



Computer Graphics 2024

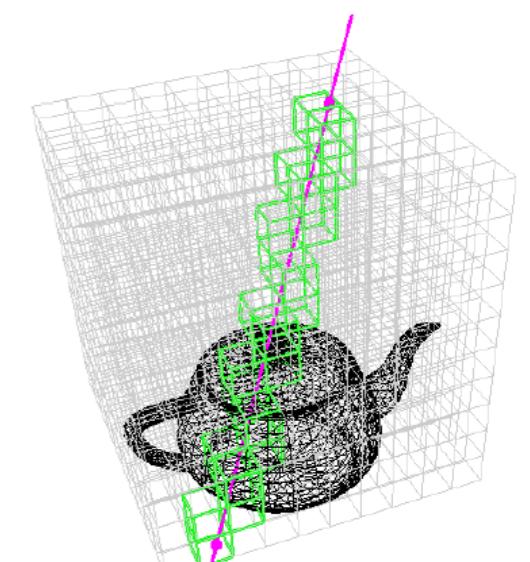
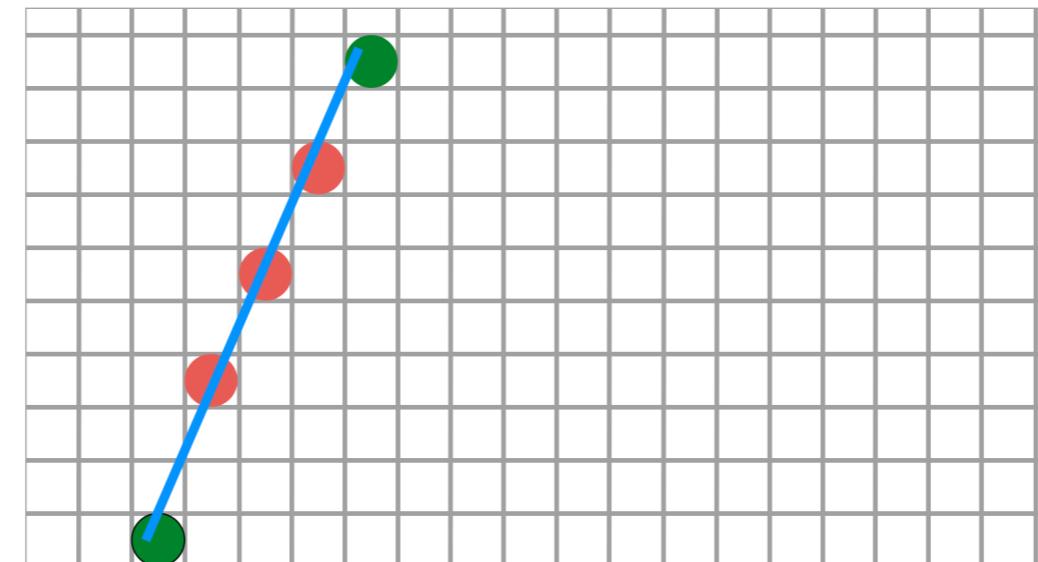
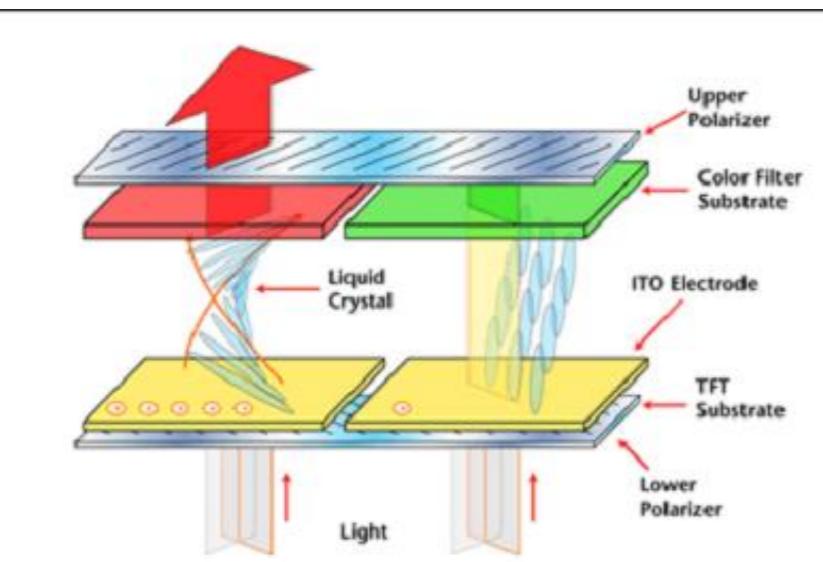
2. 2D Graphics Algorithms

张宏鑫 (Hongxin Zhang)

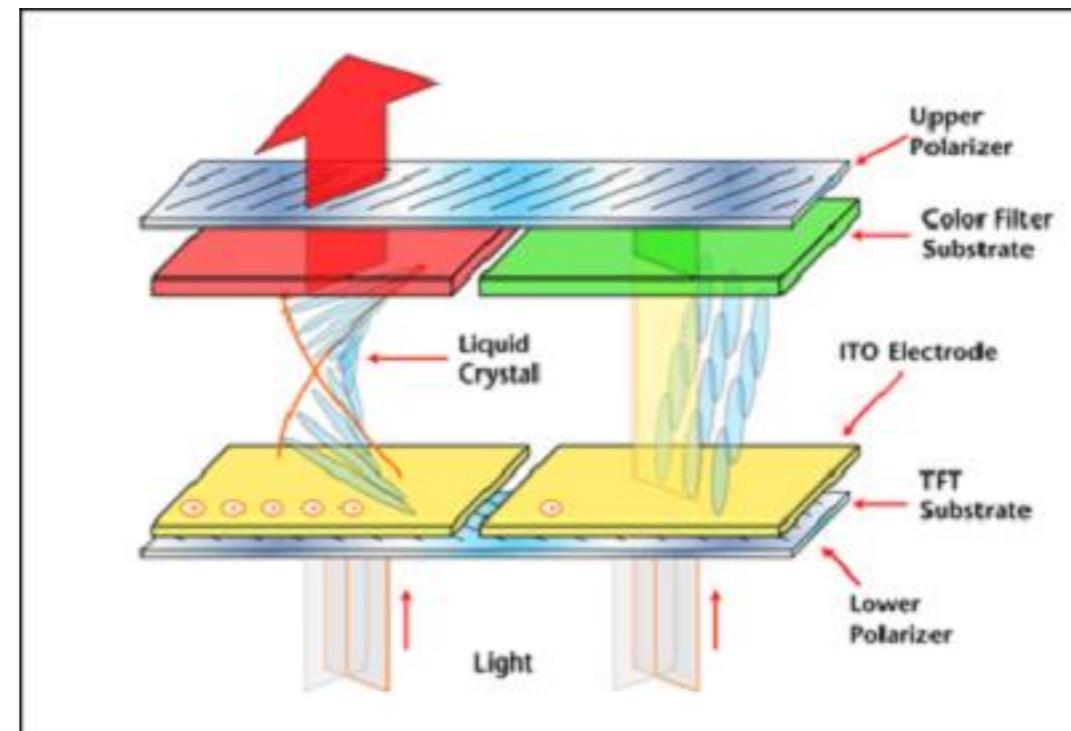
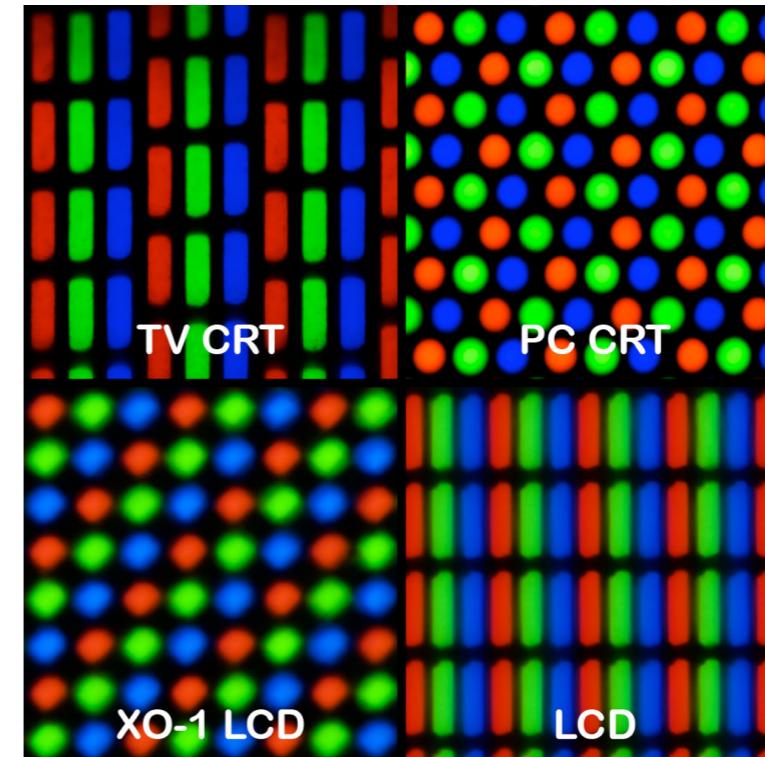
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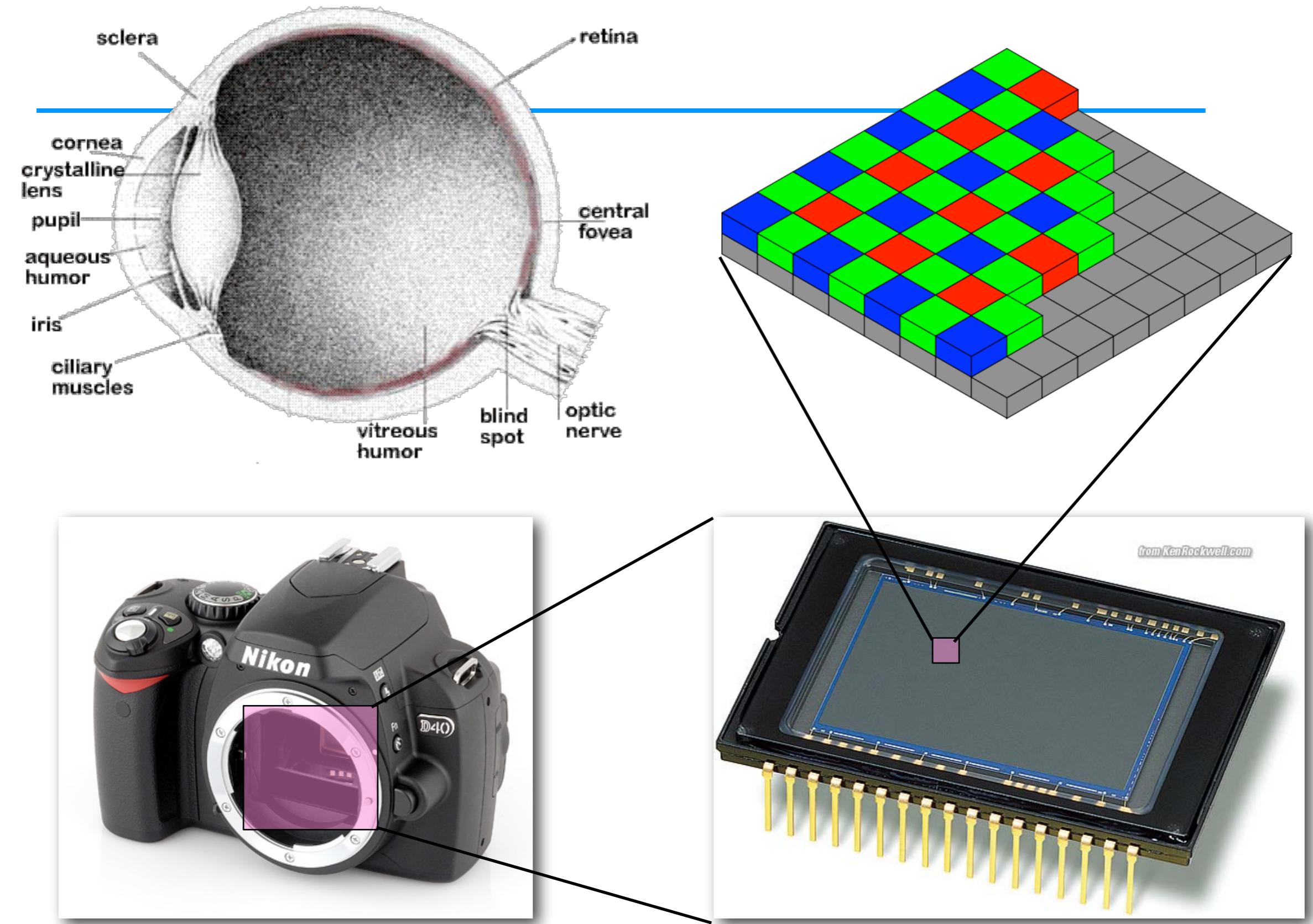
浙江大学CAD&CG全国重点实验室，计算机学院
State Key Lab of CAD&CG, Zhejiang University

2024-09-20



Screen - Linear Structure

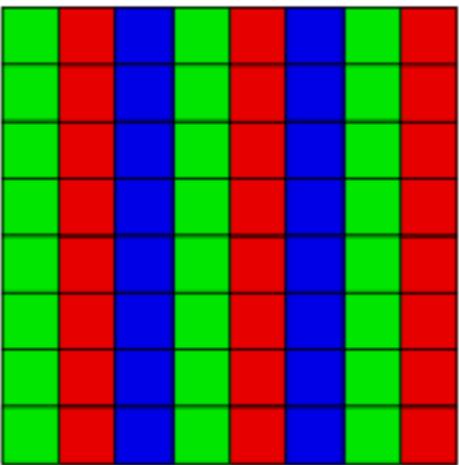




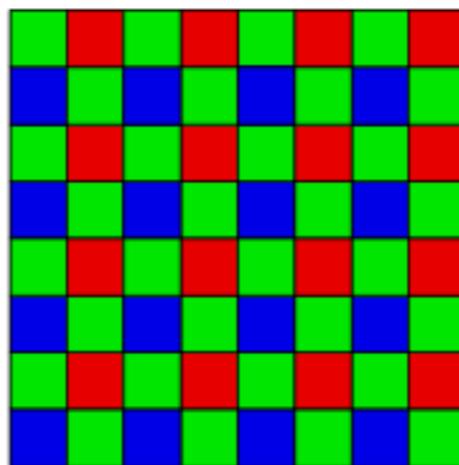
Nikon D40 Sensors

3

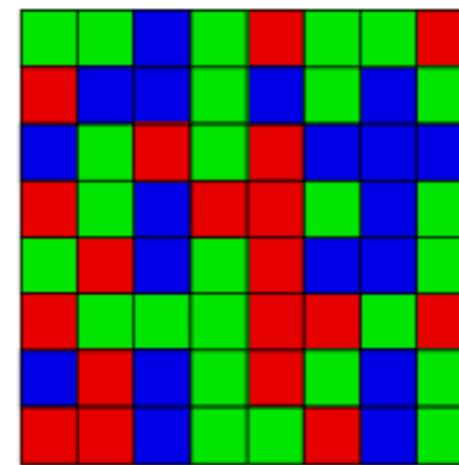
RGBW Camera Sensor



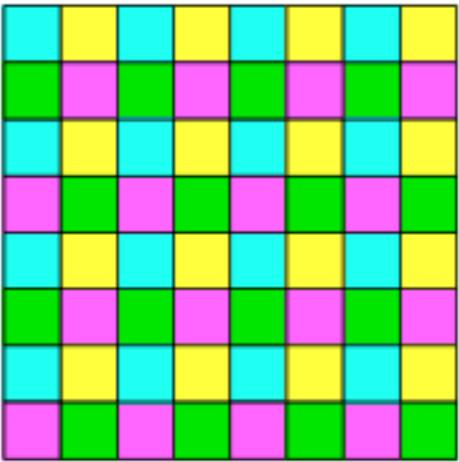
(a) Vertical stripes



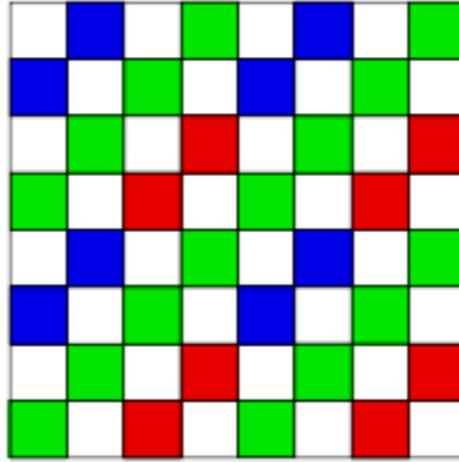
(b) Bayer



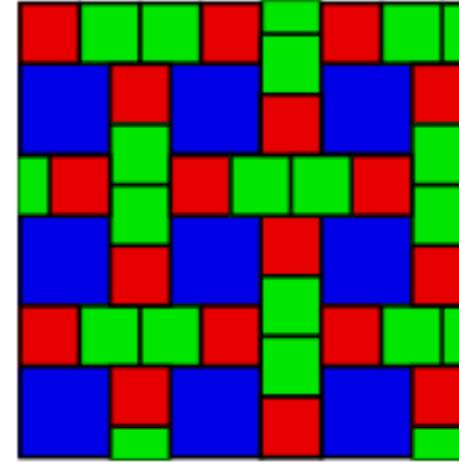
(c) Pseudo-random



(d) Complementary colors

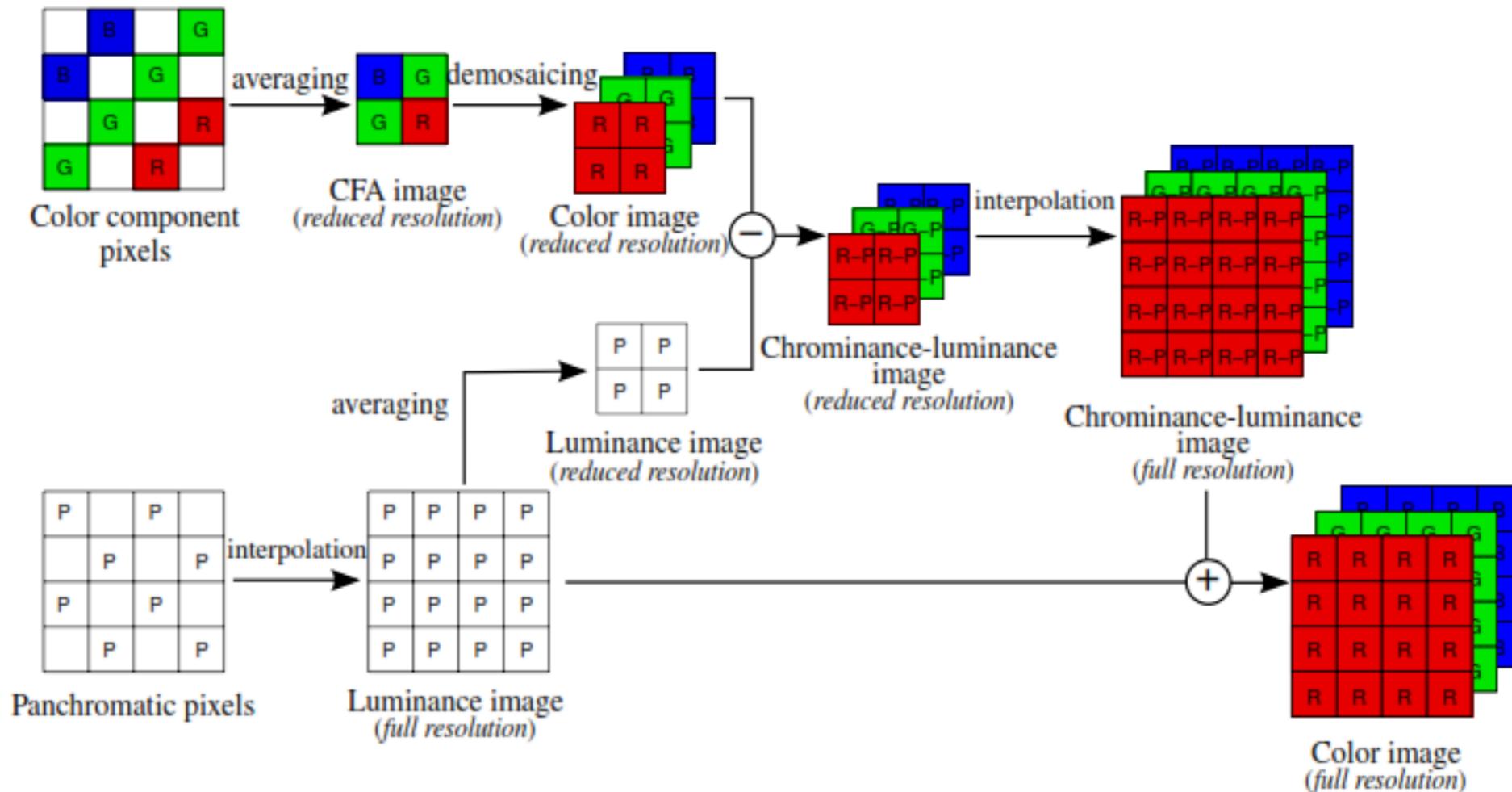


(e) “Panchromatic”, or
CFA2.0 (Kodak)

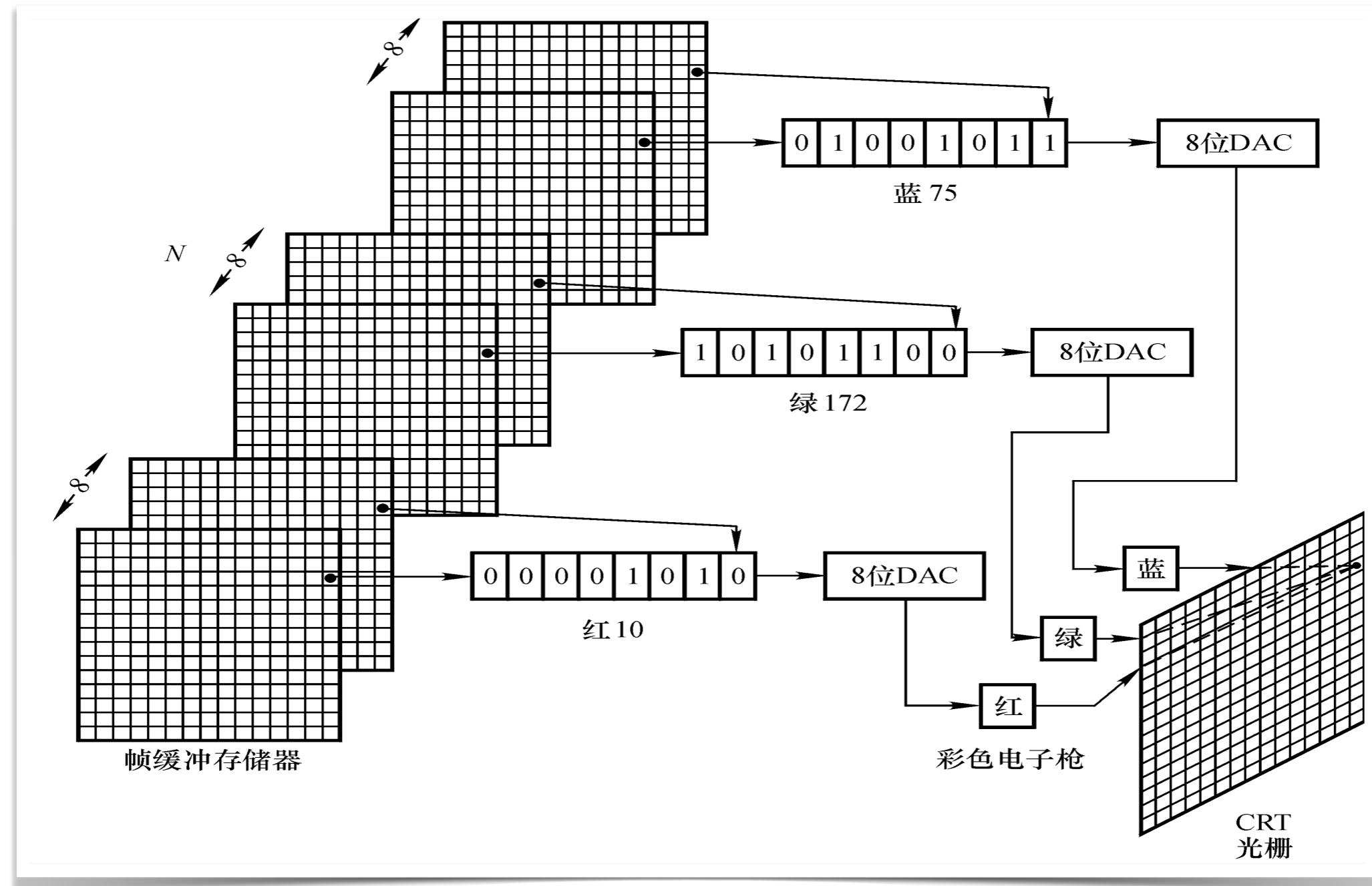


(f) “Burtoni” CFA

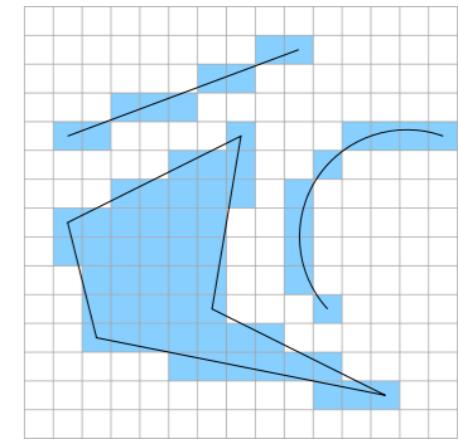
RGBW Camera Sensor



Rasterization

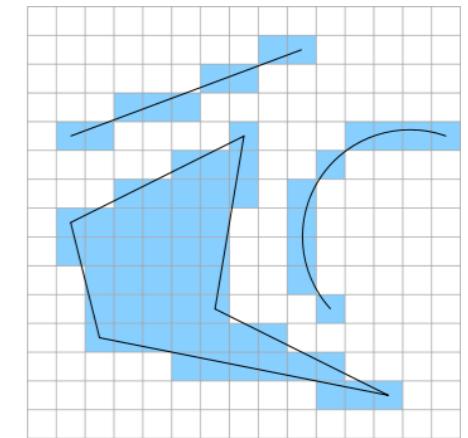


Rasterization

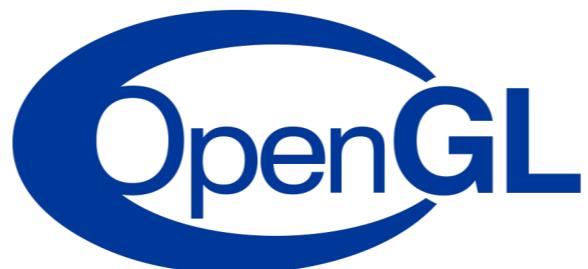


- The task of displaying a world modeled using primitives like lines, polygons, filled / patterned areas, etc. can be carried out in two steps
- **determine the pixels** through which the primitive is visible, a process called Rasterization or scan conversion
- **determine the color value** to be assigned to each such pixel.

Raster Graphics Packages

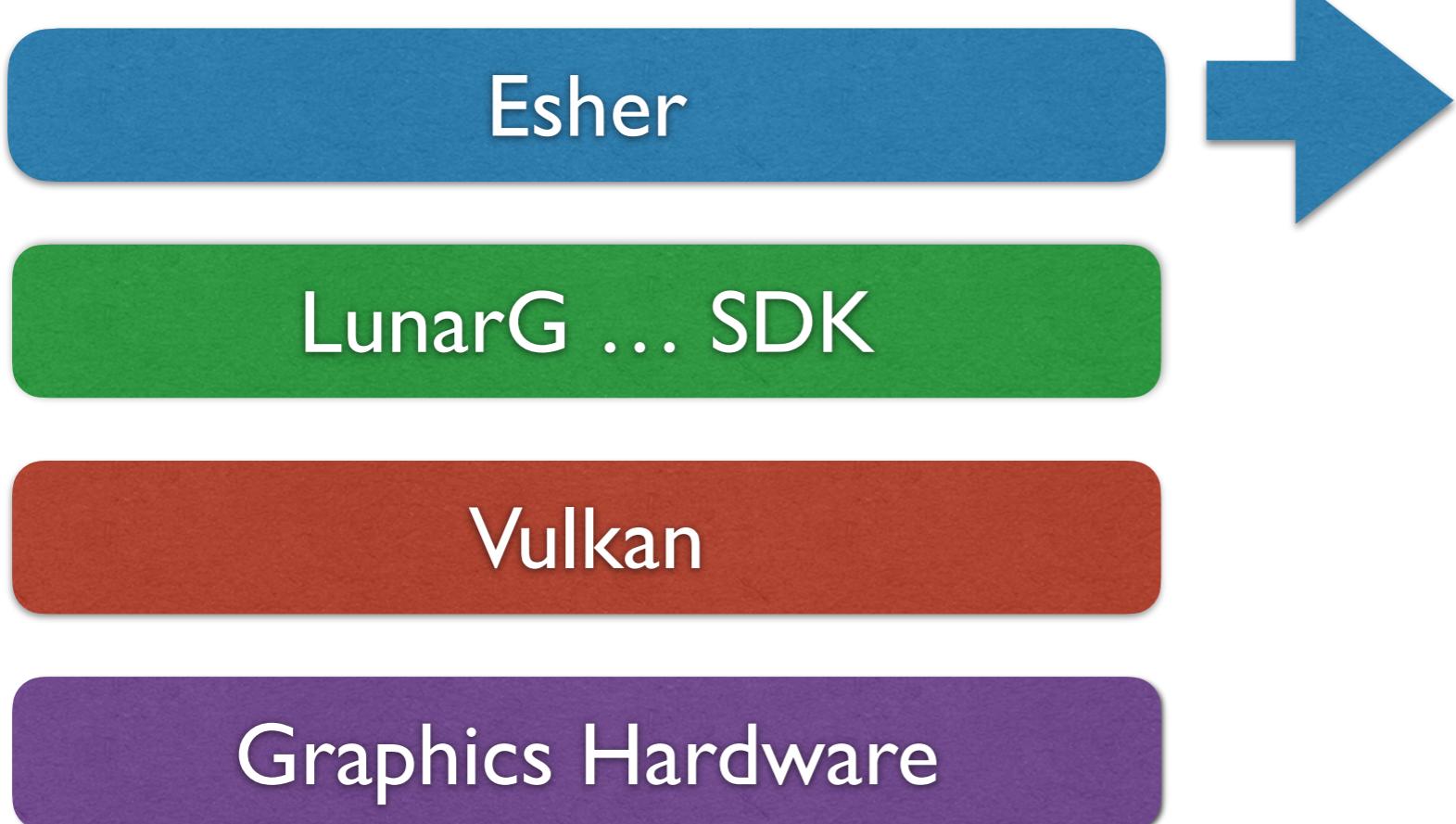


- The efficiency of these steps forms the main criteria to determine **the performance of a display**
- The raster graphics package is typically **a collection of efficient algorithms** for scan converting (rasterization) of the display primitives
- High performance graphics workstations have most of these algorithms **implemented in hardware**



More ...

- Google's New AR OS: Fuchsia



Physically Based Renderer
Volumetric soft shadows
Color bleeding
Light diffusion
Lens effect

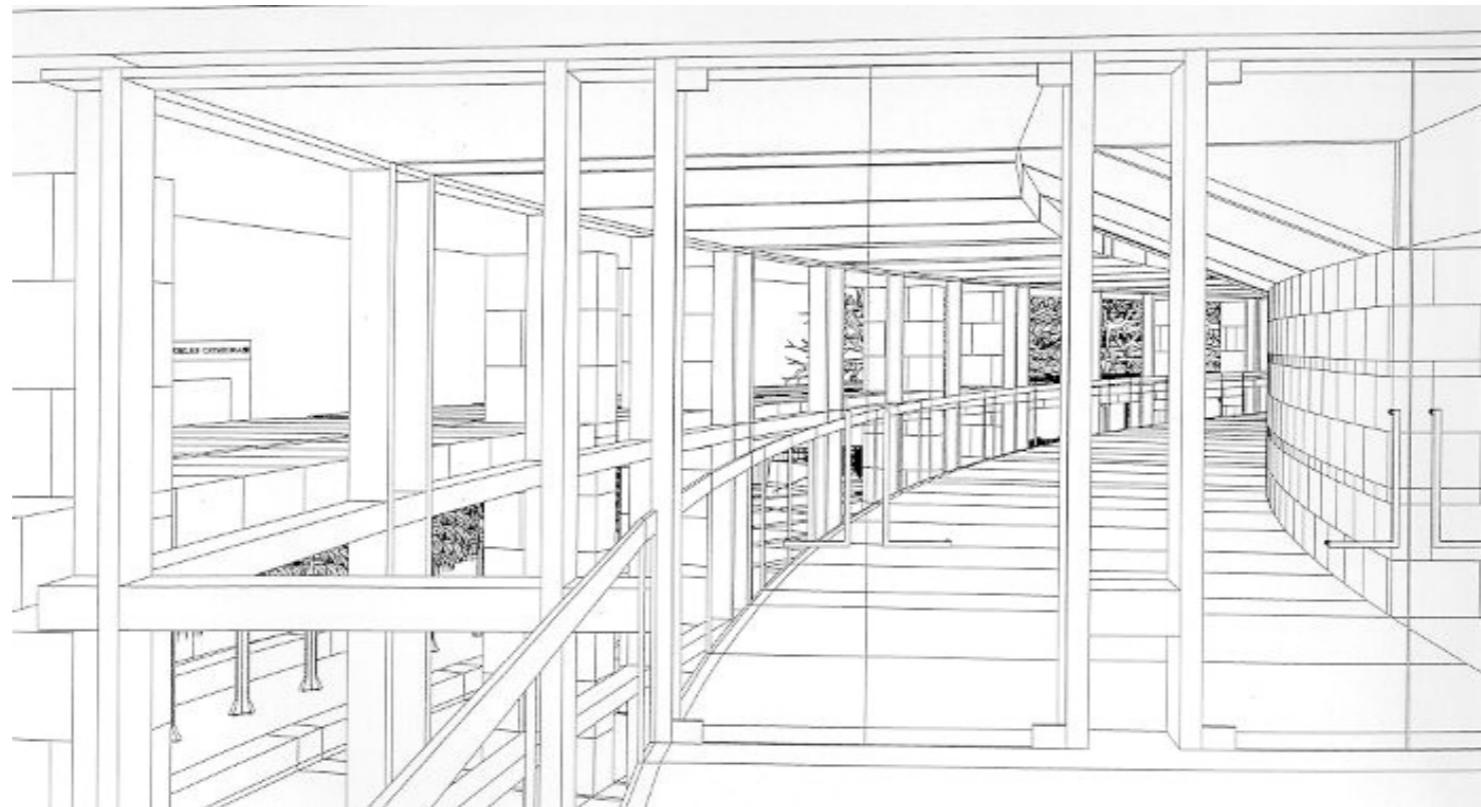
Why Study these Algorithms?

- Some of these algorithms are very good examples of clever algorithmic optimization done to dramatically improve performance using minimal hardware facilities
- Mobile graphics
- Inspiration



Scan Converting a Line Segment

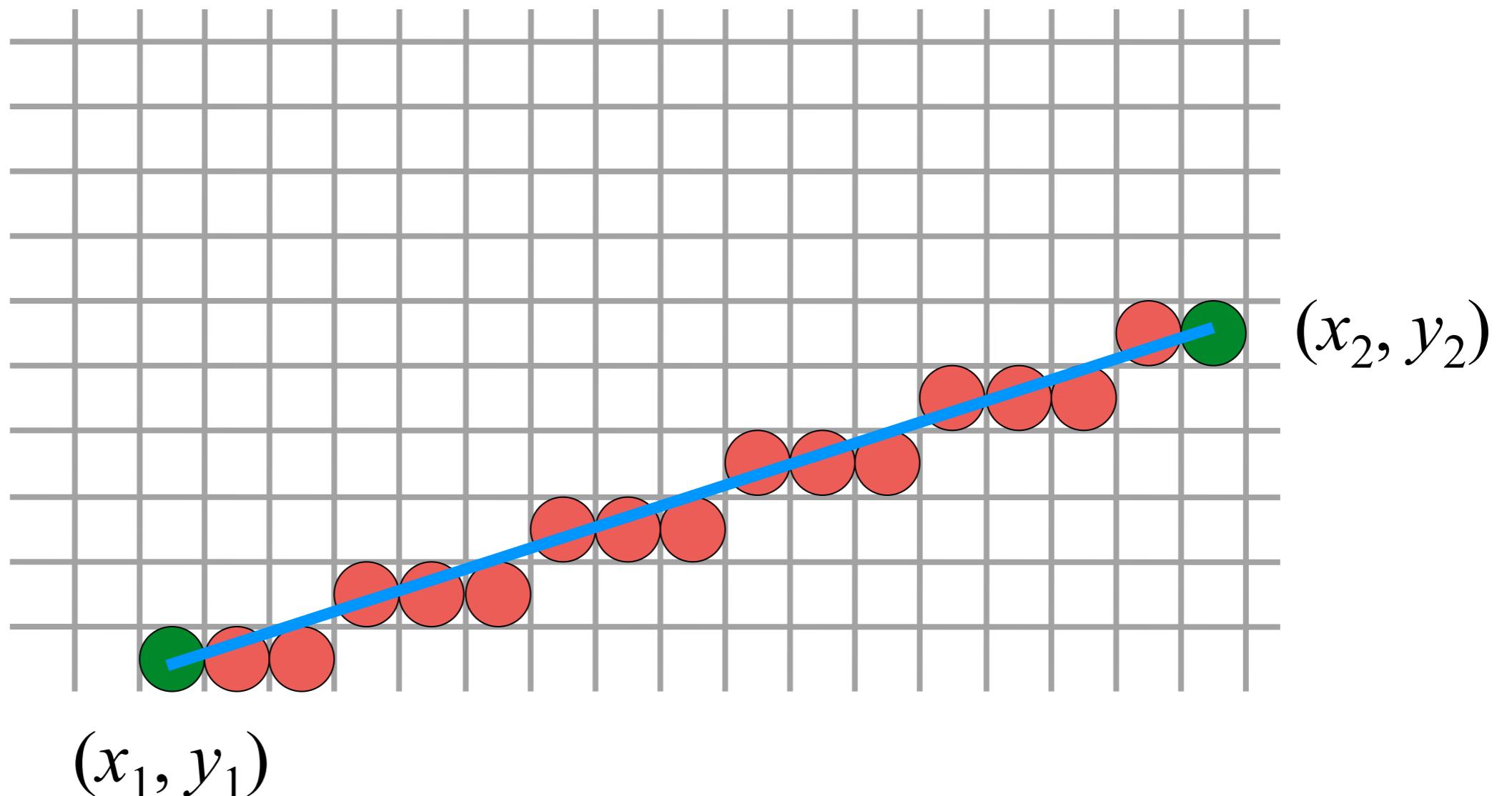
- The line is a powerful element used since the days of Euclid to model the edges in the world.



Given a line segment defined by its endpoints determine the pixels and color which best model the line segment.

Scan converting lines

start from (x_1, y_1) end at (x_2, y_2)



Scan converting lines

- Requirements
 - 理想: chosen pixels should lie as close to the ideal line as possible
 - 精确: the sequence of pixels should be as straight as possible
 - 亮度: all lines should appear to be of constant brightness independent of their length and orientation
 - 端点: should start and end accurately
 - 快速: should be drawn as rapidly as possible
 - 变化: should be possible to draw lines with different width and line styles

Question 1: How ?

$(x_1, y_1), (x_2, y_2)$



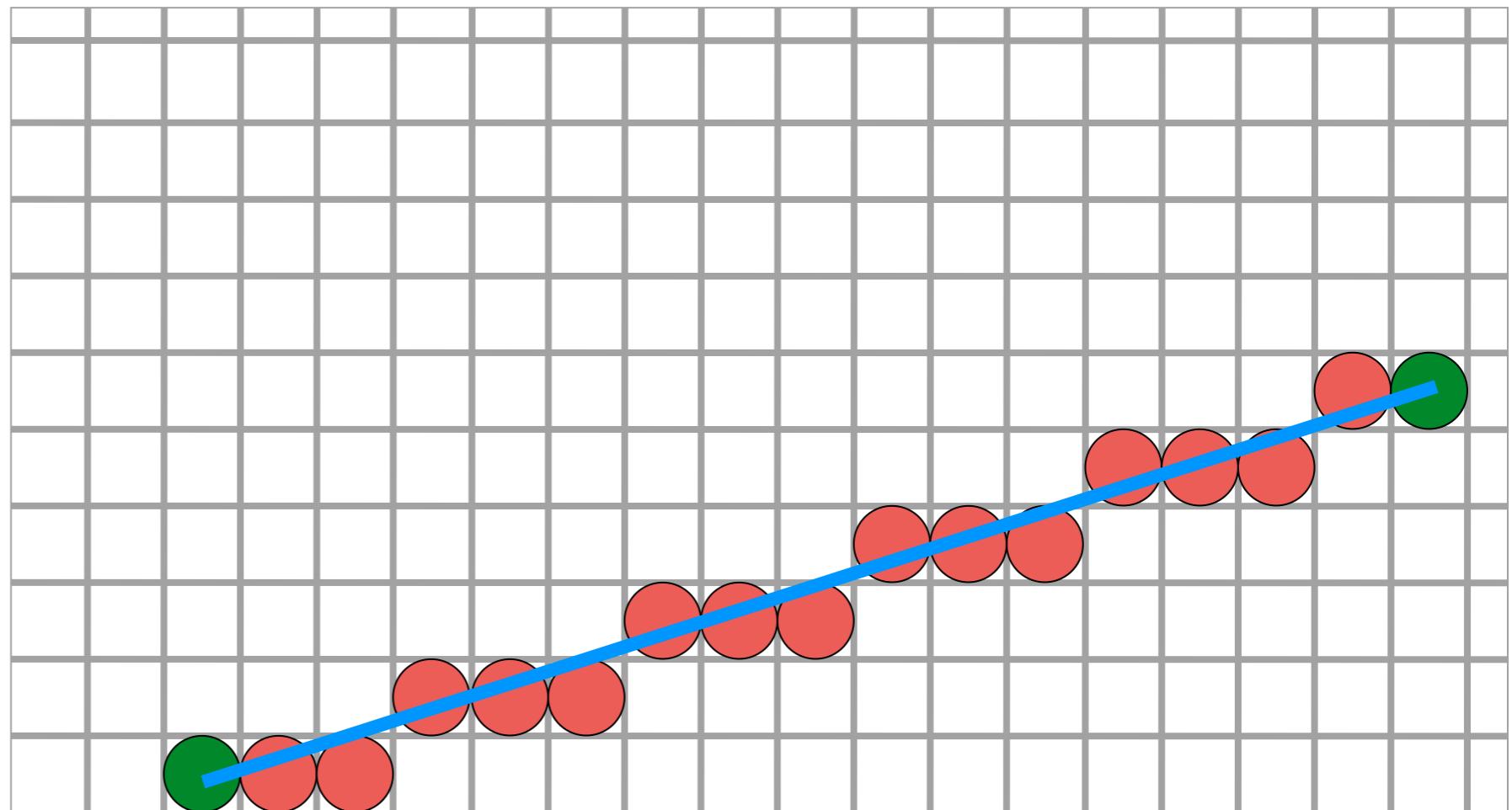
$$y = mx + b$$



$x_1 + 1 \Rightarrow y = ?, \text{ rounding}$



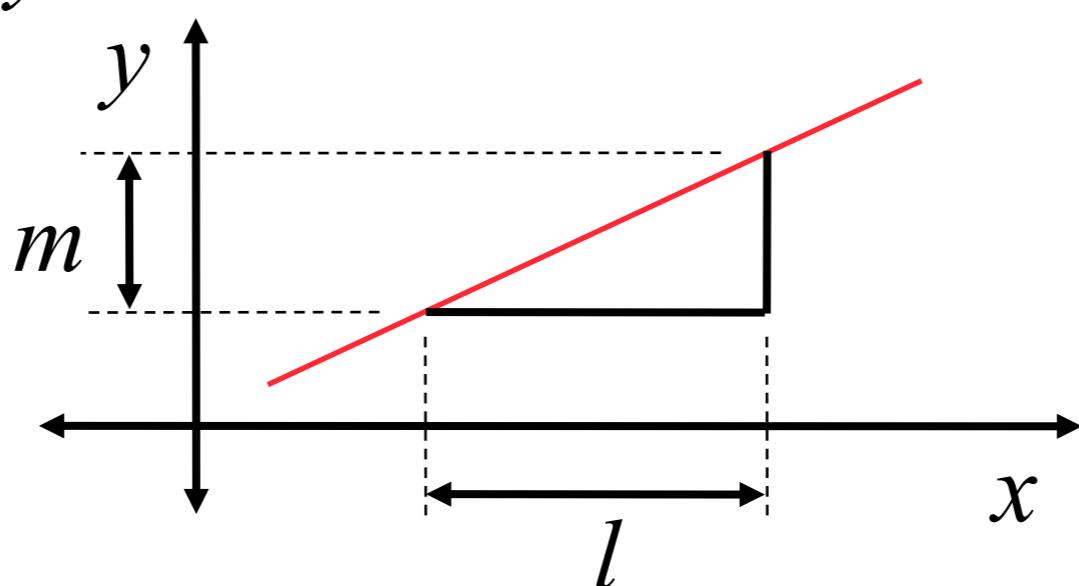
$x_1 + 2 \Rightarrow y = ?, \text{ rounding} \rightarrow x_1 + i \Rightarrow y = ?, \text{ rounding}$



Question 2: How to speed up?

Equation of a Line

- Equation of a line is $y - m \cdot x + c = 0$
- For a line segment joining points
- $P(x_1, y_1)$ and $Q(x_2, y_2)$ slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$
- Slope m means that for every unit increment in x the increment in y is m units



Digital Differential Analyzer (DDA)

- We consider the line in the first octant.
Other cases can be easily derived.
- Uses differential equation of the line

$$y_i = mx_i + c$$

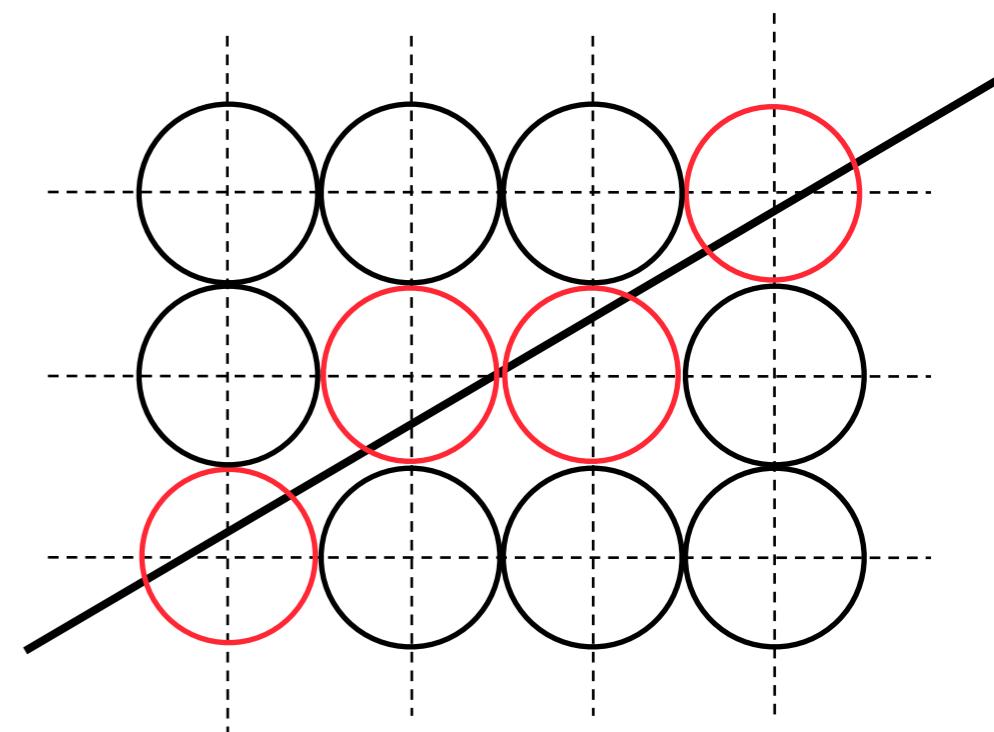
where, $m = \frac{y_2 - y_1}{x_2 - x_1}$

- Incrementing X-coordinate by 1

$$x_i = x_{i_prev} + 1$$

$$y_i = y_{i_prev} + m$$

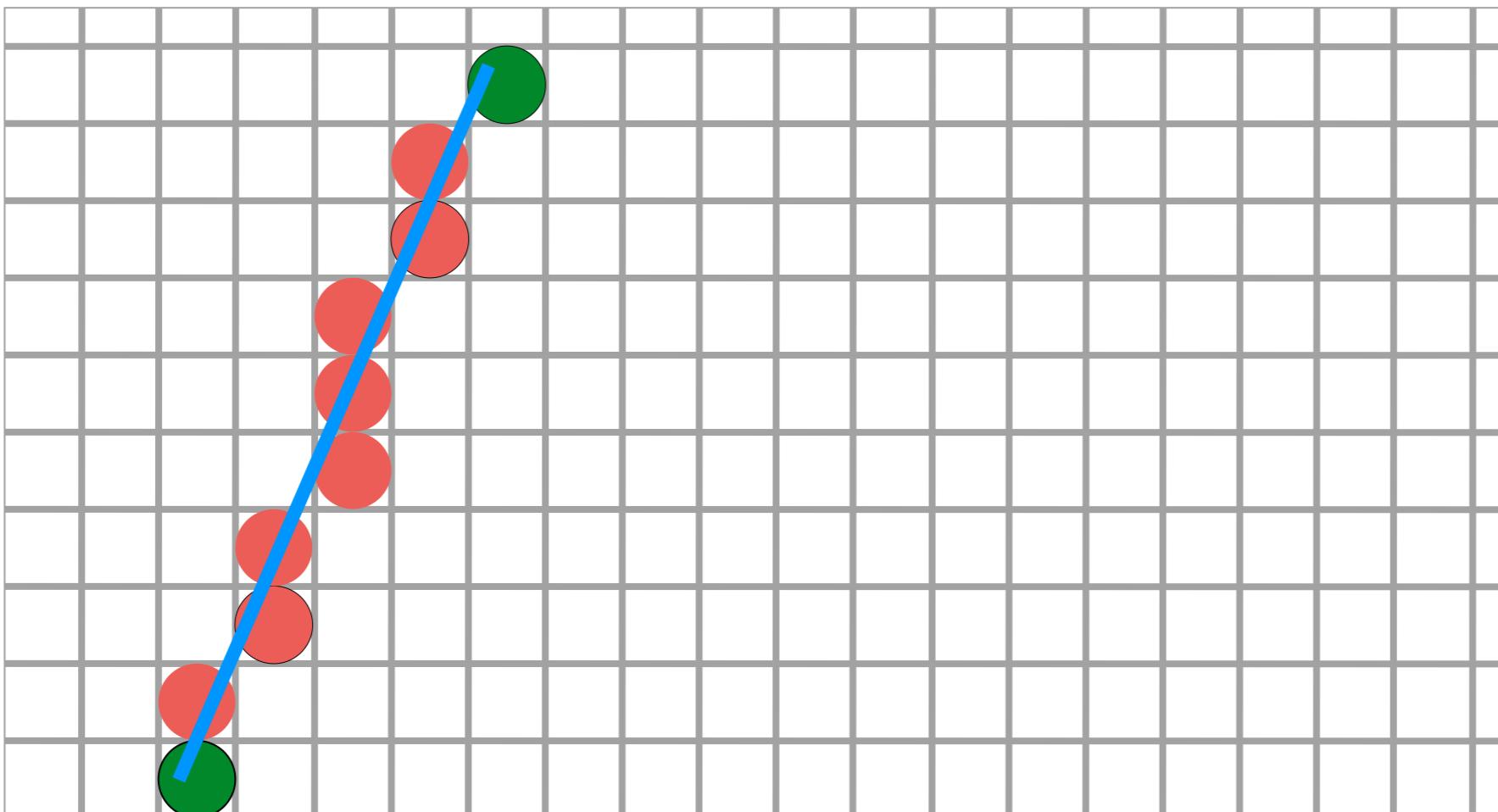
- Illuminate the pixel $[x_i, round(y_i)]$



Discussion1: What technique makes it fast?

Discussion2: Is there any problem in the algorithm?
How to avoid it?

If $\Delta x < \Delta y$



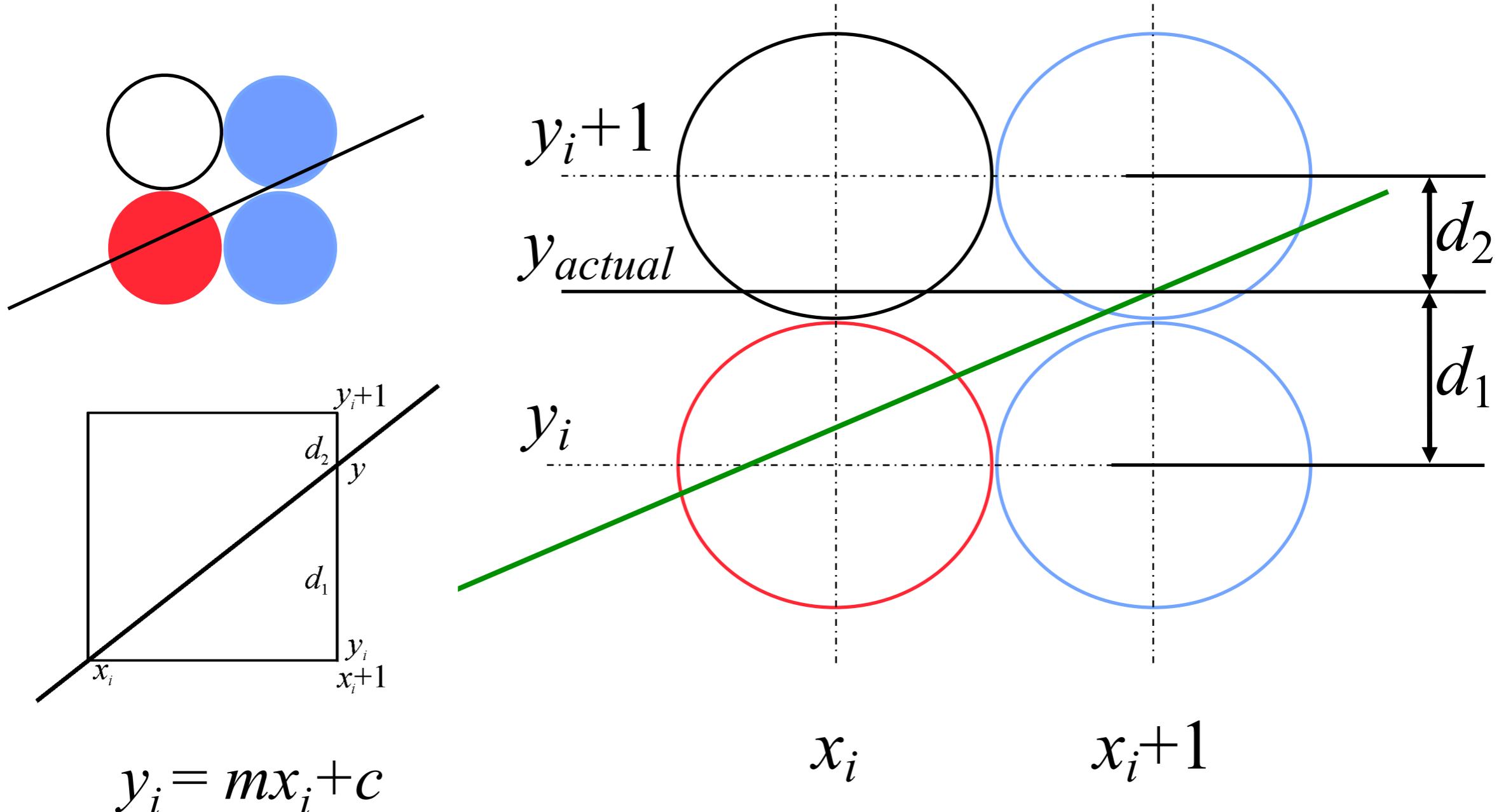
$y += 1; x += 1/m;$

Divide and conquer!

Digital Differential Analyzer

- Digital Differential Analyzer algorithm (a.k.a. DDA)
- Incremental algorithm: at each step it makes incremental calculations based on the calculations done during the preceding step
- The algorithm uses floating point operations.
- An algorithm to avoid this problem is first proposed by J. Bresenham (1937~) of IBM.
- The algorithm is well known as Bresenham's Line Drawing Algorithm (1962, when he was 25).

Bresenham Line Drawing

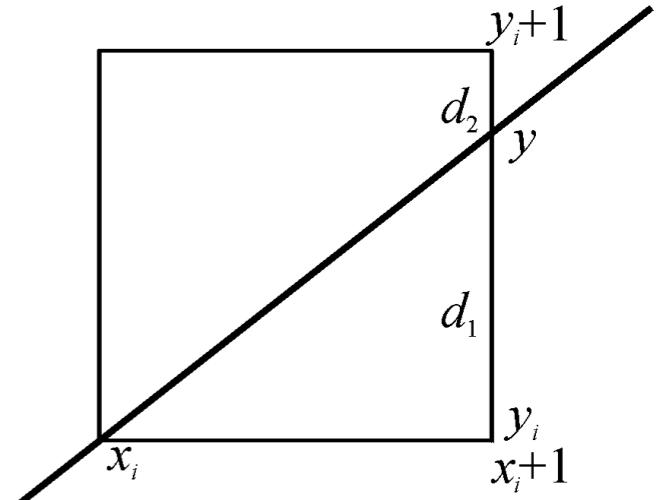


$$d_1 > d_2? \Rightarrow y_{i+1} = y_i \text{ or } y_{i+1} = y_i + 1$$

$$y = m(x_i + 1) + b \quad (2.1)$$

$$d_1 = y - y_i \quad (2.2)$$

$$d_2 = y_i + 1 - y \quad (2.3)$$



If $d_1 - d_2 > 0$, then $y_{i+1} = y_i + 1$, else $y_{i+1} = y_i$

substitute (2.1)、(2.2)、(2.3) into $d_1 - d_2$,

$$d_1 - d_2 = 2y - 2y_i - 1 = 2\frac{dy}{dx}x_i + 2\frac{dy}{dx} + 2b - 2y_i - 1$$

on each side of the equation, * dx , denote $(d_1 - d_2) dx$ as P_i , we have

$$P_i = 2x_i dy - 2y_i dx + 2dy + (2b - 1)dx \quad (2.4)$$

Because in first octant $dx > 0$, we have $\text{sign}(d_1 - d_2) = \text{sign}(P_i)$

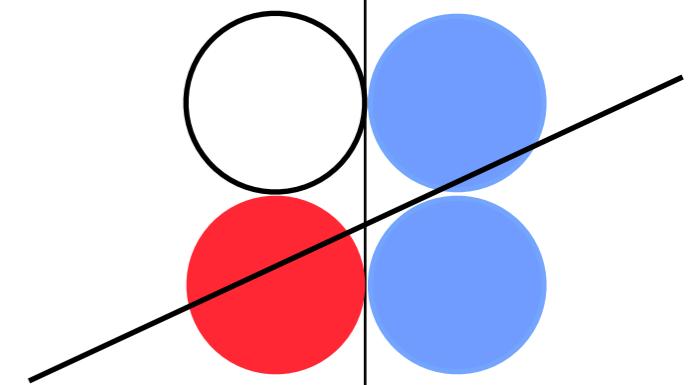
If $P_i > 0$, then $y_{i+1} = y_i + 1$, else $y_{i+1} = y_i$

$$P_{i+1} = 2x_{i+1} dy - 2y_{i+1} dx + 2dy + (2b - 1)dx, \quad \text{note that } x_{i+1} = x_i + 1$$

$$P_{i+1} = P_i + 2dy - 2(y_{i+1} - y_i) dx \quad (2.5)$$

Bresenham algorithm in first octant

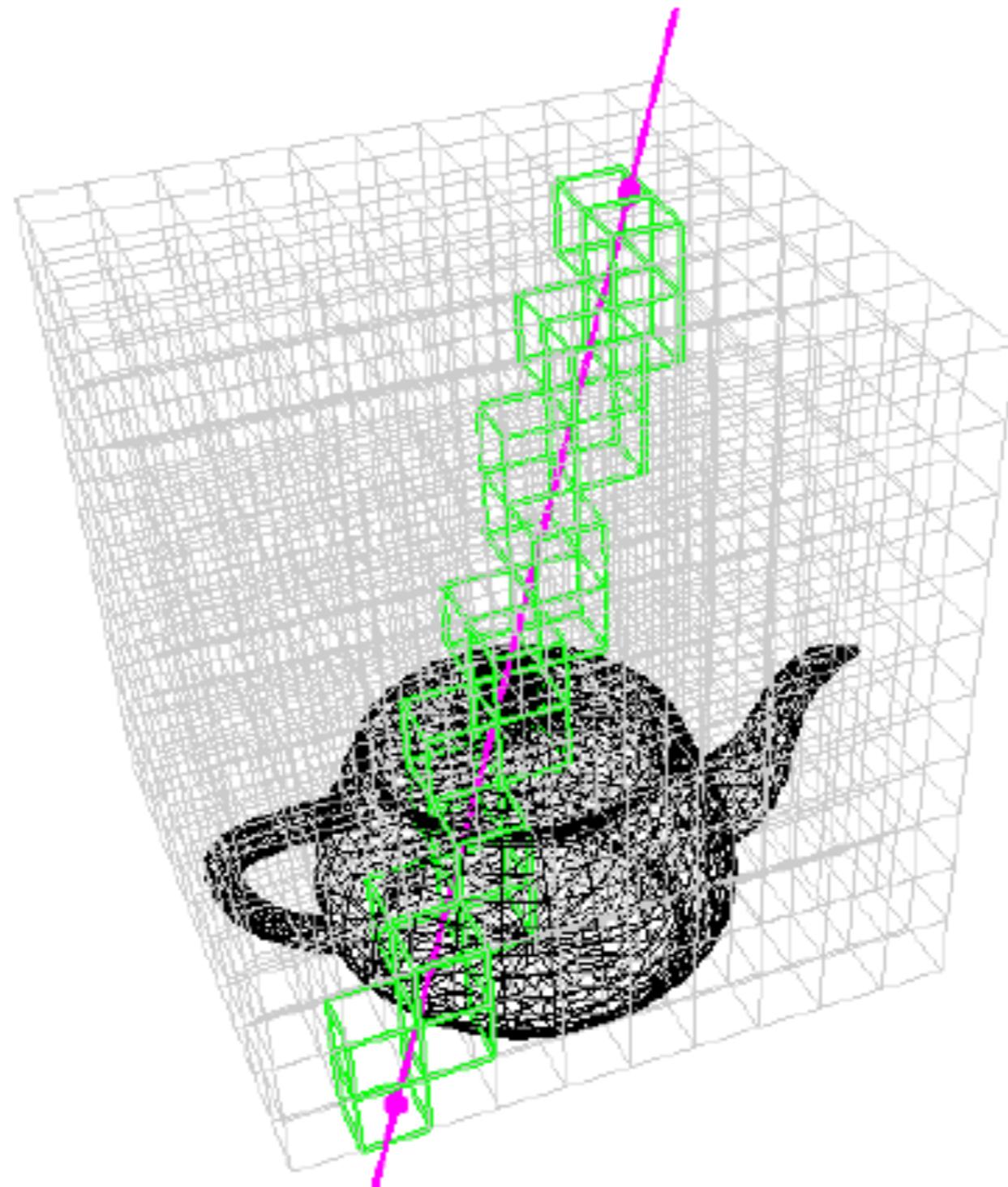
1. Initialization $P_0 = 2 dy - dx$
2. draw (x_1, y_1) , $dx=x_2-x_1$, $dy=y_2-y_1$,
Calculate $P_1=2dy-dx$, $i=1$;
3. $x_{i+1} = x_i + 1$
if $P_i > 0$, then $y_{i+1}=y_i+1$, else $y_{i+1}=y_i$;
4. draw (x_{i+1}, y_{i+1}) ;
5. calculate P_{i+1} :
if $P_i > 0$ then $P_{i+1}=P_i+2dy-2dx$,
else $P_{i+1}=P_i+2dy$;
6. $i=i+1$; if $i < dx+1$ then goto 3; else end



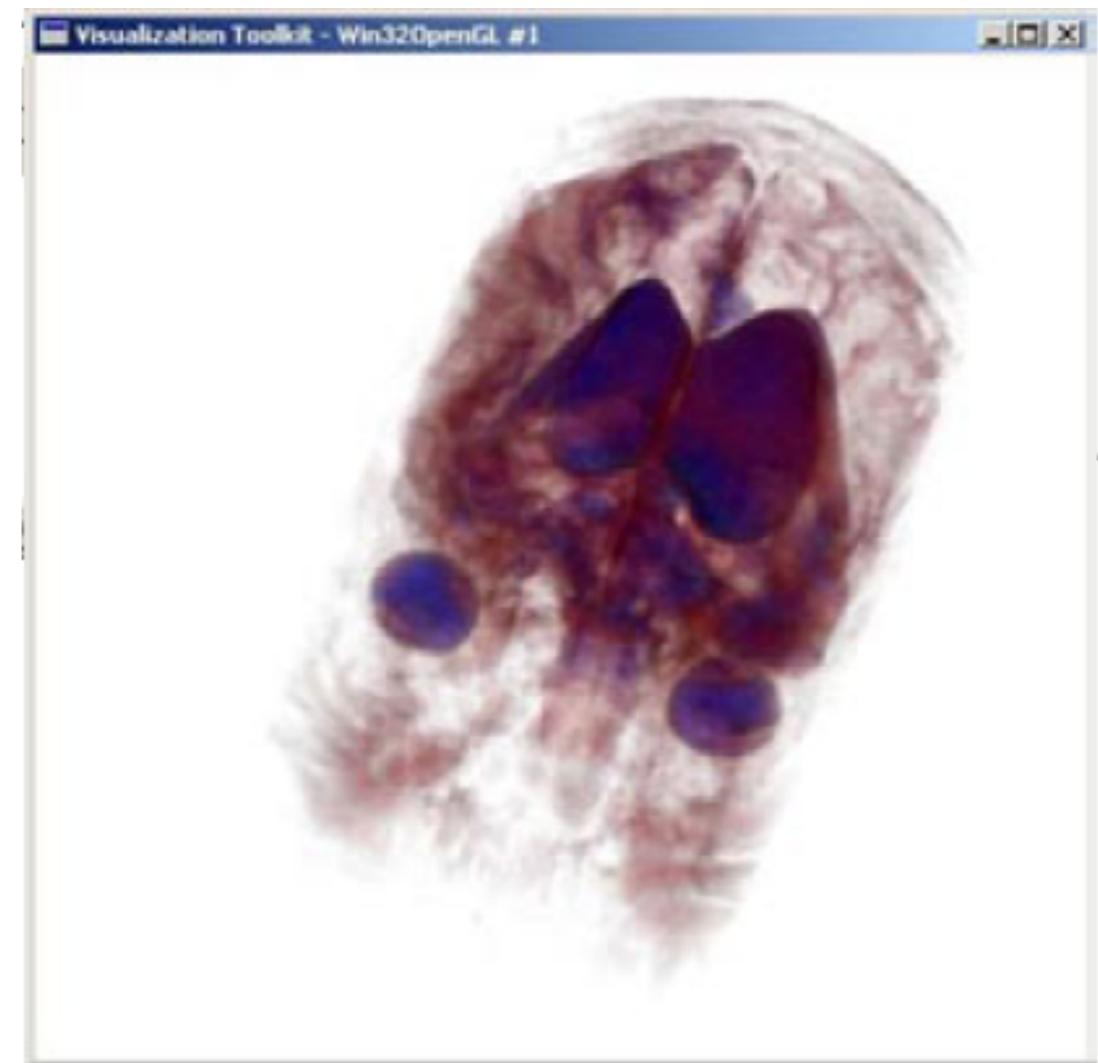
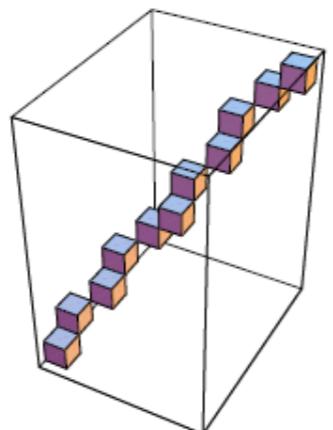
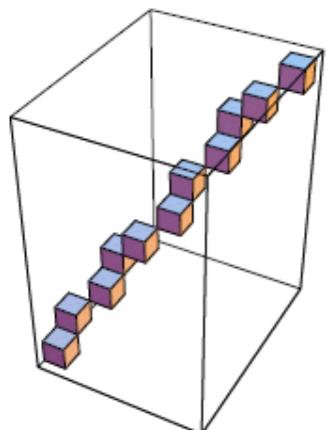
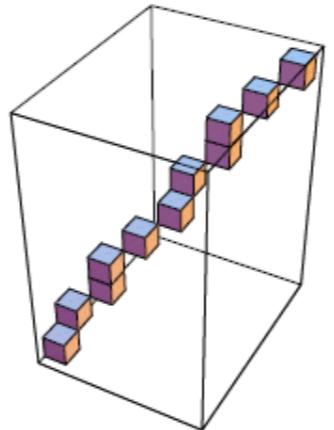
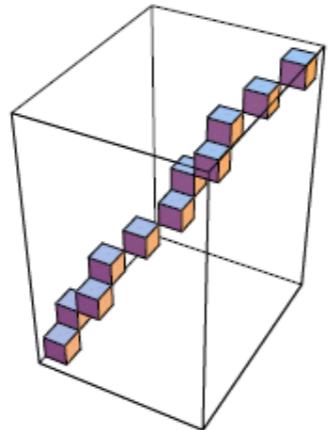
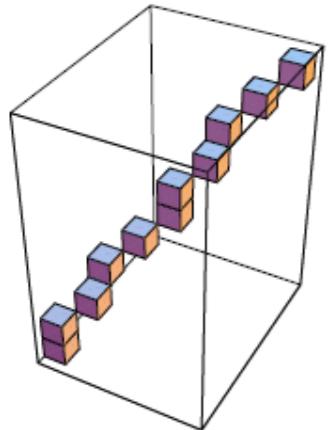
Question 3: Is it faster than DDA ?

Question 4: What technique ?

3D DDA and 3D Bresenham



3D DDA and 3D Bresenham algorithm



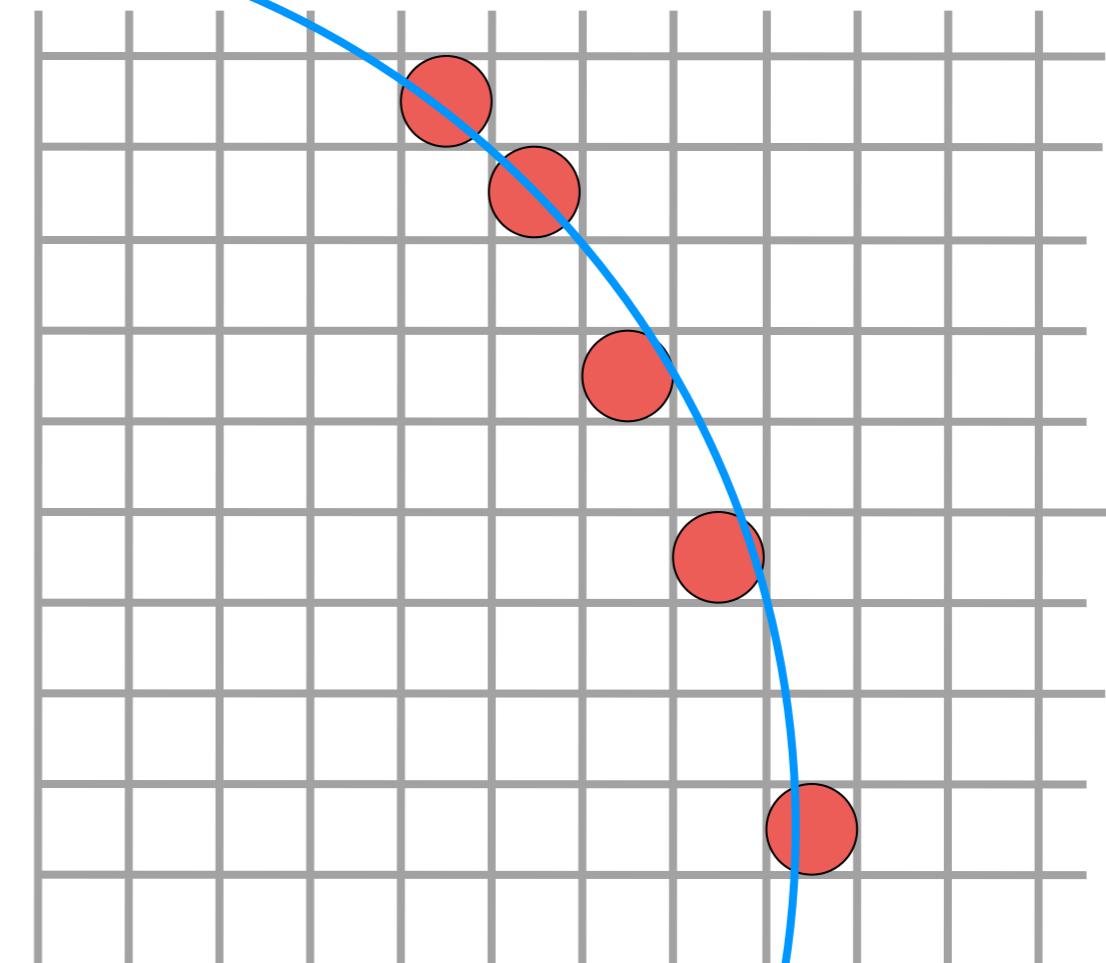
Scan converting circles

A circle with center (x_c, y_c) and radius r :

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

orthogonal coordinate

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$



polar coordinates

$$x = x_c + r \cdot \cos\theta$$

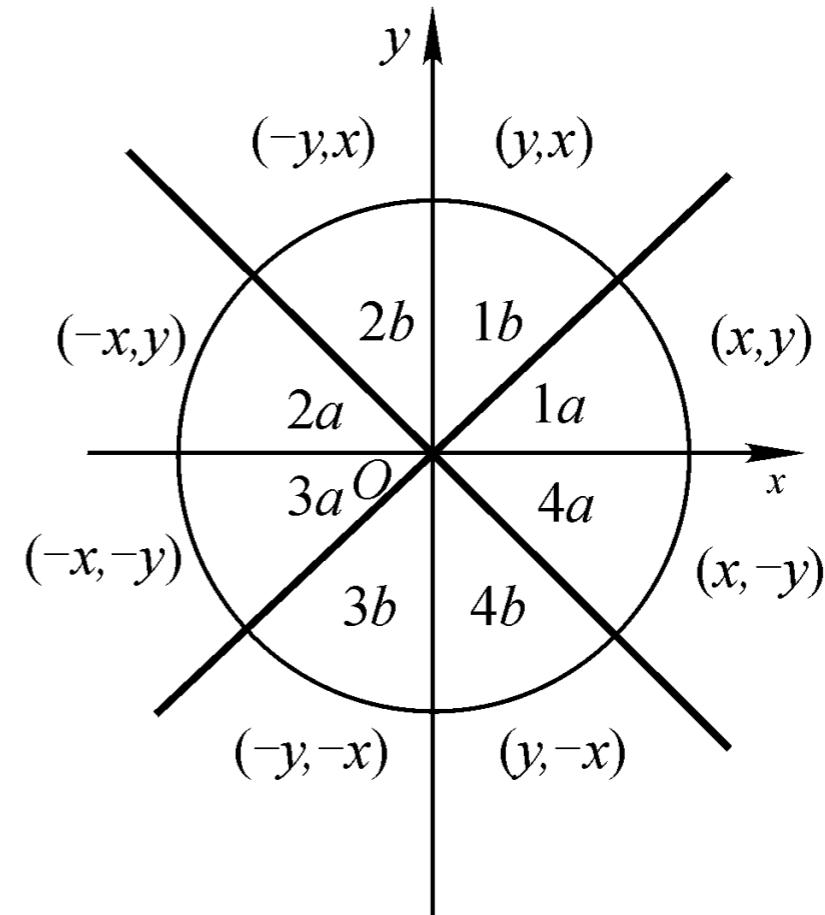
$$y = y_c + r \cdot \sin\theta$$

$$x_i = x_c + r \cdot \cos(i^* \Delta\theta)$$

$$y_i = y_c + r \cdot \sin(i^* \Delta\theta)$$

Can be accelerated by
symmetrical characteristic

$$\theta = i * \Delta\theta, \quad i=0,1,2,3,\dots$$



Discussion 3 : How to speed up?

$$x_i = r \cos\theta_i$$

$$y_i = r \sin\theta_i$$

$$x_{i+1} = r \cos(\theta_i + \Delta\theta)$$

$$= r \cos\theta_i \cos\Delta\theta - r \sin\theta_i \sin\Delta\theta$$

$$= x_i \cos\Delta\theta - y_i \sin\Delta\theta$$

Bresenham Algorithm

Homework I

Different representations

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2} \longrightarrow y = f(x) \quad x \in (x_0, x_1)$$

(explicit curve)

$$\begin{aligned} x &= x_c + r \cdot \cos\theta \\ y &= y_c + r \cdot \sin\theta \end{aligned} \longrightarrow \begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad t \in (t_0, t_1)$$

(parametric curve)

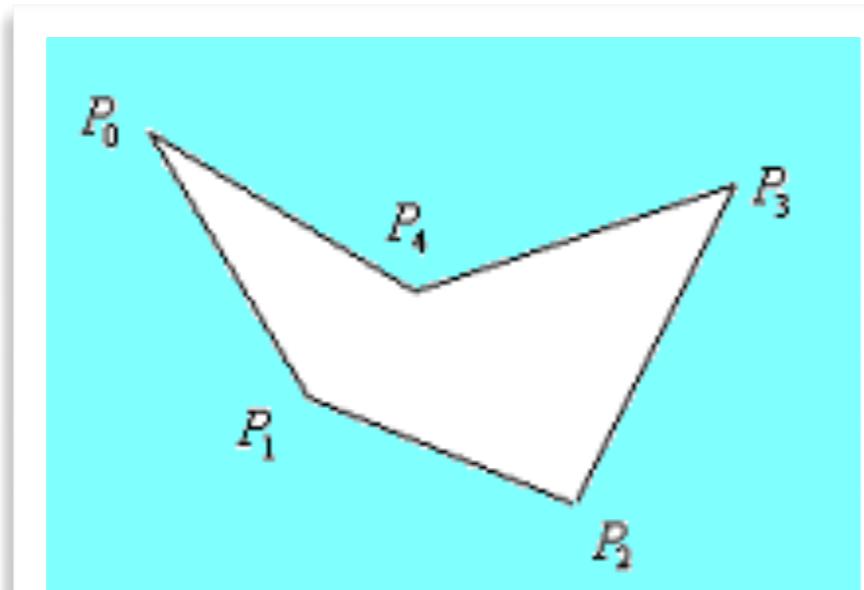
$$(x - x_c)^2 + (y - y_c)^2 = r^2 \longrightarrow g(x, y) = 0$$

(implicit curve)

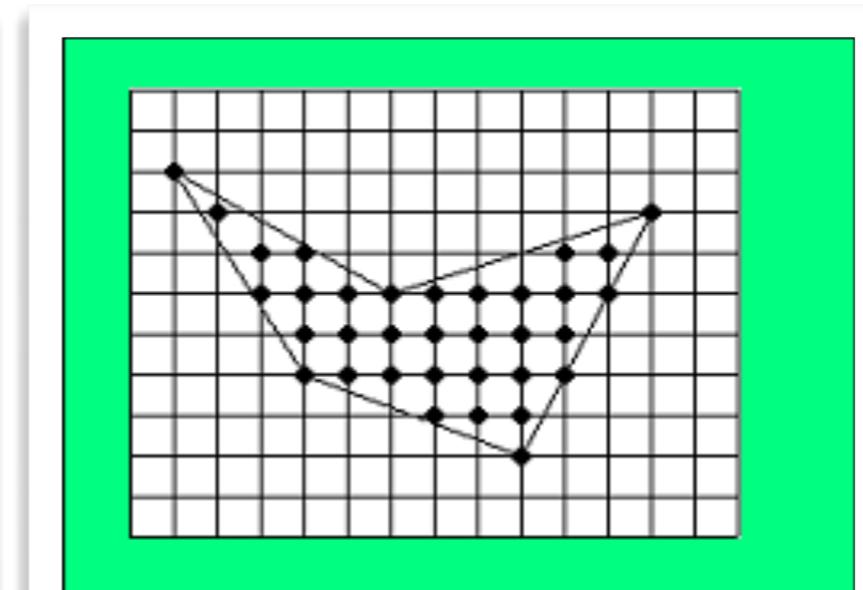
*Discussion 4 : How to display an explicit curve,
How to display a parametric curve*

Polygon filling

- Polygon representation



By vertex



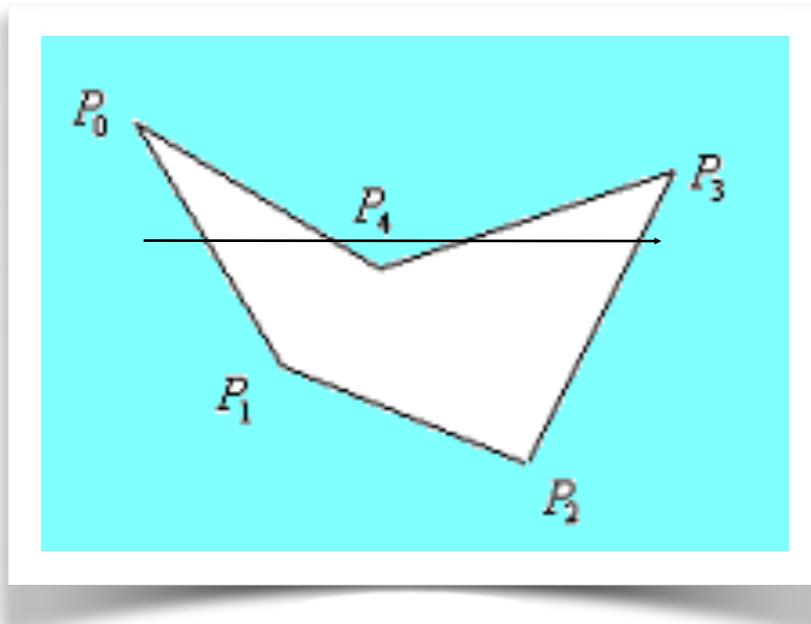
By lattice

- Polygon filling:

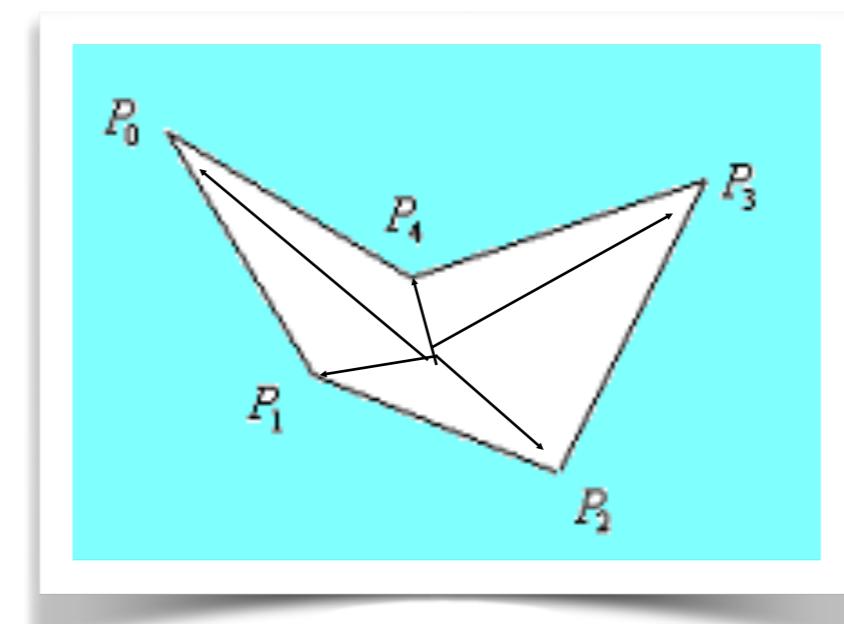
vertex representation → lattice representation

Polygon filling

- fill a polygonal area → test every pixel in the raster to see if it lies inside the polygon.



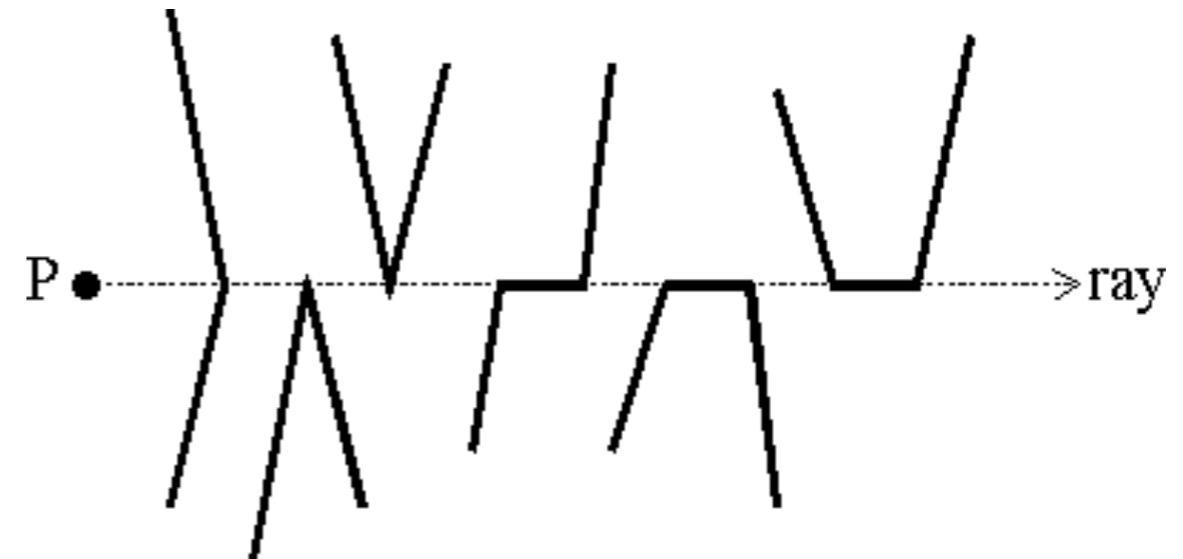
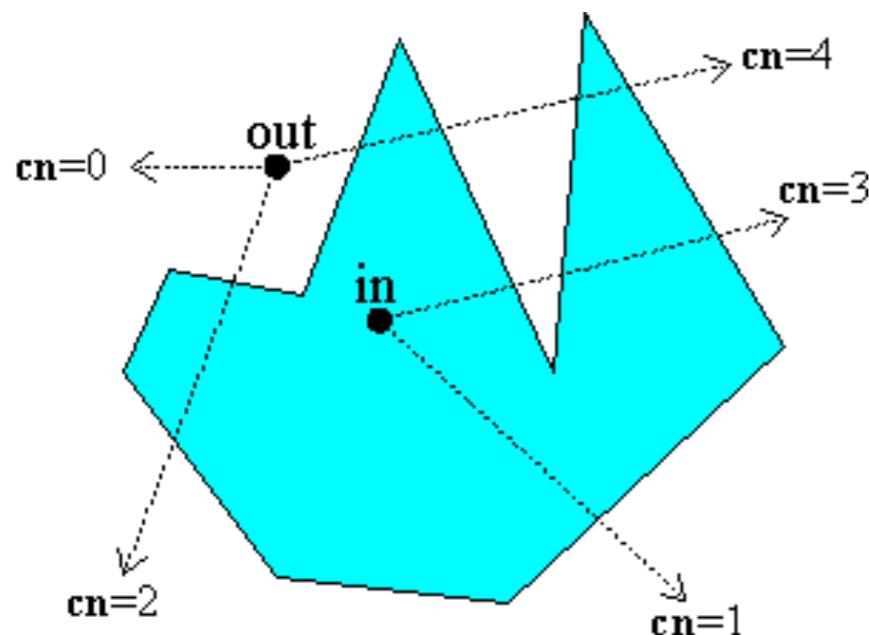
even-odd test



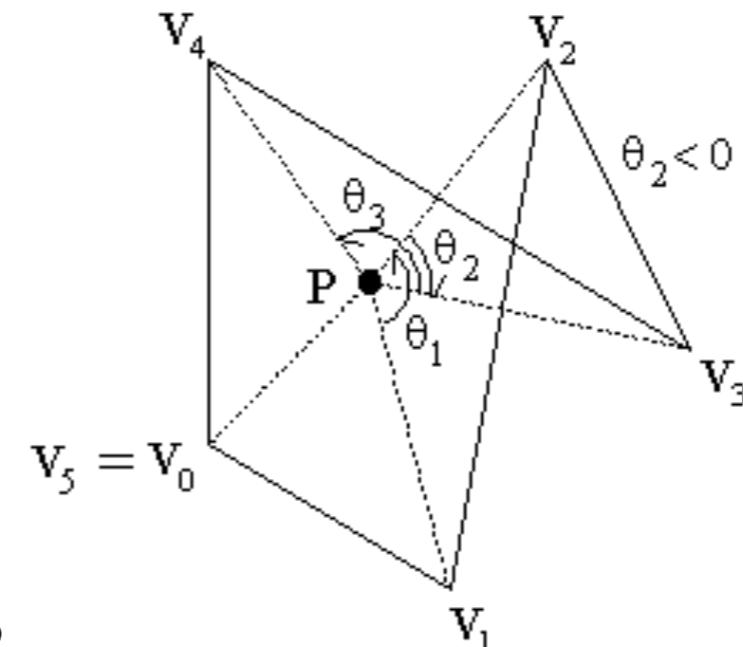
winding number test

Question5: How to Judge...?

Inside check



$$\begin{aligned}\mathbf{wn} &= \frac{1}{2\pi} \sum_{i=0}^{n-1} \theta_i \\ &= \frac{1}{2\pi} \sum_{i=0}^{n-1} \arccos \left(\frac{\mathbf{PV}_i \cdot \mathbf{PV}_{i+1}}{|\mathbf{PV}_i| |\mathbf{PV}_{i+1}|} \right)\end{aligned}$$



Question6: How to improve ...?