Limited Ability and Corporate Spin-offs

Hongyi Xu

Stockholm School of Economics

2022-06-16

Optimization in Period 1

This question can be set up like this, under the assumption that $E = 1$ is the entrepreneur's optimal action,

$$\max_{R_b(r)} \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=1) + B \quad \text{s.t.}$$

$$(\text{PC}) \quad \sum_{r\in\Omega}(r - R_b(r))\mathbf{P}(r|E=2) \geq 2(I-A)$$

$$(\text{IC}_1) \quad \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=2) \geq \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=1) + B$$

$$(\text{IC}_2) \quad \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=2) \geq \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=0) + 2B$$

Optimization in Period 1 (cts.)

And we can write out the Lagrangian as

$$\mathcal{L} = \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=1)+B+\lambda_{PC}\Big\{\sum_{r\in\Omega}(r-R_b(r))\mathbf{P}(r|E=2)-2(I-A)\Big\}$$

$$+\lambda_{IC1}\Big\{\sum_{r\in\Omega} R_b(r)\big[\mathbf{P}(r|E=2)-\mathbf{P}(r|E=1)\big]-B\Big\}$$

$$+\lambda_{IC2}\Big\{\sum_{r\in\Omega} R_b(r)\big[\mathbf{P}(r|E=2)-\mathbf{P}(r|E=0)\big]-2B\Big\}$$

FOC:

$$\frac{\partial\mathcal{L}}{\partial R_b(r)} = \mathbf{P}(r|E=1)-\lambda_{PC}\{\mathbf{P}(r|E=2)\}+\lambda_{IC1}\{\mathbf{P}(r|E=2)-\mathbf{P}(r|E=1)\}$$

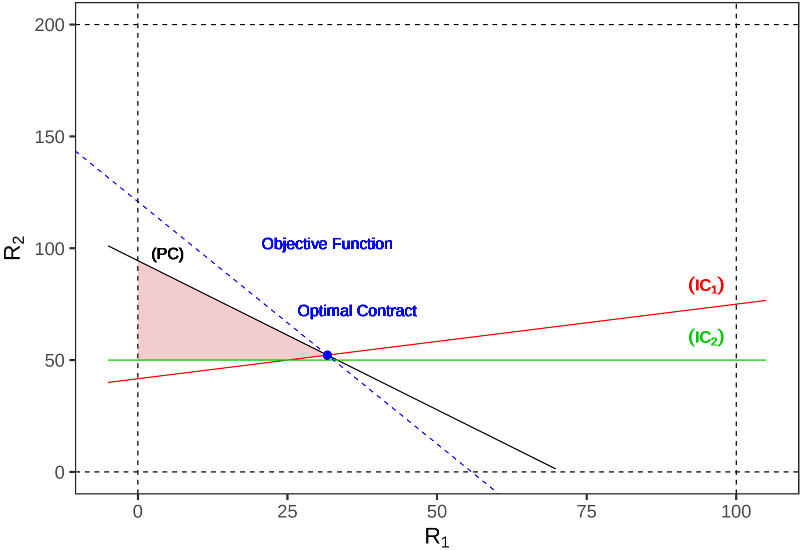
$$+\lambda_{IC2}\{\mathbf{P}(r|E=2)-\mathbf{P}(r|E=0)\}=0, \quad \forall r\in\{2R,R,0\}$$

Optimal Contract in Period 1

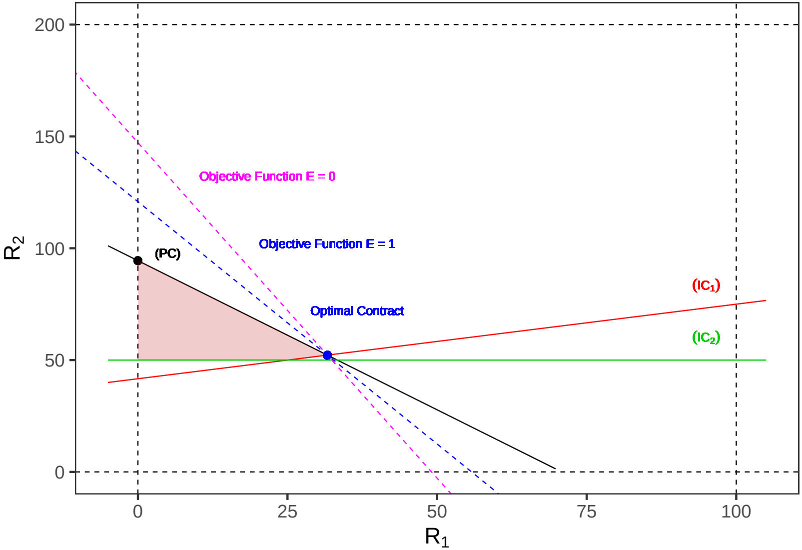
The list of parameters values:

- $p_H = 0.6$: conditional prob of success under efforts
- $p_L = 0.4$: conditional prob of success under shirking
- $R = 100$: project's cash flow under success
- $B = 5$: private benefits under shirking on each project
- $I = 50$: total investment required for each project
- $A = 7$: assets contributed by the entrepreneur in each project (*This value is different from that in the previous document.*)

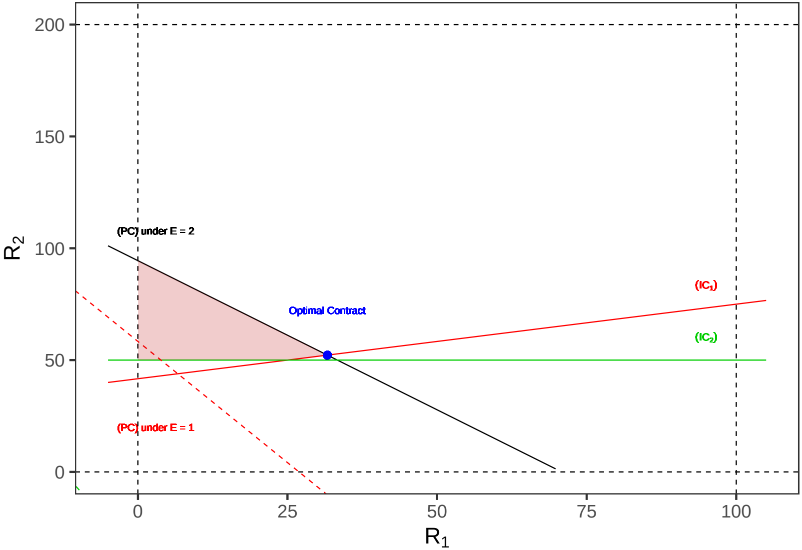
Constraints in Period 1



Optimal Contract for a Bad Entrepreneur



True PC Constraint



Investors' Belief Update

investors' belief at the beginning of period 1 is

$$\mathbb{P}(T) = \begin{cases} q & \text{, if } T = G \\ 1 - q & \text{, if } T = B, \end{cases}$$

where $q \geq \bar{q}$ and $\bar{q}$ is a threshold. This threshold $\bar{q}$ is a scaler predetermined by investors and if investors' subjective probability of the entrepreneur being a Good type is below $\bar{q}$, they will identify the entrepreneur as a Bad type.

$$\mathbb{P}(X_0 | T = G) = \begin{cases} p_H^2 & \text{, if } X_0 = 2R \\ 2p_H(1 - pH) & \text{, if } X_0 = R \\ (1 - p_H)^2 & \text{, if } X_0 = 0. \end{cases}$$

Investors' Belief Update (cts.)

Therefore, investors can use the Bayes rule to update their beliefs towards the type of the entrepreneur at the beginning of the period 2, before deciding whether to continue to finance the company. And we have

$$\mathbb{P}(T = G|X_0) = \frac{\mathbb{P}(X_0|T = G)q}{\mathbb{P}(X_0|T = G)q + \mathbb{P}(X_0|T = B)(1-q)}.$$

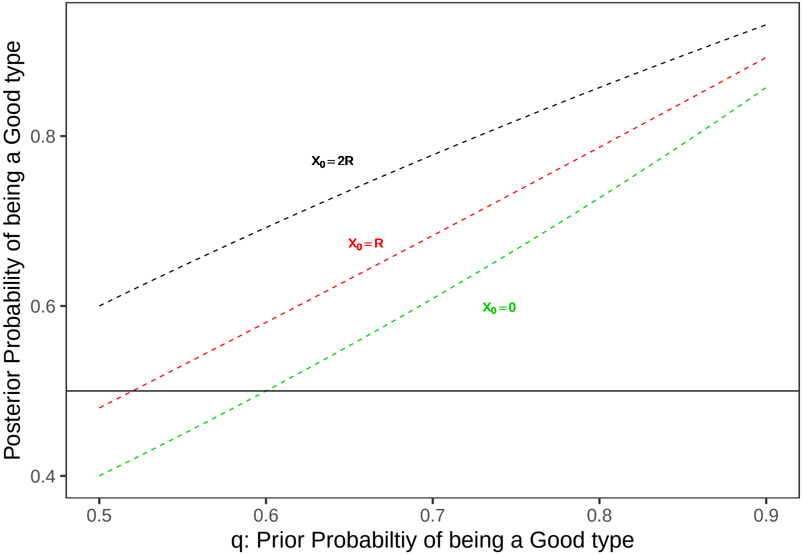
Specifically, we have

$$\mathbb{P}(T = G|X_0 = 2R) = \frac{1}{1 + \frac{p_L}{p_H}\frac{1-q}{q}} > \frac{1}{2},$$

$$\mathbb{P}(T = G|X_0 = R) = \frac{1}{1 + \frac{p_H(1-p_L)+(1-p_H)p_L}{2p_H(1-p_H)}\frac{1-q}{q}},$$

$$\mathbb{P}(T = G|X_0 = 0) = \frac{1}{1 + \frac{1-p_L}{1-p_H}\frac{1-q}{q}} > \frac{1}{2}.$$

Investors' Belief Update (cts.)



Optimal Contract in Period 2

Rewrite the optimization problem in the case that investors know the entrepreneur is a Bad type.

$$\max_{R_b(r)} \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=1) + B \quad \text{s.t.}$$

$$\text{(PC)} \quad \sum_{r\in\Omega} (r - R_b(r))\mathbf{P}(r|E=1) \geq 2(I-A)$$

$$\text{(IC)} \quad \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=1) + B \geq \sum_{r\in\Omega} R_b(r)\mathbf{P}(r|E=0) + 2B$$

$$\Rightarrow \sum_{r\in\Omega} R_b(r)\Big(\mathbf{P}(r|E=1) - \mathbf{P}(r|E=0)\Big) \geq B$$

Constraints in Period 2