Question 1

a) 3, 4 are true
b) O(n + (n^2.5)(log2 n) + n^3 + 1 + n^0.5), Θ(n^3, 2^n)

Question 2

a) f1(n) = O(n^4 log2 n)
b) f2(n) = O(n^3)
c) f3(n) = O(n^0.5)

Question 3

a) O(max((n^2), (log n), (n log n)) = O(n^2)
b) O(2^n)* O(n^2) = O(2^n*n^2)
c) O(max((n log n), (n^3))) = O(n^3)
d) O(max((n^2 log2 n^2), (m))) since n and m are independent

Question 4

To do the proof, show that there are numbers c and n0 such that 2n^3+5n^2+42 <= c*n^3 for all n>=n0.

42 <= n^3 (n >= 4)
5n^2 <= n^3 (n >=5)
5n^2 + 42 <= 2*(n^3) (n>= 5)
2n^3+5n^2+42 <= 2n^3 + 2n^3 (n >= 5)
2n^3+5n^2+42 <= 4n^3 (n >= 5)

Here we get 2n^3+5n^2+42 <= 4n^3 for all n>= 5.

Substitute c= 4 and n0= 5, we get: 2n^3+5n^2+42 <= 4n^3 for all n>= 5, which we just proved is true.
So there exist a c and n0 that makes the condition true, therefore f(n) ∈ O(n^3).

Question 5

To do the proof, show that there are number c and n0 such that
12n^2 log n^2+6n+42 <= c*(n^2 log n^2), for all n>=n0.
It's easy to see that 6n+42 <= n^2 log n^2 for all n>=10.

42 <= n^2 log n^2 (n >= 7)
6n <= n^2 log n^2 (n >= 6)
6n + 42 <= 2*(n^2 log n^2) (n >= 7)
12n^2 log n^2+6n+42 <= 12n^2 log n^2 + 2n^2 log n^2 (n>= 7)
12n^2 log n^2+6n+42 <= 14n^2 log n^2 (n >= 7)

Now add 12n^2 log n^2 to both sides and we get 12n^2 log n^2+6n+42 <= 14n^2 log n^2 for all n>= 7.

Substitute c=14 and n0=7, we get: 12n^2 log n^2+6n+42 <= 14n^2 log n^2 for all n>= 7, which we just proved is true.
So there exist a c and n0 that makes the condition true, therefore f(n) ∈ O(n^2 log n^2).

Question 6

To prove $g(n)$ is not $O(n)$, we prove by contradiction. Assuming that there are number c and n0 such that $12n^2 \log n^2+6n+42 \le c*n$, for all $n \ge n0$.

If $12n^2 \log n^2+6n+42 \le c*n$ (for all $n \ge n0$) is true
Then $12n^2 \log n^2+6n \le c*n$ (for all $n \ge n0$) is true
Then $12n^2 \log n^2 \le c*n$ (for all $n \ge n0$) is true
Then $12n \log n^2 \le c$ (for all $n \ge n0$) is true (here $n0 > 0$, $n0 < 0$ will be similar)

However,
let's take $c = 100$, then $n0=10$ will make $12n \log n^2 > c$ (for all $n \ge 10$)
let's take $c = 1000$, then $n0=100$ will make $12n \log n^2 > c$ (for all $n \ge 100$)
…
We can see whatever value c is, there is n0 with some value that makes $12n \log n^2 > c$ (for all $n \ge n0$), which cause a contradiction.

Here we cannot find a number c and n0 such that $6n + 42 \le c*n$.
Therefore, there are NO number c and n0 such that such that $12n^2 \log n^2+6n+42 \le c*n$, for all $n \ge n0$. Then $g(n)$ is not $O(n)$.

Question 7

a) cost of A.O: O(1)
   number of execution inner loop: n+1(false)
   number of execution outer loop: n+1(false)
   total number of execution: n*2n+n+1

$O(2n^2 +n +1) = O(\max(2n^2), (n), (1)) = \Theta(n^2)$

Question 8

a) Cost of A.O: O(1)
   Number of execution inner loop: (n-1)-(i+1)+1+1(false) = n-i-1
   Number of execution outer loop: n+1(false)
   Total number of execution: $\sum_{i=1}^{n-1} (2i) +(n-1) +1 +(n+1) +1= n^2 +n+2$
b) $O(n^2 +n+2) = O(n^2)$

Question 9