatrix}$$



# Bézier curve



# Bézier curve

## Rational Bézier Curve

$$\boldsymbol{R}(t) = \frac{\sum_{i=0}^{n} B_{i,n}(t)\omega_i \boldsymbol{P}_i}{\sum_{i=0}^{n} B_{i,n}(t)\omega_i} = \sum_{i=0}^{n} R_{i,n}(t)\boldsymbol{P}_i$$

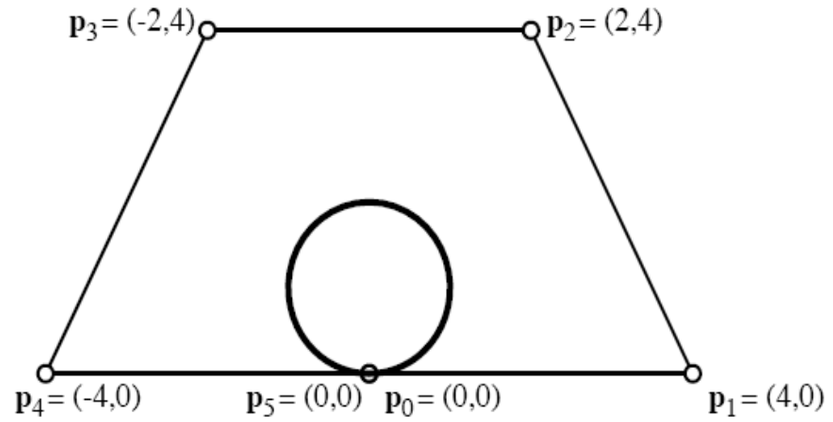



Figure 2.19: Circle as Degree 5 Rational Bézier Curve.

where $B_{i,n}(t)$ is Bernstein basis, $\omega_i$ is the weight at $p_i$ .

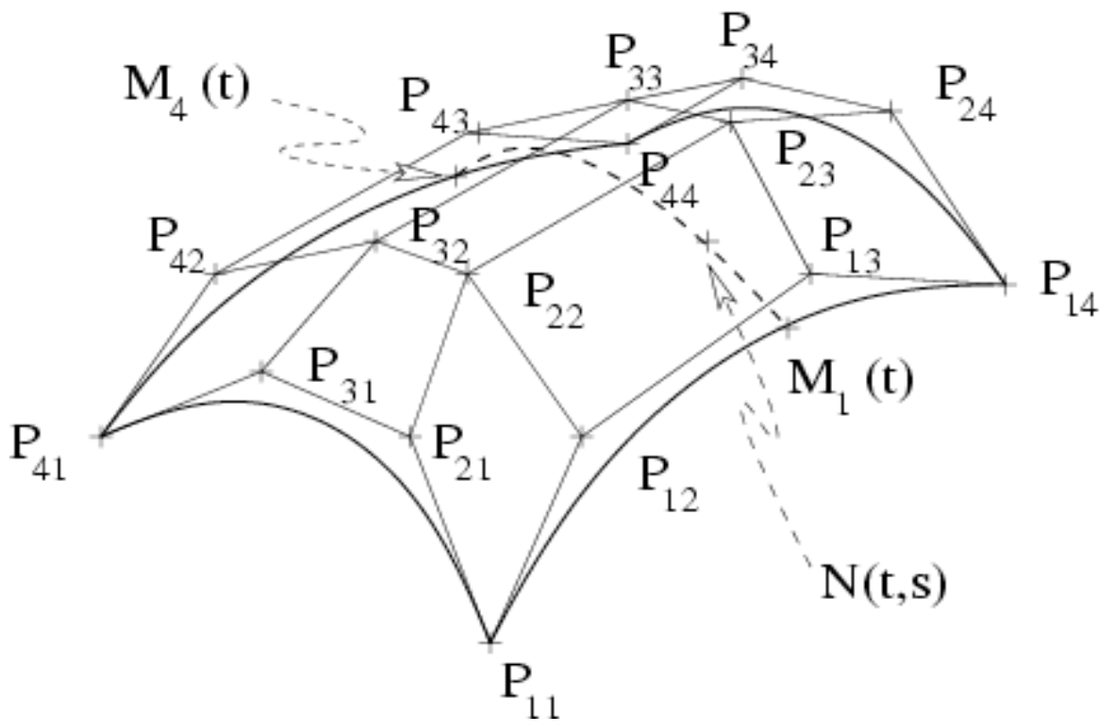
It's a generalization of Bézier curve, which can express more curves, such as circle.

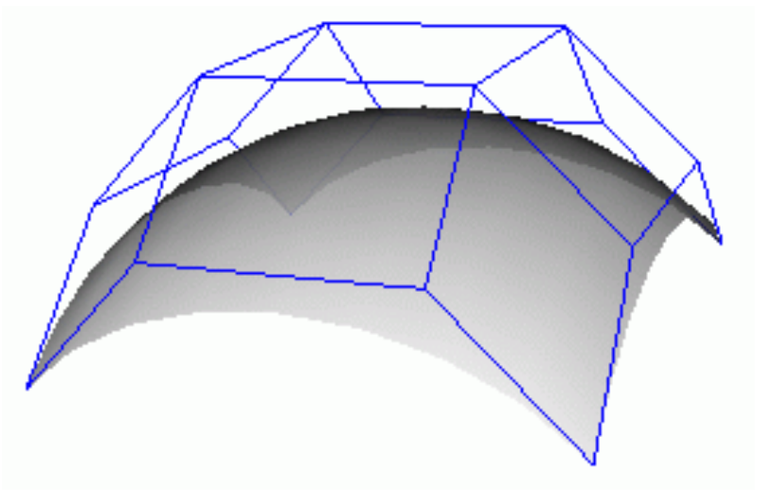
## Bézier surface

Bézier surface:

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} \boldsymbol{P}_{ij} B_{i,n}(u) B_{j,m}(v), \qquad 0 \le u, v \le 1$$

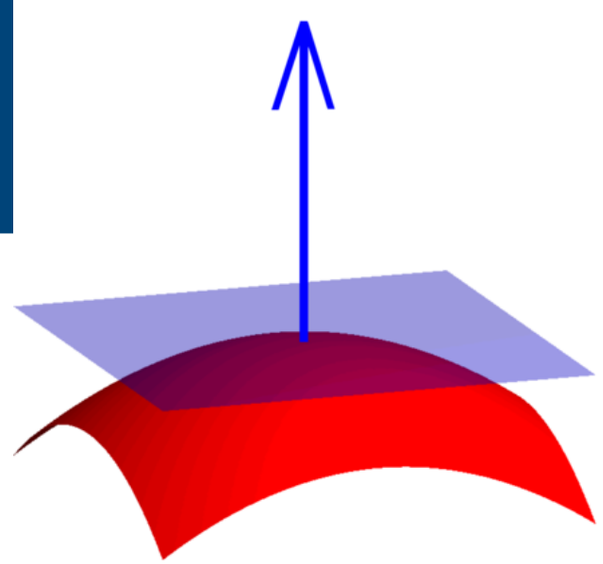
where $B_{i,n}(u)$和$B_{j,m}(v)$ Bernstein basis with n degree and m degree, respectively, $(n+1)\times(m+1)$ $\boldsymbol{P}_{i,j}(i=0,1,\ldots,n; j=0,1,\ldots,m)$ construct the control meshes.



# Bézier surface

## normal vector of Bézier surface



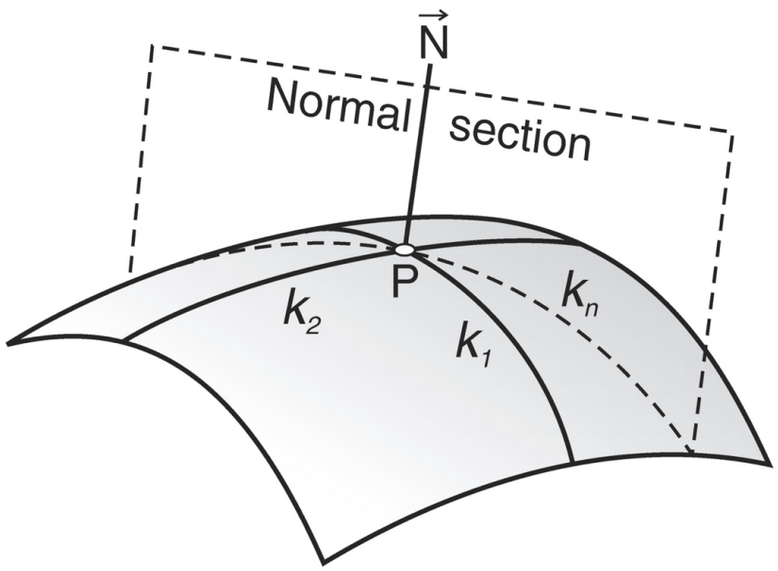
partial derivation of Bézier surface $\boldsymbol{S}(u,v)$:

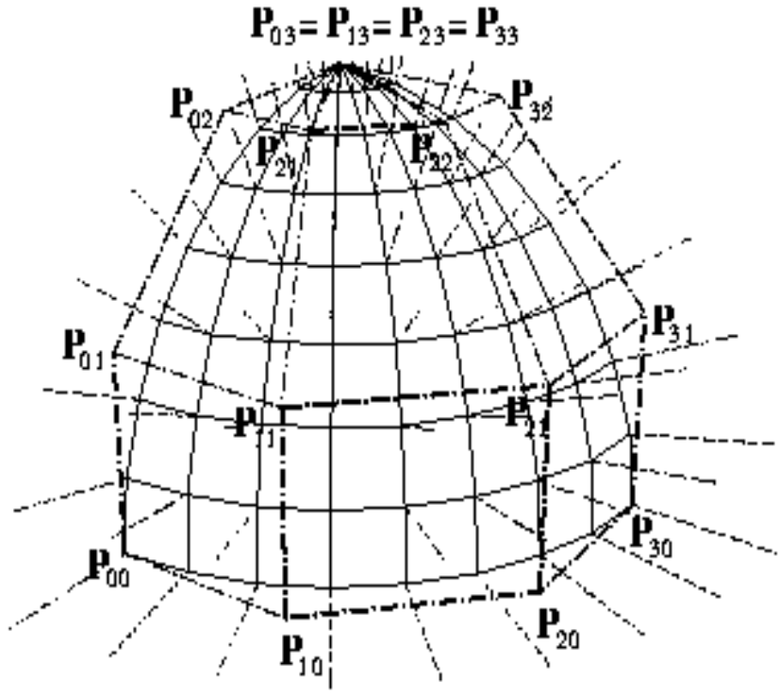
$$\frac{\partial}{\partial u}\boldsymbol{S}(u,v)=\frac{\partial}{\partial u}\sum_{i=0}^{n}\sum_{j=0}^{m}\boldsymbol{P}_{ij}B_{i,n}(u)B_{j,m}(v)=n\sum_{i=0}^{n-1}\sum_{j=0}^{m}(\boldsymbol{P}_{i+1,j}-\boldsymbol{P}_{ij})B_{i,n-1}(u)B_{j,m}(v)$$

$$\frac{\partial}{\partial v}\boldsymbol{S}(u,v)=\frac{\partial}{\partial v}\sum_{i=0}^{n}\sum_{j=0}^{m}\boldsymbol{P}_{ij}B_{i,n}(u)B_{j,m}(v)=m\sum_{i=0}^{n}\sum_{j=0}^{m-1}(\boldsymbol{P}_{i,j+1}-\boldsymbol{P}_{ij})B_{i,n}(u)B_{j,m-1}(v)$$

normal $\boldsymbol{N}(u,v)$ :

$$N(u,v)=\frac{\partial \boldsymbol{S}(u,v)}{\partial u}\times\frac{\partial \boldsymbol{S}(u,v)}{\partial v}$$



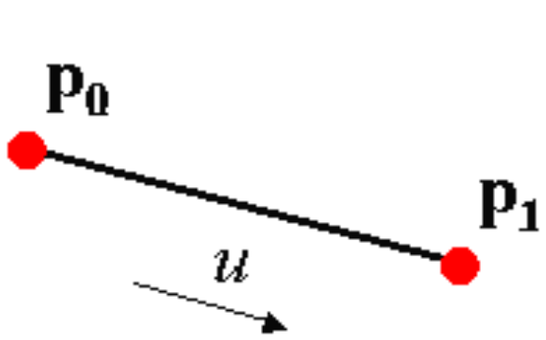



# NURBS curve

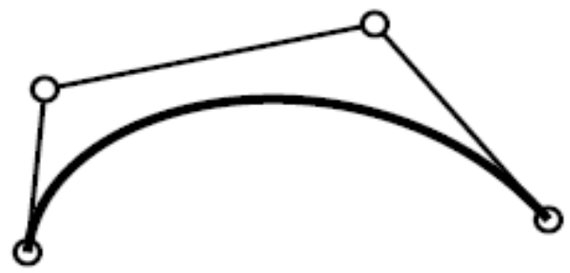
## B-spline curve

- disadvantages of Bézier curve:

1. control points  determine the degree of the curve.  many control points means high degree.
2. It's global. A control point influences the whole curve.




Degree 2



Degree 3

de Boor et al. replaced Bernstein basis with B-spline basis  to generate  B-spline curve.

# NURBS curve

## B-spline curve:

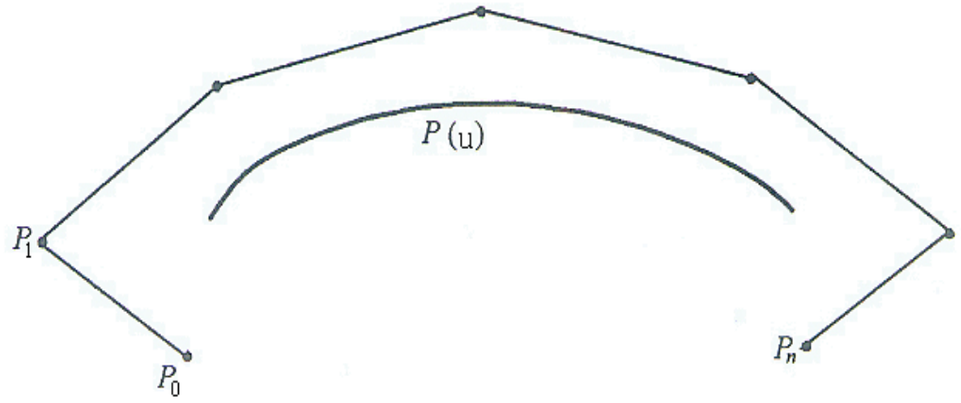
$$C(u) = \sum_{i=0}^{n} P_i N_{i,p}(u) \qquad a \le u \le b$$

Where $\boldsymbol{P}_0, \boldsymbol{P}_1, \ldots, \boldsymbol{P}_n$ are control points, $\mathbf{u}=[u_0=a, u_1, \ldots, u_i, \ldots, u_{n+k+1}=b]$.

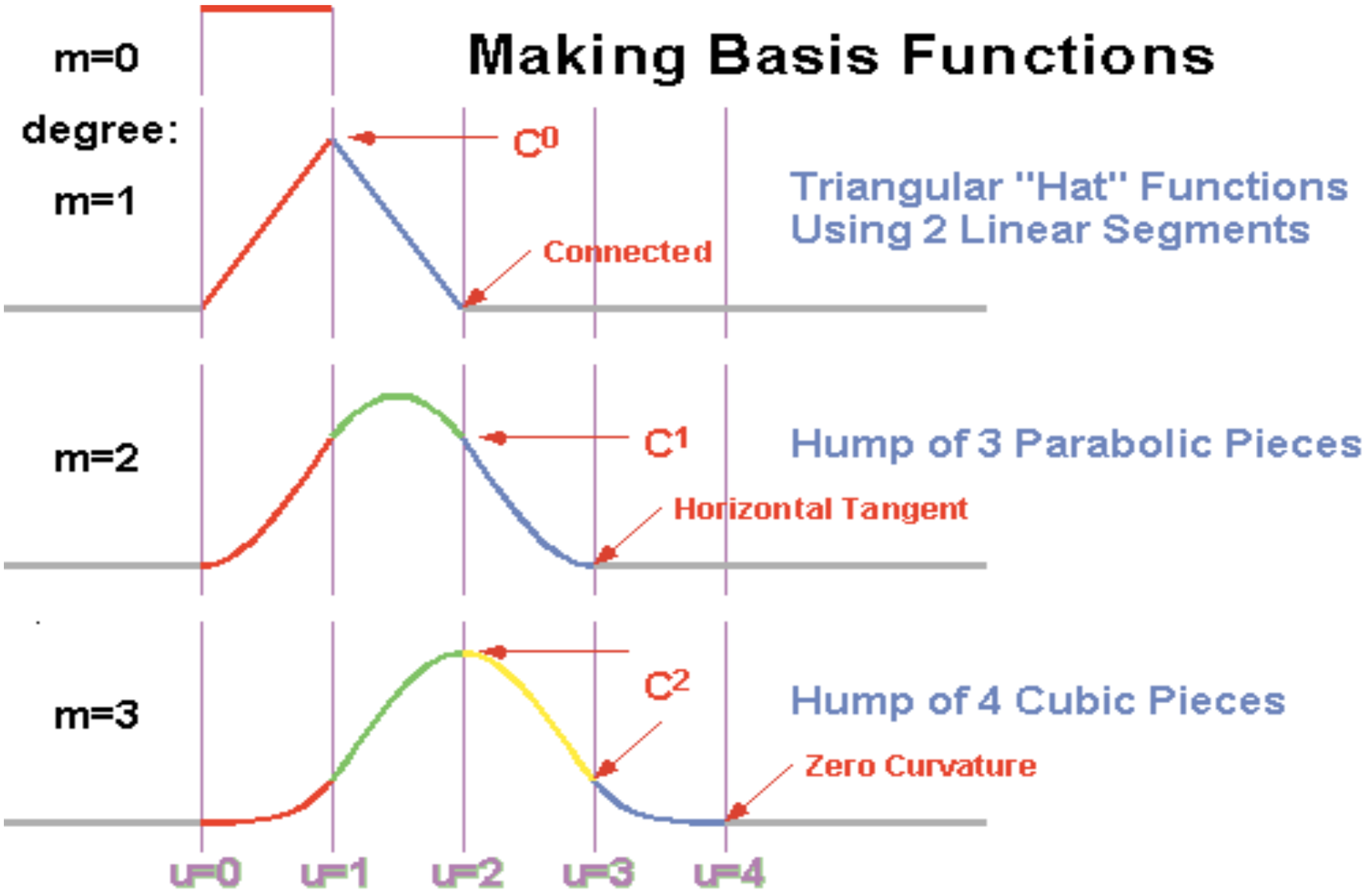
$$N_{i,0}(u) = \begin{cases} 1 & u_i \le u < u_{i+1} \\ 0 & otherwise \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u),$$

$$\frac{0}{0} = 0$$



# B-spline basis



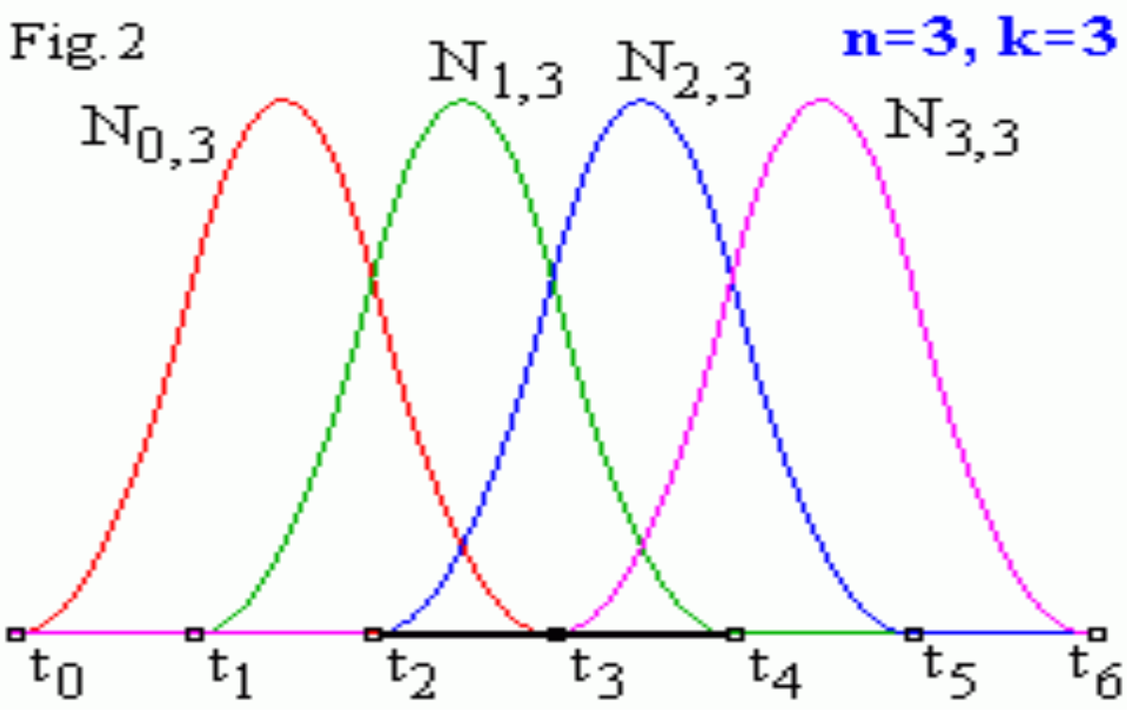

$$U = \{u_i\}_{i=-\infty}^{\infty}$$

$$N_{i,0}(u) = \begin{cases} 1 & u_i \le u < u_{i+1} \\ 0 & else \end{cases}$$

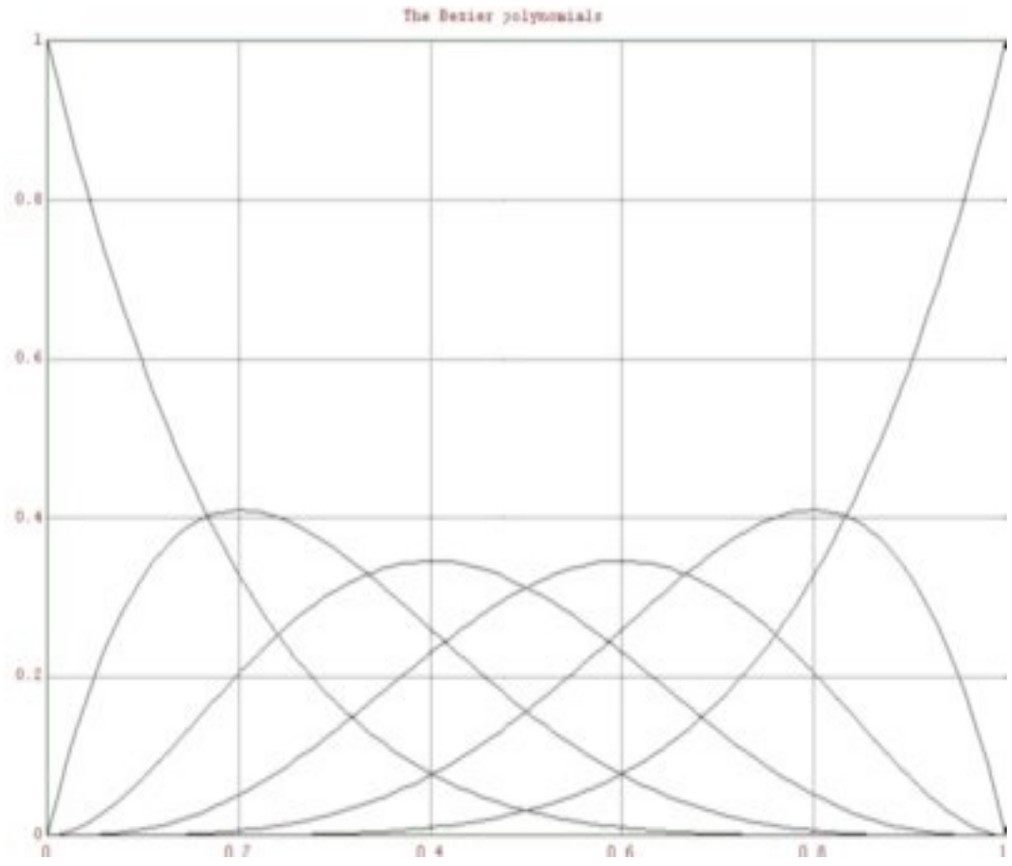
$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u),$$

$$\frac{0}{0} = 0$$

# B-spline basis v.s. Bernstein ~




$$B_{i,n}(t) = (1-t)B_{i,n-1}(t) + tB_{i-1,n-1}(t),\ i = 0,1,...,n.$$




$$N_{i,0}(u) = \begin{cases} 1 & u_i \le u < u_{i+1} \\ 0 & else \end{cases}$$

$$U = \{u_i\}_{i=-\infty}^{\infty}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u),$$

$$\frac{0}{0} = 0$$

# NURBS curve

## properties of B-spline basis

1. localization: $N_{i,p}(u)$ >0 only when u∈$[u_i,u_{i+p+1}]$ .

$$N_{i,p}(u) = \begin{cases} > 0, & u_i \le u < u_{i+p+1} \\ = 0, & u < u_i \text{ 或 } u > u_{i+p+1} \end{cases}$$

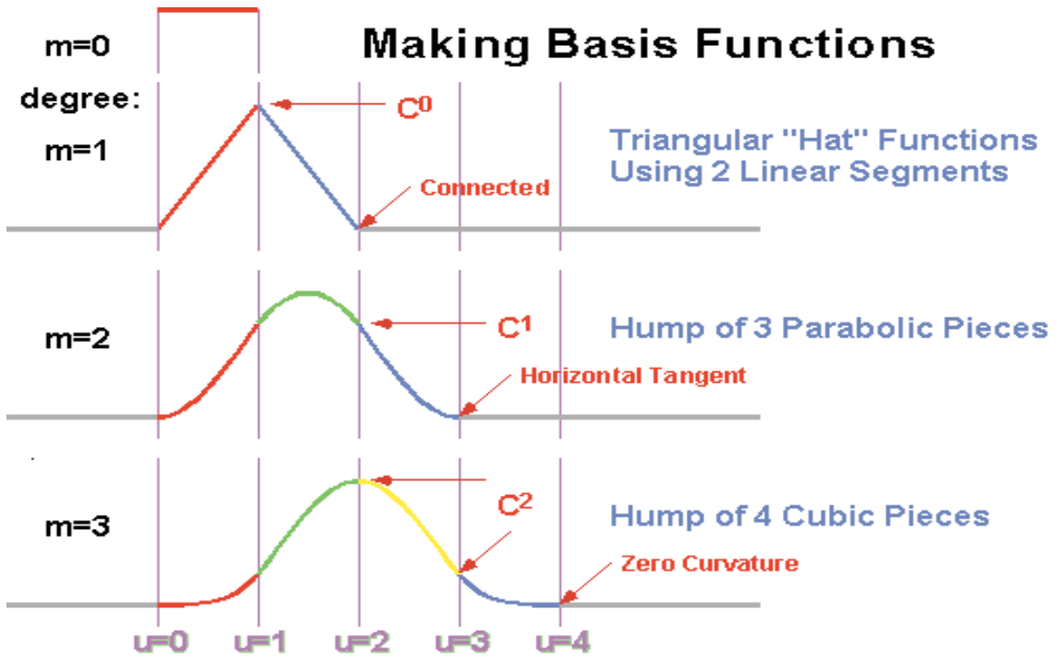
2. normalization:

$$\sum_{j=-\infty}^{\infty} N_{j,p}(u) = \sum_{j=i-p}^{i} N_{j,p}(u) = 1,\ u \in [u_i, u_{i+1})$$

3. piecewise polynomial: $N_{i,p}(u)$ is a polynomial with degree < p, in every $[u_j,u_{j+1})$

4. differential:

$$N'_{i,p}(u) = \frac{p}{u_{i+p} - u_i} N_{i,p-1}(u) - \frac{p}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$



# NURBS curve

## Properties of B-spline curve:

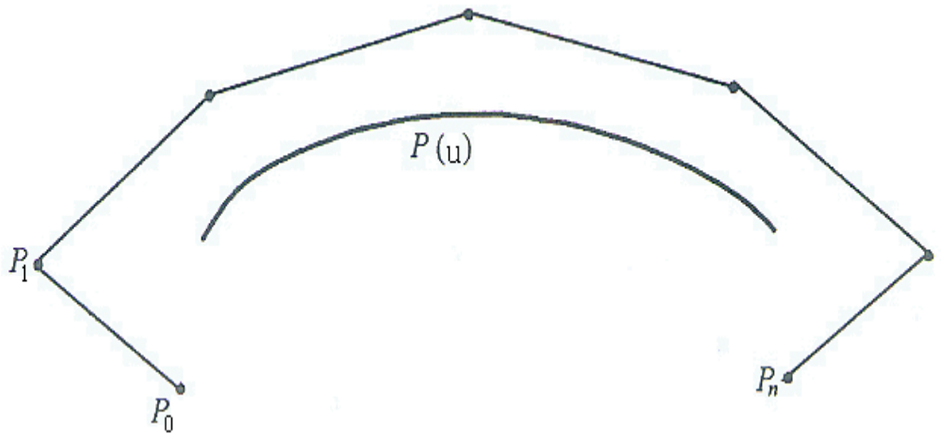
1. **Convex Hull Property**

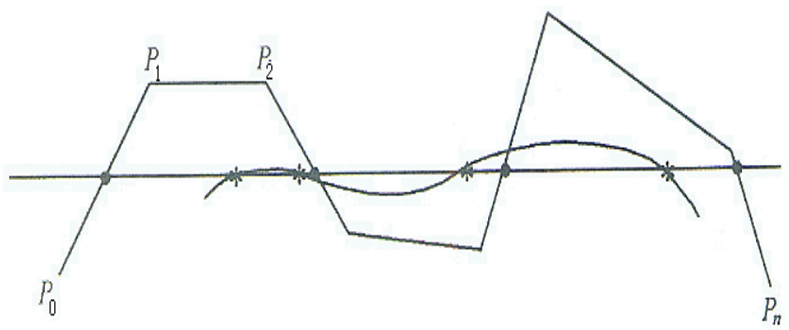
2. **variation diminishing property.**

3. **Affine Invariance**

4. **local**

5. **piecewise polynomial**



# NURBS curve

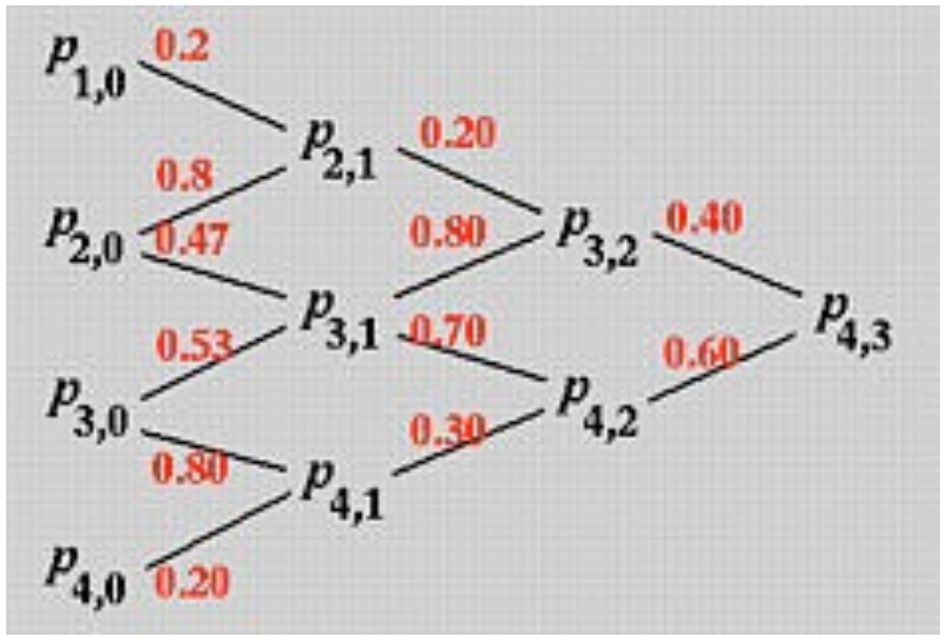
## B-spline---de Boor algorithm

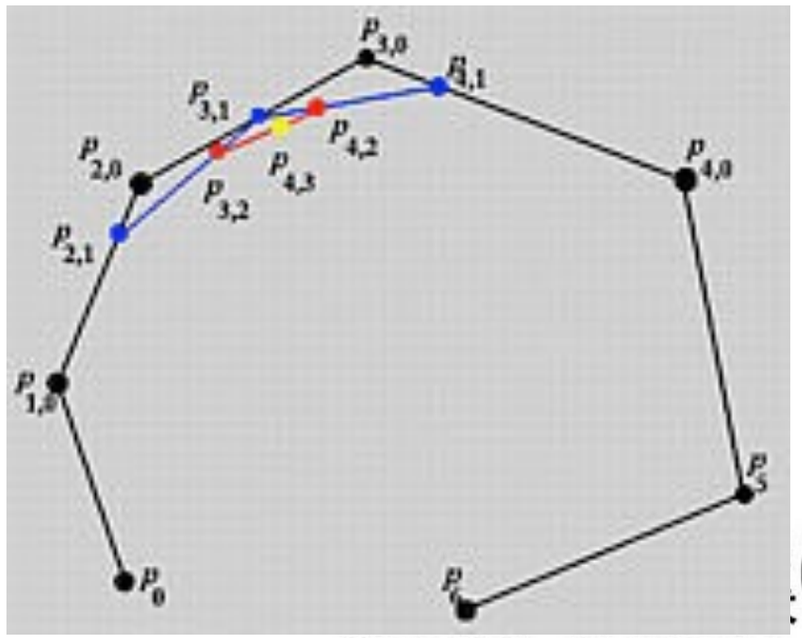
to calculate the point of B-spline curve $\boldsymbol{C}(u)$ at $u$:

1. find the interval where $u$ lies in : $u\in[u_j,u_{j+1})$;
2. curve in $u\in[u_j,u_{j+1})$ is only determined by $\boldsymbol{P}_{j-p}, \boldsymbol{P}_{j-p+1}, \ldots, \boldsymbol{P}_j$;
3. calculate

$$\boldsymbol{P}_i^r(u)=\begin{cases}\boldsymbol{P}_i & r=0, i=j-p;\ j-p+1,\mathrm{L}\ ,j;\\ \dfrac{u-u_i}{u_{i+k-r}-u_i}\boldsymbol{P}_i^{r-1}(u)+\dfrac{u_{i+k-r}-u}{u_{i+k-r}-u_i}\boldsymbol{P}_{i-1}^{r-1}(u), & r=1,2,\mathrm{L}\ k-1;\ i=j-p+r, j-p+r+1,\mathrm{L}\ ,j.\end{cases}$$

4. $\boldsymbol{P}_j^{k-1}(u)=C(u)$



# NURBS curve

## Catmull-Clark and Doo-Sabin subdivision

Start from

$$P^i = \left( \mathrm{L} \ , p^i_{-1}, p^i_0, p^i_1, p^i_2, \mathrm{L} \ \right)$$

Catmull-Clark rules

$$p^{i+1}_{2j} = \frac{1}{8} p^i_{j-1} + \frac{6}{8} p^i_j + \frac{1}{8} p^i_{j+1}$$

$$p^{i+1}_{2j+1} = \frac{4}{8} p^i_j + \frac{4}{8} p^i_{j+1}$$

Doo-Sabin rules:

$$p^{i+1}_{2j} = \frac{3}{4} p^i_j + \frac{1}{4} p^i_{j+1}$$

$$p^{i+1}_{2j+1} = \frac{1}{4} p^i_j + \frac{3}{4} p^i_{j+1}$$

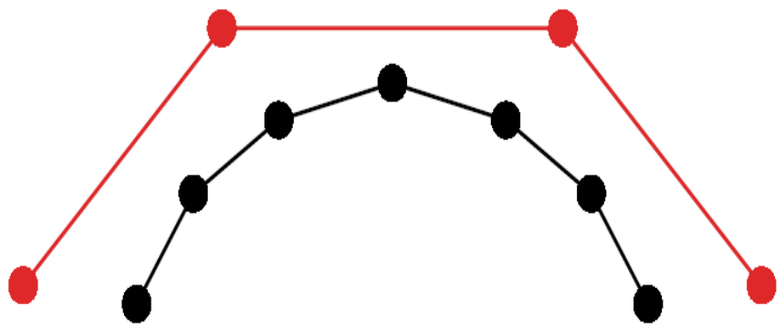


Figure 3: Subdividing an initial set of control points (upper, red) results in additional control points (lower, black), that more closely approximate a curve.

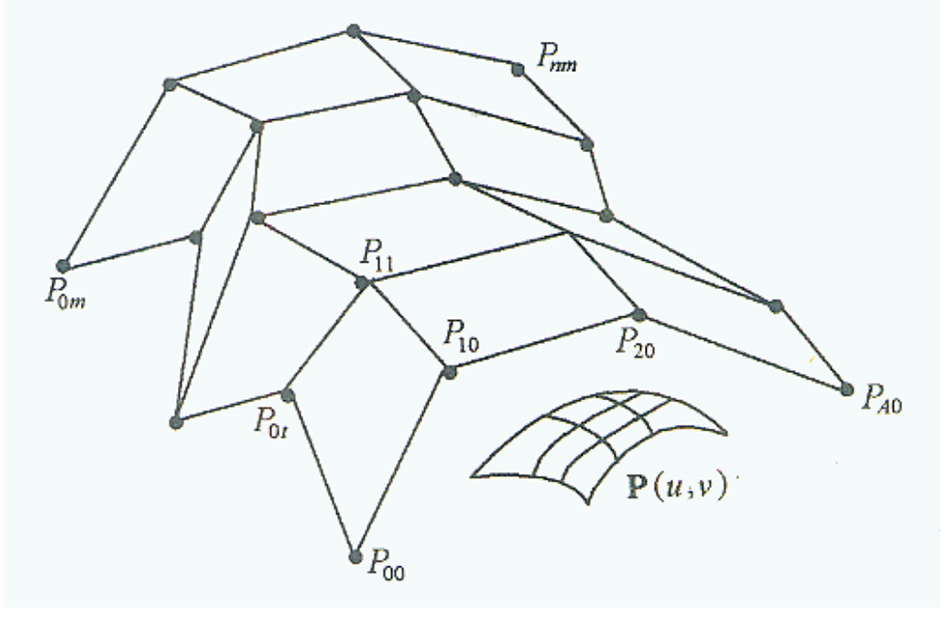
# B-spline surface

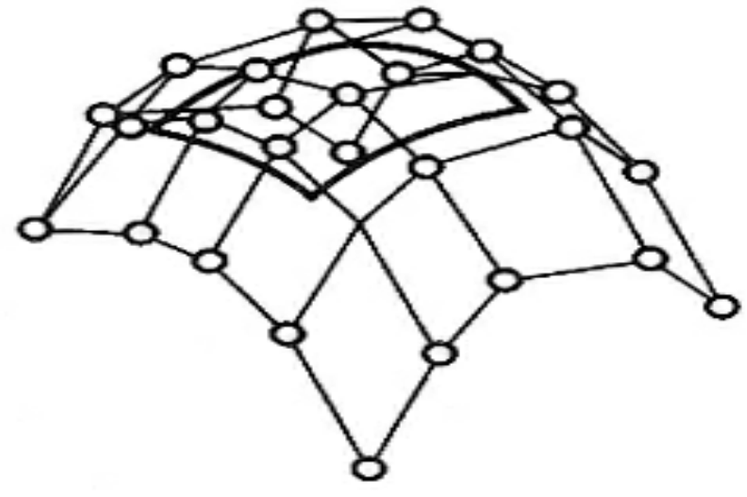
(n+1)×(m+1) control points: $\boldsymbol{P}_{i,j}$ (Degrees of $u, v$: $p, q$);

nodes: $U=[u_0,u_1,\ldots,u_{n+p+1}]$, $V = [v_0,v_1, \ldots, v_{m+q+1}]$,

Then a tensor B-spline surface with degree $p\times q$ :

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\boldsymbol{P}_{i,j}$$



# NURBS surface

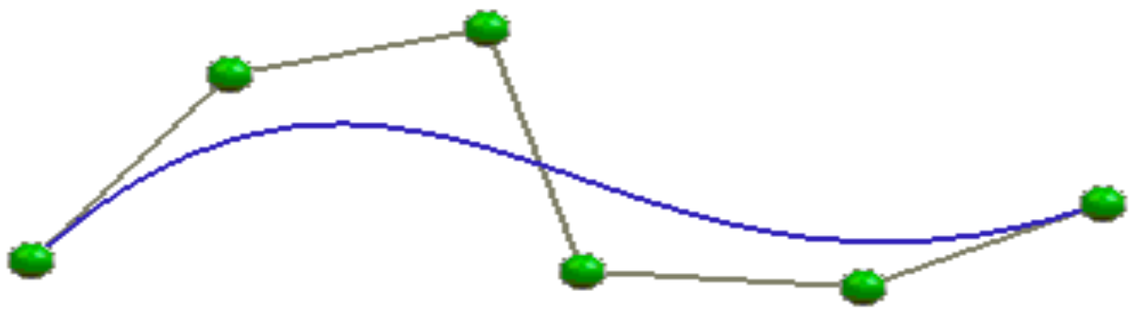
## NURBS (Non-uniform **Rational** B-spline)

NURBS curves:

$$\boldsymbol{C}(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u)\omega_i \boldsymbol{P}_i}{\sum_{i=0}^{n} N_{i,p}(u)\omega_i}, \quad a \le u \le b$$

$$U = \{\underbrace{a,\ldots,a}_{p+1}, u_{p+1},\ldots,u_{m-p-1}, \underbrace{b,\ldots,b}_{p+1}\}$$

重节点



# NURBS surface

## NURBS surface

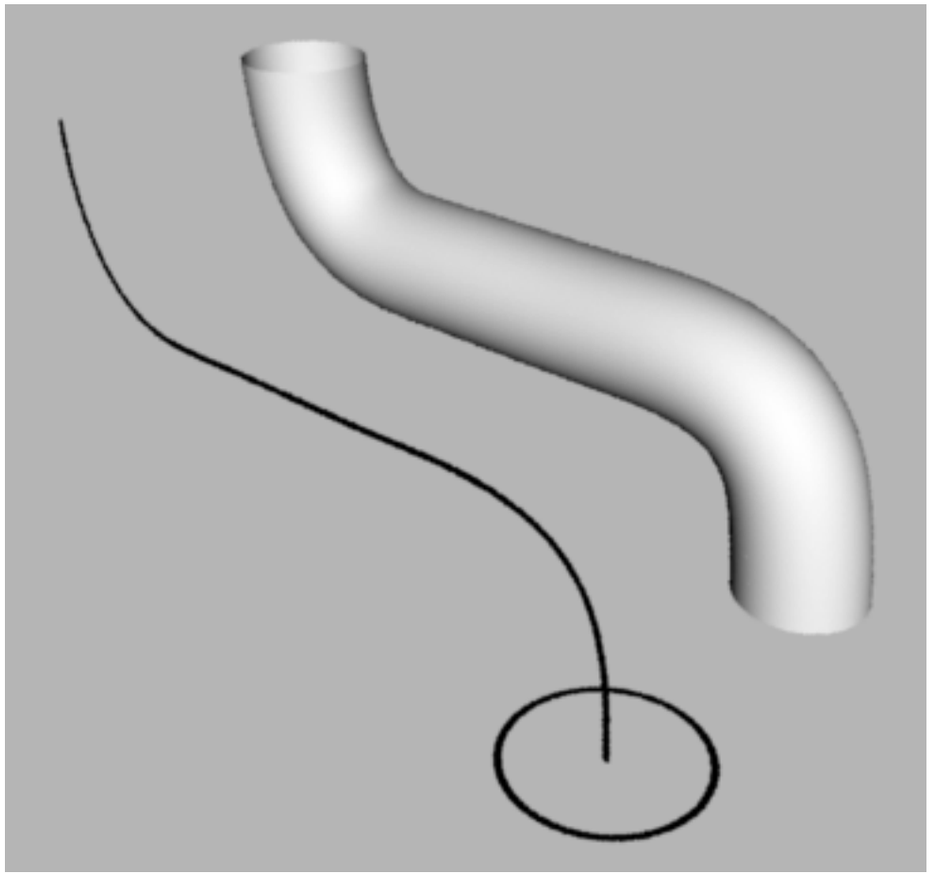
$$\boldsymbol{S}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\omega_{i,j}\boldsymbol{P}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\omega_{i,j}} \qquad 0 \le u, v \le 1$$

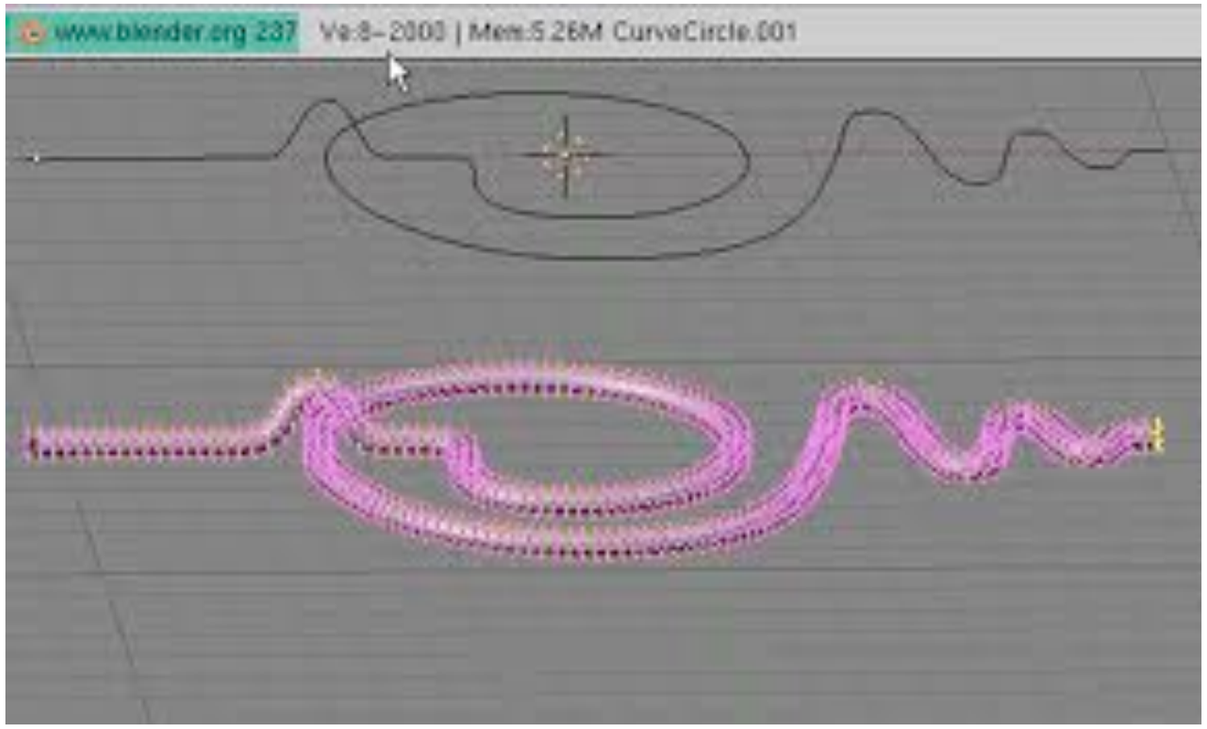
$\omega_{ij}$: weights

$$U = \{\underbrace{0,\ldots,0}_{p+1}, u_{p+1},\ldots,u_{r-p-1}, \underbrace{1,\ldots,1}_{p+1}\}$$

$$V = \{\underbrace{0,\ldots,0}_{q+1}, v_{q+1},\ldots,v_{s-q-1}, \underbrace{1,\ldots,1}_{q+1}\}$$

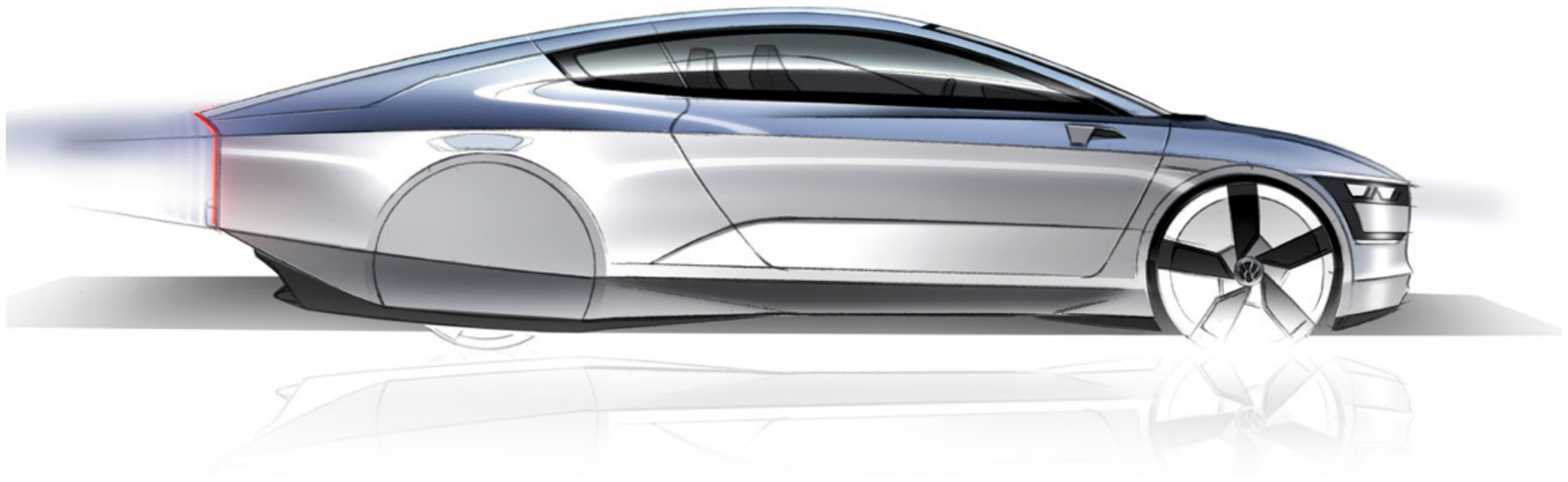
# Sweeping





# Homework 4

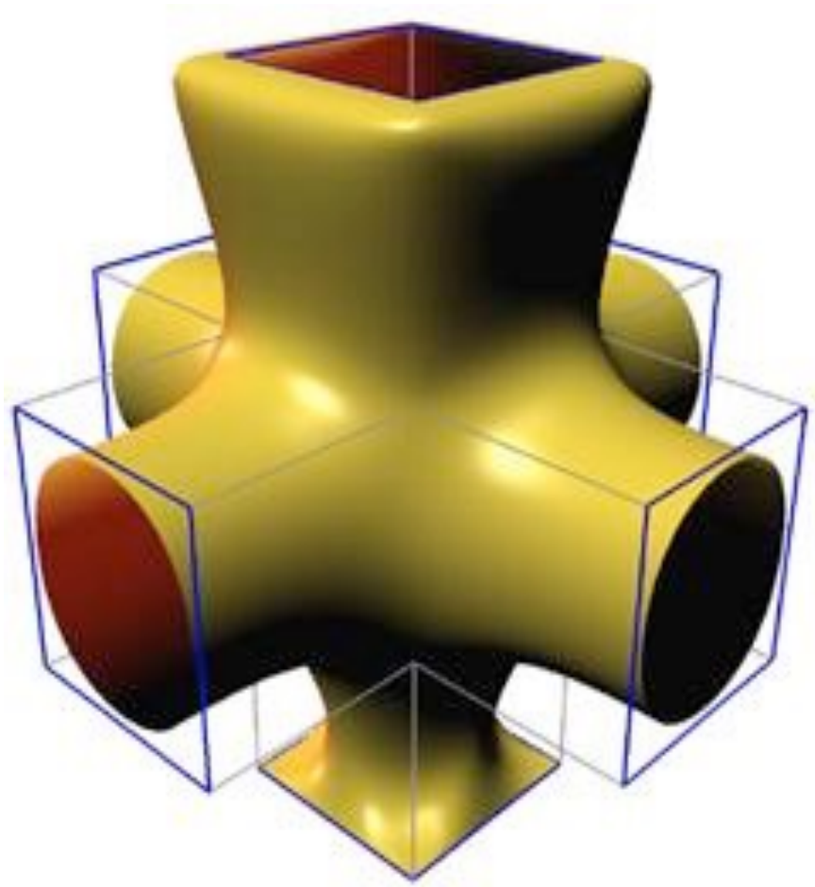
- Draw a circle by Cubic Rational Bezier Curves
  - Please give a detailed implementation in OpenGL/ WebGL
  - Key: to understand what is weight in the rational spline representations

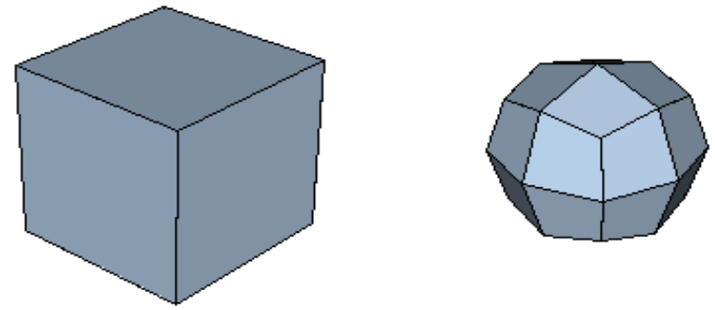


## CS3621 Introduction to Computing with Geometry Notes

- http://pages.mtu.edu/~shene/COURSES/cs3621/NOTES/

# More complex objects?



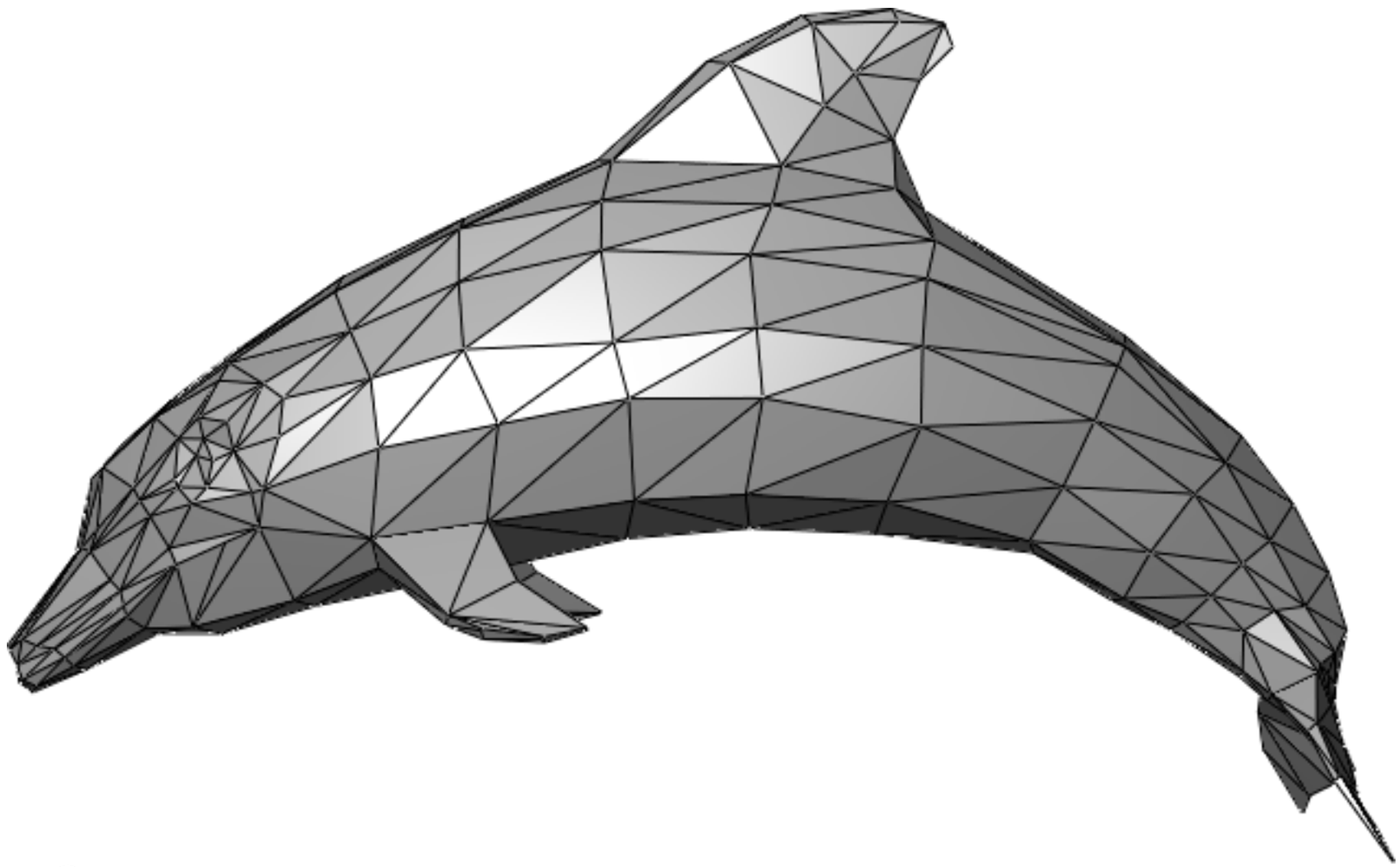


Challenge: surface continuity?

# Polygonal Mesh

# What is polygonal mesh?

- A polygon mesh is a collection of vertices, edges and faces that defines the shape of a polyhedral object in 3D computer graphics and solid modeling



# What is polygonal mesh?

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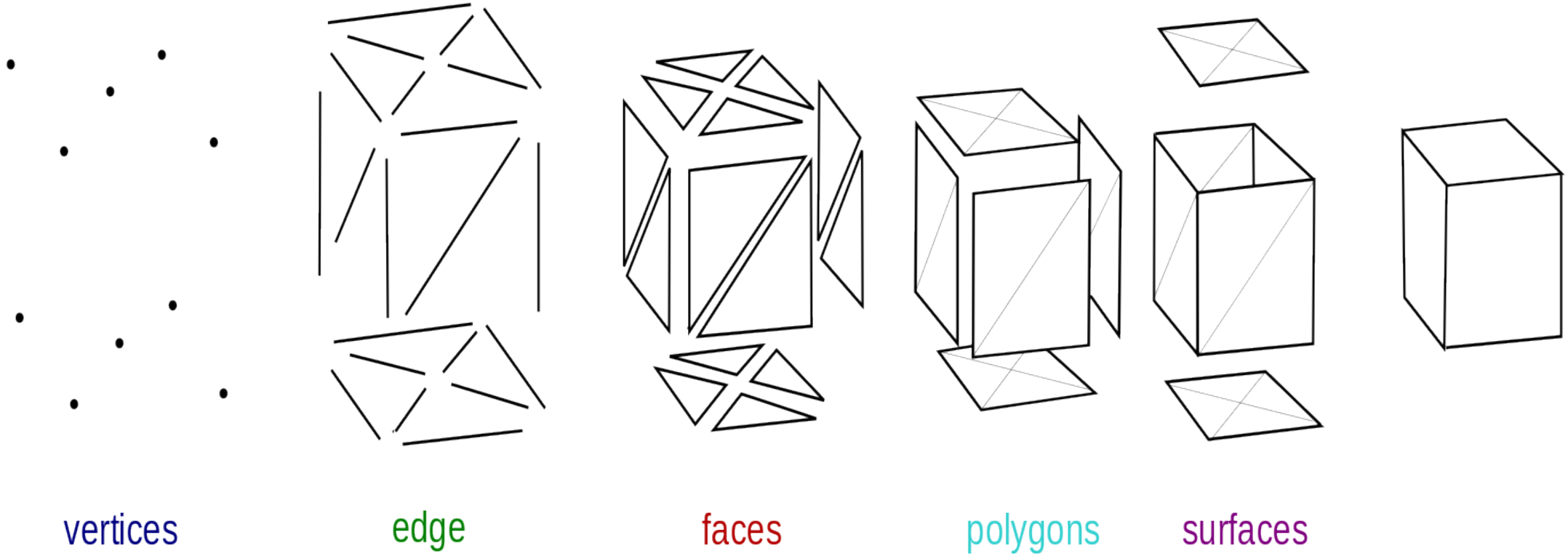
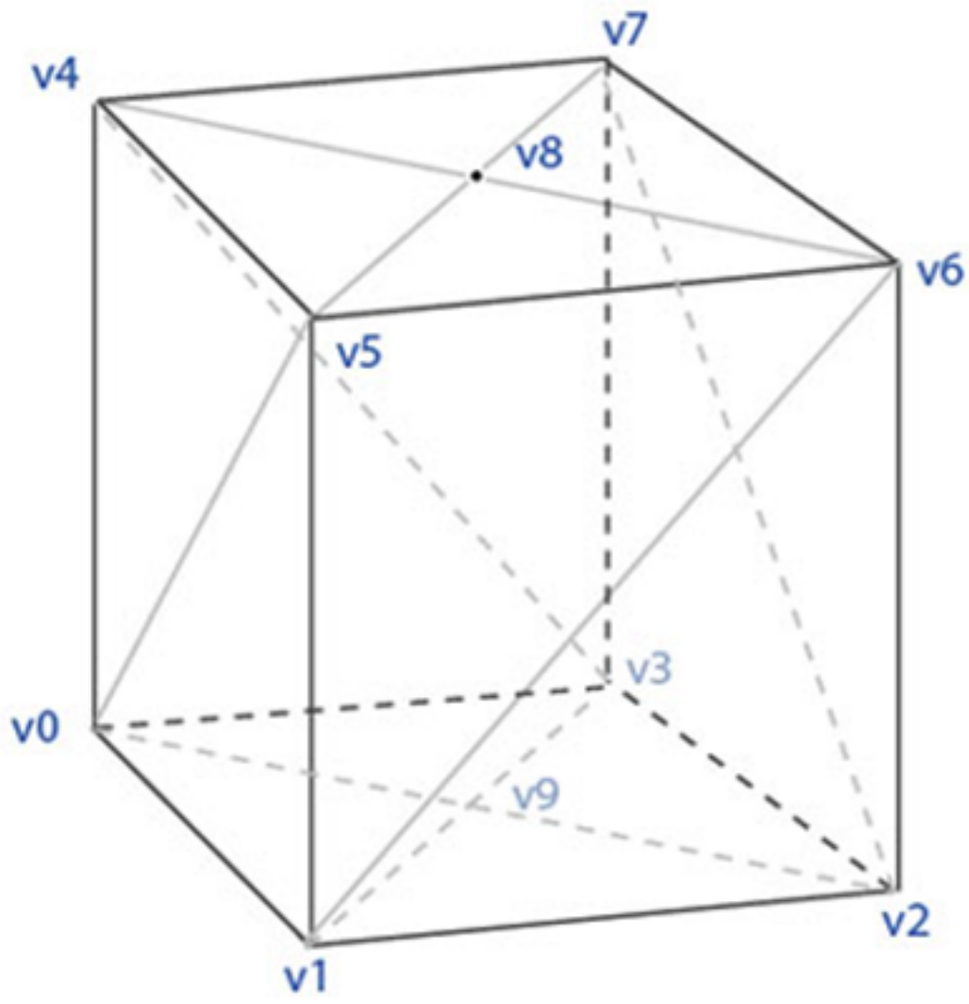


# polygonal mesh representations

## Vertex-Vertex Meshes (VV)

Vertex List

| | | |
|---|---|---|
| v0 | 0,0,0 | v1 v5 v4 v3 v9 |
| v1 | 1,0,0 | v2 v6 v5 v0 v9 |
| v2 | 1,1,0 | v3 v7 v6 v1 v9 |
| v3 | 0,1,0 | v2 v6 v7 v4 v9 |
| v4 | 0,0,1 | v5 v0 v3 v7 v8 |
| v5 | 1,0,1 | v6 v1 v0 v4 v8 |
| v6 | 1,1,1 | v7 v2 v1 v5 v8 |
| v7 | 0,1,1 | v4 v3 v2 v6 v8 |
| v8 | .5,.5,1 | v4 v5 v6 v7 |
| v9 | .5,.5,0 | v0 v1 v2 v3 |



# polygonal mesh representations

## Face-Vertex Meshes

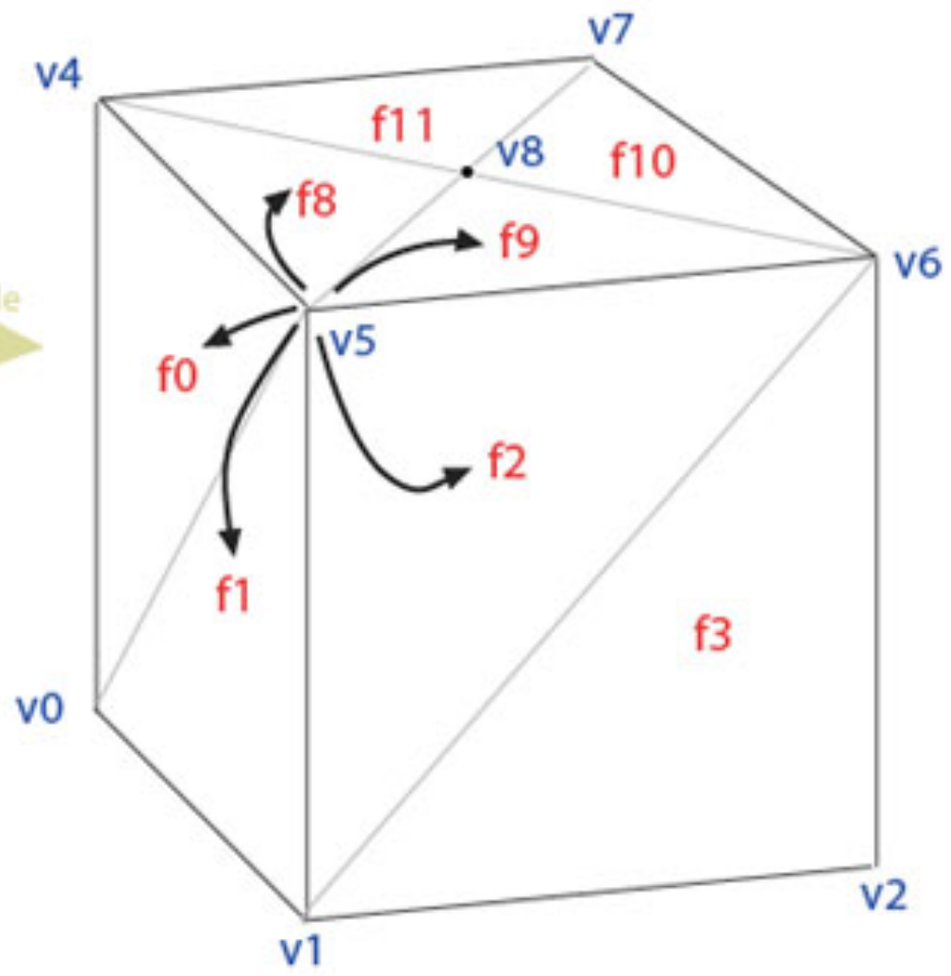
Face List

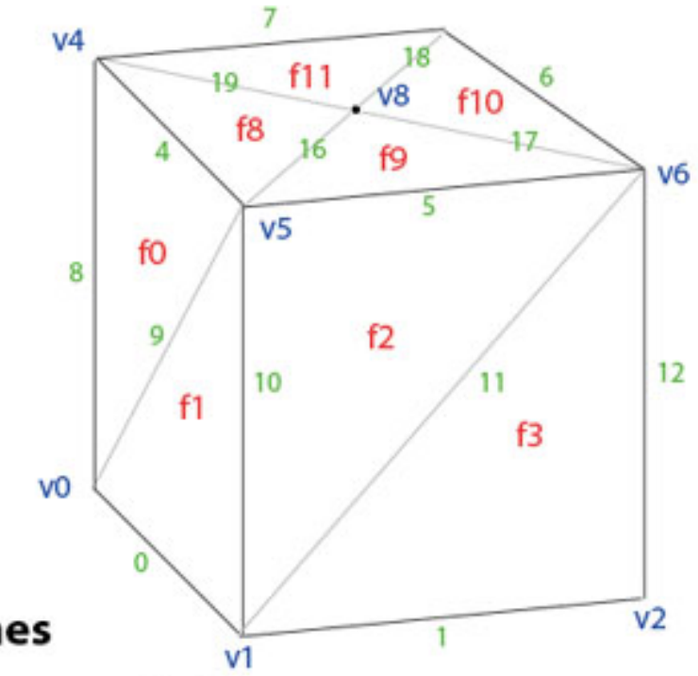
| | |
|---|---|
| f0 | v0 v4 v5 |
| f1 | v0 v5 v1 |
| f2 | v1 v5 v6 |
| f3 | v1 v6 v2 |
| f4 | v2 v6 v7 |
| f5 | v2 v7 v3 |
| f6 | v3 v7 v4 |
| f7 | v3 v4 v0 |
| f8 | v8 v5 v4 |
| f9 | v8 v6 v5 |
| f10 | v8 v7 v6 |
| f11 | v8 v4 v7 |
| f12 | v9 v5 v4 |
| f13 | v9 v6 v5 |
| f14 | v9 v7 v6 |
| f15 | v9 v4 v7 |

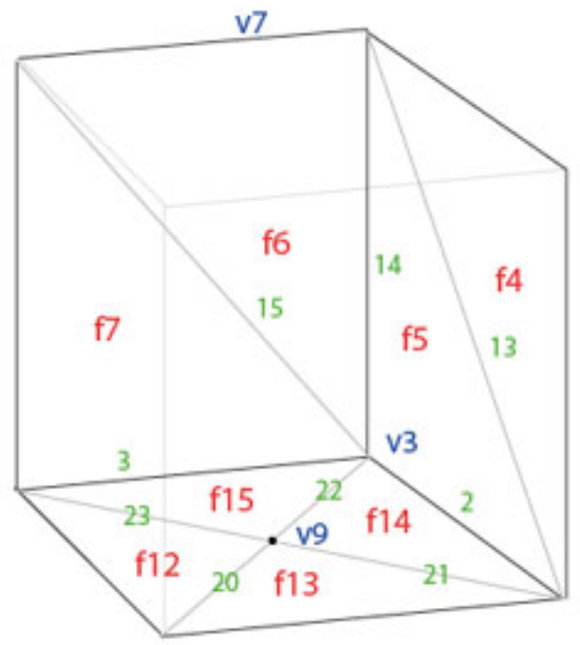
Vertex List

| | | |
|---|---|---|
| v0 | 0,0,0 | f0 f1 f12 f15 f7 |
| v1 | 1,0,0 | f2 f3 f13 f12 f1 |
| v2 | 1,1,0 | f4 f5 f14 f13 f3 |
| v3 | 0,1,0 | f6 f7 f15 f14 f5 |
| v4 | 0,0,1 | f6 f7 f0 f8 f11 |
| v5 | 1,0,1 | f0 f1 f2 f9 f8 |
| v6 | 1,1,1 | f2 f3 f4 f10 f9 |
| v7 | 0,1,1 | f4 f5 f6 f11 f10 |
| v8 | .5,.5,0 | f8 f9 f10 f11 |
| v9 | .5,.5,1 | f12 13 14 15 |

example






# Winged-Edge Meshes

Face List

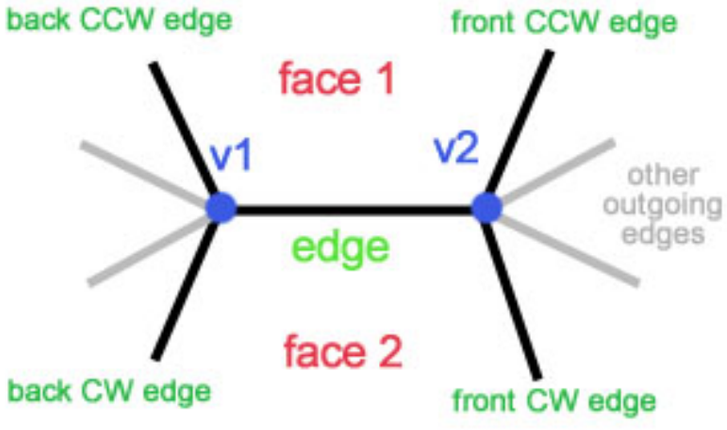
| | |
|---|---|
| f0 | 4 8 9 |
| f1 | 0 10 9 |
| f2 | 5 10 11 |
| f3 | 1 12 11 |
| f4 | 6 12 13 |
| f5 | 2 14 13 |
| f6 | 7 14 15 |
| f7 | 3 8 15 |
| f8 | 4 16 19 |
| f9 | 5 17 16 |
| f10 | 6 18 17 |
| f11 | 7 19 18 |
| f12 | 0 23 20 |
| f13 | 1 20 21 |
| f14 | 2 21 22 |
| f15 | 3 22 23 |

Edge List

| | | | |
|---|---|---|---|
| e0 | v0 v1 | f1 f12 | 9 23 10 20 |
| e1 | v1 v2 | f3 f13 | 11 20 12 21 |
| e2 | v2 v3 | f5 f14 | 13 21 14 22 |
| e3 | v3 v0 | f7 f15 | 15 22 8 23 |
| e4 | v4 v5 | f0 f8 | 19 8 16 9 |
| e5 | v5 v6 | f2 f9 | 16 10 17 11 |
| e6 | v6 v7 | f4 f10 | 17 12 18 13 |
| e7 | v7 v4 | f6 f11 | 18 14 19 15 |
| e8 | v0 v4 | f7 f0 | 3 9 7 4 |
| e9 | v0 v5 | f0 f1 | 8 0 4 10 |
| e10 | v1 v5 | f1 f2 | 0 11 9 5 |
| e11 | v1 v6 | f2 f3 | 10 1 5 12 |
| e12 | v2 v6 | f3 f4 | 1 13 11 6 |
| e13 | v2 v7 | f4 f5 | 12 2 6 14 |
| e14 | v3 v7 | f5 f6 | 2 15 13 7 |
| e15 | v3 v4 | f6 f7 | 14 3 7 15 |
| e16 | v5 v8 | f8 f9 | 4 5 19 17 |
| e17 | v6 v8 | f9 f10 | 5 6 16 18 |
| e18 | v7 v8 | f10 f11 | 6 7 17 19 |
| e19 | v4 v8 | f11 f8 | 7 4 18 16 |
| e20 | v1 v9 | f12 f13 | 0 1 23 21 |
| e21 | v2 v9 | f13 f14 | 1 2 20 22 |
| e22 | v3 v9 | f14 f15 | 2 3 21 23 |
| e23 | v0 v9 | f15 f12 | 3 0 22 20 |

Vertex List

| | | |
|---|---|---|
| v0 | 0,0,0 | 8 9 0 23 3 |
| v1 | 1,0,0 | 10 11 1 20 0 |
| v2 | 1,1,0 | 12 13 2 21 1 |
| v3 | 0,1,0 | 14 15 3 22 2 |
| v4 | 0,0,1 | 8 15 7 19 4 |
| v5 | 1,0,1 | 10 9 4 16 5 |
| v6 | 1,1,1 | 12 11 5 17 6 |
| v7 | 0,1,1 | 14 13 6 18 7 |
| v8 | .5,.5,0 | 16 17 18 19 |
| v9 | .5,.5,1 | 20 21 22 23 |




Winged Edge Structure

# Wavefront .obj file

```
# List of Vertices, with (x,y,z[,w]) coordinates, w is optional and defaults to 1.0.
  v 0.123 0.234 0.345 1.0
  v ...
  ...
  # Texture coordinates, in (u ,v [,w]) coordinates, these will vary between 0 and 1, w is optional and
default to 0.
  vt 0.500 1 [0]
  vt ...
  ...
  # Normals in (x,y,z) form; normals might not be unit.
.
  vn 0.707 0.000 0.707
  vn ...
  ...
  # Parameter space vertices in ( u [,v] [,w] ) form; free form geometry statement ( see below )
  vp 0.310000 3.210000 2.100000
  vp ...
  ...
  # Face Definitions (see below)
  f 1 2 3
  f 3/1 4/2 5/3
  f 6/4/1 3/5/3 7/6/5
  f ...
  ...
```

# Wavefront .obj file

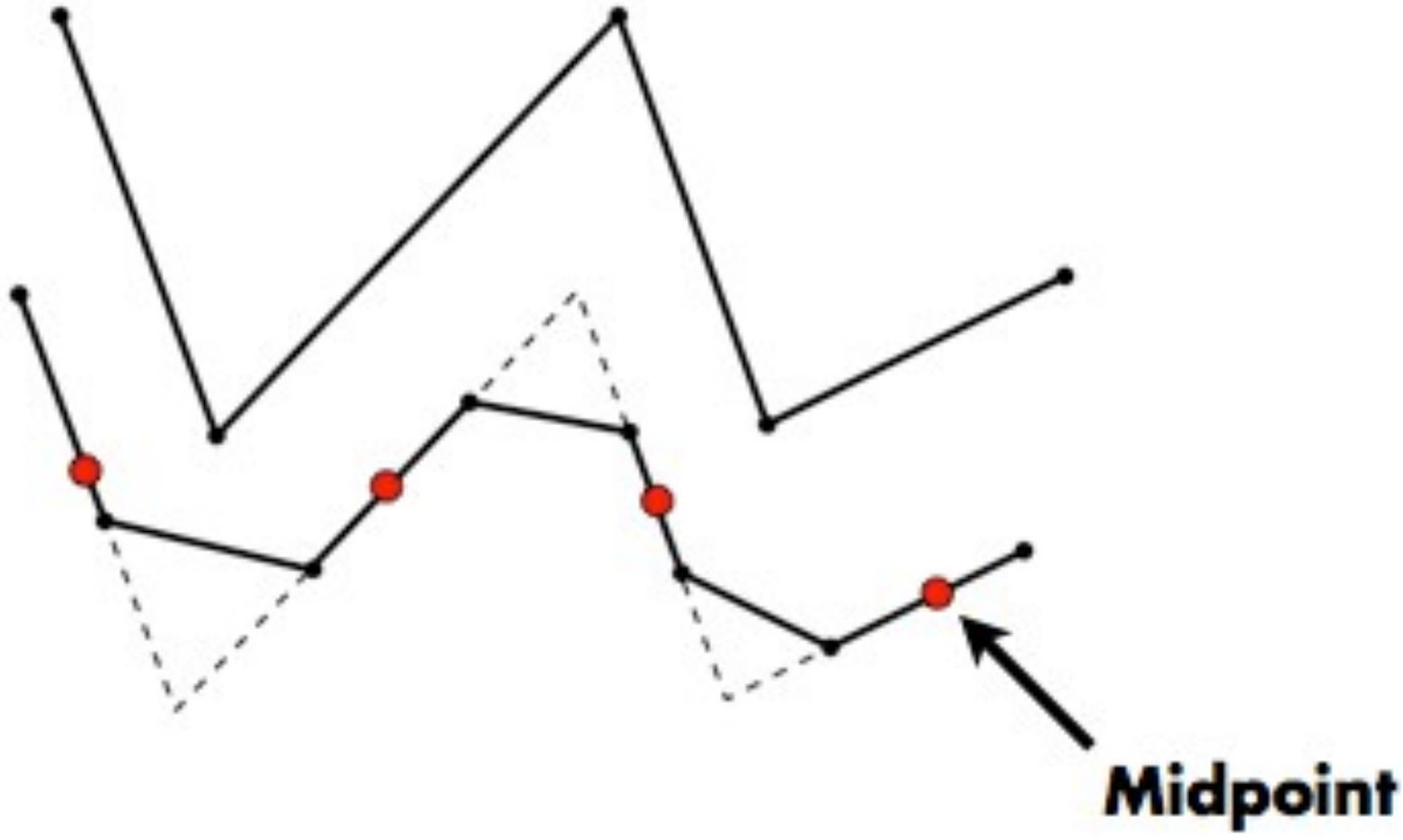
- Vertex positions
  - v, vt, vn
- Face definitions
  - f v1 v2 v3 v4 ...
  - f v1/vt1 v2/vt2 v3/vt3 ...
  - f v1/vt1/vn1 v2/vt2/vn2 v3/vt3/vn3 ...
  - f v1//vn1 v2//vn2 v3//vn3 ...
- Referencing materials
  - mtllib [external .mtl file name]
  - usemtl [material name]

# Wavefront .obj file

- Named objects and polygon groups are specified via the following tags.
  - o [object name]
  - g [group name]
- Smooth shading across polygons is enabled by smoothing groups.
  - s 1
  - ...
  - # Smooth shading can be disabled as well.
  - s off
  - ...

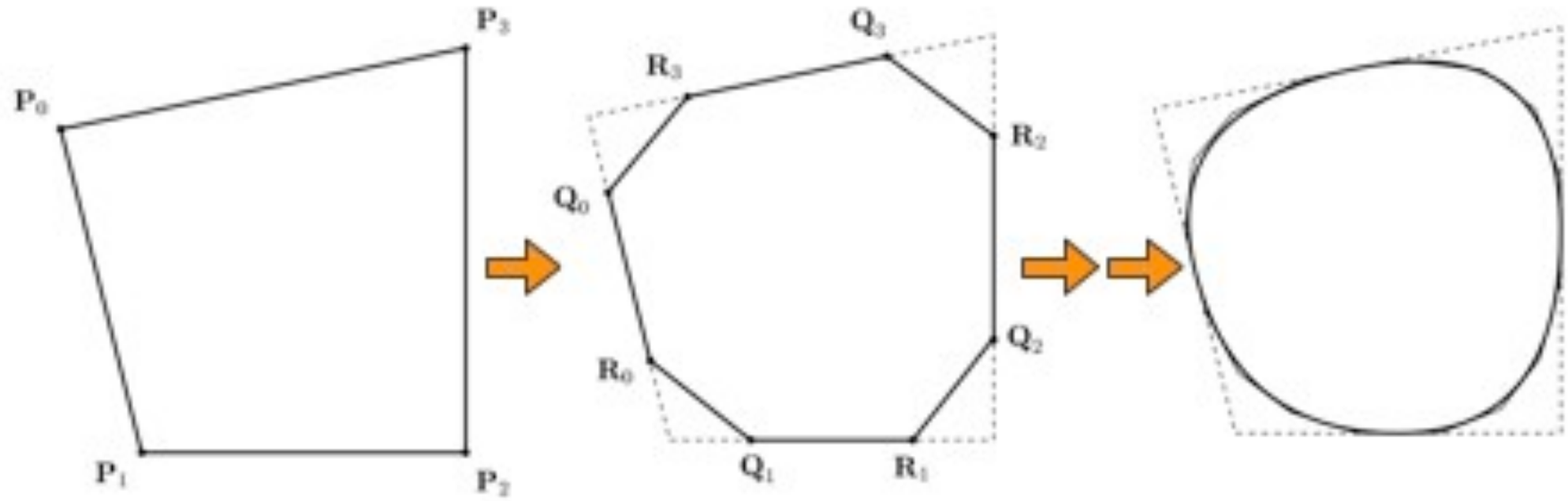
# Subdivision surfaces

# Corner Cutting Algorithm




Chaiken (1974)

# Procedural Curve

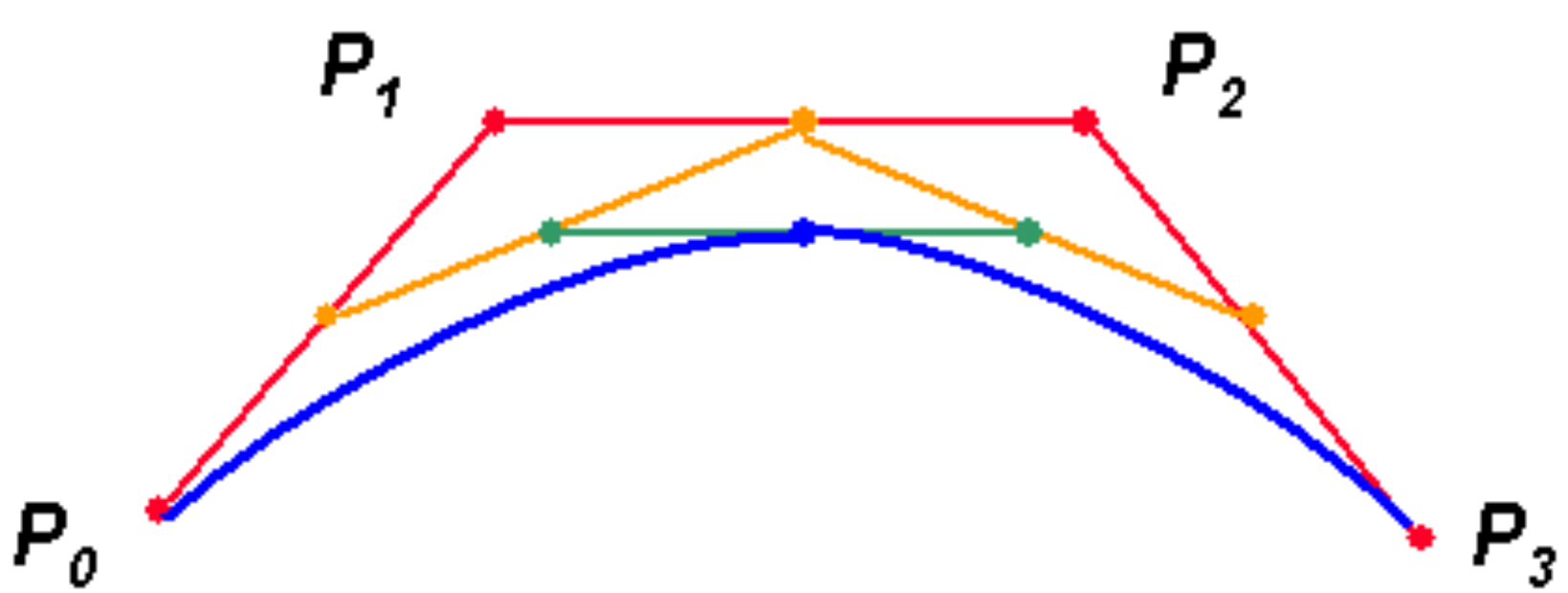



**Repeatedly cutting corners generates a limit curve**
**1. Interpolates midpoints**
**2. Tangent preserved at midpoints**

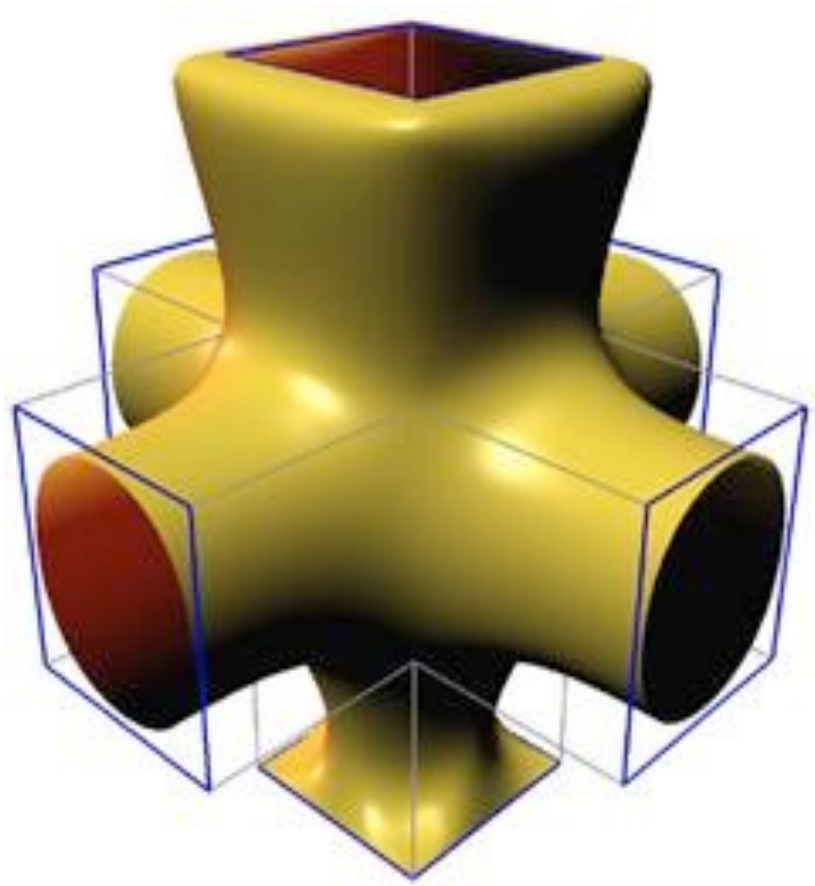
# subdivision surface

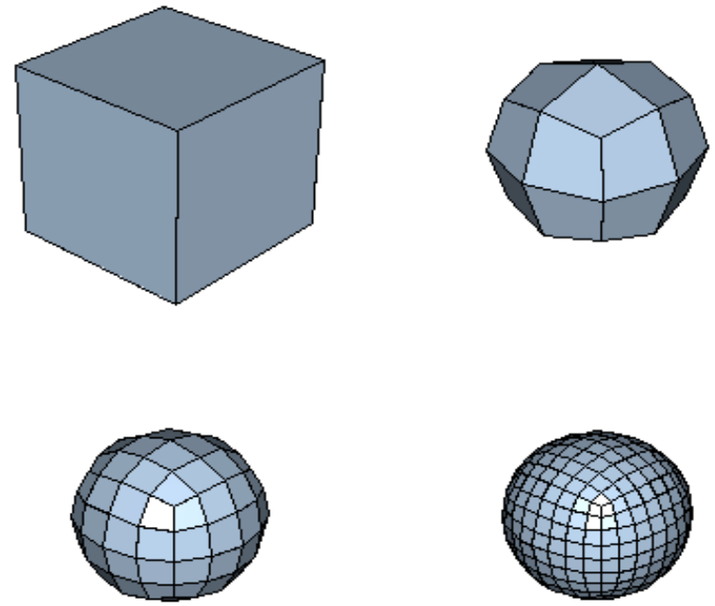
subdivision curves:

- starting from a set of points, generate new points in every step under some rules, when such step goes on infinitely, the points will be convergent to a smooth curve.



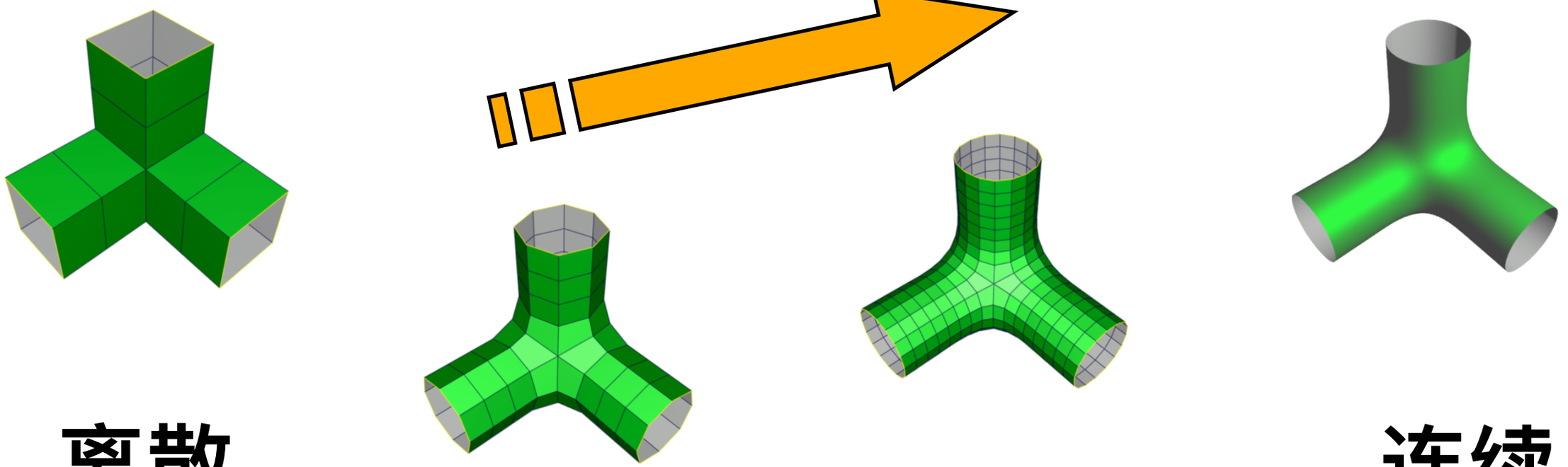
# subdivision surface





# 细分曲面的基本概念

- 是一种过程式（迭代）的曲面构造方法
  - 输入： 多边形网格 => 控制网格
  - 输出： a. 加密网格 => 有限次迭代 [用于绘制]
    b. 连续曲面 => 极限曲面 [用于分析]

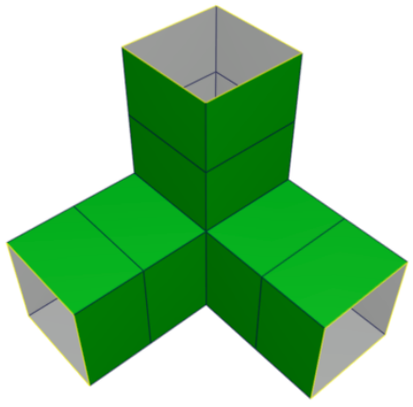


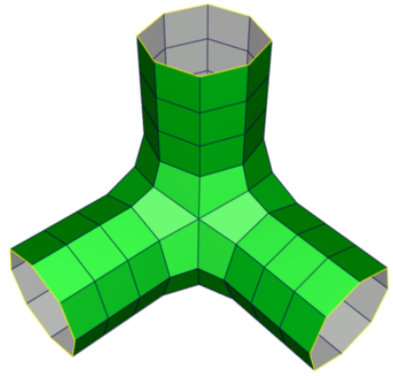
离散

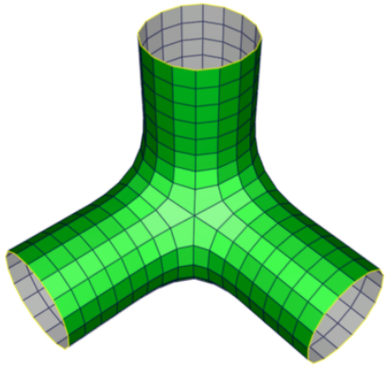
连续

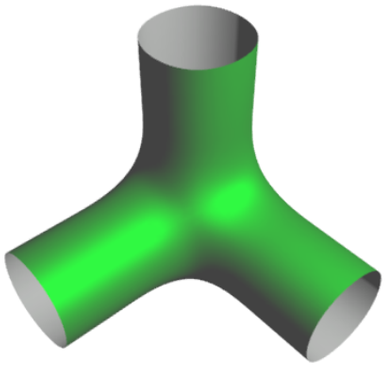
# Classical subdivision schemes

- Catmull-Clark surface. [Catmull 1978]
- Doo-Sabin surface. [Doo 1978]
- Loop's subdivision schemes. [Loop 1987]
- Butterfly subdivision schemes. [Dyn 1990]









**离散**

**连续**

# advantage of subdivision surface

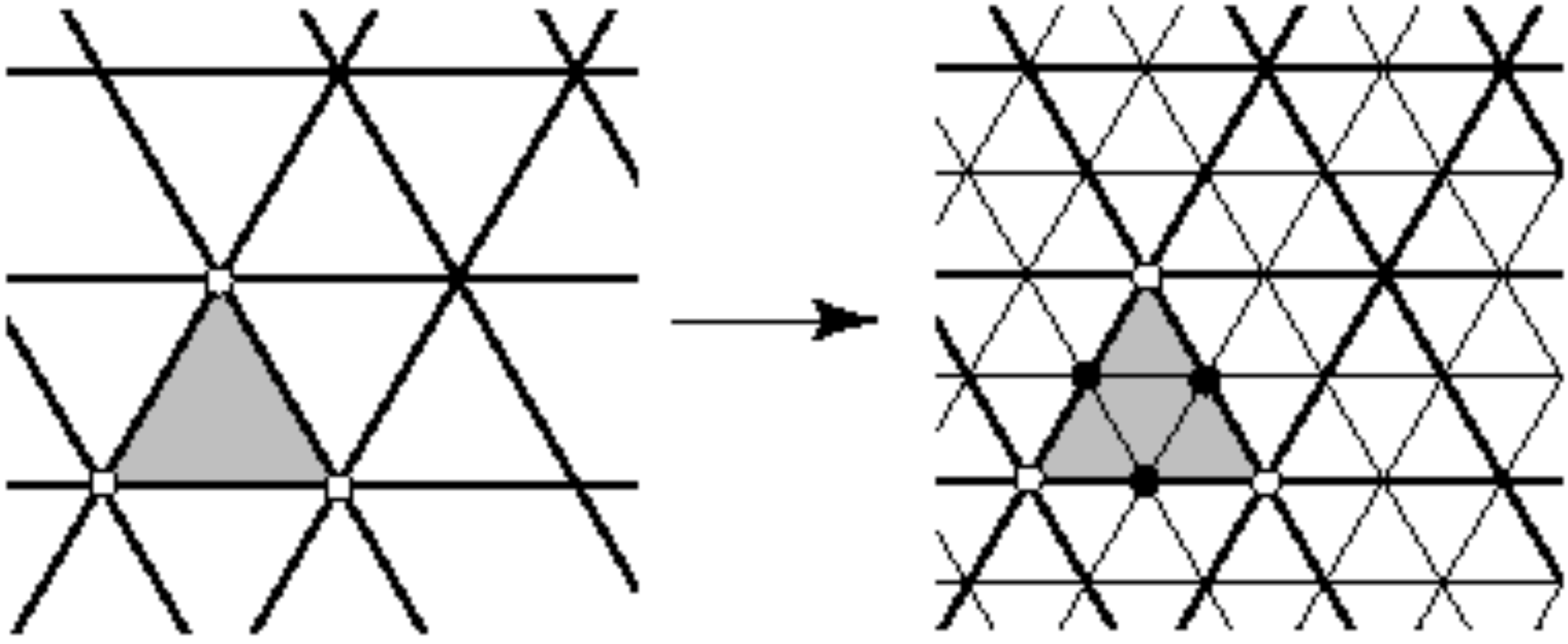
- **topologically complex shape**
- **stable;**
- **easy to implement;**
- **no need to merge between th surfaces**
- **LOD**

# 细分曲面的基本步骤

- 细分操作：
  - 拓扑规则：加密采样，重新构造网格
  - 几何规则：光顺网格

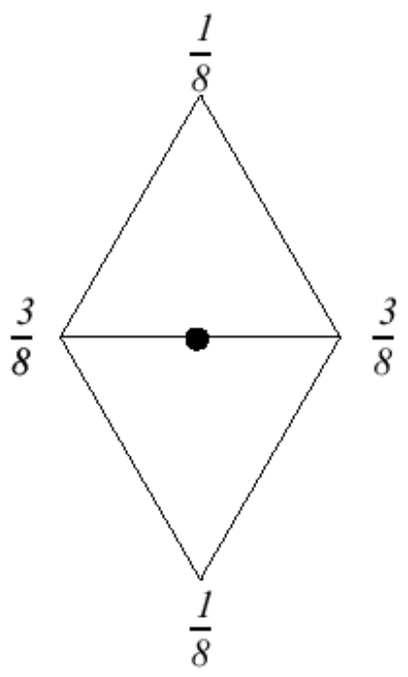
# Loop subdivision surface

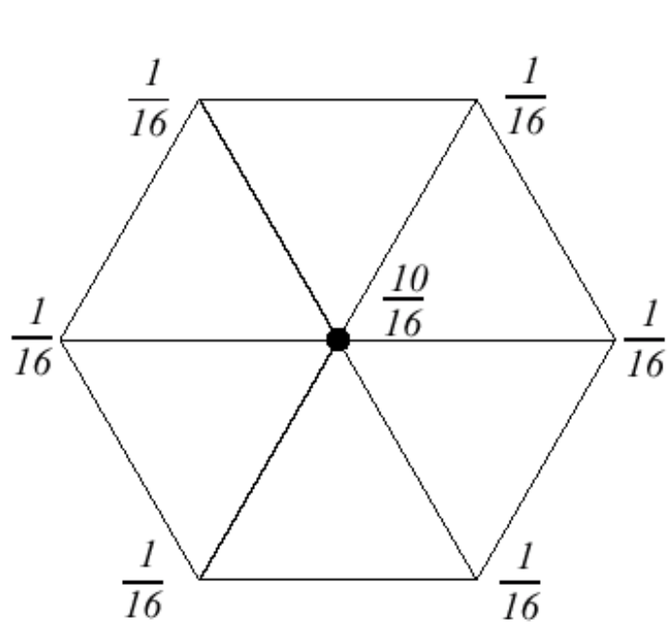
- 拓扑规则：怎样加密三角网格?
  - –对分每条边，并连接新顶点
  - –将每个三角形剖分成四个更小的三角形



# Loop subdivision surface

- 几何规则：怎样放置新顶点的位置?
  - –利用原始网格中与新顶点相邻的顶点来做**加权平均**



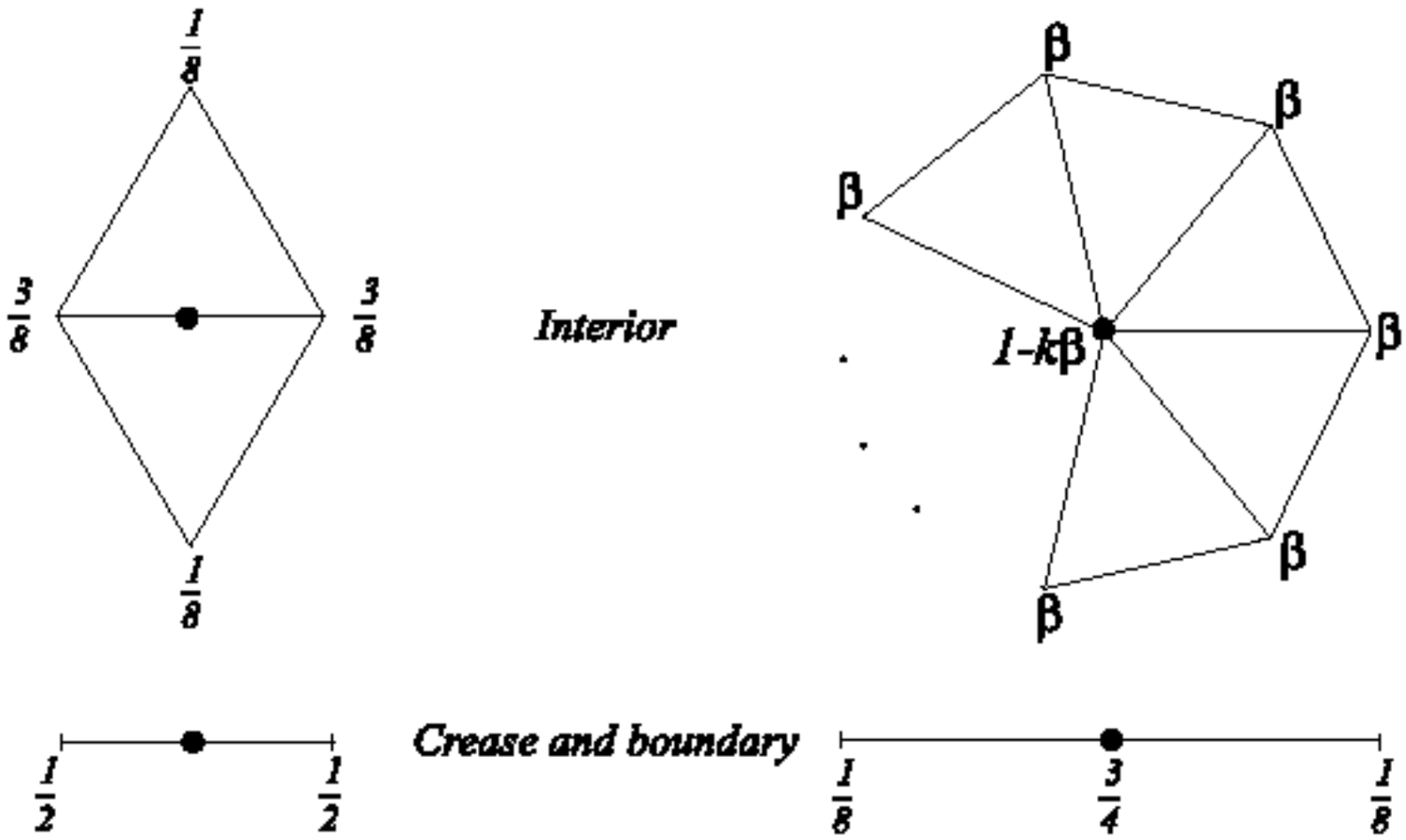




如果顶点的连接度不是6该怎么办?

# Loop subdivision surface

- 几何规则：怎样放置新顶点的位置?

  –奇异顶点处的规则




a. *Masks for odd vertices*

b. *Masks for even vertices*

# Loop subdivision surface

- 怎样选择系数Beta?

  –分析极限曲面性质

  –与曲面的连续性和光滑性相关

–需要计算相关（细分系数）矩阵的特征结构

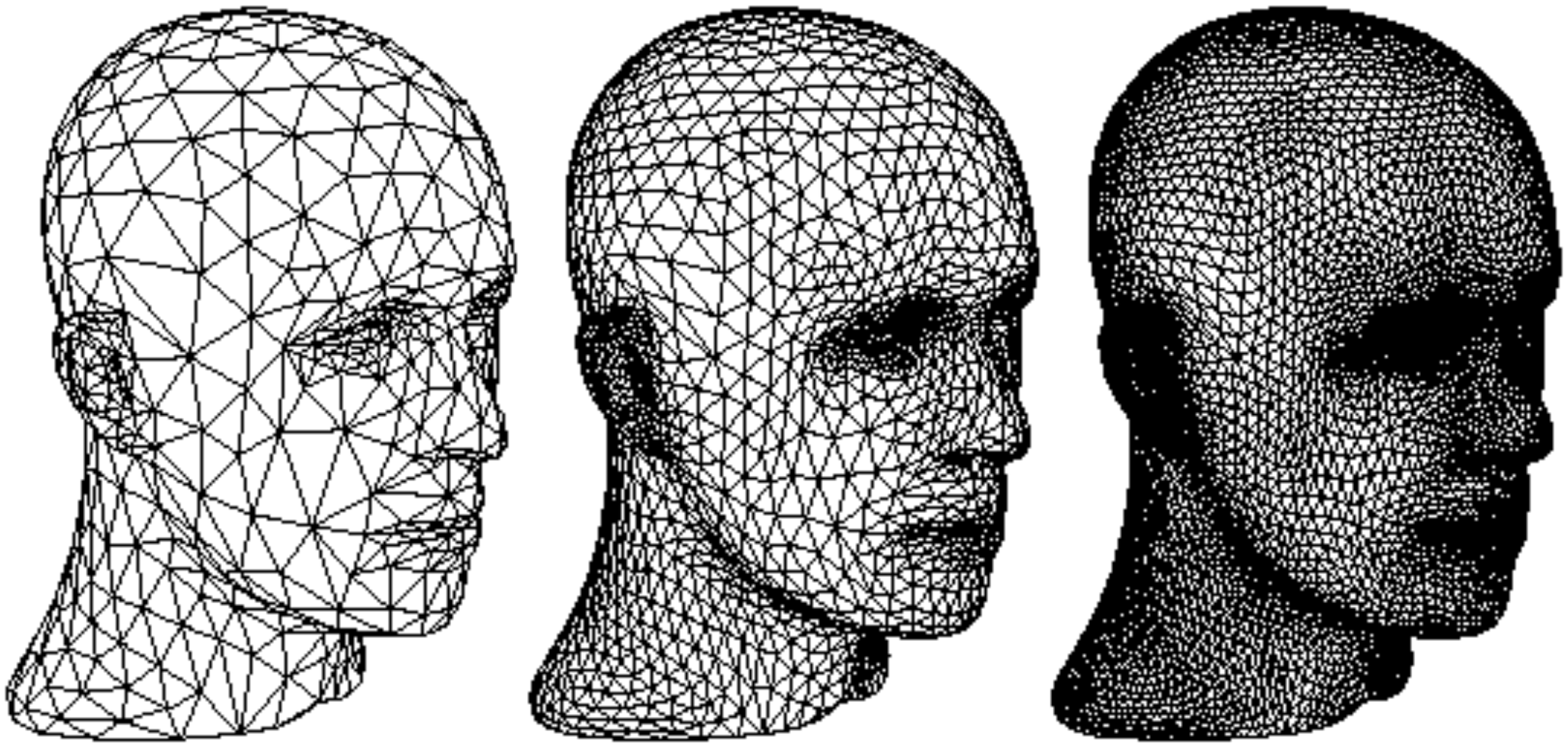
» Original Loop

$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

» Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

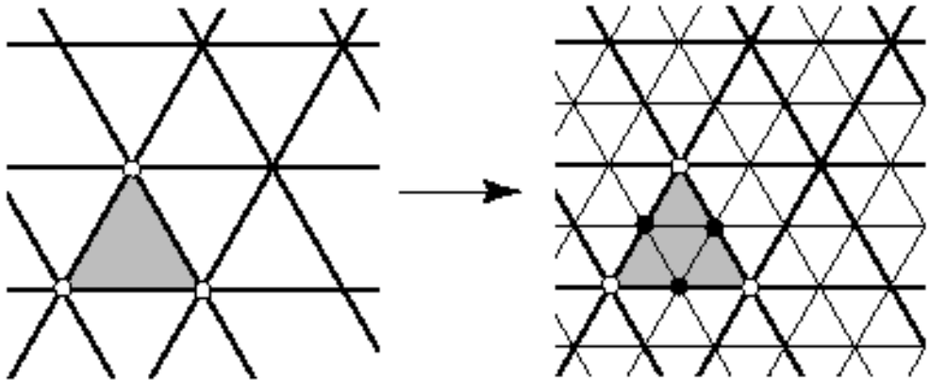
# Loop subdivision surface



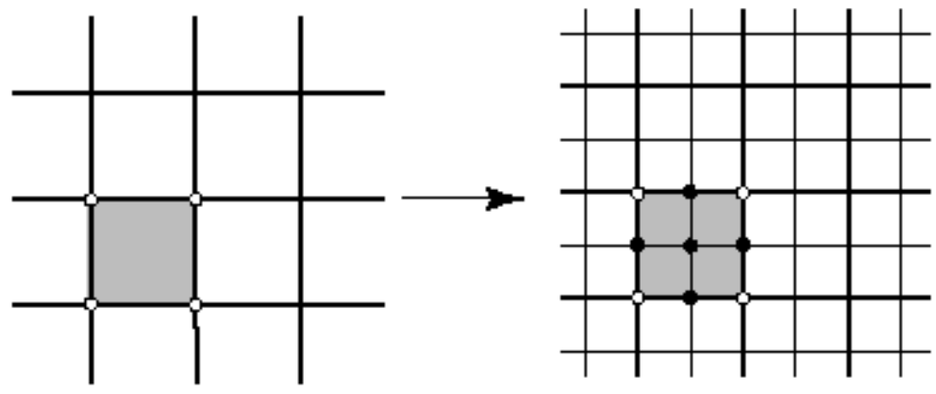
细分结果可达到较好的连续性光顺性！

# Subdivision schemes

- 各种不同的细分方法
  - 不同的处理拓扑加密方法
  - 不同的布置顶点位置方法
    - 逼近 vs. 插值



*Face split for triangles*

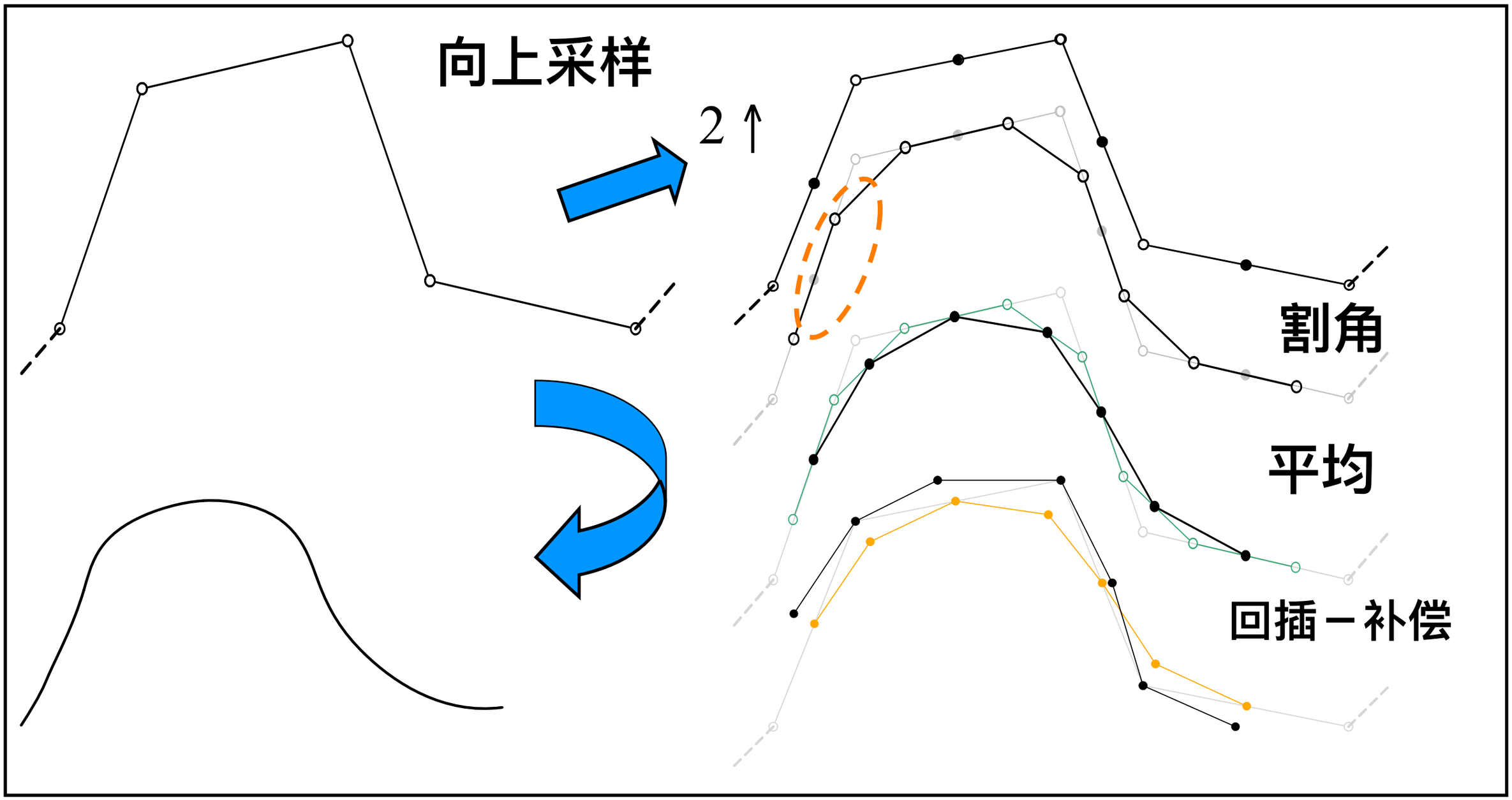


*Face split for quads*

| Face split | | |
|---|---|---|
| | *Triangular meshes* | *Quad. meshes* |
| *Approximating* | Loop ($C^2$) | Catmull-Clark ($C^2$) |
| *Interpolating* | Mod. Butterfly ($C^1$) | Kobbelt ($C^1$) |

| Vertex split |
|---|
| Doo-Sabin, Midedge ($C^1$) |
| Biquartic ($C^2$) |

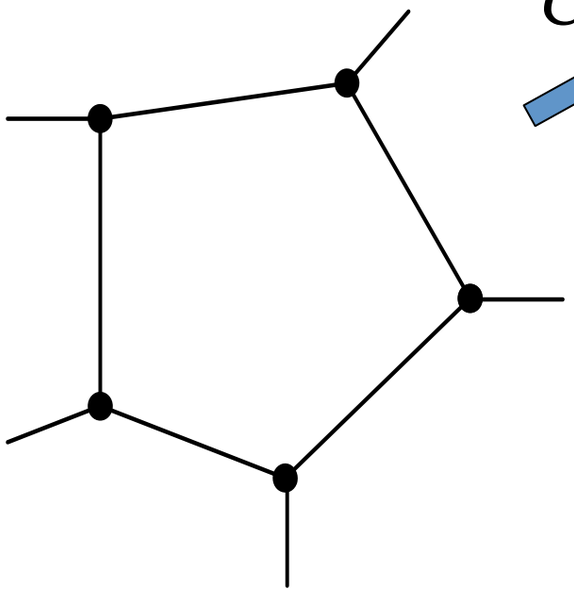
# 细分曲面的基本思想



# 四边网格上的拓扑规则

$C(\alpha_n) := D(0)D(\alpha_n)$

Catmull-Clark 型



Doo-Sabin 型

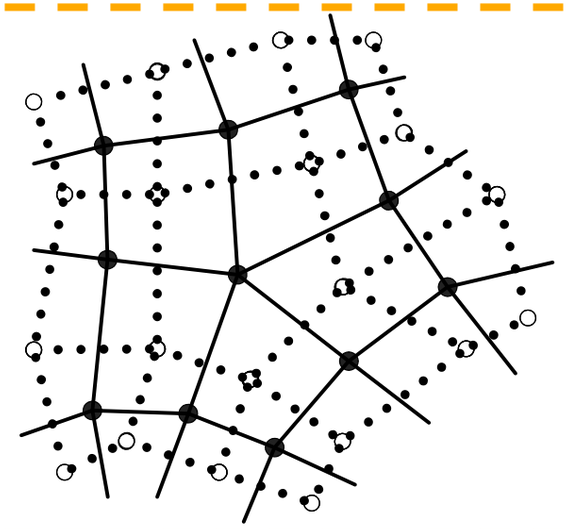
$U$

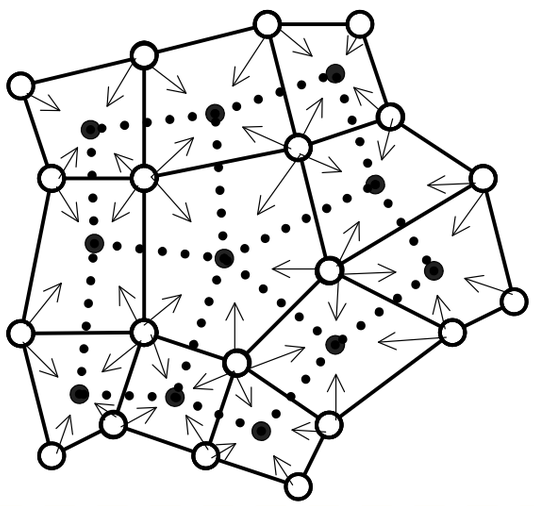
$D(\alpha_n)$

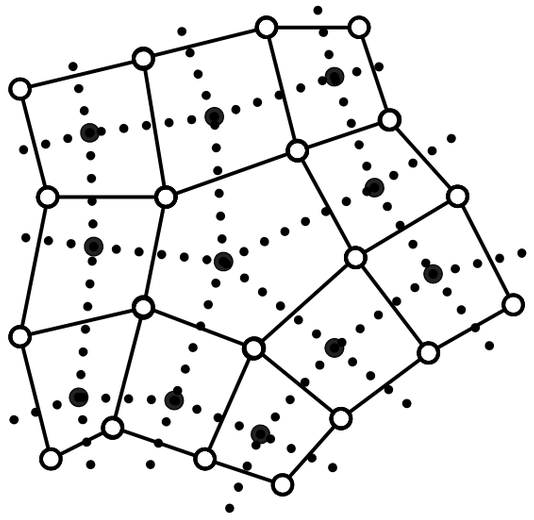
$D(0)$

Primal

Dual



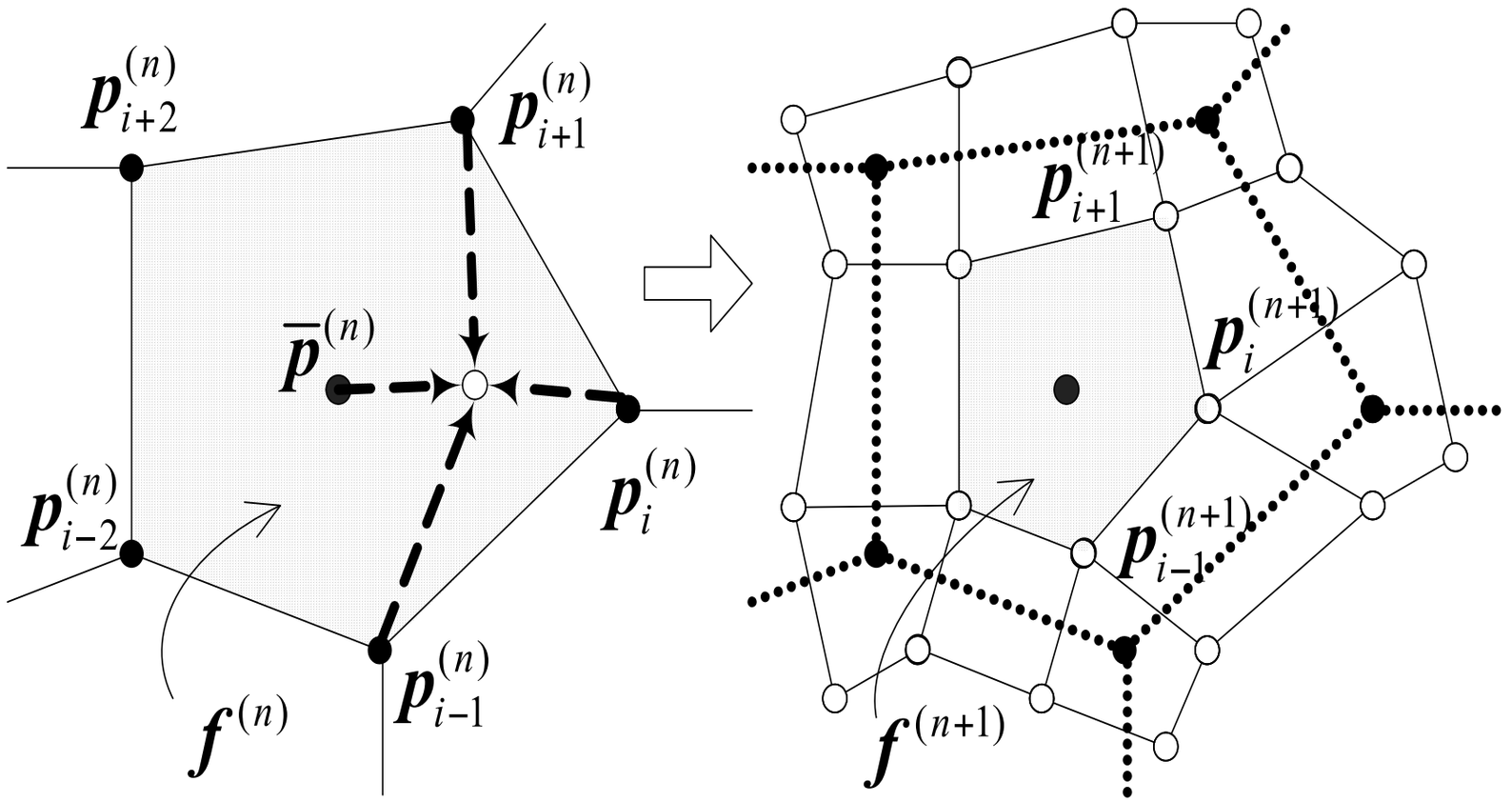




$D := D(f(.);\alpha_n);$

$\alpha_n := 2^{-n}\alpha_0.$

# Doo-Sabin型细分曲面




$$\boldsymbol{p}_i^{(n+1)} = \frac{f_n}{(1+f_n)}\boldsymbol{p}_i^{(n)} + \frac{f_n}{2(1+f_n)^2}(\boldsymbol{p}_{i-1}^{(n)} + \boldsymbol{p}_{i+1}^{(n)}) + \frac{1}{N(1+f_n)^2}\sum_{i=1}^{N}\boldsymbol{p}_i^{(n)}.$$

# subdivision surface

## Catmull-Clark and Doo-Sabin subdivision

start from

$$P^i = \left(\mathrm{L}\ , p^i_{-1}, p^i_0, p^i_1, p^i_2, \mathrm{L}\ \right)$$

Catmull-Clark rules

$$p^{i+1}_{2j} = \frac{1}{8} p^i_{j-1} + \frac{6}{8} p^i_j + \frac{1}{8} p^i_{j+1}$$

$$p^{i+1}_{2j+1} = \frac{4}{8} p^i_j + \frac{4}{8} p^i_{j+1}$$

Doo-Sabin rules:

$$p^{i+1}_{2j} = \frac{3}{4} p^i_j + \frac{1}{4} p^i_{j+1}$$

$$p^{i+1}_{2j+1} = \frac{1}{4} p^i_j + \frac{3}{4} p^i_{j+1}$$

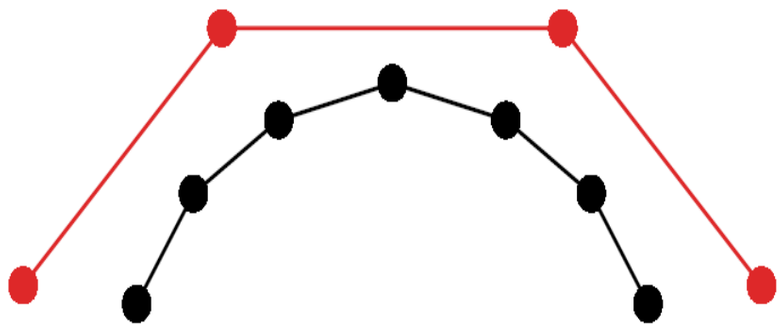
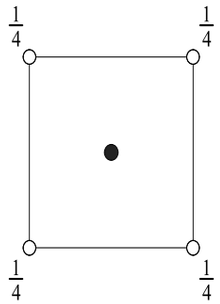
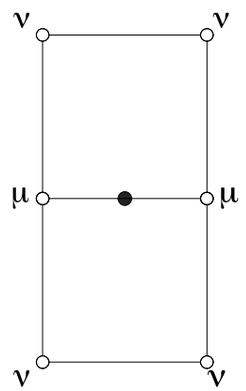


Figure 3: Subdividing an initial set of control points (upper, red) results in additional control points (lower, black), that more closely approximate a curve.

# Catmull-Clark subdivision

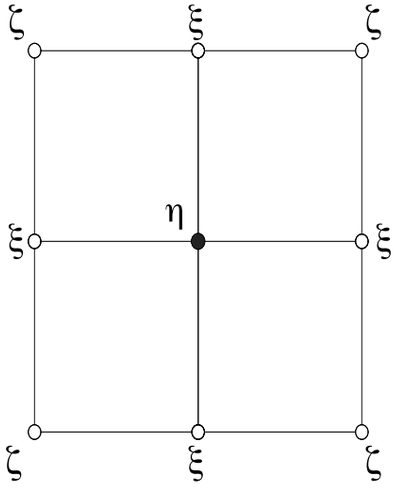


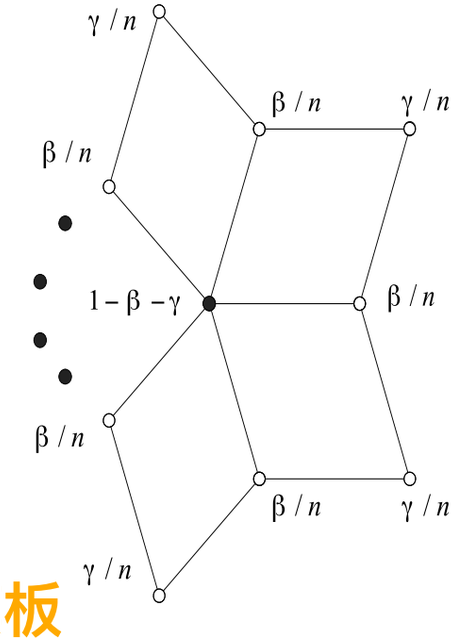

面点模板




边点模板

$$\begin{cases} f_j = f(2^{-j-1}\alpha), \\ \mu_j = \dfrac{2f_j+1}{4f_j+4}, \quad \nu_j = \dfrac{1}{8f_j+8}, \\ \eta_j = \dfrac{(2f+1)^2}{(2f+2)^2}, \quad \xi_j = \dfrac{4f+2}{(4f+4)^2}, \quad \zeta_j = \dfrac{1}{(4f+4)^2}, \\ \beta_j = 4\xi_j, \quad \gamma_j = 4\zeta_j. \end{cases}$$




顶点模板

# subdivision surface

## Catmull-Clark subdivision surface:

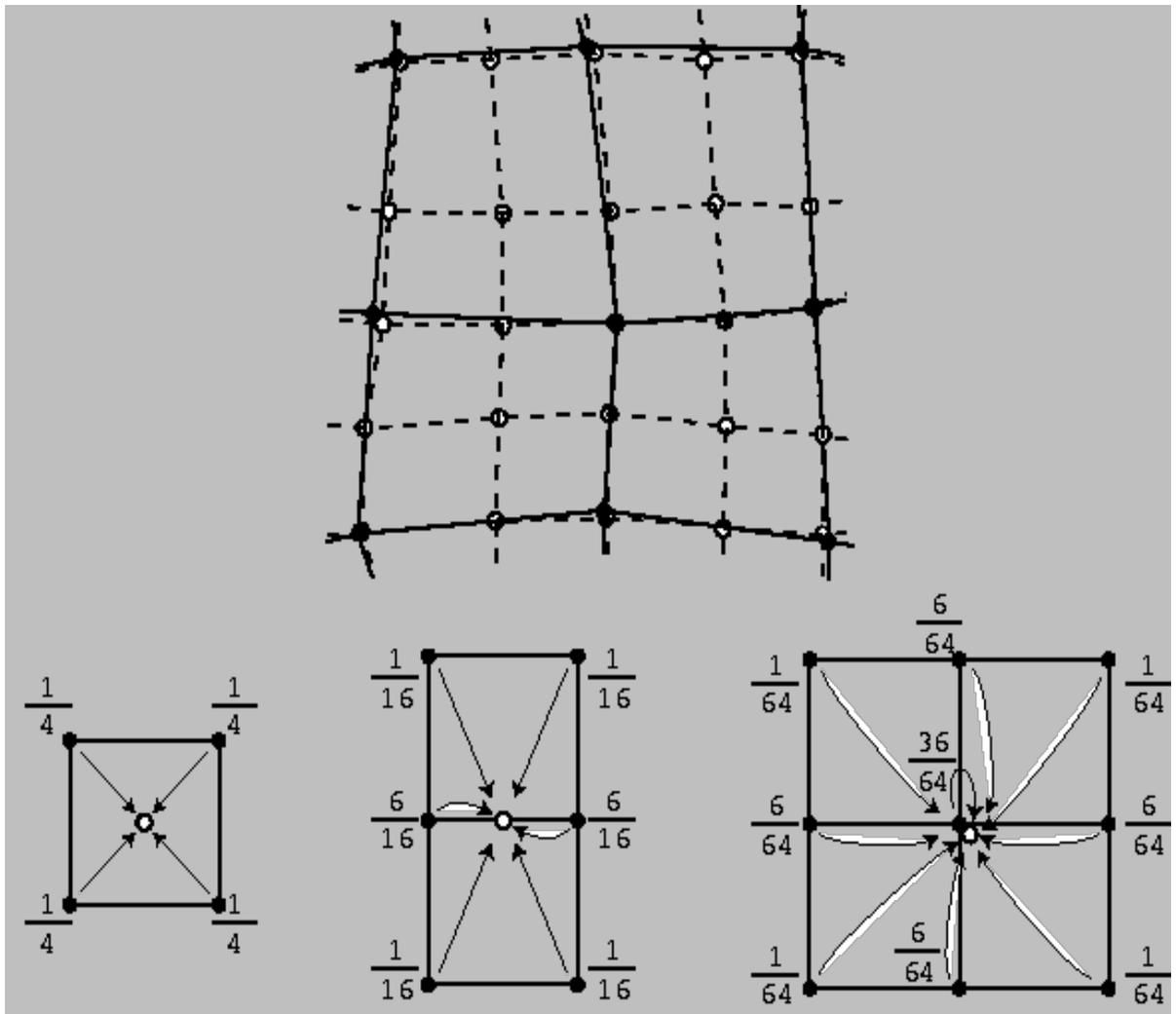
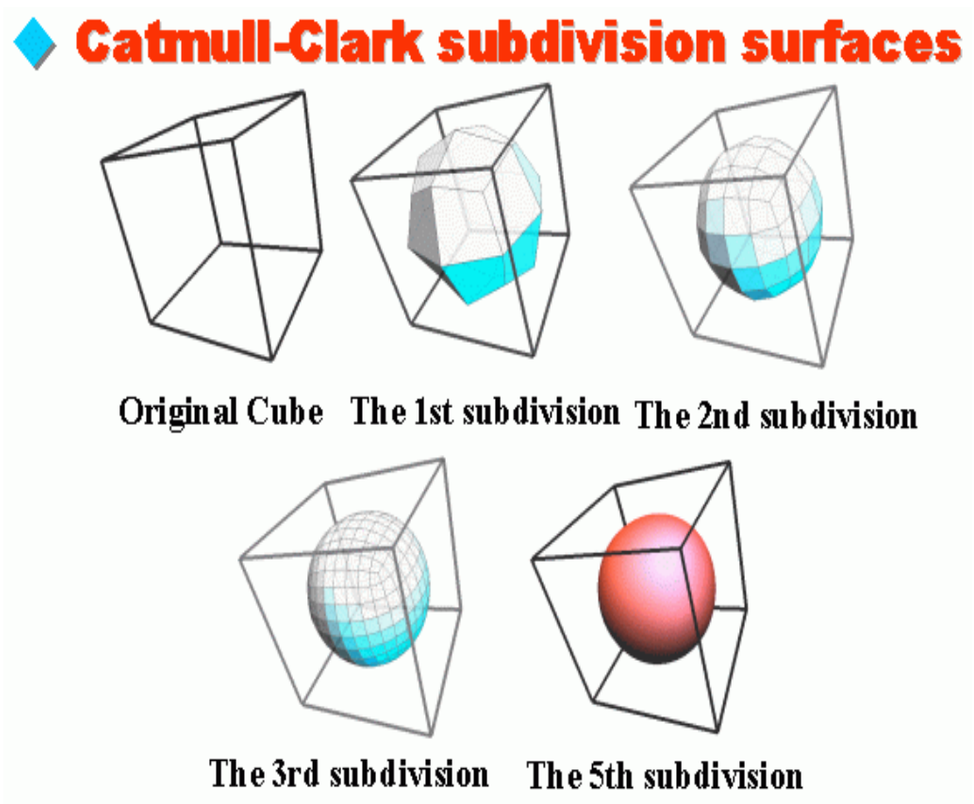



Figure 3: Catmull-Clark subdivision. Above a mesh (solid dots and solid lines) that has been refined (open dots and dashed lines). Below the weights used to generated each type of refined vertex: centre, edge, and modified old vertex.



# subdivision surface

## Doo-Sabin surface

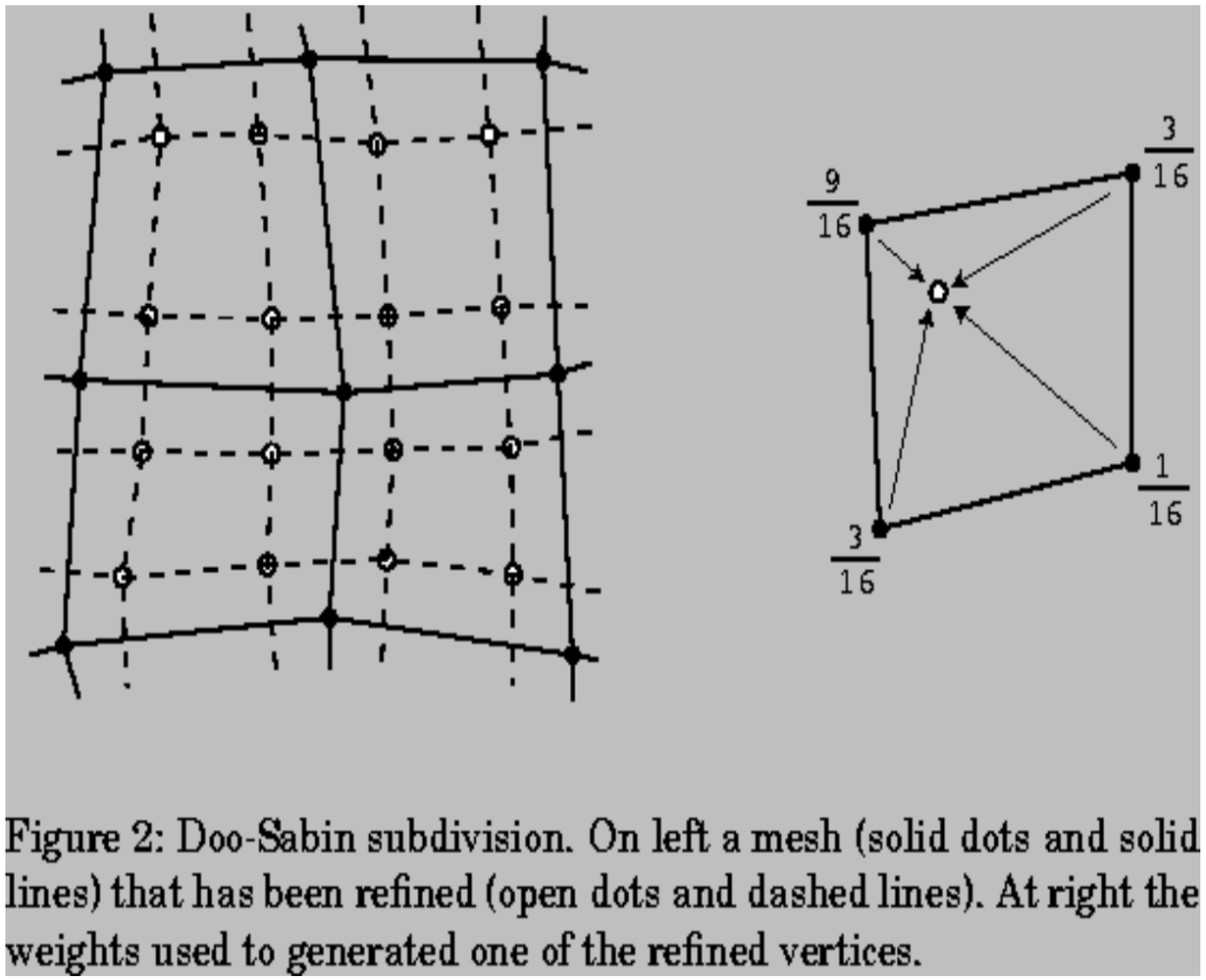
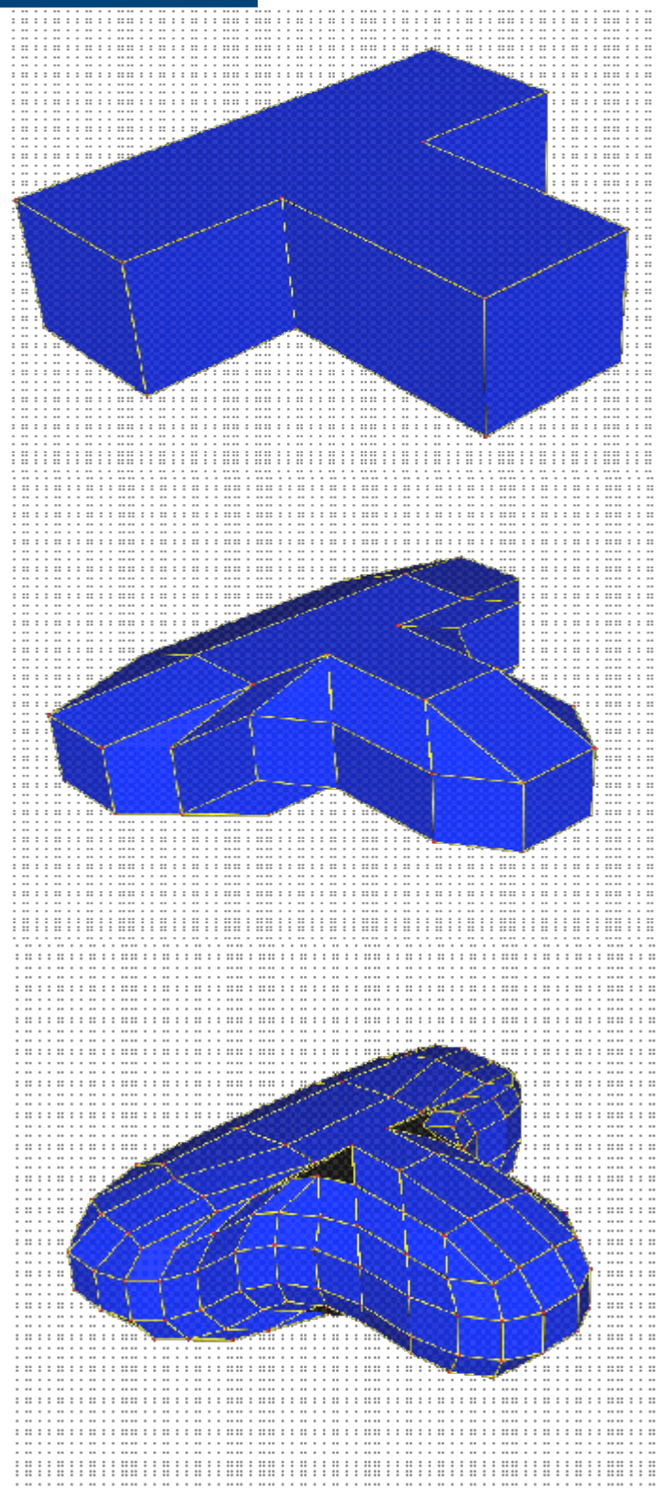
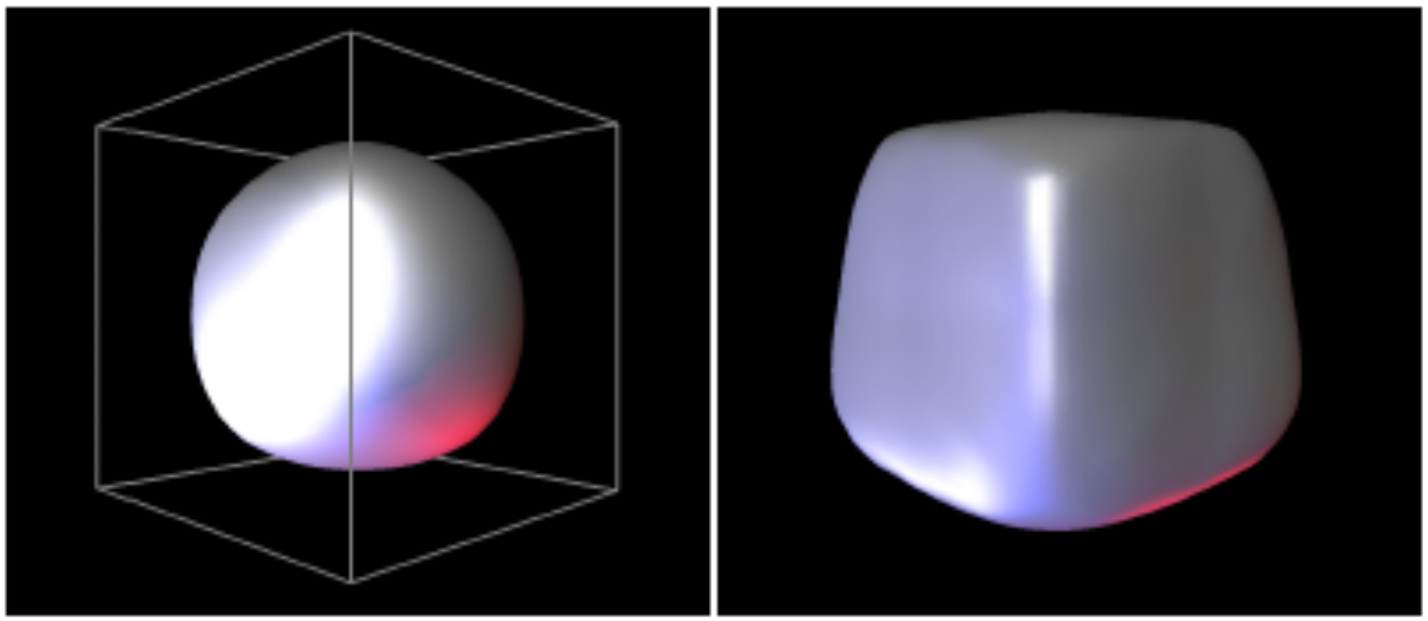



Figure 2: Doo-Sabin subdivision. On left a mesh (solid dots and solid lines) that has been refined (open dots and dashed lines). At right the weights used to generated one of the refined vertices.



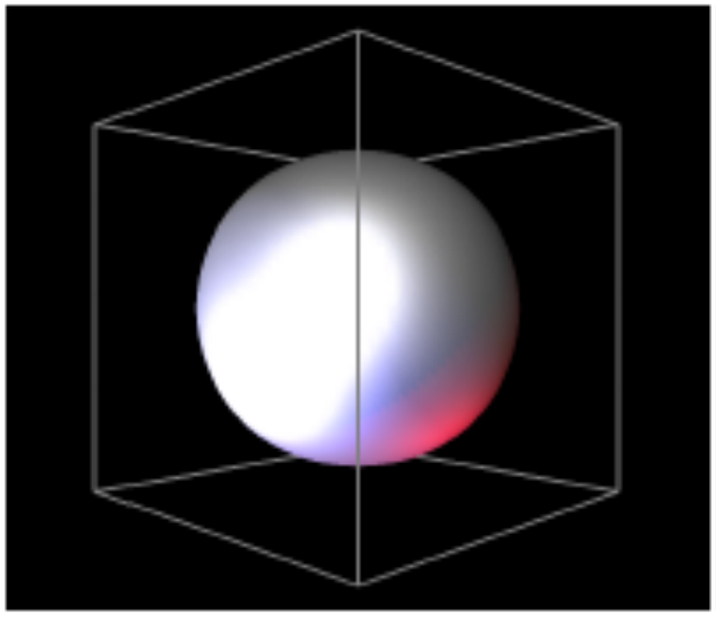
# 不同细分曲面的比较



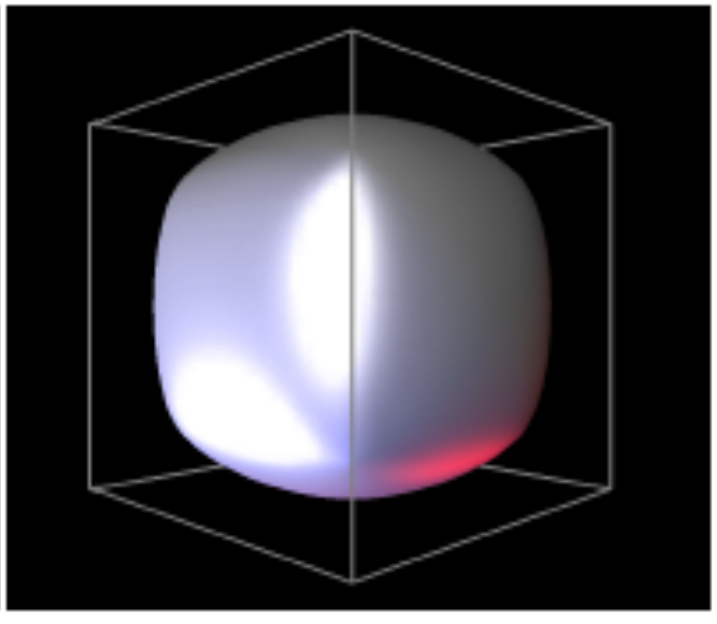
*Loop*



*Butterfly*

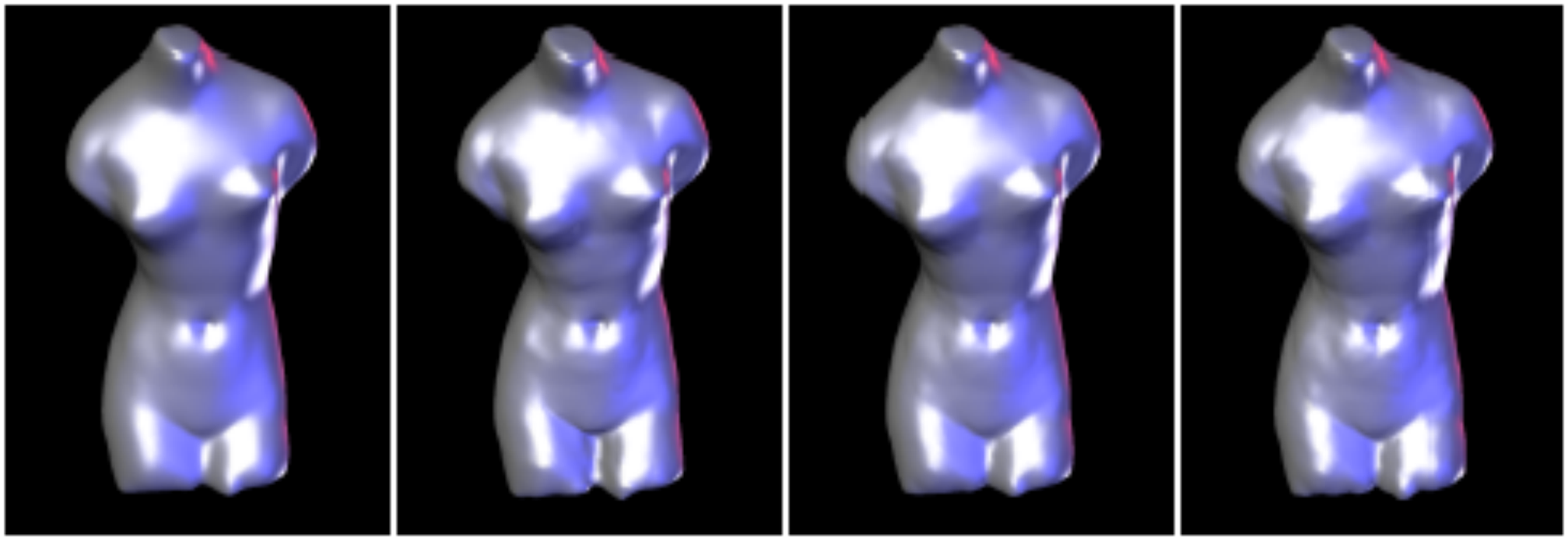


*Catmull-Clark*



*Doo-Sabin*

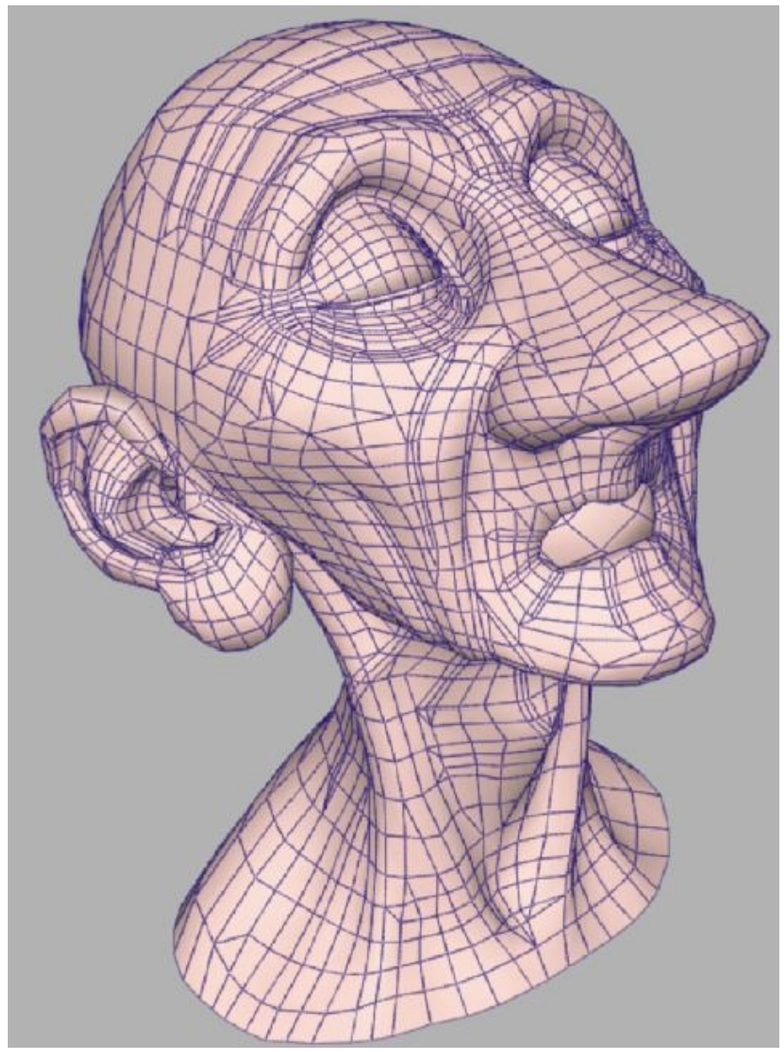
# 不同细分曲面的比较



*Loop* *Butterfly* *Catmull-Clark* *Doo-Sabin*

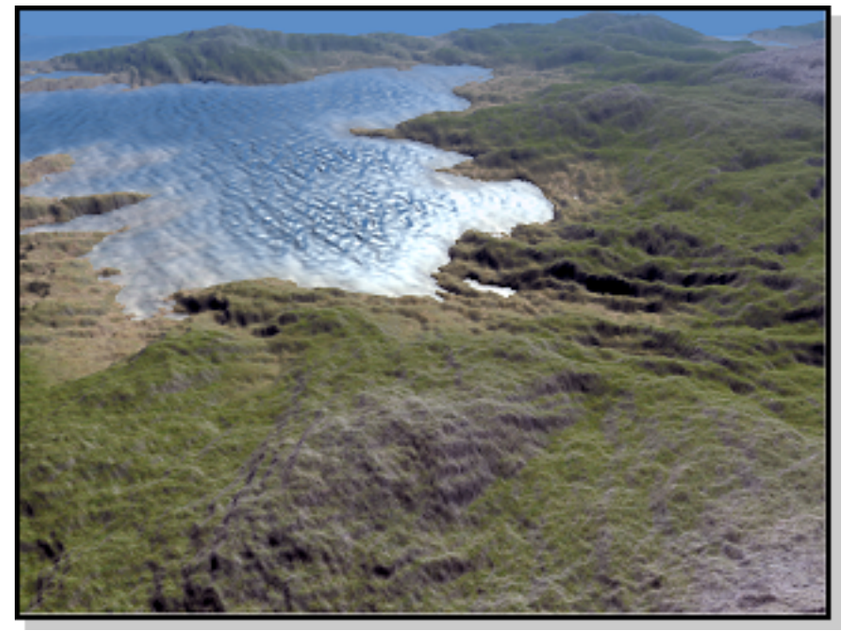
# 参考资料

- 普林斯顿大学的课件
- 半静态回插细分

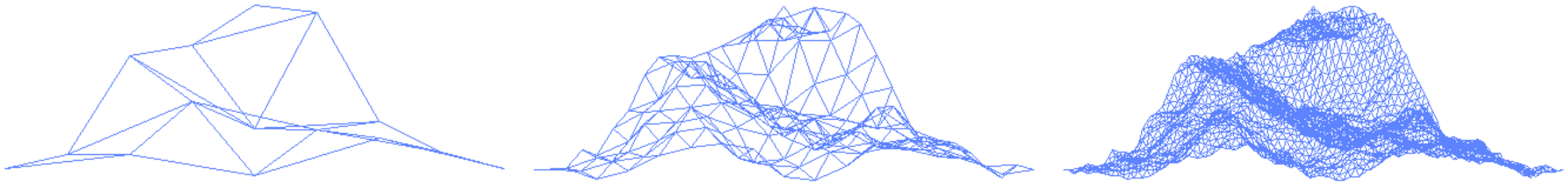


From Pixar, Geri’s Game

# Geri's Game (youku link)



subdivision + random

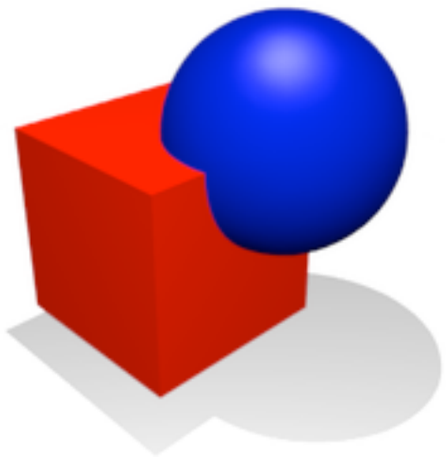


# terrain generation

# terrain generation

# More complex 3D

# Boolean operation

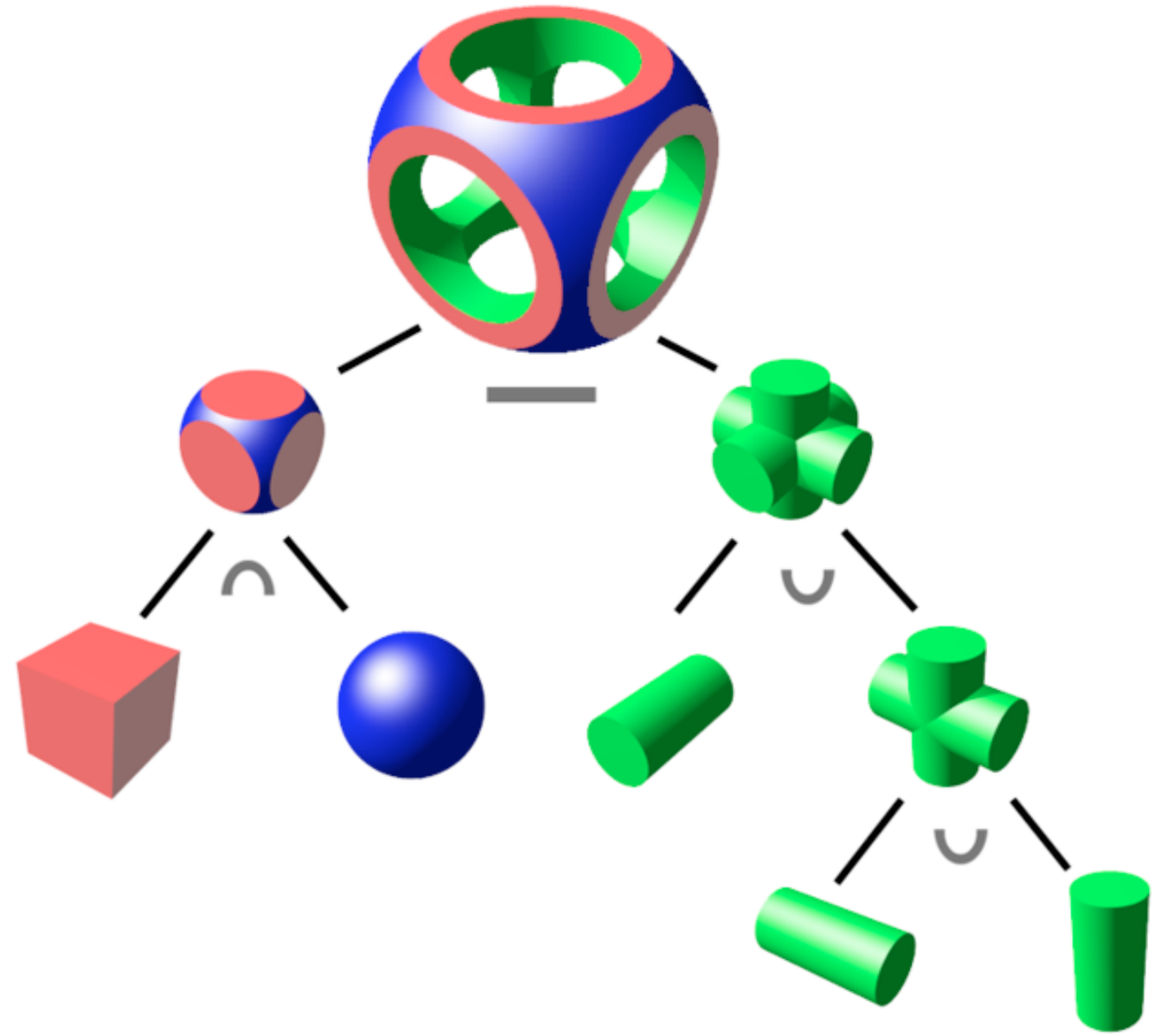


Boolean
union

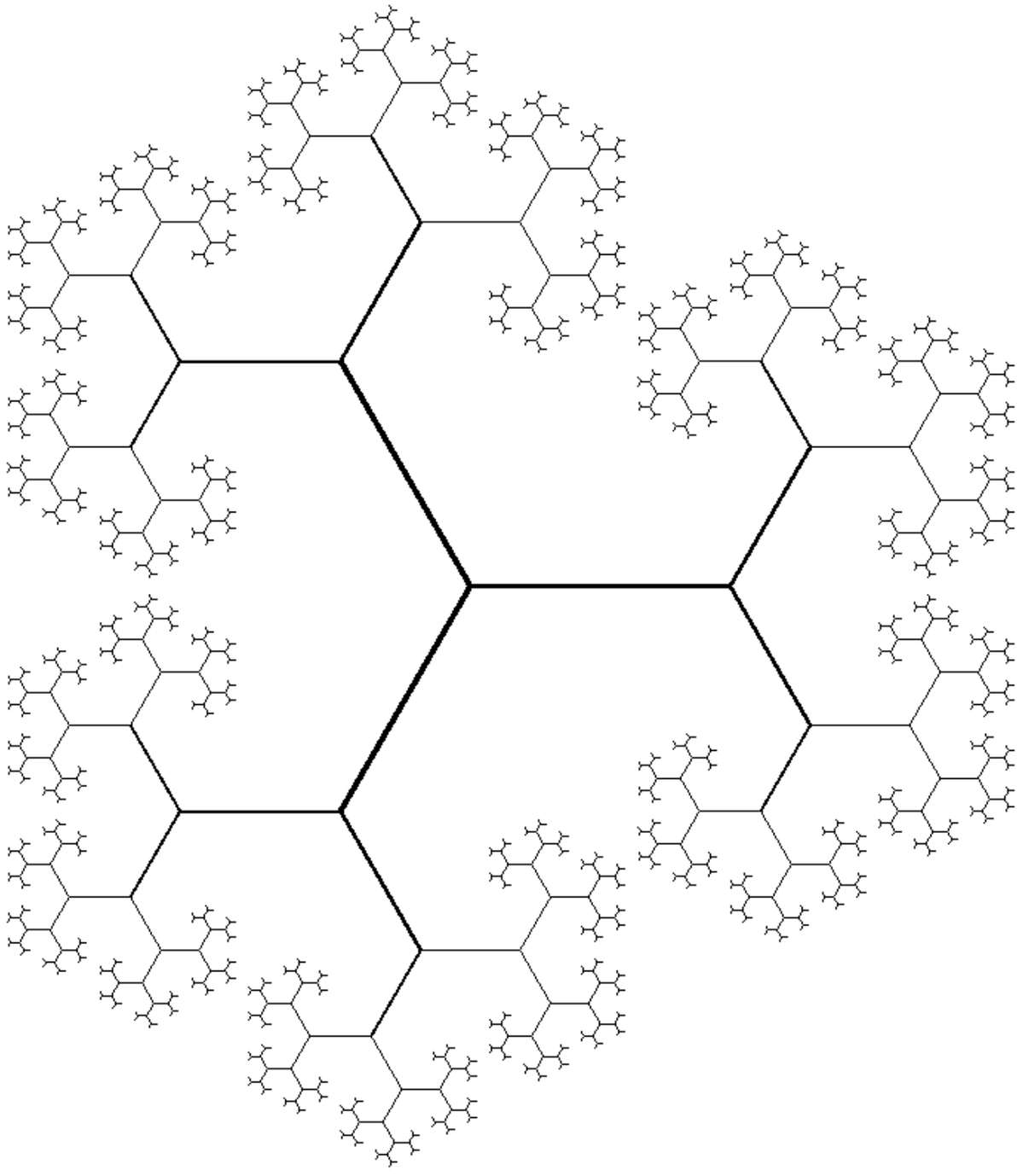
Boolean
difference

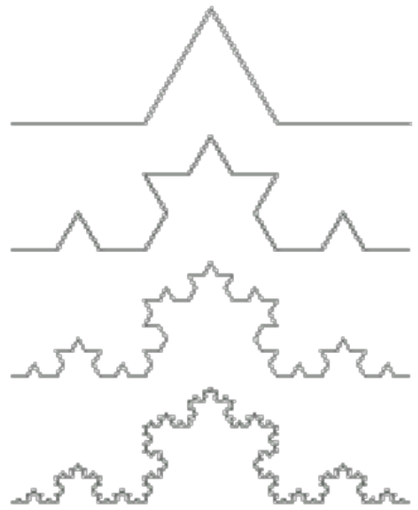
Boolean
intersection

# **C**onstructive **S**olid **G**eometry



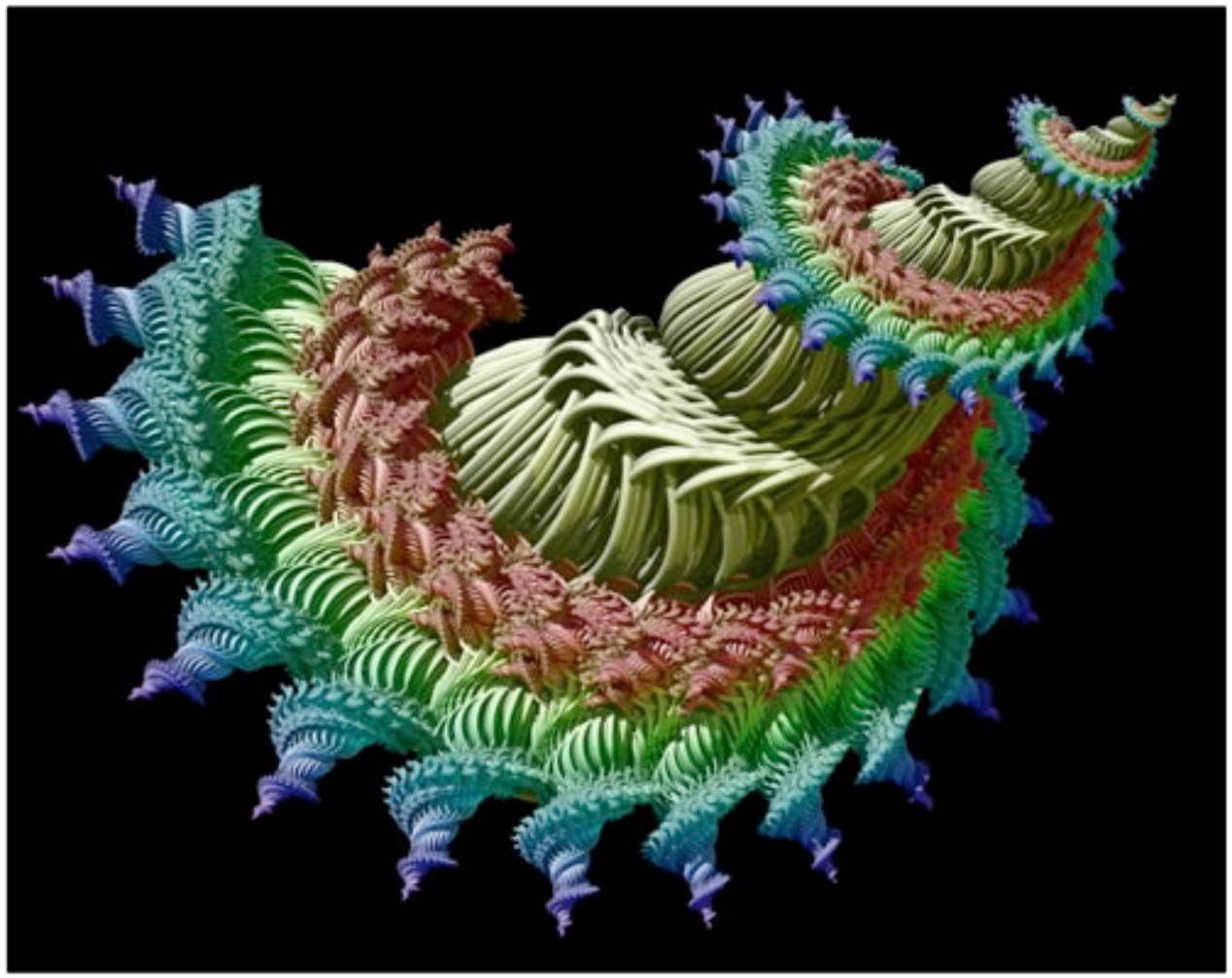
# **C**onstructive **S**olid **G**eometry

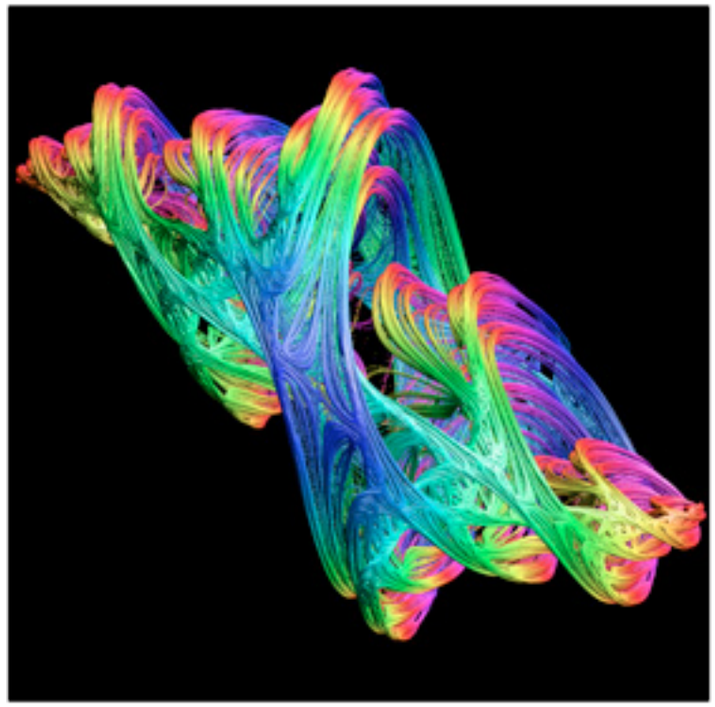




Self-similar
Self-affine
Invariant fractal set

# Fractal





Julia Set

# Fractal

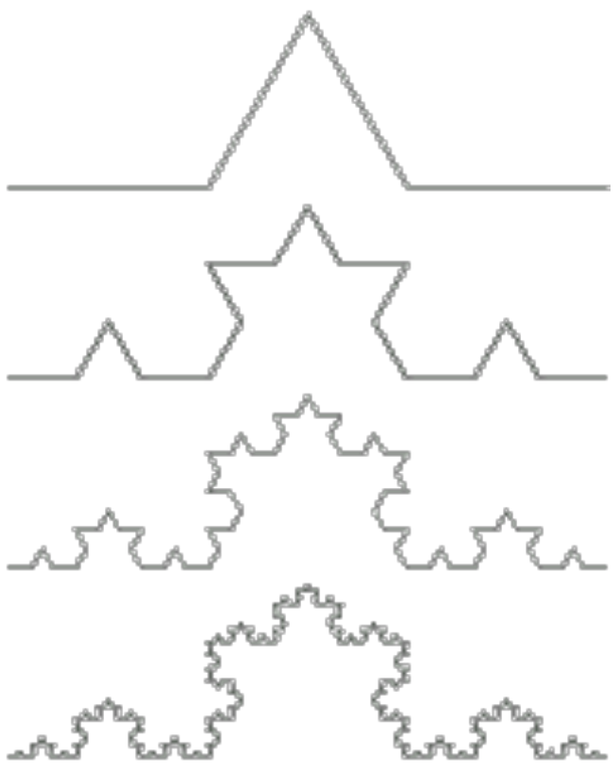
字符 : F

常数 : +, −

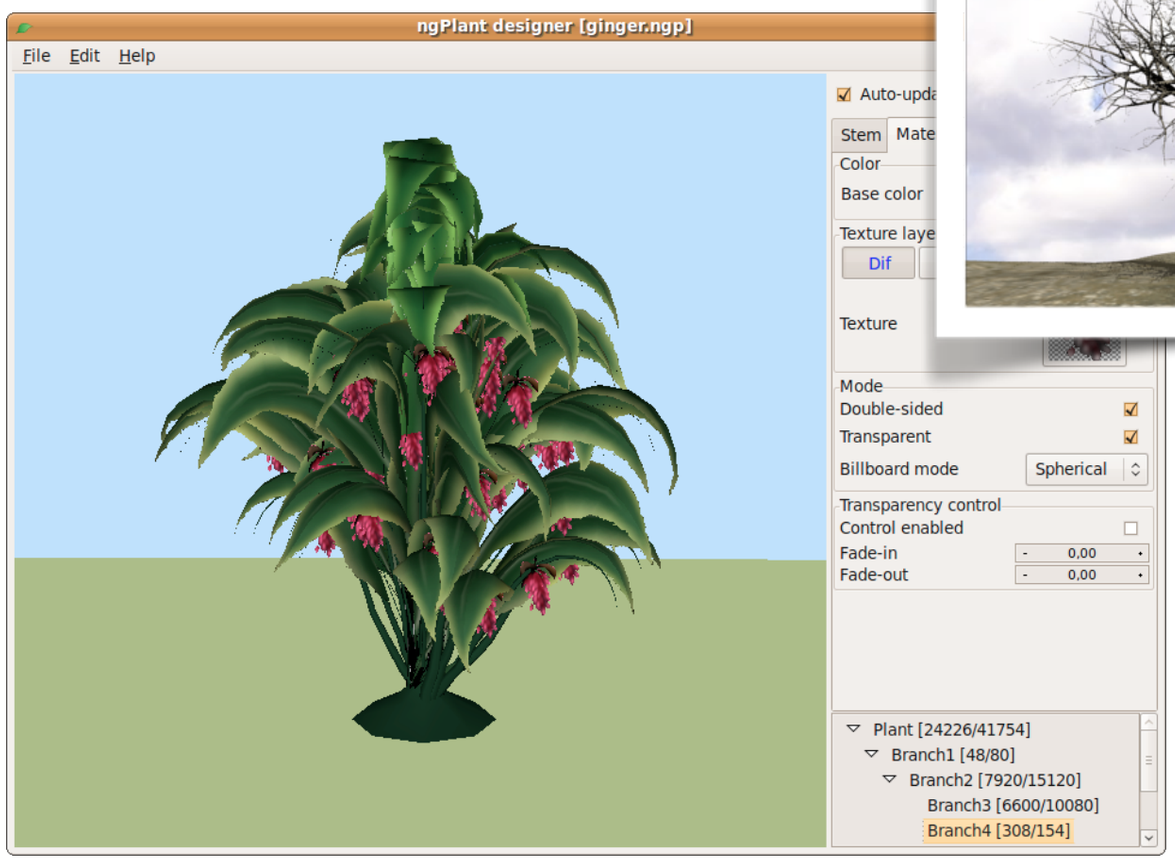
公理 : F++F++F

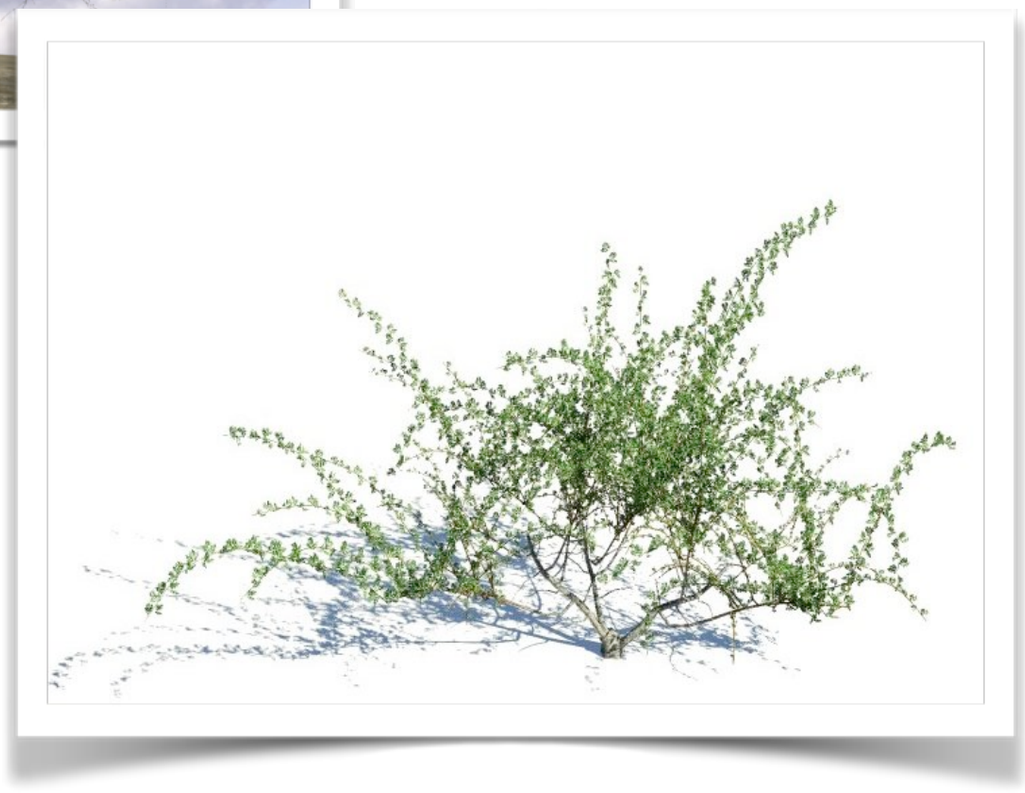
规则: F → F−F++F−F

- F： 向前
- - ： 左转60°
- + ： 右转60°



# L-system



# L-system for tree

# V-Plant: A Variational Approach for Interactive Tree Modeling

*Online Submisson ID: 355*

# Sketch Tree Demo

# procedure (script) modeling

- A lot of research
  - Natural scene modeling
  - City modeling (shape grammar)

- Amazing software
  - nodebox v.s. processing

# Particle systems