http://bburl/tpp-slides-p3
http://bburl/tpp-lab-p3

Modeling Irregular Time Series

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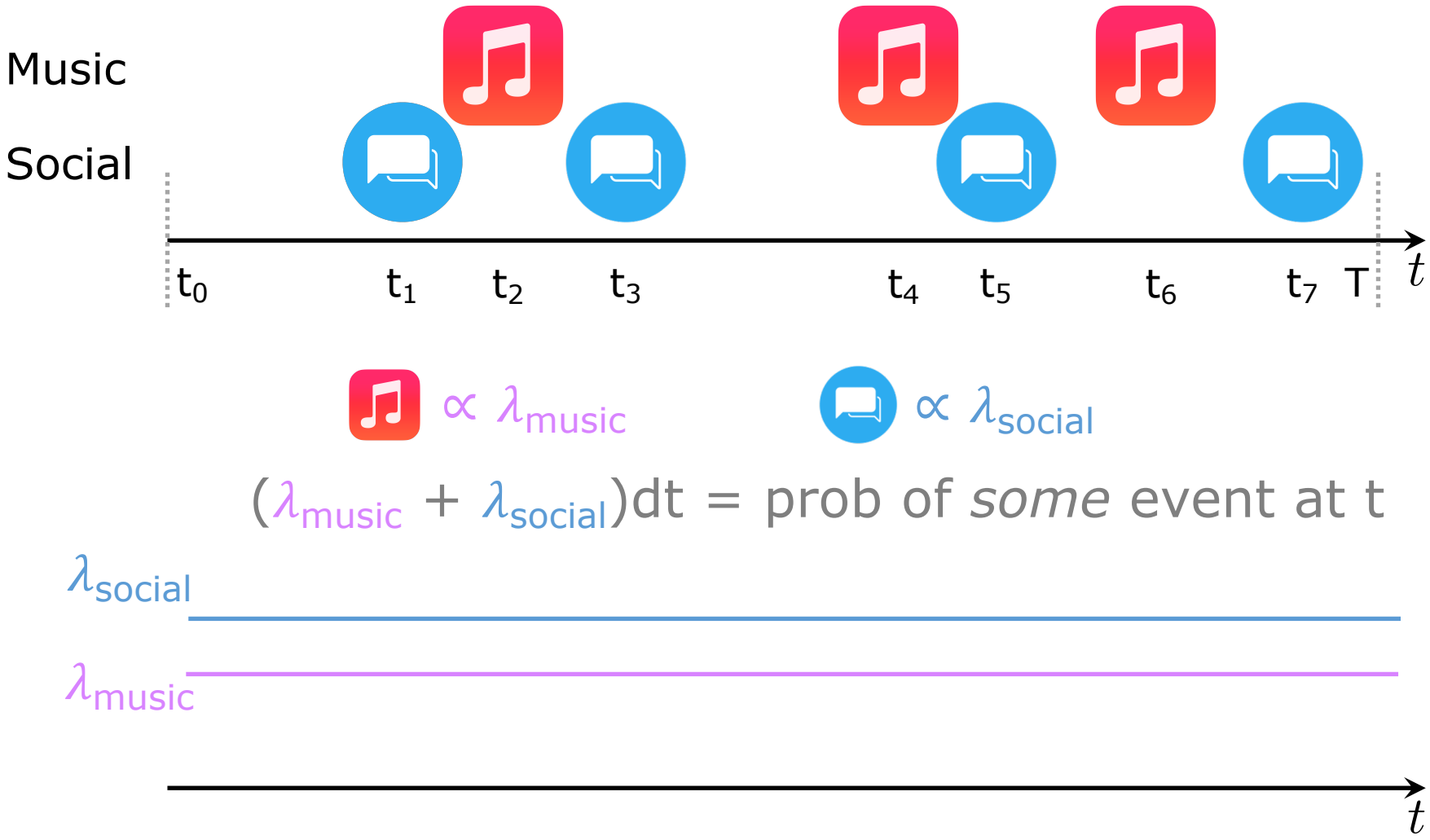
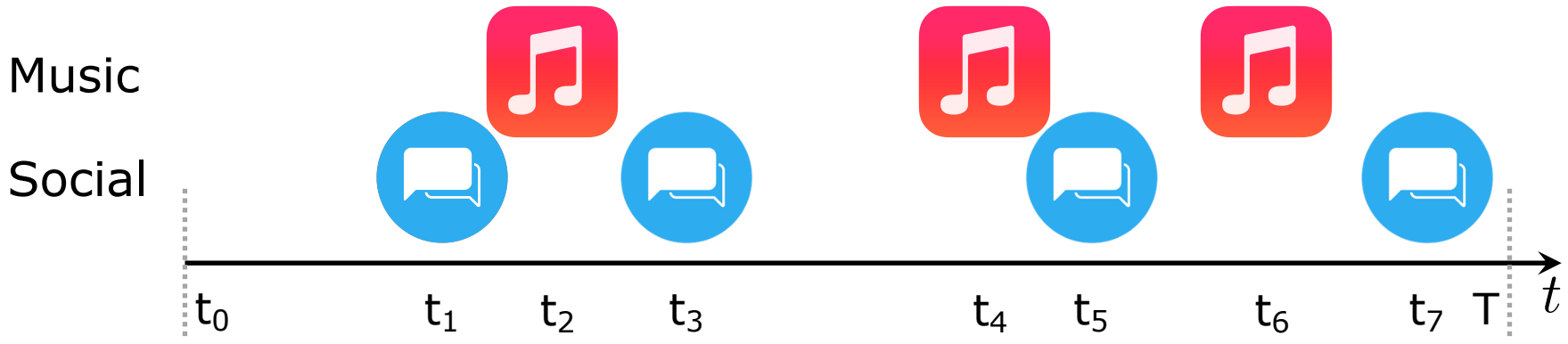
Multivariate Time Series



Multivariate Time Series: MLE



$$\lambda_{social} \exp(-\lambda (t_1 - t_0))$$
$$\times \lambda_{music} \exp(-\lambda (t_2 - t_1))$$
$$\times \lambda_{social} \exp(-\lambda (t_3 - t_2))$$
$$\dots \dots$$
$$= \lambda_{social}^4 \lambda_{music}^3 \exp(-\lambda (T - t_0))$$

$$(1-\lambda dt) \dots (1-\lambda dt)$$
$$\lambda = \lambda_{music} + \lambda_{social}$$

$$\lambda_{social} = 4 / (T - t_0)$$
$$\lambda_{music} = 3 / (T - t_0)$$

# of events of *that* type

Multivariate Time Series: MBR

Music

Social

$\lambda_{social} = 4 / (T - t_0)$

$\lambda_{music} = 3 / (T - t_0)$

t_0 s t

time

$t_{pred} = \int_s^\infty t\, p(t)\, dt \quad p(t) = \lambda \exp(-\lambda (t - s))$

$\lambda = \lambda_{music} + \lambda_{social} = 7 / (T - t_0)$

type?

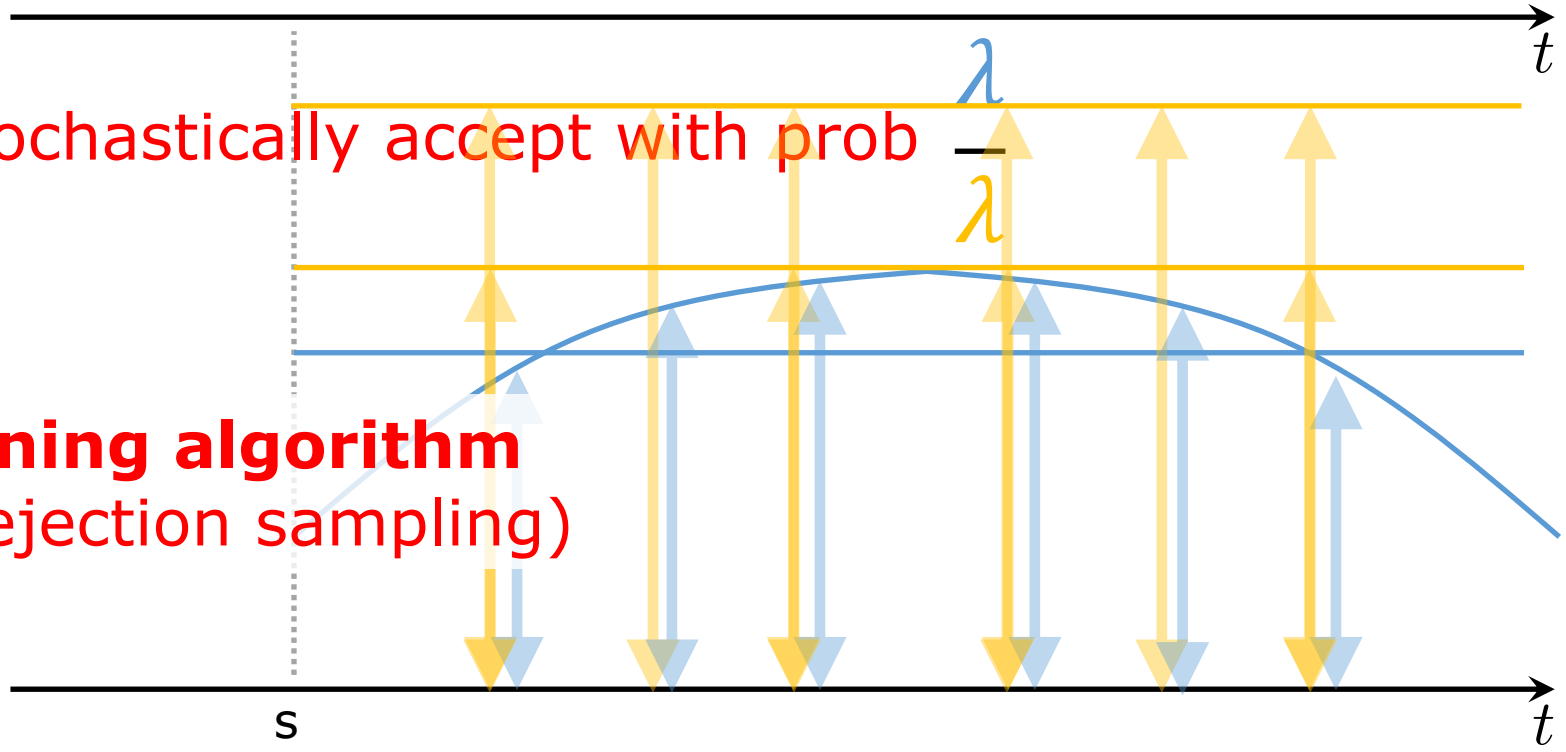
$k = \text{argmax}_k\, p(k \mid t) = \text{argmax}_k\, \lambda_k$

$\text{argmin}_k \sum_j p(j \mid t)\, \mathbb{I}(k \neq j) \quad j, k \in \{\text{music}, \text{social}\}$

choose type k that makes $\sum_{j \neq k} p(j \mid t)$ small

When intensities are not constant...

F^{-1} too complicated!!!

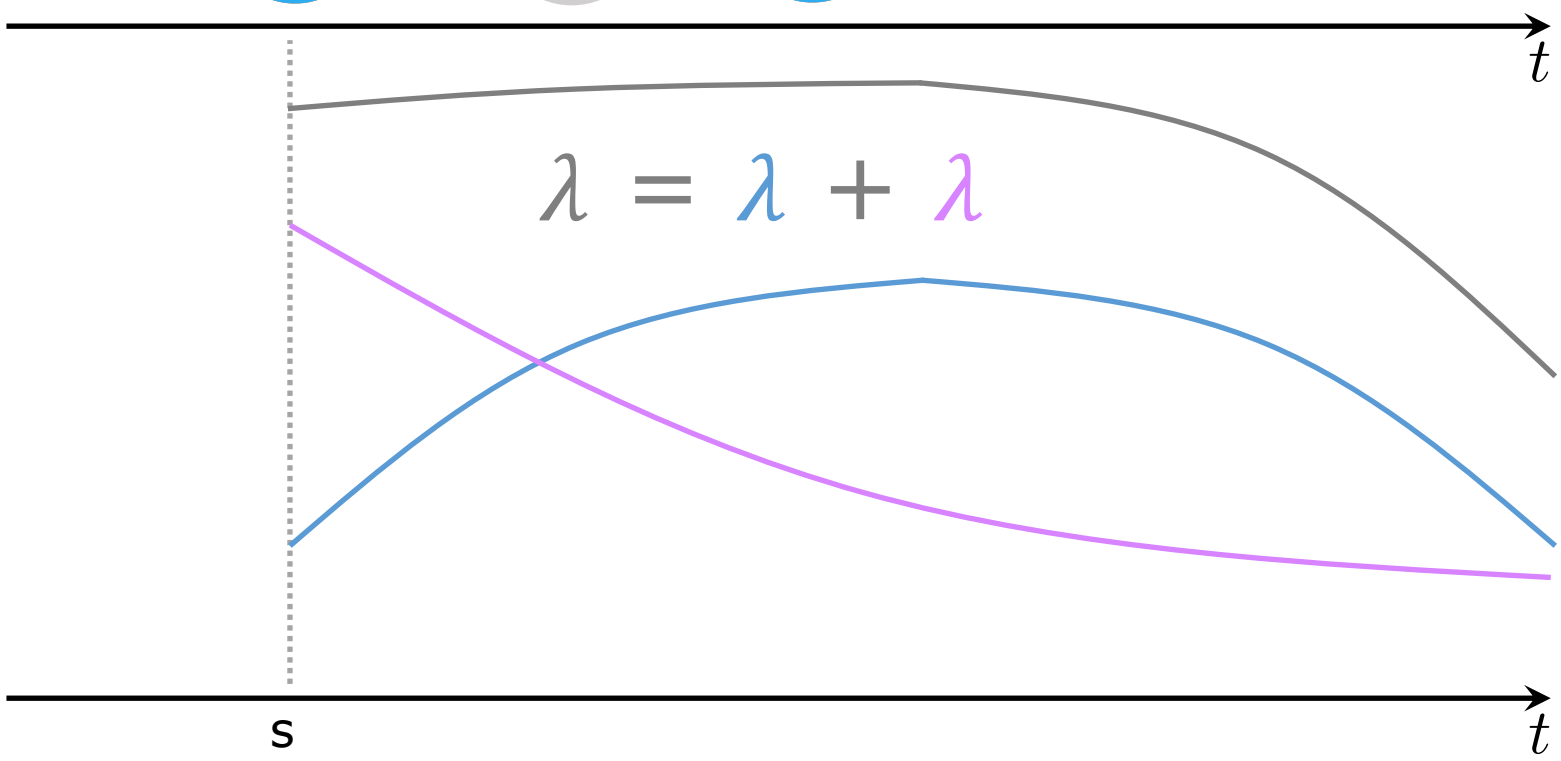


thinning algorithm
(c.f. rejection sampling)

Different types *race* against one another

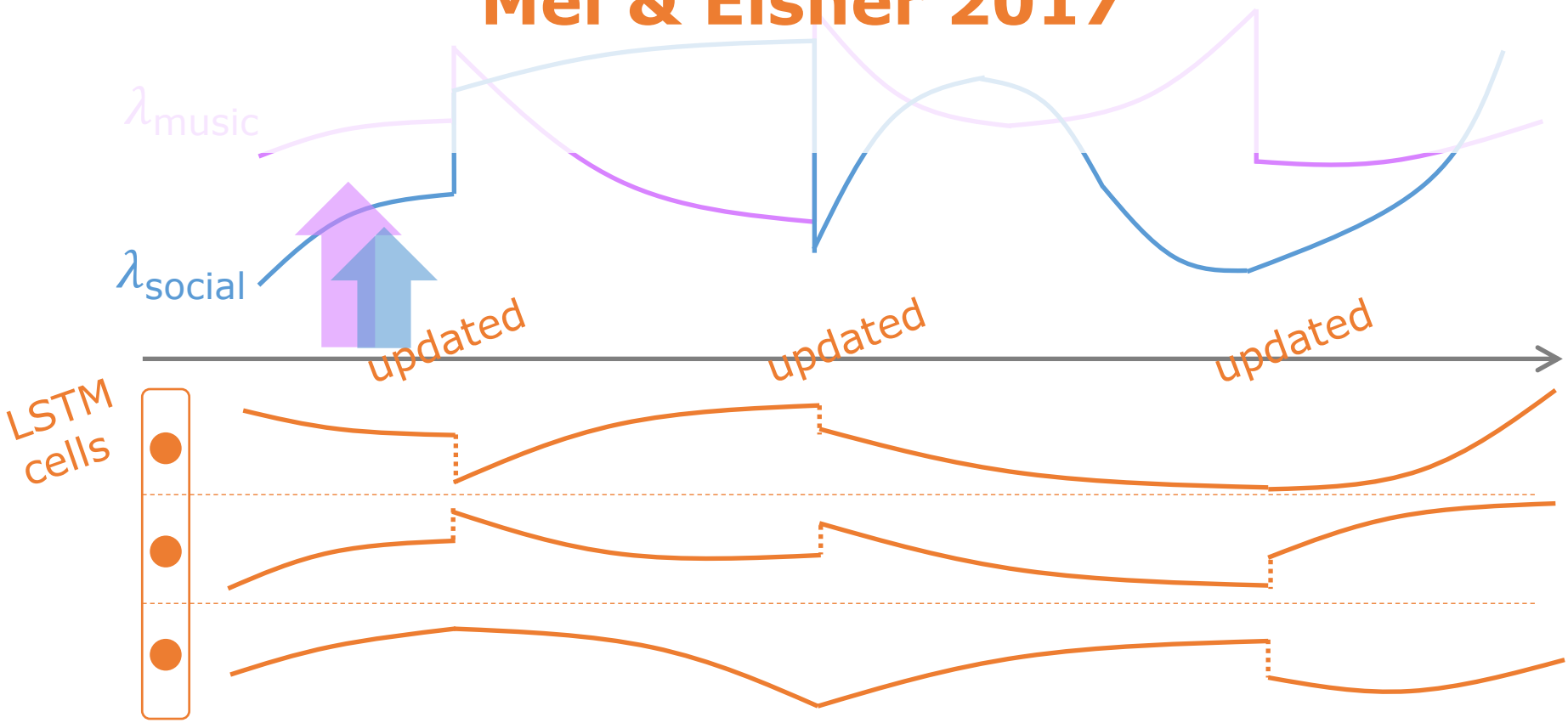
draw an event for each type

keep the earliest



Let LSTM state *evolve* with time

Neural Hawkes Process (NHP)
Mei & Eisner 2017



Lecture Structure

- Lecture-1: concepts
  - Intensity, point process, MLE, thinning, …
- Lecture-2: fancy models
  - Hawkes, NHP, Neural ODE, …
- Lecture-3: training + advanced topics
  - MLE & NCE
  - Large event space + domain knowledge

MLE: Max log prob of *data*

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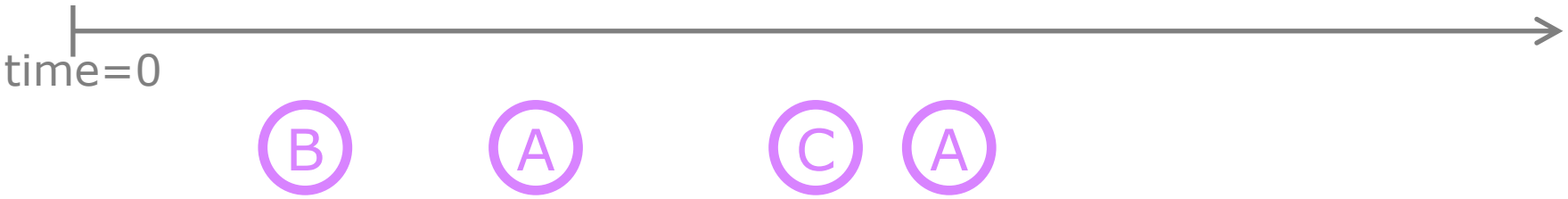

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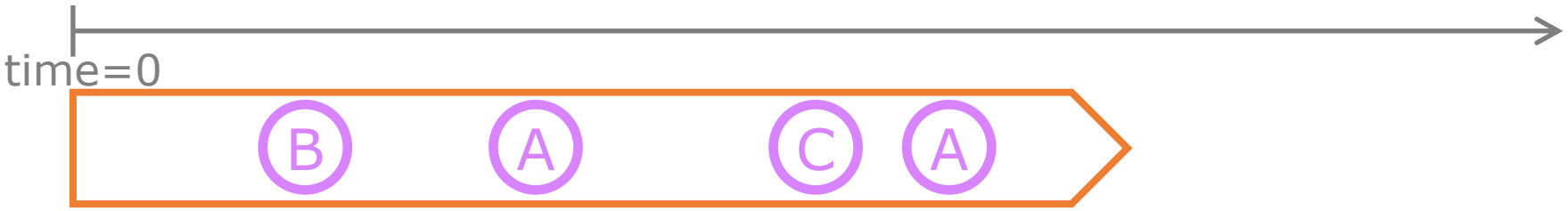

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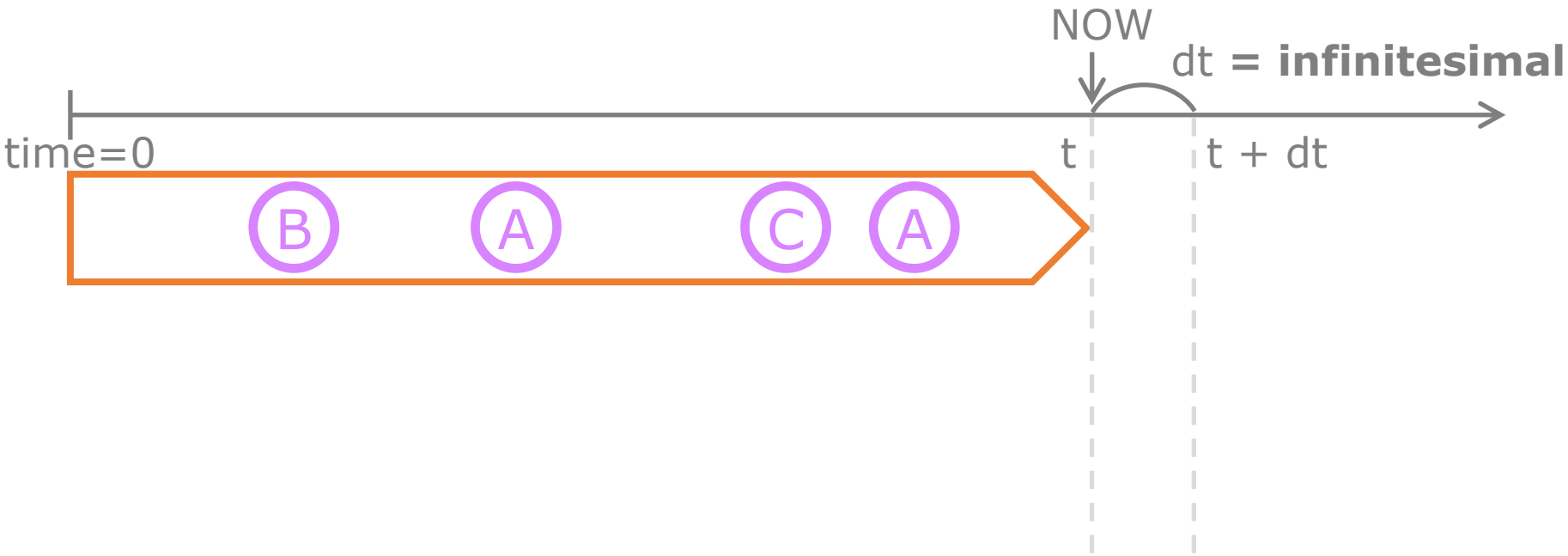

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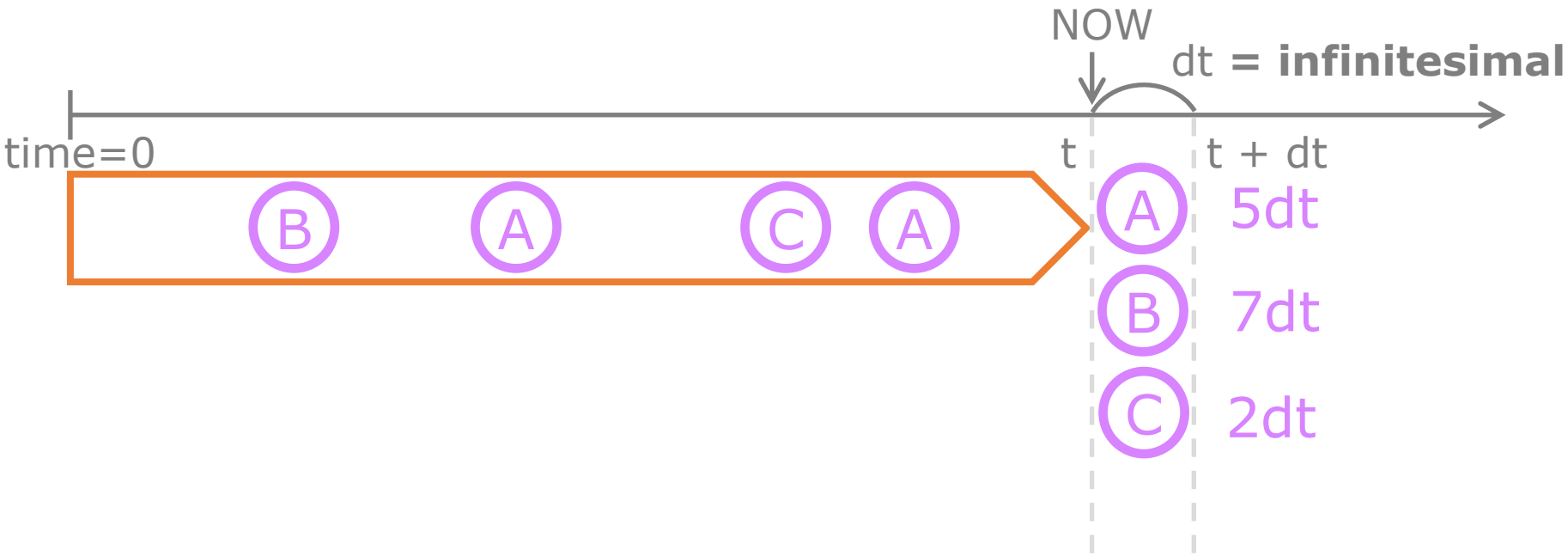

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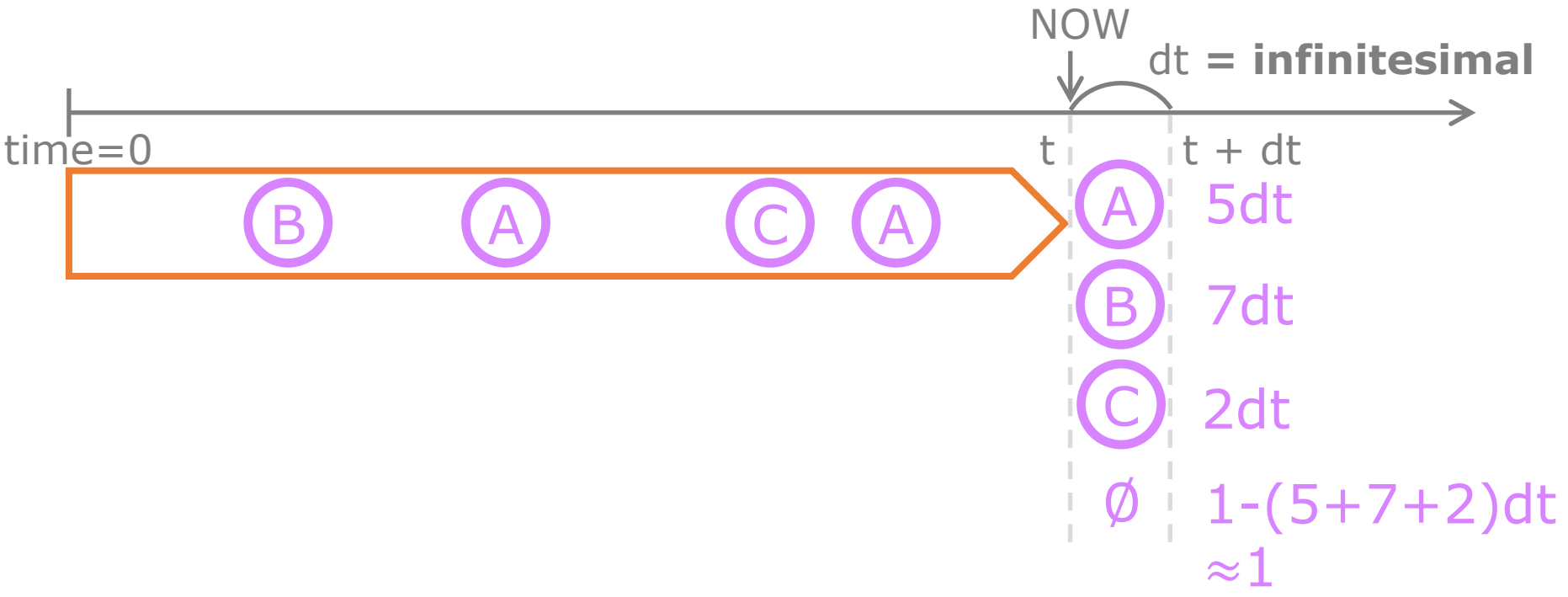

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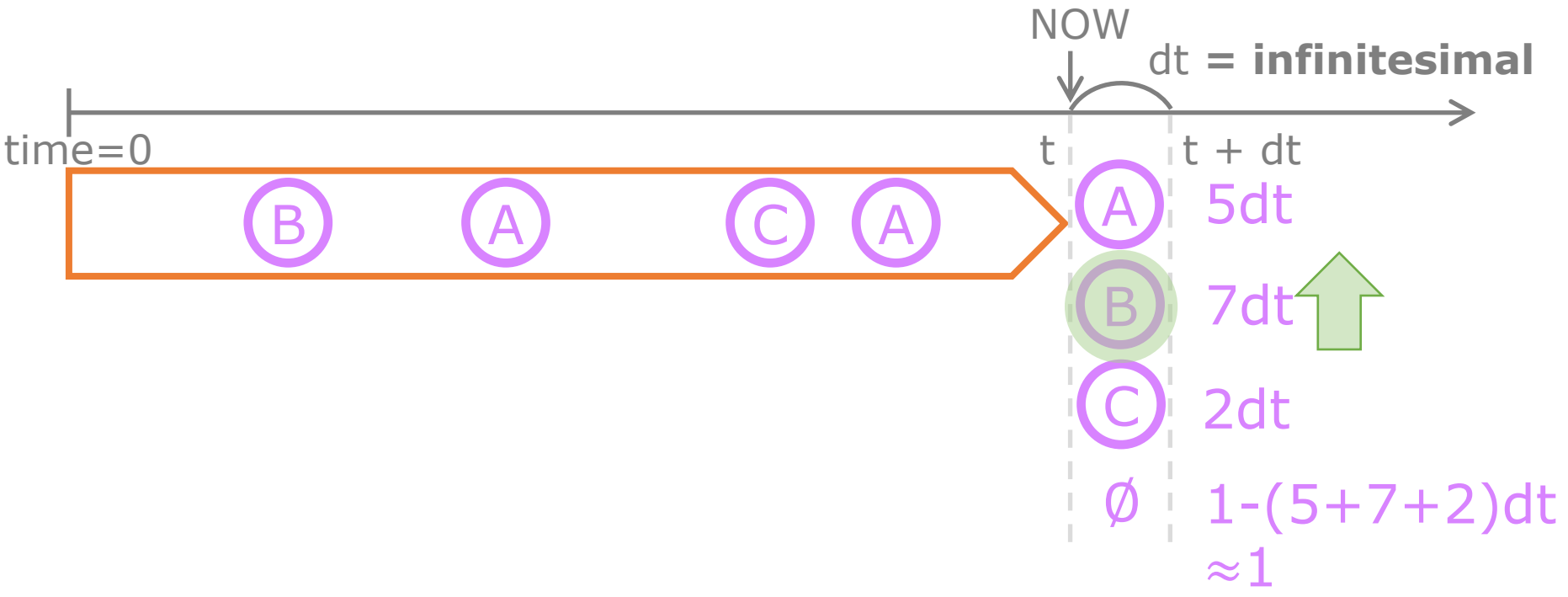

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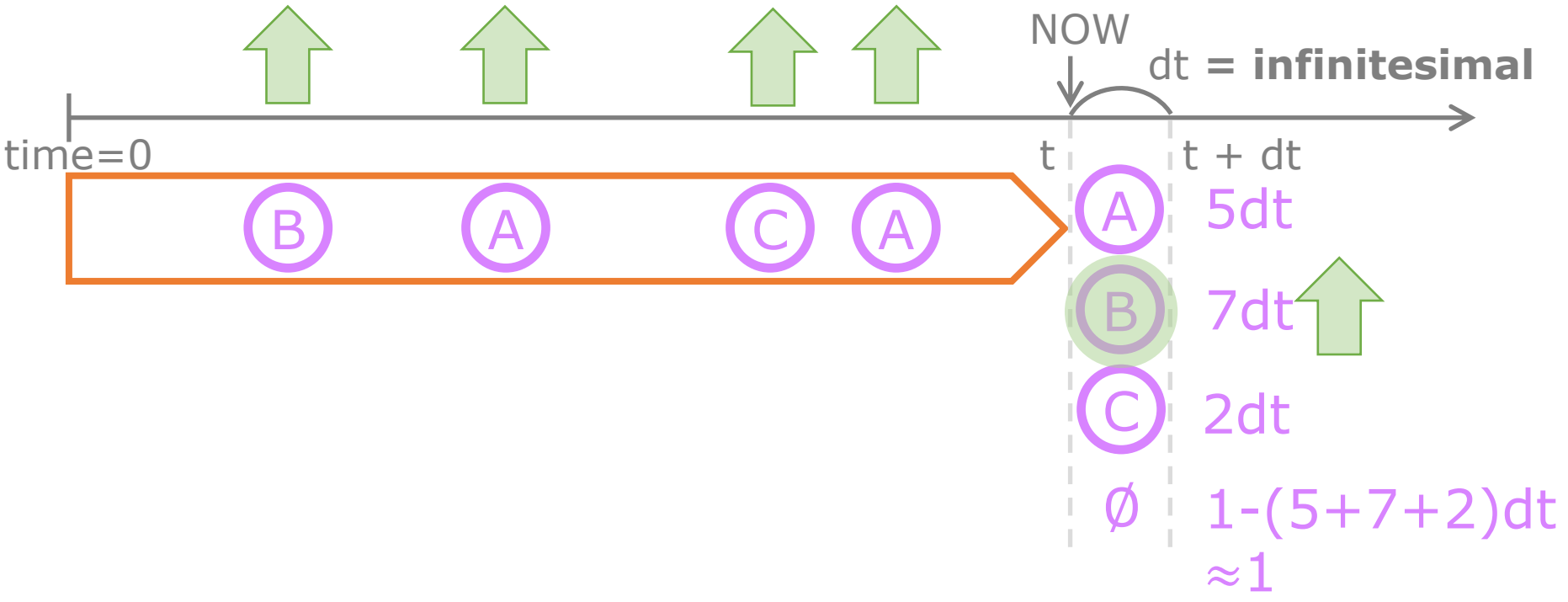

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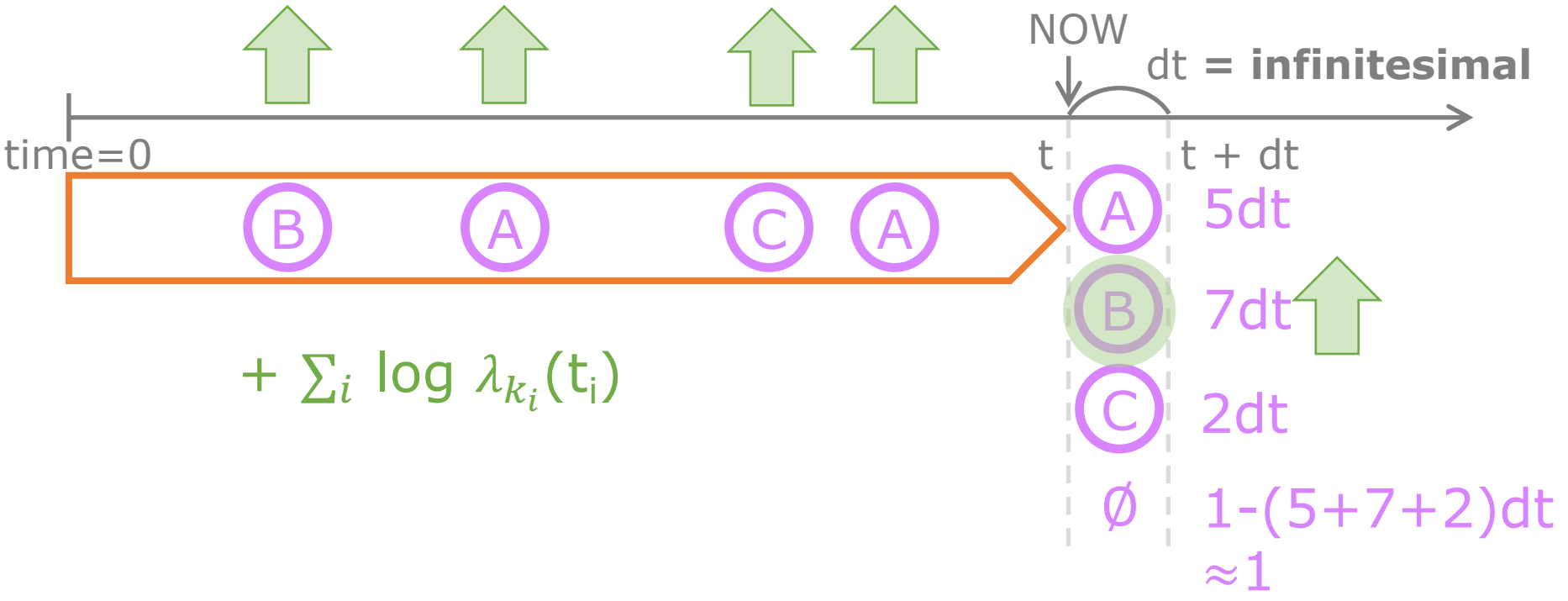

$+ \sum_i \log \lambda_{k_i}(t_i)$

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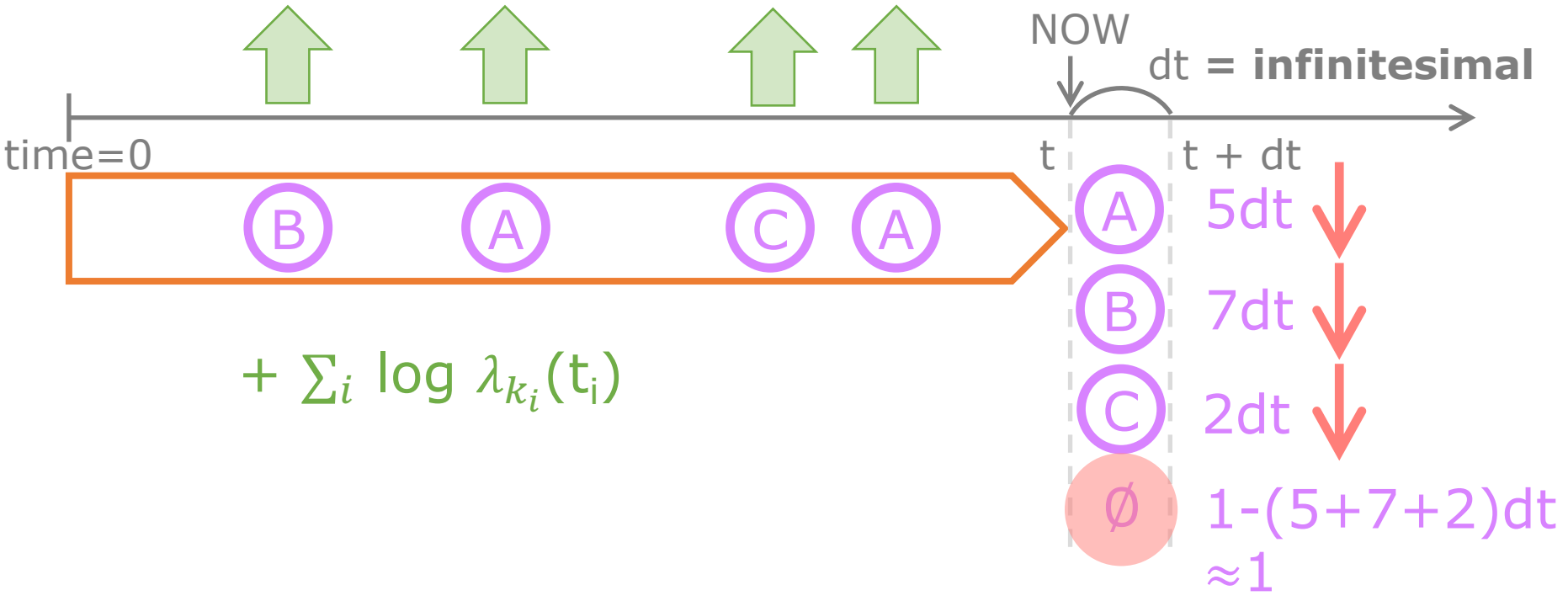

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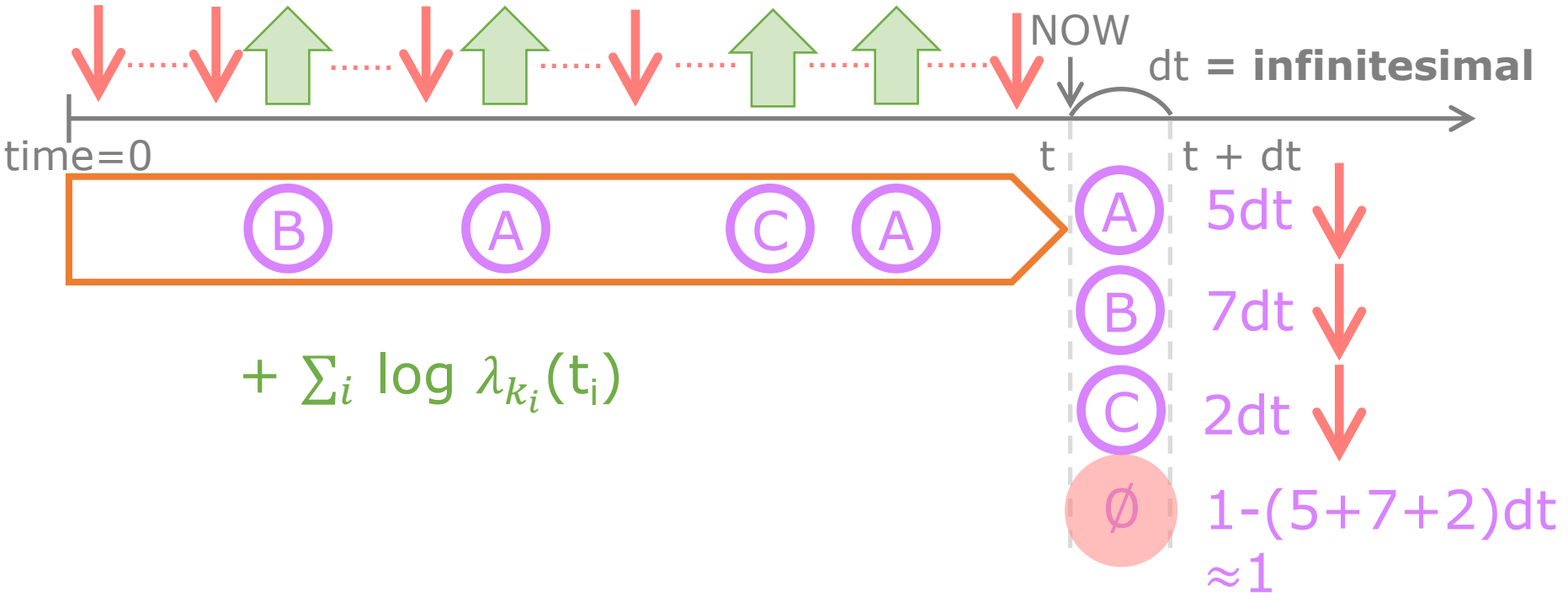

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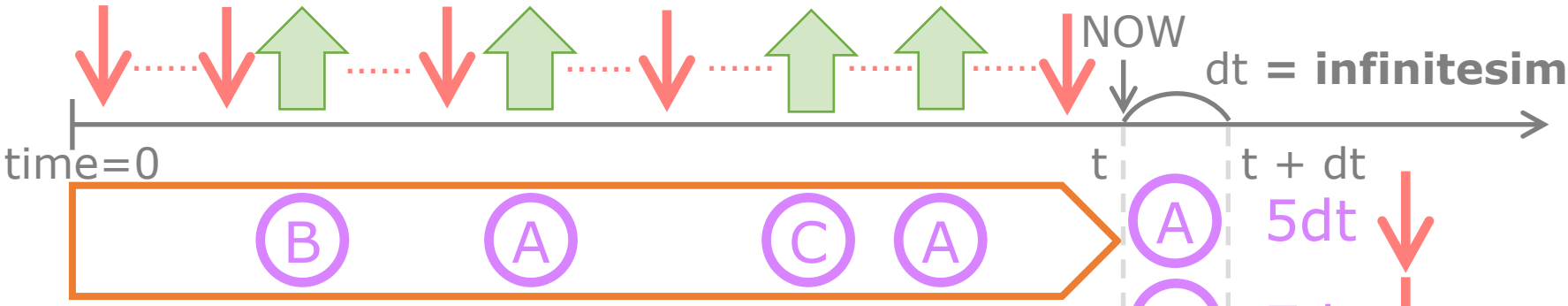

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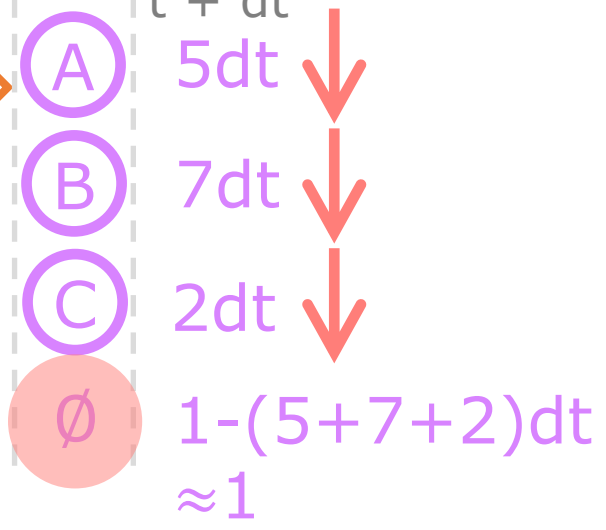
$$- \int_0^T \sum_k \lambda_k(t)\, dt$$



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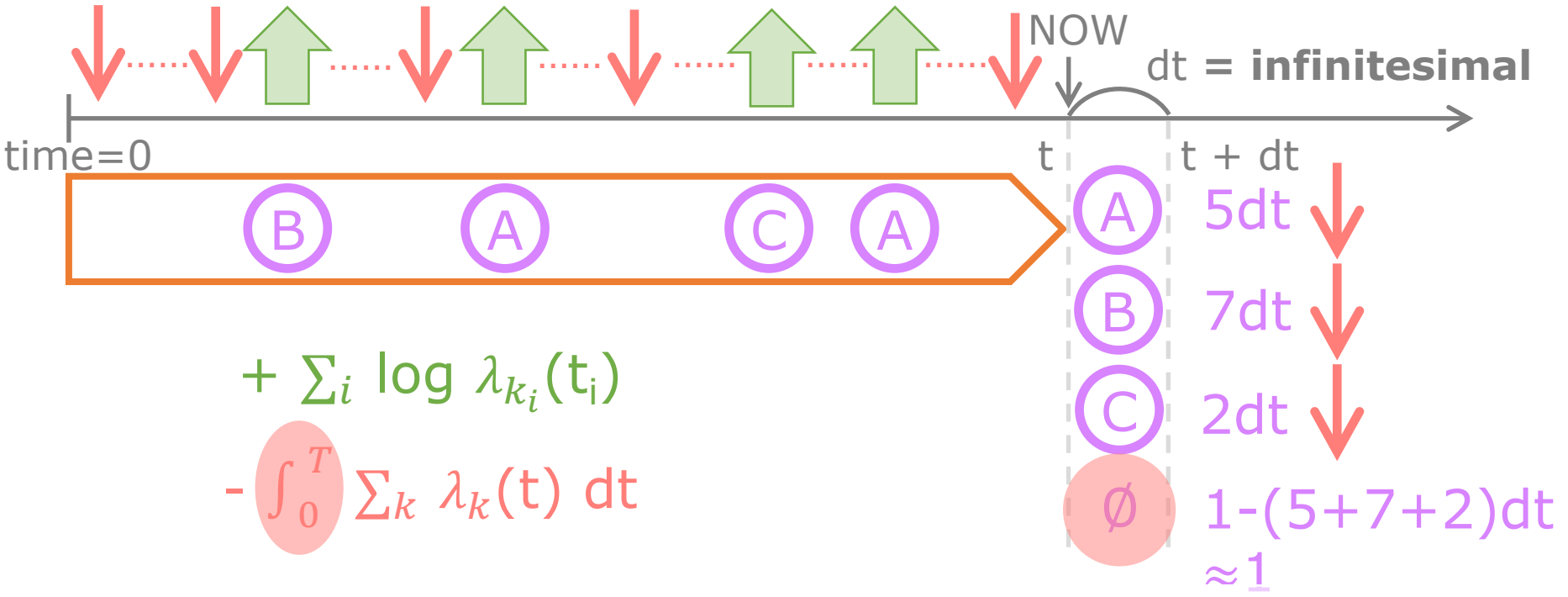

Integrate over <u>infinitely</u> many *non*-events!

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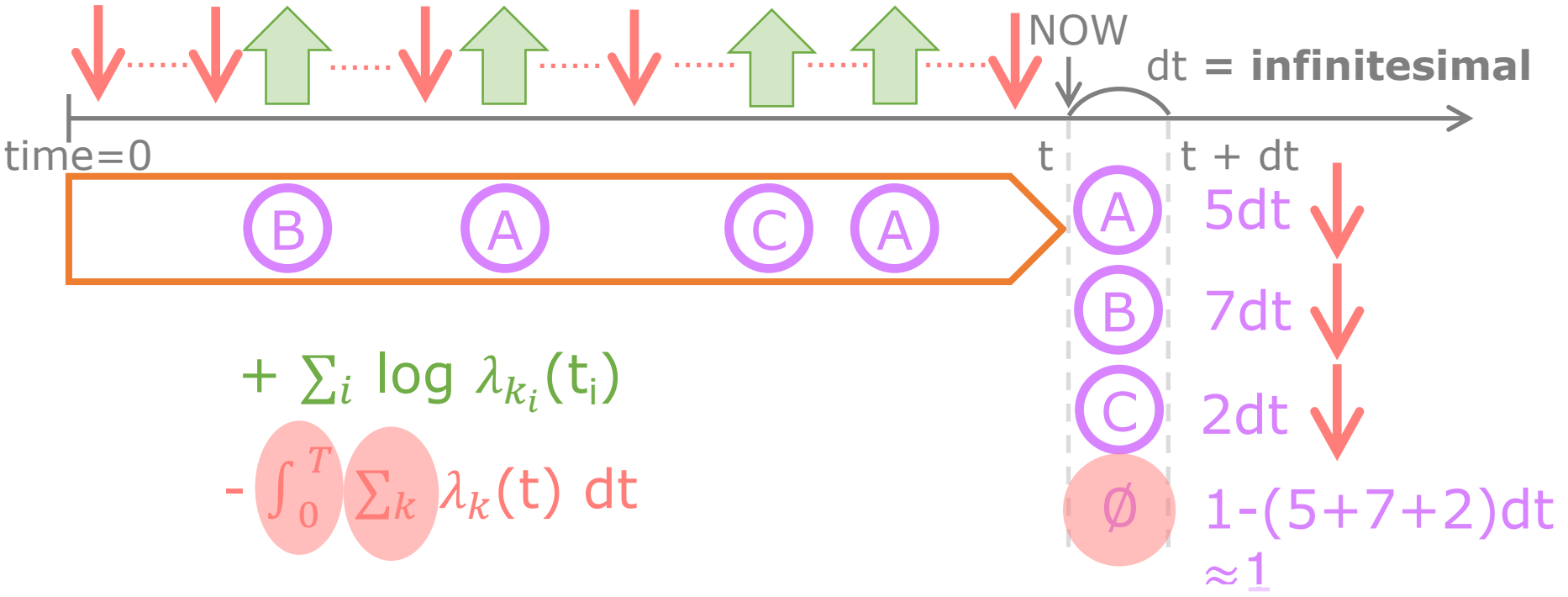

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Loop over <u>all</u> event types!

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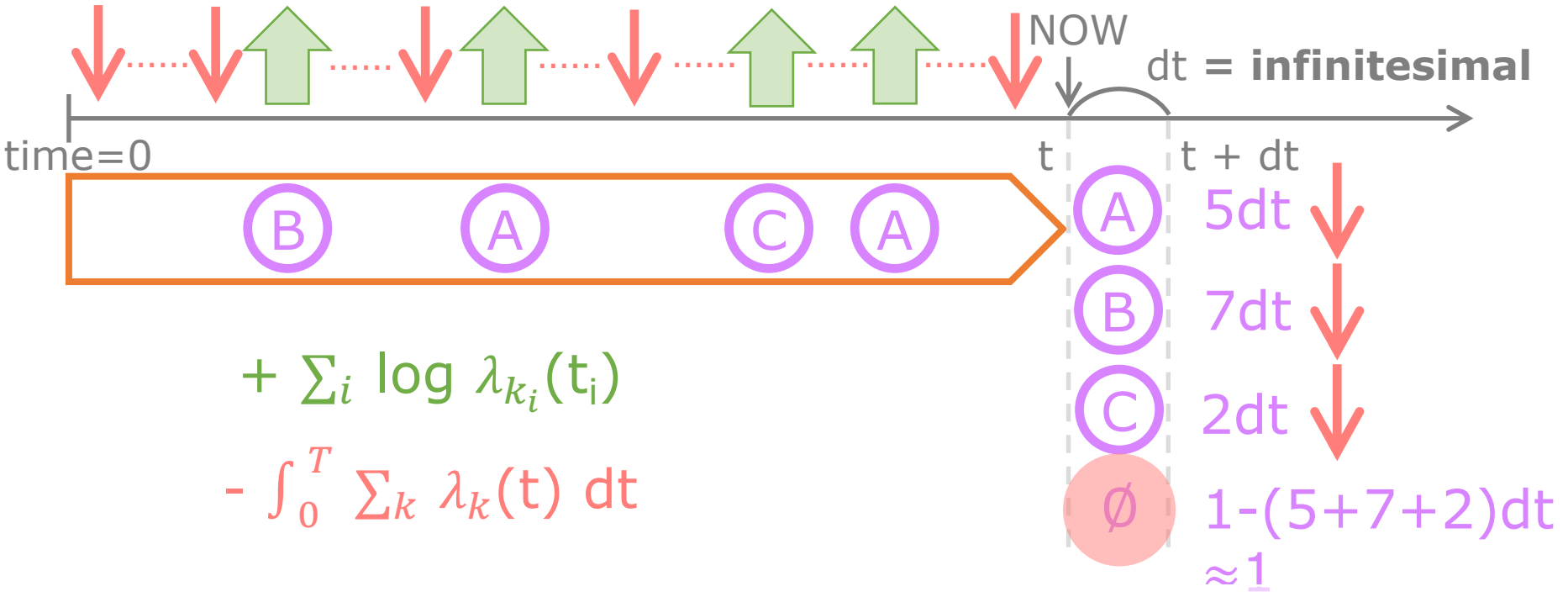

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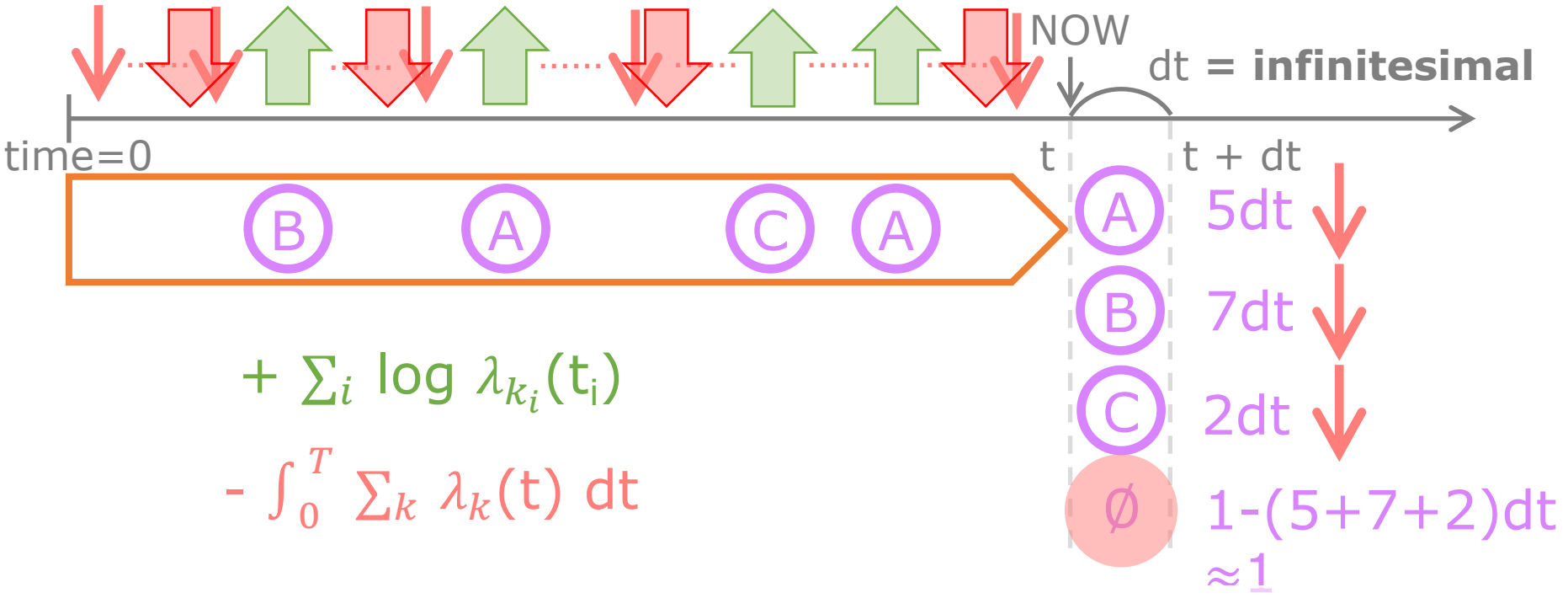

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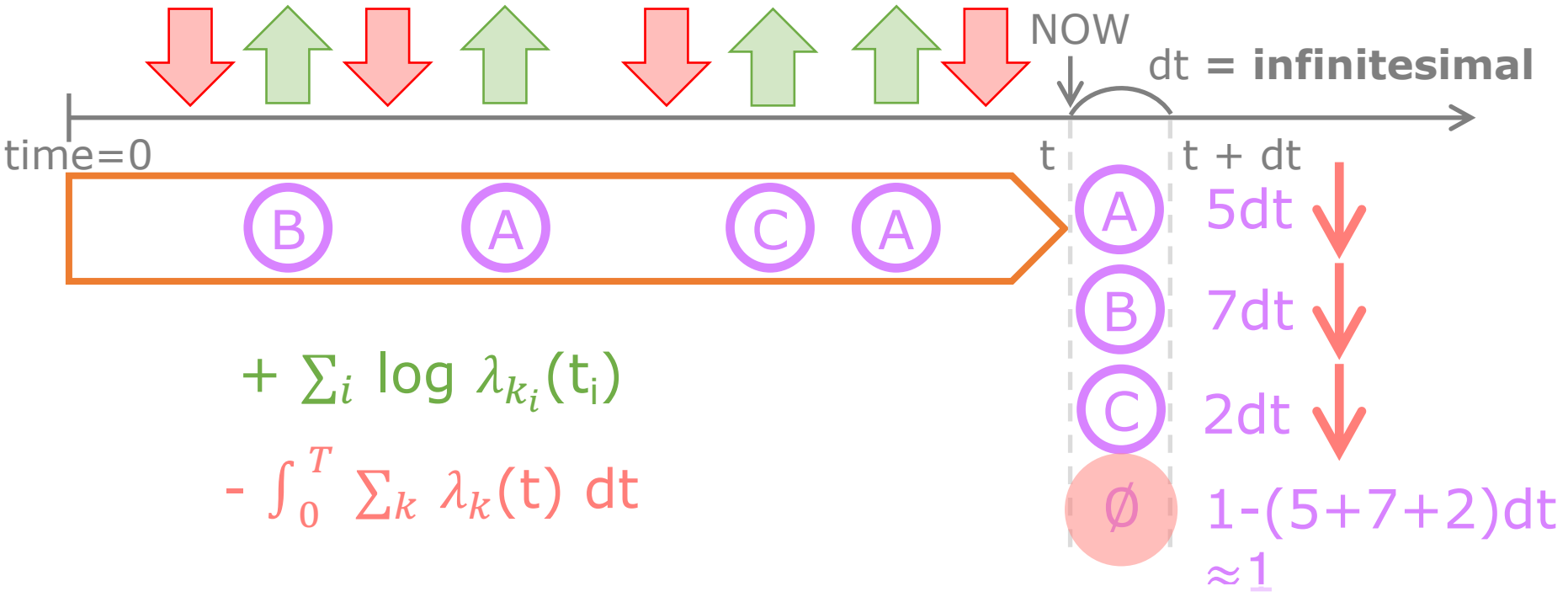

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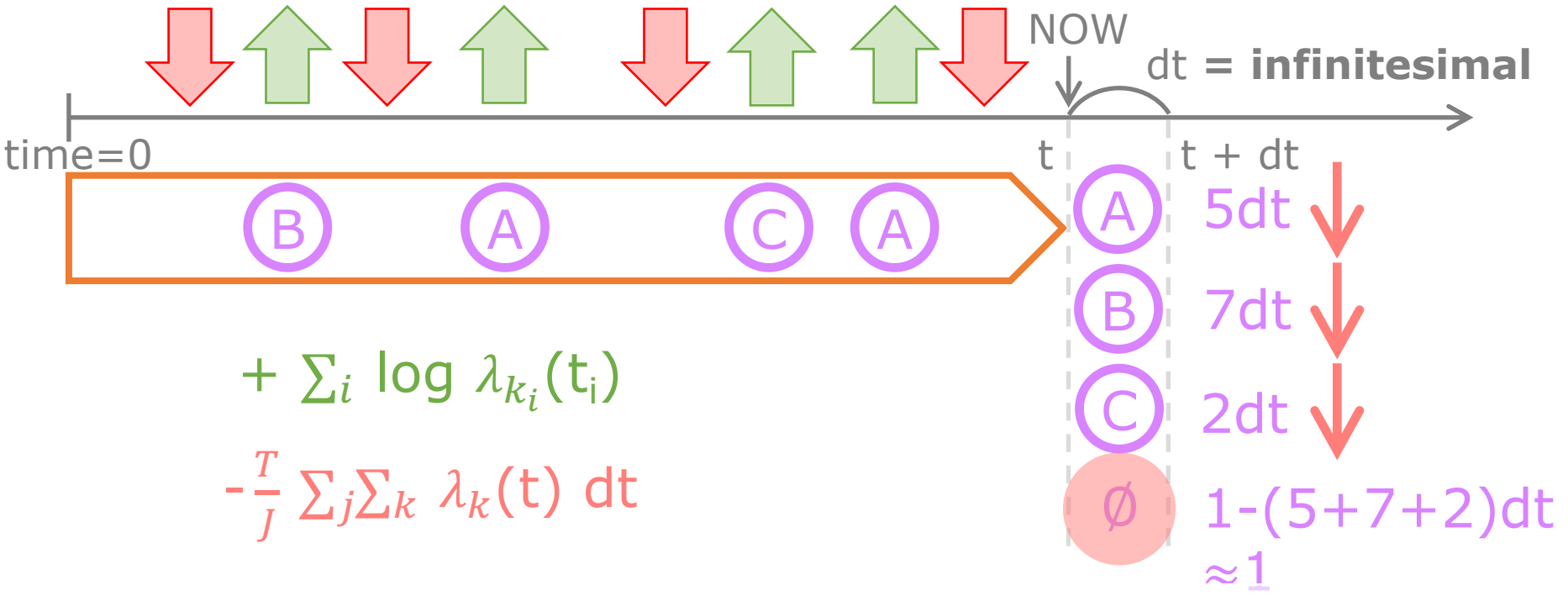

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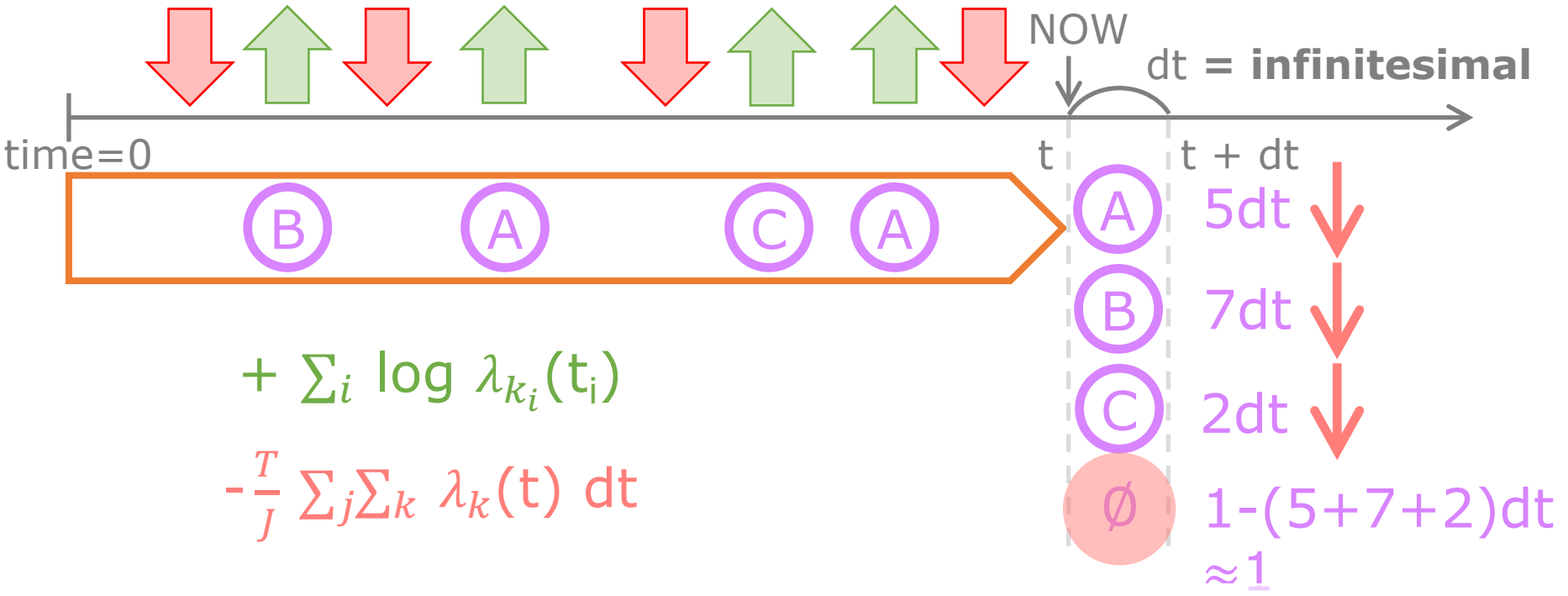

Approx by sampling

Loop over <u>all</u> event types!

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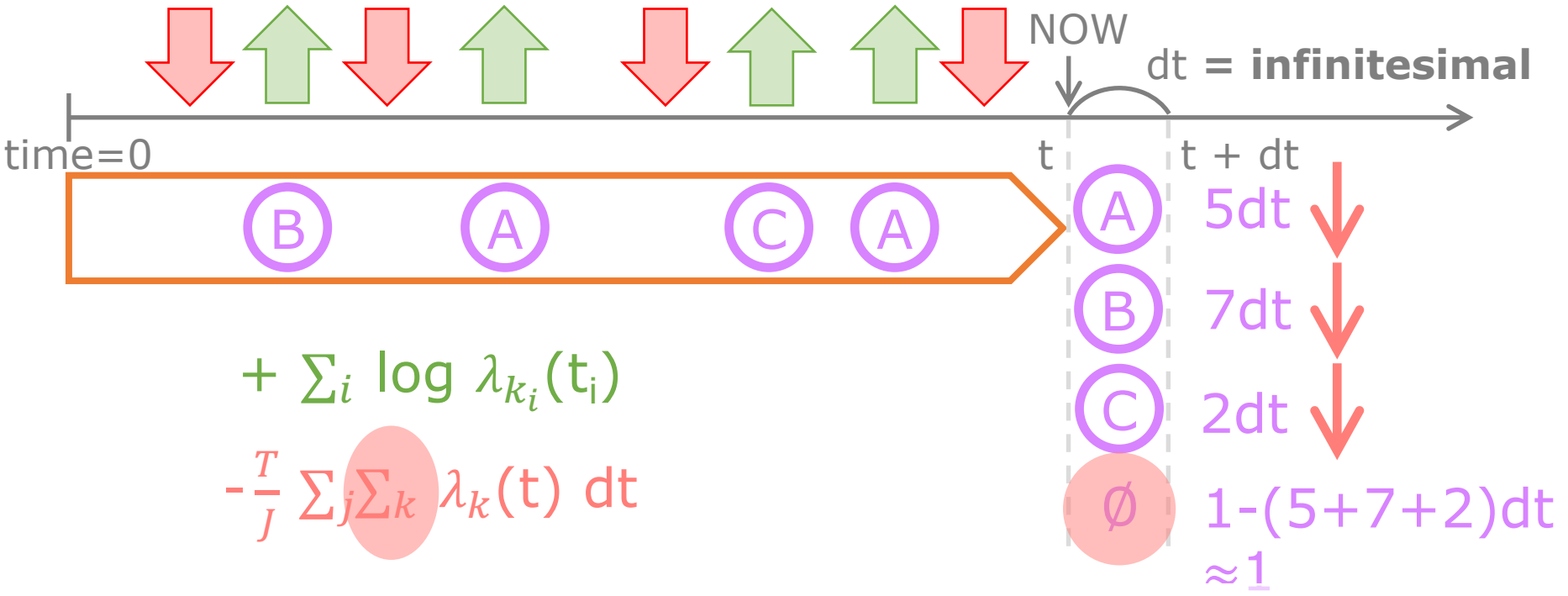

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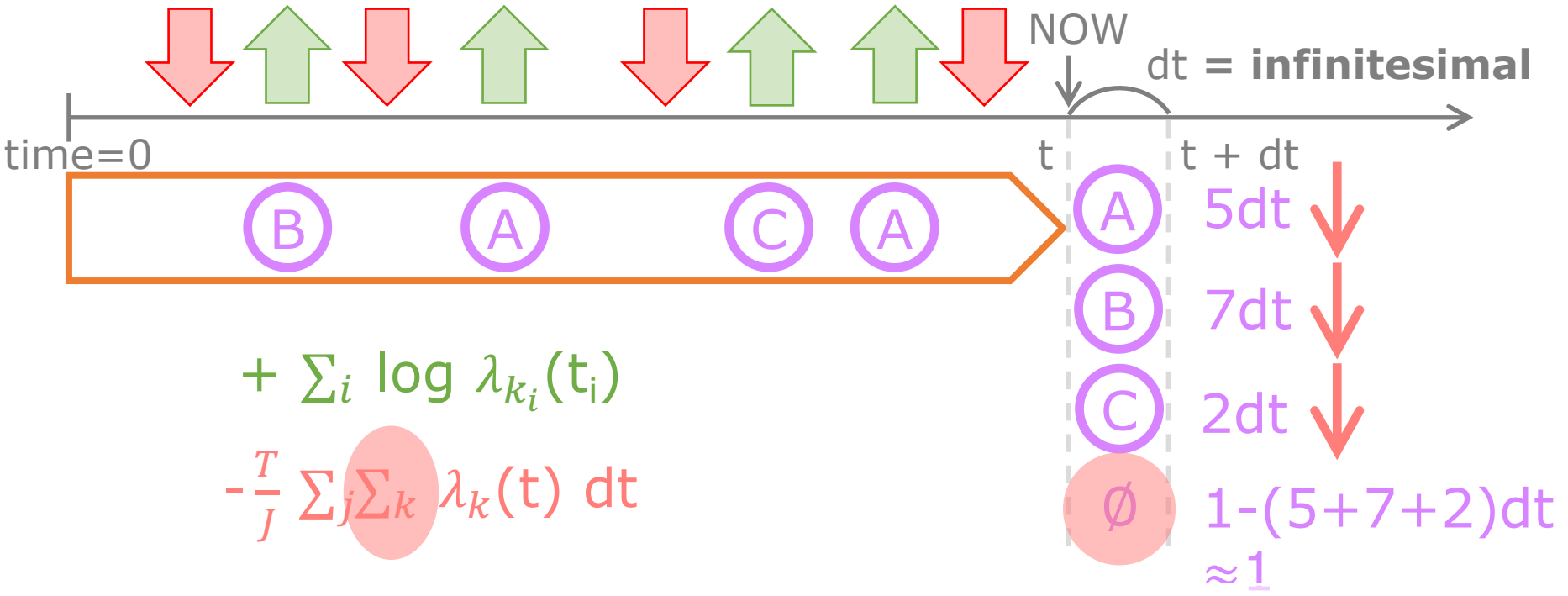

Approx by sampling

Still **Loop over <u>all</u> event types!**

Any Questions?

http://bburl/tpp-slides-p3

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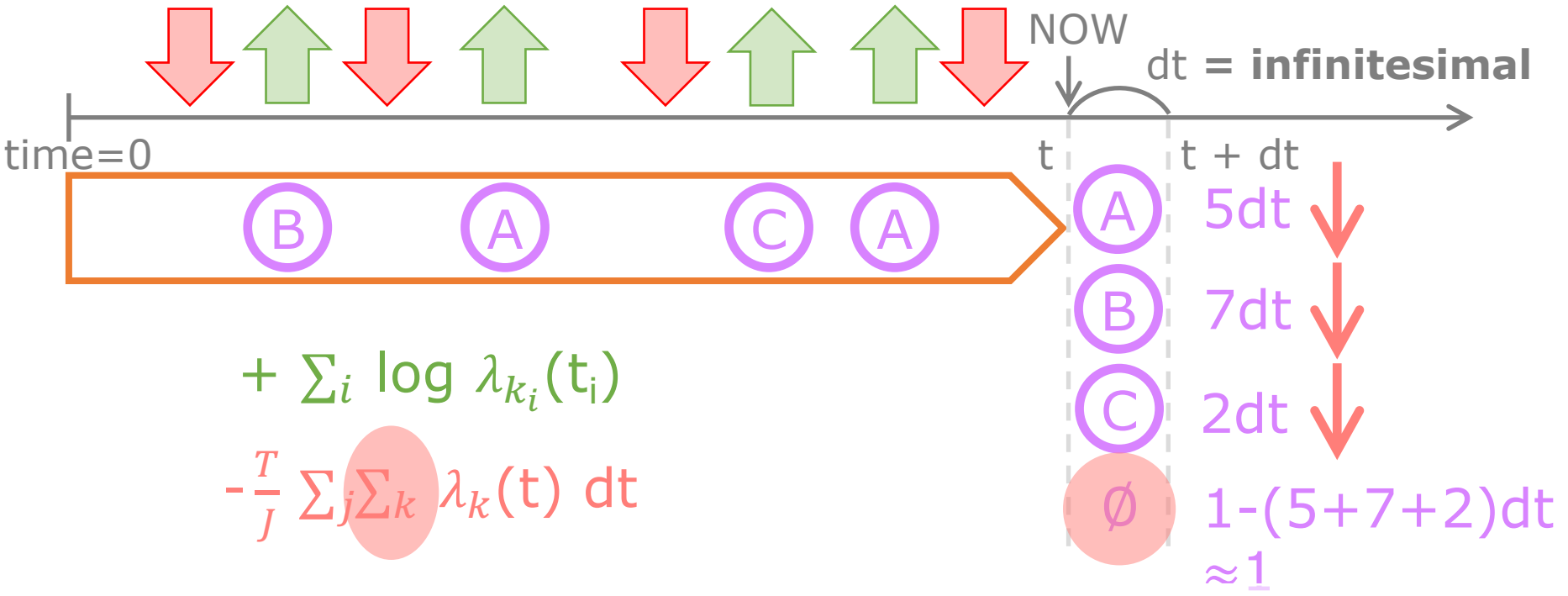

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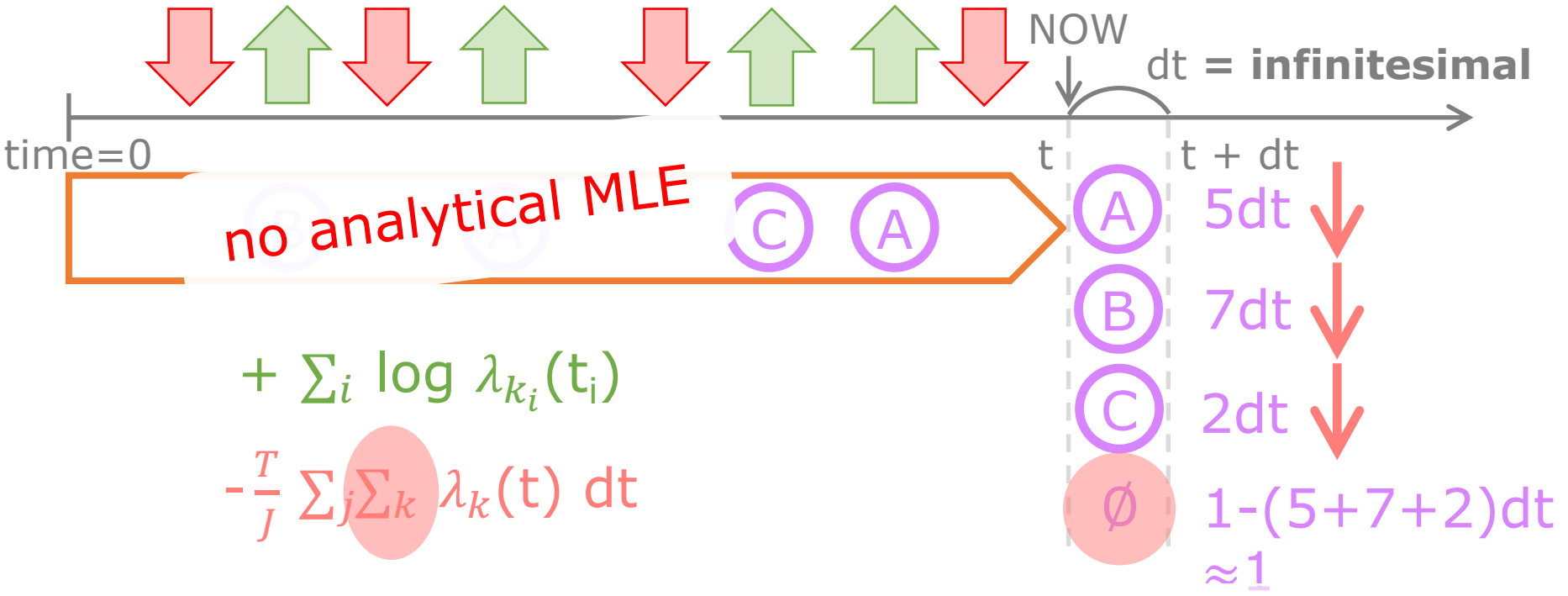

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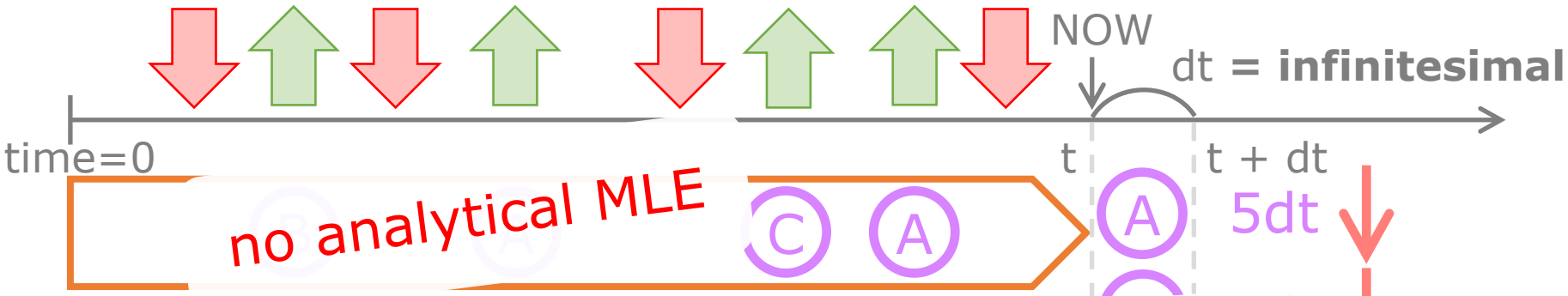


Stochastic Gradient Descent

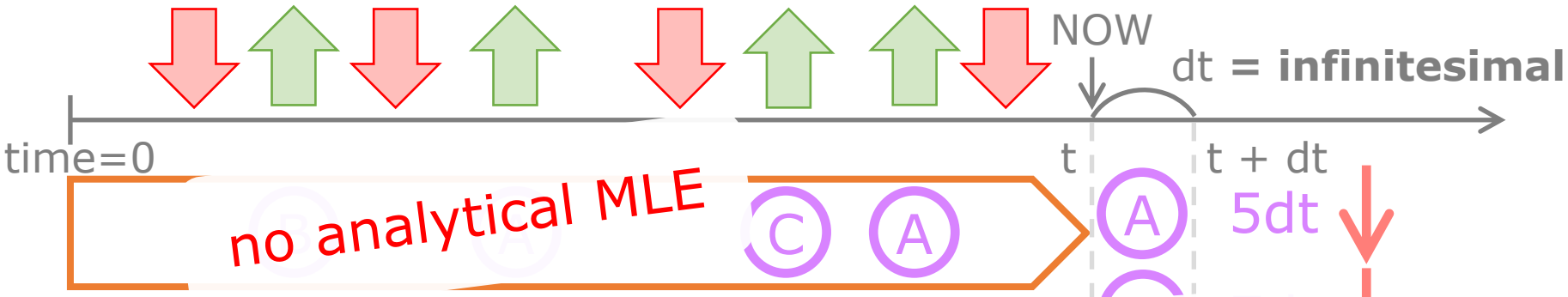
while not converge:

sequence ~ data corpus

S Still Loop over all event types!

$\theta \leftarrow \theta + \nabla_\theta$ log-likelihood

MLE: Max log prob of *data*



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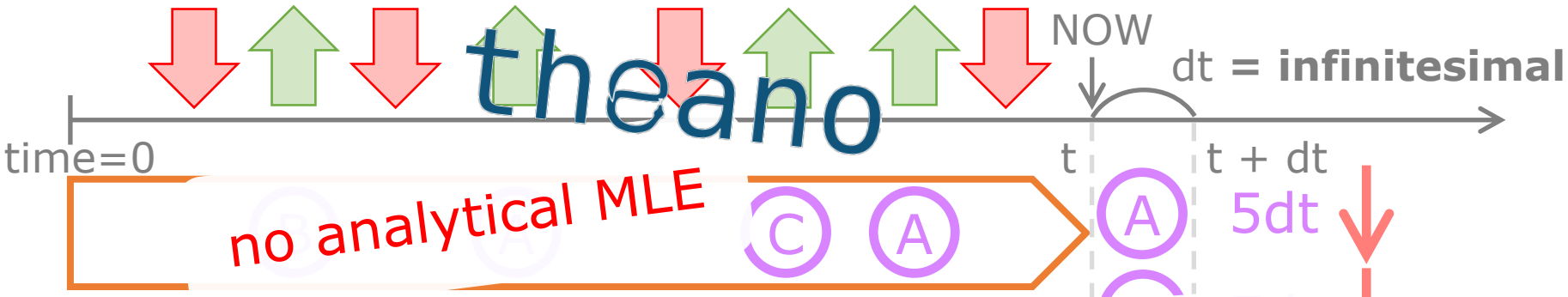

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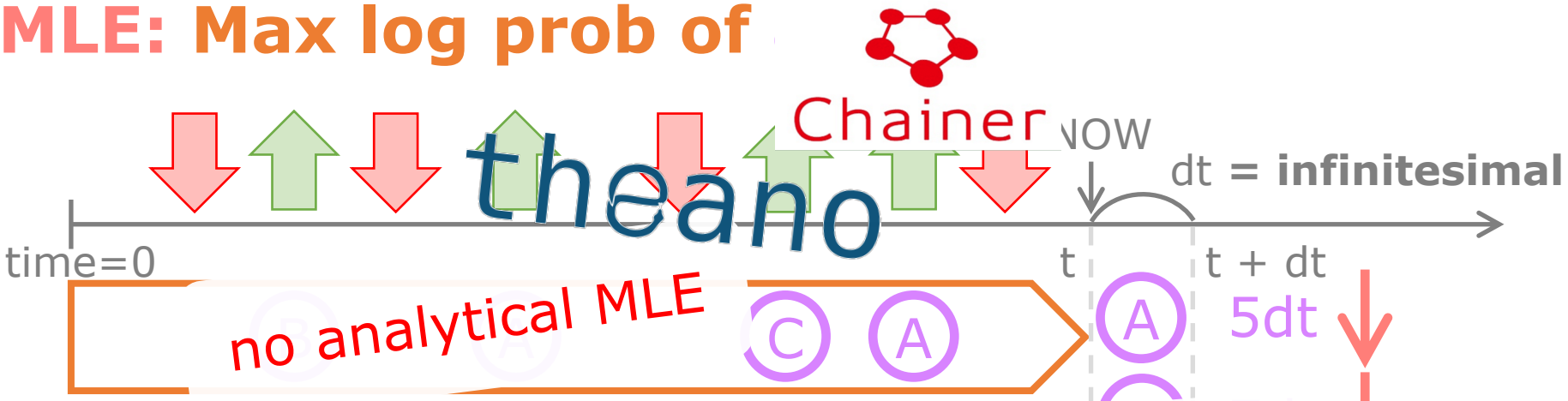

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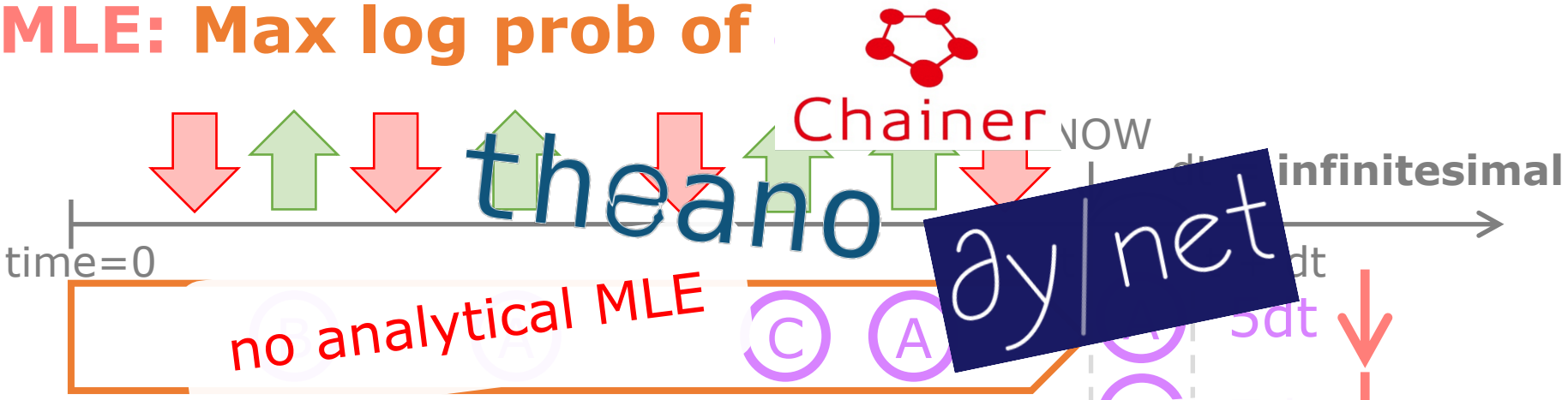

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MLE: Max log prob of

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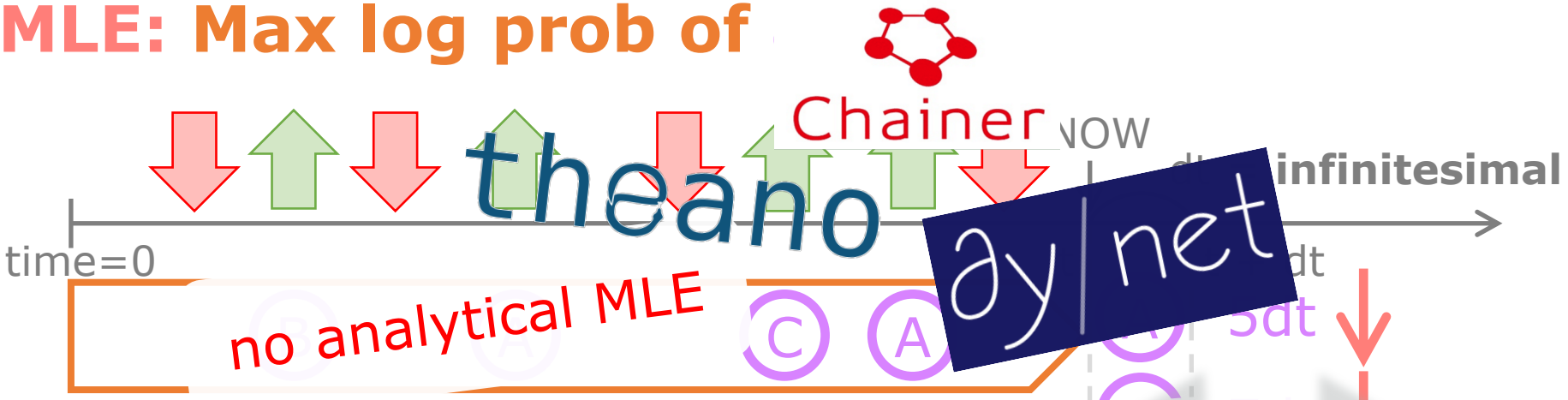

Stochastic Gradient Descent

while not converge:

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$\theta \leftarrow \theta + \nabla_\theta$ log likelihood

The Incredible PYTORCH

Let’s write some code

http://bburl/tpp-lab-p3

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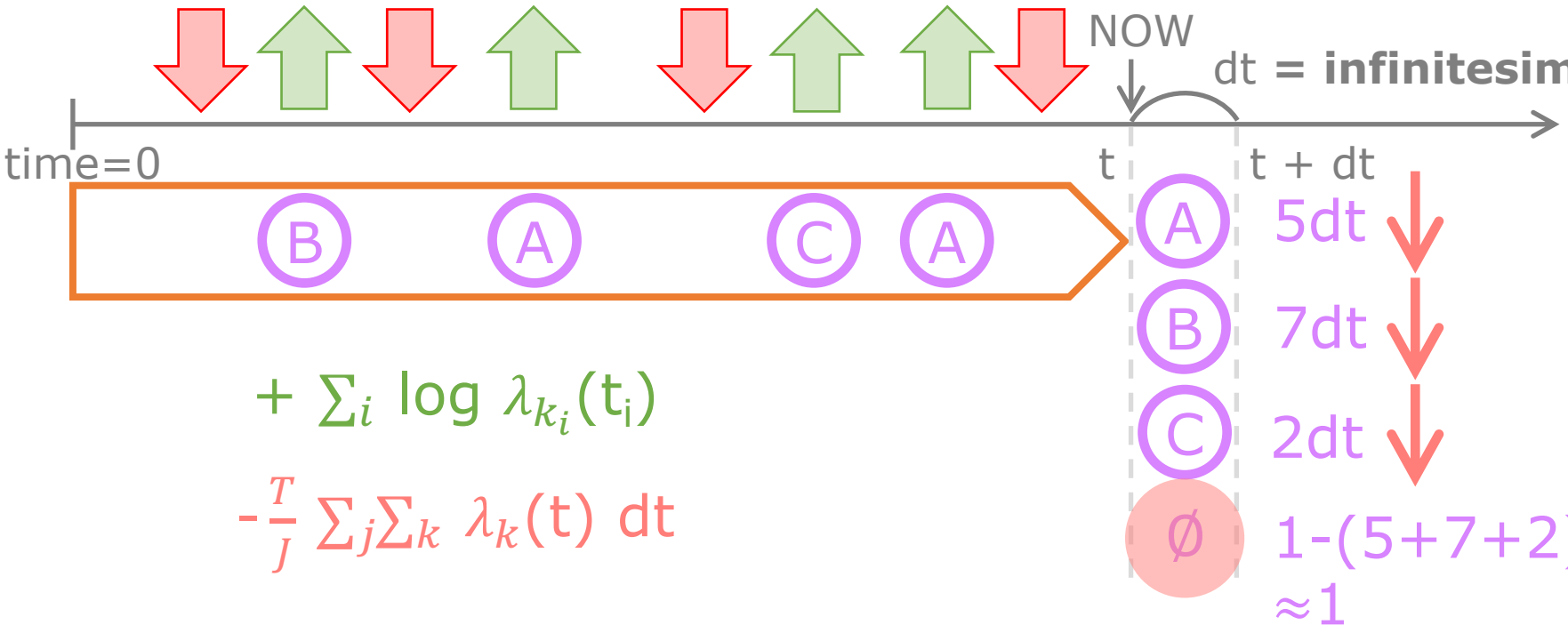

$+ \sum_i \log \lambda_{k_i}(t_i)$

$-\frac{T}{J} \sum_j \sum_k \lambda_k(t)\, dt$

Loop over <u>all</u> event types!

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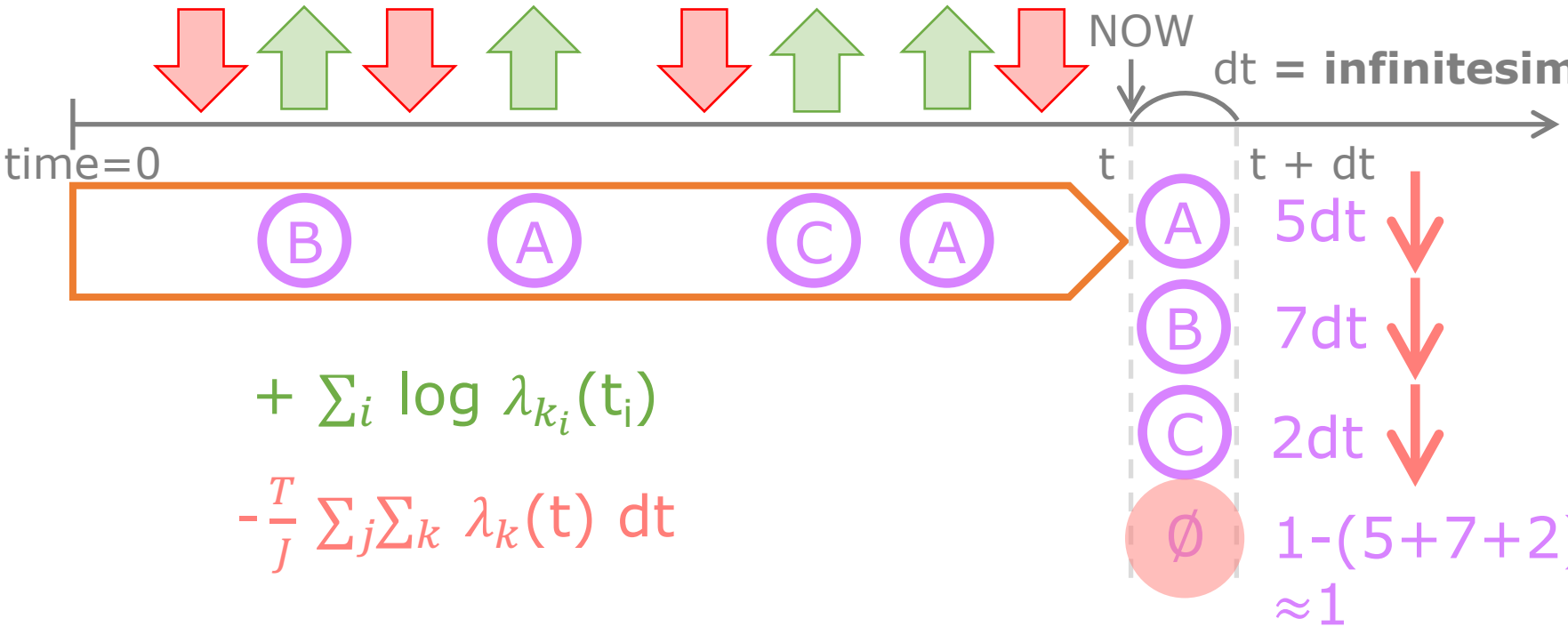

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= 50000?

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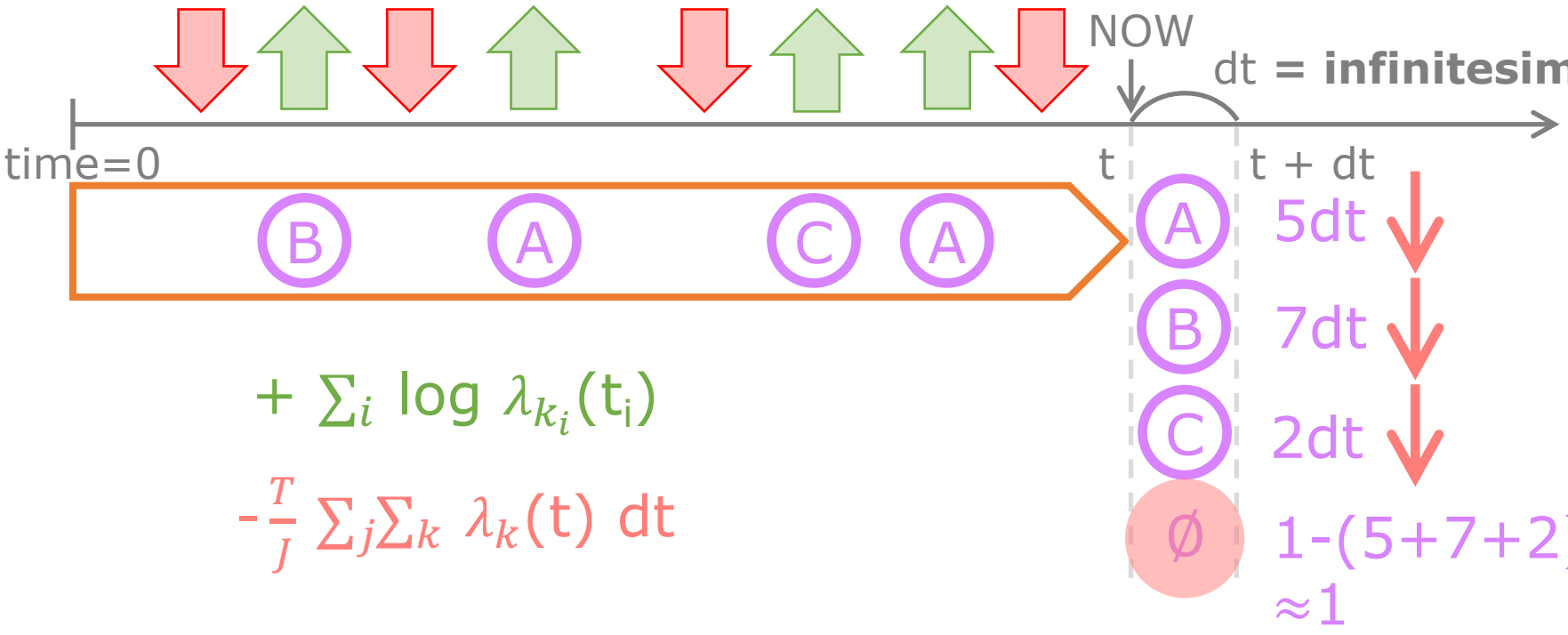

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SLOW!

= 50000?

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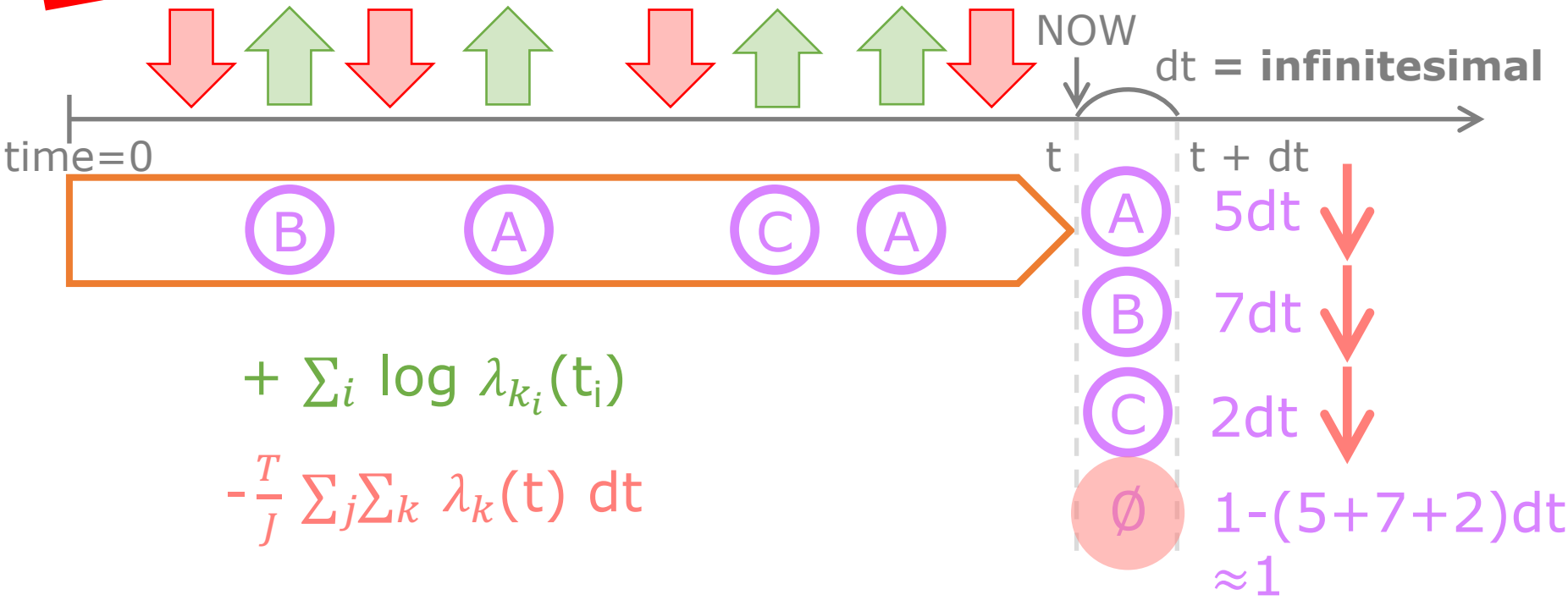

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SLOW!

= 50000?

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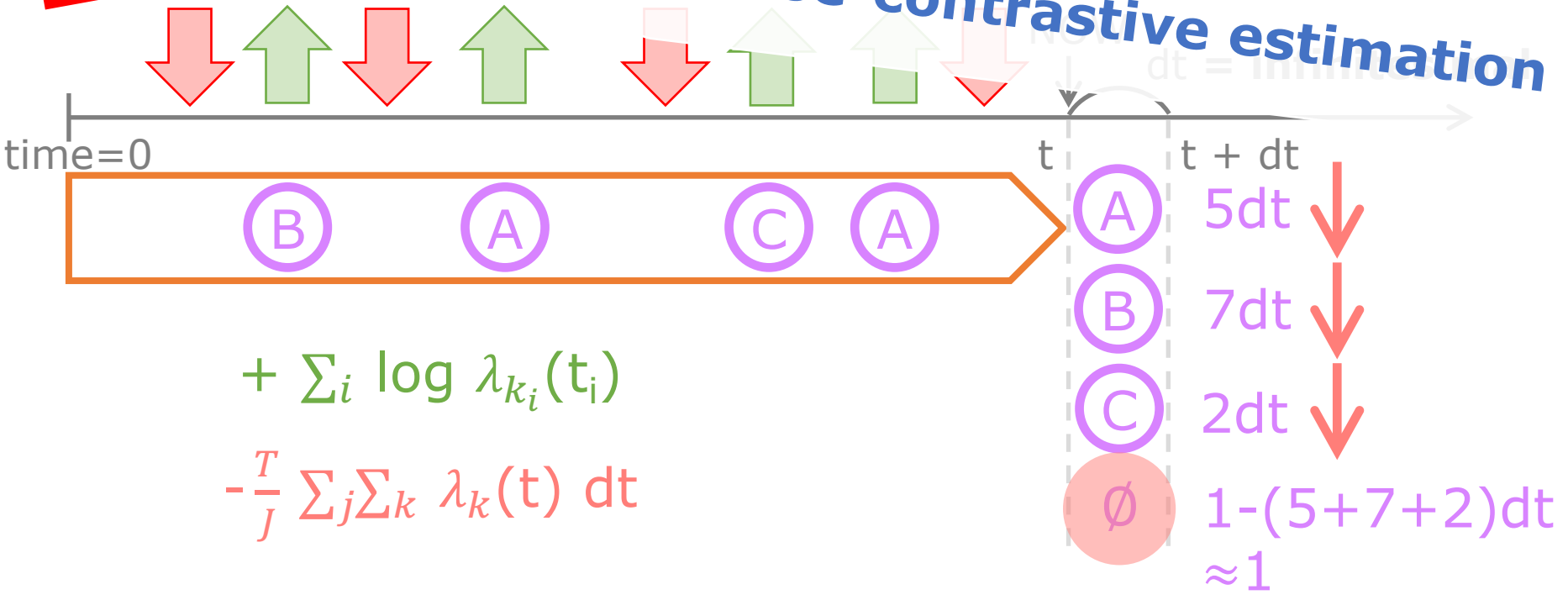
~~MLE: Max log prob of data~~

alternative: noise-contrastive estimation



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SLOW!

= 50000?

Loop over <u>all</u> event types!

NCE: Max log prob of *correct discrimination*

NCE: Max log prob of *correct discrimination*

NCE: Max log prob of *correct discrimination*

log p(A)

NCE: Max log prob of *correct discrimination*

~~$\log p(\cdot)$~~

A

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A

A

Is it Real or Noise?

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A

A

Is it Real or Noise?

A

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

Is it Real or Noise?

p(A)

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

Is it Real or Noise?

p(A)

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

Is it Real or Noise?

p(A)

q(A)

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A

A

Is it Real or Noise?

A p(A)

A q(A)

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A

A $\log \dfrac{p(A)}{p(A) + q(A)}$

Is it Real or Noise?

A $p(A)$

A $q(A)$

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A

A $\log \dfrac{p(A)}{p(A) + q(A)}$

Is it Real or Noise?

A $p(A)$ 

A $q(A)$

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A

A $\log \dfrac{p(A)}{p(A) + q(A)}$

B

Is it Real or Noise?

A $p(A)$ 

A $q(A)$

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A A $\log \frac{p(A)}{p(A) + q(A)}$ B B

Is it Real or Noise?

A $p(A)$ 

A $q(A)$

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A A $\log \frac{p(A)}{p(A) + q(A)}$ B B

Is it Real or Noise?

A p(A) B

A q(A)

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A A $\log \frac{p(A)}{p(A) + q(A)}$ B B

Is it Real or Noise?

A p(A)  B p(B)

A q(A)

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A A $\log \frac{p(A)}{p(A) + q(A)}$ B B

Is it Real or Noise?

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A p(A)  B p(B)

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~~log p(A)~~

A A $\log \frac{p(A)}{p(A) + q(A)}$

B B $\log \frac{q(B)}{p(B) + q(B)}$

Is it Real or Noise?

A p(A) 

B p(B)

A q(A)

B q(B)

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

A A $\log \frac{p(A)}{p(A) + q(A)}$

B B $\log \frac{q(B)}{p(B) + q(B)}$

Is it Real or Noise?

A p(A) ↑

A q(A)

B p(B) ↓

B q(B)

q = a cheaper model

NCE: Max log prob of *correct discrimination*

~~log p(A)~~

Binary-Classification Version
Gutmann & Hyvarinen 2010

A A $\log \frac{p(A)}{p(A) + q(A)}$

B B $\log \frac{q(B)}{p(B) + q(B)}$

Is it Real or Noise?

A p(A) ↑

B p(B) ↓

A q(A)

B q(B)

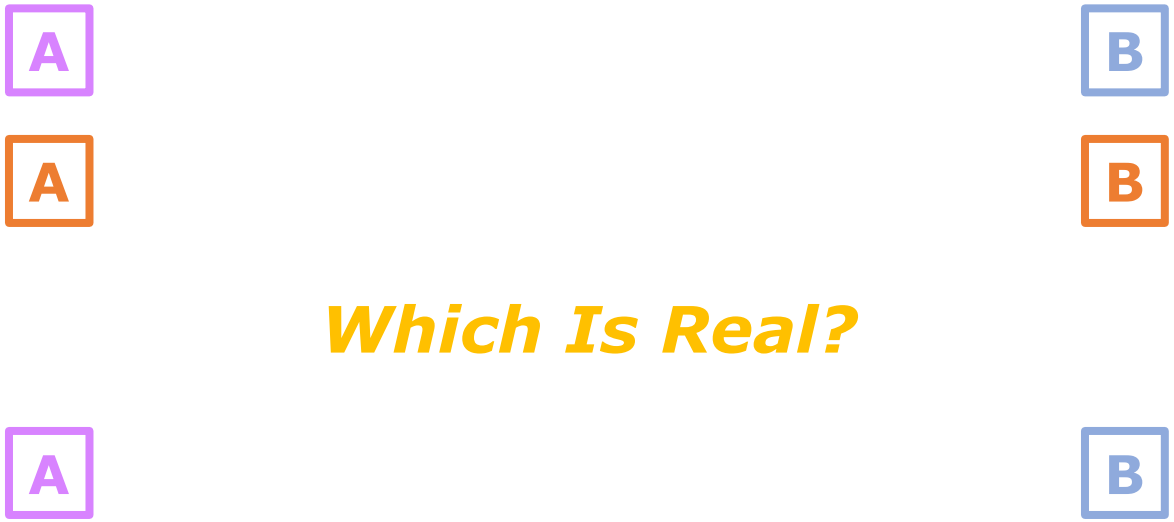
q = a cheaper model

NCE: Max log prob of *correct discrimination*

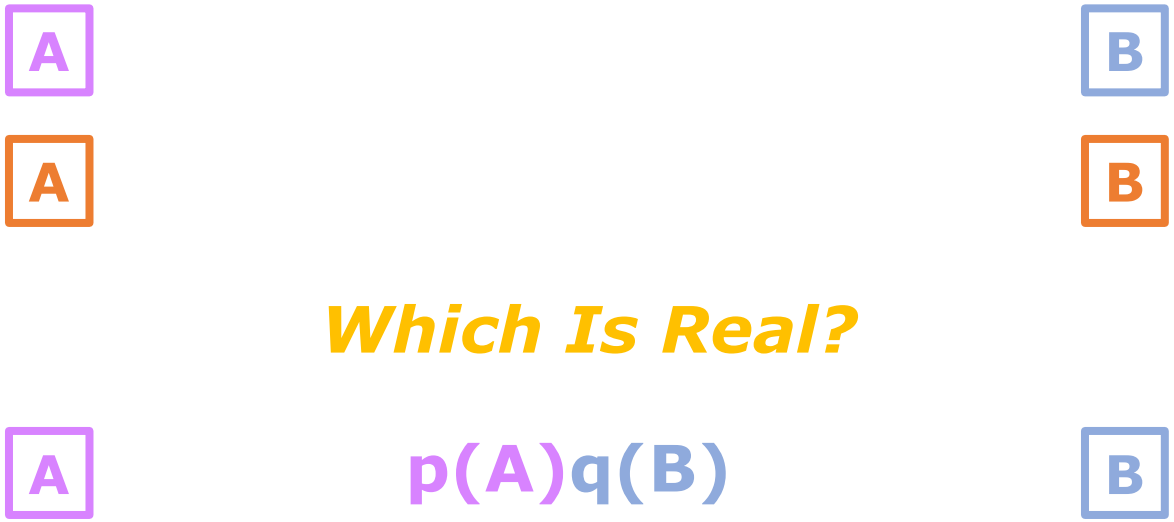
NCE: Max log prob of *correct discrimination*

Which Is Real?

NCE: Max log prob of *correct discrimination*



NCE: Max log prob of *correct discrimination*



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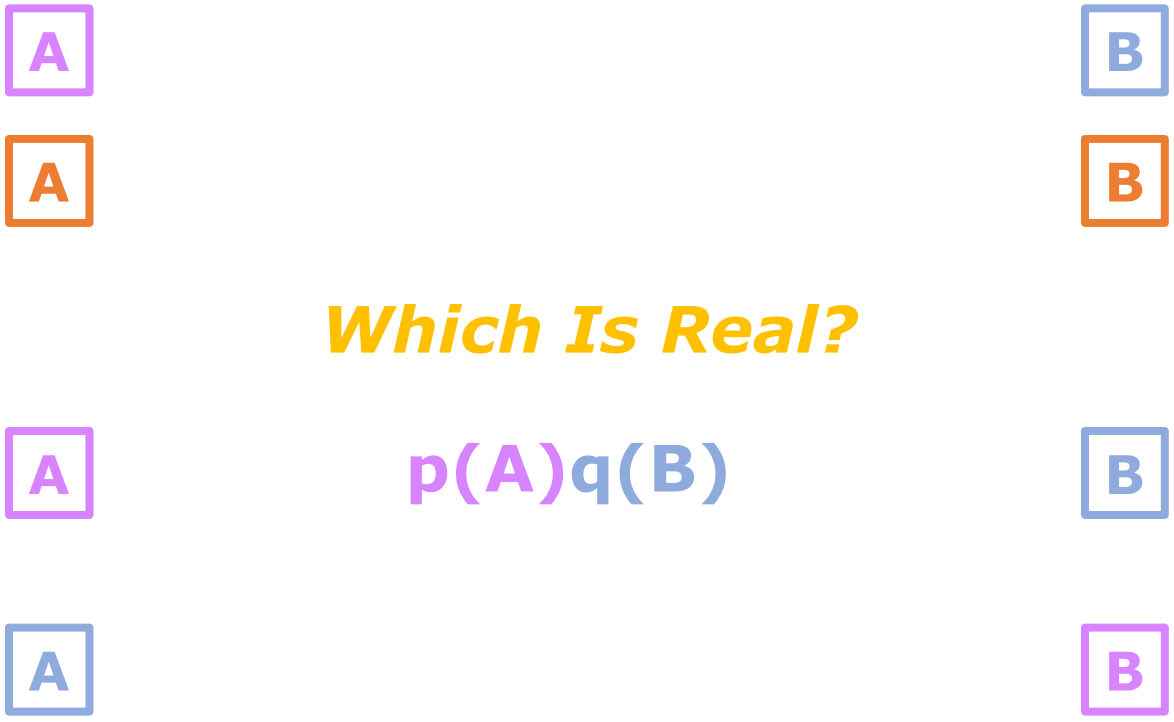

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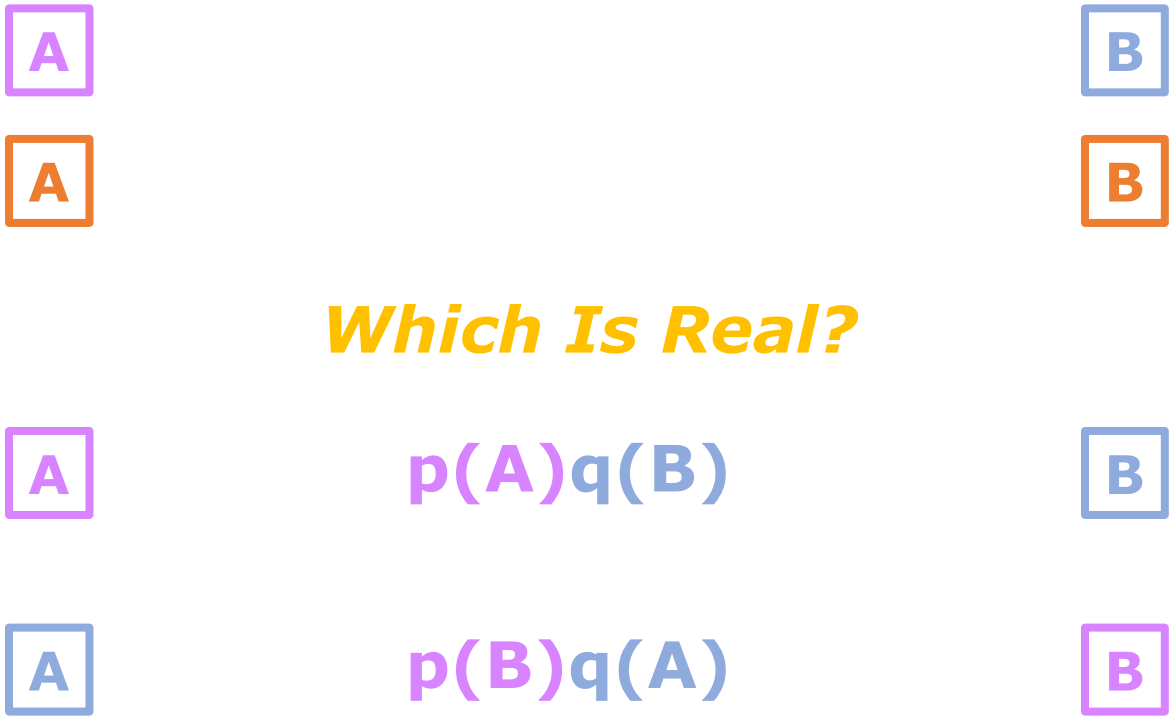

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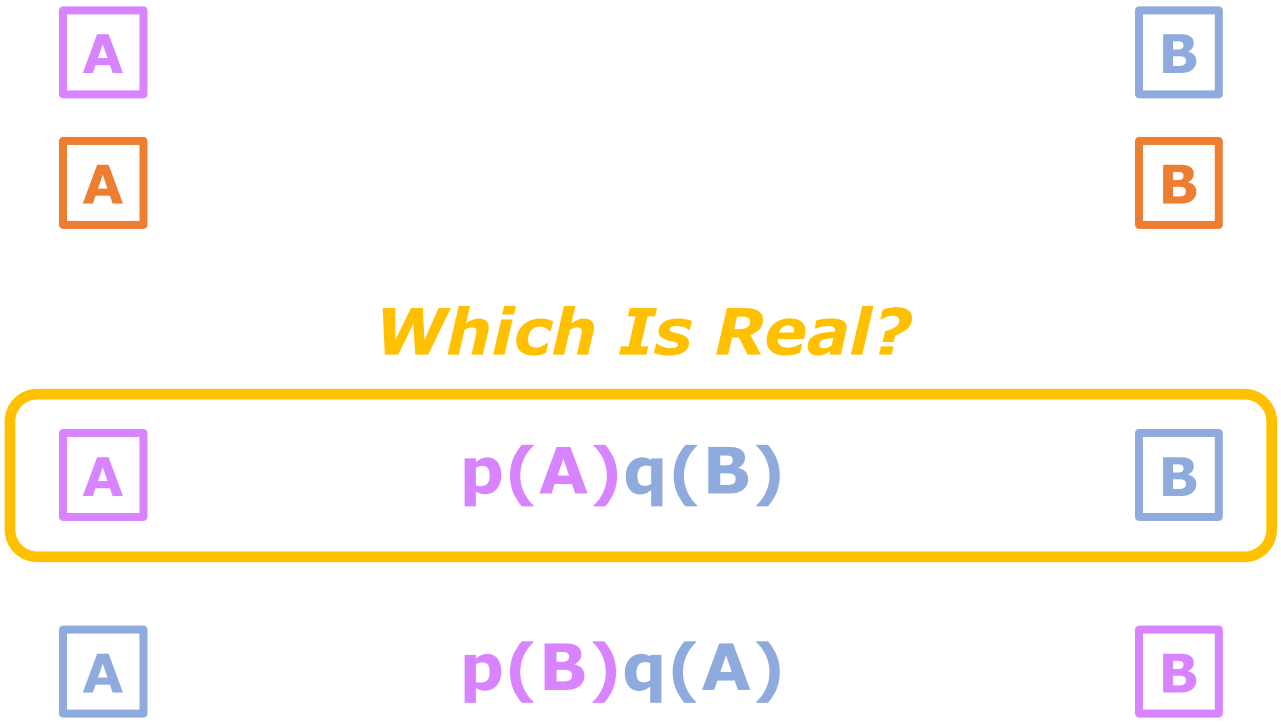

NCE: Max log prob of *correct discrimination*

$$\log \frac{p(A)q(B)}{p(A)q(B) + p(B)q(A)}$$

A A B B

Which Is Real?

A p(A)q(B) B

A p(B)q(A) B

NCE: Max log prob of *correct discrimination*

A A $\log \frac{p(A)q(B)}{p(A)q(B) + p(B)q(A)}$ B B

p(A) ↑

Which Is Real?

A p(A)q(B) B

A p(B)q(A) B

NCE: Max log prob of *correct discrimination*

$$\log \frac{p(A)q(B)}{p(A)q(B) + p(B)q(A)}$$

A A B B

Which Is Real?

A p(A)q(B) B

A p(B)q(A) B

p(A) ↑

p(B) ↓

NCE: Max log prob of *correct discrimination*

Ranking Version
Ma & Collins 2018

A A $\log \frac{p(A)q(B)}{p(A)q(B) + p(B)q(A)}$ B B

p(A) ↑

p(B) ↓

Which Is Real?

A p(A)q(B) B

A p(B)q(A) B

NCE: Max log prob of *correct discrimination*

Consistency under *weaker* assumptions

Ranking Version Ma & Collins 2018

A A $\log \frac{p(A)q(B)}{p(A)q(B) + p(B)q(A)}$ B B

p(A) ↑

p(B) ↓

Which Is Real?

A p(A)q(B) B

A p(B)q(A) B

Any Questions?

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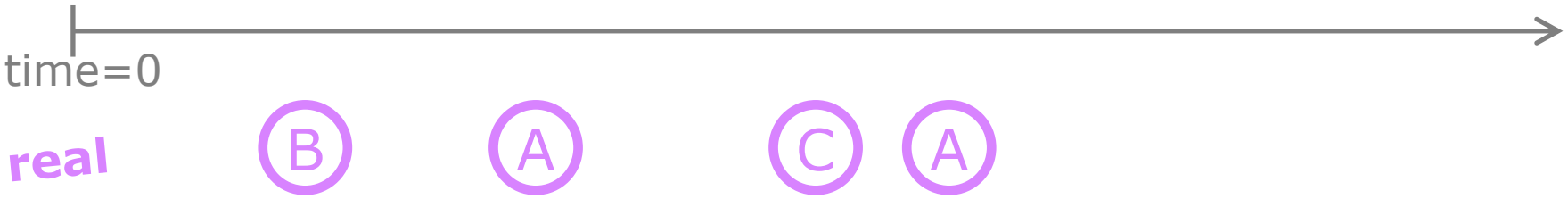
NCE: Max log prob of *correct discrimination*



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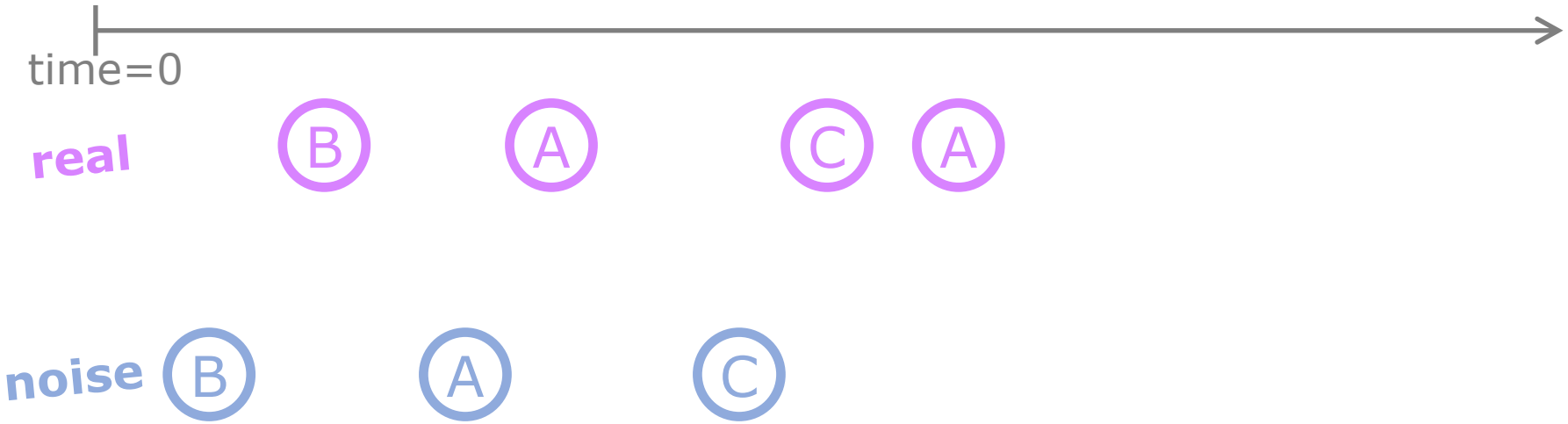

NCE: Max log prob of *correct discrimination*

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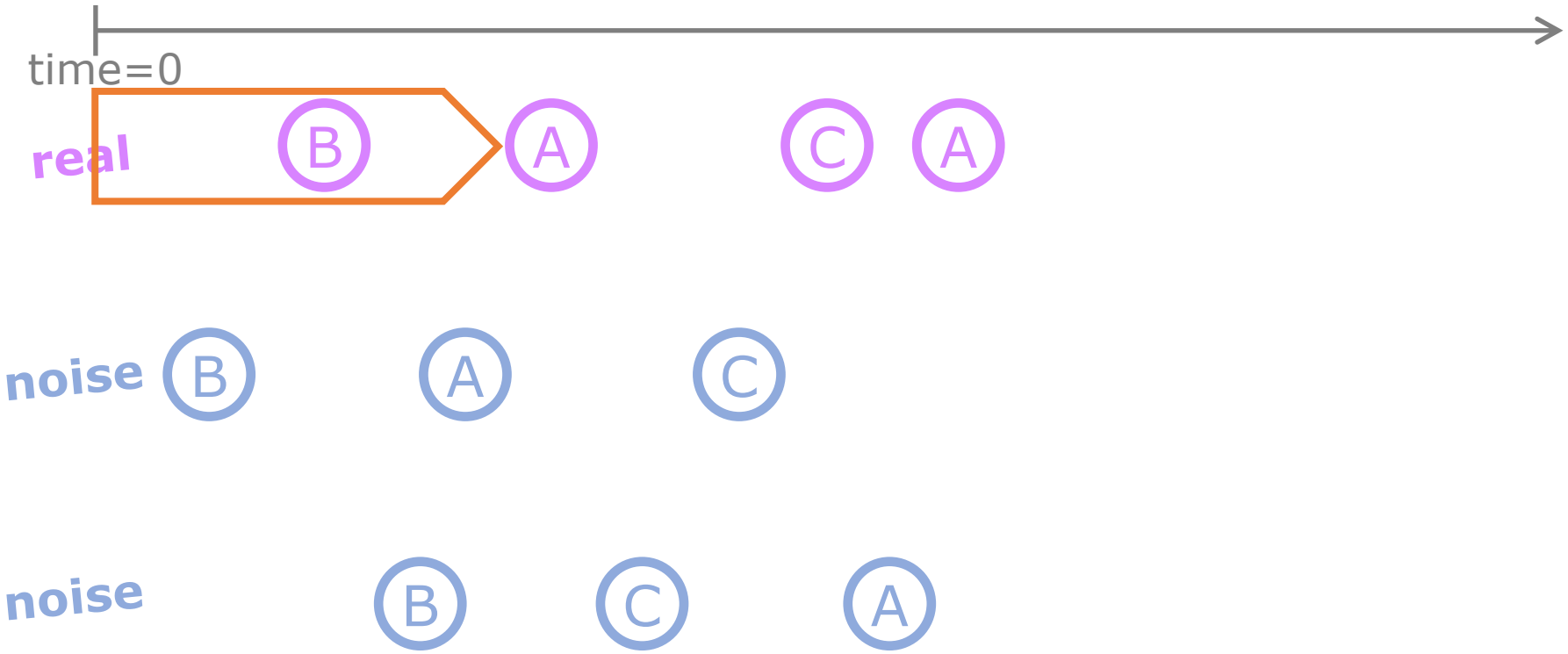

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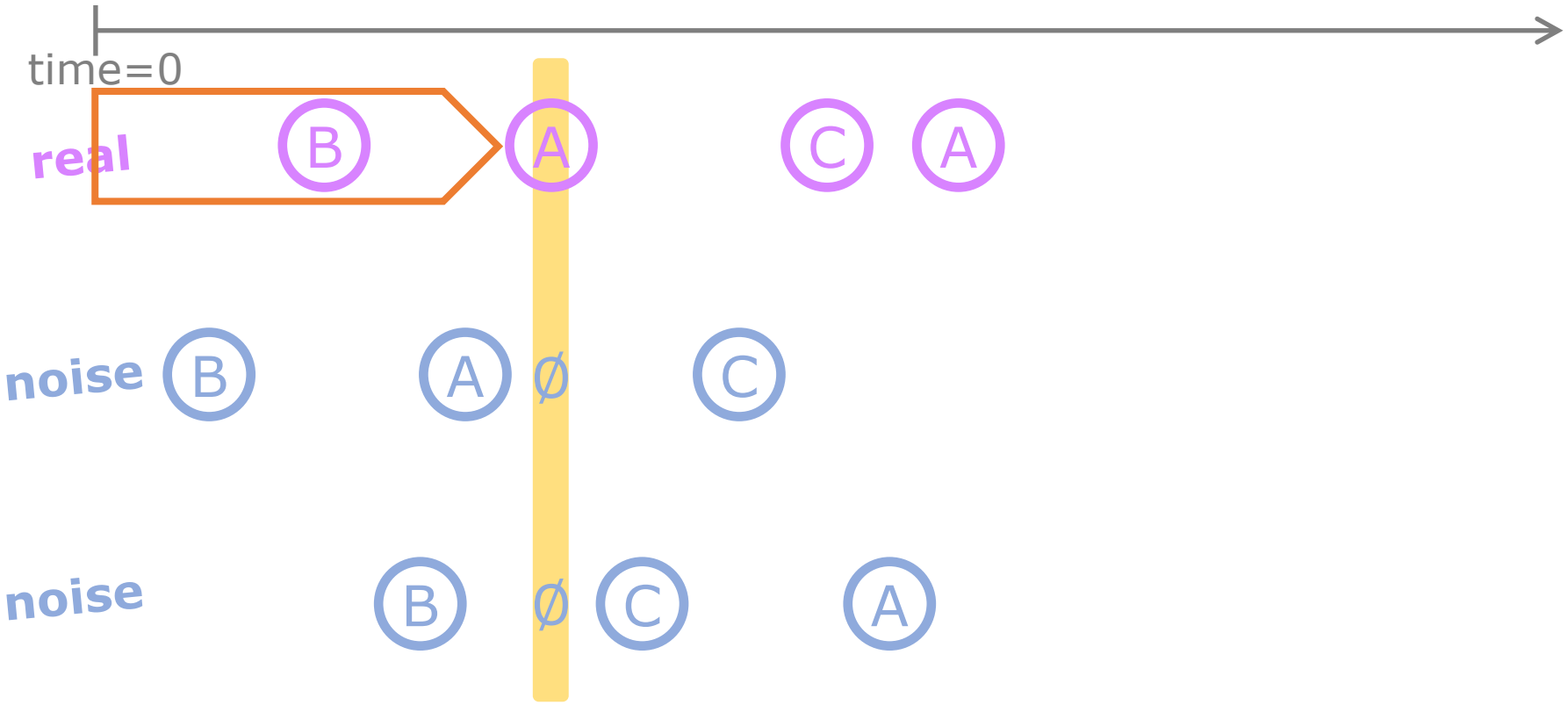

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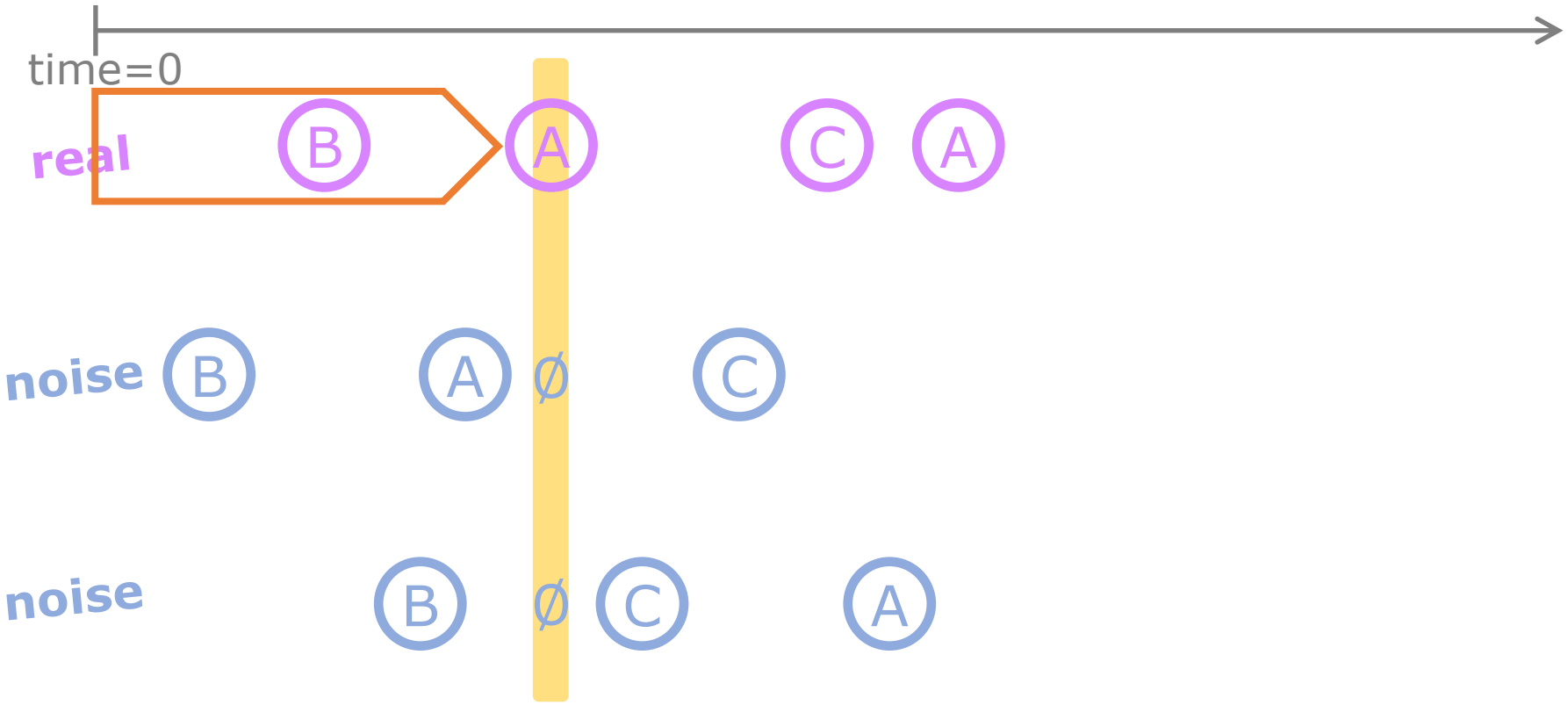
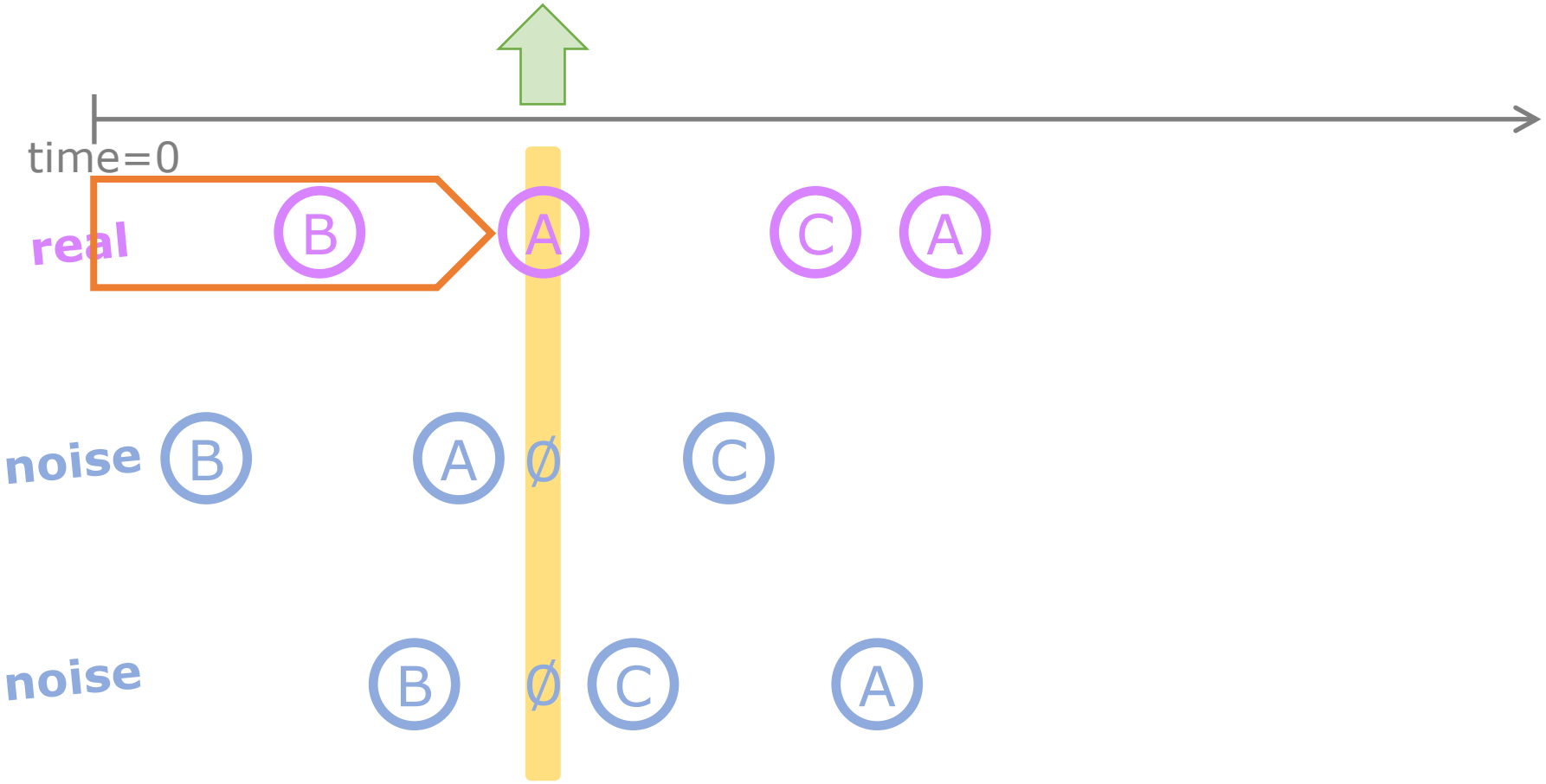


Which Is Real?

NCE: Max log prob of *correct discrimination*



Which Is Real?

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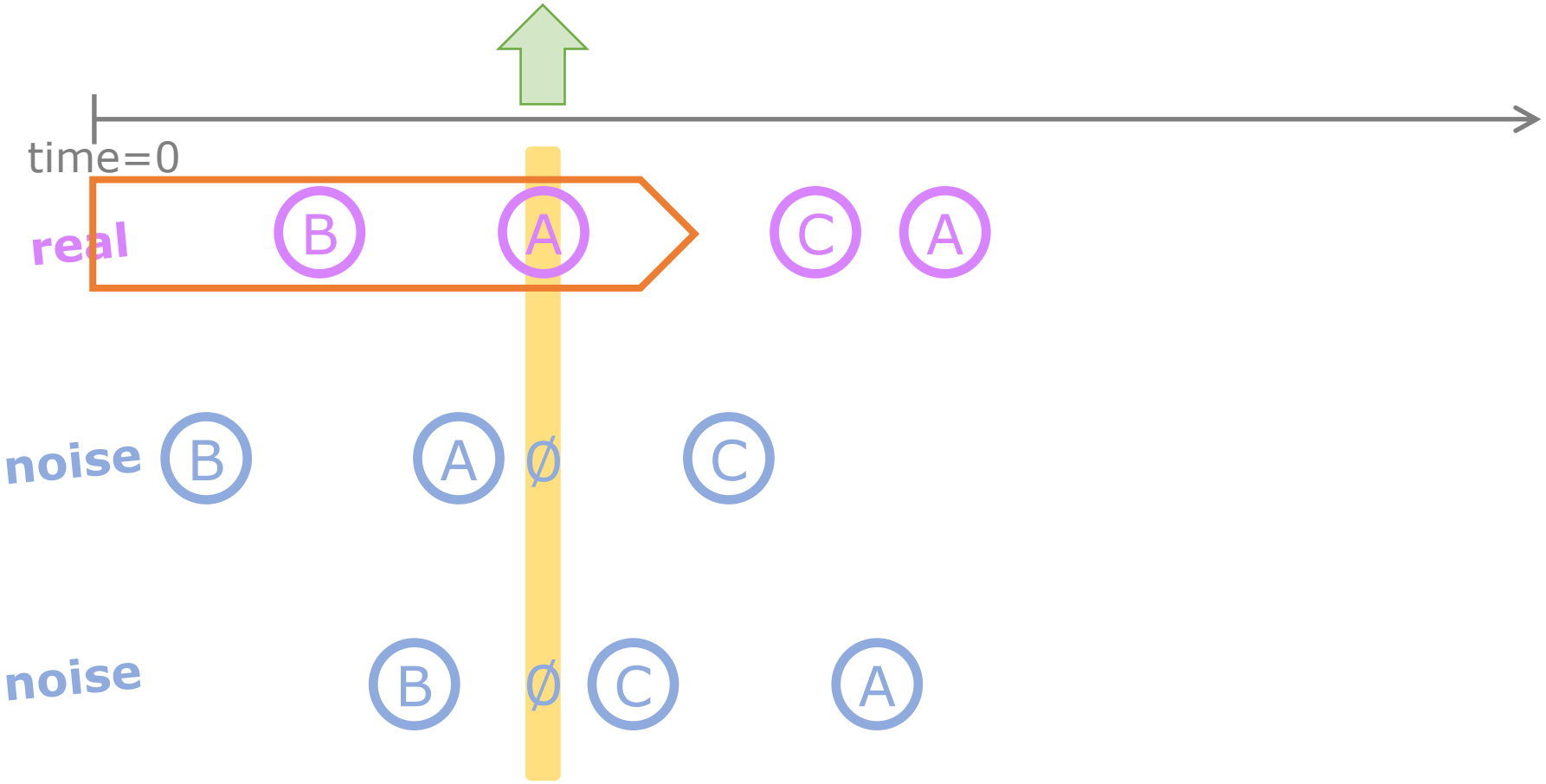

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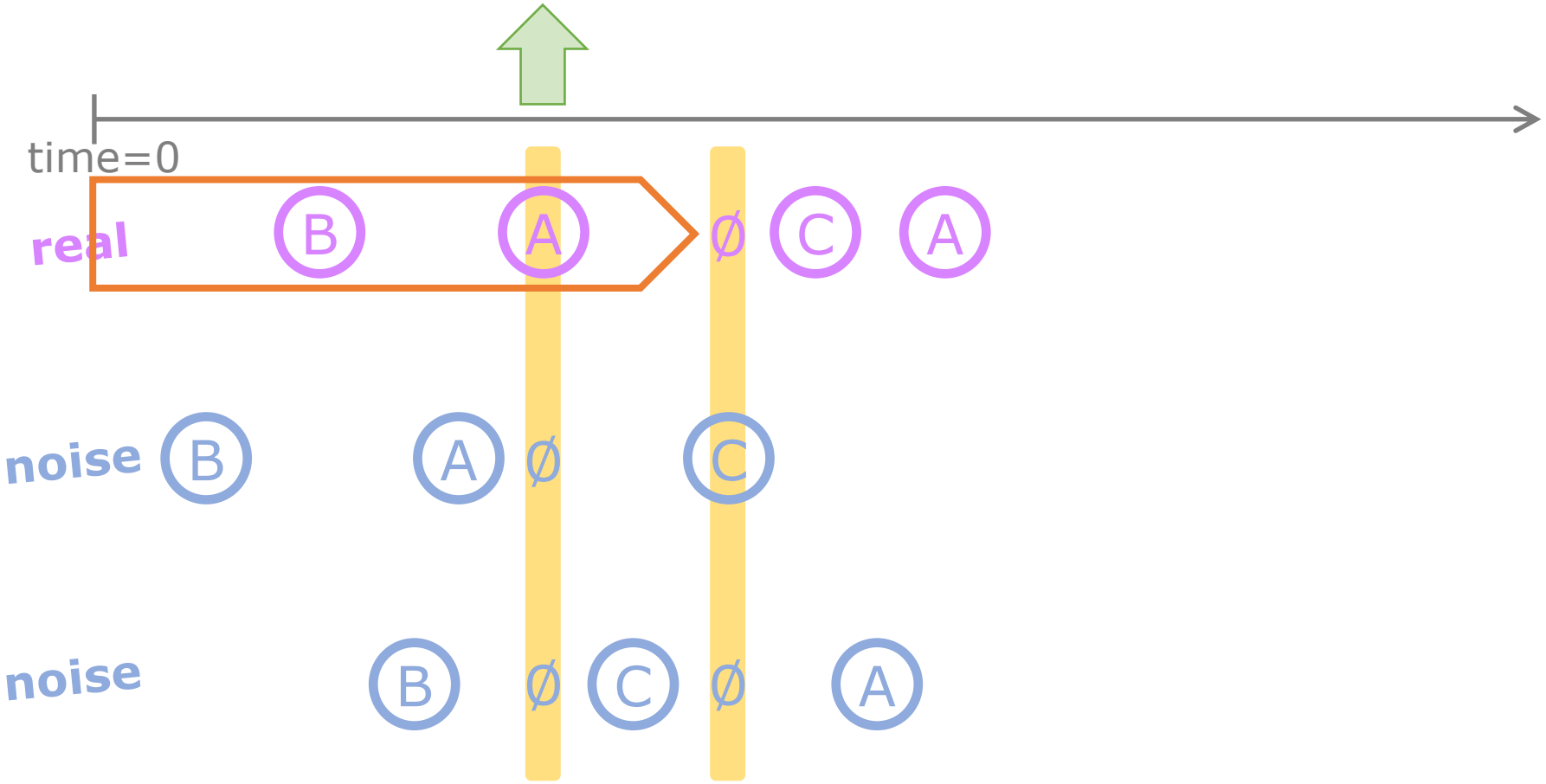

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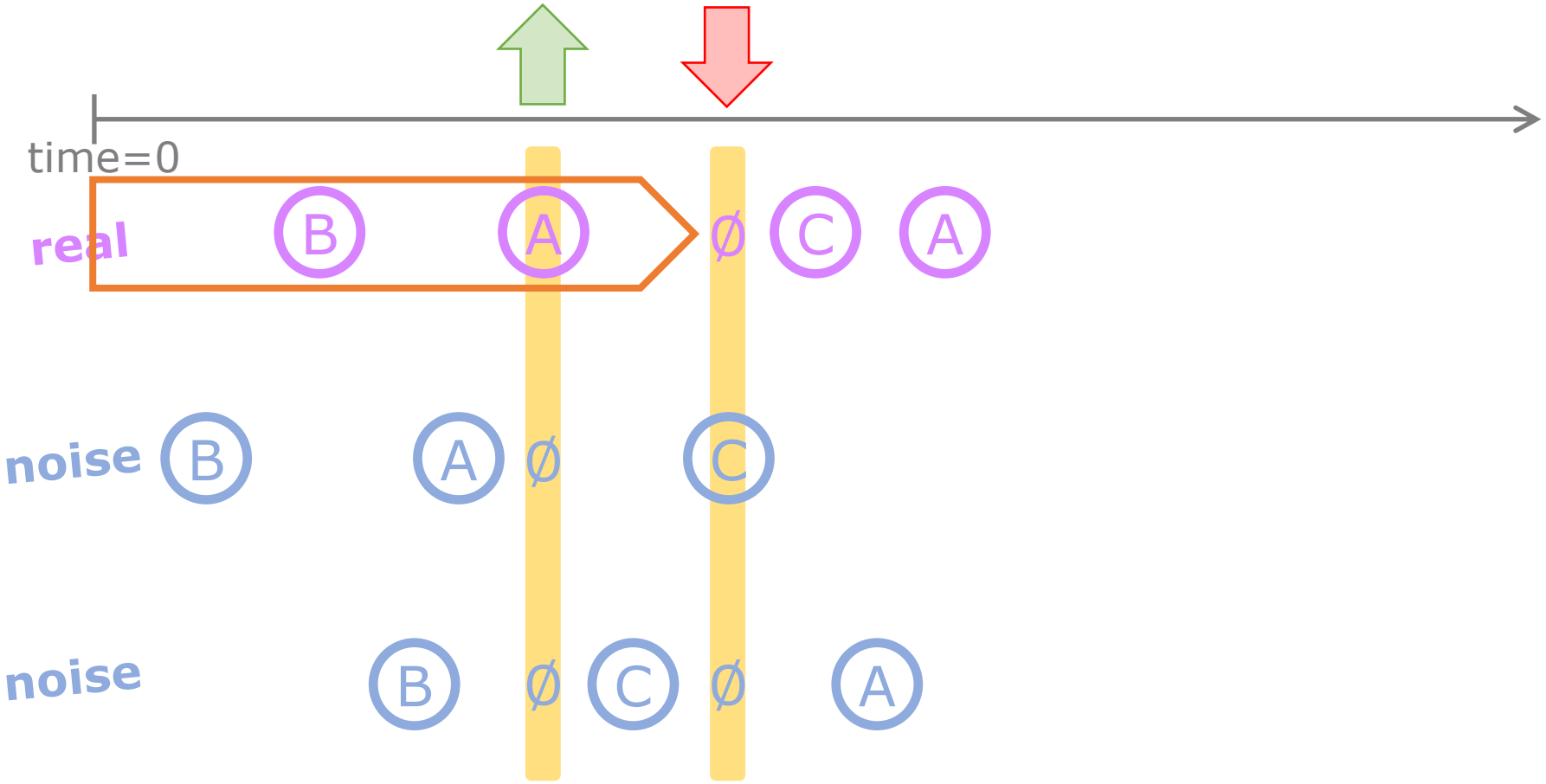
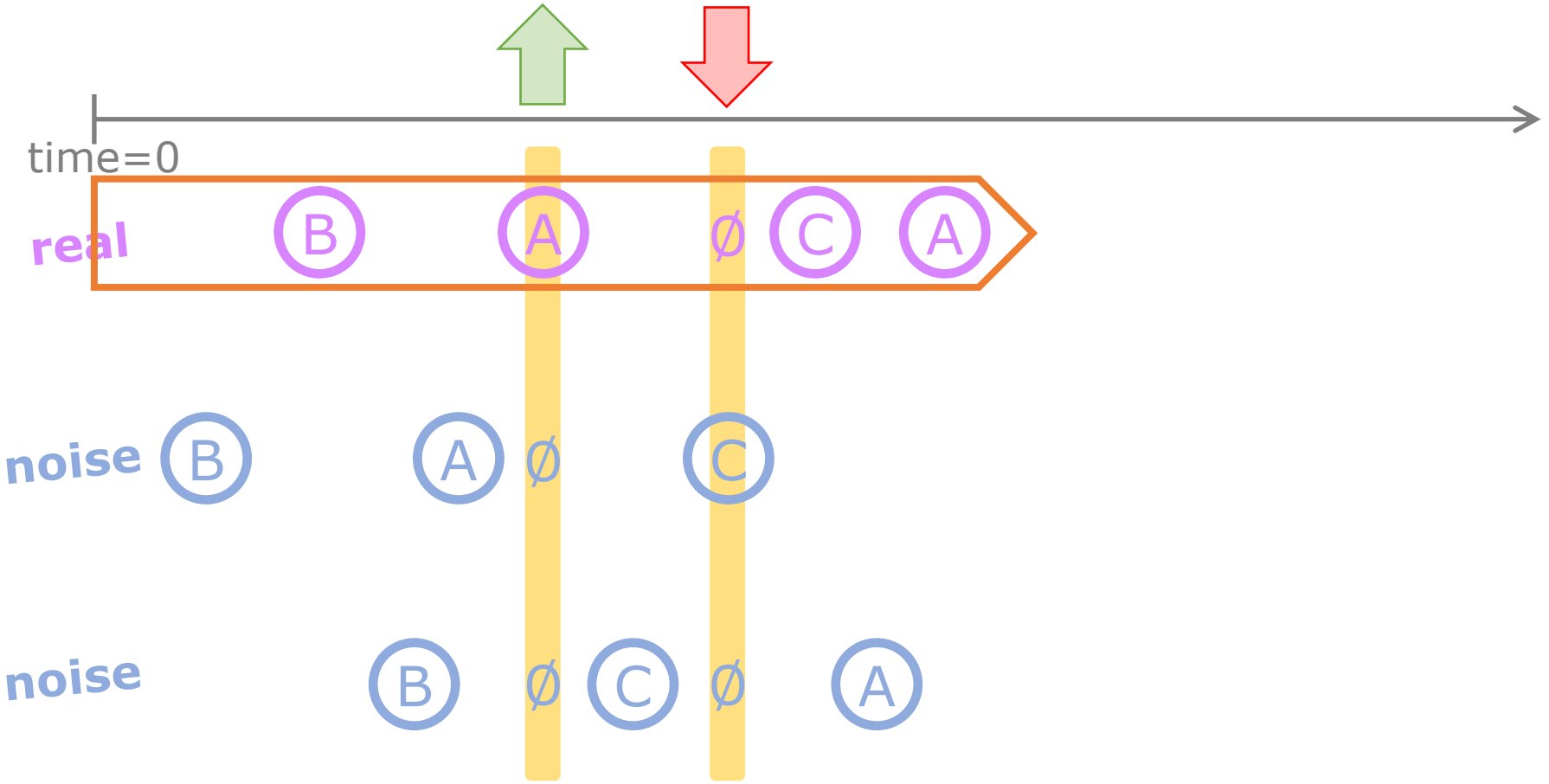


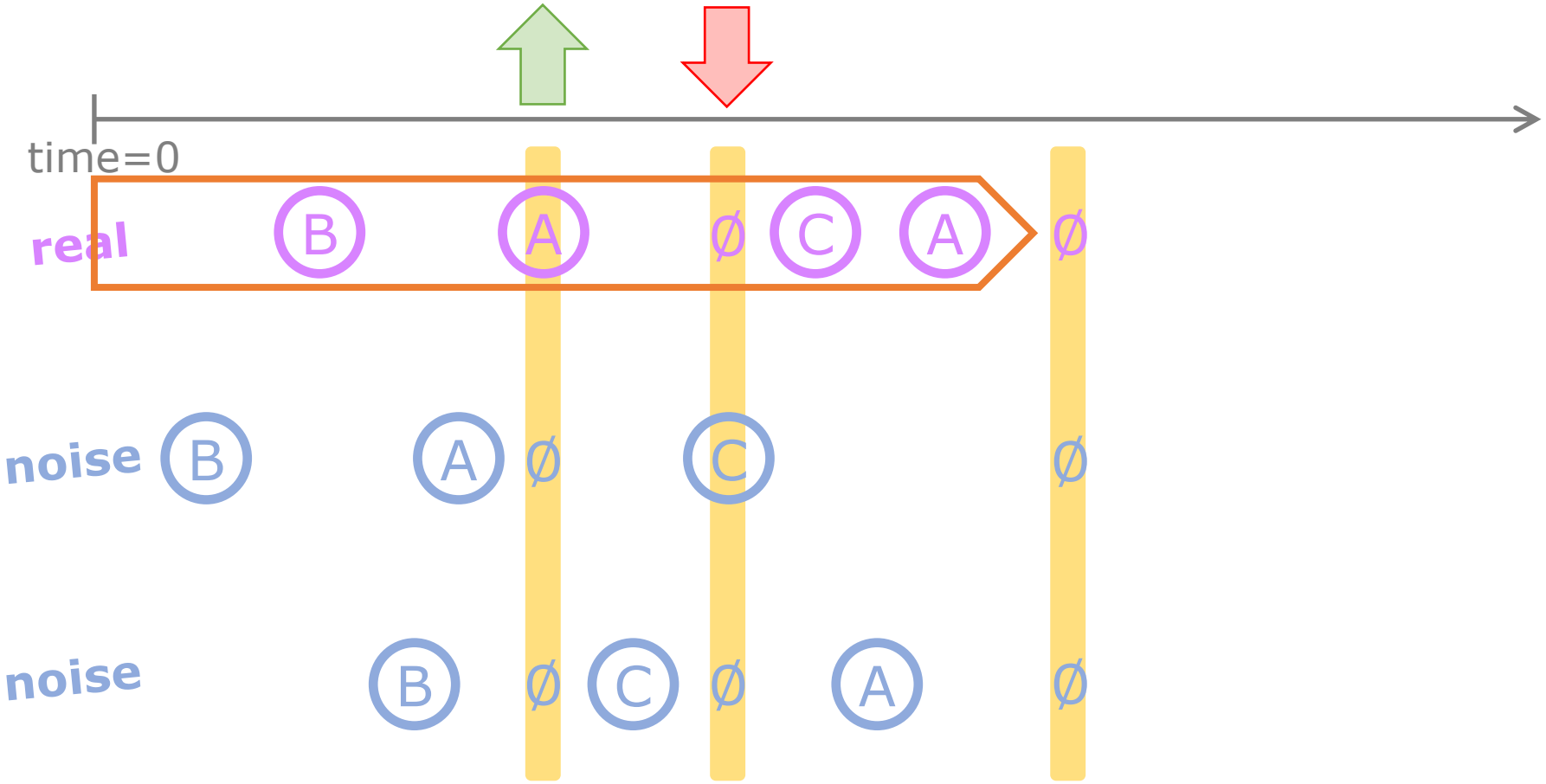
Which Is Real?

NCE: Max log prob of *correct discrimination*



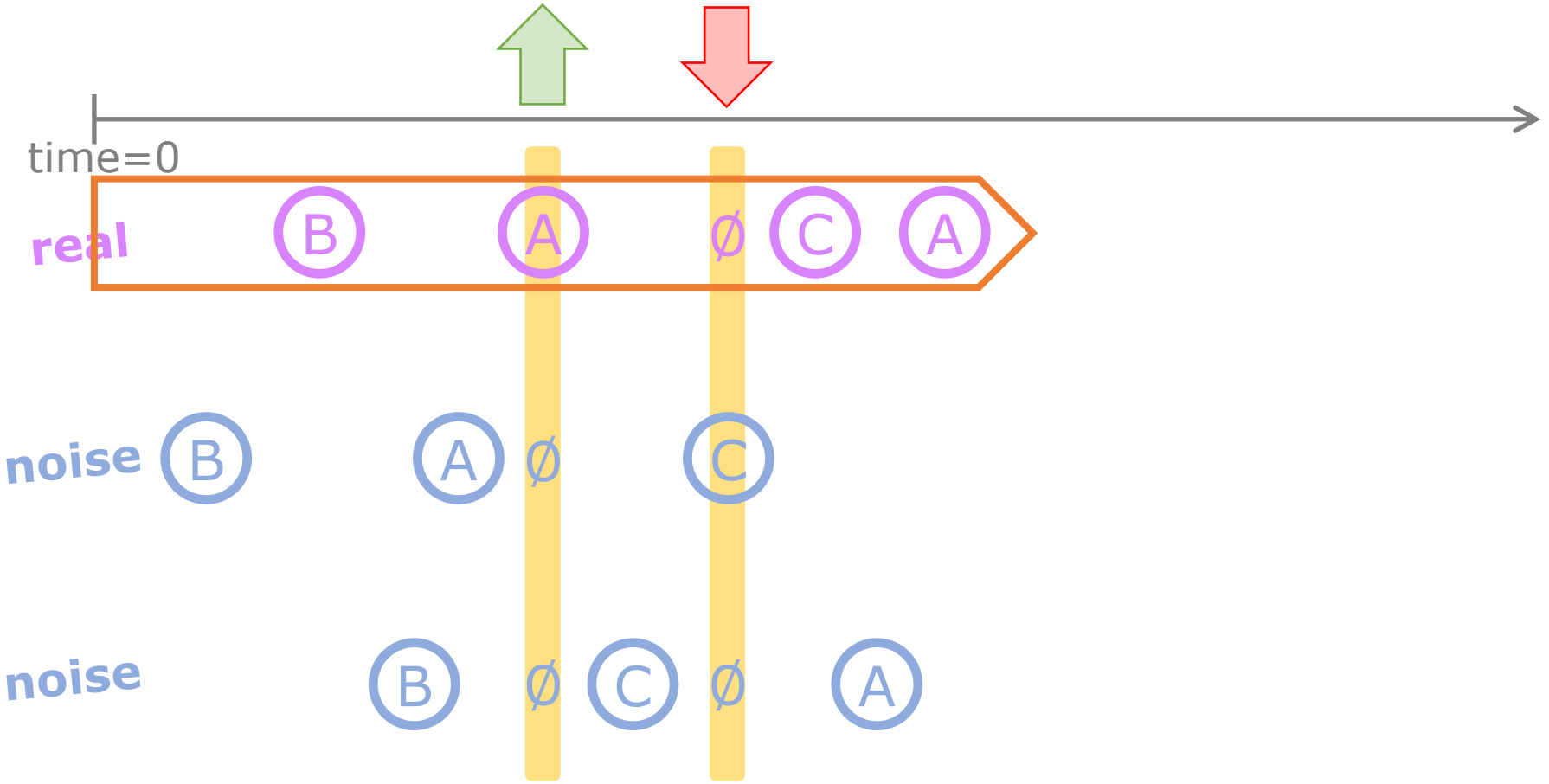
Which Is Real?

NCE: Max log prob of *correct discrimination*



Which Is Real?

NCE: Max log prob of *correct discrimination*



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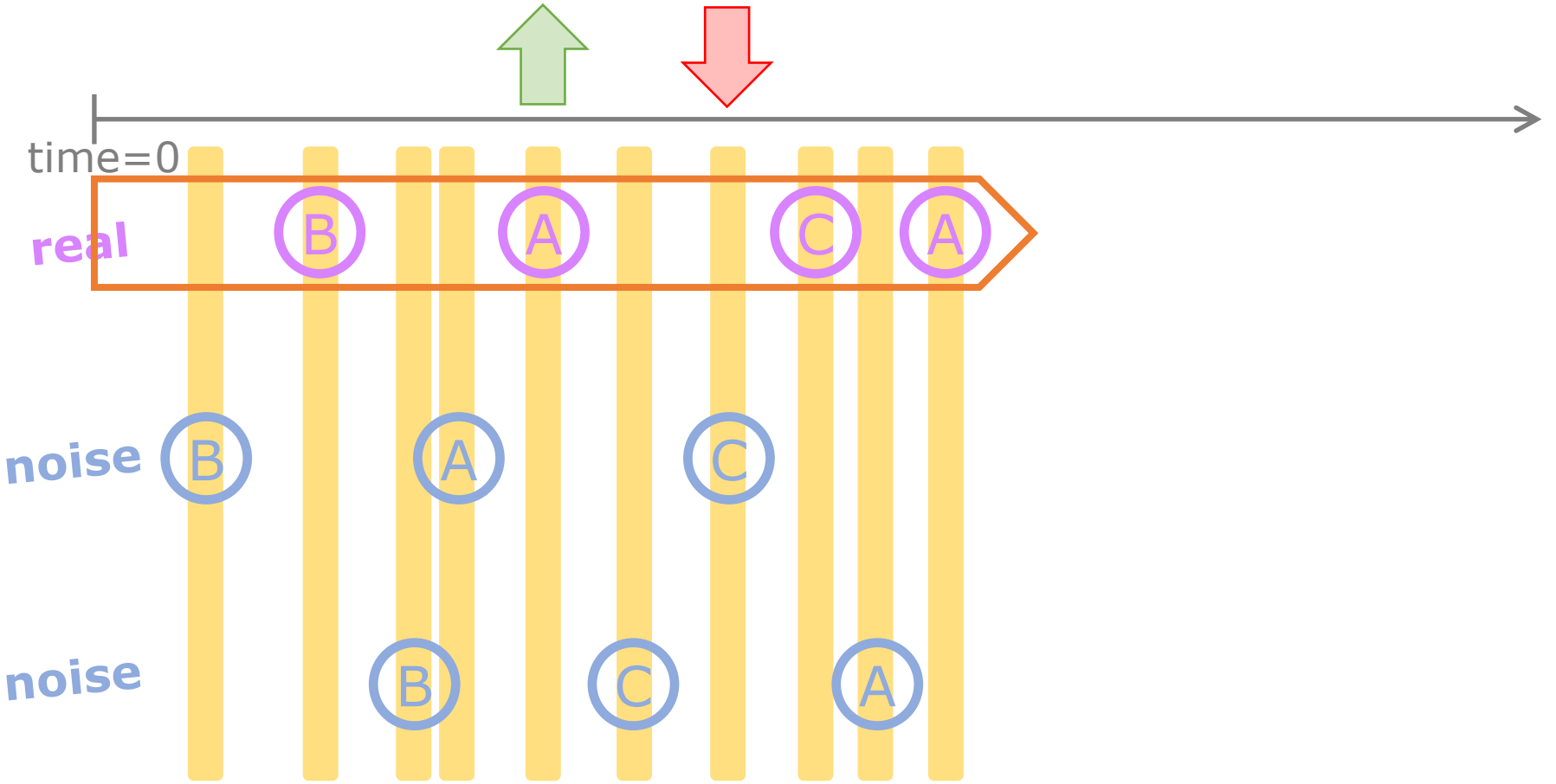
NCE: Max log prob of *correct discrimination*



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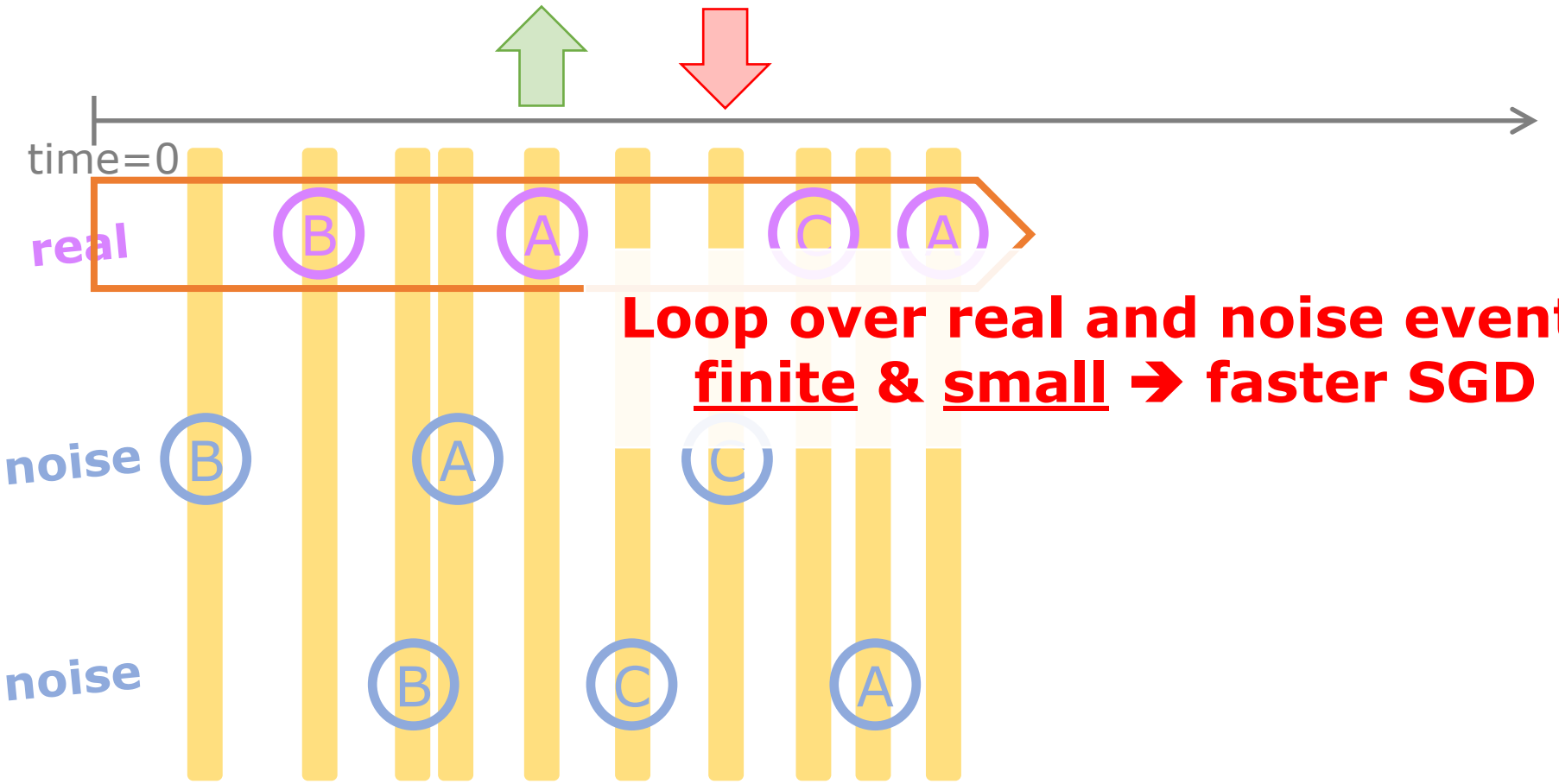
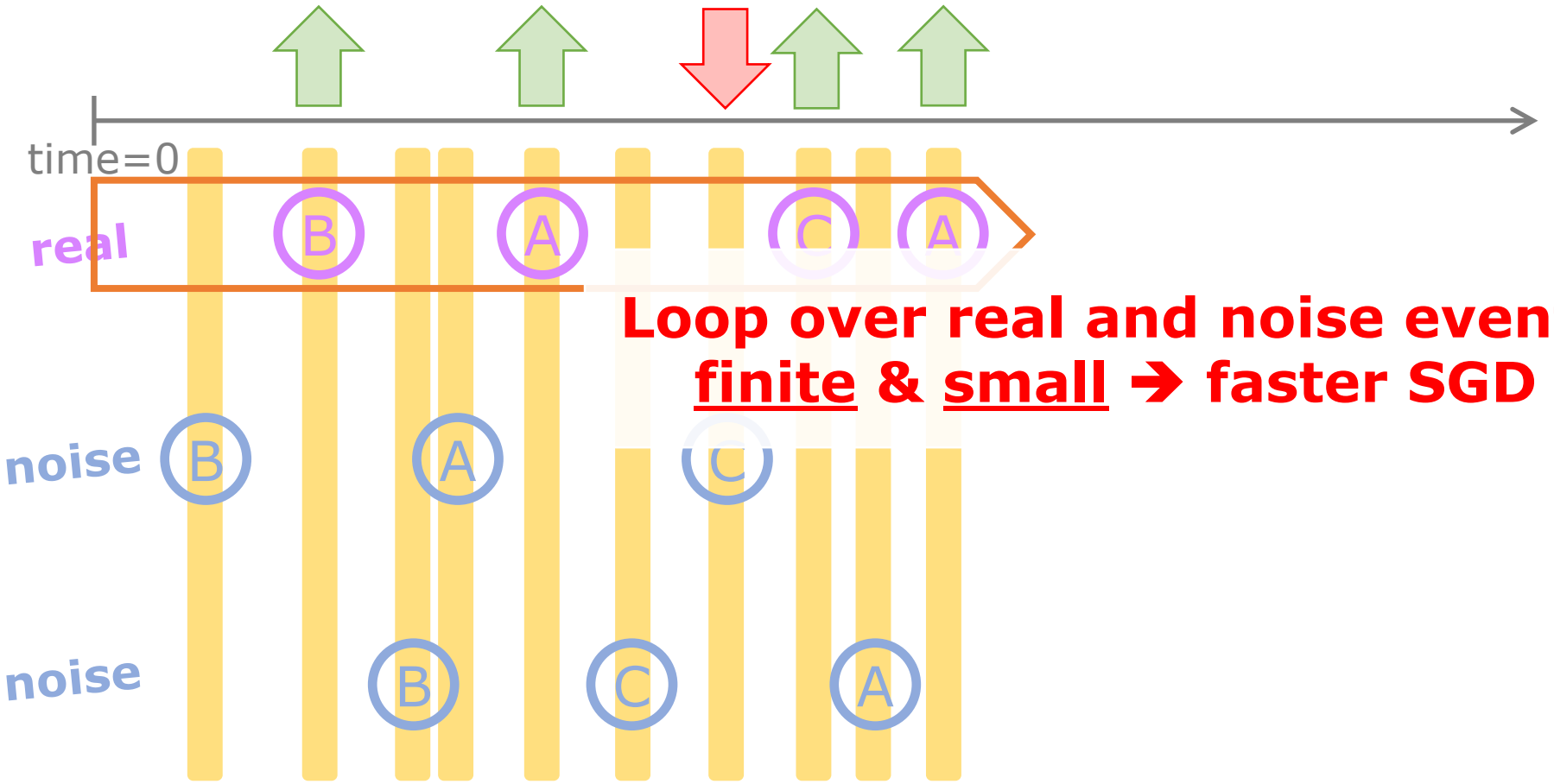
NCE: Max log prob of *correct discrimination*



Loop over real and noise events
finite & small ➔ faster SGD

Which Is Real?

NCE: Max log prob of *correct discrimination*



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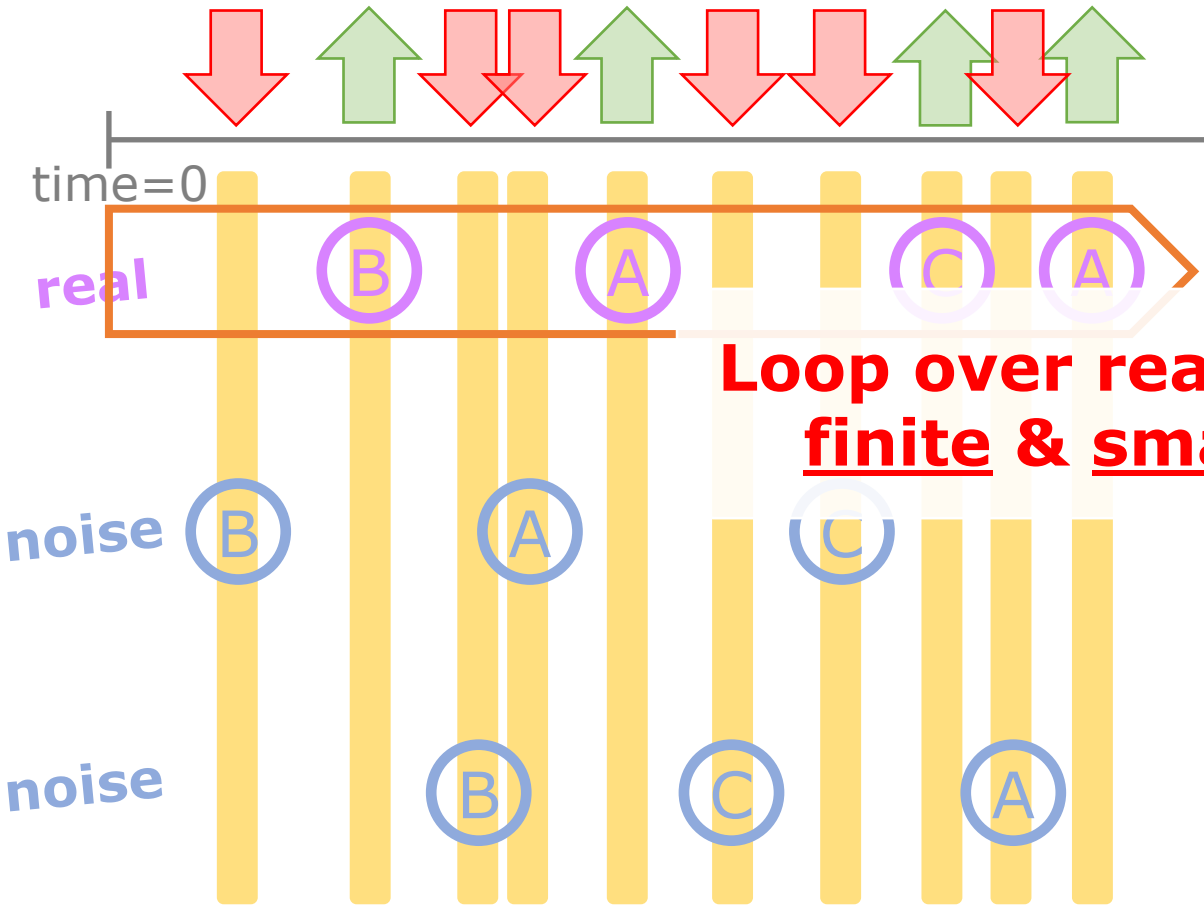

Loop over real and noise events
finite & small ➔ faster SGD

Which Is Real?

Let’s write some code

http://bburl/tpp-lab-p3

NCE vs MLE: consistent & efficient & faster

NCE vs MLE: consistent & efficient & faster

MLE NCE

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?		

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?		

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …		

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise
fewer func evals?		

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise
fewer func evals?		yes

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise
fewer func evals?		yes
lower runtime?		

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http://bburl/tpp-lab-p3

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise
fewer func evals?		yes
lower runtime?		yes

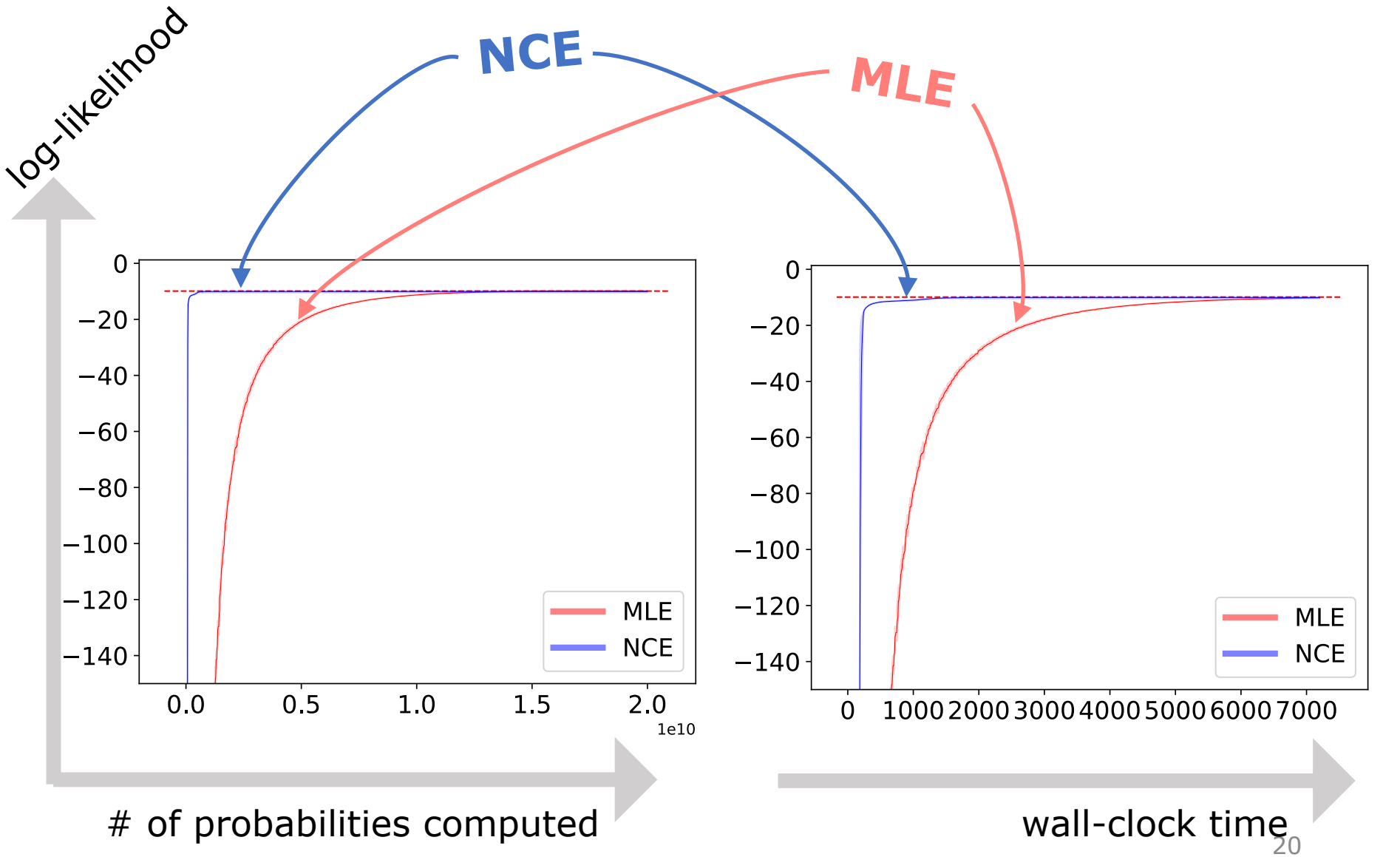
NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise
fewer func evals?		yes
lower runtime?		yes
better learning curve?		

NCE vs MLE: consistent & efficient & faster

	MLE	NCE
consistent?	yes	yes
sample-efficient?	yes	(yes)
sampling for …	integration	noise
fewer func evals?		yes
lower runtime?		yes
better learning curve?		yes

NCE vs MLE: what it typically looks like



NCE: More in Mei et al. NeurIPS 2020

NCE: More in Mei et al. NeurIPS 2020

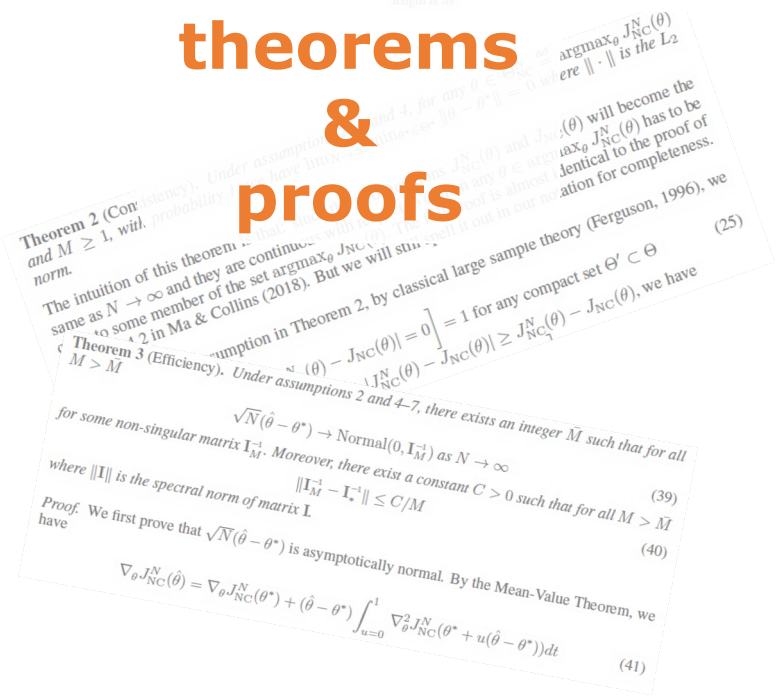
Theorem 1 (Optimality). *Under assumptions 1 and 2, $\theta \in \operatorname{argmax}_\theta J_{\mathrm{NC}}(\theta)$ if and only if $p_\theta = p^\star$.*

We first need to highlight the key insight that $H_\theta(k, t, x^0_{[0,t)})$ in equation (20) is the negative cross-entropy between the following two discrete distributions over $\{\varnothing, 1, \ldots, K\}$:

$$[\frac{\lambda^\star_k(t|x^0_{[0,t)})}{\Delta^\star_k(t|x^0_{[0,t)})}, \frac{\lambda^q_k(t|x^0_{[0,t)})}{\Delta^\star_k(t|x^0_{[0,t)})}, \ldots, \frac{\lambda^q_k(t|x^0_{[0,t)})}{\Delta^\star_k(t|x^0_{[0,t)})}] \tag{21a}$$

$$[\frac{\lambda_k(t|x^0_{[0,t)})}{\Delta_k(t|x^0_{[0,t)})}, \underbrace{\frac{\lambda^q_k(t|x^0_{[0,t)})}{\Delta_k(t|x^0_{[0,t)})}, \ldots, \frac{\lambda^q_k(t|x^0_{[0,t)})}{\Delta_k(t|x^0_{[0,t)})}}] \tag{21b}$$

theorems & proofs



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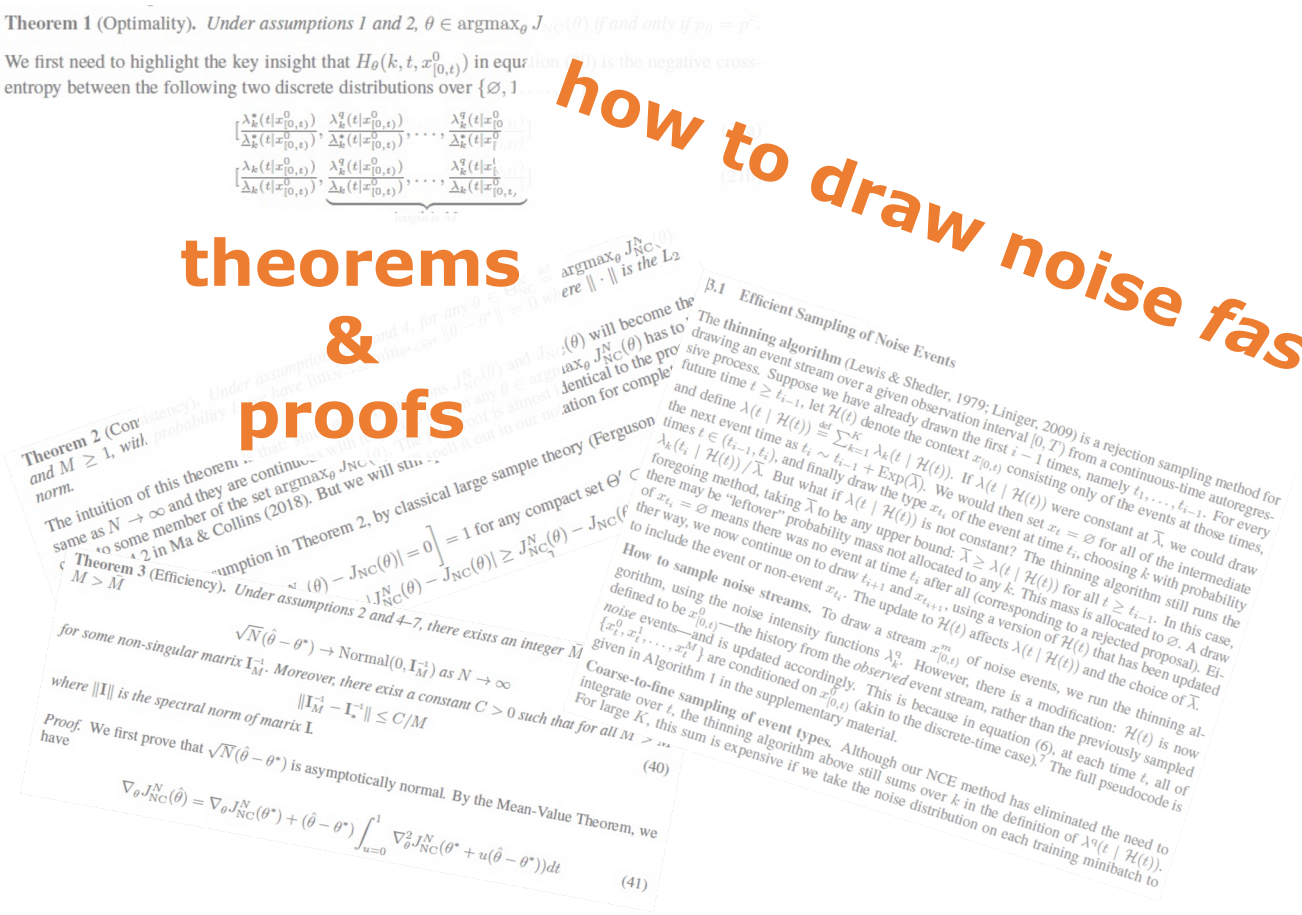

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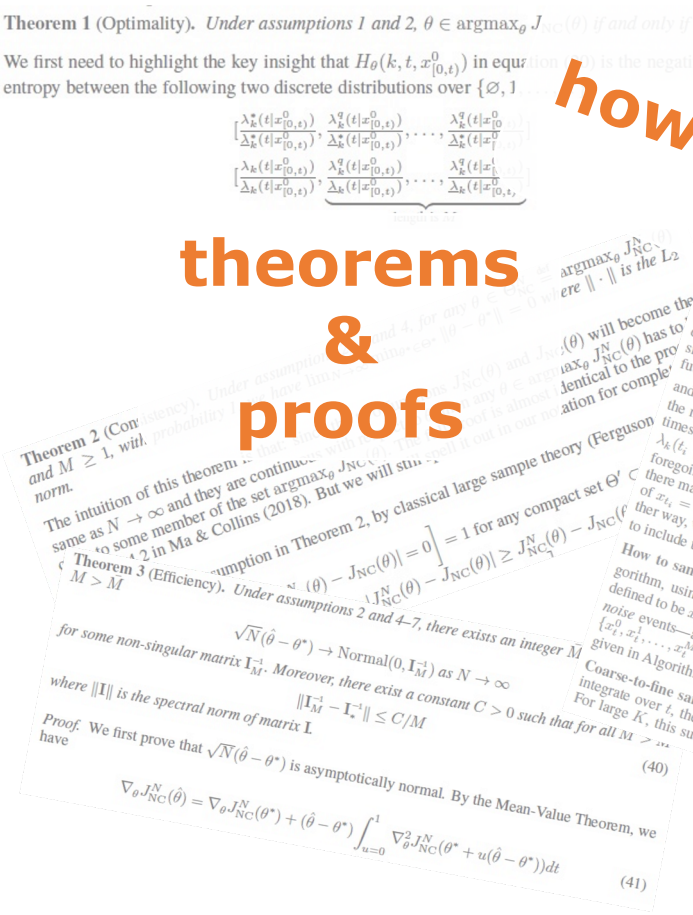
http://bburl/tpp-lab-p3

NCE: More in Mei et al. NeurIPS 2020

how to draw noise

theorems & proofs

more results & analysis



More Topics

More Topics

Domain Knowledge

KownEvolve: Trivedi et al. ICML 2017

DyRep: Trivedi et al. ICLR 2019

More Topics

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KownEvolve: Trivedi et al. ICML 2017
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Policy-based: Upadhyay et al. NeurIPS 2018

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Granger Causality over event types?
CAUSE: attribution methods, Zhang et al. ICML 2020

More Topics

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Variational Autoencoder (VAE)
NHP as decoder: Boyd et al. NeurIPS 2020

Neural Datalog Through Time

Hongyuan Mei[1], Guanghui Qin[1], Minjie Xu[2], Jason Eisner[1]

[1]Johns Hopkins University

[2]Bloomberg

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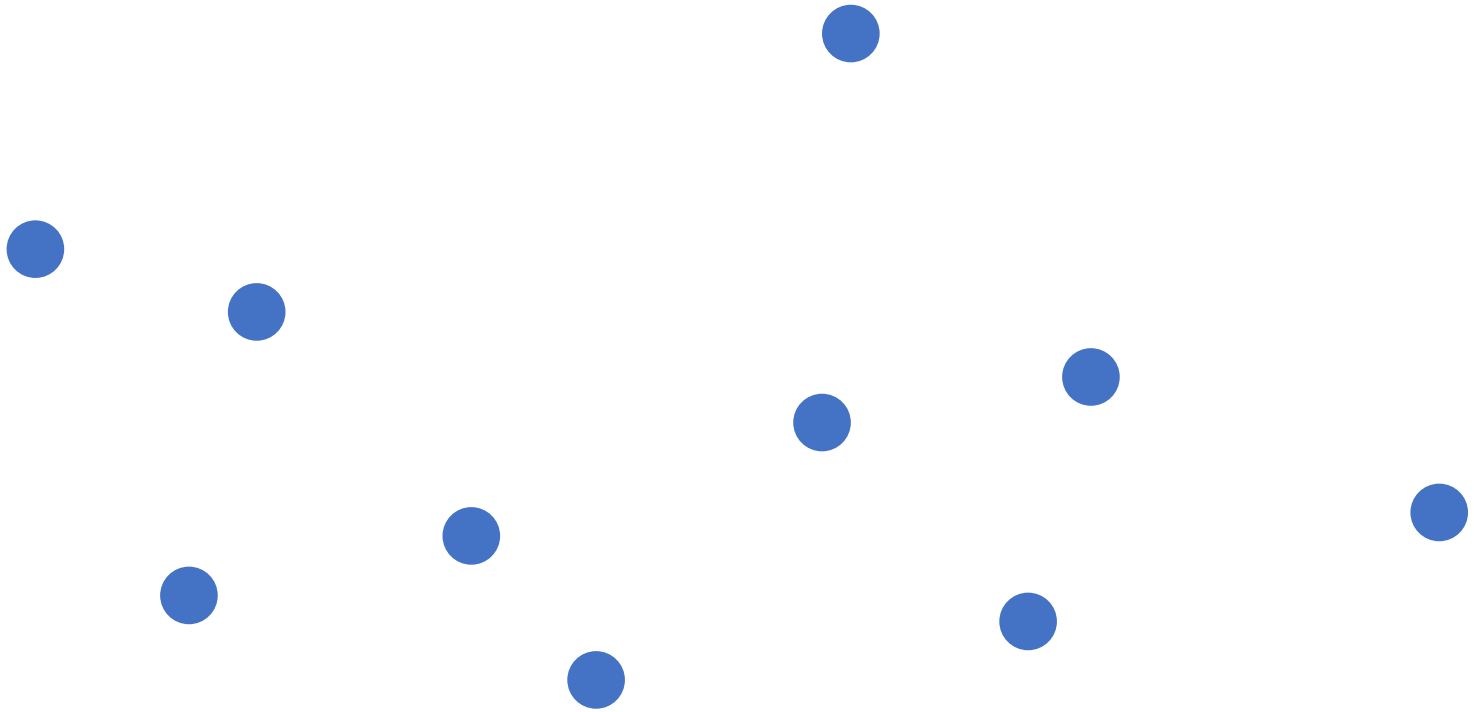
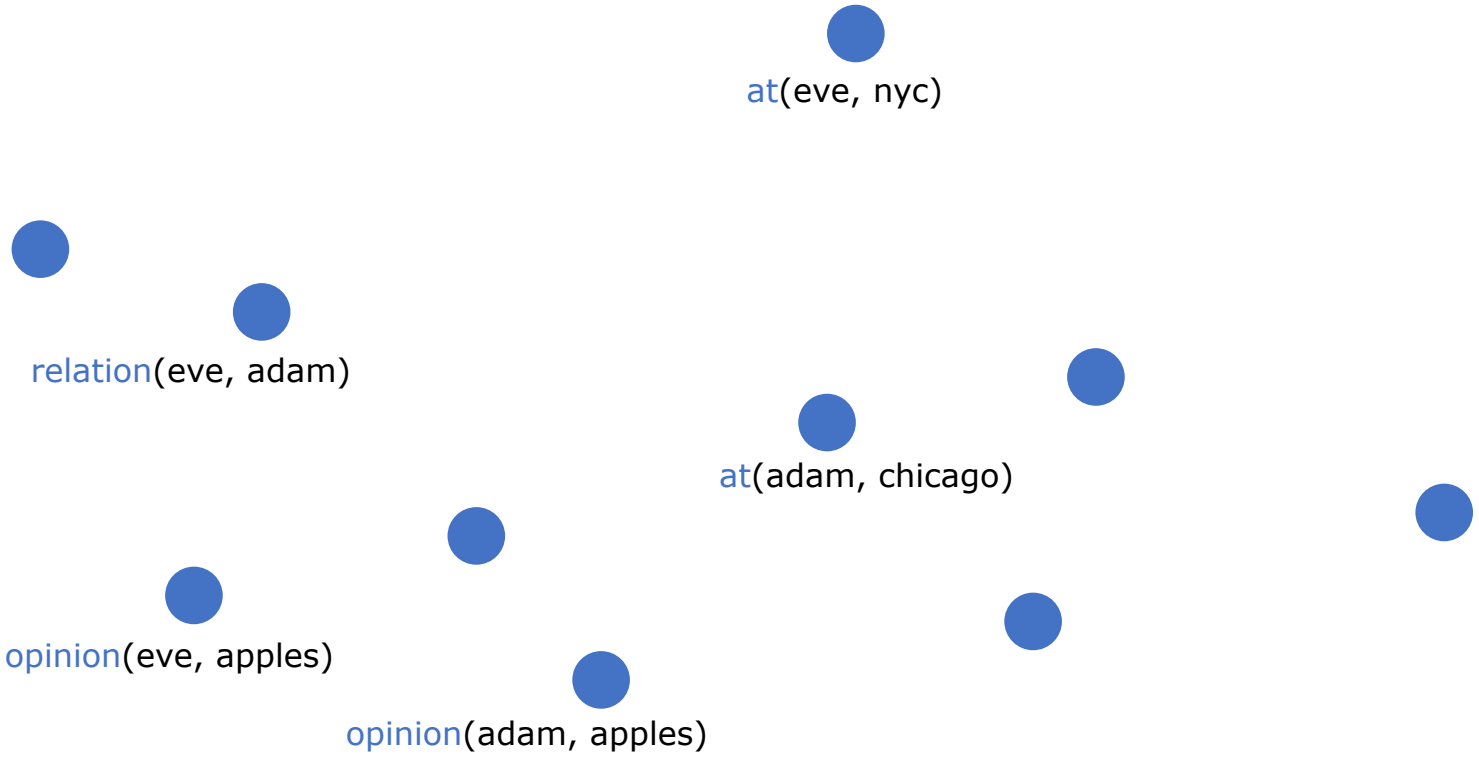
http://bburl/tpp-lab-p3

Model How a Database Changes Over Time



Model How a Database Changes Over Time

200,000 facts right now



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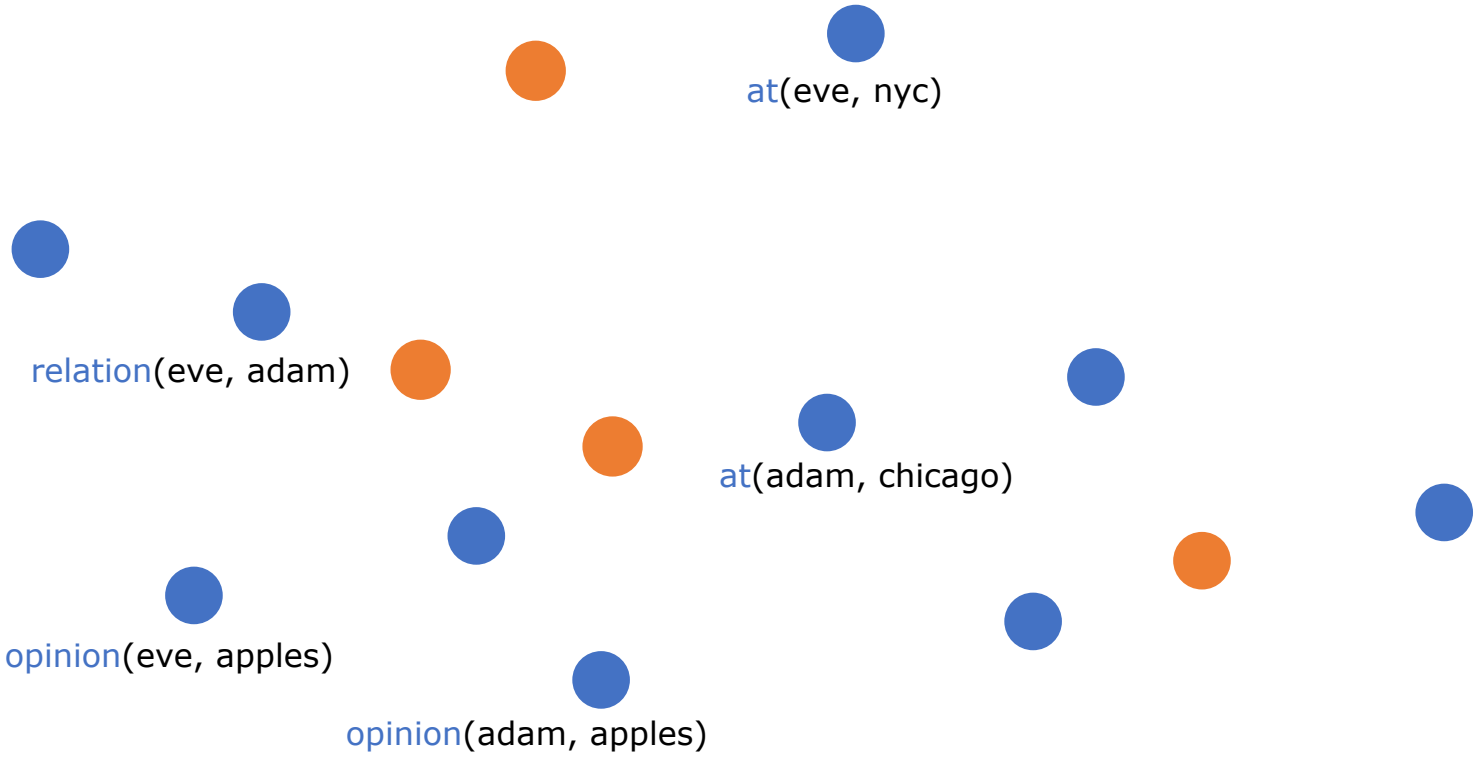
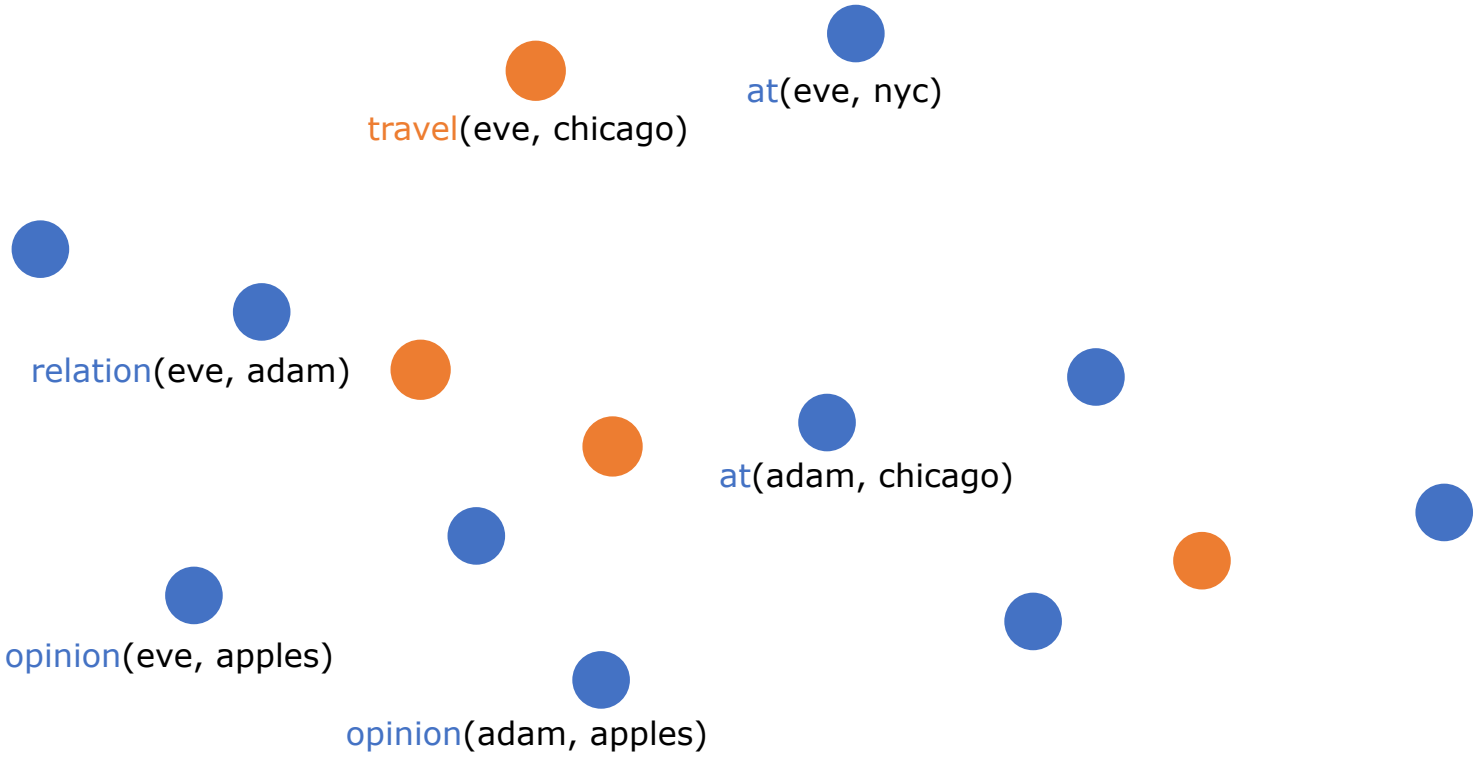
200,000 facts right now



Model How a Database Changes Over Time

200,000 facts right now

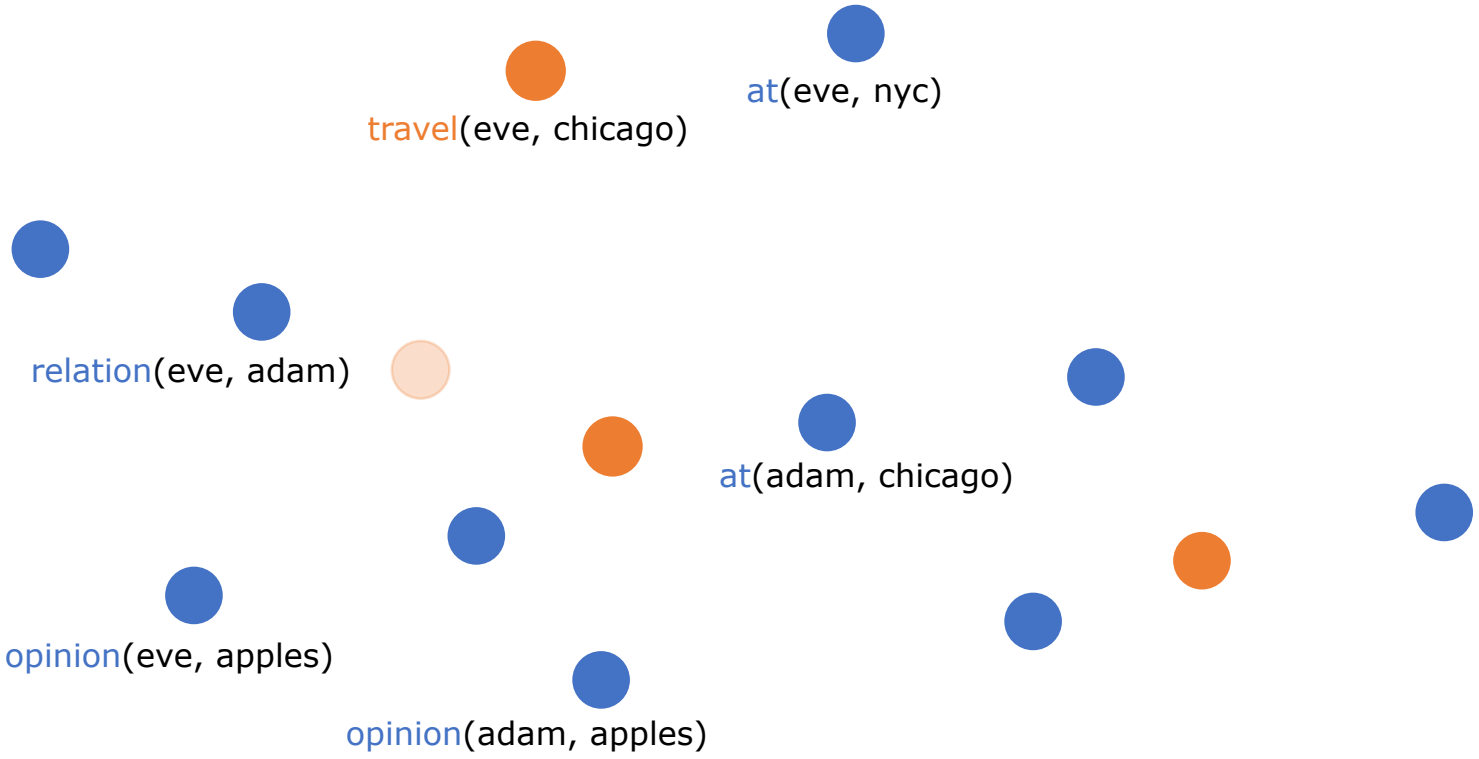
50,000 possible events right now



Model How a Database Changes Over Time

200,000 facts right now

50,000 possible events right now



Model How a Database Changes Over Time

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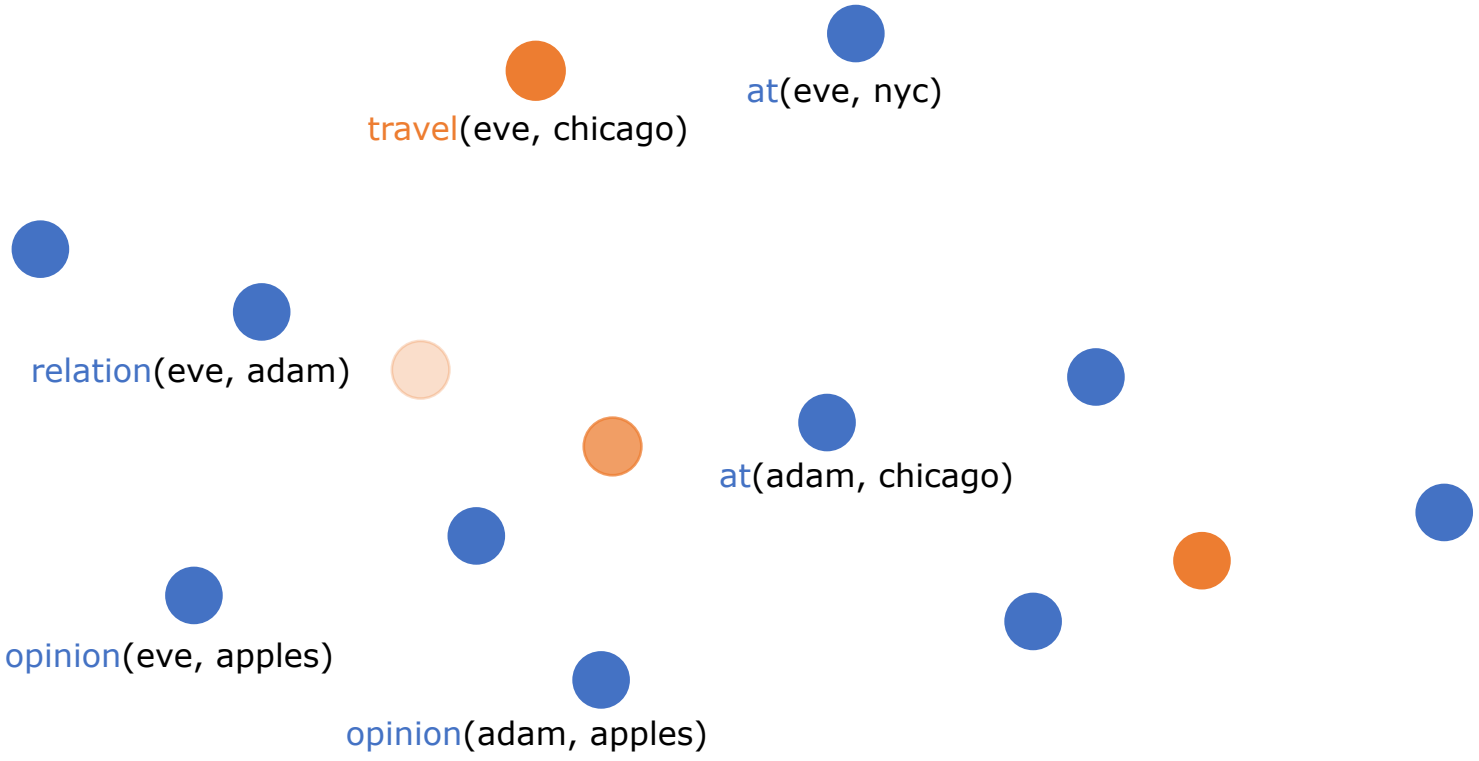
little language
to specify a generative model
of event sequences



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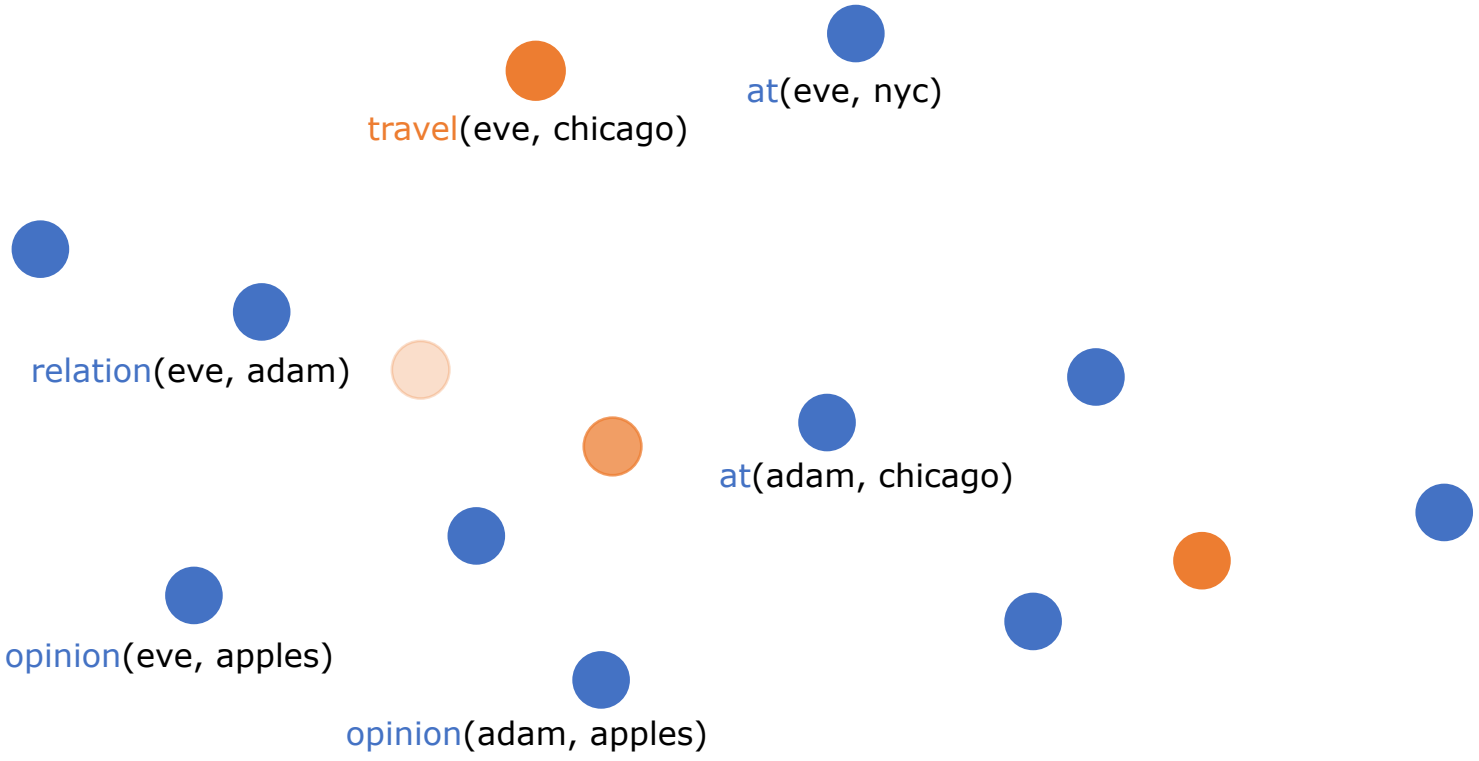

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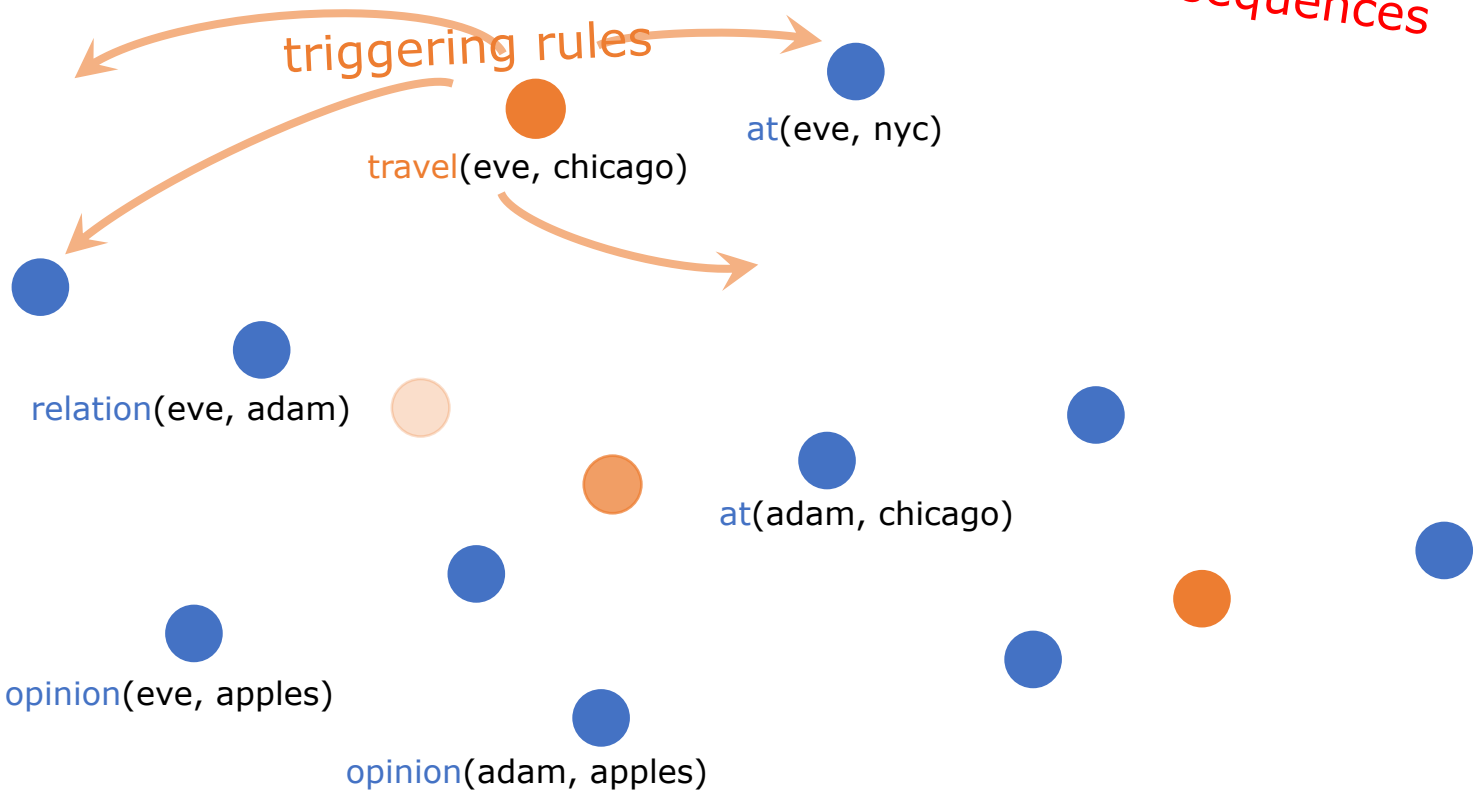

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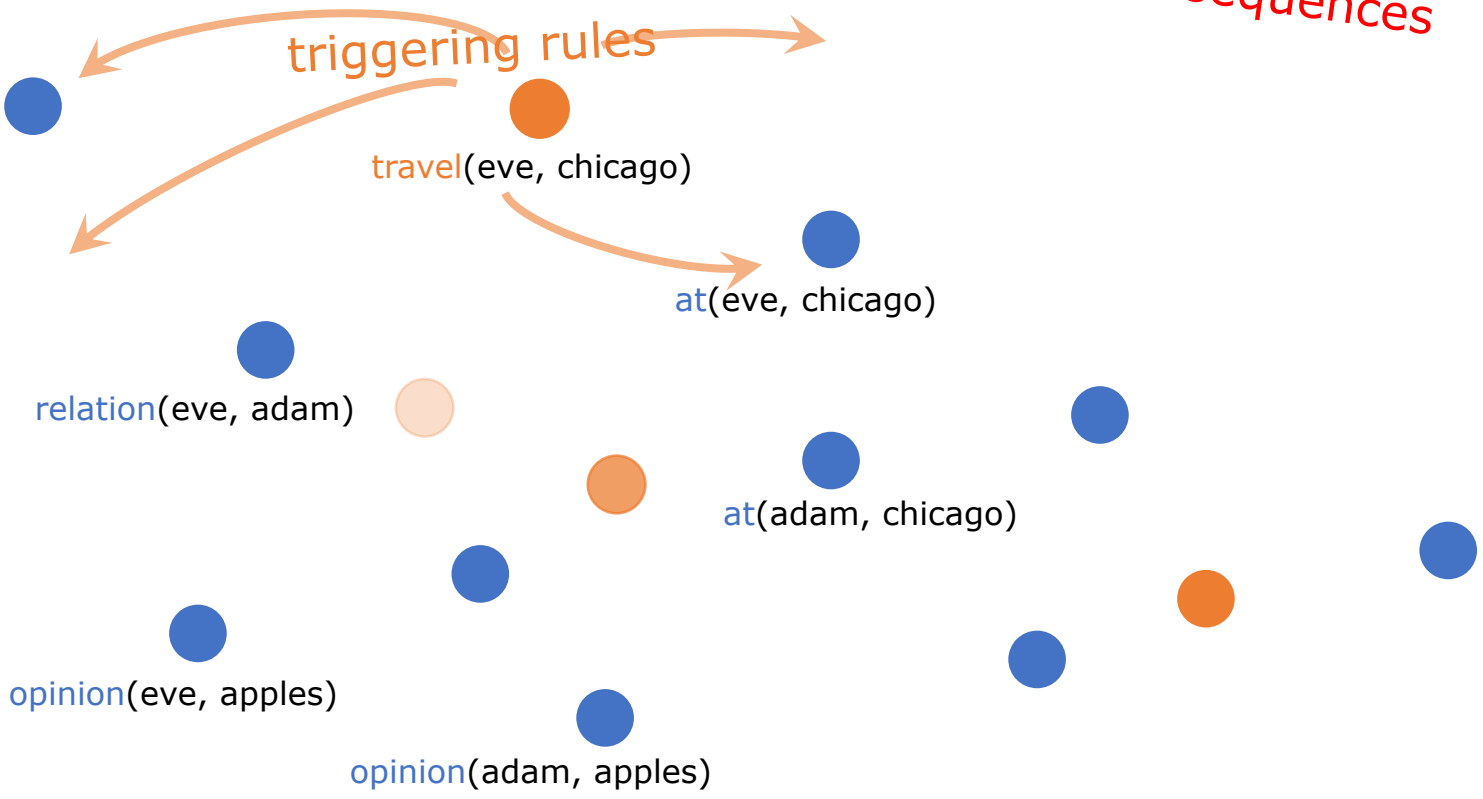

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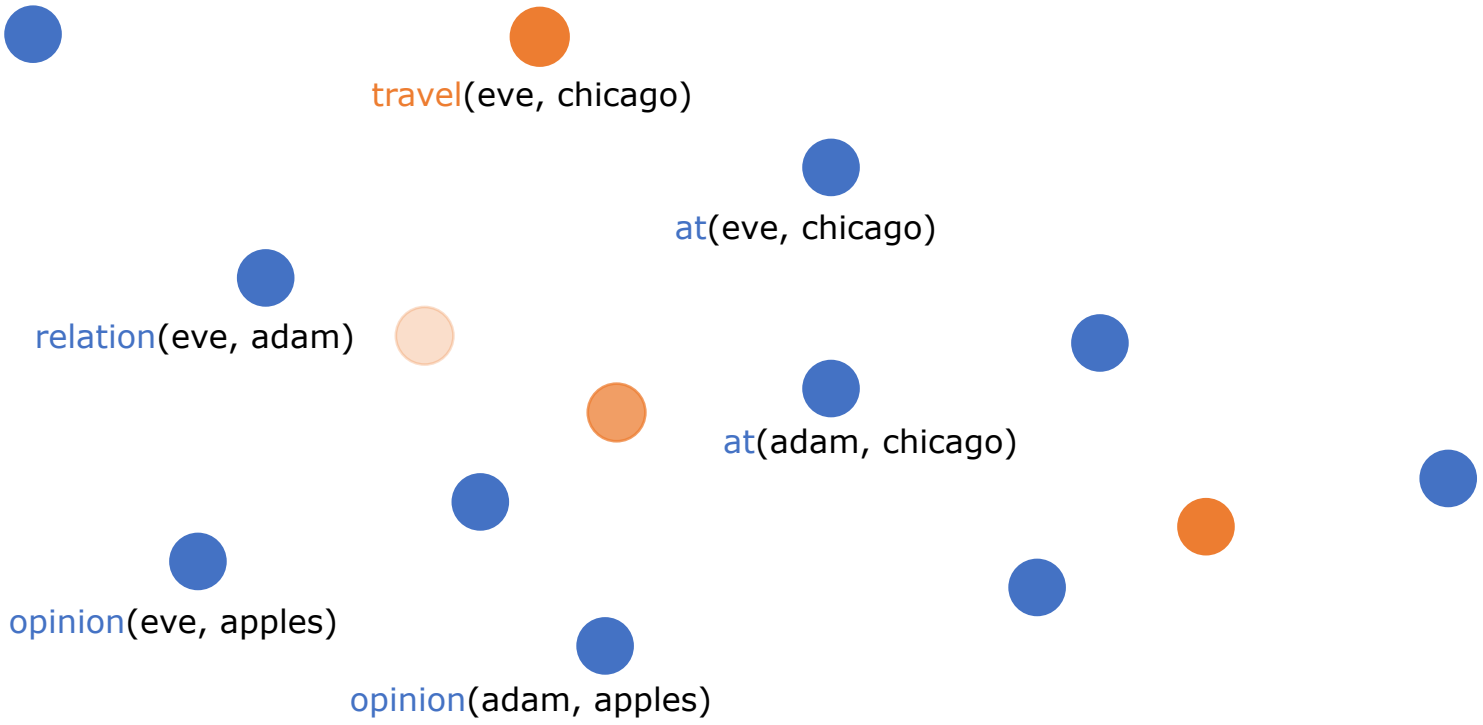

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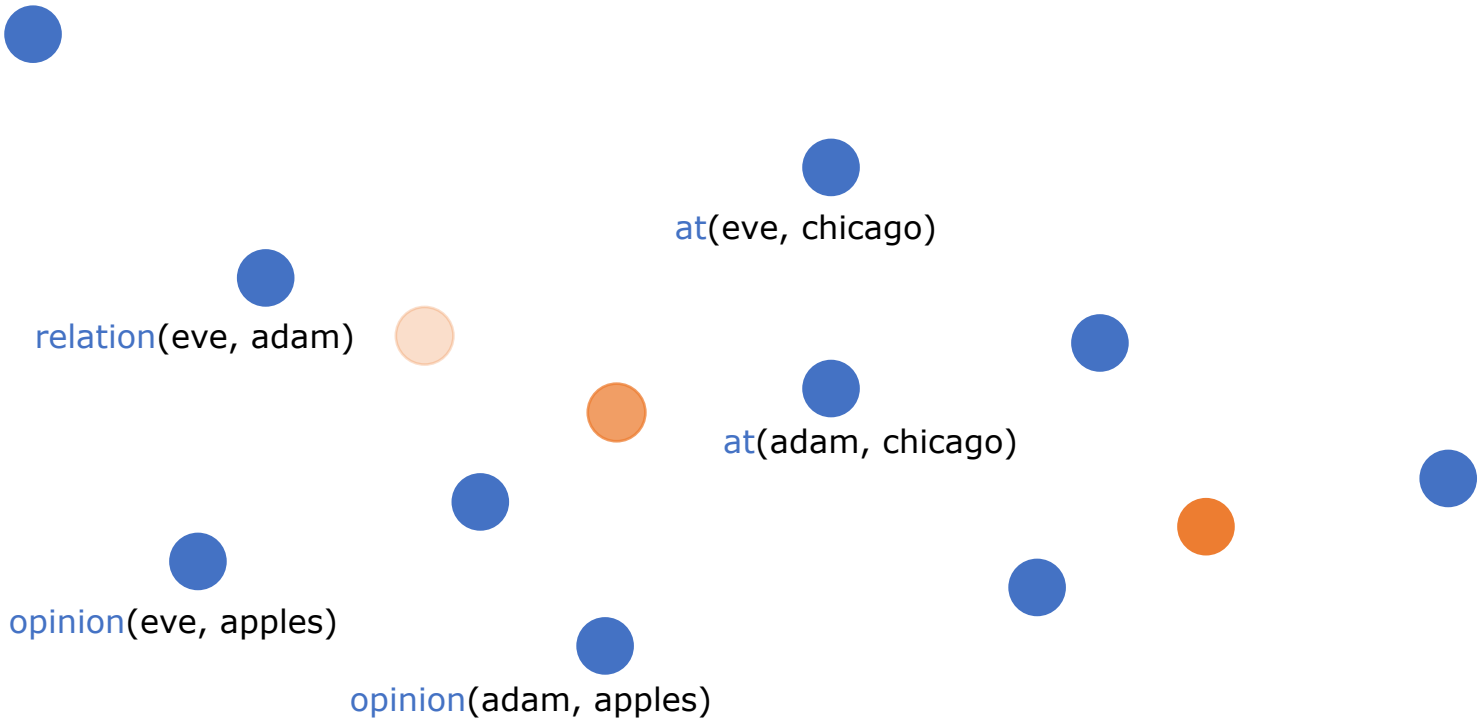

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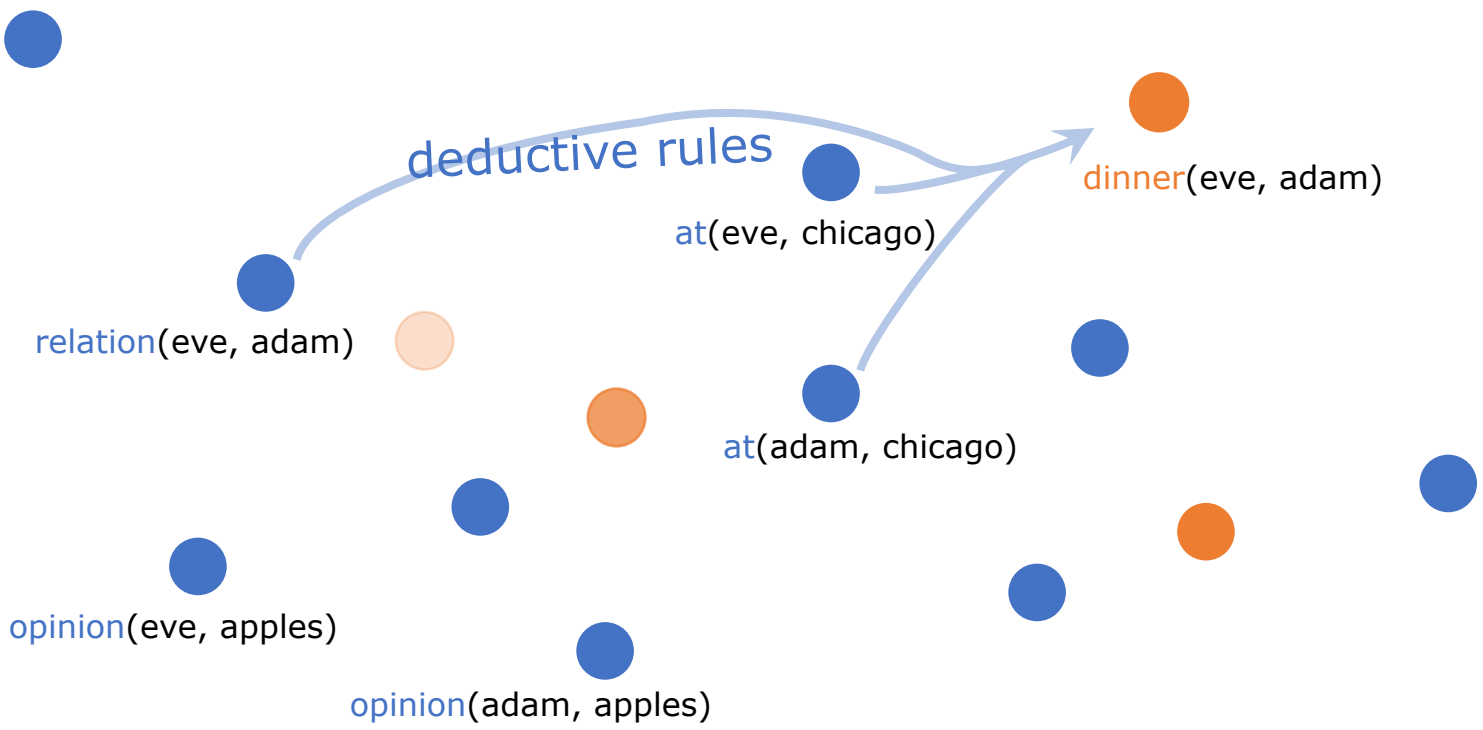

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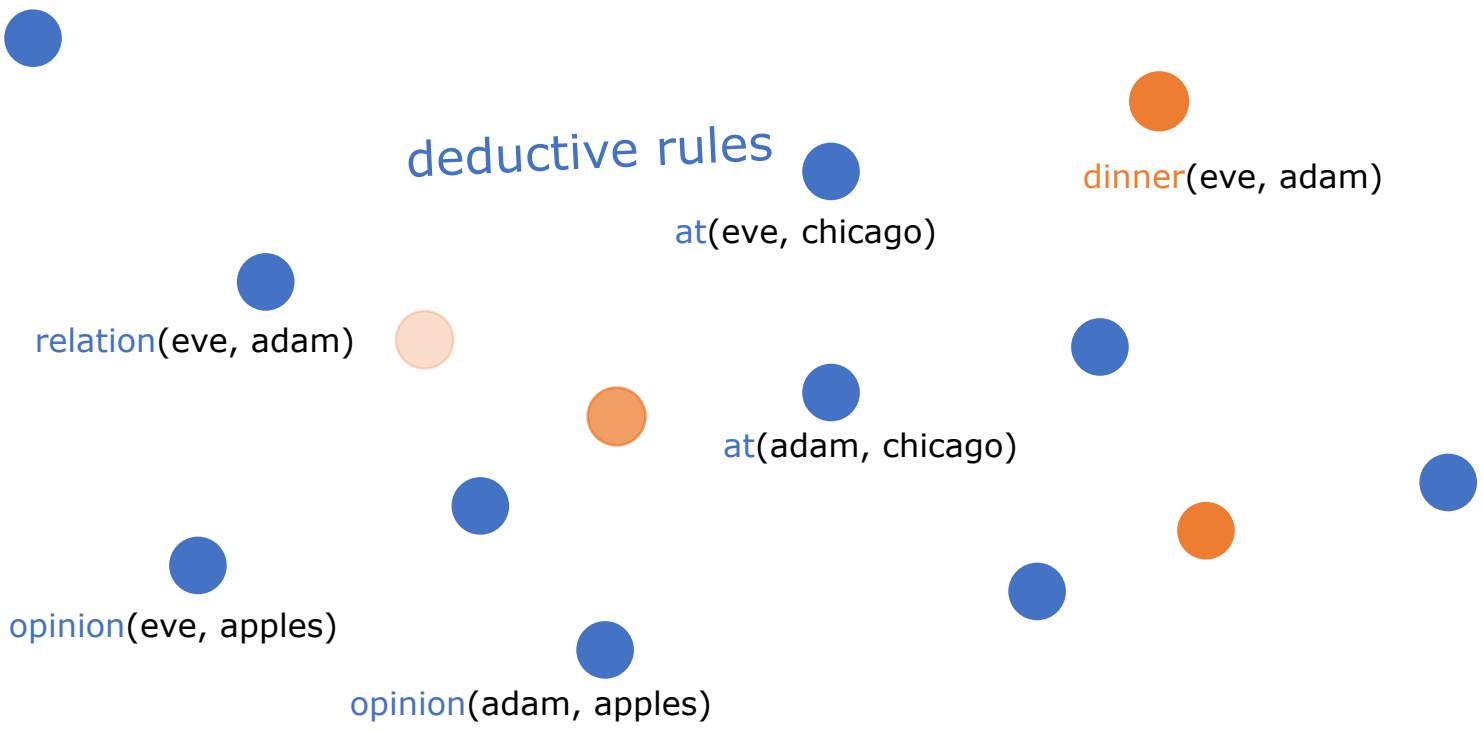

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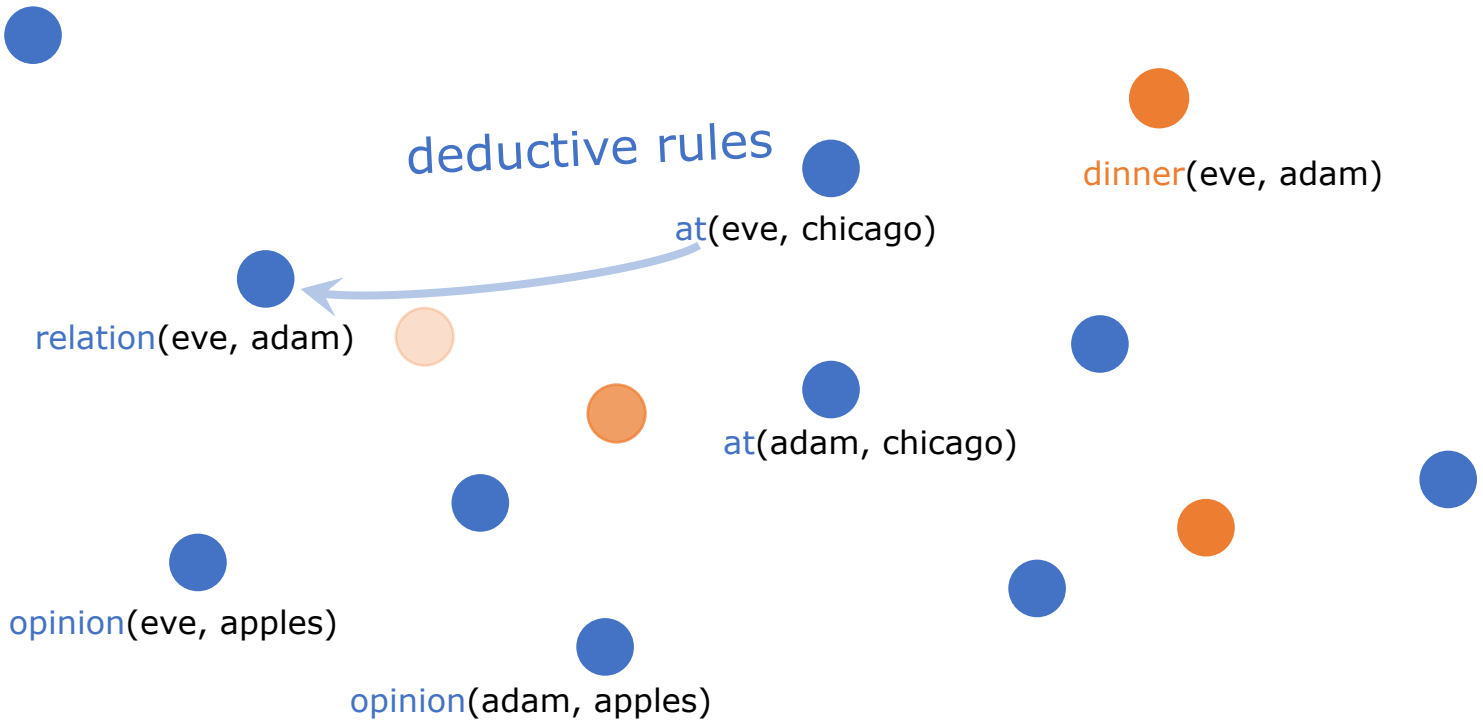

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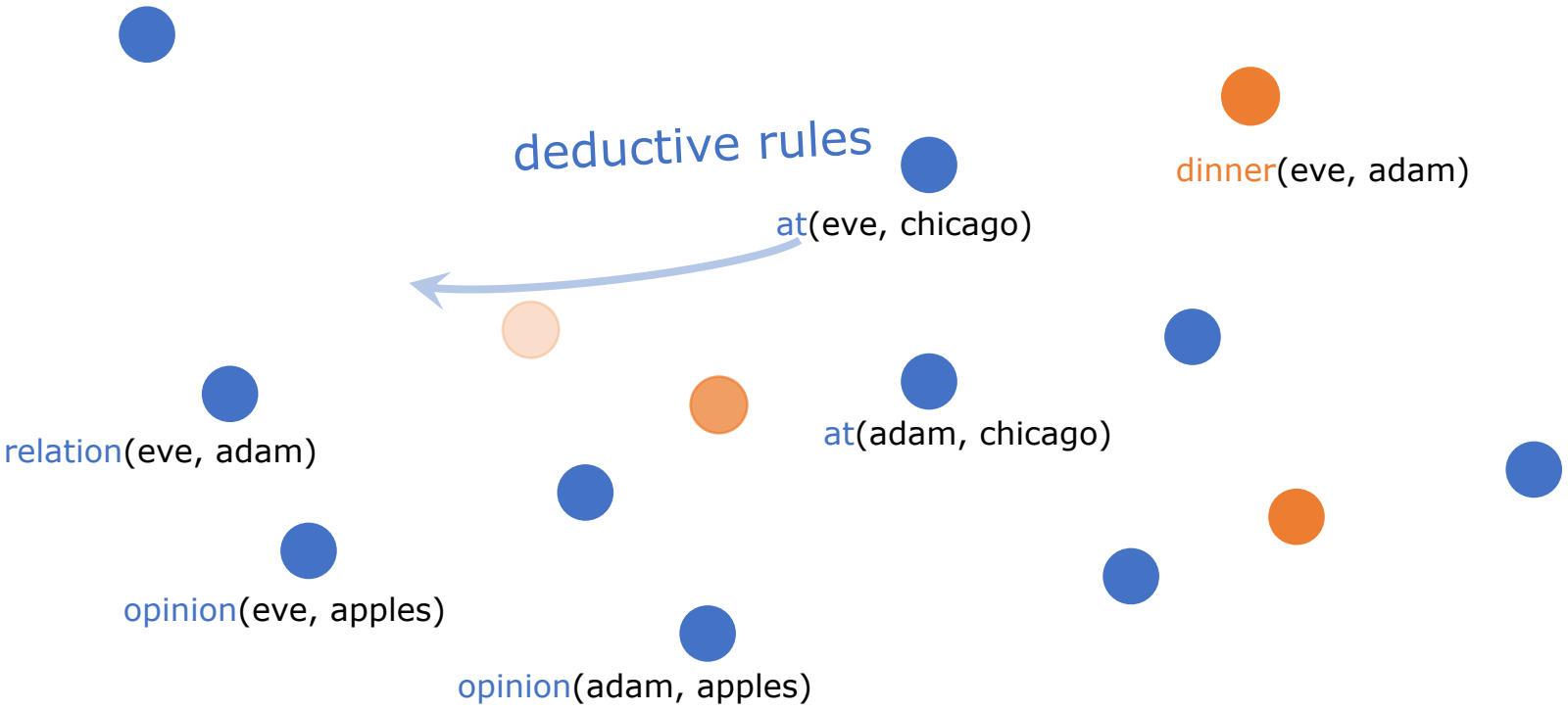

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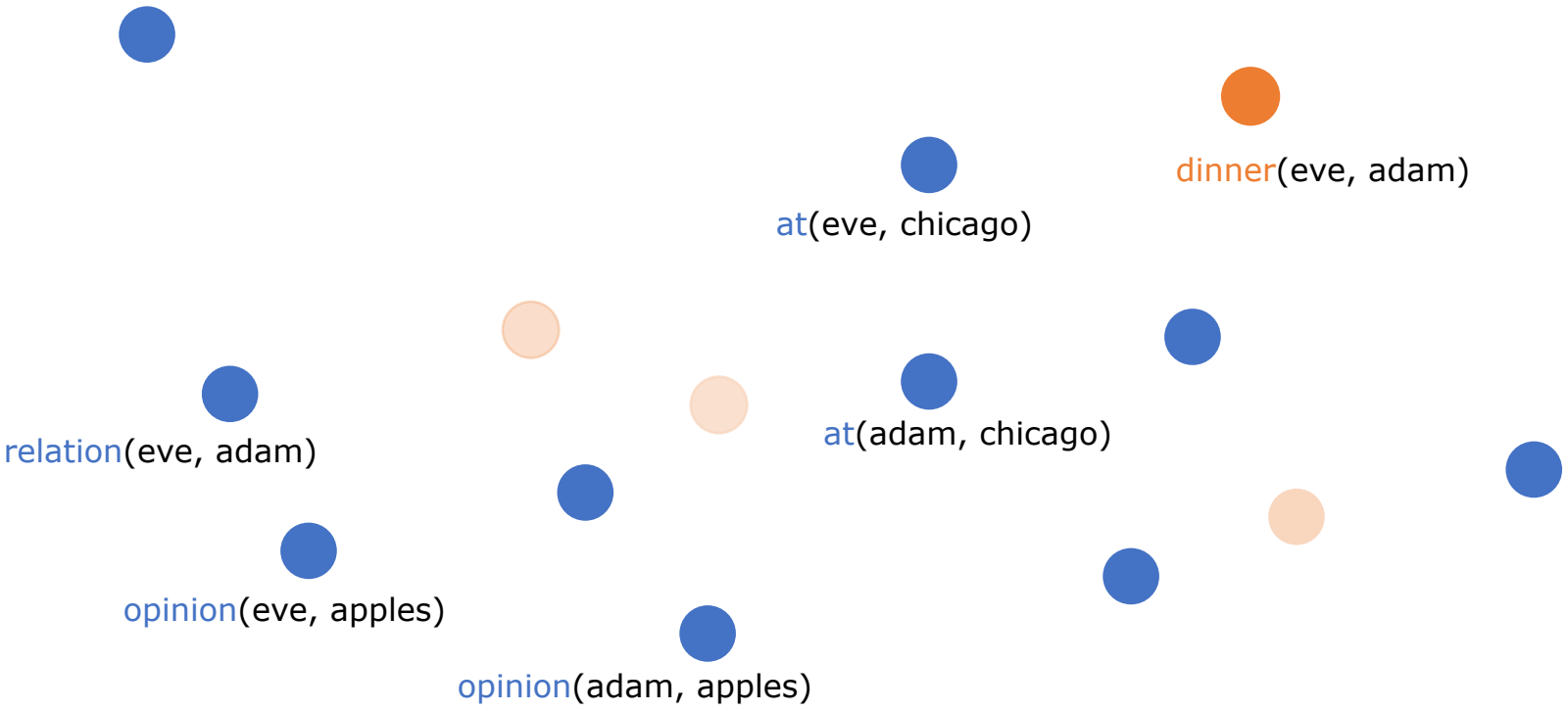
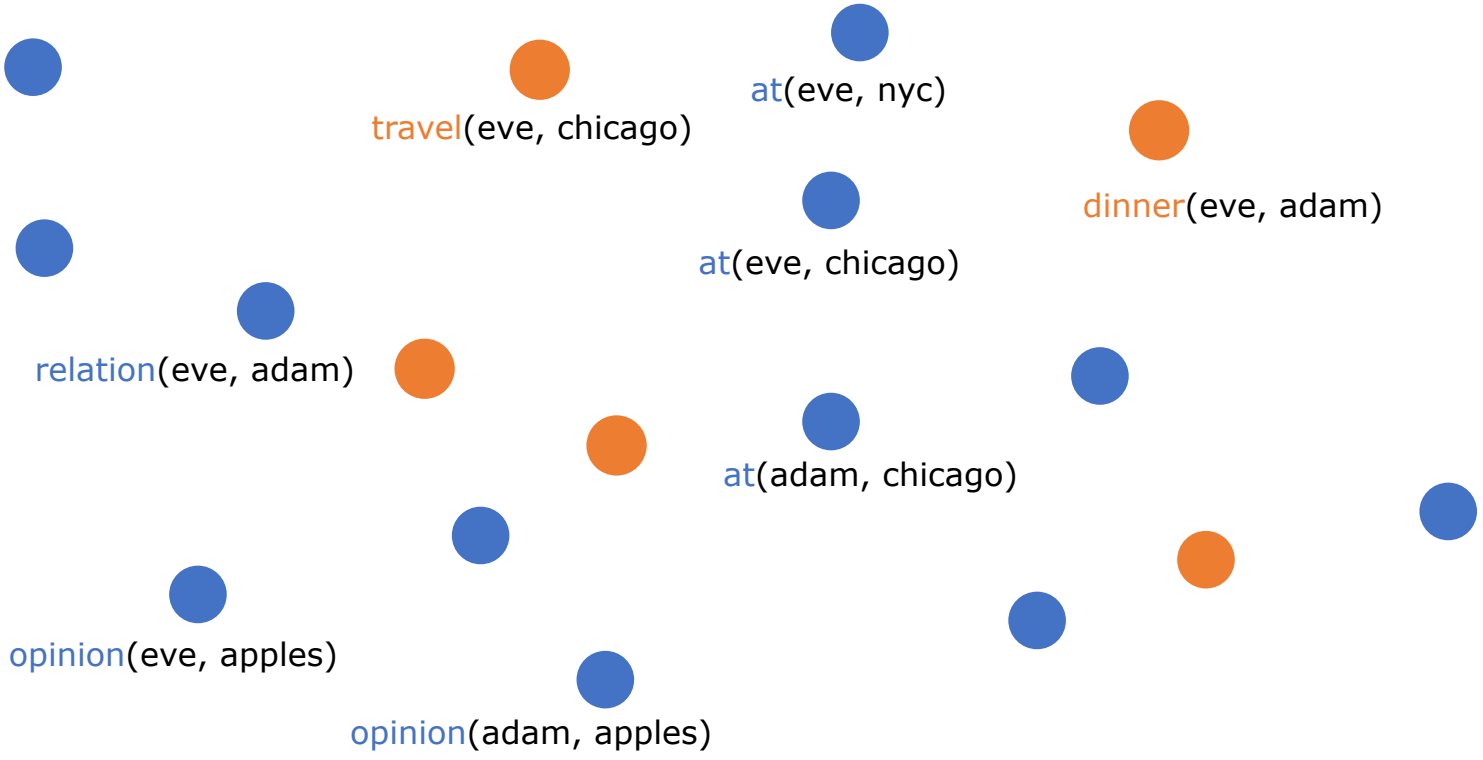
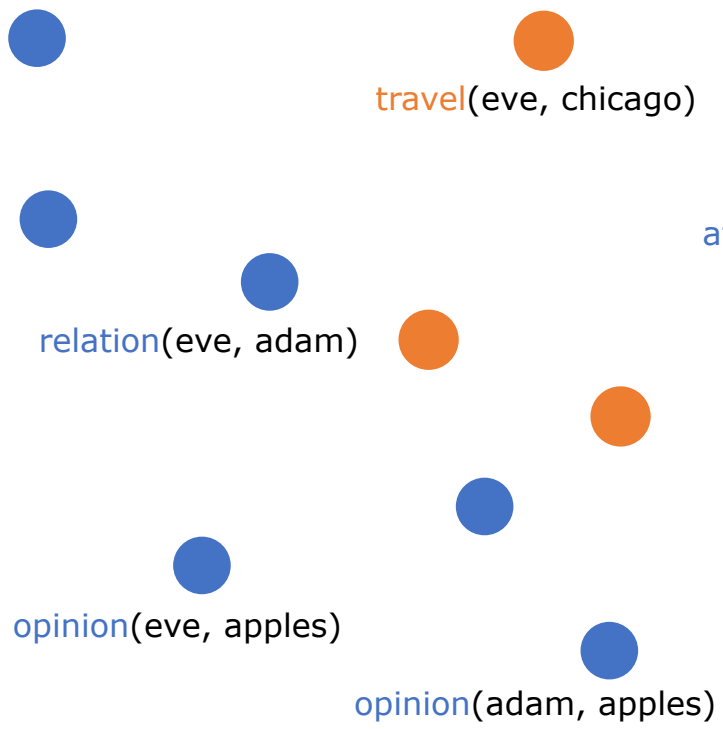


Deductive Rules, Triggering Rules



Deductive Rules, Triggering Rules




```
relation(X, Y)
    :- opinion(X, U), opinion(Y, U).

travel(X, P)
    :- relation(X, Y), at(Y, P).

!at(X, Q)
    ← travel(X, P), at(X, Q), P != Q.

at(X, P)
    ← travel(X, P).

dinner(X, Y)
    :- relation(X, Y), at(X, P), at(Y, P).

relation(X, Y)
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```
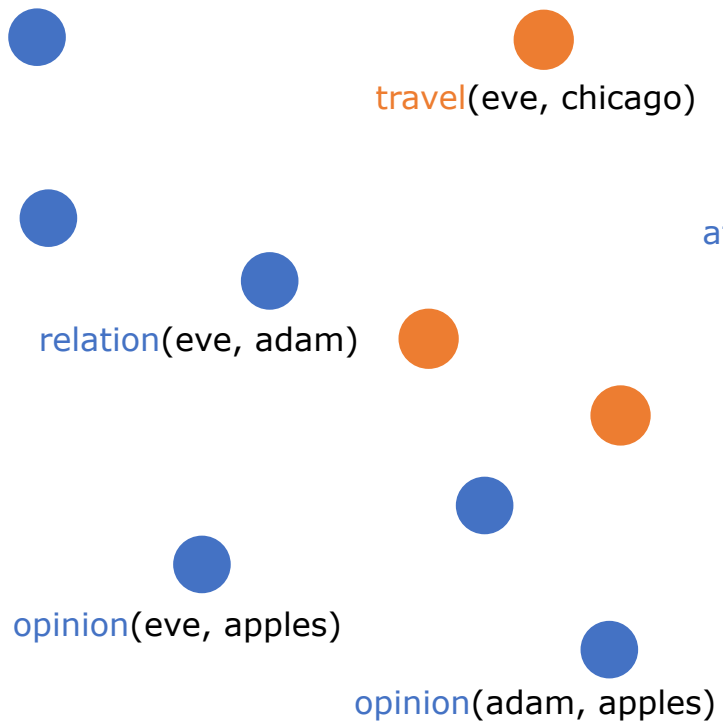

Deductive Rules, Triggering Rules

logic!

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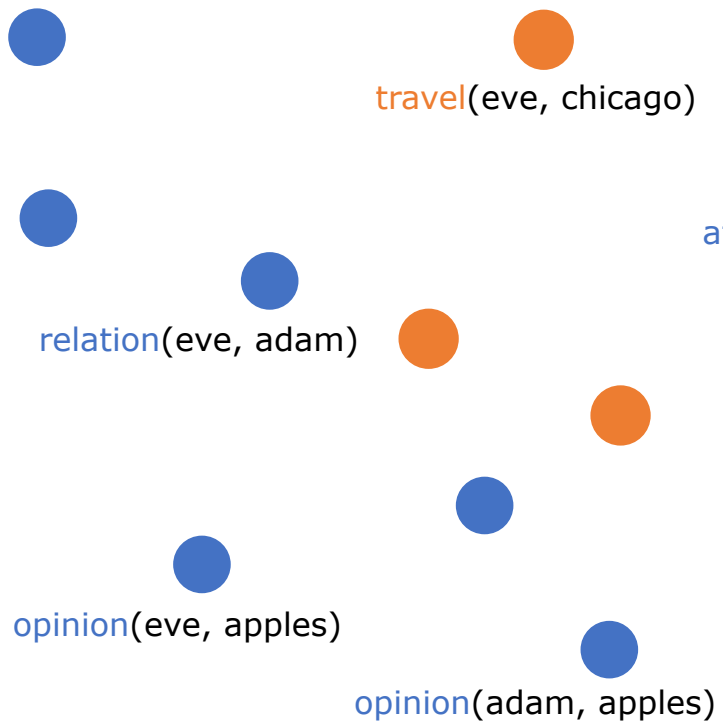
Deductive Rules, Triggering Rules

which facts are in the database



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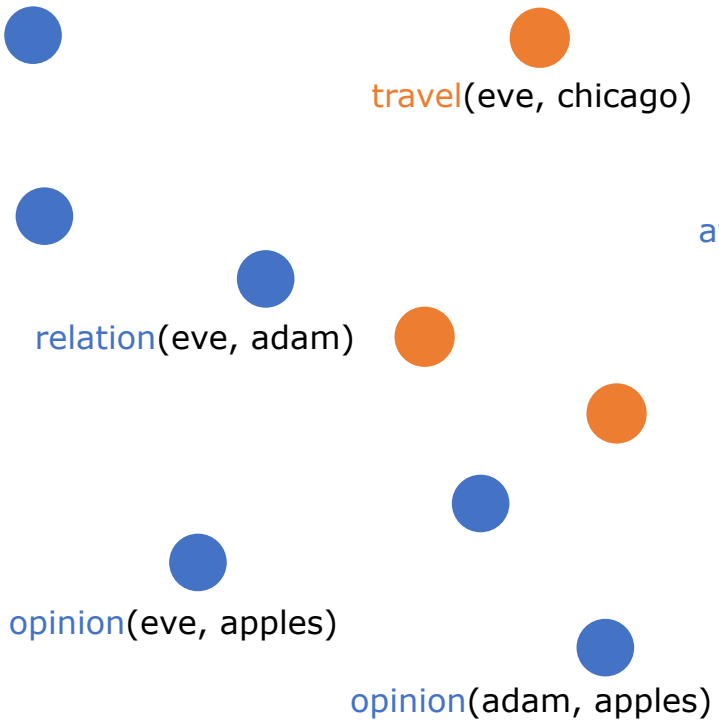
Deductive Rules, Triggering Rules

which facts are in the database
define a trainable neural architecture

logic!

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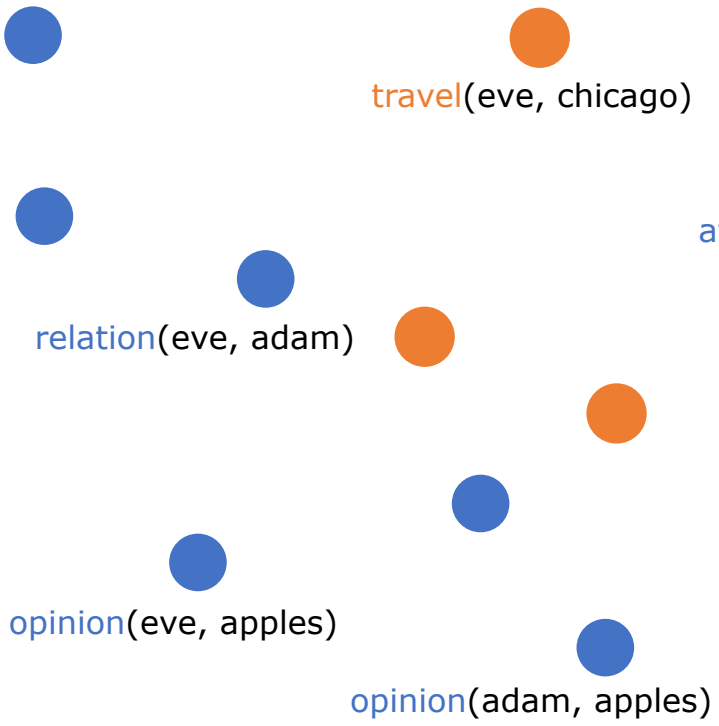
Deductive Rules, Triggering Rules

which facts are in the database
define a trainable neural architecture
that computes embeddings of the facts

logic!

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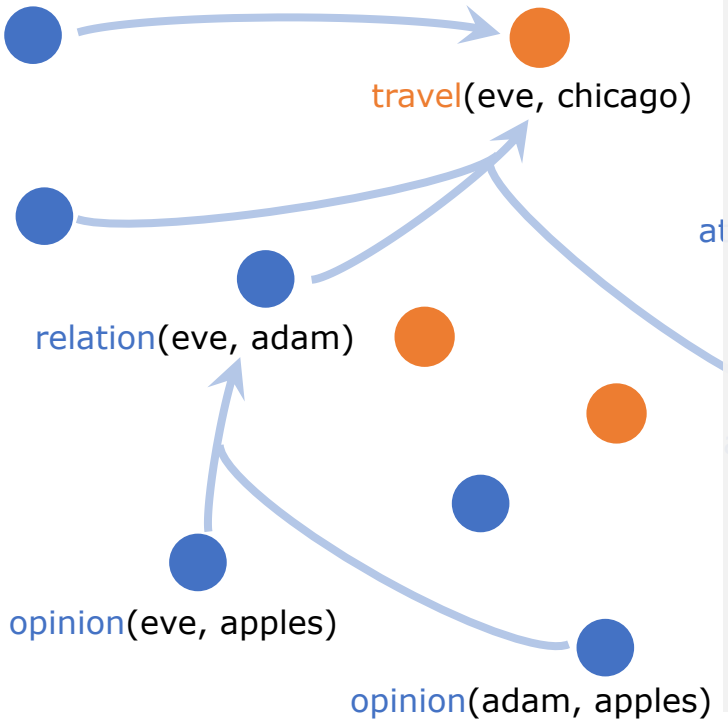


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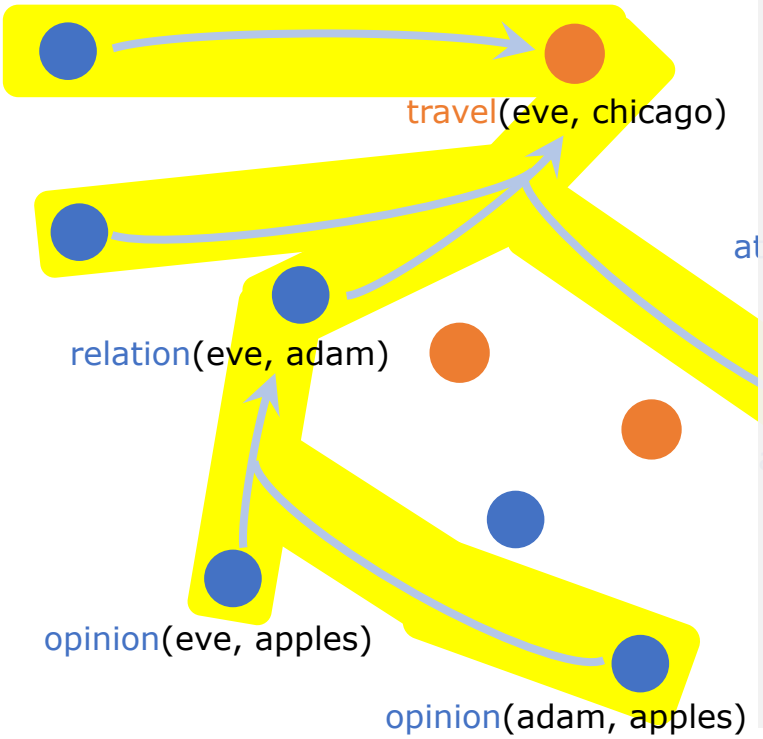


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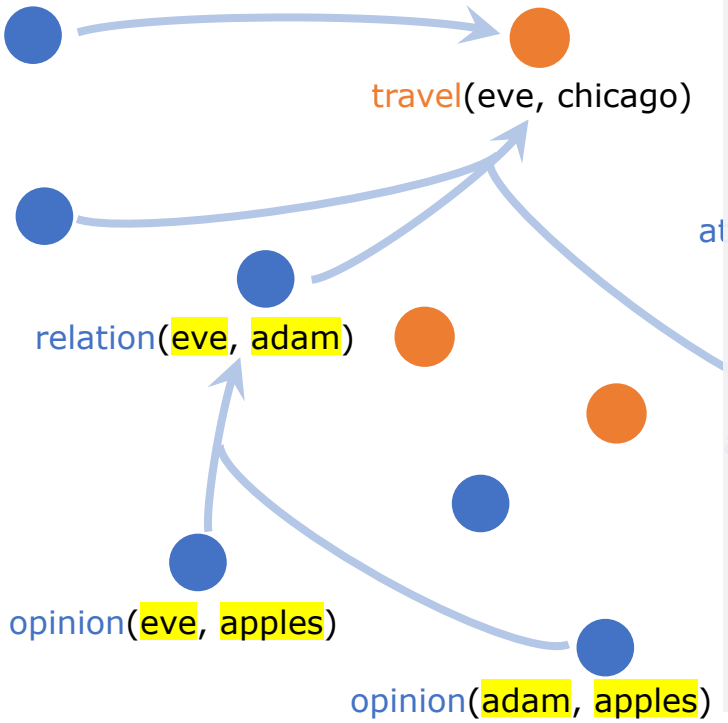


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Datalog ➔ Neural Datalog Through Time

Datalog ➔ Neural Datalog Through Time

deductive rule

new fact :- old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database

new fact :- old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if

new fact :- old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, ...

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, ...

likes(X, U),

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, ...

likes(X, U), likes(Y, U)

Datalog ➔ Neural Datalog Through Time

deductive rule

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:- likes(X, U), likes(Y, U)

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if these are in database

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compatible(X, Y) :- likes(X, U), likes(Y, U)

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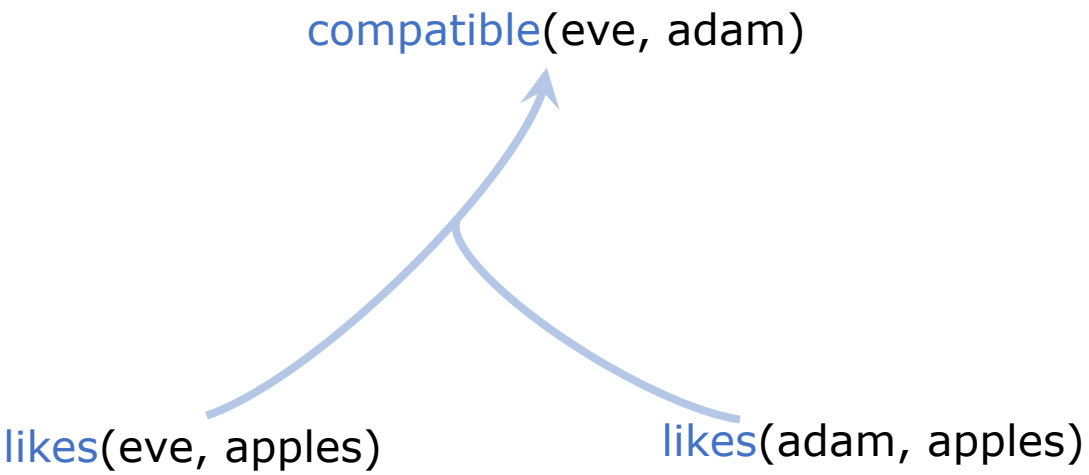

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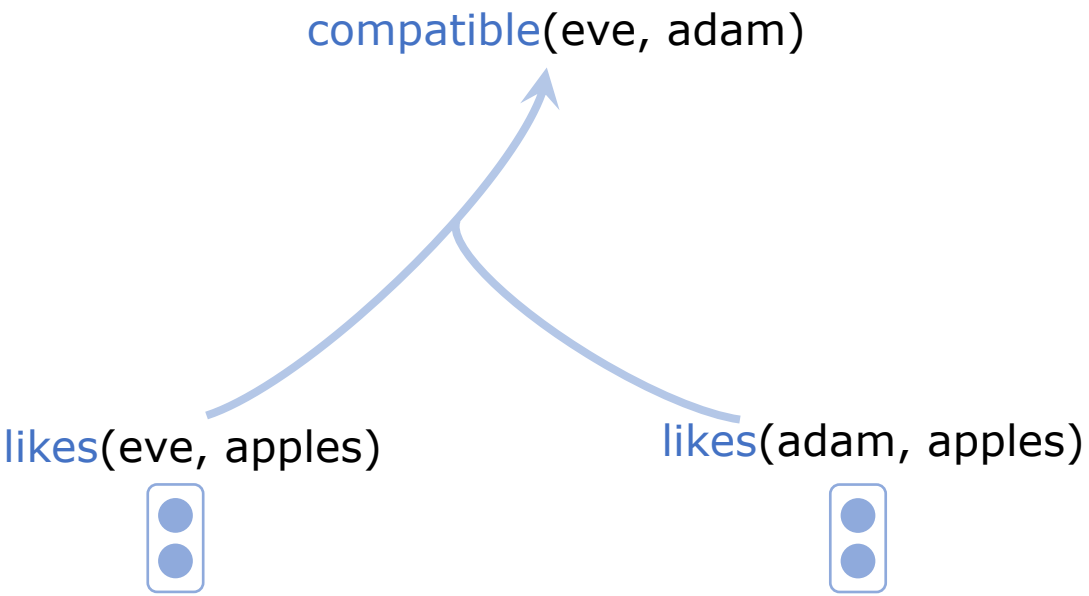

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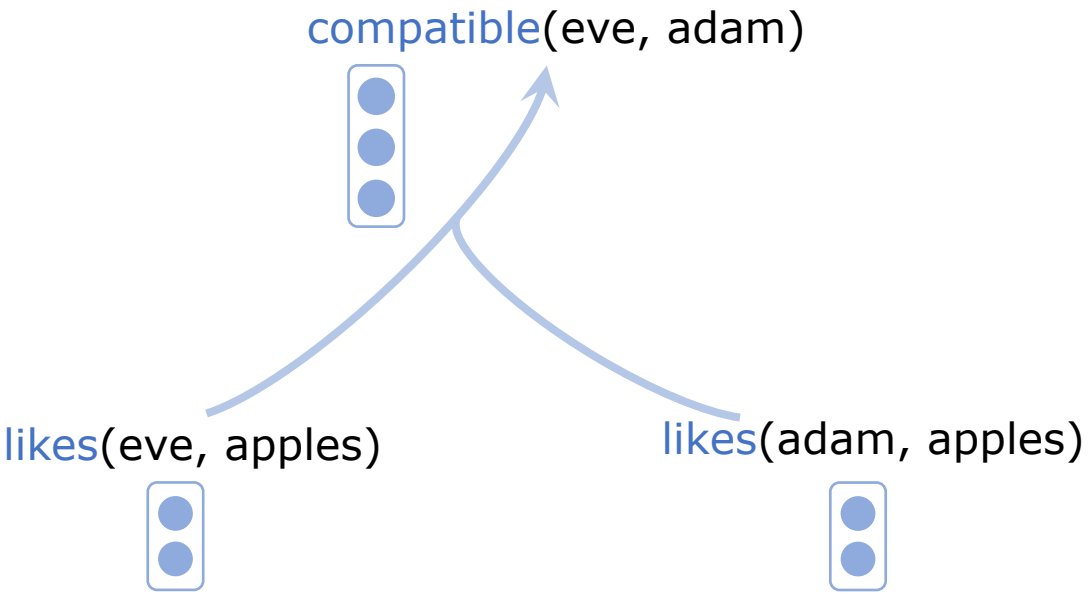

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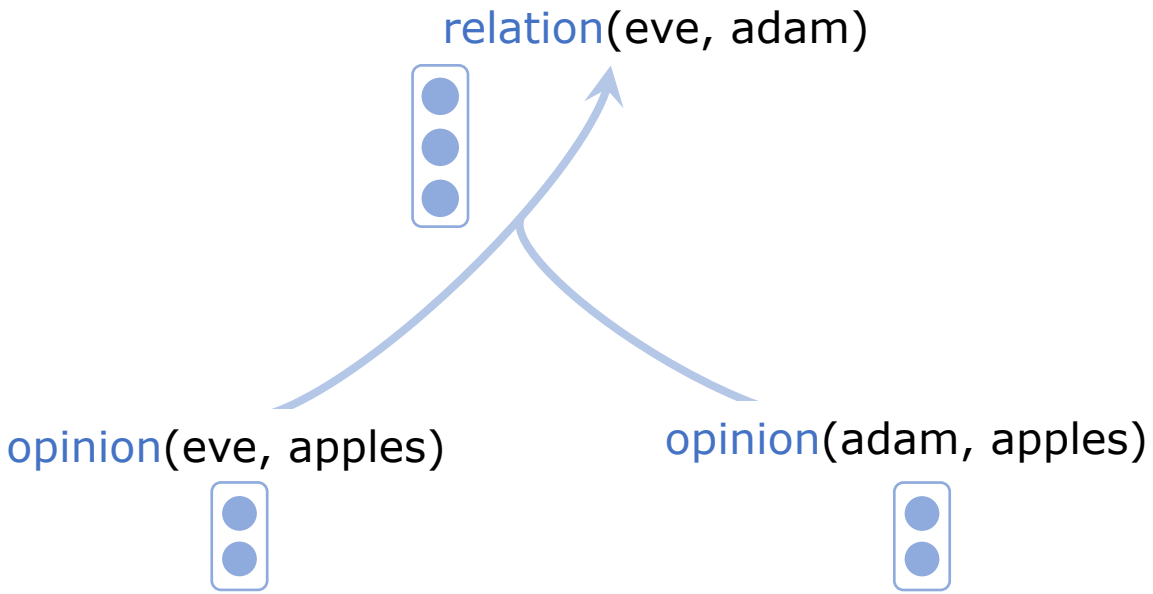

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

new fact ← event, old fact $_1$, old fact $_2$, …

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

when

new fact ← event, old fact $_1$, old fact $_2$, …

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

when this happens

new fact ← event, old fact $_1$, old fact $_2$, …

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

when this happens while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

add to database when this happens while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

Datalog → Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

add to database when this happens while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

! old fact ← event, old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

add to database when this happens while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

when

! old fact ← event, old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database — if — these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

add to database — when this happens — while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

when this happens

! old fact ← event, old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

add to database when this happens while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

when this happens while these are in database

! old fact ← event, old fact $_1$, old fact $_2$, …

Datalog ➔ Neural Datalog Through Time

deductive rule

add to database if these are in database

new fact :- old fact $_1$, old fact $_2$, …

triggering rule

add to database when this happens while these are in database

new fact ← event, old fact $_1$, old fact $_2$, …

delete when this happens while these are in database

! old fact ← event, old fact $_1$, old fact $_2$, …

Computing the Embeddings

Computing the Embeddings

relation(eve, adam)

Computing the Embeddings

relation(X, Y) :- opinion(X, U), opinion(Y, U)

relation(eve, adam)

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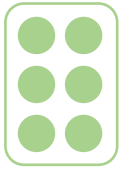

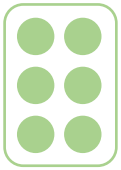

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Computing the Embeddings

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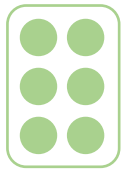

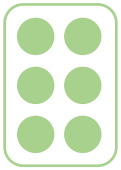

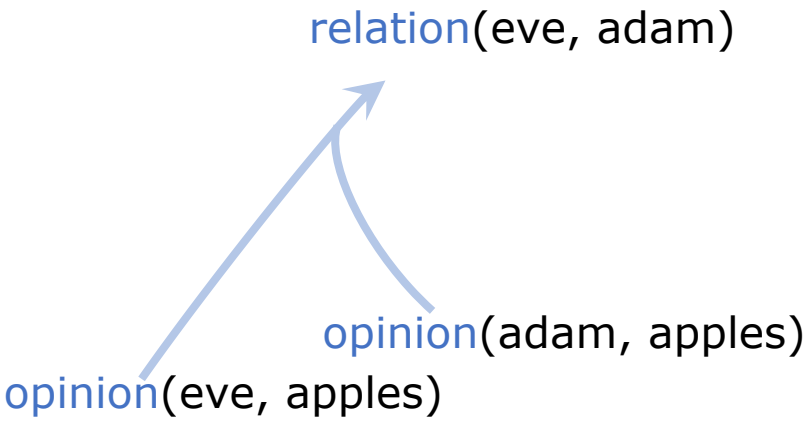

Computing the Embeddings

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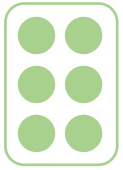

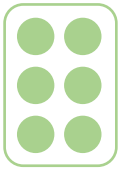

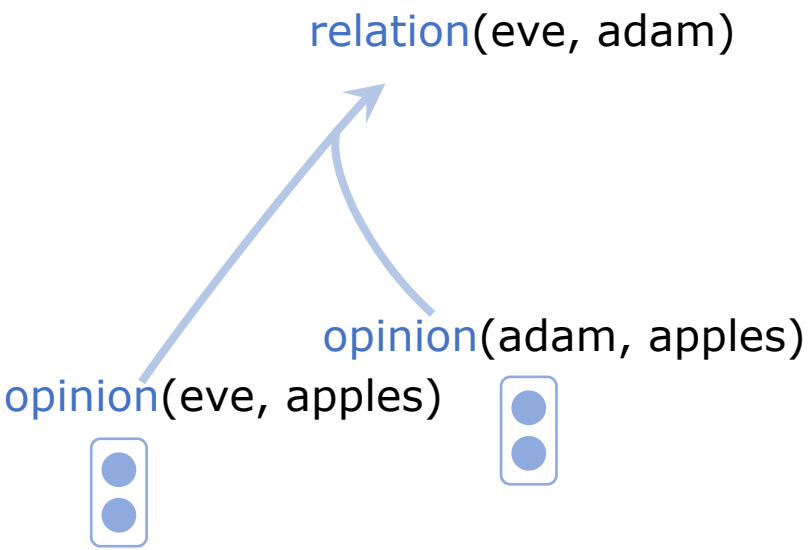

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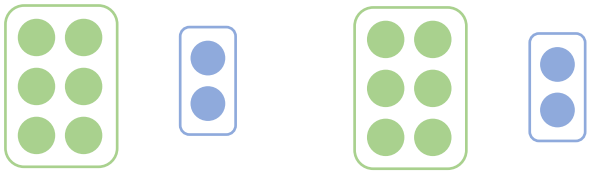

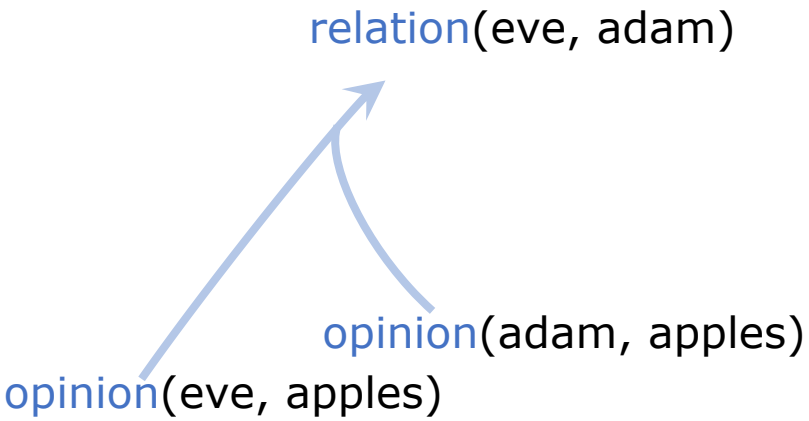

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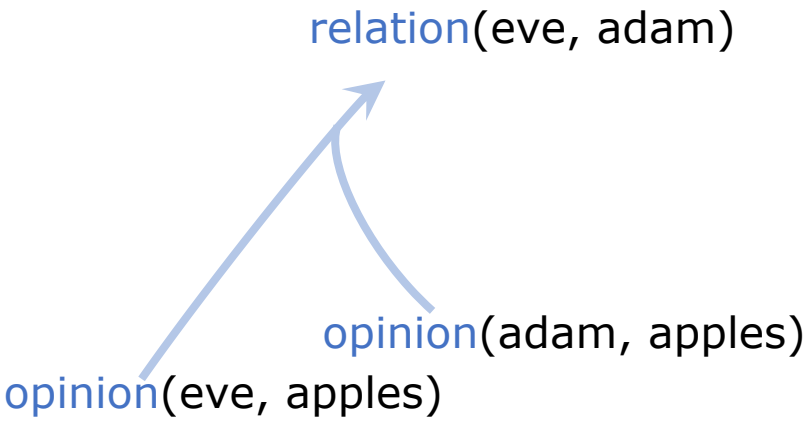

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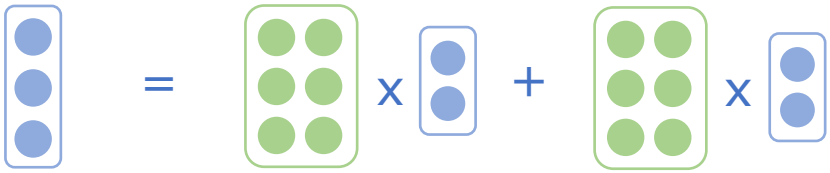

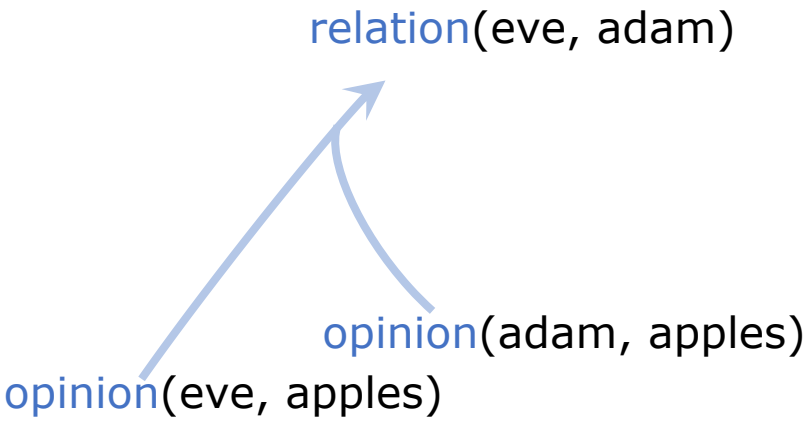

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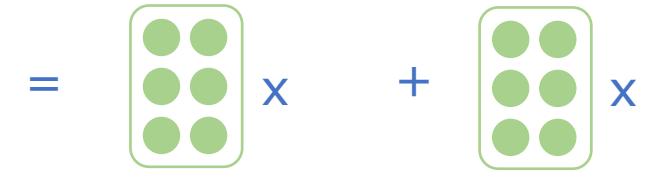

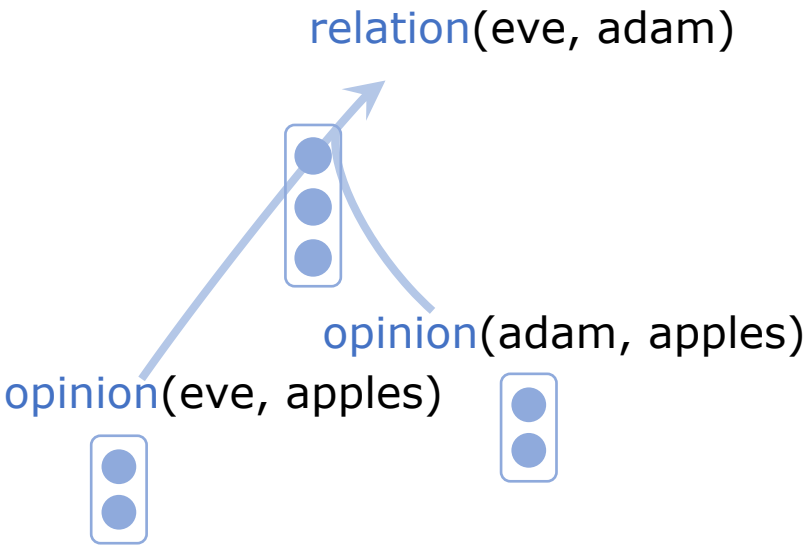

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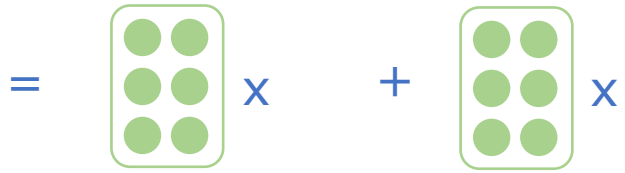

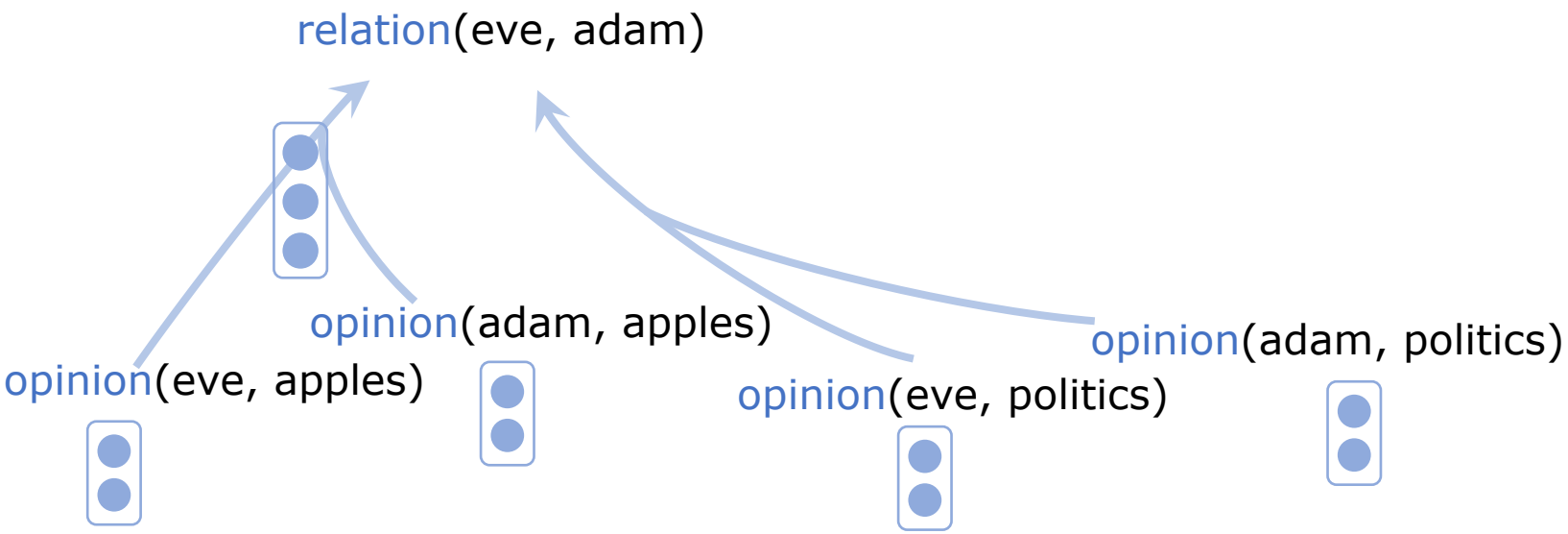

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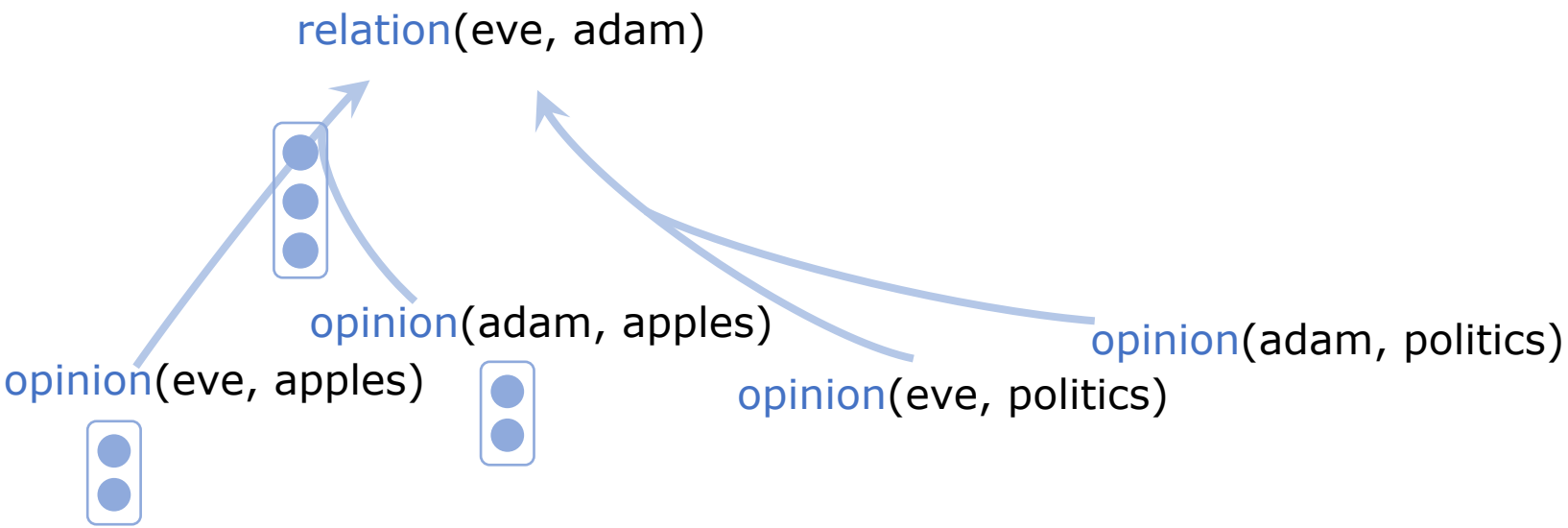

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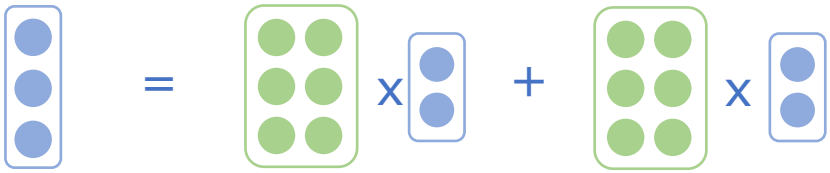

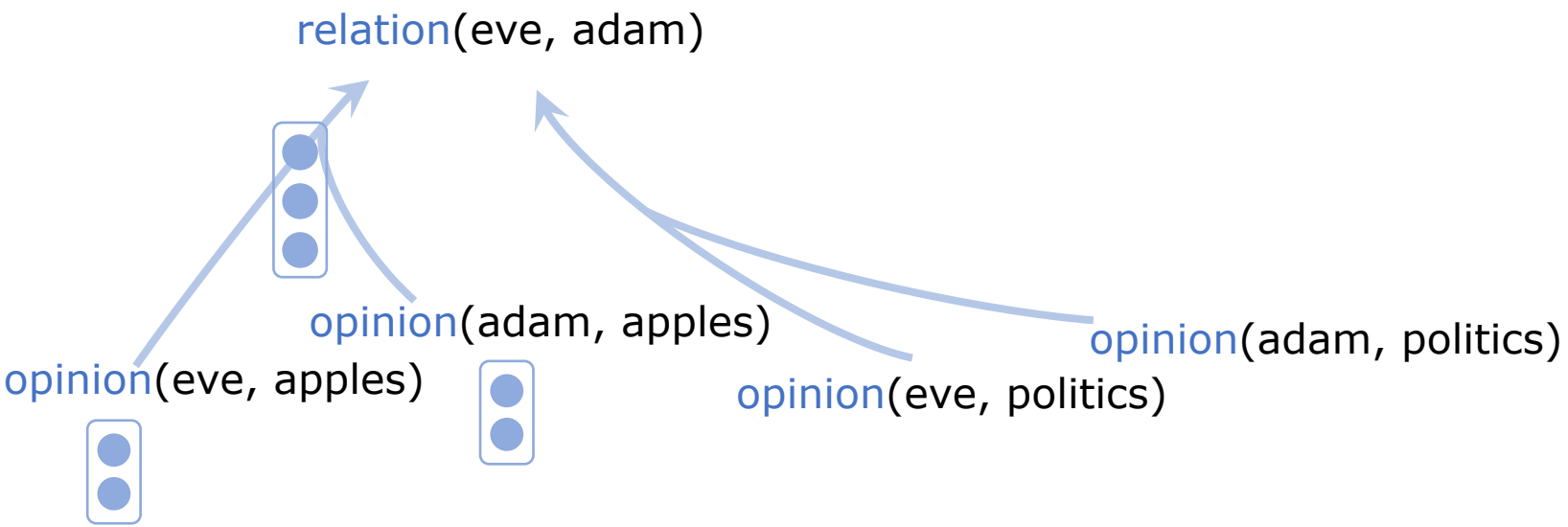

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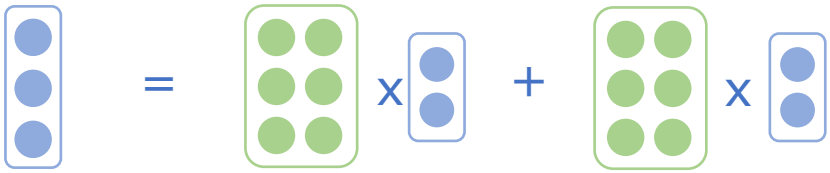
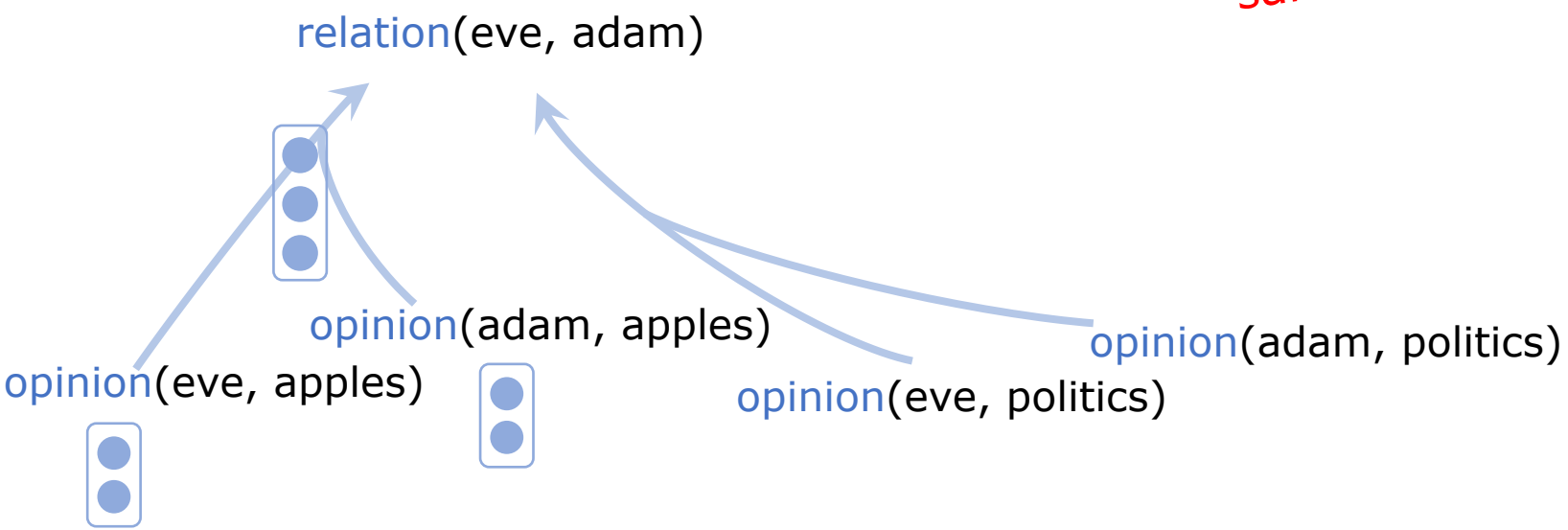


different inputs

same params



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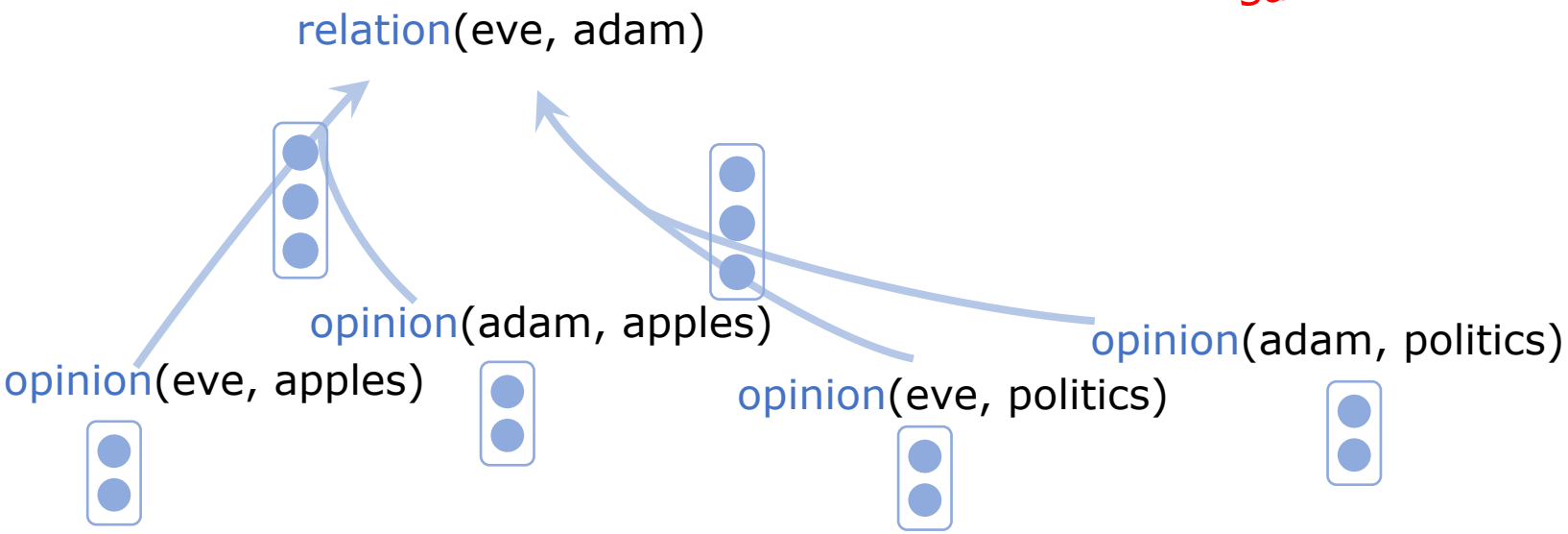

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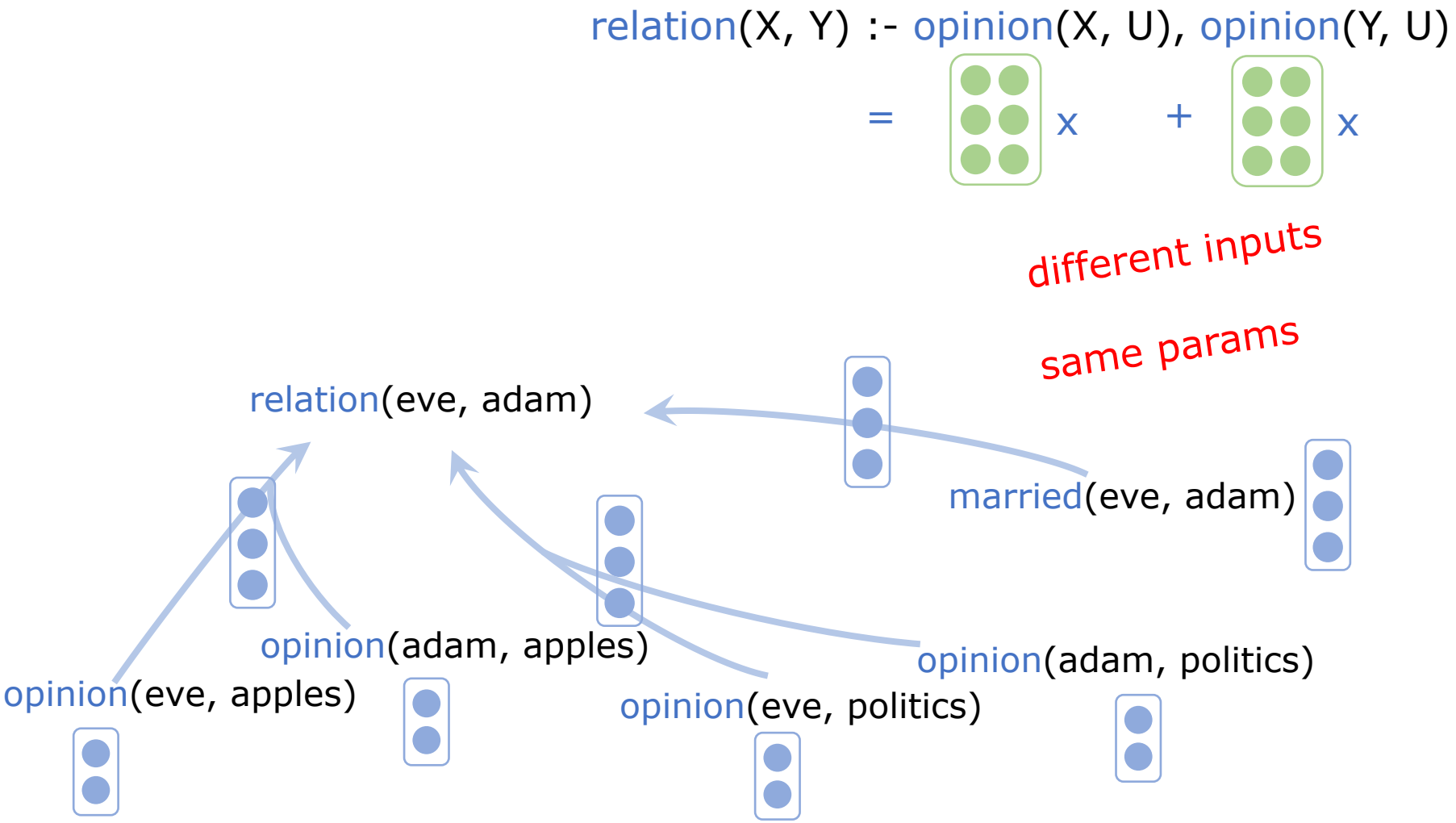

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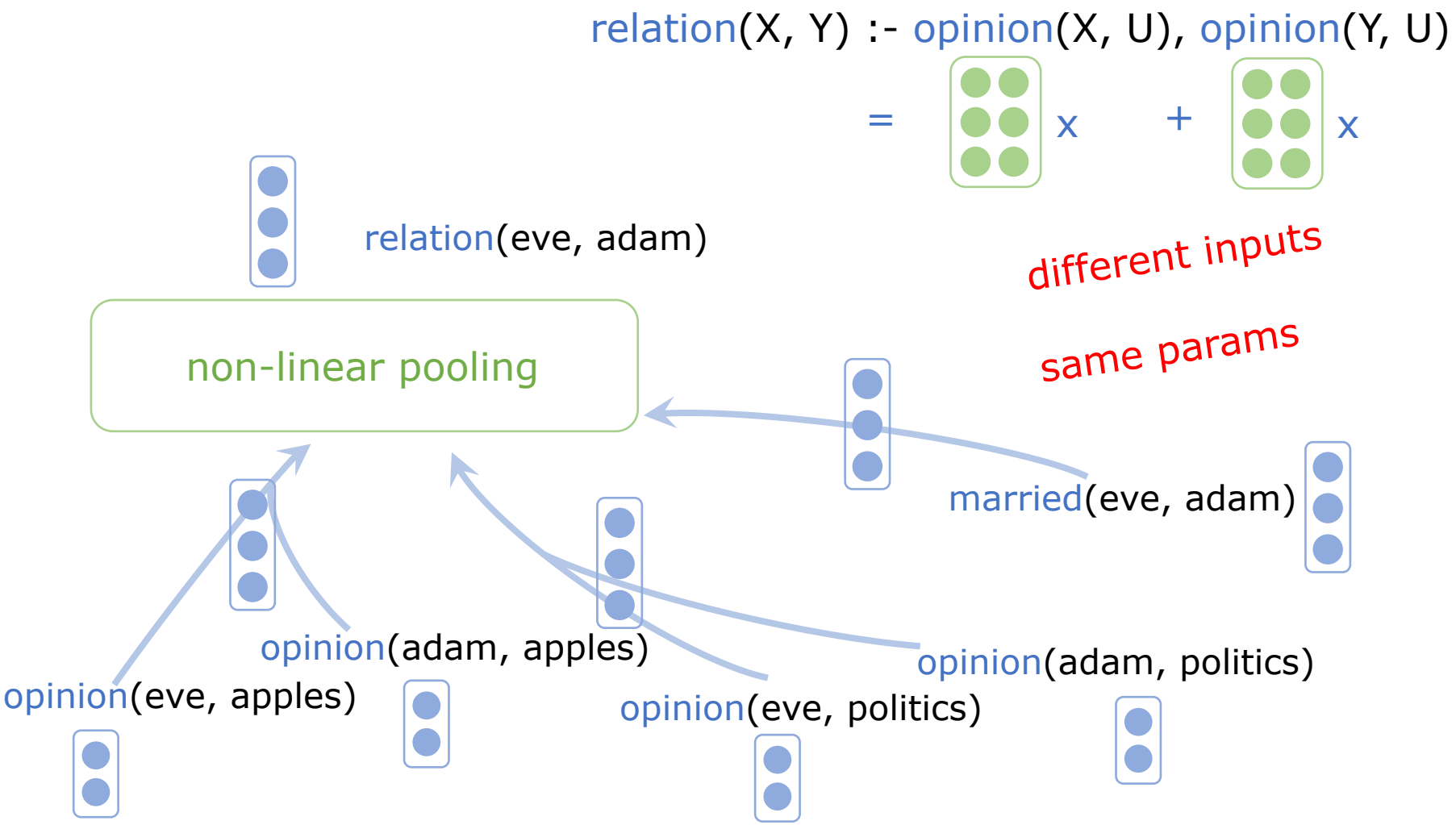

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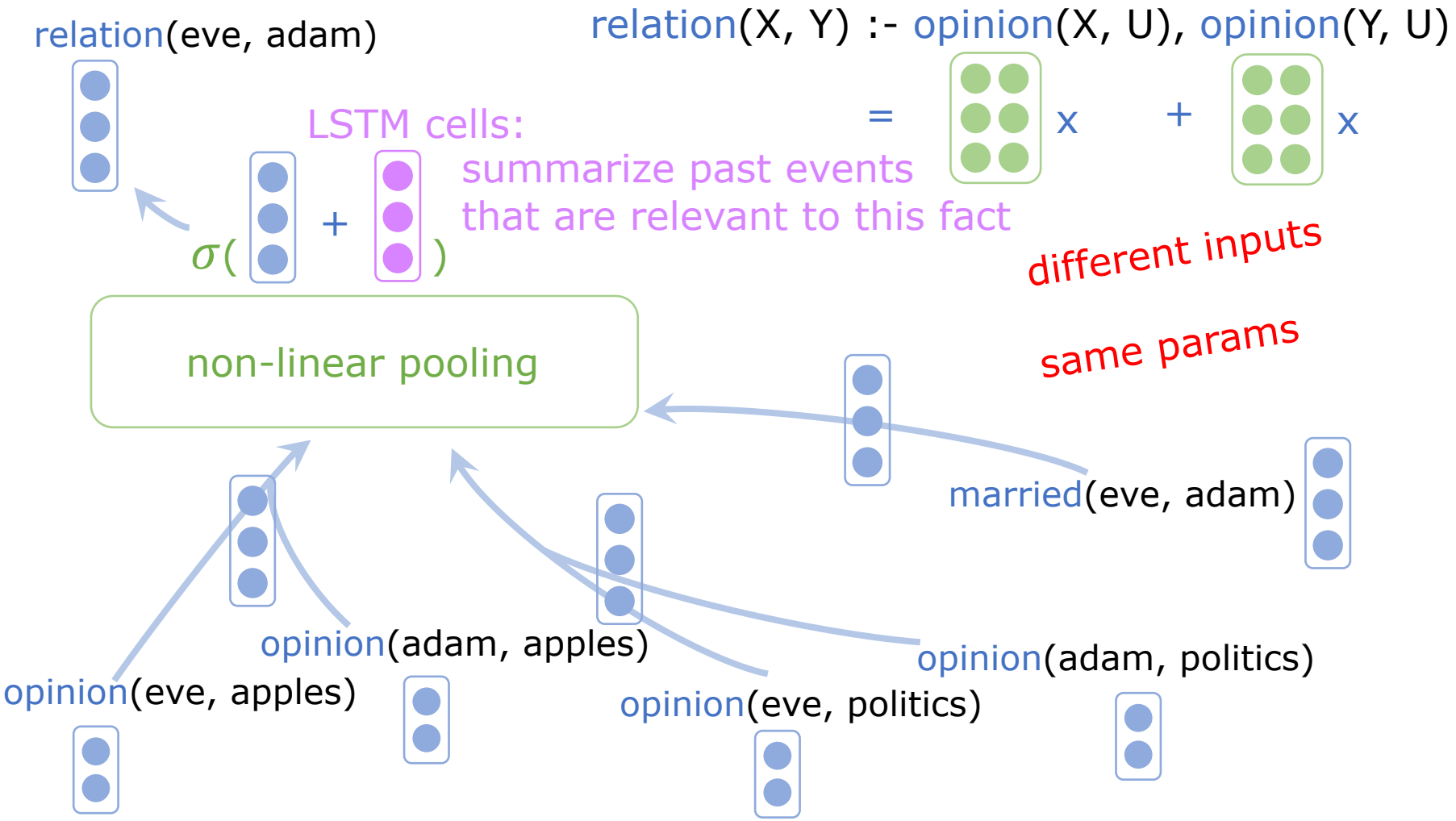

Computing Embeddings & Probabilities

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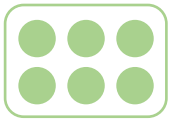
...

relation(eve, adam)

relation(eve, cain)

Computing Embeddings & Probabilities

travel(X, P) :- relation(X, Y), at(Y, P).



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Computing Embeddings & Probabilities

travel(X, P) :- relation(X, Y), at(Y, P).

...

...

relation(eve, adam)

at(adam, chicago)

relation(eve, cain)

at(cain, chicago)

Computing Embeddings & Probabilities

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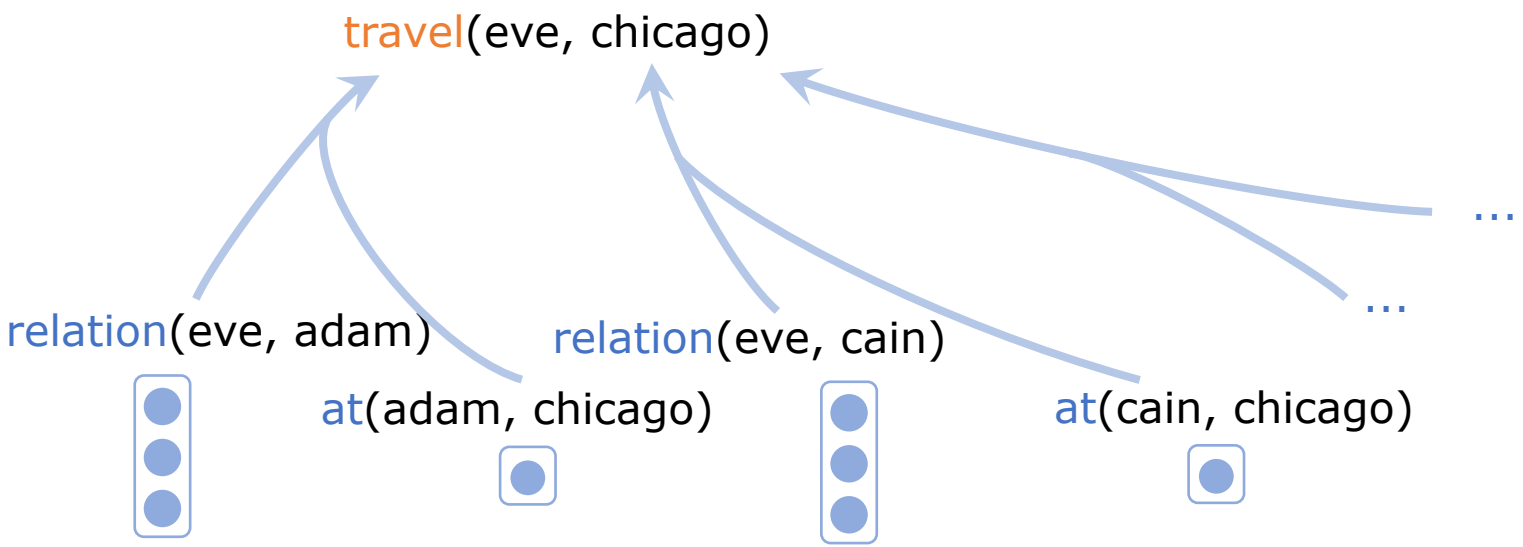

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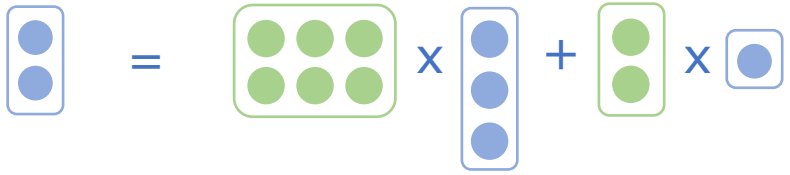
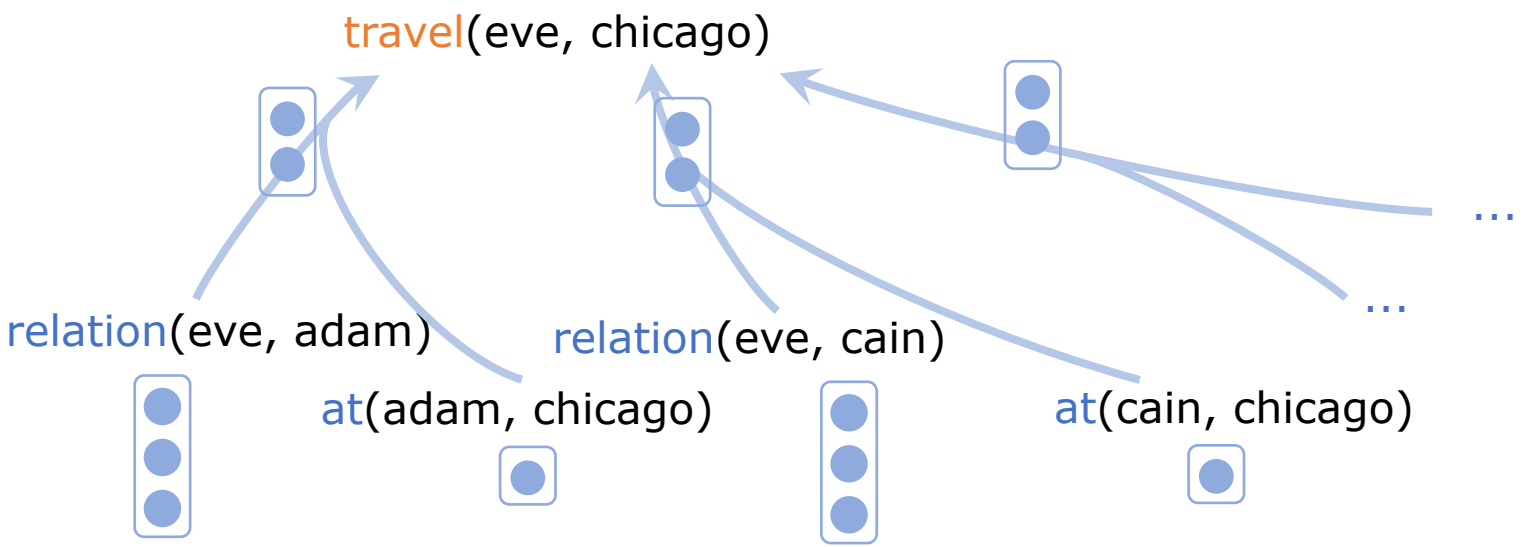
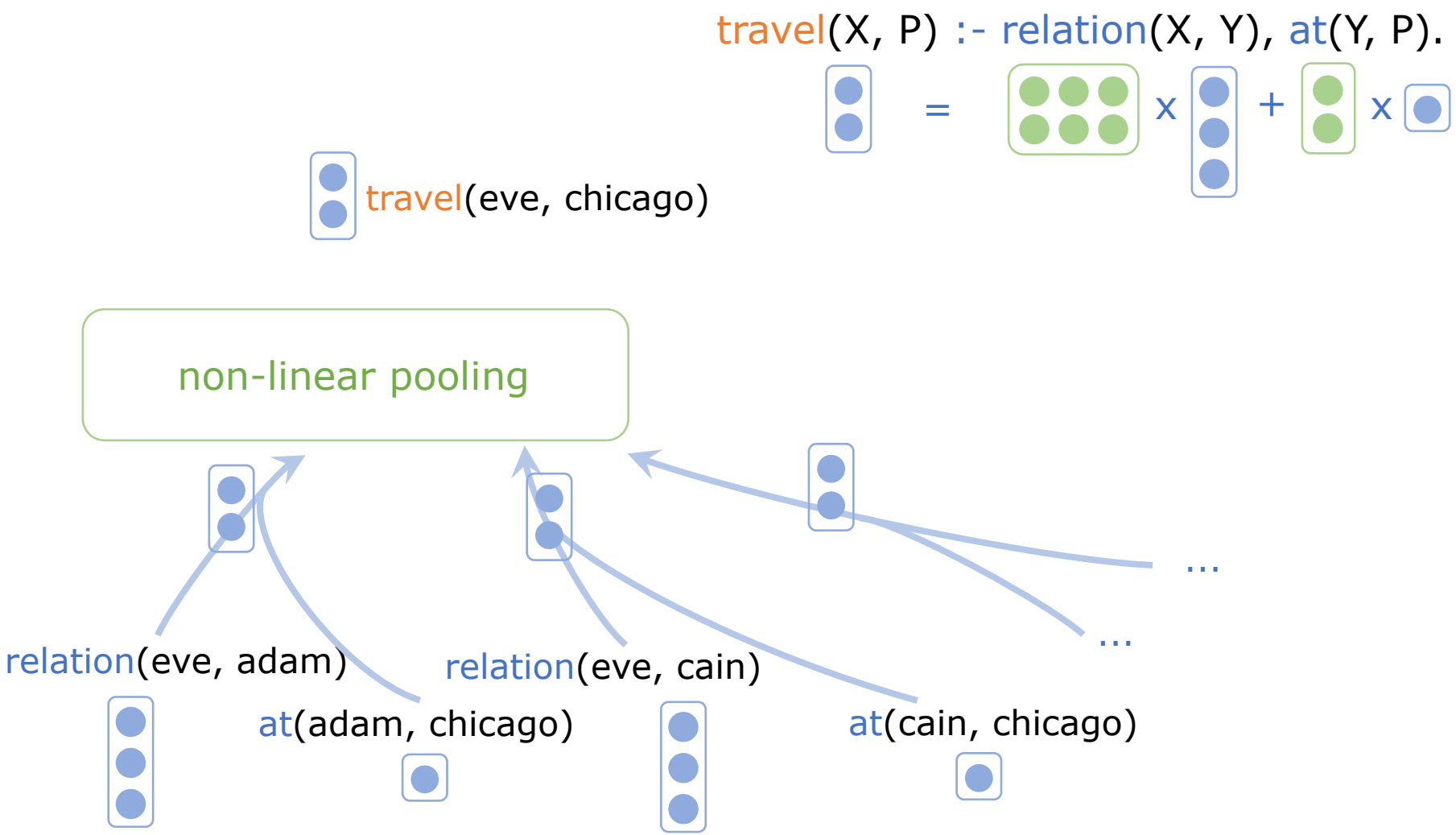
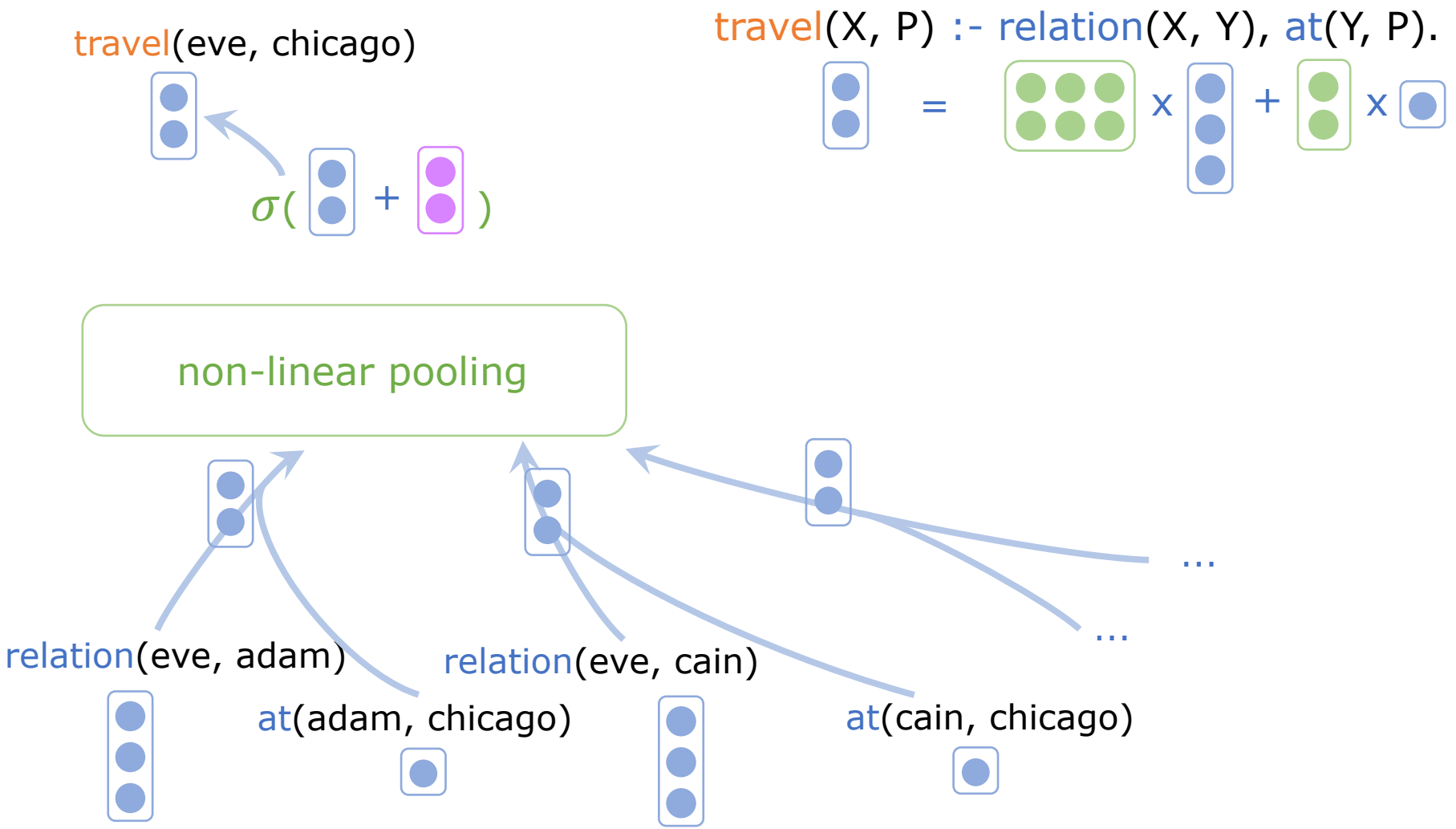




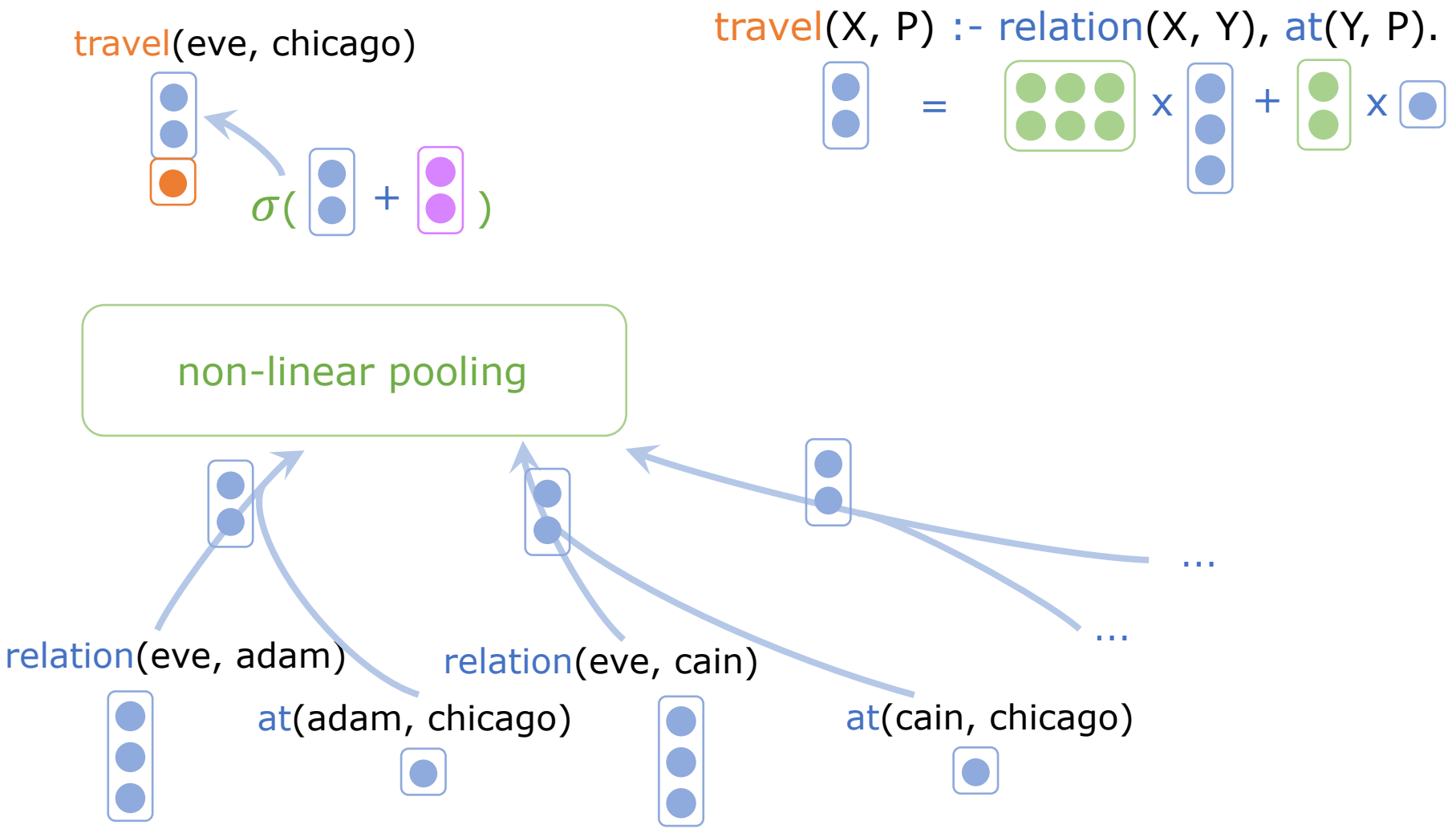
Computing Embeddings & Probabilities



Computing Embeddings & Probabilities



Computing Embeddings & Probabilities



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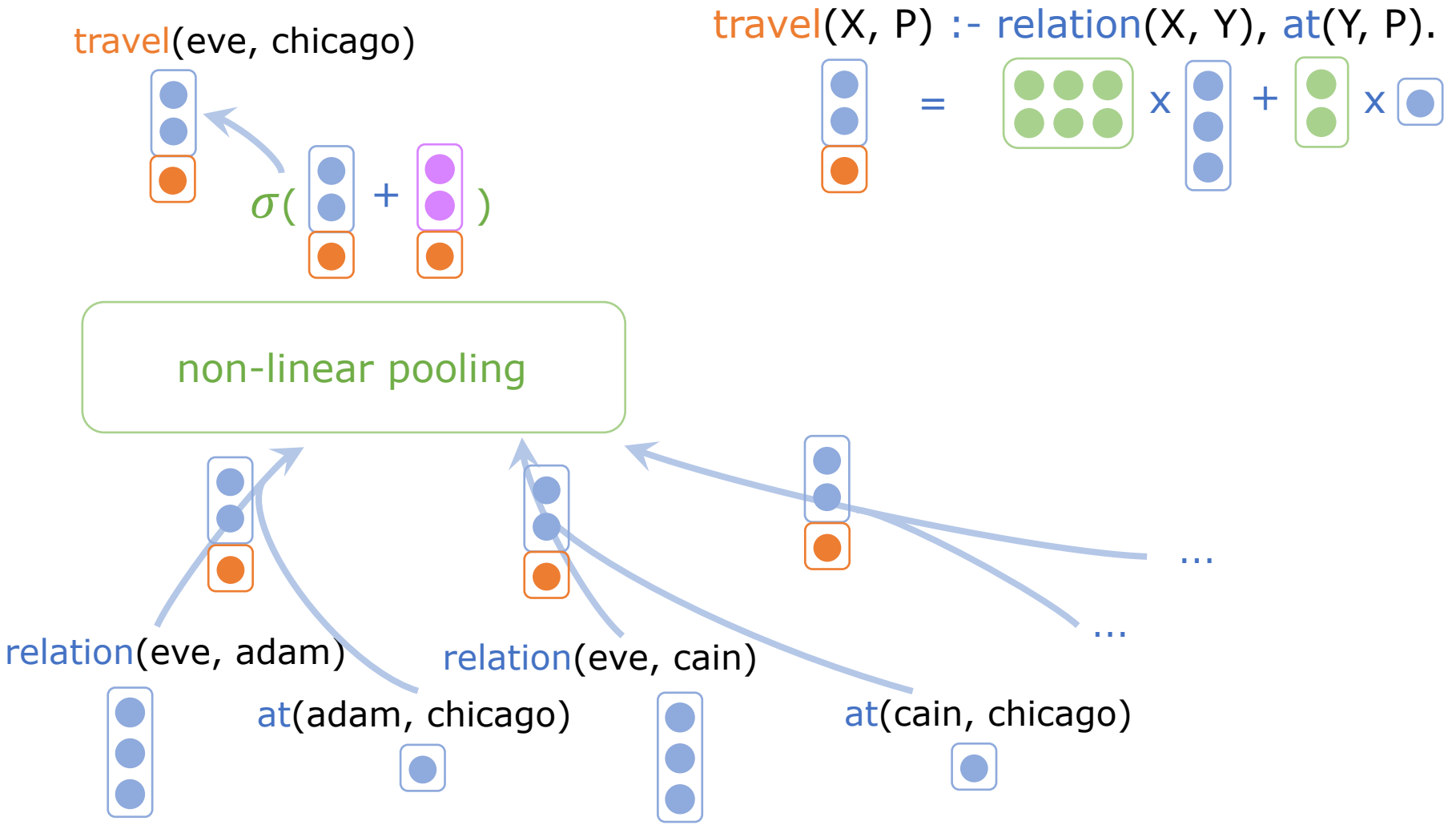

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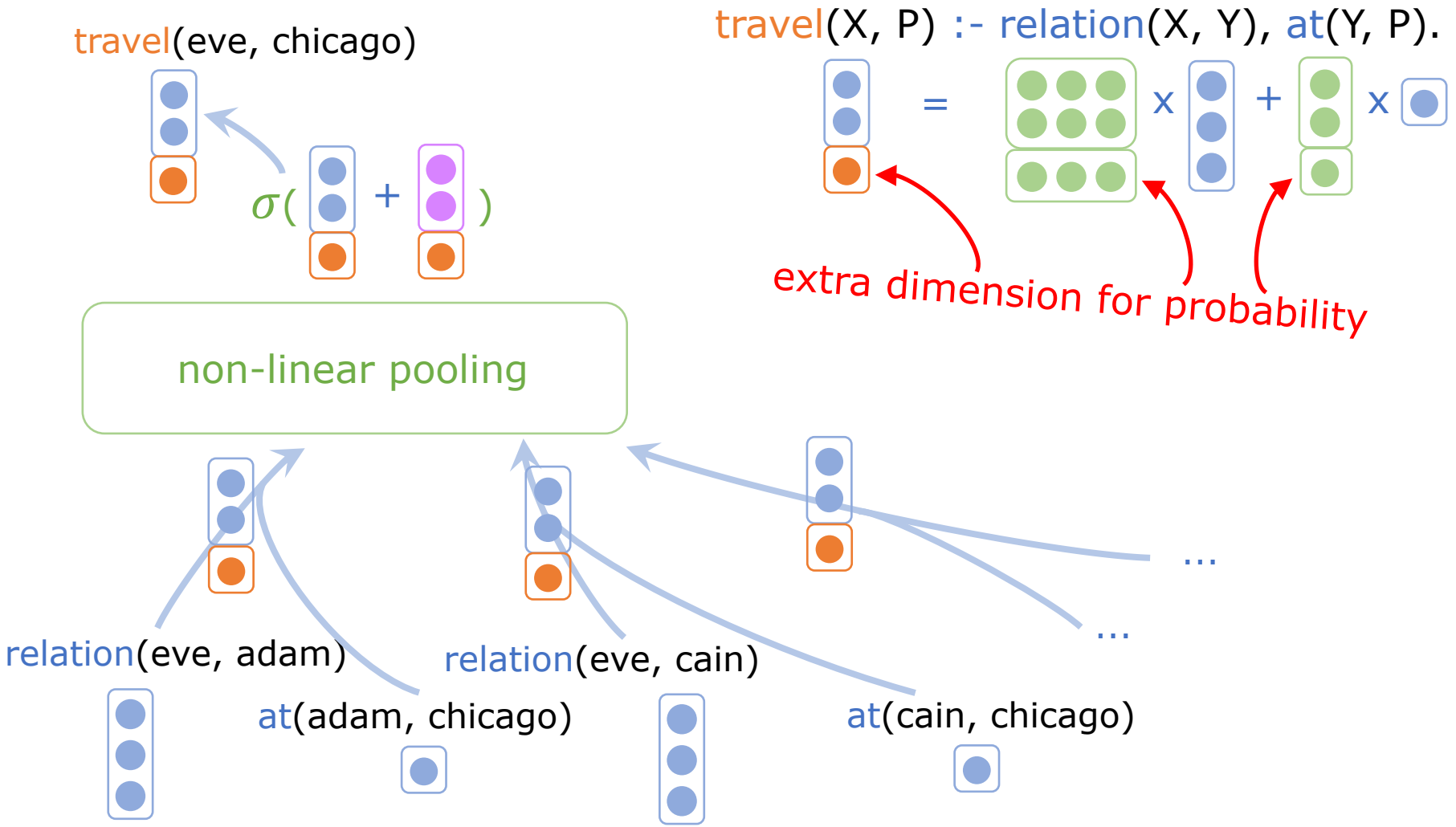

Rules ➔ Deep Recurrent Neural Net

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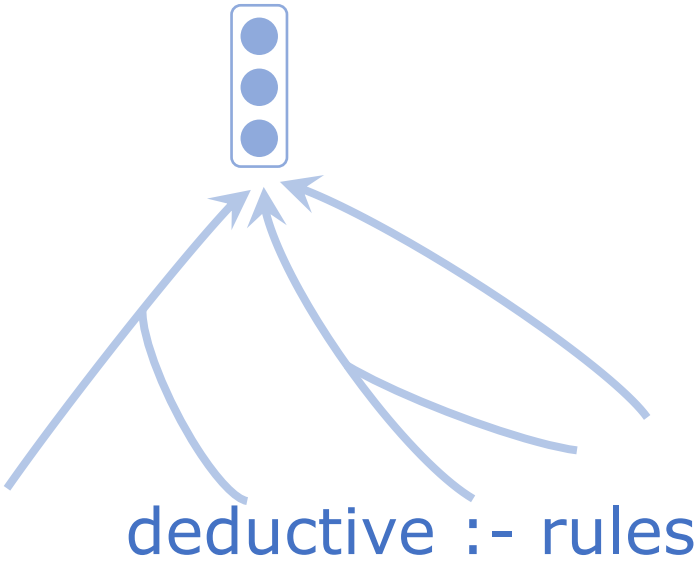

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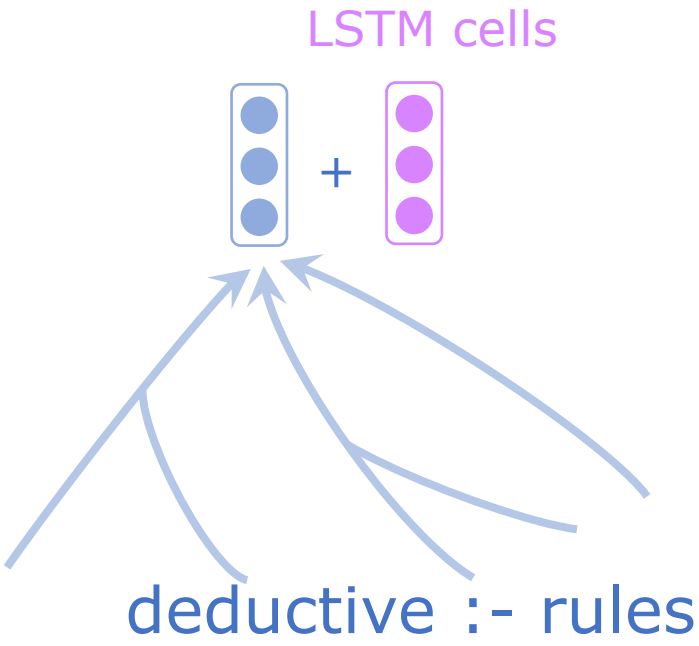

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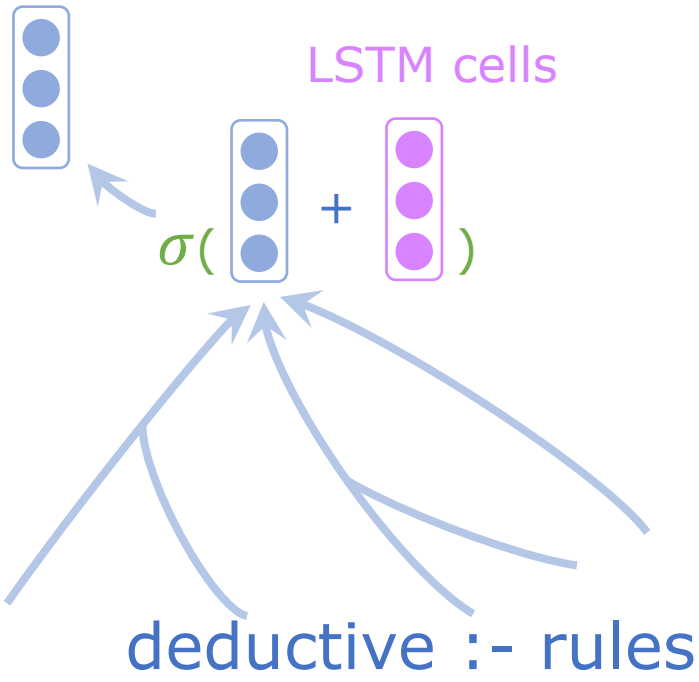

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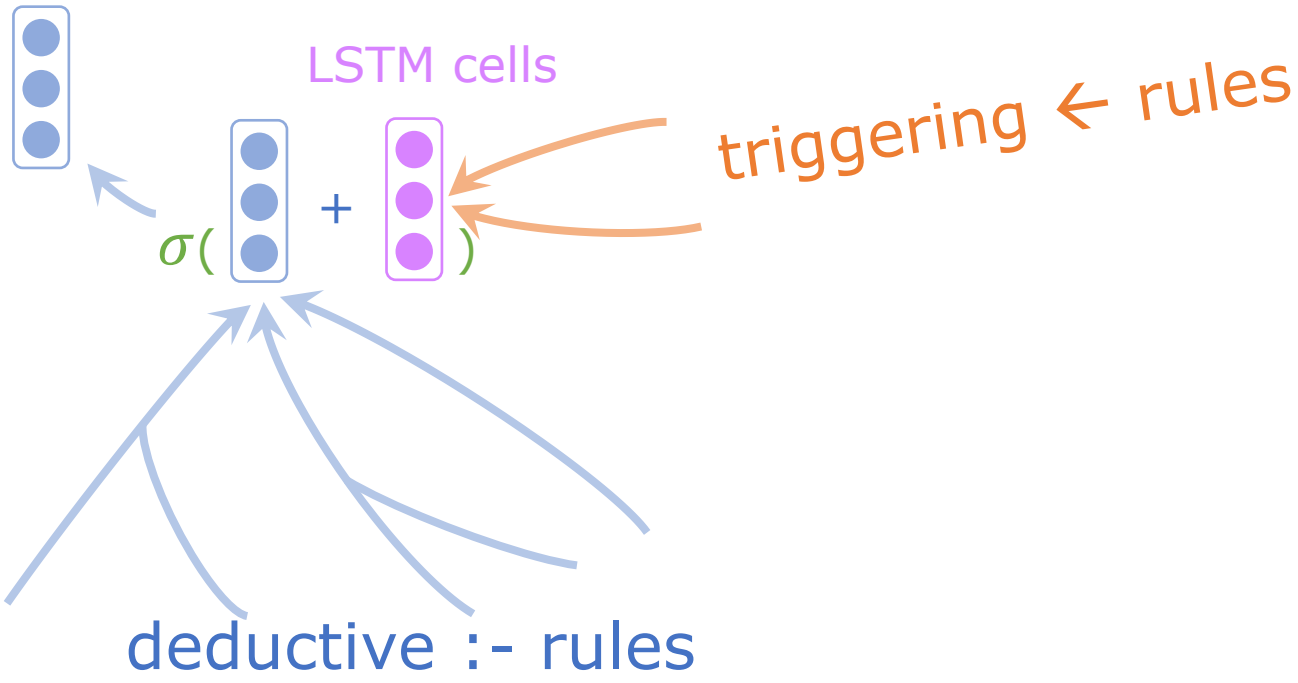

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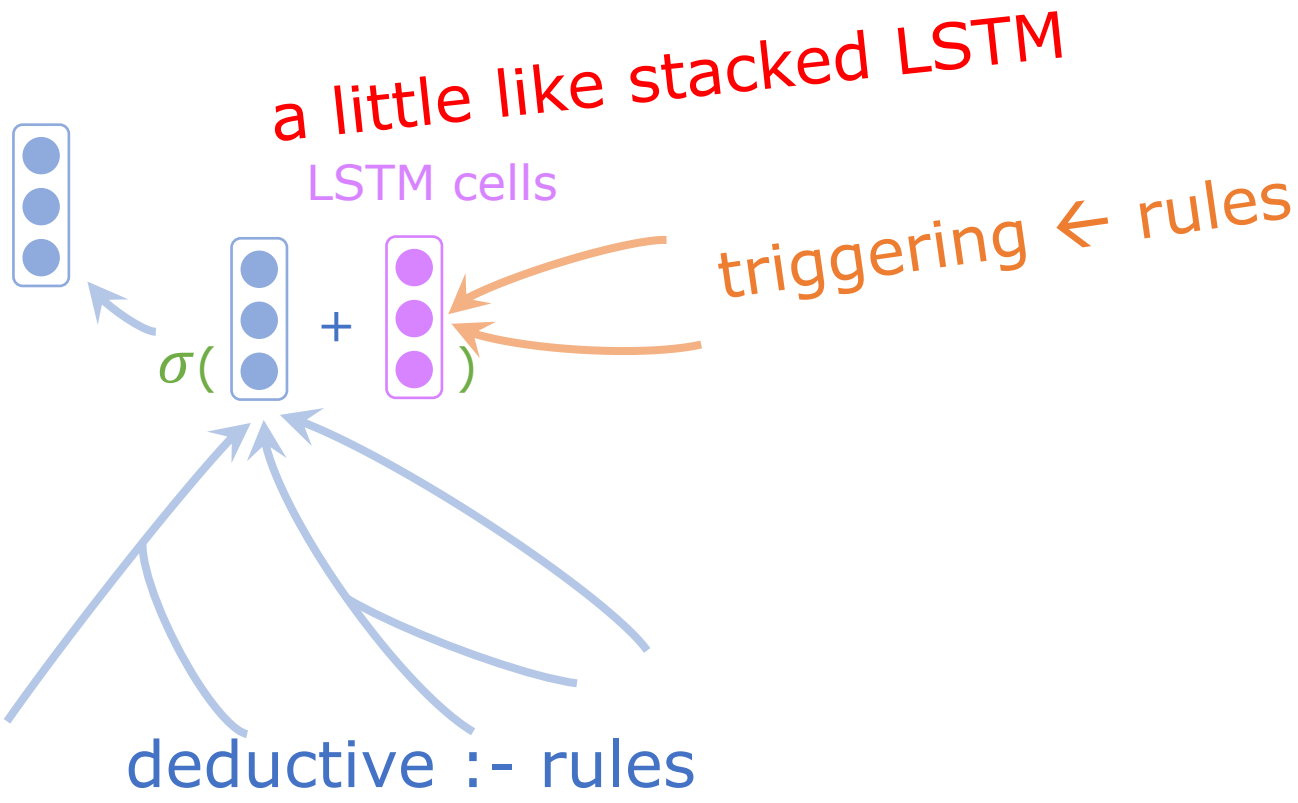

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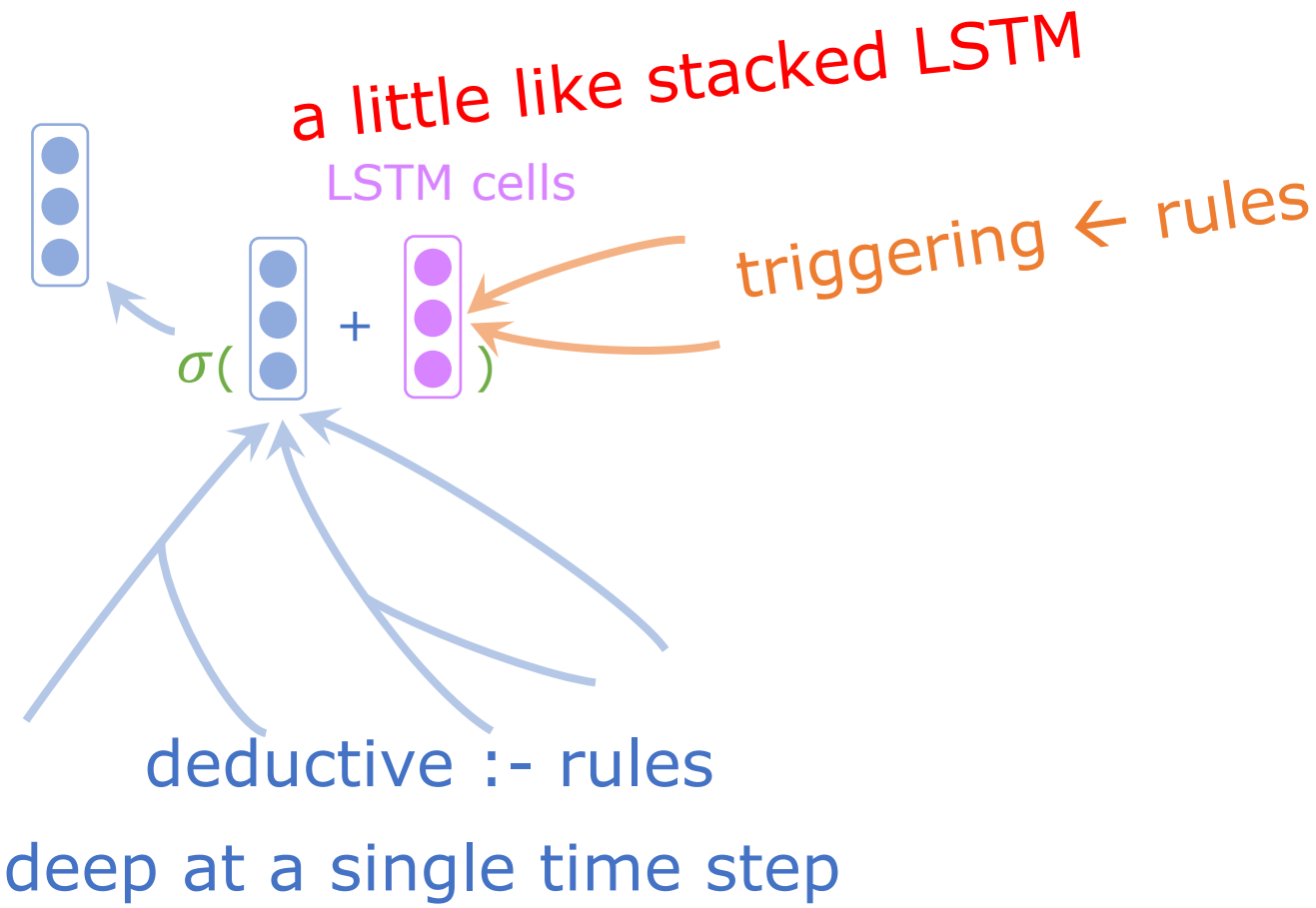

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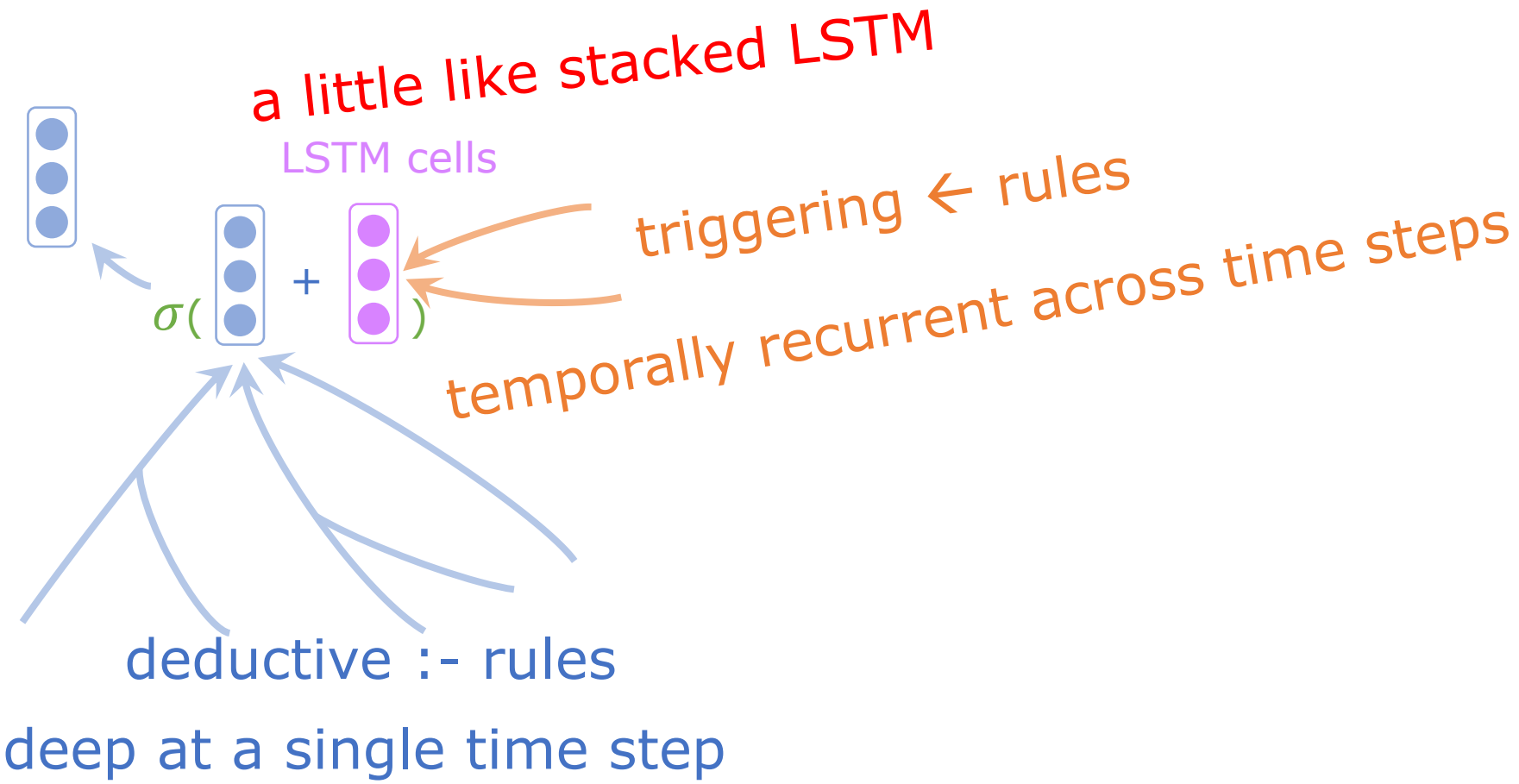

Life Story of a Fact

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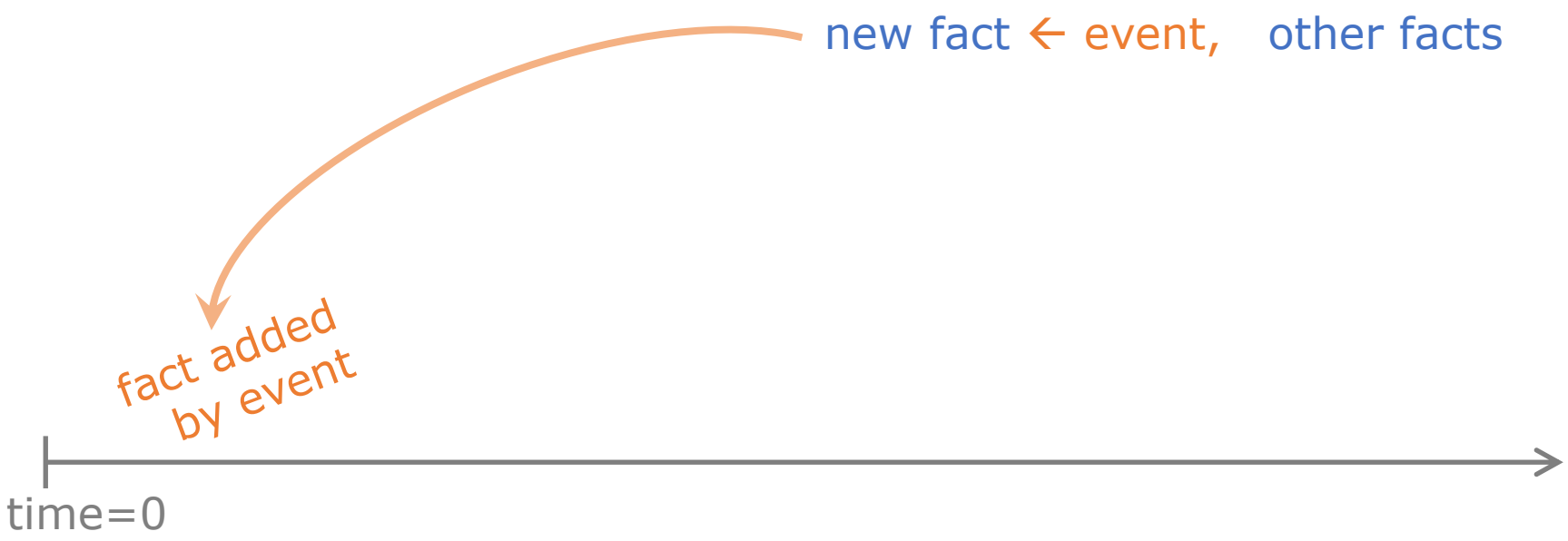
new fact ← event, other facts

Life Story of a Fact



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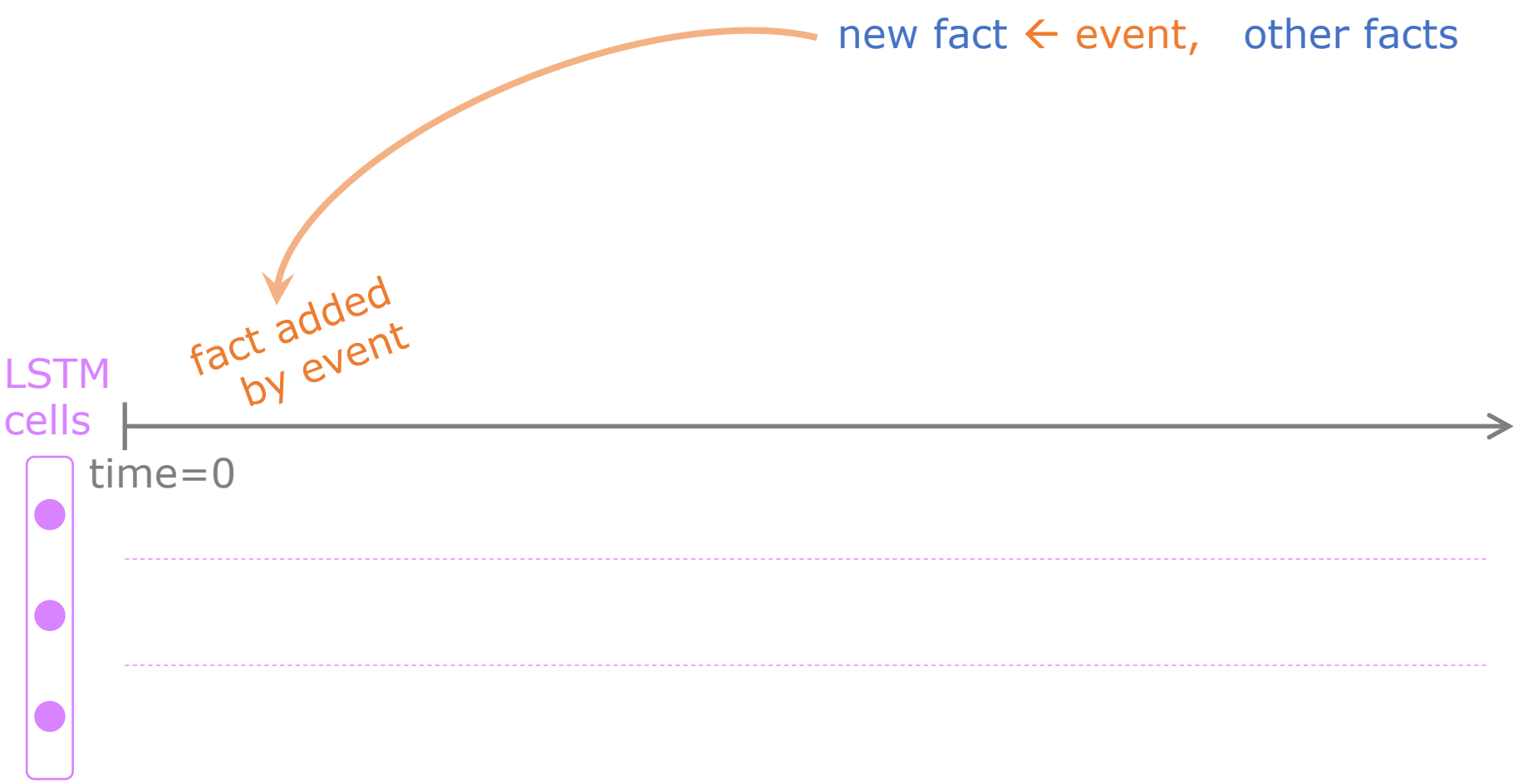

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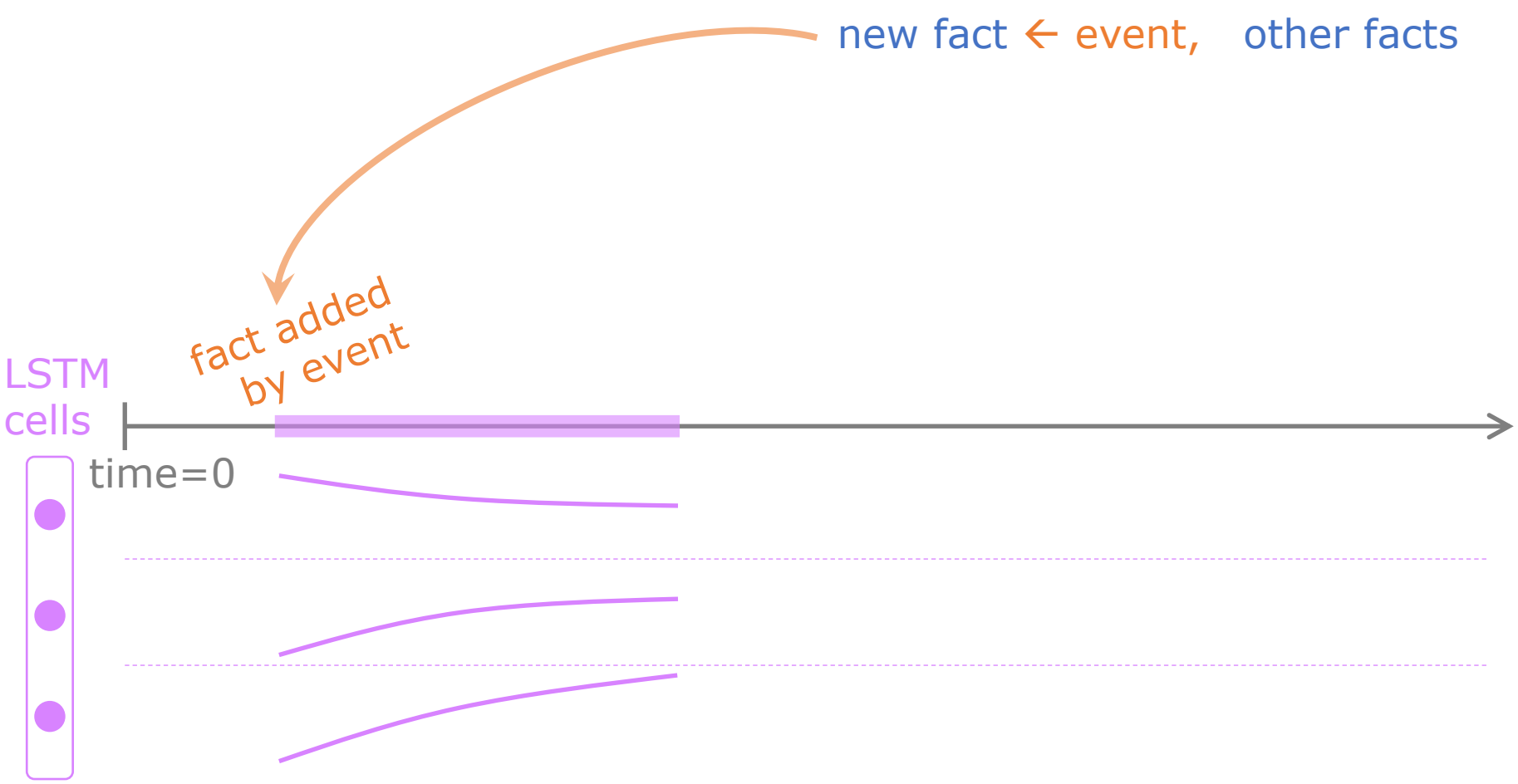

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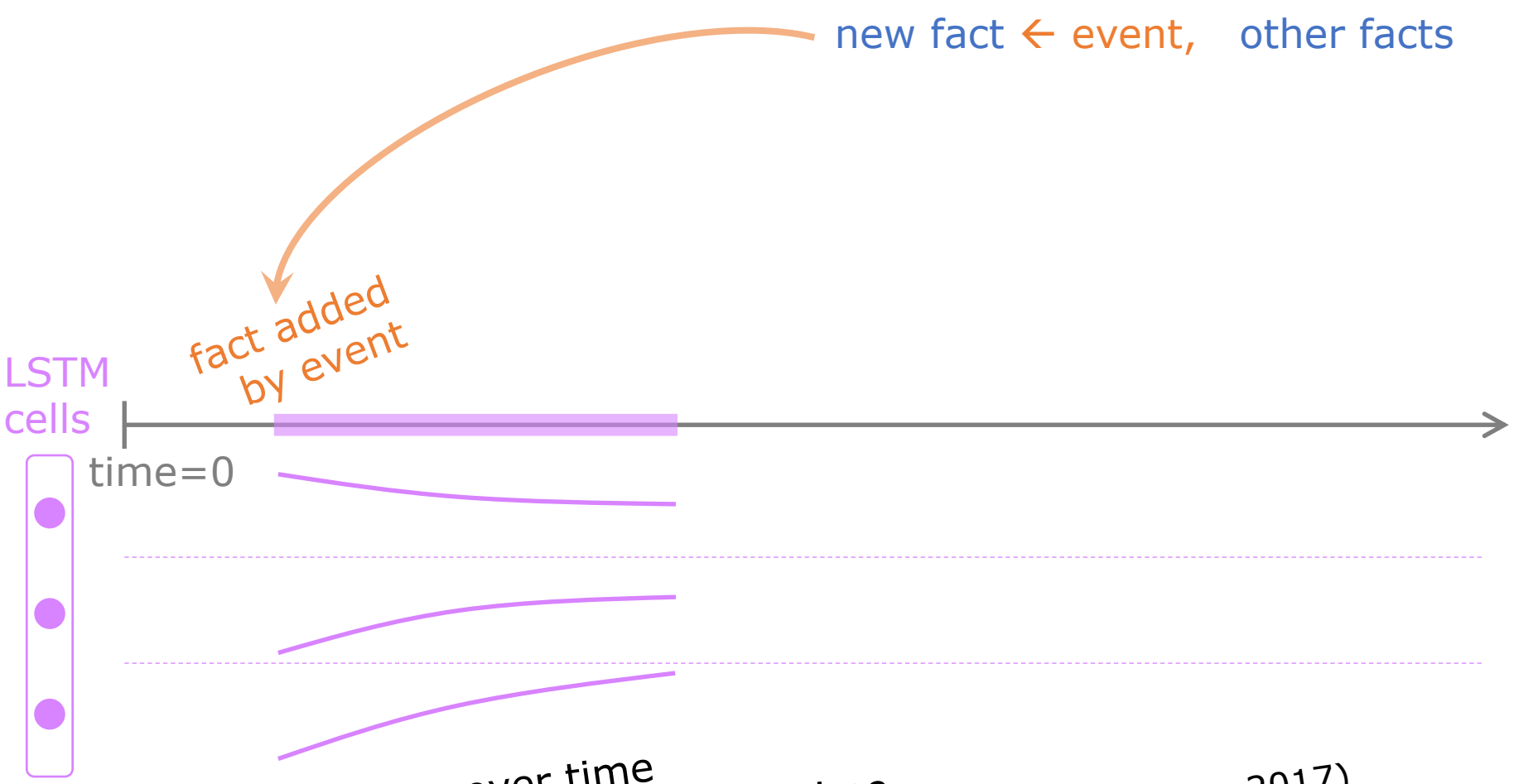
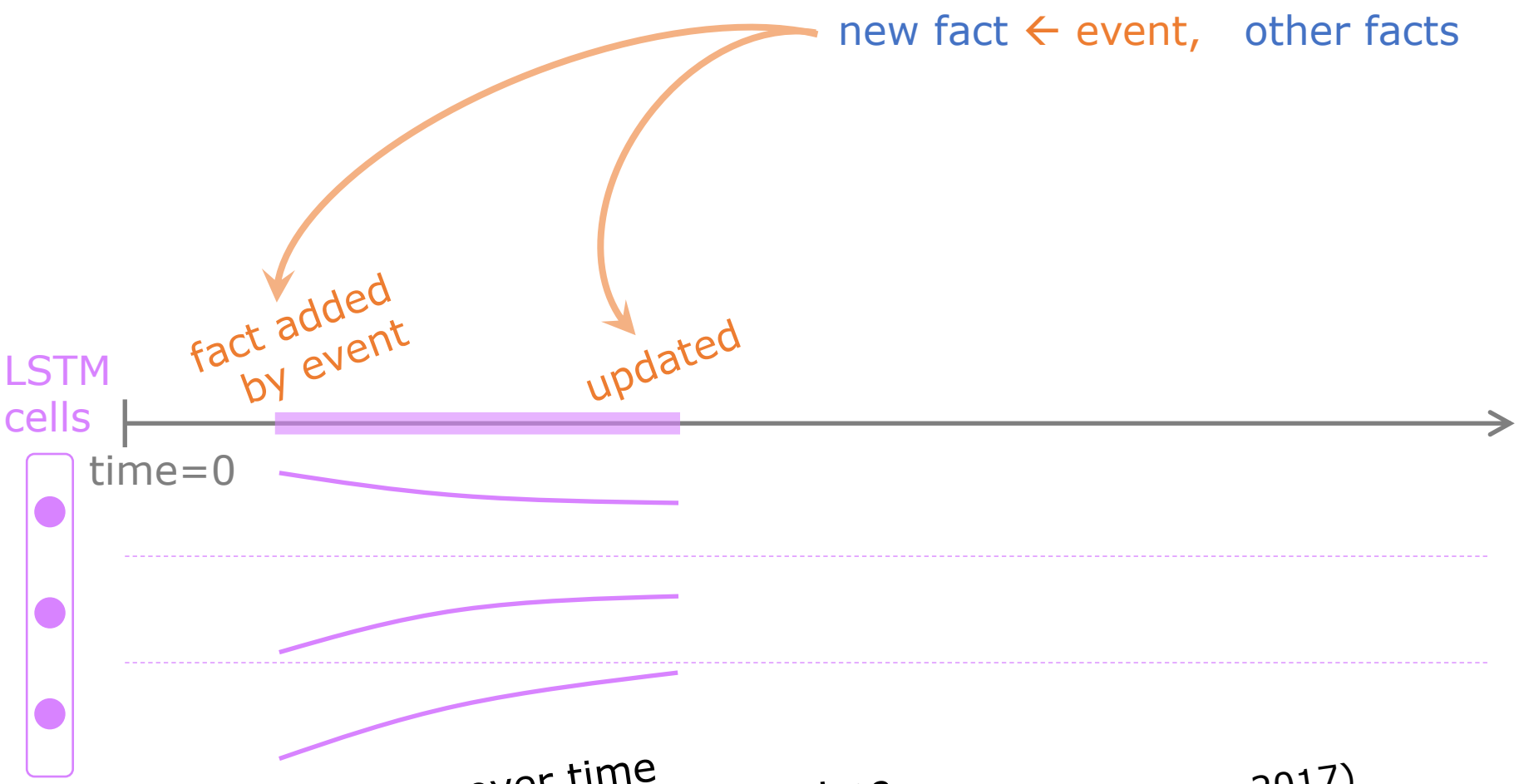


embedding changes over time
probability changes over time
≈ neural Hawkes process (Mei & Eisner, 2017)

Life Story of a Fact



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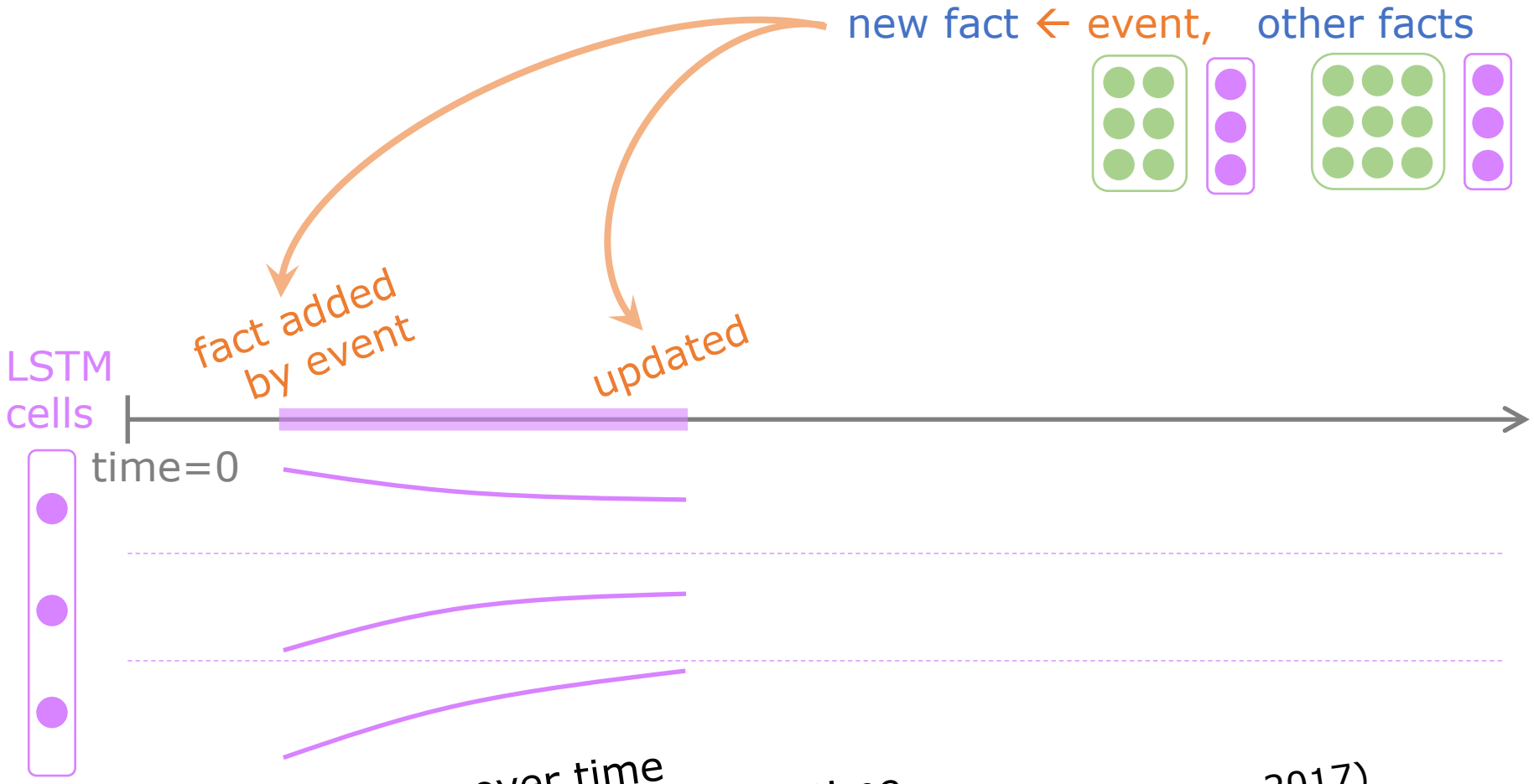

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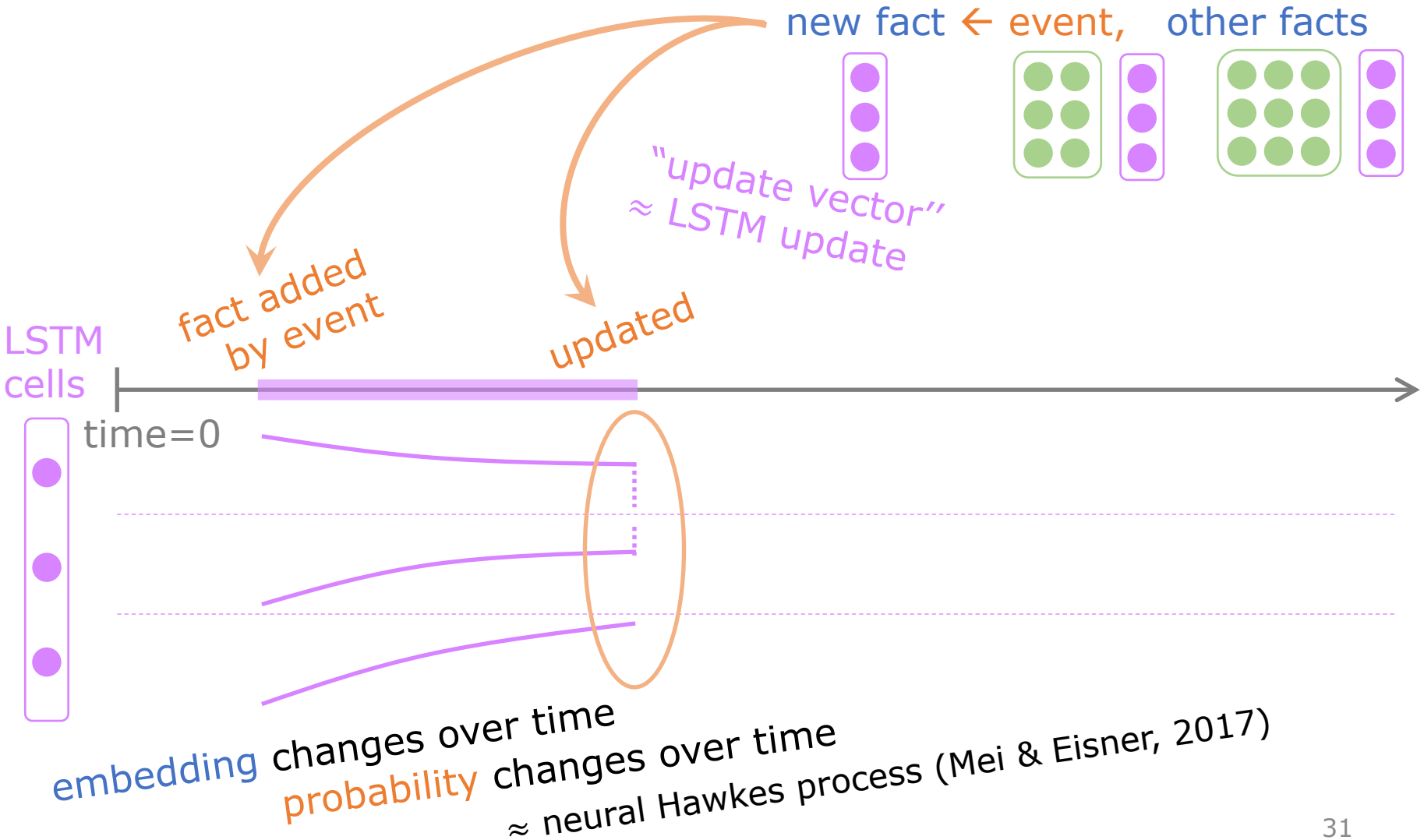

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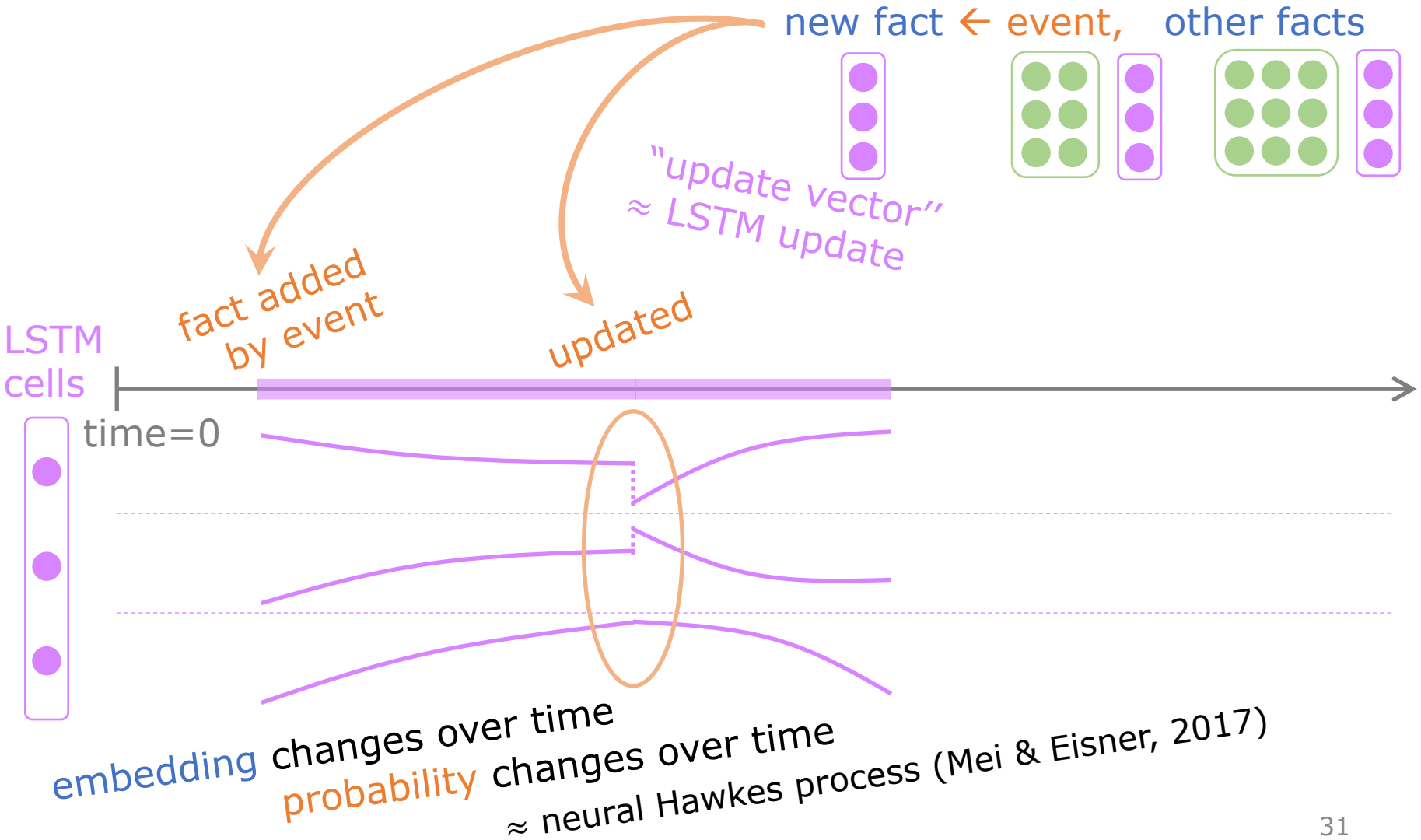
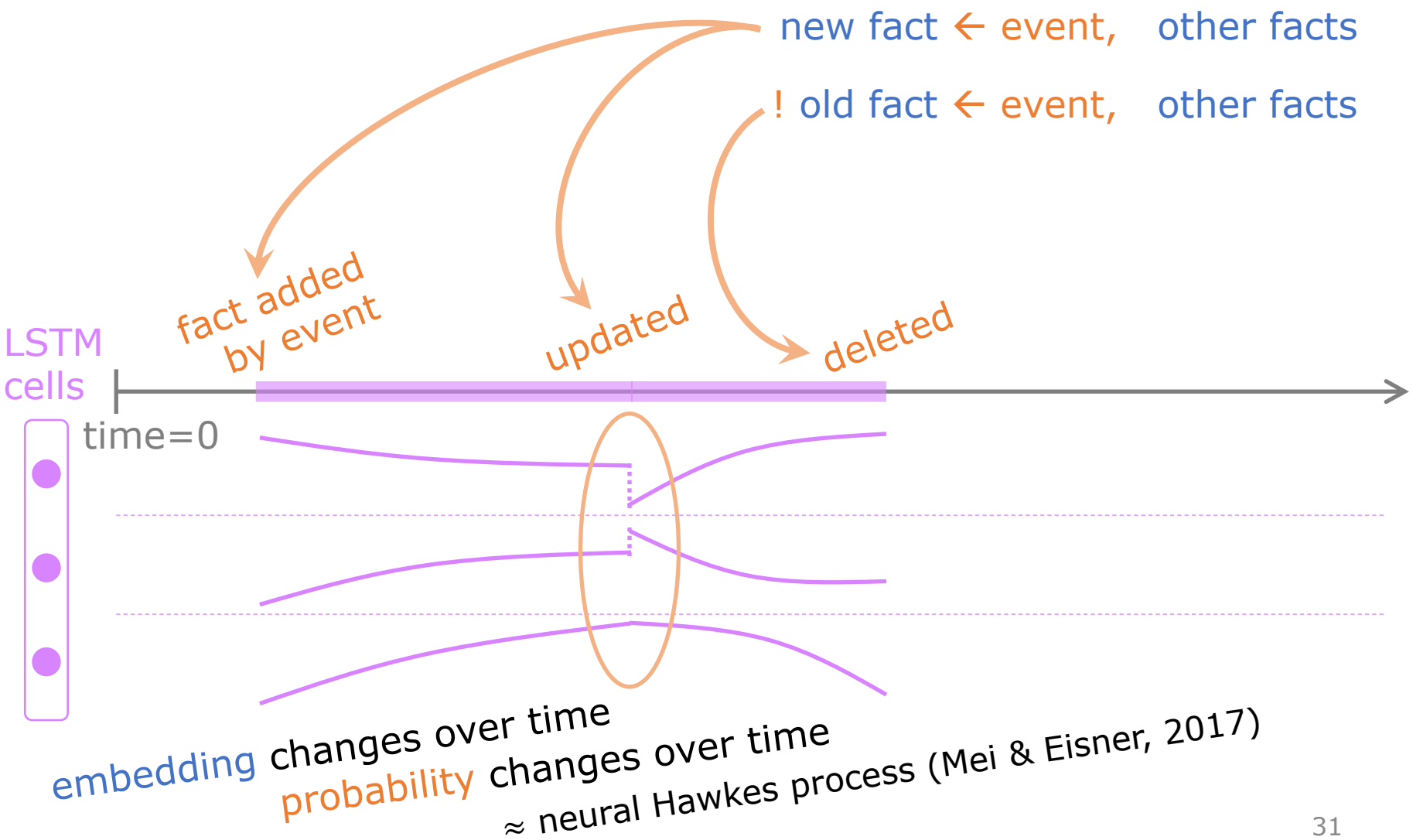


Life Story of a Fact



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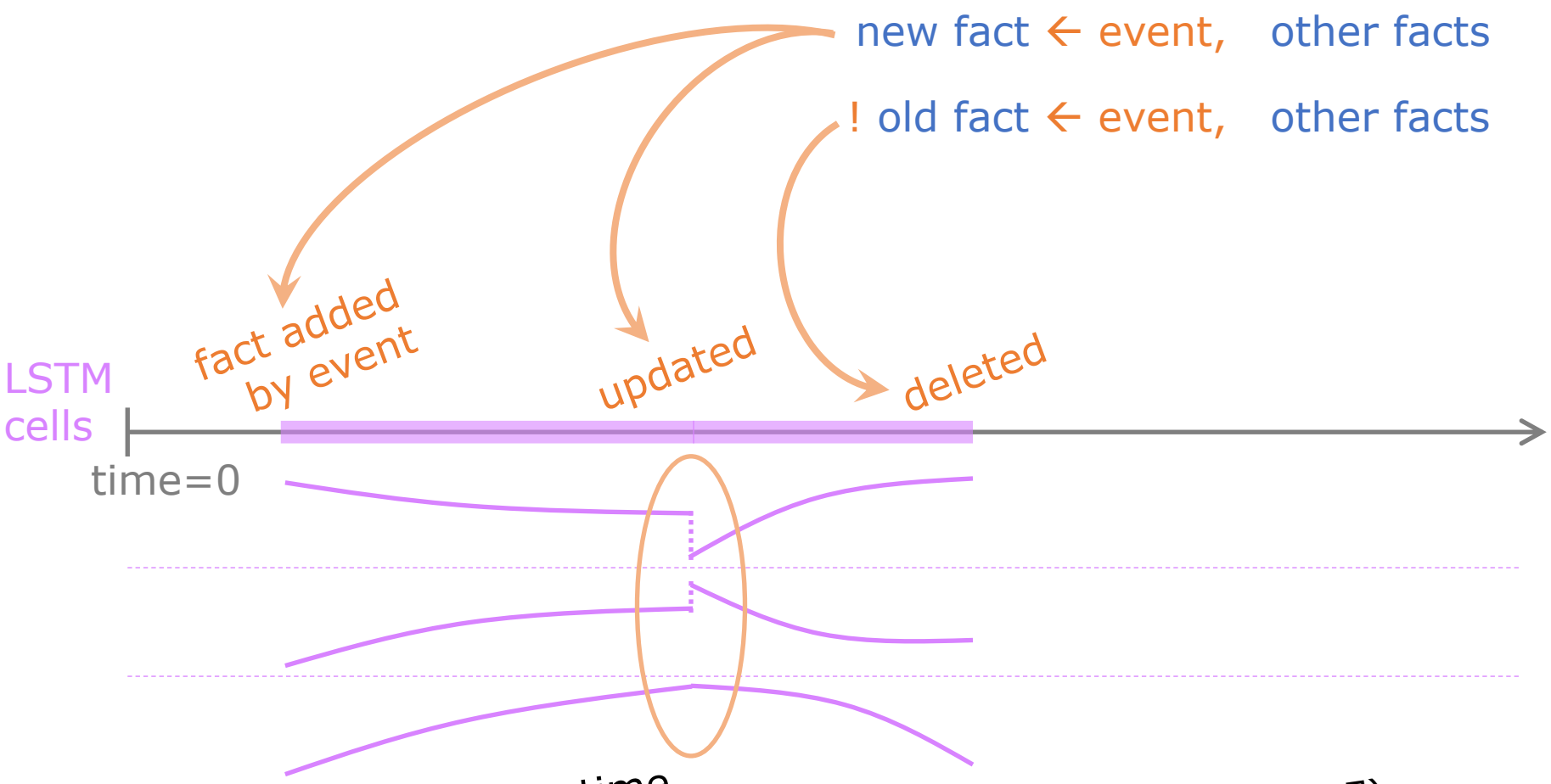

embedding changes over time
probability changes over time
≈ neural Hawkes process (Mei & Eisner, 2017)

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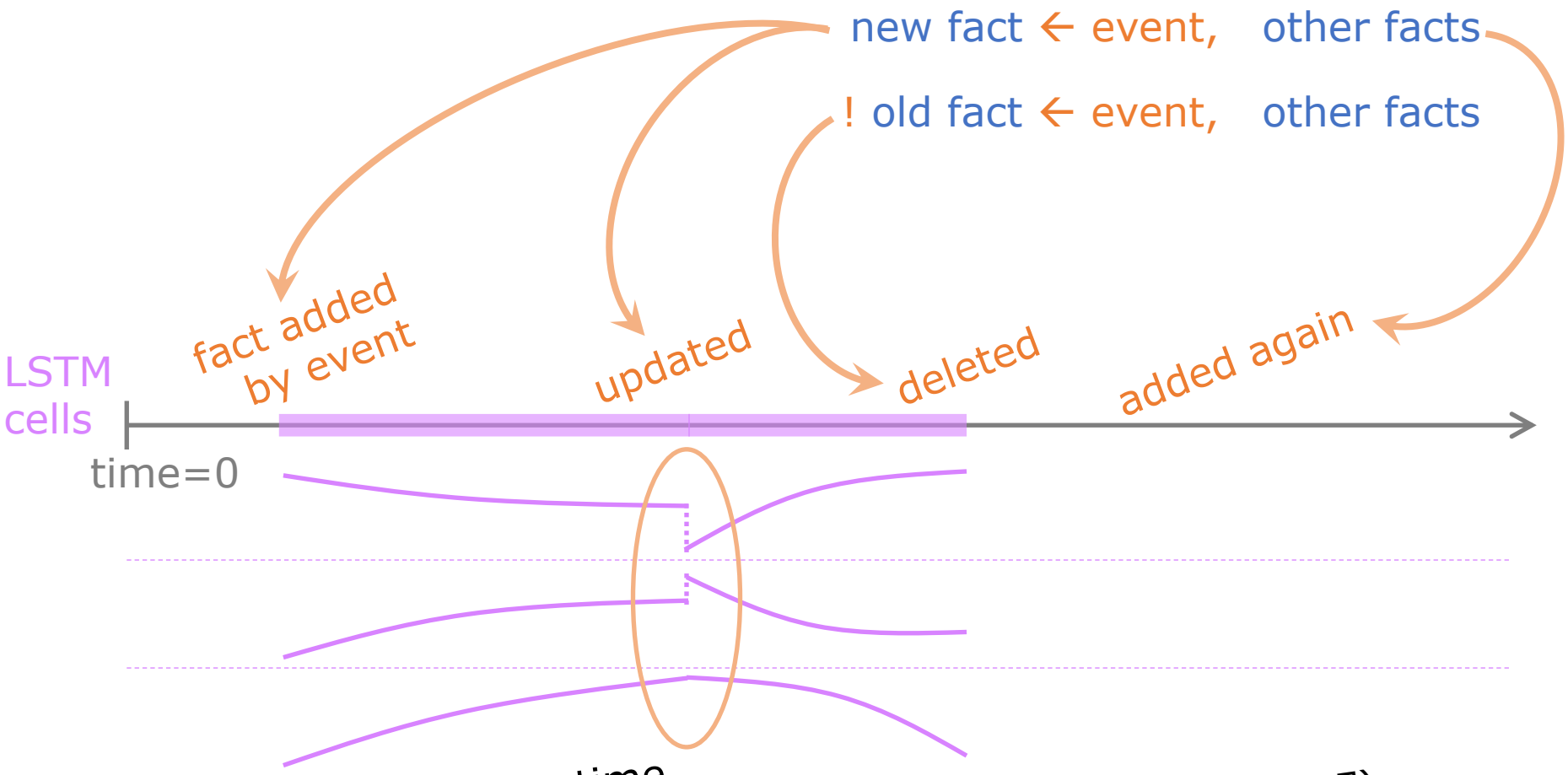

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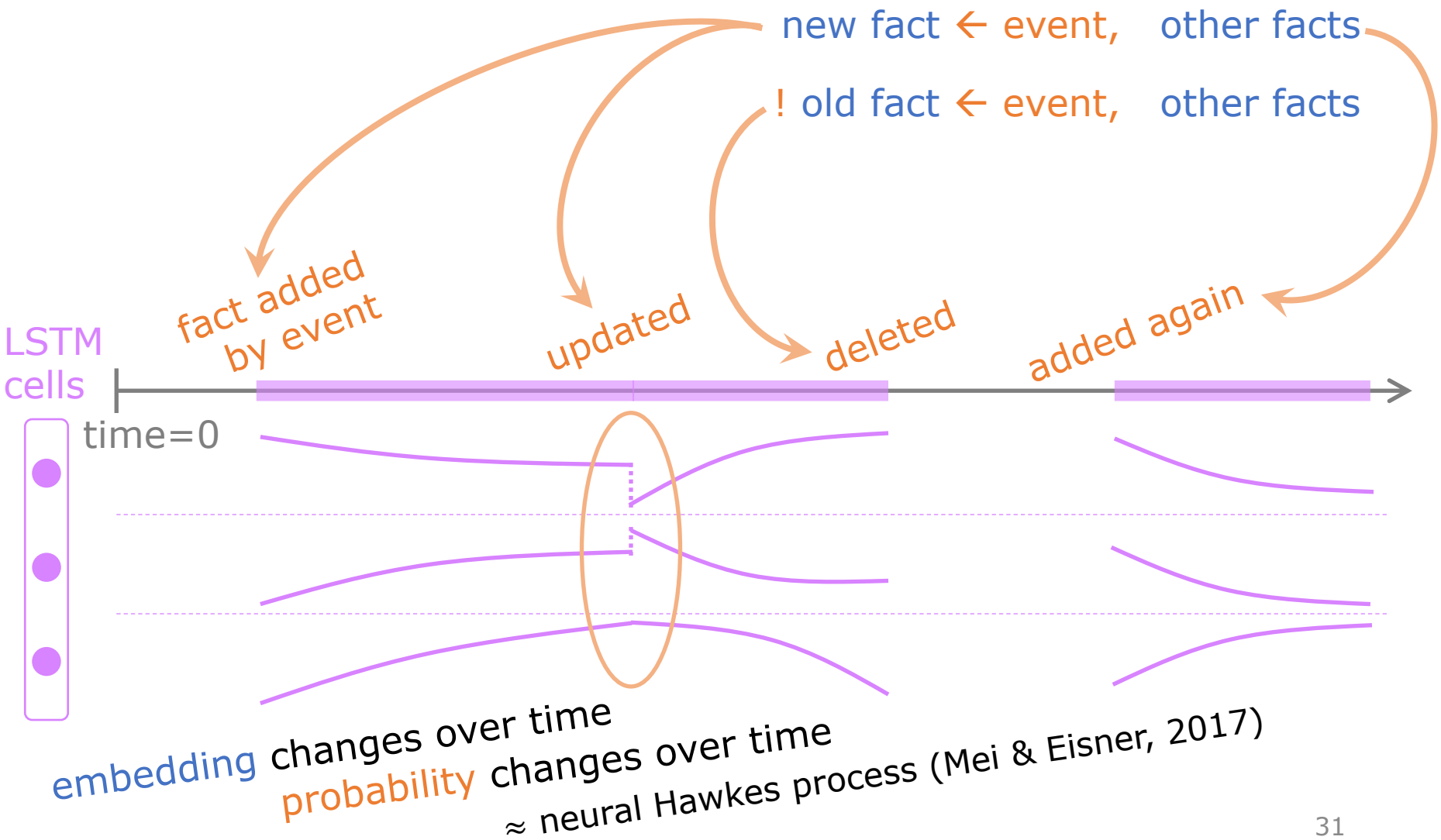


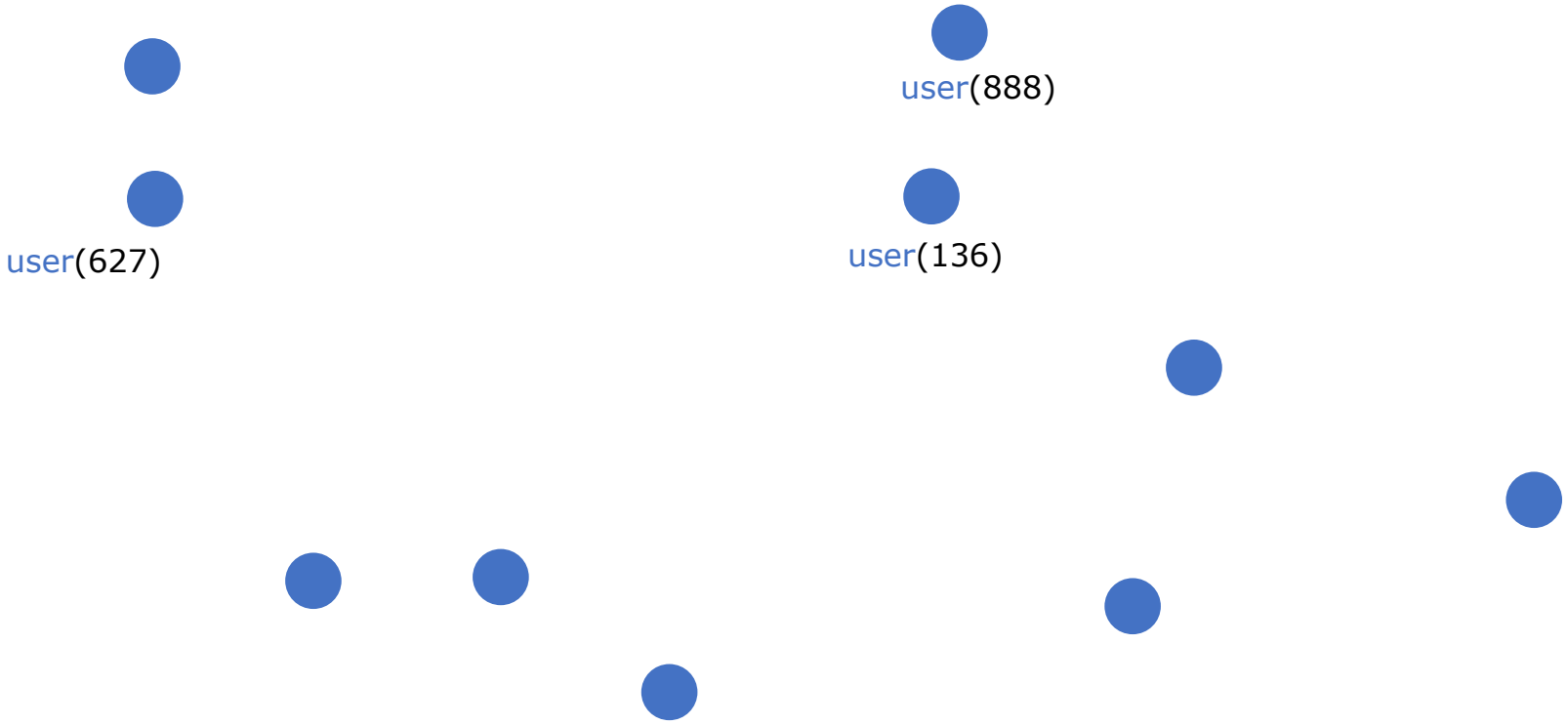
Experiment: Users watch TV programs

collaborative filtering problem with timing
who watches what and when?

Experiment: Users watch TV programs

collaborative filtering problem with timing
who watches what and when?

1000 users



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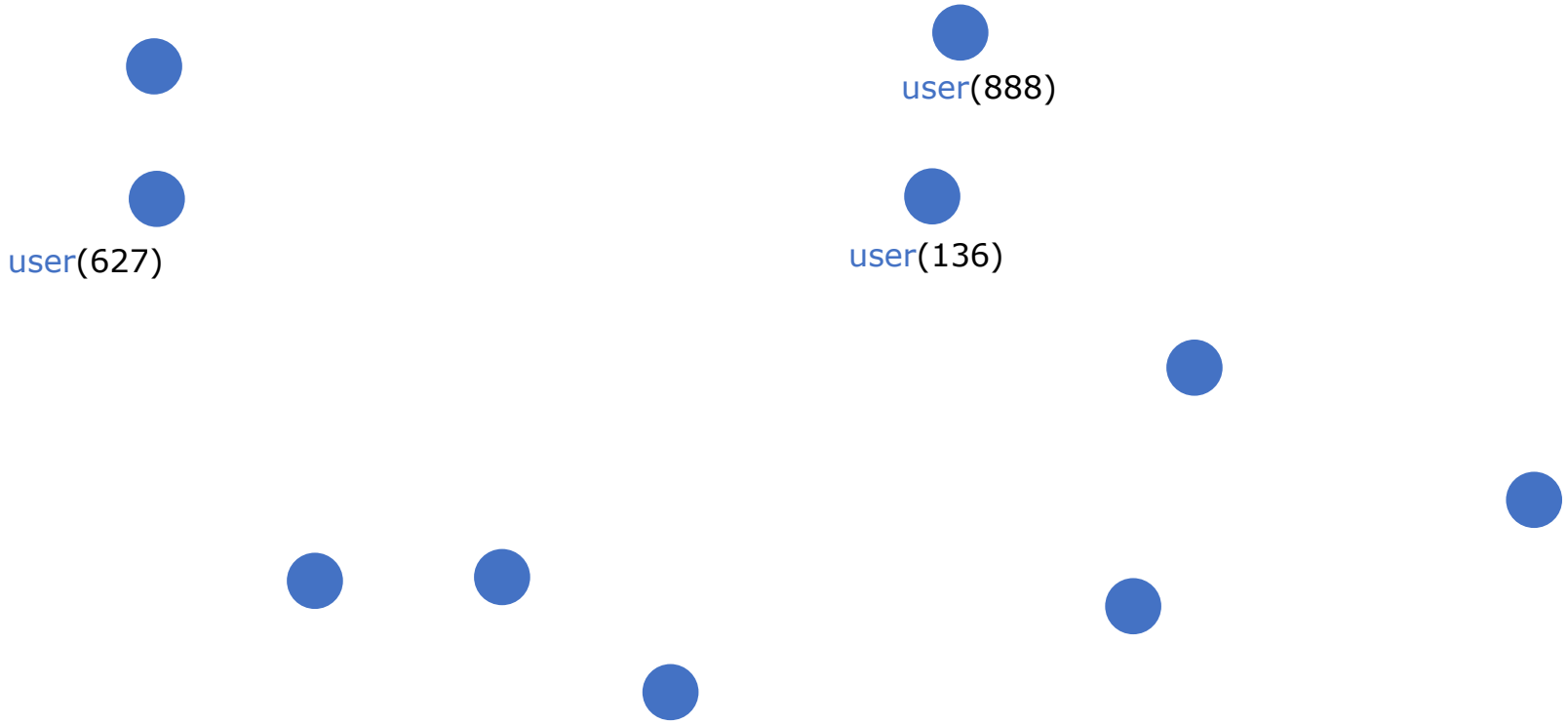
1000 users 49 TV programs to be released



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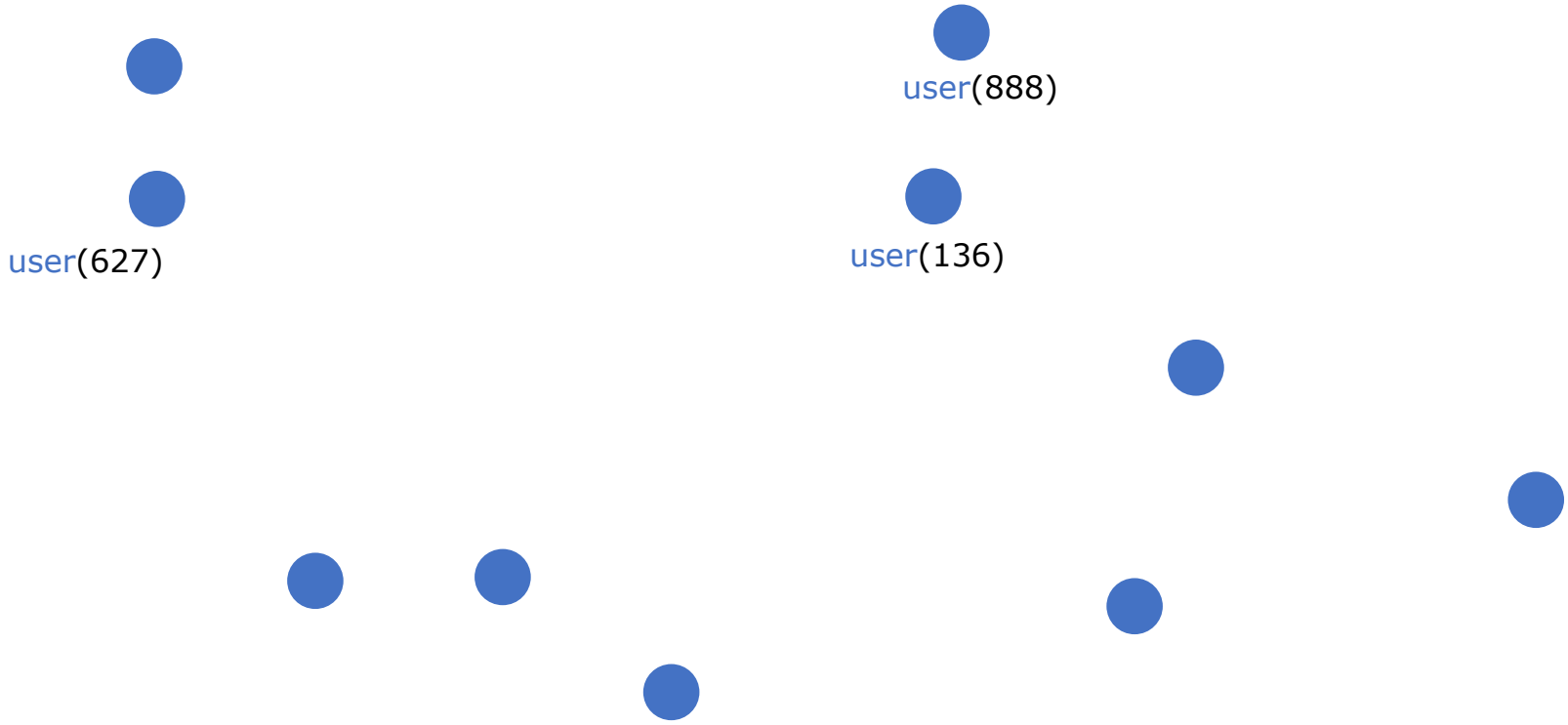
1000 users 49 TV programs to be released

49000 possible watch events



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can not watch it
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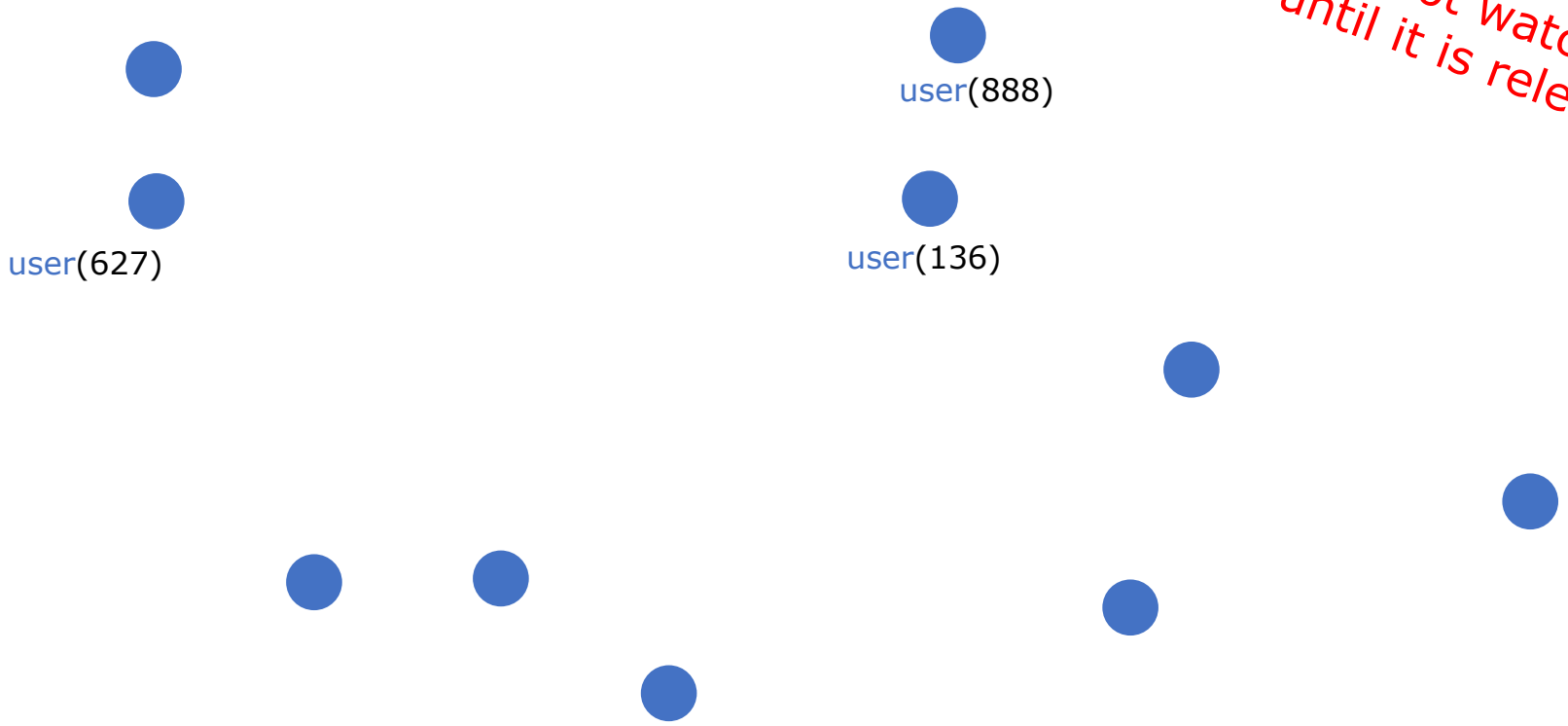

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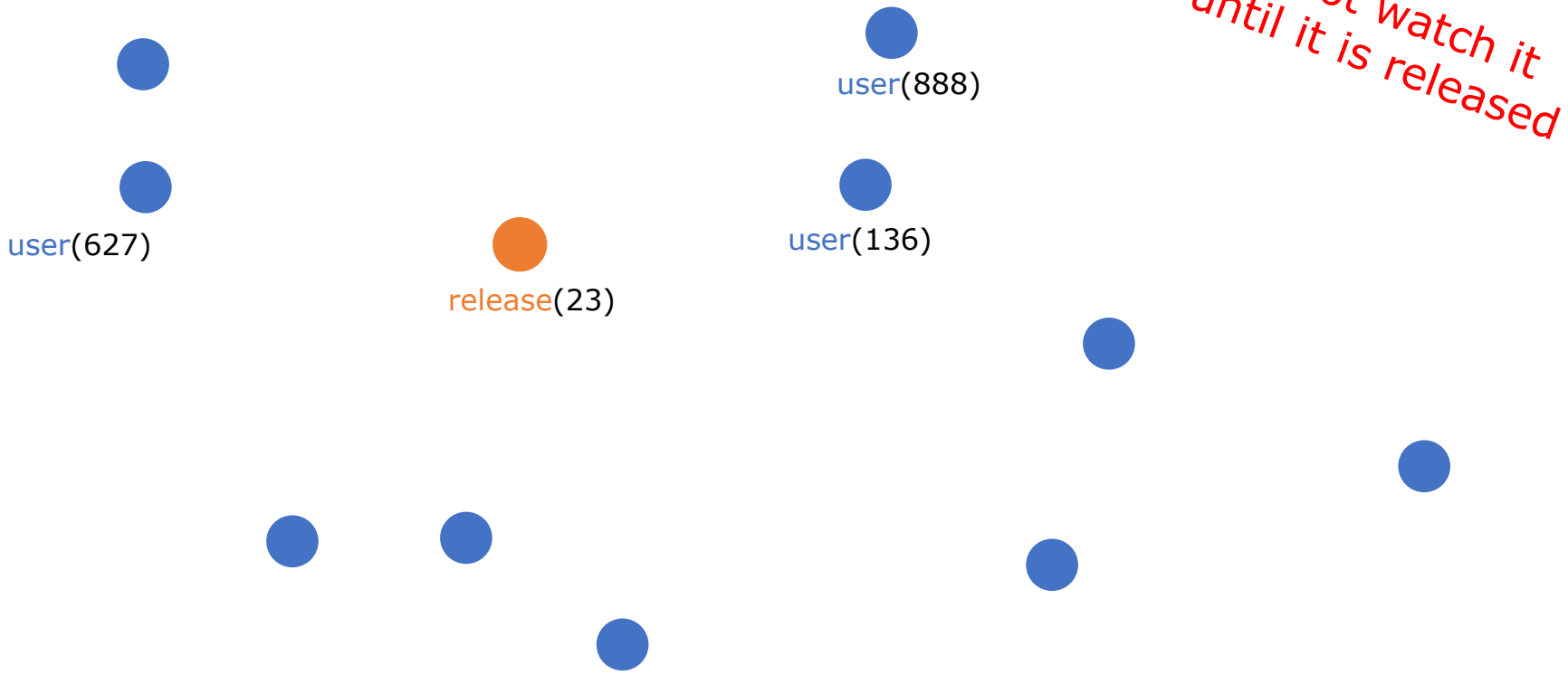

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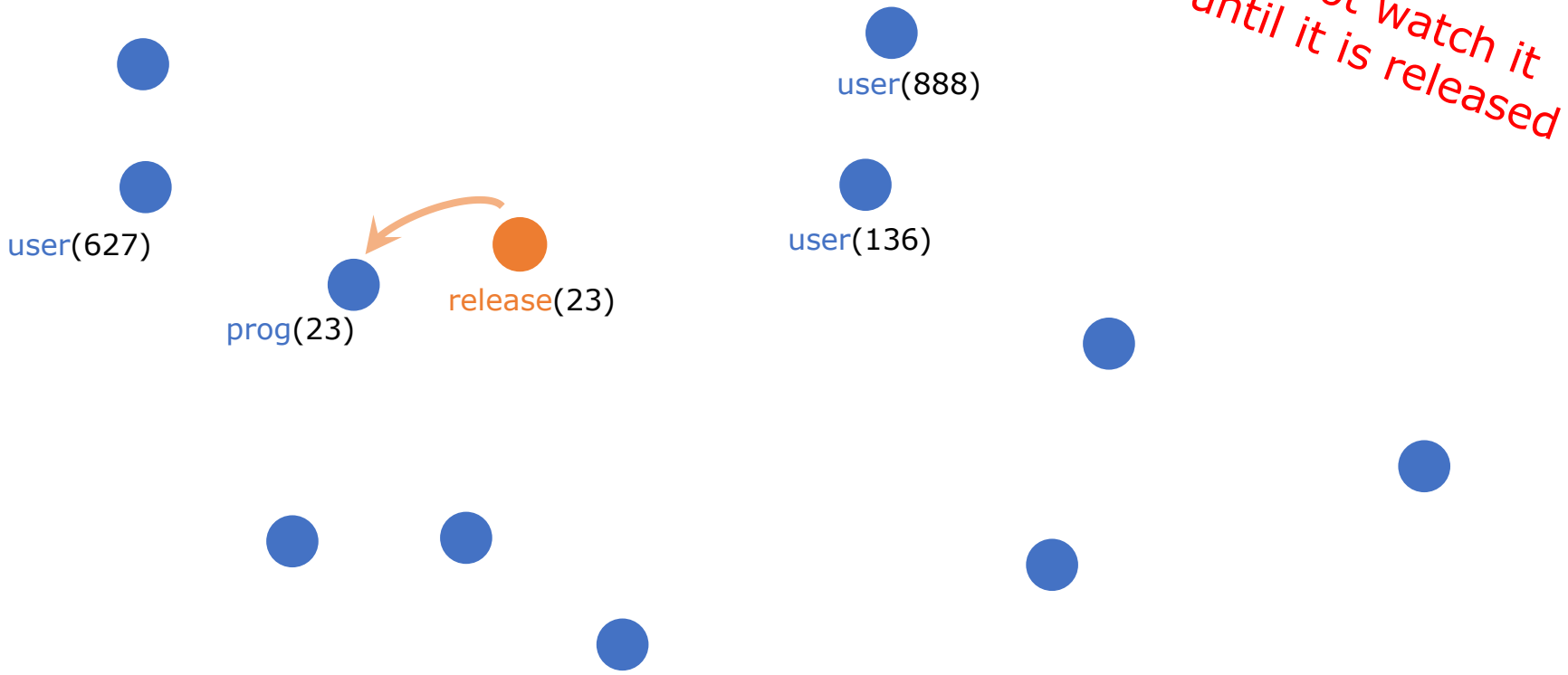

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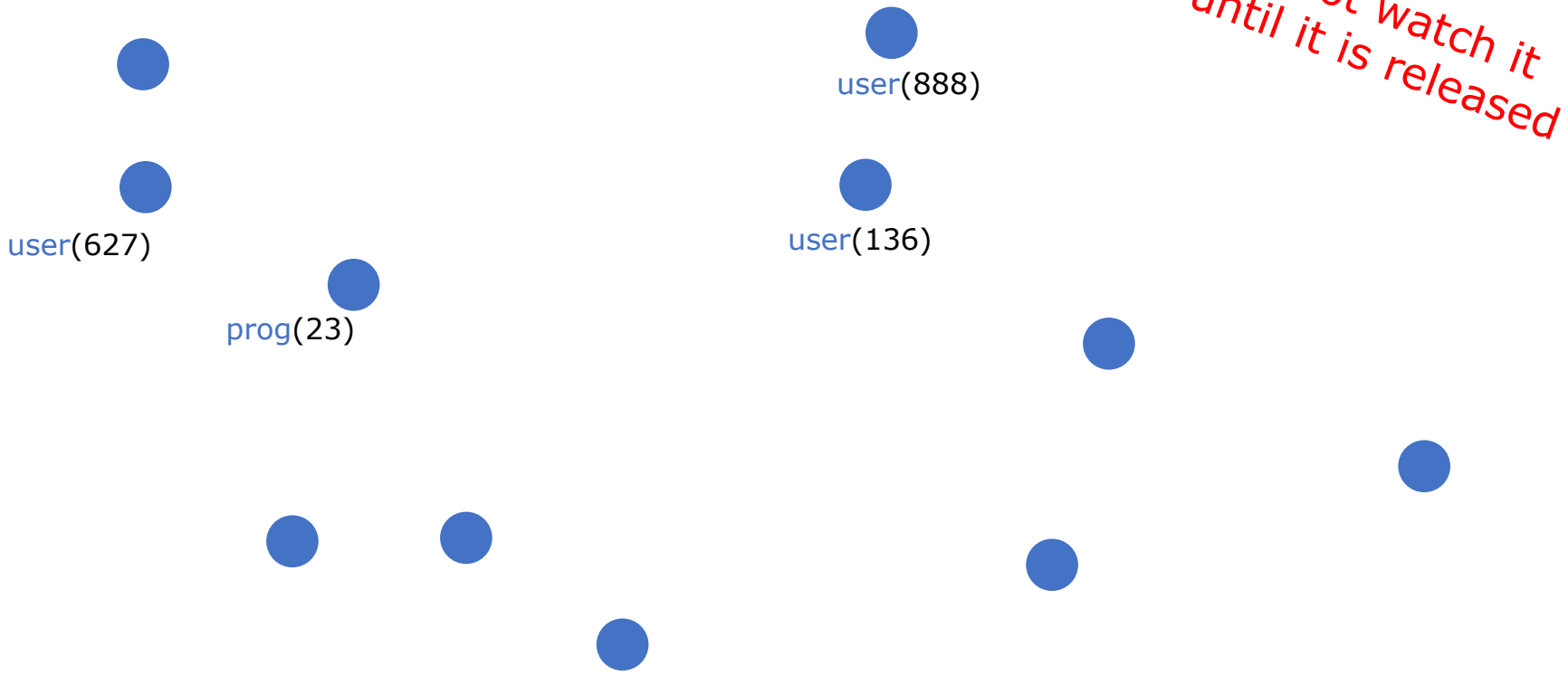

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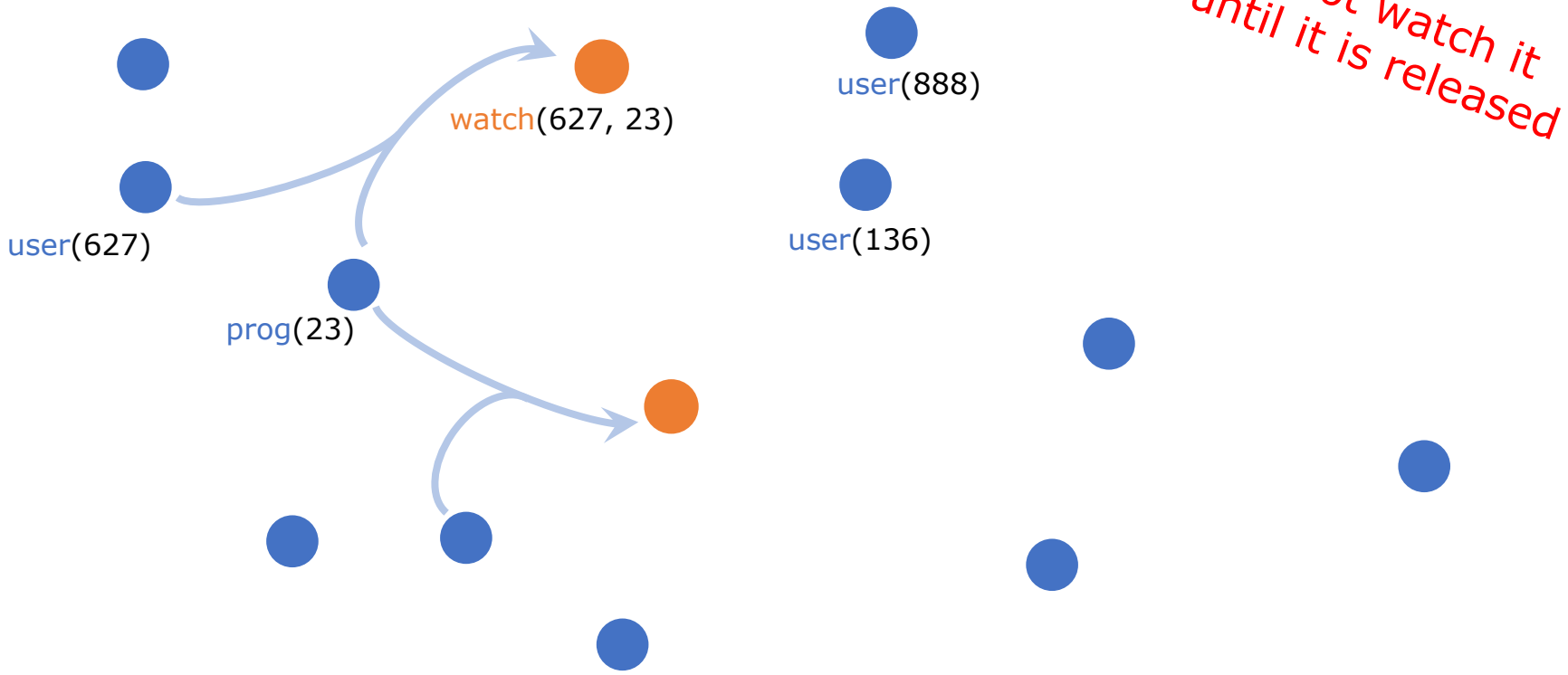

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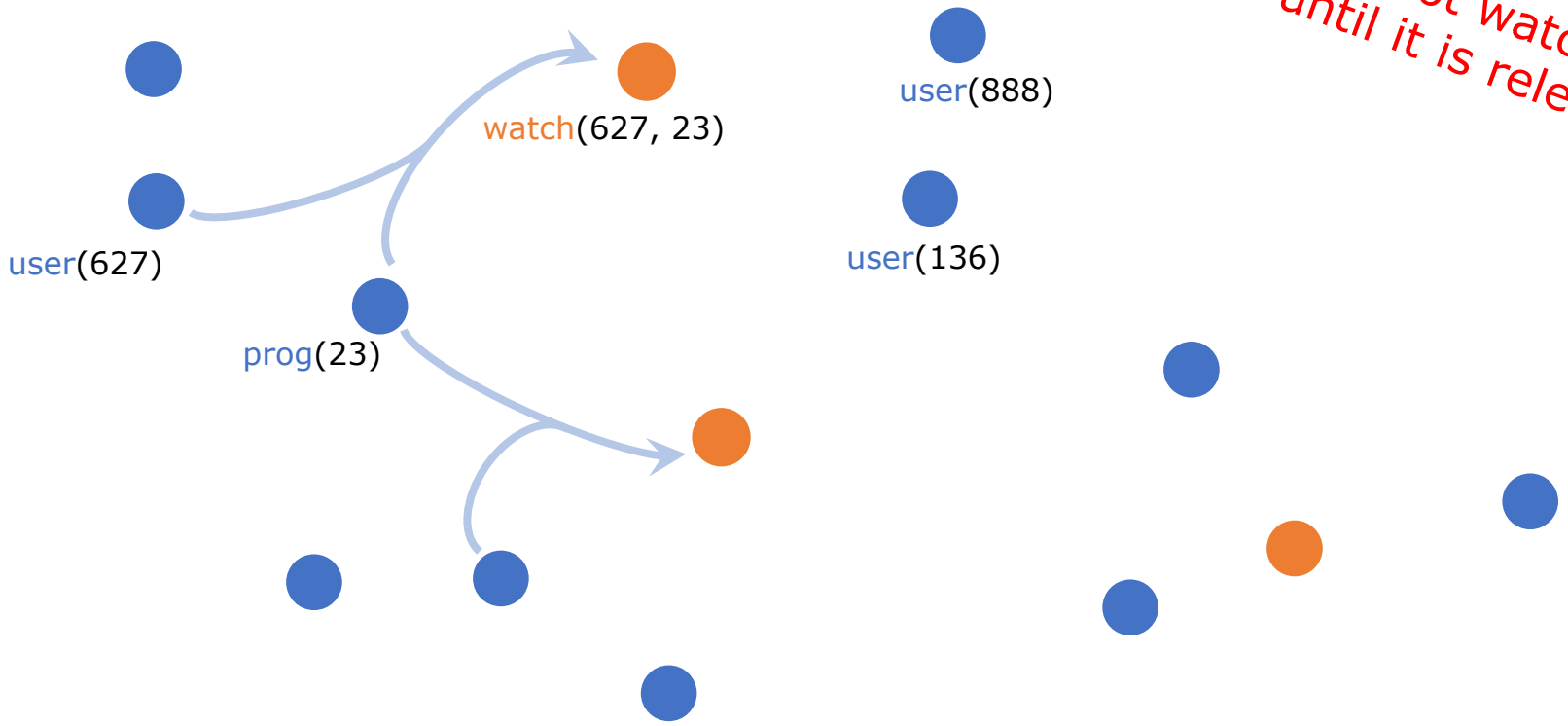

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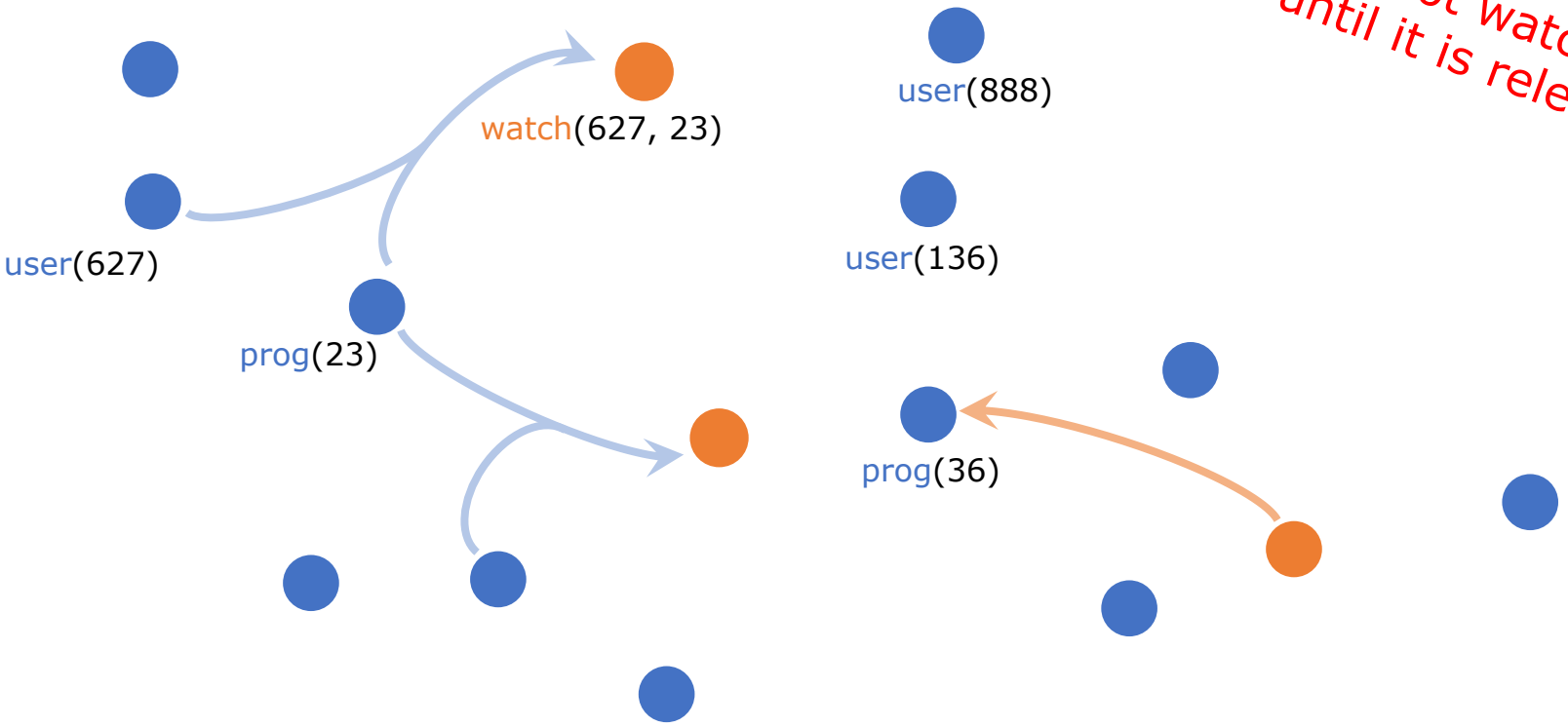

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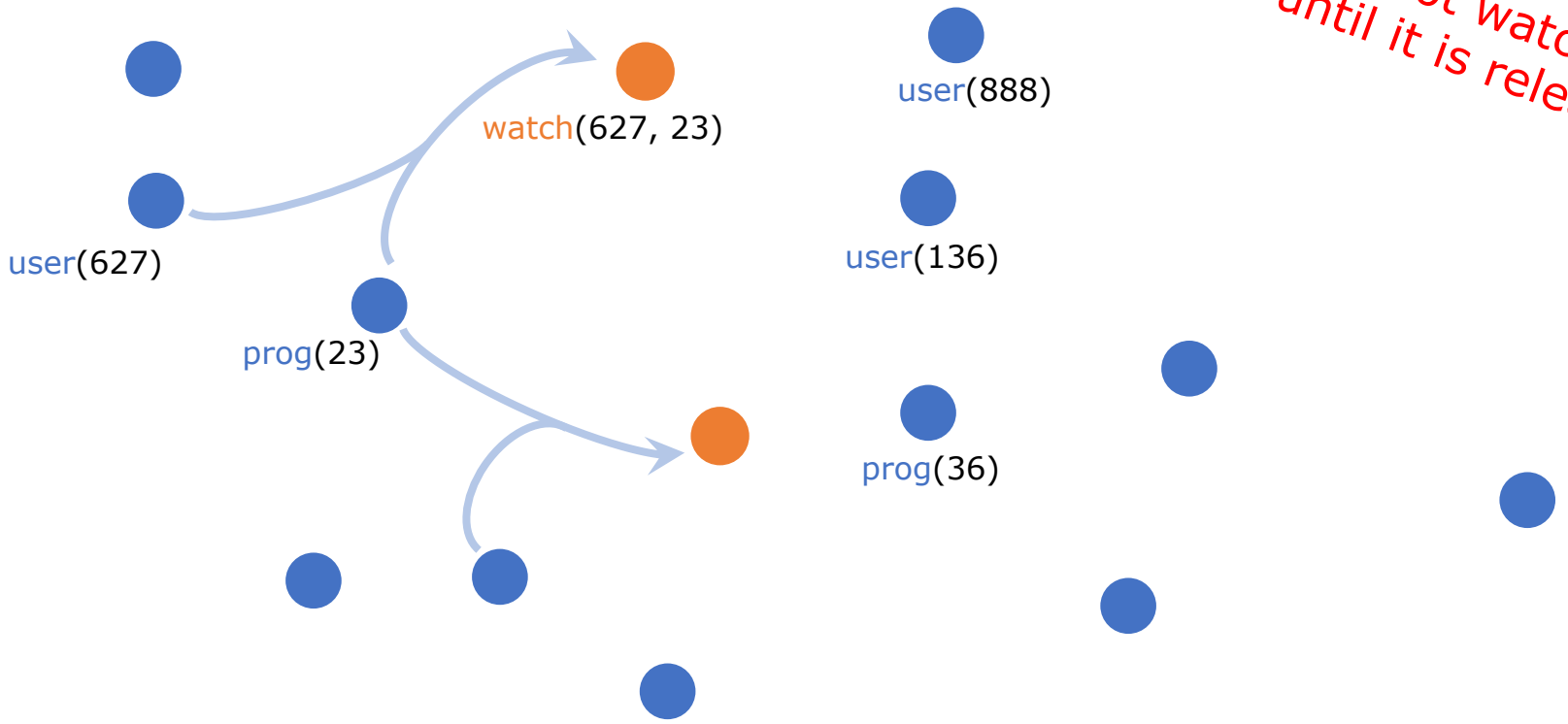

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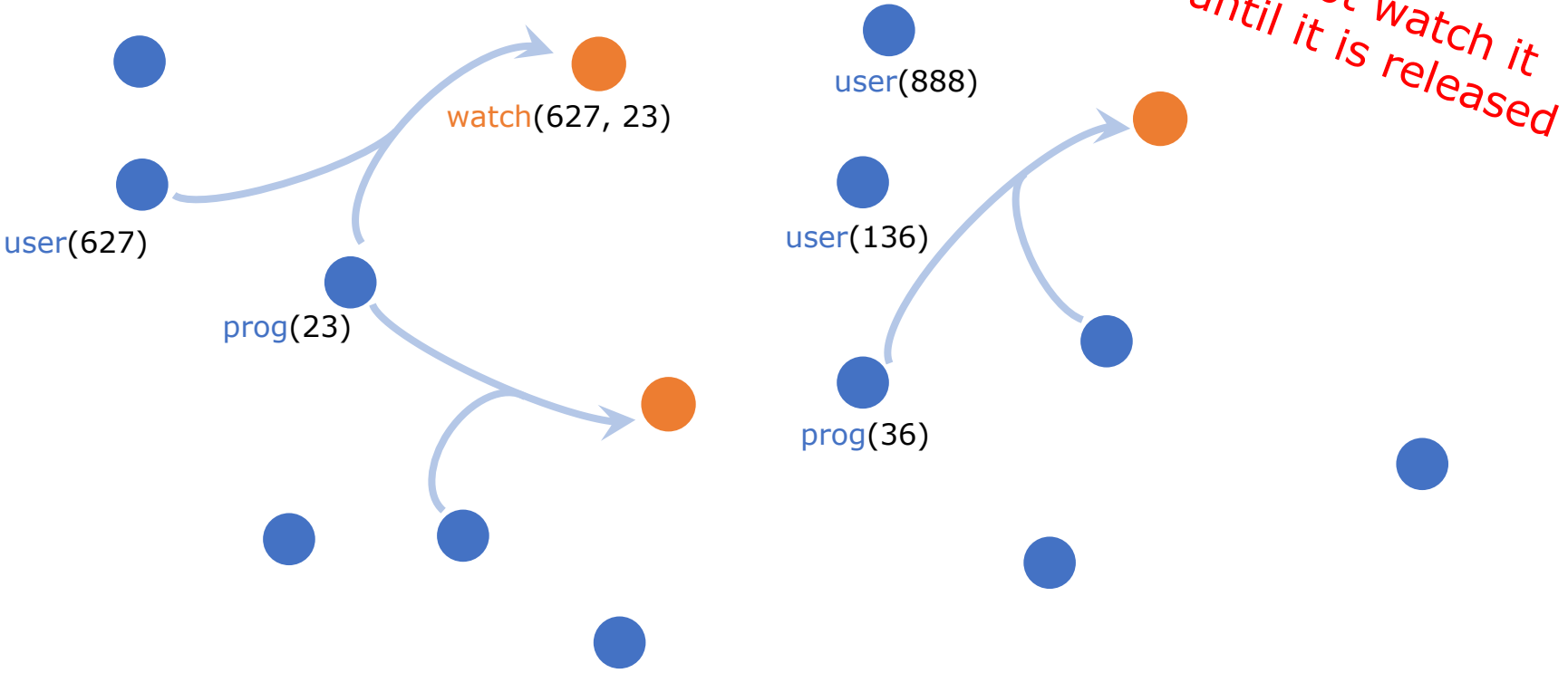

Experiment: Robots Kick/Pass/Steal

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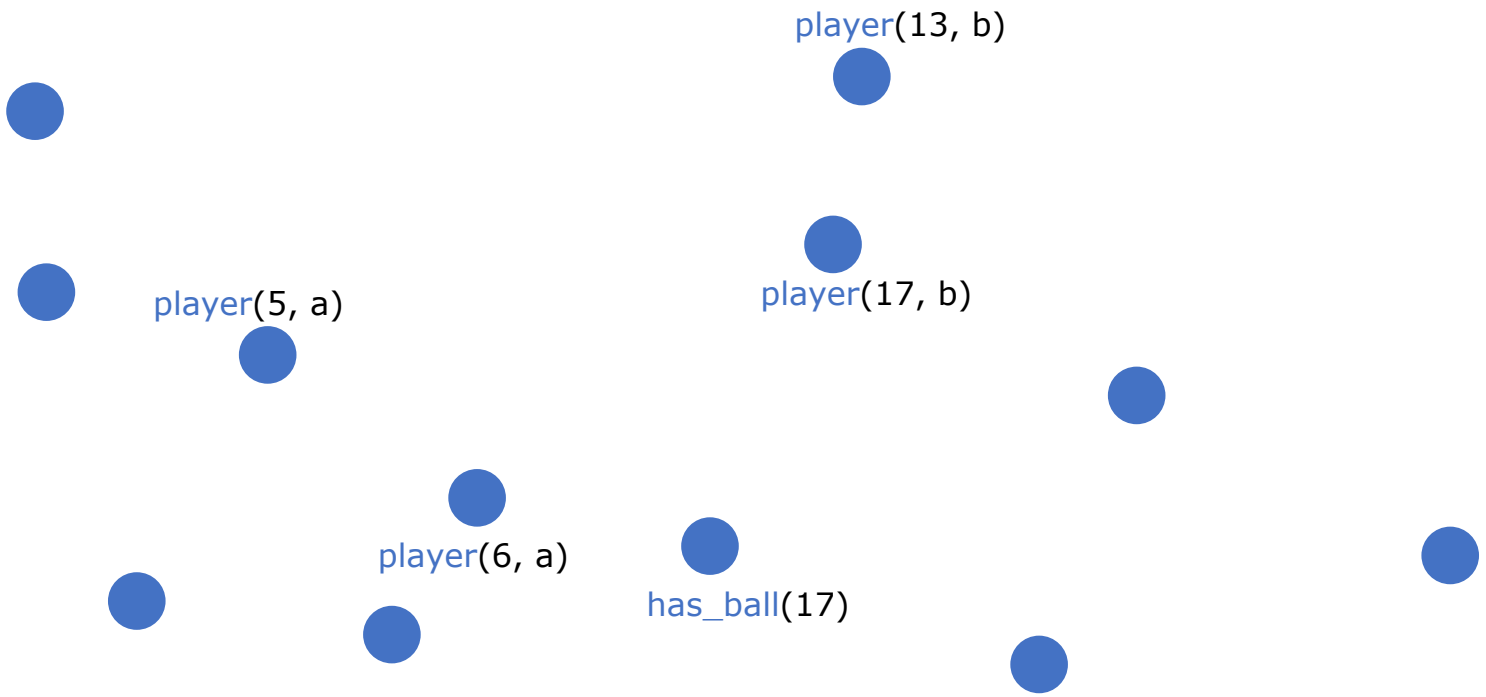
22 robot soccer players

player(Number, Team)



Experiment: Robots Kick/Pass/Steal

kick

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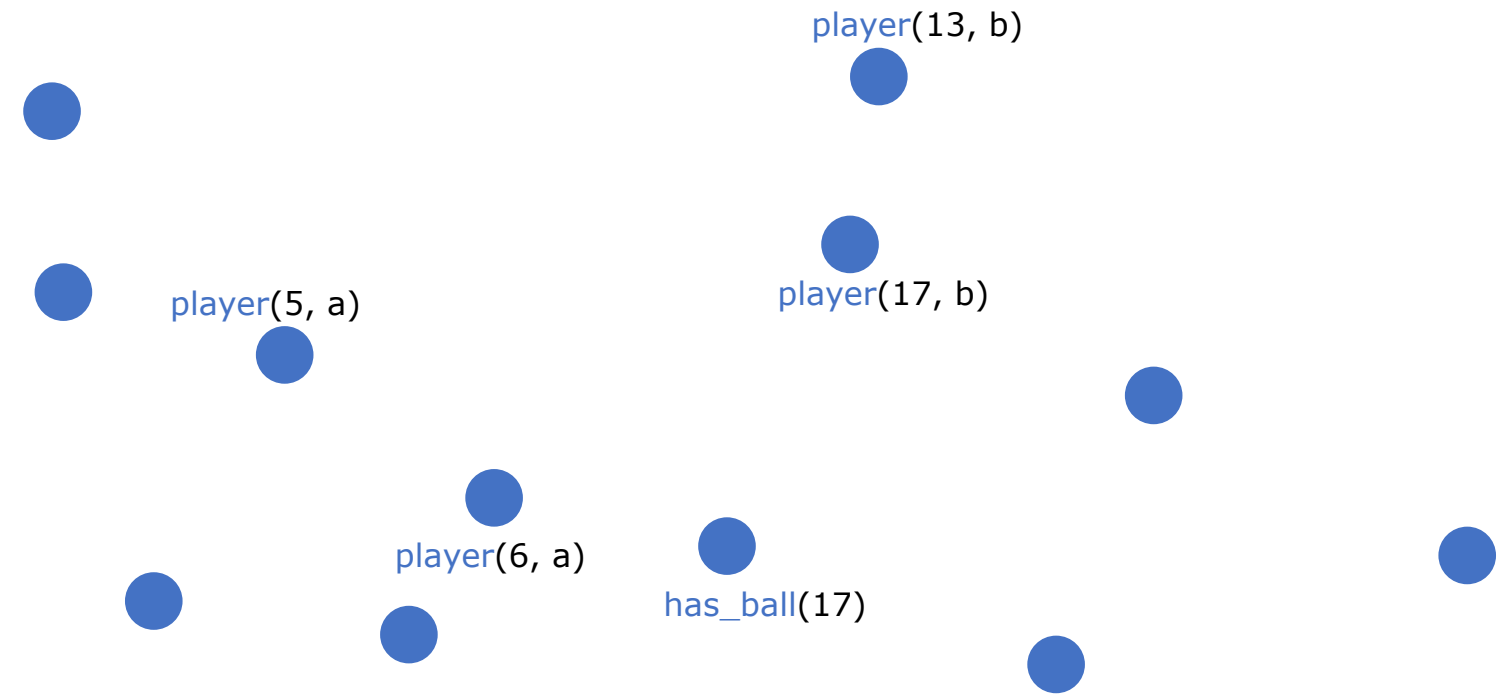

Experiment: Robots Kick/Pass/Steal

kick only if has ball

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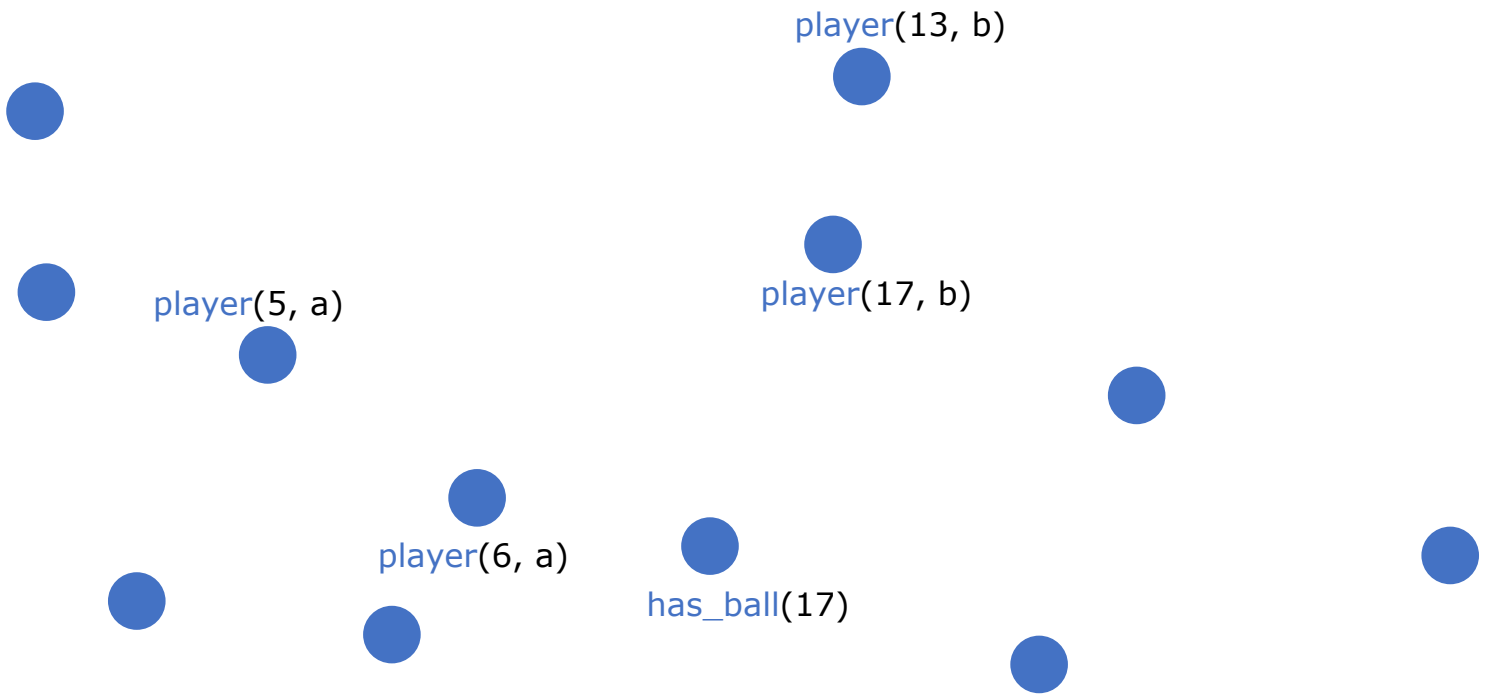

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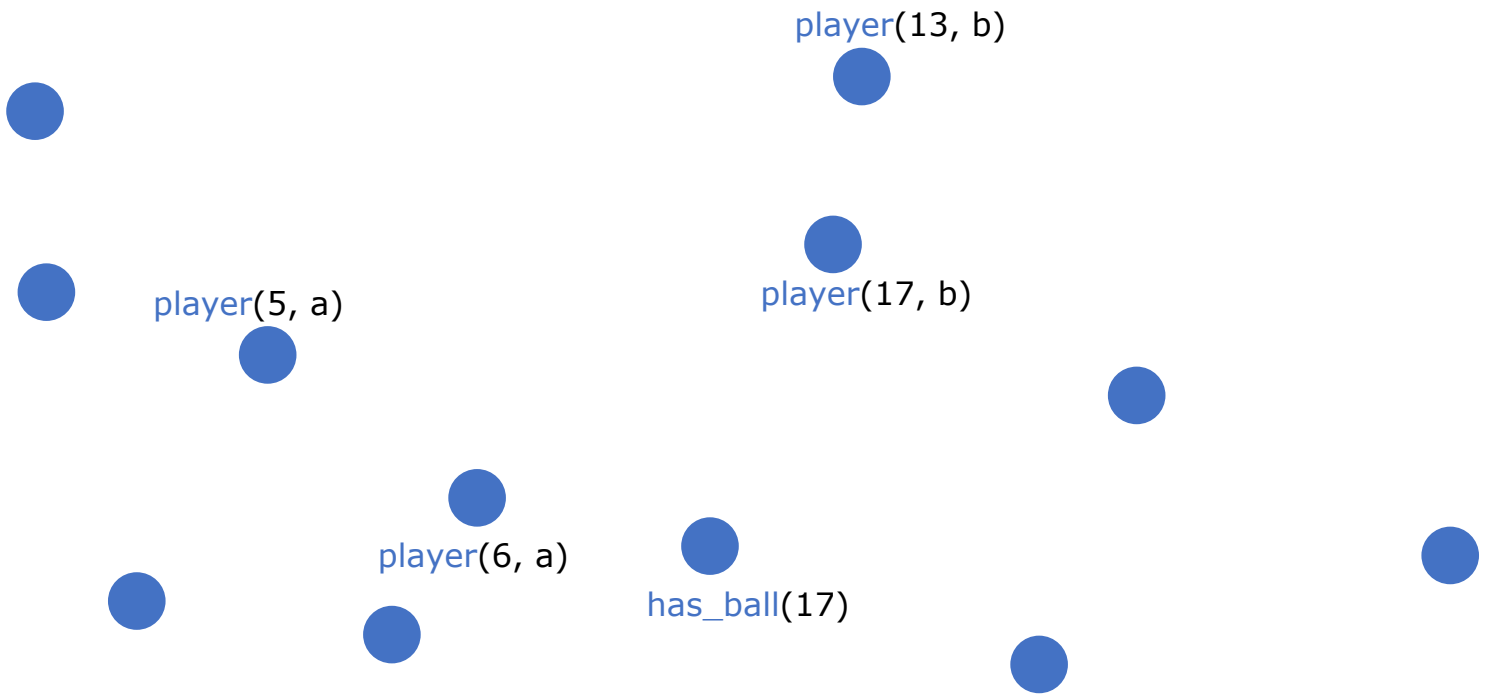
kick only if has ball

pass



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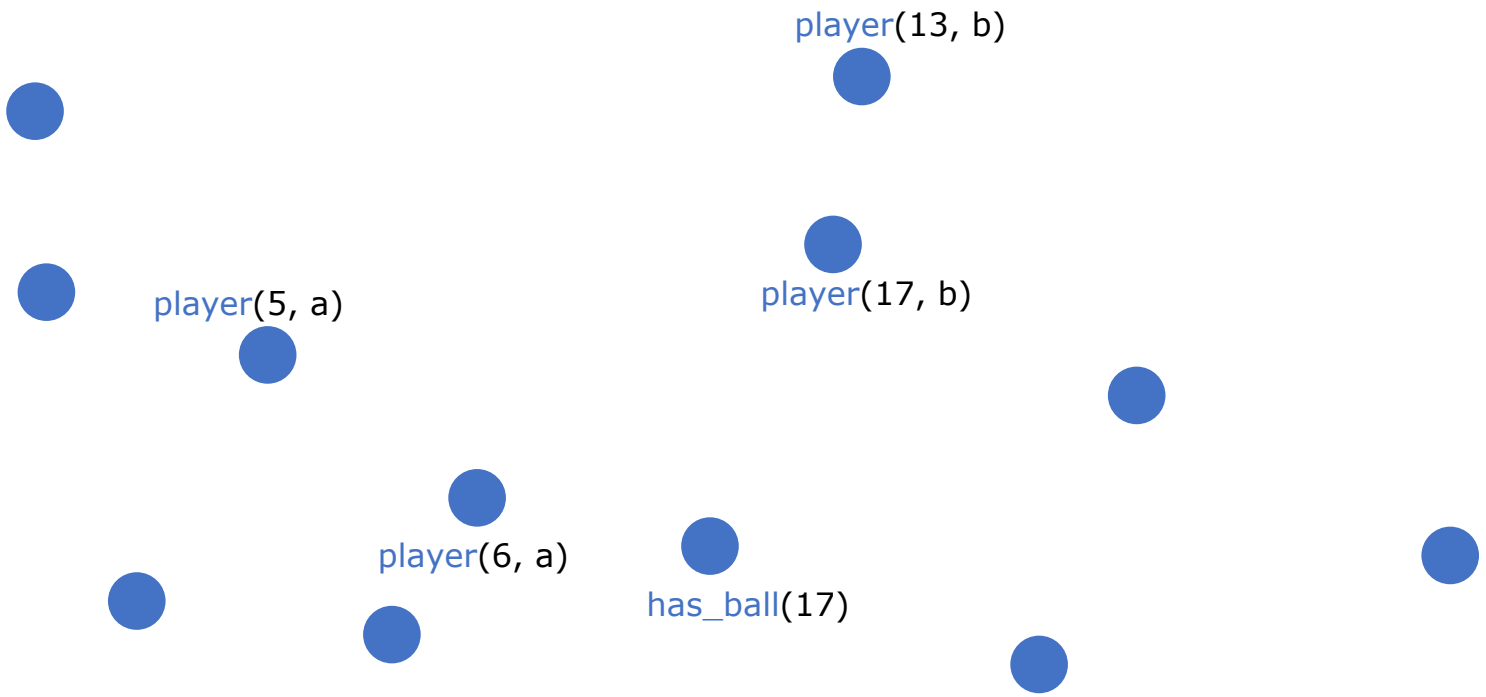
22 robot soccer players

player(Number, Team)

kick only if has ball

pass only to a teammate



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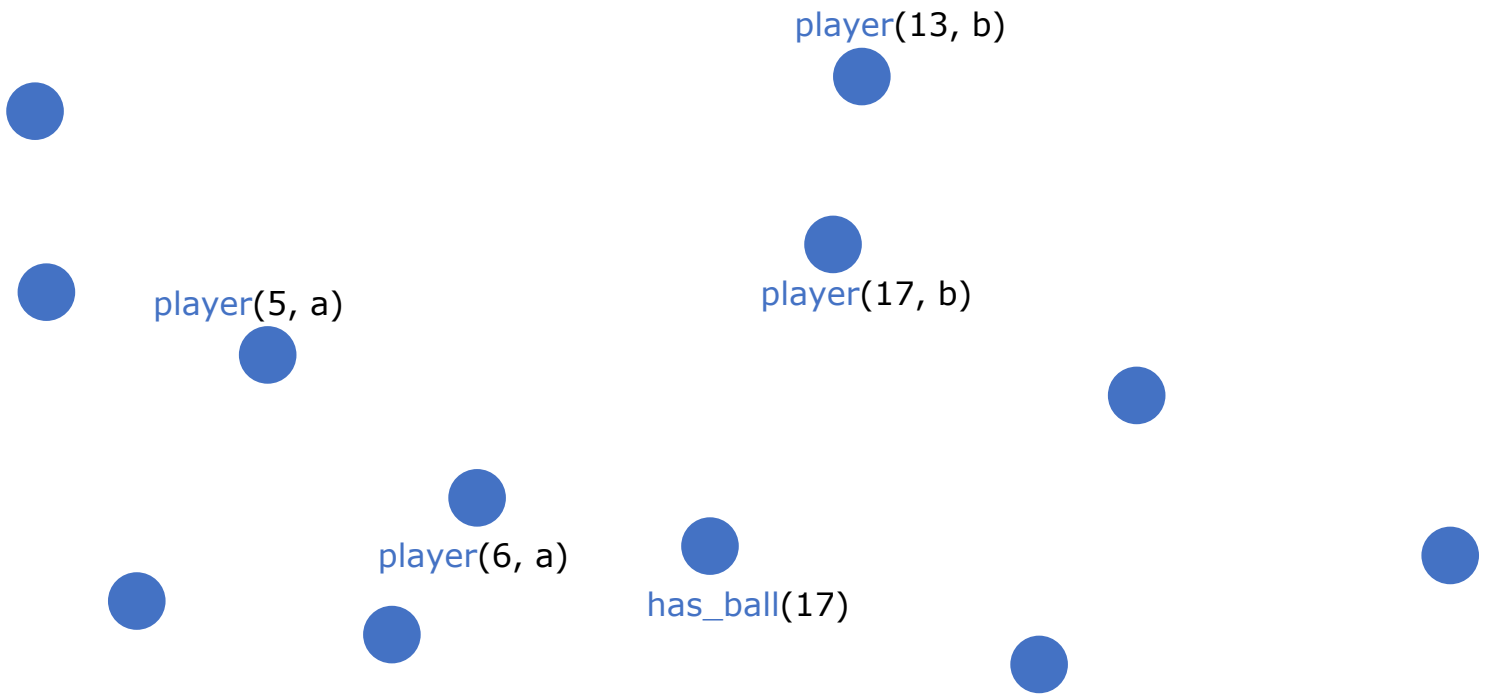

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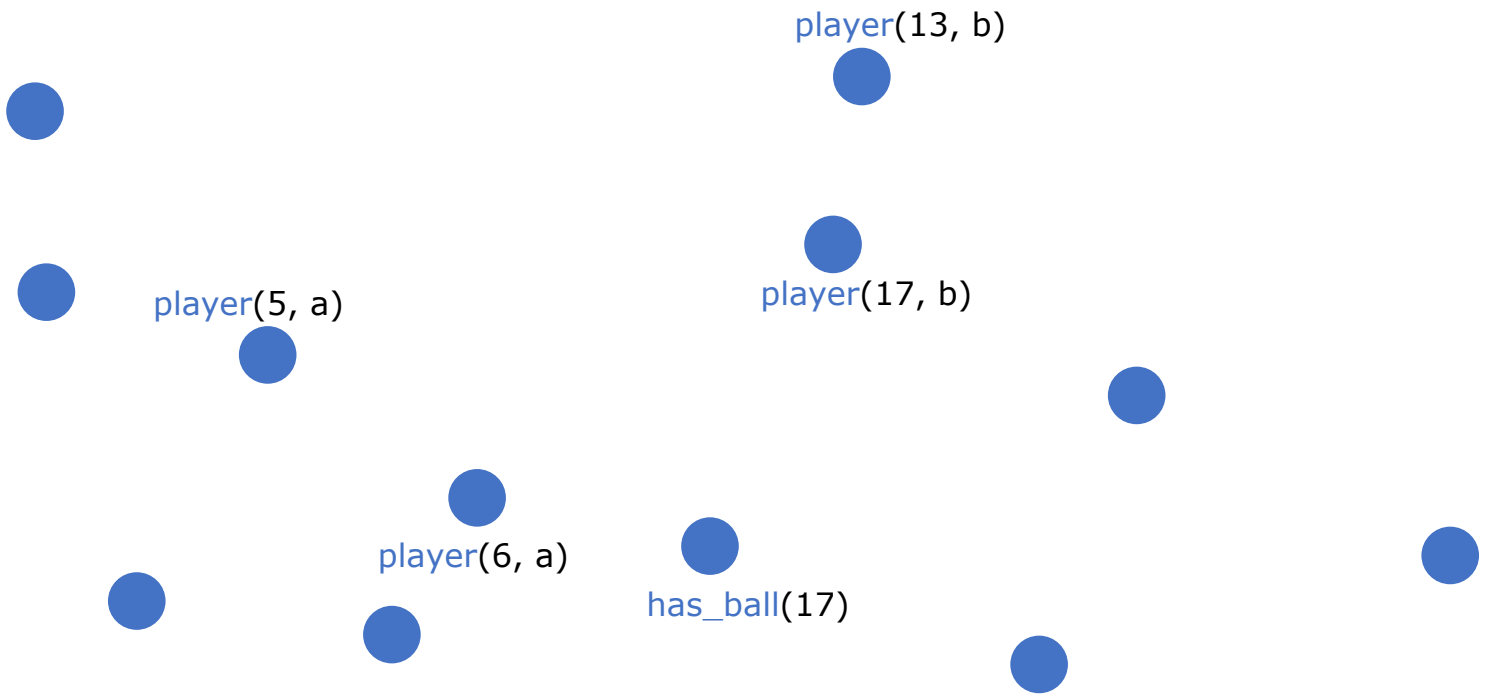

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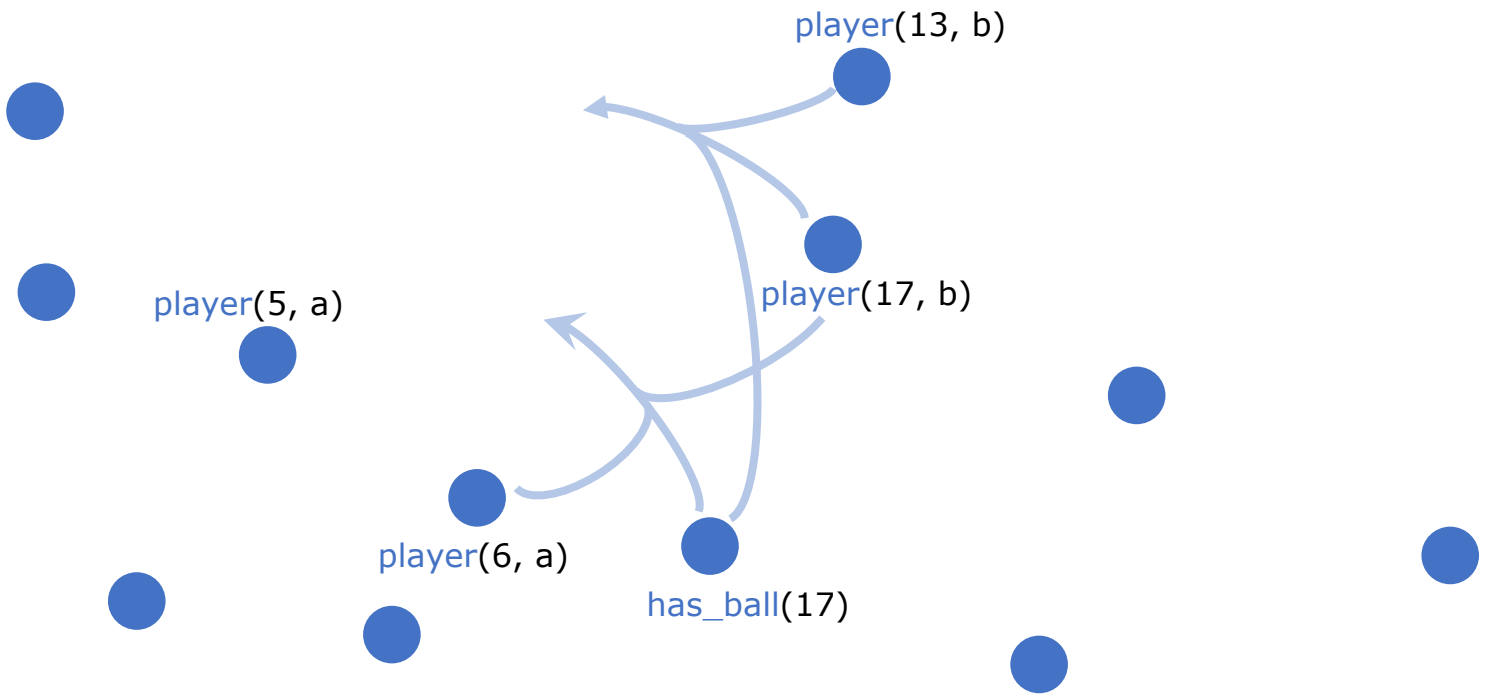

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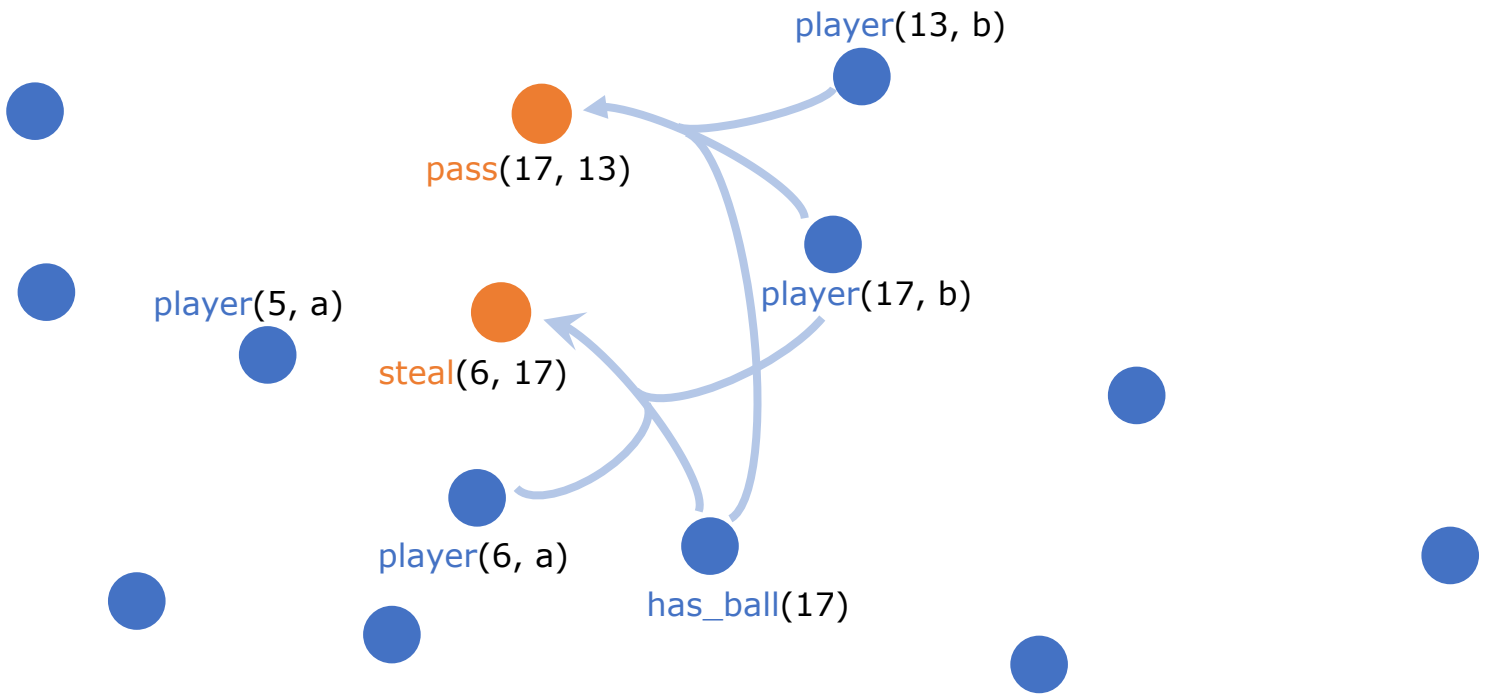

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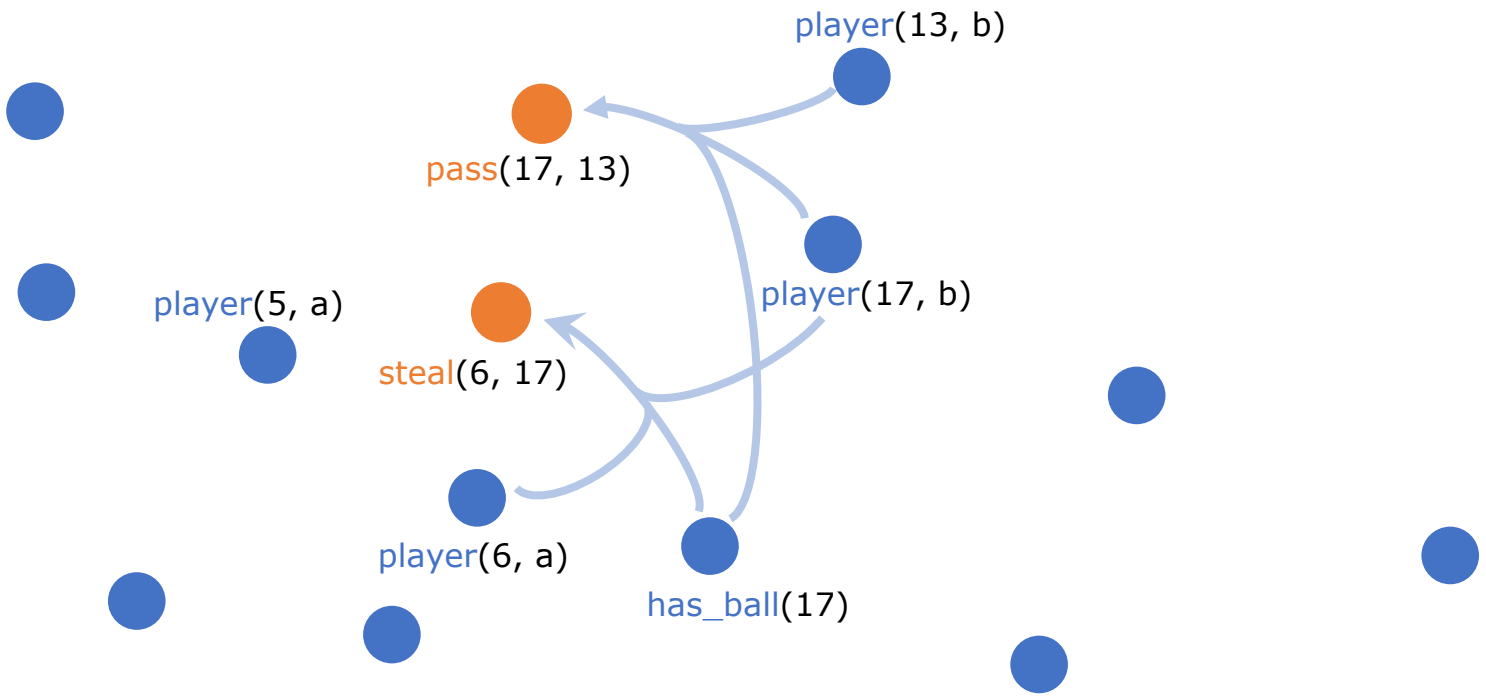

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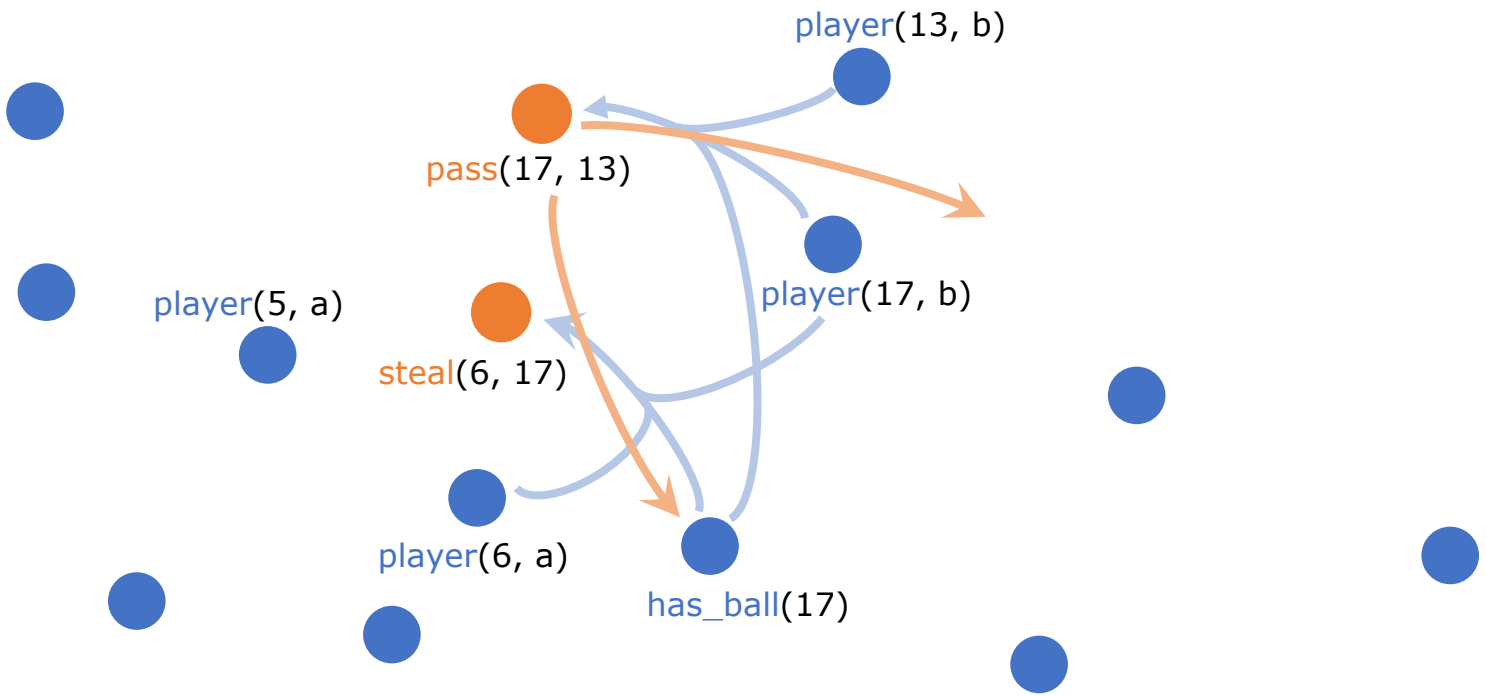

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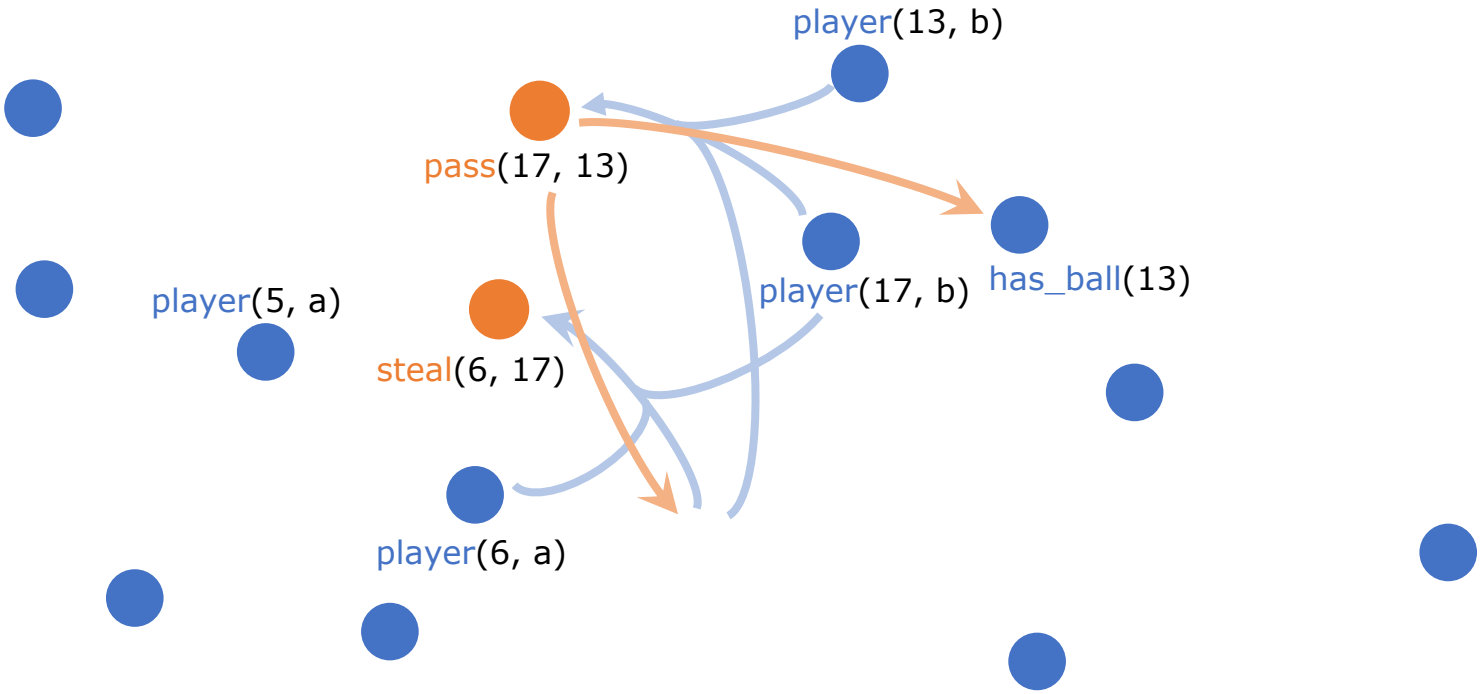

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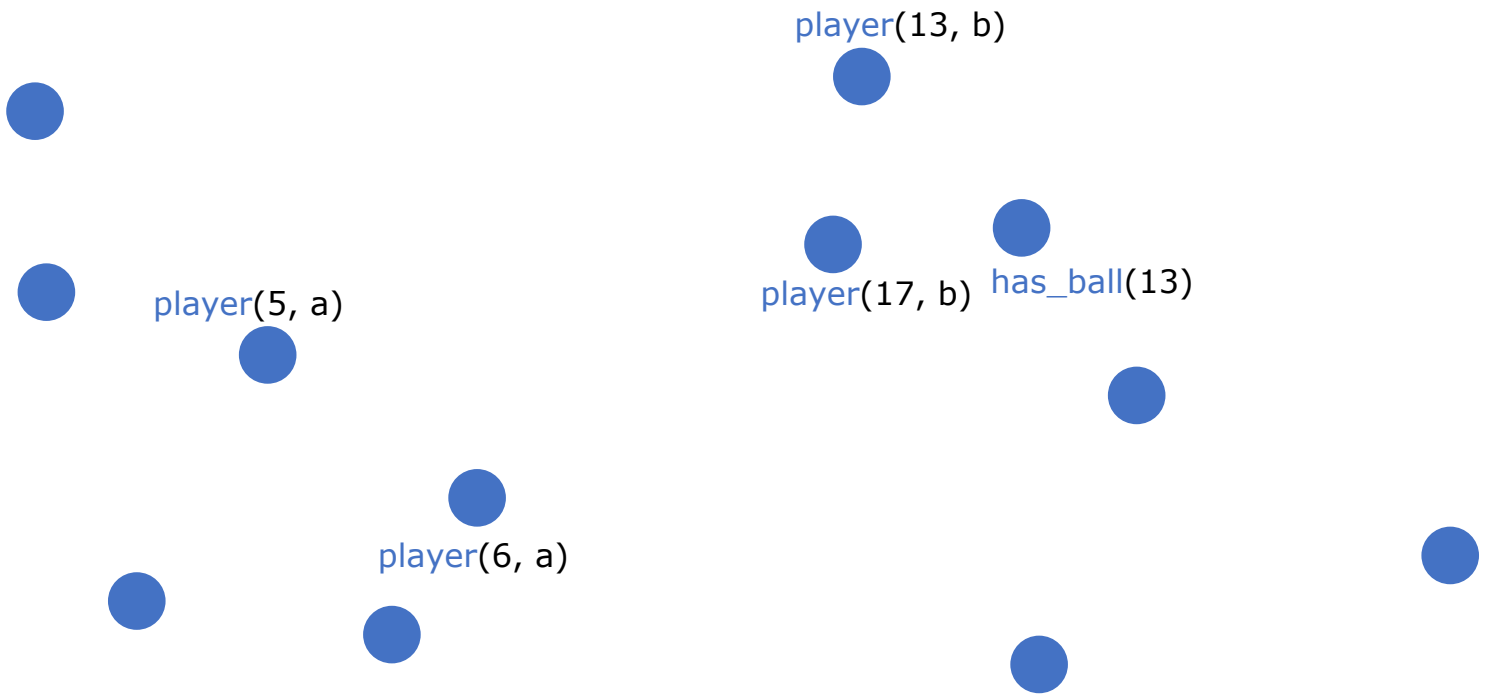

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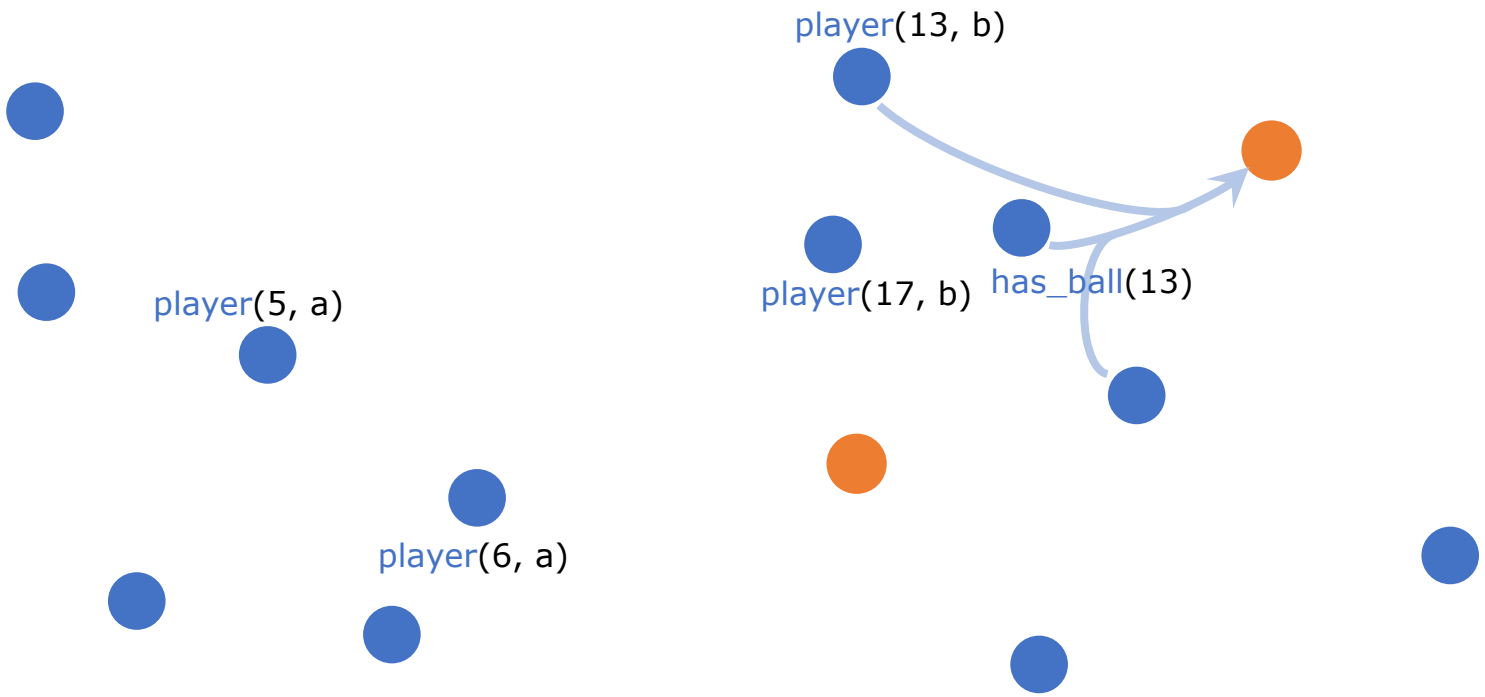
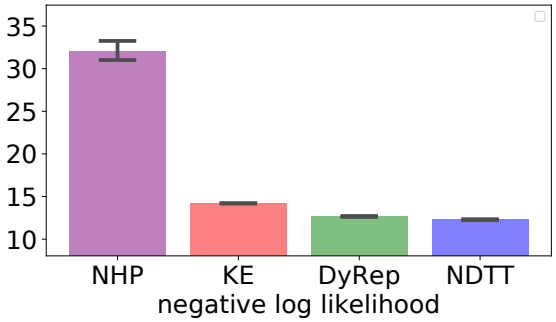
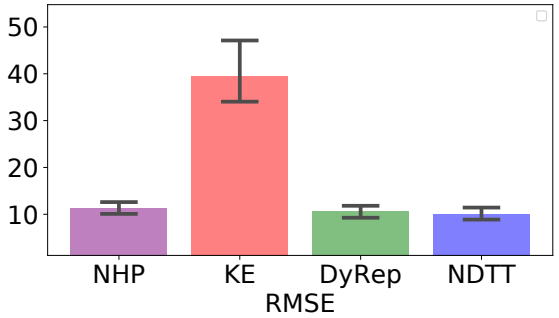
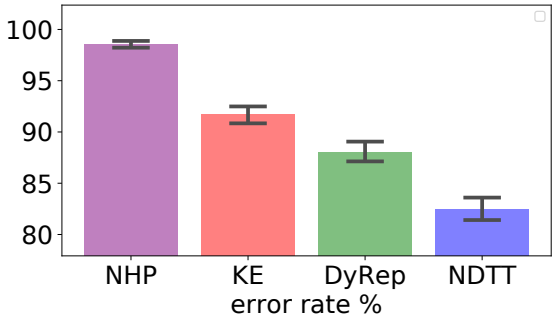


Results: NDTT > Competitors

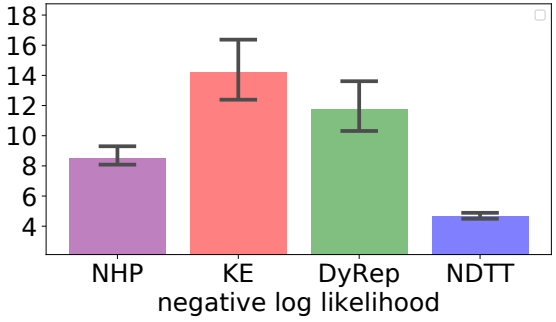
3 error metrics (in 3 columns): smaller is better

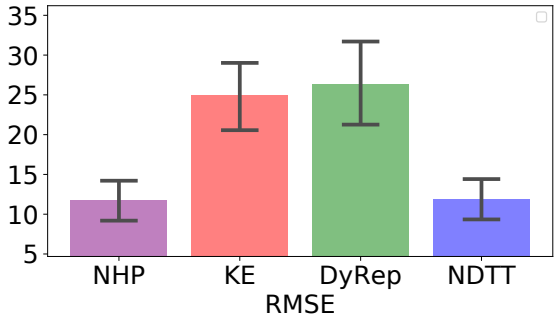


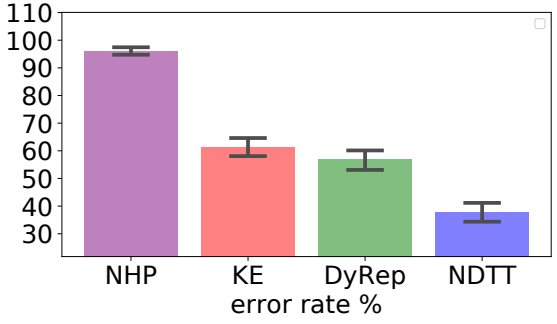




users watch TV programs

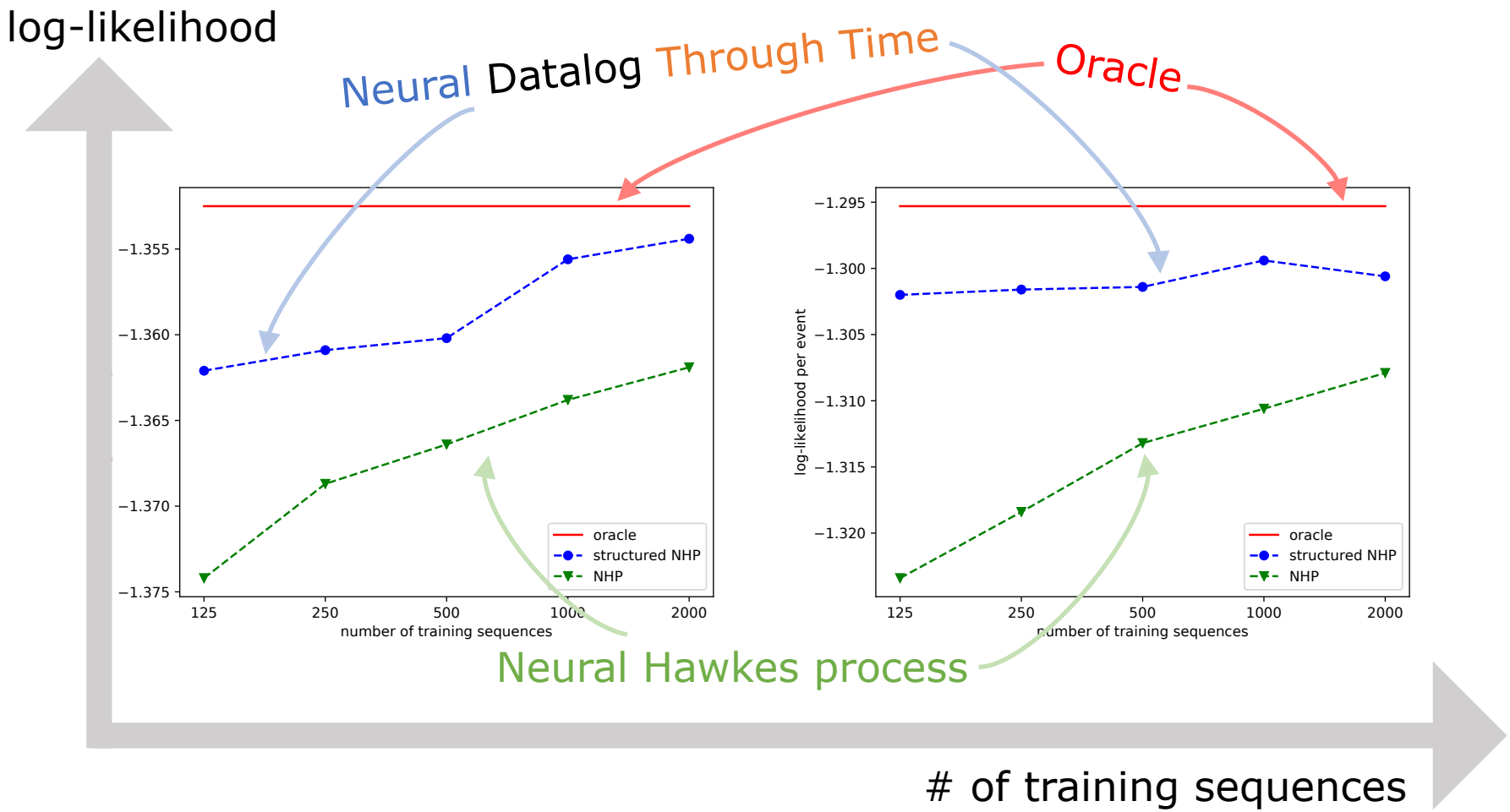






robots kick/pass/steal soccer ball

Good Generalization with Less Data



Summary: Deep Recurrent Net

Summary: Deep Recurrent Net

e.g., RNN
LSTM

discrete-time

Summary: Deep Recurrent Net

e.g., RNN discrete-time
LSTM
neural Hawkes process continuous-time

Summary: Deep Recurrent Net

hidden
system
state

e.g., RNN discrete-time
LSTM
neural Hawkes process continuous-time

Summary: Deep Recurrent Net

hidden system state

$h \in \mathbb{R}^{300}$

e.g., RNN discrete-time
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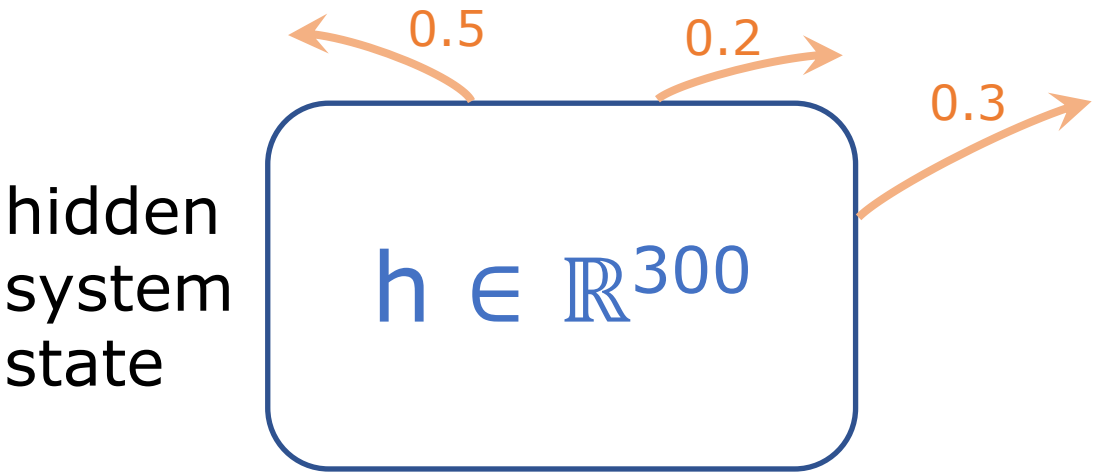
neural Hawkes process continuous-time

Summary: Deep Recurrent Net



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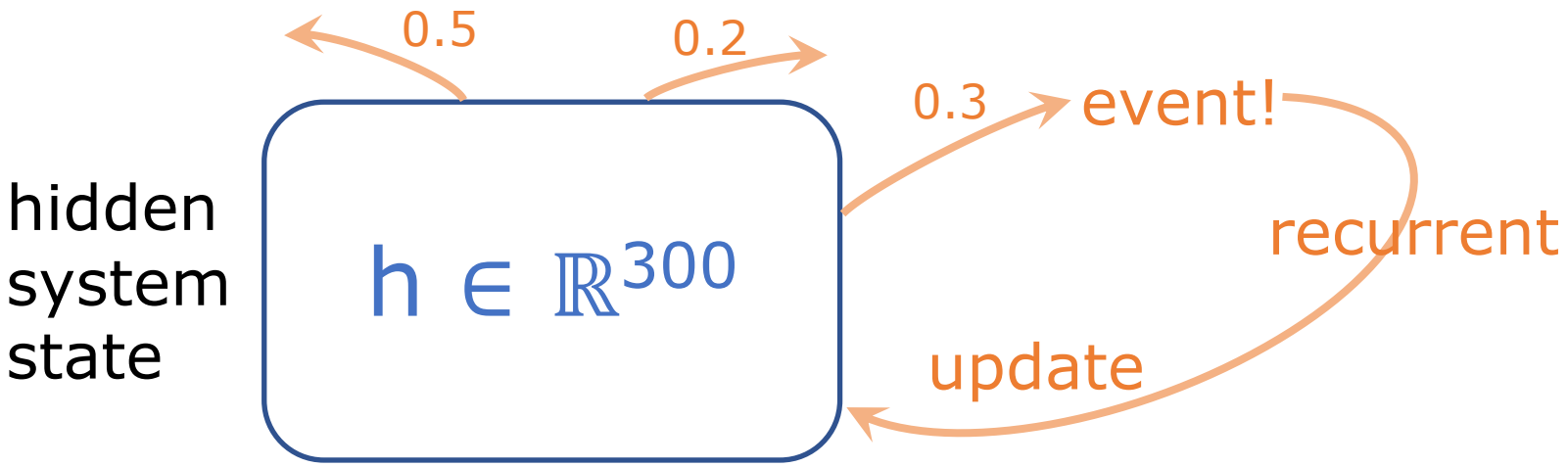

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Summary: Logic ➔ Deep Recurrent Net

hidden system state

Summary: Logic ➔ Deep Recurrent Net

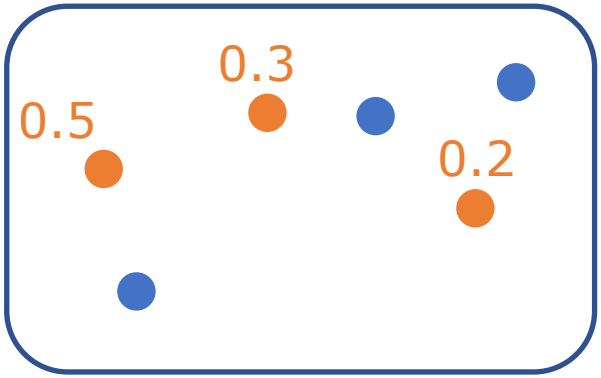
distributed

hidden
system
state

Summary: Logic ➔ Deep Recurrent Net

distributed

hidden
system
state



Summary: Logic ➔ Deep Recurrent Net

distributed

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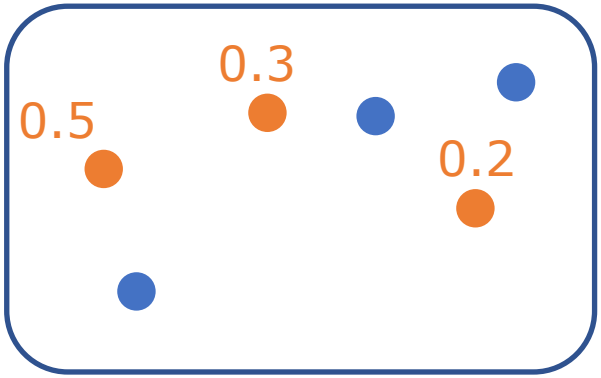
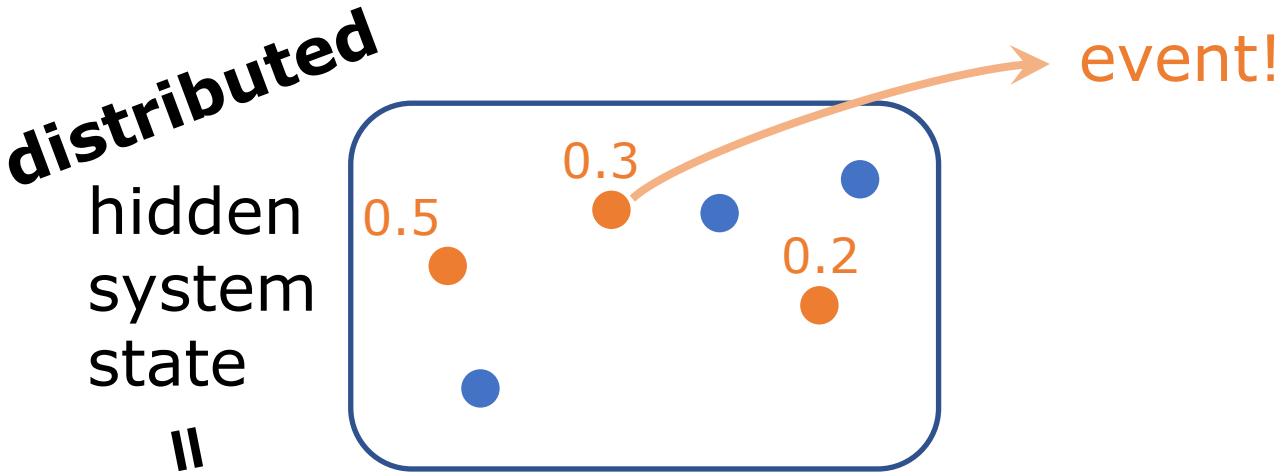


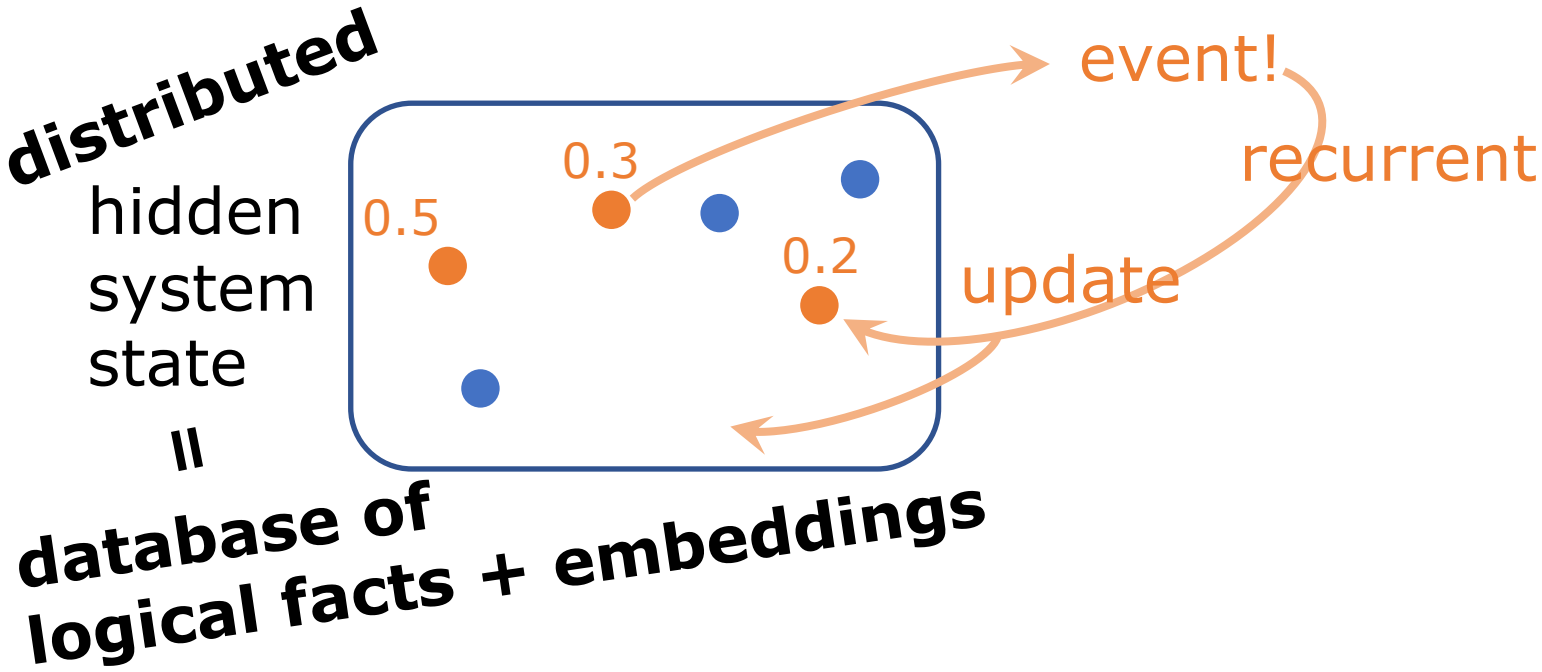
II

database of
logical facts + embeddings

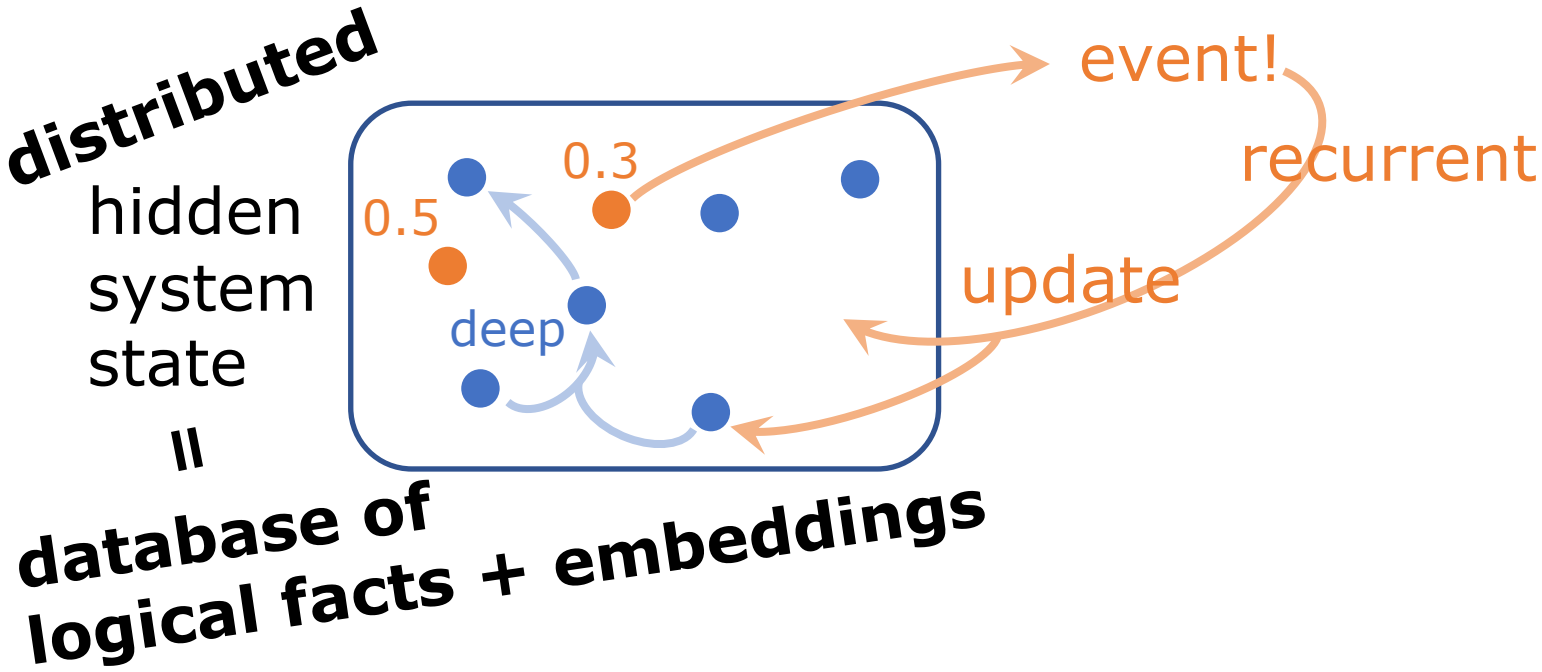
Summary: Logic ➔ Deep Recurrent Net



Summary: Logic ➔ Deep Recurrent Net

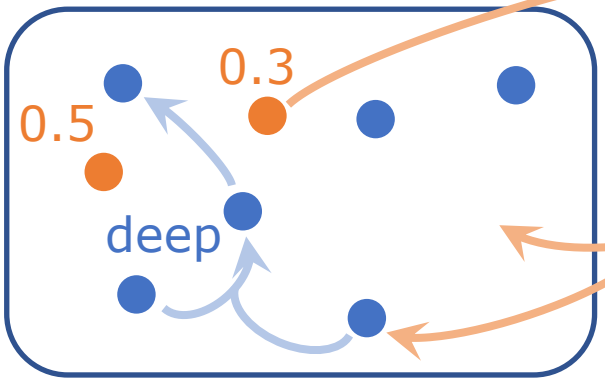


Summary: Logic ➔ Deep Recurrent Net



Summary: Logic ➔ Deep Recurrent Net

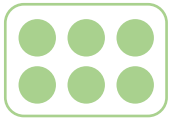
distributed
hidden
system
state
=
database of
logical facts + embeddings



event!
recurrent
update

update derived from
small rule set +
learned low-dim matrices

new fact :- old fact $_1$, …



new fact ← event, …

! old fact ← event, …

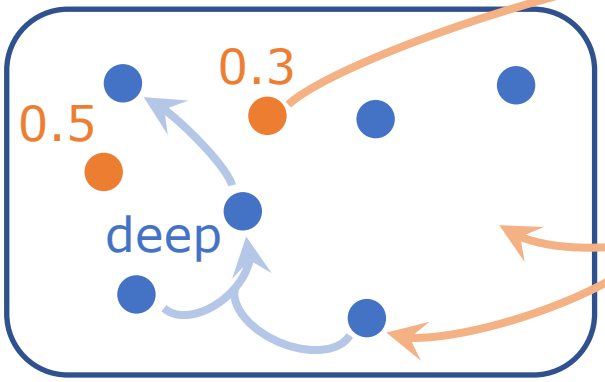
Summary: Logic → Deep Recurrent Net

distributed

hidden
system
state

=

database of logical facts + embeddings



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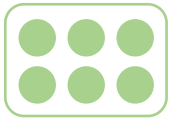

new fact ← event, …

! old fact ← event, …

try our code in your domain!

http://bburl/tpp-slides-p3
http://bburl/tpp-lab-p3

Neural Datalog Through Time

Bloomberg PhD Fellowship
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ICLR reviewers
Rakshit Trivedi
ICML reviewers
MARCC
Hongteng Xu
JHU-CLSP
Argo Lab
NVIDIA

Thank You

Any Questions?

Thank You

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