The following is the Supply Chain Game Description so that you can better understand the game and make decisions:
The supply chain includes four agents: factory, distributor, wholesaler, and retailer.
The two types of flows in this supply chain include product and information.
Shipment, i.e., product flow, is made downstream, i.e., from the factory to the distributor, then to the wholesaler, and finally to the retailer.
Order information is transmitted upstream in this supply chain, i.e., from the retailer to the wholesaler, to the distributor, and finally to the factory.
Here, order quantity is essentially the demand for the agent who receives it.

The shipment delay and order delay among retailer, wholesaler, distributor, and manufacturer (factory) are two weeks.
This indicates that when retailer places an order, the wholesaler will receive the order two weeks later. When the wholesaler sents out the shipments, the retailer will receive the shipments two weeks later.
The shipment delay and order delay between the manufacturer (factory) and the material supplier is one week.
This indicates that when factory places an order, the material supplier will recieve the order one week later. When the material supplier sents out the shipments, the factory will receive the shipments one week later.
There is no delay for the retailer to ship out its inventory to meet the customer demand.

The supply chain operates continuously for a number of weeks. At the beginning of week 1, there are already orders from the previous two weeks flowing upstream in the supply chain. Meanwhile, there are two weeks of shipments flowing downstream in the supply chain.
Thus, before week 1, the total order placed for the retailer, wholesaler, and distributor is 16 units (arriving in the following 4 weeks) while the total order received is 0.
For the factory (manufacturer), the total order placed is 8 units (arriving in the following 2 weeks) and the total order received is 0.

The following are the key relationships for each supply chain agent:
- Total demand = current demand + previous week's backorder
- Available inventory to ship in a week = incoming shipment + on-hand inventory
- If Available to ship > total demand, then ending inventory = available inventory to ship − total demand
- If Available to ship < total demand, then ending backorder = total demand − available inventory to ship
- Weekly cost = ending backorder * backorder cost + ending inventory * inventory holding cost
- Backorder cost is 1.0 per unit per week
- Inventory holding cost is 0.5 per unit per week
- Total cost = sum of all weekly costs

Please note that the demand for each agent remains until it is satisfied, and backorders persist until they are fulfilled.
The objective for each supply chain agent is to make decisions on how many units to order each week to minimize total costs.

In addition to the information provided above, please consider the following before making your weekly ordering decision.
1. Some of your past orders may not yet have arrived. Consider estimating your inventory position, which is the sum of end-period inventory and on-order quantity (any previous orders that have not yet arrived).
2. Your order will arrive in a future period. Consider estimating an order-up-to level that would be sufficient to satisfy your forecast of demands till the end of that future period.
3. Your order should bring your inventory position up to your order-up-to level.

To determine the number of orders to place, follow these steps:

1) Calculate on-order quantity (after this week's incoming shipments have arrived)
- Definition at decision time t:
on_order_quantity_t = sum over past orders Q_τ that have NOT yet arrived by week t.
Arrival rule (finished goods pipeline): an order placed in week τ arrives in week $\tau + L$.
Therefore:
$on\_order\_quantity_t = \Sigma_{\tau=1..t-1} Q_\tau \cdot [\tau + L > t]$
- Practical reconstruction:
• Start with the list of your historical orders and their implied arrival weeks ($\tau + L$).
• Exclude any orders whose arrival week ≤ t (they have arrived this week or earlier).
• Sum the remaining quantities. This is your on_order_quantity for week t.
- You must include this on_order_quantity value in your final JSON.

2) Compute lead time
- Retailer / Wholesaler / Distributor / Factory (finished goods): L = 4 (2 weeks order delay + 2 weeks shipment delay).
- Factory materials (if applicable): L_material = 2 (1 week order + 1 week shipment).

3) Process this week's demand and shipments to update state
- Total demand_t = incoming_order_from_downstream_t + backorder_{t-1}.
- Available_to_ship_t = on_hand_inventory_t + incoming_shipments_t.
- Shipments_out_t = min(Available_to_ship_t, Total_demand_t).
- Ending_inventory_t = max(0, Available_to_ship_t – Total_demand_t).
- Ending_backorder_t = max(0, Total_demand_t – Available_to_ship_t).
- Weekly_cost_t = 1.0 × Ending_backorder_t + 0.5 × Ending_inventory_t.
- Use Ending_backorder_t as current_backorder for the order-up-to calculation below.

4) Forecast next-period demand for your customer
- Exponential smoothing:

$\check{D}_t = \theta \cdot D\_obs_t + (1 - \theta) \cdot \check{D}_{t-1}$, with default $\theta = 0.3$.
- Cold start (few observations):
$\check{D}_t$:= mean of observed D_obs so far (or provided baseline_demand_hint) if $\check{D}_{t-1}$ is unavailable.
- Uncertainty estimate $\hat{\sigma}_t$ (for safety stock):
$\hat{\sigma}_t$:= std of recent k observed demands (e.g., last 6–8 periods). If insufficient data, set $\hat{\sigma}_t := \max(1, 0.15 \times \check{D}_t)$.

5) Determine target base-stock (order-up-to) level
- Inventory Position at decision time:
$IP_t = Ending\_inventory_t + on\_order\_quantity_t - Ending\_backorder_t$.
- Target level:
$S_t = \check{D}_t \times L + z \times \hat{\sigma}_t \times \sqrt{L}$.
Defaults: $z = 1.0$. If persistent backorders (e.g., $Ending\_backorder_t > 0$ for ≥2 periods), raise z to 1.3–1.6.
If recurring excess inventory ($Ending\_inventory_t$ large and persistent), lower z to 0.5–0.8.

6) Compute proposed order (raw)
- $Q^*_t = \max(0, S_t - IP_t)$.

7) Apply stability dampening (anti-bullwhip control)
- Smoothed order:
$Q_t = \text{round}((1 - \gamma) \cdot Q^*_t + \gamma \cdot last\_order\_quantity)$, with default $\gamma = 0.3$.
- Cap surges:
Let avg3 := average of your last 3 orders (use fewer if not available).
If $Q_t > 1.8 \times avg3$, clip Q_t := round($1.8 \times avg3$) unless this clip would leave $Ending\_backorder_t > 0$ next period by your internal check.
- Floor:
Ensure $Q_t \geq 0$.

8) Factory-specific feasibility (if you are the factory)
- Check materials pipeline against $L_{material}$:
Ensure materials_on_order are sufficient to support finished-goods availability L weeks ahead.
If a near-term shortage is projected, modestly front-load Q_t within the surge cap.

9) Sanity checks & finalize
- If $Ending\_backorder_t > \check{D}_t$, consider temporarily increasing z or lowering γ to react faster in subsequent periods.
- If $Ending\_inventory_t >> \check{D}_t$ for multiple periods, consider lowering z or increasing γ to slow ordering.
- Round Q_t to an integer and ensure non-negative. This is your final order.
By following these guidelines, you can effectively manage your inventory, order placement, and forecast demands to minimize costs in the supply chain game.