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Improvement of MEMS-IMU/GPS performance using fuzzy modeling

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Abstract Performance improvement of integrated Inertial Measurement Units (IMU) utilizing micro-electro-mechanical-sensors (MEMS) and GPS is described in this paper. An offline pre-defined Fuzzy model is employed to improve the system performance. The Fuzzy model is used to predict the position and velocity errors, which are the inputs to a Kalman Filter (KF) during GPS signal outages. The proposed model has been verified on real MEMS inertial data collected in a land vehicle test. A number of 30-s GPS outages were simulated during the data processing at different times and under different vehicle dynamics. Performance of the suggested Fuzzy model was compared to that of the traditional KF particularly during the simulated GPS outages. The test results indicate that the proposed Fuzzy model can efficiently compensate for GPS updates during short outages.

Keywords MEMS · INS/GPS · Takagi-Sugeno systems · Fuzzy logic modeling · Navigation

Introduction

The last two decades have seen an increasing trend in the use of integrated IMU and GPS for a variety of positioning and navigation applications. The use of low cost (e.g., MEMS) IMU systems include robotics, integrated navigation systems and Unmanned Autonomous Vehicle (UAV) applications (Shin and El-Sheimy 2002a).

The implementation of MEMS-based IMUs in navigation applications is promising. It has been shown in Da (1997) that low cost Inertial Navigation Systems (INS) can provide reliable navigation accuracy with the aid of external updating measurements (e.g., GPS-derived position and velocity). The integration of GPS with an IMU can be implemented using a KF and different schemes such as loosely, tightly, and ultra-tightly coupled integration and can be implemented either in a closed-loop or open loop architecture. Current MEMS sensors have very high drift rates that forestall the system from functioning properly in a stand-alone mode. Therefore, the performance of an integrated MEMS IMU/GPS system will dramatically degrade during GPS signal outages.

In the last decade, Takagi-Sugeno (TS) Fuzzy systems have been used to successfully model nonlinear systems and have proven to be a good representation of dynamic systems (Wang et al. 1966; Babuska et al. 1998). In these studies, a nonlinear plant was represented by a set of linear models interpolated by membership functions of a TS Fuzzy model. A noise characteristic modified Fuzzy logic model was proposed by Sasiadek et al. (2000) to help Extended Kalman Filtering (EKF) for tactical-grade INS/GPS data fusion in guidance, navigation and control of airborne vehicles. They investigated their proposed Fuzzy adaptive KF for guidance and control of mobile robots. The results have illustrated the effectiveness of the suggested model in reducing the number of states and improving the navigation accuracy. In Tiano et al. (2001), the parameters of a KF were converted to linguistic variables to produce a neuro-Fuzzy KF. The resulting neuro-Fuzzy KF was combined with a map matching method to enhance the real-time performance of a navigation system fusing GPS and low-cost hardware sensors. The experimental validation of the proposed combined system showed better performance when estimating the position of a car inside a city route under normal traffic conditions. Kaygisiz et al. (2003) have proposed a model for integrating a feed-forward multilayer perceptron to intelligently aid the GPS/INS coupled navigation system in the absence of GPS signals. The used IMU in this model was a tactical grade unit utilizing a triad of fiber optic gyros. The proposed enhanced GPS and INS architecture was tested in a highly dynamic environment where the results indicated a significant improvement in the achieved positioning accuracy.

In this paper, an offline extracted Fuzzy model is used to estimate the position and velocity errors of an integrated GPS/IMU system. The Fuzzy model is extracted from input/output patterns. During the model extraction process, the input to the Fuzzy model is the output of the INS mechanization equations and the desired outputs of the Fuzzy model are position and velocity errors during various simulated GPS outages. At the end of the training process the best estimates of the Fuzzy model parameters are achieved. Therefore, the model should be able to provide corrections or compensatory update measurements to the KF predicted positions and velocities during real-time GPS outages. This will consequently keep the filter always working in update mode, even if the real GPS update measurements are unavailable. The proposed model has been assessed using data collected from a MEMS IMU/GPS integrated system mounted in a passenger vehicle traveling on road. This field test included kinematic and static mode scenarios.

Section II summarizes the mathematical background of the newly developed integration model. Section III introduces the fundamentals of Fuzzy inference systems. Section IV presents detailed explanations of the proposed model. Section V presents the experimental results and analysis of the Fuzzy-KF integrated model.

Mathematical background

IMU mechanization

IMU mechanization is the process of determining the navigation states (position, velocity and attitude) from raw inertial measurements through solving the differential equations describing the system motion. IMU measurements include three angular rate components provided by the gyroscopes and denoted by the 3×1 vector ω^{b}_{ib} as well as three linear acceleration components provided by the accelerometers and denoted by the 3×1 vector f^{b}. The subscript (b) refers to the sensor body frame where all inertial measurements typically take place, while the subscript (i) denotes the inertial frame. This means that the angular velocities, ω^{b}_{ib}, of the *body* frame are measured with respect to the inertial frame and the superscript (b) indicates that the inertial measurements are expressed as seen by an observer in the *body* frame.

Mechanization is usually expressed by a set of differential equations and typically performed in the local level frame defined by the local east, north and ellipsoid normal (Wong 1988; El-Sheimy 2002):

$$\begin{pmatrix} \dot{r}^{\ell} \\ \dot{v}^{\ell} \\ \dot{R}^{\ell}_{\mathrm{b}} \end{pmatrix} = \begin{pmatrix} D^{-1}\, v^{\ell} \\ R^{\ell}_{\mathrm{b}} f^{\mathrm{b}} - (2\Omega^{\ell}_{\mathrm{ie}} + \Omega^{\ell}_{\mathrm{e}\ell}) v^{\ell} + g^{\ell} \\ R^{\ell}_{\mathrm{b}} (\Omega^{\mathrm{b}}_{\mathrm{ib}} - \Omega^{\mathrm{b}}_{\mathrm{i}\ell}) \end{pmatrix} \tag{1}$$

where

$\dot{r}^{\ell}$	the position vector [φ(latitude), λ (longitude), h (height)] in the local level frame,
$\dot{v}^{\ell}$	the velocity vector ($v_{\mathrm{east}} v_{\mathrm{north}} v_{\mathrm{up}}$) in the local level frame,
$\dot{R}^{\ell}_{\mathrm{b}}$	the transformation matrix from body to local frame as a function of attitude components,
e	the earth fixed frame,
g^{ℓ}	the gravity vector in the local level frame,
$\Omega^{\mathrm{b}}_{\mathrm{ib}}$,$\Omega^{\mathrm{b}}_{\mathrm{il}}$	the skew-symmetric matrices of the angular velocity vectors $\omega^{\mathrm{b}}_{\mathrm{ib}}$, $\omega^{\mathrm{b}}_{\mathrm{il}}$ respectively, and
D^{-1}	a 3×3 matrix whose non-zero elements are functions of the user's latitude φ and ellipsoidal height (*h*).

For more details about the solution and numerical implementation of Eq. 1, the reader is referred to El-Sheimy (2002).

Inertial state dynamic error model and Kalman filtering

The navigation algorithm integrates the IMU mechanization equations (Eq. 1) to yield the parameters on the left-hand side, namely the position, velocity, and attitude (PVA) of the vehicle. The algorithm takes into account the Earth's rotation rate and gravity. If this navigation algorithm is applied in a standalone mode, it will not be sufficient since the inertial sensor errors (mainly sensors biases) and the fixed-step integration errors will cause the PVA solution to diverge quickly. Therefore, the navigation algorithm must be augmented with an adequate error model that accounts for these error sources in order to correct the estimated PVA. The most common estimation algorithm used in integrated IMU/GPS is the KF. The KF exploits a powerful synergy between GPS and IMU measurements. This synergy is possible in part because the IMU and GPS measurements have complementary error characteristics. Short-term navigation errors from the IMU are relatively small, but they degrade without bounds over time (Mohinder et al. 2001). On the other hand, GPS position and velocity errors are not as good over the short term, but they do not degrade with time. The KF is able to take advantage of these characteristics to provide a common integrated navigation implementation with performance superior to that of either the GPS or INS sub-systems (Jay and Matthew 1999). In this KF integration, the GPS derived positions and velocities are used as the update measurements for the IMU derived PVA. The KF error state vector in this case includes the navigation parameters as well as the accelerometer and gyroscope error states.

The dynamic error model of the navigation parameters (i.e., PVA) can be determined through the linearization of the mechanization model given in Eq. 1. A simplified form is then obtained by neglecting the insignificant terms in the resultant linearized model (Bar-Itzhack and Berman 1988):

$$\begin{aligned} \delta\dot{r}^{\ell} &= D^{-1}\delta v^{\ell} \\ \delta\dot{v}^{\ell} &= -(2\Omega_{\mathrm{ie}}^{\ell} + \Omega_{\mathrm{e}\ell}^{\ell}) \times \delta v^{\ell} - \delta R_{\mathrm{b}}^{\ell} f^{\mathrm{b}} + R_{\mathrm{b}}^{\ell}\delta f^{\mathrm{b}} + \delta g^{\ell} \\ \delta\dot{A}^{\ell} &= E\delta v^{\ell} + R_{\mathrm{b}}^{\ell}\delta\omega^{\mathrm{b}} \end{aligned} \tag{2}$$

where

- $\delta\dot{r}^{\ell}$ the position error state vector in local level frame,
- $\delta\dot{v}^{\ell}$ the velocity error state vector in local level frame,
- $\delta\dot{A}^{\ell}$ the attitude error state vector in local level frame,
- δg^{ℓ} the error in the computed gravity vector in local level frame,
- δf^{b}, $\delta\omega^{\mathrm{b}}$ accelerometer bias and gyro drift vectors in the body frame respectively,
- E a 3×3 matrix whose non-zero elements are functions of the vehicle's latitude and the Earth's radii of curvatures.

Adding the various inertial sensor residuals to the KF, the state vector to be solved consists of 15 states; the nine navigation parameters, three accelerometer biases, and three gyro drift terms. Based on the mathematical expressions in Eq. 2 and the commonly used Gauss-Markov representation of inertial sensor errors, the inertial state dynamic error model can be written in the following form:

$$\dot{x} = Fx + Gw \tag{3}$$

where

- x the error state vector of inertial navigation whose elements include three position errors, three velocity errors, three attitude errors, three accelerometer biases, and three gyro drift terms,
- F the state transition matrix, see Mohinder and Angus (2001) for more details,
- G a rectangular matrix whose elements may be time variant,
- W a zero-mean Gaussian white noise vector, and
- Gw the covariance matrix of the applied state model.

The KF estimates these elements by using a form of feedback control. Equations of the KF are divided into two groups: *time prediction* and *measurement update* (Mohinder et al. 2001). The *time prediction* equations are responsible for the forward time transition of the current epoch (k-1) states to the next epoch (k) states and are given by:

$$\hat{x}_k(-) = \mathbf{F}_{k,k-1}\hat{x}_{k-1}(+) \tag{4}$$

$$P_k(-) = F_{k,k-1}P_{k-1}(+)F_{k,k-1}^{\mathrm{T}} + Q_{k-1} \tag{5}$$

where

- P the estimated variance–covariance matrix of inertial states,
- Q the system noise matrix,
- $(-)$ the estimated value after prediction, and
- $(+)$ the estimated value after updating.

The *measurement update* equations utilize new measurement into the a-priori state estimate to obtain an optimized a-posteriori state estimate. The measurement update equations are given by:

$$K_k = P_k(-)H_k^{\mathrm{T}}\left[H_kP_k(-)H_k^{\mathrm{T}} + R_k\right]^{-1} \tag{6}$$

$$\hat{x}_k(+) = \hat{x}_k(-) + K_k(Z_k - H_k\hat{x}_k(-)) \tag{7}$$

$$P_k(+) = P_k(-) - K_kH_k^{\mathrm{T}}P_k(-) \tag{8}$$

where

- K the Kalman gain matrix,
- Z the vector of updating measurements of position and velocity,
- R the measurements variance–covariance matrix, and
- H the design matrix that relates the measurements to the state vector.

The KF update engine is triggered at every GPS measurement epoch using the difference between the GPS and the INS mechanization solutions as input. Hence, KF generates an updated estimate to diminish the INS errors using measurement update equations. Whenever the GPS signal is missing, the KF works in a time prediction mode to estimate the error state vector. A functional block diagram of the integration approach is shown in Fig. 1. KF equations require the statistical properties of the system to be stationary and well defined which cannot be guaranteed especially in a dynamic environment.

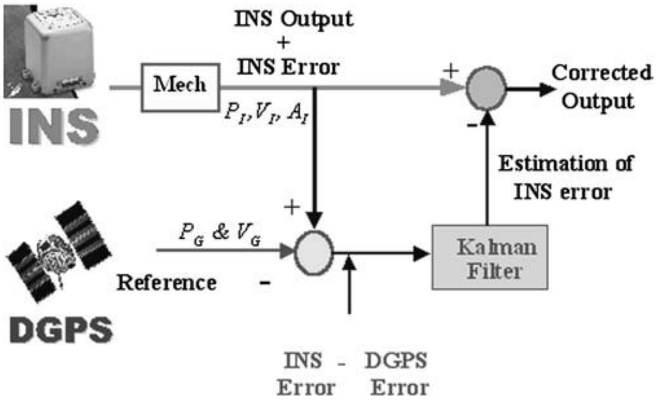

Fig. 1 Functional block diagram of KF-based IMU/GPS integration

Fuzzy modeling

Fuzzy set theory is a mathematical concept proposed by Zadeh (1984). Fuzzy logic uses graded or quantified statements rather than rules that are strictly true or false. The Fuzzy sets allow objects to have grades of membership of values ranging from zero to one. These sets, represented by linguistic variables, are used to represent a particular Fuzzy set in a given problem, such as "large", "medium" and "small". For instance, if U is a collection of objects denoted generically by $\{u\}$, which could be discrete or continuous, U is called the universe of discourse while u represents the generic element of U.

A Fuzzy set F in a universe of discourse U is characterized by a membership function μ_f that takes on values in the interval [0,1]. A Fuzzy set F in U may be represented as a set of ordered pairs of a generic element u and its grade of membership function as follows:

$$\begin{aligned} F &= \{(u, \mu_F(u)) | u \in U\} \\ F &= \int_U \frac{\mu_F(u)}{u} \mathrm{d}u \\ F &= \sum_{i=1}^{n} \frac{\mu_F(u_i)}{u_i} \end{aligned} \tag{9}$$

Fuzzy sets have some operations that can be applied to crisp sets, as a subset of Fuzzy sets. The set theory operation of union, intersection, complement and other relations of Fuzzy sets are defined by their membership function. For example, if A and B are two Fuzzy sets in U with membership function μ_A and μ_B, respectively then, the membership function $\mu_{A \cup B}$ of the union A∪B is point-wise defined for all $u \in U$ by:

$$\mu_{A\cup B}(u) = \max \mu_A(u), \mu_B(u) \tag{10}$$

while the membership function $\mu_{A \cap B}$ of the intersection $A \cap B$ is point-wise defined for all u ∈ U by

$$\mu_{A\cap B}(u) = \min \mu_A(u), \mu_B(u) \tag{11}$$

The Fuzzy Inference System (FIS) is the process of formulating the mapping from a given input to an output using Fuzzy logic. The dynamic behavior of a FIS is characterized by a set of linguistic description rules based on expert knowledge as follows:

IF (a set of antecedent conditions is satisfied) THEN (a set of consequences can be inferred).

Since the antecedents and the consequents of these IF-THEN rules are associated with Fuzzy concepts (linguistic terms), they are often called Fuzzy conditional statements (Lee 1990). In the case of a Two-Input-Single-Output Fuzzy system, the Fuzzy rules have the form:

$$\begin{aligned} &R_1: \text{ if } (x) \text{ is } A_1 \text{ and } (y) \text{ is } B_1 \quad \text{then } (z) \text{ is } C_1, \\ &R_2: \text{ if } (x) \text{ is } A_2 \text{ and } (y) \text{ is } B_2 \quad \text{then } (z) \text{ is } C_2, \\ &\qquad \ldots \quad \ldots \quad \ldots \quad \ldots \\ &R_n: \text{ if } (x) \text{ is } A_n \text{ and } (y) \text{ is } B_n \quad \text{then } (z) \text{ is } C_n, \end{aligned}$$

where x, y, and z are linguistic variables representing a two input process state and one output variable, respectively. The terms A_i and B_i are linguistic values of the linguistic variables x, y in the universe of discourse U and V, respectively, with $i = 1,2,..., n$. The C_i terms are linguistic values of the linguistic variables z in the universe of discourse W in case of Mamdani FIS (Mamdani 1974) and mapping functions of the inputs in case of TS FIS (Takagi and Sugeno 1985).

Generally, the TS FIS model has four stages:

1. The fuzzification interface involves the following functions:
   - Measuring the values of input variables,
   - Performing a scale mapping that transfers the range of values of input variables into corresponding universes of discourse,
   - Performing the function of fuzzification that converts input data into suitable linguistic values that may be viewed as labels of Fuzzy sets.
2. The knowledge base comprises knowledge of the application domain and the attendant control goals. It consists of a "database" and a "linguistic (Fuzzy) rule base", where:
   - The database provides the necessary definitions, which are used to define linguistic rules and Fuzzy data manipulation in the FIS.
   - The rule base characterizes the goals and policy of the experts' domain by means of a set of linguistic rules.
3. The decision-making logic is the kernel of the FIS. It has the capability of simulating human decision-making based on Fuzzy concepts, implication and the rules of inference in Fuzzy logic.

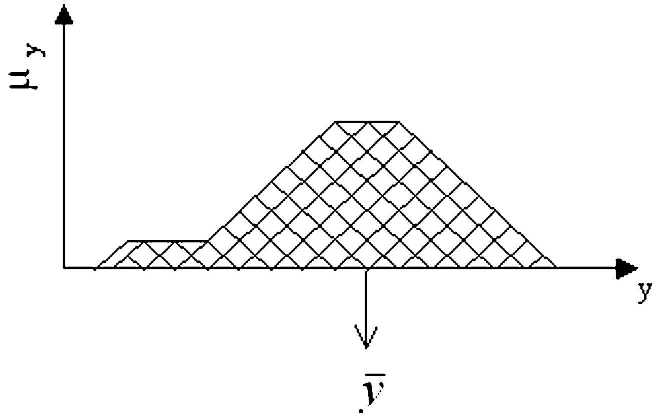

Fig. 2 The centroid method

4. The defuzzification inference converts an inferred Fuzzy action into a crisp one using the following defuzzification strategies:
   - Scale mapping for converting the range of values of output variables into corresponding universe of discourse.
   - Defuzzification for yielding a non-Fuzzy action from an inferred action. One of the commonly used methods herein is the Centroid, which generates the center of gravity of the area bounded by the membership function curve as shown in Fig. 2.

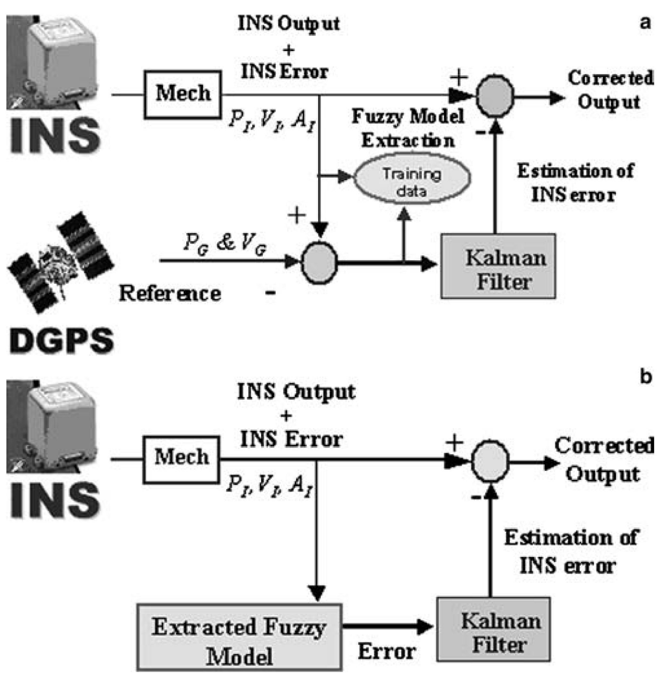

Fig. 3 **a** Block diagram shows data collection for Fuzzy model extraction. **b** Block diagram of proposed MEMS INS/GPS integration during GPS signal outage

Proposed model

The KF is a recursive algorithm designed to compute corrections to a system based on external measurements. In inertial navigation, this can be accomplished by using an external navigation reference, such as GPS. As long as GPS measurements are available, the KF solution of INS/GPS integration works efficiently and provides accurate estimate of the navigation states. Nevertheless, during GPS signal outages, stand-alone IMU (specifically MEMS) is affected by the rapid accumulation of drift rate errors. As a result, the overall performance of a MEMS IMU/GPS integrated system is dramatically degraded.

The proposed solution for this problem is a Fuzzy model extracted by INS mechanization output data which acts as input to the model. Meanwhile, the error measurements provided by GPS act as the desired model output during the training process as shown in Fig. 3a. Whenever the GPS signal is blocked, the Fuzzy model can alternatively generate position and velocity error measurements. Hence, the KF can continue to utilize the update measurement equations as shown in Fig. 3b.

The proposed Fuzzy model utilizes a TS-FIS that has specific properties as it can interpolate local linear models by overlapping local basis functions.

The proposed TS-FIS of the applied Fuzzy model can be written in a general form as:

$$R_i: \text{ if } x_k \text{ is } A_i(x_k) \quad \text{then } \hat{y}_i = a_i^{\mathrm{T}} \phi_k + b_i, \qquad i = 1, \ldots, c \tag{12}$$

where

$A_i(x_k)$	a Gaussian membership function that describes the Fuzzy set A_i,
a_i and b_i	the consequent parameters θ_i of the Fuzzy model that describe the local linear model,
φ_k and x_k	subsets of the input sequence,
R_i	the rule number i,
c	the total number of rules,
$\hat{y}_i$	the estimated local linear model output.

The antecedent proposition "x_k is A_i (x_k)" can be defined for the individual components of x in the conjunction form:

$$R_i: \text{ if } x_{1,k} \text{ is } A_{i,1}(x_{1,k}) \text{ and} \ldots \text{and } x_{n,k} \text{ is } A_{i,n}(x_{n,k}) \quad \text{then } \hat{y}_i = a_i^{\mathrm{T}} \phi_k + b_i \tag{13}$$

The components of x are the nine input states, which include three velocity components v^{ℓ}, three velocity increments Δv^{ℓ} and three attitude components $\Delta\omega^{\ell}$. The selection of these parameters as inputs to the Fuzzy model is convenient (see Eq. 2) since they are the main factors affecting the prospected outputs of the Fuzzy model (position and velocity error components).

Moreover, these states are all determined in the local level frame and easily obtained from the INS mechanization process.

Four clusters have been assigned iteratively to every single input state where a Gaussian function has been implemented to represent the degree of membership to each cluster. Based on the number of inputs (9) and the number of clusters (4) the initial possible number of rules is $5^9 = 1953125$. The estimated model outputs, $\hat{y}$, are the position and the velocity error state vectors, δr^ℓ and δv^ℓ, in the local level frame and can be computed from the FIS in Eq. 12 as follows:

$$\hat{y} = \frac{\sum_1^c \beta_i(x_k)(a_i\phi_k + b_i)}{\sum_1^c \beta_i(x_k)} \tag{14}$$

where

$$\beta_i = \prod_{j=1}^{n} A_{i,j}(x_{j,k}) = \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\frac{(x_{j,k} - V_{i,j})^2}{\sigma_{i,j}^2}\right)$$

and $V_{i,j}$, $\sigma^2{}_{i,j}$ represents the center and the variance of the Fuzzy Gaussian membership functions respectively.

To identify the FIS of the Fuzzy model, the antecedent parameters, the parameters $V_{i,j}$ and $\sigma^2{}_{i,j}$ of Fuzzy membership sets, and the consequent parameters θ_i need to be determined. Abonyi et al. (2001) proposed a method to identify TS-Fuzzy models through clustering and Orthogonal Least Squares Plus (OLS^+). In this method, the antecedent parameters are achieved through a Fuzzy C-means (FCM) algorithm while the consequent parameters are estimated by utilizing the OLS^+ method. Clustering involves the task of dividing data points into homogeneous classes or clusters. Hence, items in the same class are as similar as possible while items in different classes are as dissimilar as possible. FCM algorithm clusters the data by minimizing the total distance (D) of each data point to the cluster centers. This algorithm, aims at partitioning the identification data (**Z**) into (c) clusters from a given training set and hence produces the nonlinear parameters of the membership functions, by minimizing the basic c-means objective function (J) (Turčan et al. 2003):

$$J(\mathbf{Z}; \mathbf{U}, \mathbf{V}) = \sum_{i=1}^{c}\sum_{k=1}^{N} (\mu_{ik})^m \|\mathbf{z}_k - \mathbf{v}_i\|_{\mathbf{A}}^2 = \sum_{i=1}^{c}\sum_{k=1}^{N} (\mu_{ik})^m D_{ik}^2 \tag{15}$$

where

μ_{ik}	the degree of membership,
$\mathbf{U} = [\mu_{ik}] \in M_{fc}$	the Fuzzy partition matrix of Z,
$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_c]$, $\mathbf{v}_i \in \mathbf{R}^n$	a vector of cluster initials,
$D^2{}_{ik} = \|\|\mathbf{z}_k - \mathbf{v}_i\|\|^2$	the dissimilarity measure between the sample $\mathbf{z}_k$ and the center $\mathbf{v}_i$ of the specific cluster of the specific cluster i (Euclidean distance) and
$m \in (1,\infty)$	a weighting component, that determines the fuzziness of the resulting clusters.

The minimization of the objective function J can be achieved by finding the cluster centers iteratively (Billings et al. 1988):

$$\mathbf{v}_i^{(l)} = \frac{\sum_{k=1}^{N} (\mu_{ik})^m \mathbf{z}_k}{\sum_{k=1}^{N} (\mu_{ik})^m},\ \sigma_{i,j}^{2(l)} = \frac{\sum_{k=1}^{N} (\mu_{ik})^m (x_{j,k} - \mathbf{v}_{j,k})^2}{\sum_{k=1}^{N} (\mu_{ik})^m},\quad 1 \le i \le c \tag{16}$$

where,

$$\mu_{ik} = \begin{cases} \frac{1}{\sum_{j=1}^{c} (D_{ik}/D_{jk})^{2/(m-1)}} & \text{if } D_{ik} > 0 \\ 0 \end{cases},$$

$$\mu_{ik} \in \langle 0, 1\rangle,\quad \sum_{i=1}^{c} \mu_{ik} = 1$$

After estimating the center of each cluster using Eqs. 15 and 16, the consequent parameters of the Fuzzy model θ_i can be obtained through the minimization of the objective function F (Abonyi et al. 2001):

$$F = \|y - \phi_e\theta_i\|^2\beta_i \tag{17}$$

where $\|y - \varphi_e\theta_i\|^2$ represents the difference between the actual and the estimated Fuzzy output. $\varphi_e = [\varphi \quad 1]$ denotes the extended regression matrix obtained by adding a unit column to φ, $\varphi = [\varphi_1 \quad \varphi_2 \quad \varphi_3 \ldots \varphi_N]^T$, $y = [y_1 \quad y_2 \quad y_3 \ldots\ y_N]^T$, and

$$\beta_i = \begin{bmatrix} \mu_{i,1} & 0 & \ldots & . & 0 \\ 0 & \mu_{i,2} & \ldots & . & . \\ . & . & \ldots & . & . \\ . & . & \ldots & . & . \\ 0 & 0 & \ldots & . & \mu_{i,N} \end{bmatrix}.$$

Applying the OLS^+ method to minimize the objective function F (Eq. 17) results in an estimate for the consequent parameters *theta*$_i$:

$$\theta_i = \beta^{+T} y \sqrt{\beta_i} \tag{18}$$

where β^+ denotes the Moore-Penrose pseudo inverse of $\phi_e\sqrt{\beta_i}$.

Antecedent variables selection is usually necessary since it results in an effective reduction of the number of applied Fuzzy rules. A modified version of the fischer interclass separability method (Roubos et al. 2001), which is based on statistical properties of the data, has been applied for feature selection of the premise Fuzzy clusters. The interclass separability criterion is based on the F_B between-class and the F_W within-class covariance matrices that sum up to the total covariance of the training data F_T, where:

$$F_W = \sum_{i=1}^{c} p(\eta_i) F_i, \quad F_B = \sum_{i=1}^{c} p(\eta_i)(v_i - v_o)^T (v_i - v_o) \qquad (19)$$

with $p(\eta_i) = \frac{1}{N}\sum_{k=1}^{N} \mu_{i,k}$, $v_o = \sum_{i=1}^{c} p(\eta_i)$, and F_i is a diagonal matrix that contains the variances of the Gaussian membership functions.

The feature interclass separability selection criterion is a trade-off between the F_W and F_B:

$$J = \frac{\det(F_B)}{\det(F_W)} \qquad (20)$$

The importance of a feature (antecedent Fuzzy cluster) is measured by leaving out the feature and calculating J for the reduced covariance matrices. The feature selection is made iteratively by leaving out the least needed feature (Jang 1996).

Testing the proposed model

In order to examine the effectiveness of the proposed Fuzzy model and its impact on the accuracy of the estimated navigation parameters (particularly position), field test data provided by the Applanix Corporation, incorporating MEMS-based IMU and DGPS sensors in a land vehicle, are used. The MEMS-based IMU implements Analog Devices MEMS gyros and Colibrys MEMS accelerometers. In addition, a POS-LV-320 system, which integrates a tactical grade LTN200 IMU and a DGPS, was also included in the same test to provide the reference. Both DGPS and LTN200 trajectories were available throughout the entire test so that they can be used as an accurate reference for the low cost MEMS IMU. The MEMS IMU and the DGPS data were processed through the Mobile Multi-Sensor Systems research group, University of Calgary, INS Tool Box KF software (Shin and El-Sheimy 2002b). A number of intentionally introduced GPS outages of 30 s were simulated at different locations along the test trajectory (see Fig. 4). The GPS outages were created by removing the measurements during the intervals selected. Thus, the GPS measurement accuracy and satellite geometry were normal immediately before and after the outages. The outage periods were carefully selected to consider different dynamics and different motion types throughout the test trajectory.

The extracted Fuzzy model is identified using the methods described in Sect. IV. The antecedent parameters are calculated using the FCM algorithm and the consequent parameters are estimated utilizing the OLS + method. Moreover, the number of the considered Fuzzy rules is effectively reduced to 120 with the implementation of the interclass separability method (Eqs. 19 and 20). The membership functions of three normalized inputs (V_E, δV_U, and $\delta\omega_x$) are shown in Fig. 5a, b. For brevity, only three representative GPS outages results are presented. These outage periods are denoted as outage

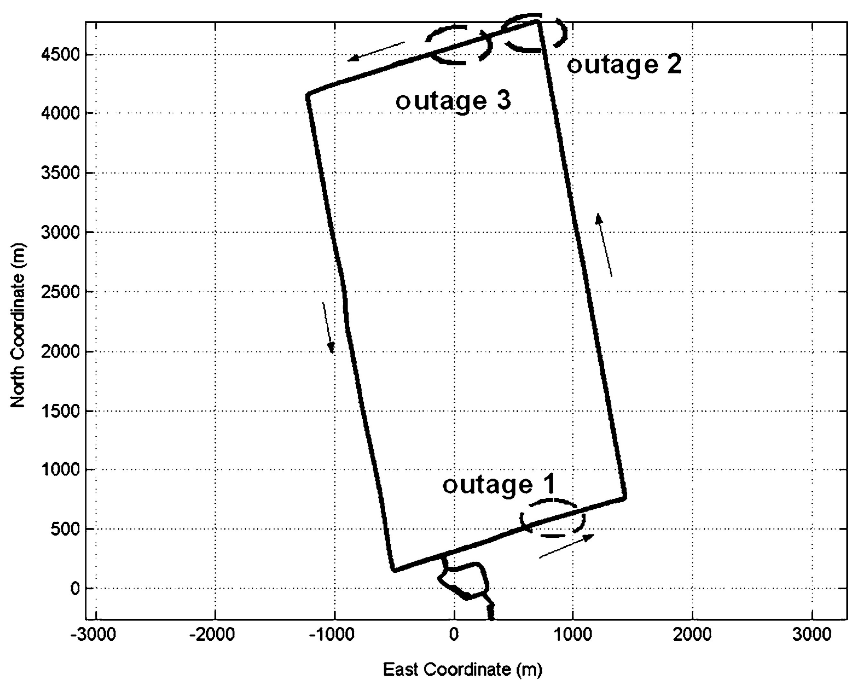

Fig. 4 Test trajectory showing the location of the simulated GPS outages

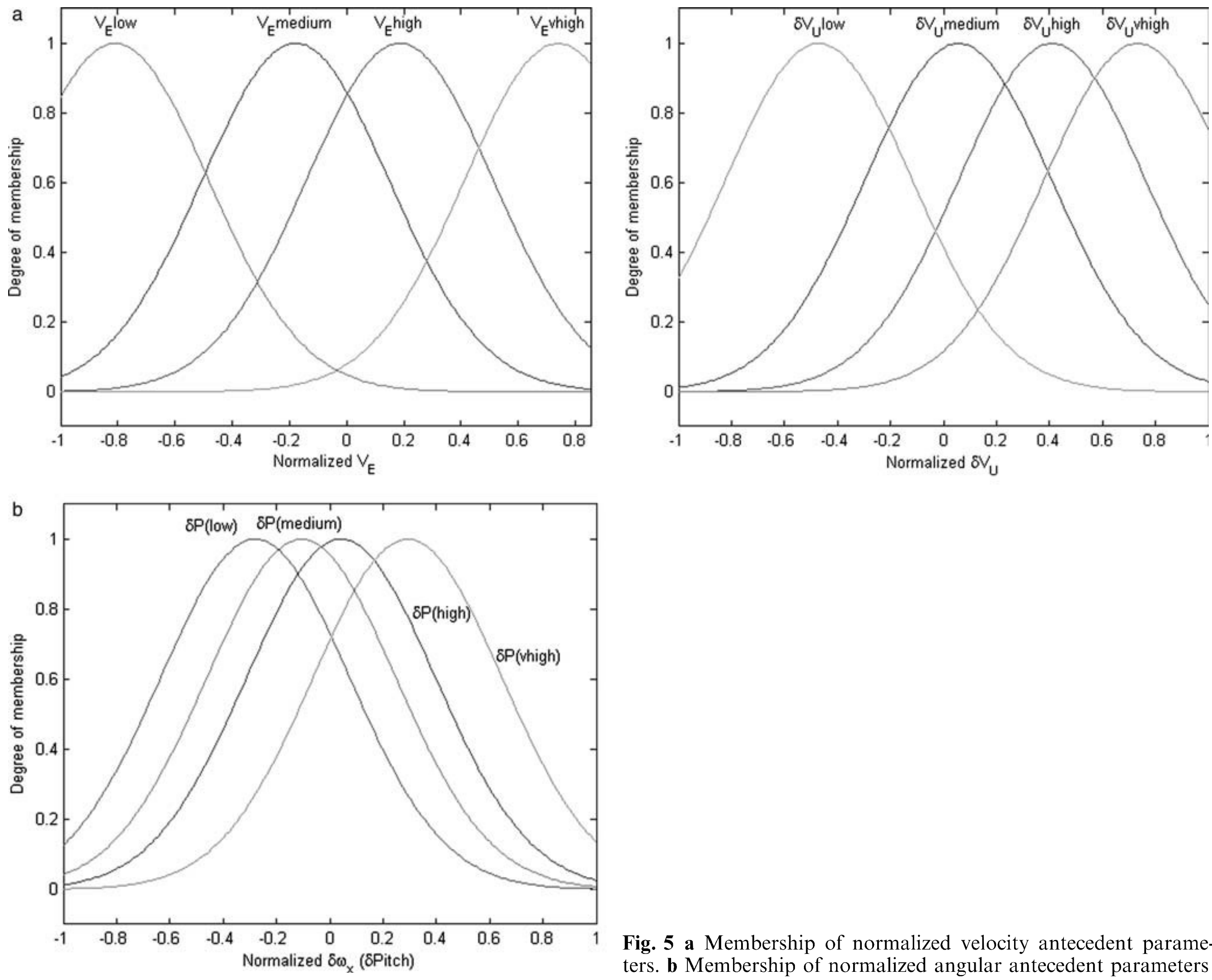

Fig. 5 **a** Membership of normalized velocity antecedent parameters. **b** Membership of normalized angular antecedent parameters

1 (time: 5630–5660), outage 2 (time: 5930–5960) and outage 3 (Time: 5960–5990). During these simulated outages, the KF will work in prediction mode and therefore the position and velocity errors are expected to increase with time. On the other hand, the proposed model will work in update mode where the update measurements are obtained from the Fuzzy model. Figures 6, 7, and 8 show the position and velocity error drifts during the three simulated GPS outages before and after applying the proposed Fuzzy model.

The trajectory computed with the Applanix POS-LV-320 system was used as a reference for calculating the estimated position and velocity errors presented in the latter figures. The figures clearly indicate that significant improvement in the positioning and velocity accuracy can be achieved after applying the proposed integrated Fuzzy-KF integrated model.

It is apparent from the above figures that the maximum errors in the position and velocity, which typically happens at the end of the simulated outages, were effectively reduced. Also, similar accuracy enhancements were obtained during the other simulated GPS outages.

Tables 1, 2 and 3 summarize the Root-Mean-Square-Error (RMSE) and the maximum errors of both position and velocity estimates for the traditional KF solution compared to the proposed Fuzzy-KF model throughout each of the three simulated GPS outages. The results clearly indicate that a sub-meter positioning accuracy level was achieved through the application of the integrated Fuzzy-KF model. These attainable sub-meter positioning accuracy levels might be a little bit worse in other simulated GPS blockage periods particularly if maneuvers like 90° left or right turns are included. In this case, the positioning accuracy might be degraded and reach a few meters. Also, in an actual GPS outage situation, satellite geometry and measurement accuracy degradations are likely to be more gradual and affect performance adversely. This is still a significant accuracy

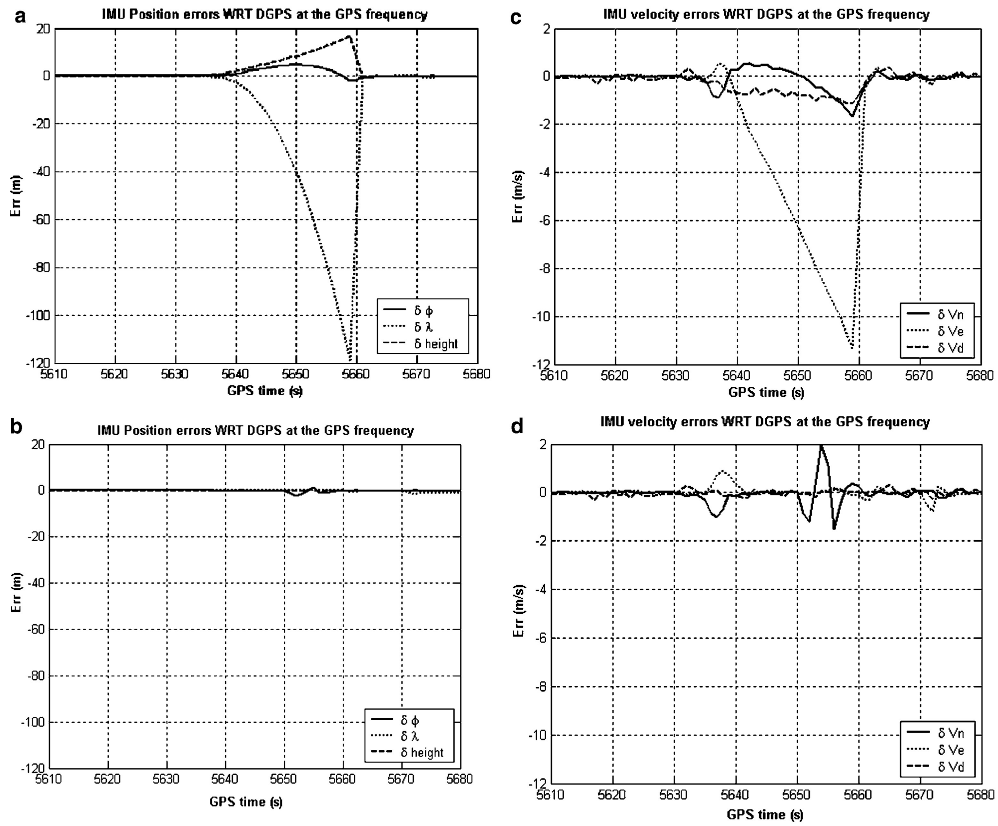

Fig. 6 **a** Kalman estimated position drift during outage (1). **b** Fuzzy OLS estimated position drift during outage (1). **c** Kalman estimated velocity drift during outage (1). **d** Fuzzy OLS estimated velocity drift during outage (1)

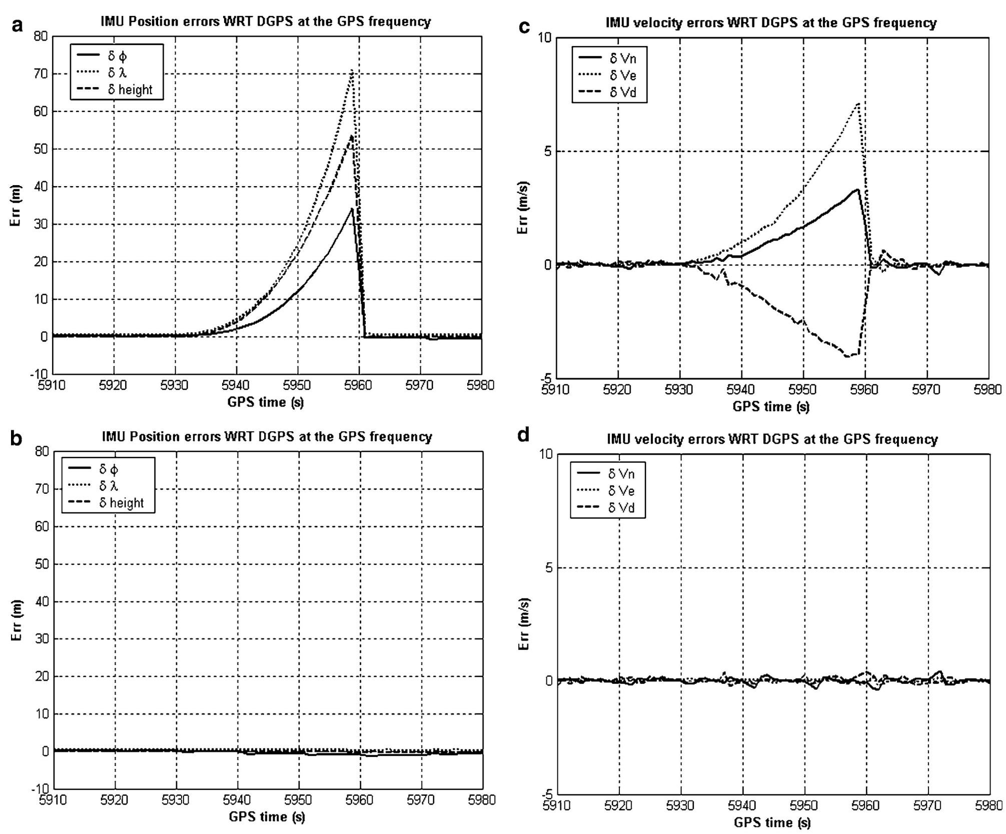

Fig. 7 **a** Kalman estimated position drift during outage (2). **b** Fuzzy OLS estimated position drift during outage (2). **c** Kalman estimated velocity drift during outage (2). **d** Fuzzy OLS estimated velocity drift during outage (2)

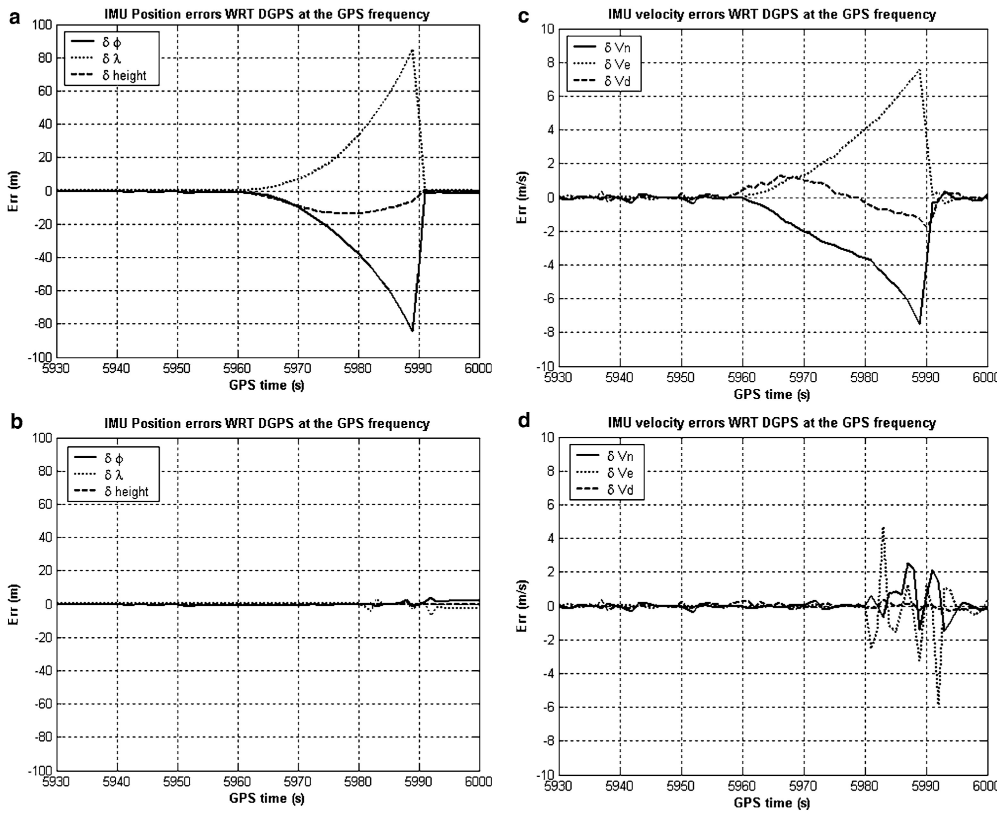

Fig. 8 **a** Kalman estimated position drift during outage (3). **b** Fuzzy OLS estimated position drift during outage (3). **c** Kalman estimated velocity drift during outage (3). **d** Fuzzy OLS estimated velocity drift during outage (3)

Table 1 Position and velocity error values at the end of first GPS outage (time: 5630–5660)

	KF only		Proposed model	
	RMSE	Maximum error	RMSE	Maximum error
phi(m)	2.6690	−1.72	0.69	−0.29
λ(m)	47.489	−118.96	0.12	0.19
h(m)	7.93	16.92	0.11	−0.14
V_E(m/s)	0.60	−1.66	0.64	0.38
V_N(m/s)	5.57	−11.31	0.27	−0.09
V_U(m/s)	0.69	−1.15	0.12	0.07

Table 2 Position and velocity error values at the end of first GPS outage (time:5930–5960)

	KF only		Proposed model	
	RMSE	Maximum error	RMSE	Maximum error
φ(m)	13.75	34.20	0.60	−0.86
λ(m)	28.43	70.83	0.42	0.20
h(m)	23.14	53.68	0.15	−0.30
V_E(m/s)	1.57	3.31	0.13	−0.09
V_N(m/s)	3.26	7.13	0.06	−0.21
V_U(m/s)	2.29	−3.99	0.14	0.22

improvement when compared to the results obtained from KF only, which reaches 120 m in positioning accuracy during outage one.

Table 3 Position and velocity error values at the end of first GPS outage (time: 5960–5990)

	KF only		Proposed model	
	RMSE	Maximum error	RMSE	Maximum error
φ(m)	13.27	−34.41	0.83	−0.56
λ(m)	10.65	29.50	0.36	0.32
h(m)	7.50	−13.62	0.23	−0.28
V_E(m/s)	1.71	−3.48	0.13	−0.04
V_N(m/s)	1.53	3.70	0.05	−0.06
V_U(m/s)	0.66	−0.02	0.14	0.07

Conclusions

This paper has presented a new model for integrating MEMS based IMU and GPS using Kalman filtering and Fuzzy modeling. The new model allows for Kalman filter to continuously work in update mode even if GPS signal is not available. The proposed model was tested using a MEMS IMU data collected in a land vehicle. The experimental results demonstrated the ability of the proposed model to provide consistent accuracy even during GPS signal outages. Without the use of the proposed model, the solution of Kalman filter is unacceptable and the system has a degraded performance during GPS outages. By applying the proposed model, the simulation results show total improvement of the system. Further work is currently being developed on the identification of a neuro-Fuzzy prediction model that replaces the off-line Fuzzy model. The aim of this work is the achievement of effective decrease in training time in addition to real-time capabilities.

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