

A Priority Queue ADT

Name: PriorityQueue <G>

Set: PQ: set of Priority queues containing items from G

G: set of items that can be in the queue

B: {true, false}

N: set of non-negative integers

Signatures:

newPriorityQueue<G>(n): \longrightarrow PQ

PQ.insert(g): G \nrightarrow PQ

PQ.isEmpty: \longrightarrow B

PQ.isFull: \longrightarrow B

PQ.maxItem: \nrightarrow G

PQ.minItem: \nrightarrow G

PQ.deleteMax: \nrightarrow PQ

PQ.deleteAllMax: \nrightarrow PQ

PQ.deleteMin: \nrightarrow PQ

PQ.frequency: G \nrightarrow N

Preconditions:

For all $pq \in PQ, g \in G, n \in \mathbb{N}$

`newPriorityQueue<G>(n)`: none

`pq.insert`: `pq` is not full

`pq.isEmpty`: none

`pq.isFull`: none

`pq.maxItem`: `pq` is not empty

`pq.minItem`: `pq` is not empty

`pq.deleteMax`: `pq` is not empty

`pq.deleteAllMax`: `pq` is not empty

`pq.deleteMin`: `pq` is not empty

`pq.frequency`: `pq` is not empty

Semantics:

For $pq \in PQ, g \in G, n \in \mathbb{N}$

`newPriorityQueue<G>(n)`: make a new queue of item from G with capacity n

`pq.insert(g)`: inserts an element g with a certain priority

`pq.isEmpty`: return true if `pq` is empty, false otherwise

`pq.isFull`: return true if `pq` is full, false otherwise

`pq.maxItem`: obtain the item in the `pq` with the highest priority

`pq.minItem`: obtain the item in the `pq` with the lowest priority

`pq.deleteMax`: remove from the `pq` the item with the highest priority

`pq.deleteAllMax`: remove from the `pq` all items that are tied for the highest priority

`pq.deleteMin`: remove from the `pq` the item with the lowest priority

`pq.frequency`: obtain the number of times a certain item occurs in the `pq` (with any priority)

Question2

a) Algorithm insert(H, e)

Cost of A.O: $O(1)$

Let's assume it is a heap with total number of n elements, and k level of height, therefore In this heap, the worst case will be searching from top to bottom, which is k time.

Then the total number of elements $n = 2^0 + 2^1 + 2^2 + \dots + 2^{(k-1)}$

$$n = 2^k - 1 \quad (\text{sum of } 2^0 \text{ to } 2^{(n-1)} \text{ is } (2^n)-1)$$

$$k = \log(n+1)$$

So number of execution of loop: $\log(n+1)$

Total number of execution: $1+1+\log(n+1) = 2+\log(n+1)$

$$O(2+\log(n+1)) = O(\max(2, (\log n+1))) = O(\log(n+1)) = O(\log(n))$$

b) Algorithm deleteItem(H)

Since a) & b) are both heap methods, the answer of b) will be the same as a), which is $O(\log(n))$.