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Étude 4 Desert Crossing

You are planning to cross a desert 2413 km wide in a special desert vehicle.

- The vehicle can travel 12 km on 1 litre of fuel.
- It has a 60 litre tank and can carry at most four 20 litre cans.
- The only source of fuel and cans is at the base at the start of the desert. Fuel and cans at the base are not limited.

Task

Write a report giving your answers to the following:

1. Using the vehicle without refuelling, how far into the desert can you safely go?
2. Describe a procedure whereby you could cross the desert in the vehicle.
3. Describe a procedure whereby you could cross the desert and return in the vehicle.
4. Describe a procedure whereby you could cross the desert in the vehicle using the minimum amount of fuel.
5. Describe a procedure whereby you could cross the desert and return in the vehicle using the minimum amount of fuel.

Report

A format that described procedures from previous years goes as followed:

- T23.4 – pick up 23.4 litres and put them into the tank
- C23.4 – load cans containing 23.4 litres onto the back of the truck
- F23.4 – drive forward (towards the end of the desert) 23.4 kilometres
- D23.4 – dump cans containing 23.4 litres at the current position in the desert
- B23.4 – drive backwards (towards the base) 23.4 kilometres

We will be using this to aid in how we describe the procedures.

Question 1 (Max vehicle distance)

The vehicle can go 360 km into the desert safely. This is assuming that firstly that the vehicle must travel safely into the desert and get back to the base camp and secondly that the vehicle doesn't refuel using any cans.

This value was calculated by taking half the amount of the tank which is 30 litres and multiplying by 12 km per litre. The reason we are using half the tank is because that's the max amount that we can travel into the desert with enough fuel to travel back.

With aid from the format above the procedure would be described as followed:

- T60
- F360
- B360

Question 2 (To the end of the desert)

When designing a procedure whereby you could cross the desert in the vehicle, we decided that dropping cans off at intervals equal to the distance the vehicle could travel using a can would be a good way to simplify the problem while also remaining reasonably efficient. We also concluded that the further from the start a tank is placed, the more valuable it is, since it costs more to place it there.

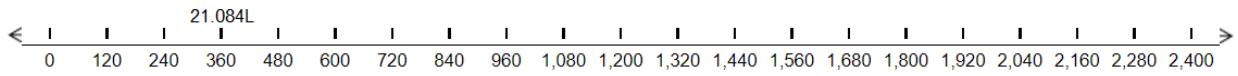
Using these assumptions we devised a way to cross the desert using **461.084 litres** of fuel.

With aid from the format above the procedure would be described as followed:

Place 2 tanks (21.084L) at 360 km: **Cost 81.084L**

- C21.084
- F360
- D21.084
- B360

State of desert after this step:

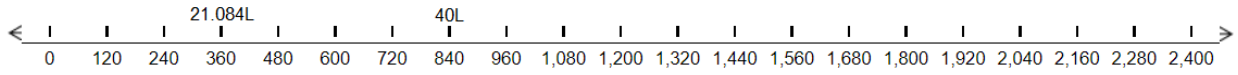


Place 2 tanks (40L total) at 840 km: **Cost 240L**

- C40
- F360
- D40
- B360
- C80
- F360
- T30
- F480
- D40
- T20
- B240
- T20
- B240

- T10
- B360

State of desert after this step:



Last trip: **Cost 140L**

- C80
- F360
- T21.084
- F480
- T40
- F480
- T40
- F480
- T40
- F613

Total cost to cross the desert: **461.084 Litres**

Question 3 (Cross and return)

We approached this problem in a similar way to the previous question, spacing out the drop zones by a minimum of 240 km to simplify the problem. Answering this question made it very clear that more research and a new plan was needed in order to try and tackle optimisation.

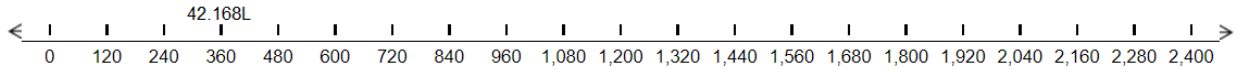
Using these assumptions we devised a way to cross the desert and return using **7,722.168 litres** of fuel.

With aid from the format above the procedure would be described as followed:

Place 3 tanks (42.168L) at 360 km: **Cost 102.168L** (Step 1)

- C42.168
- F360
- D42.168
- B360

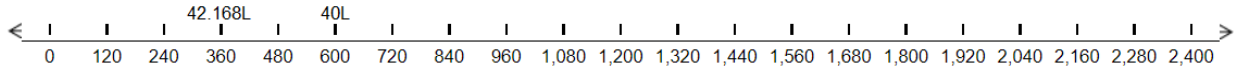
State of desert after this step:



Place 2 tanks (40L) at 600 km: **Cost 140L** (Step 2)

- C80
- F360
- T20
- F240
- D40
- B360
- T20
- B240

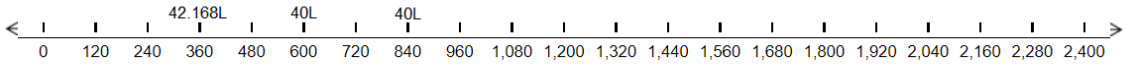
State of desert after this step:



Place 2 tanks (40L) at 840 km: **Cost 240** (Step 3)

- 40L at 360 km: Repeat Step 1 but with 40L instead of 42.168L. (Cost 100L)
- C80
- F360
- T20
- F240
- T20
- F240
- D40
- T20
- B480
- T20
- B360

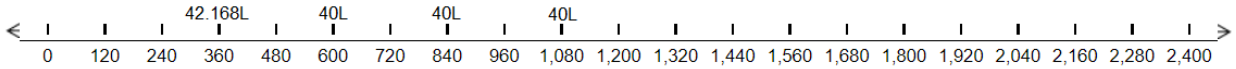
State of desert after this step:



Place 2 tanks (40L) at 1080 km: **Cost 720L** (Step 4)

- 40L at 360 km: Repeat Step 1 but with 40L instead of 42.168L. (Cost 100L)
- 40L at 600 km: Repeat Step 2 (Cost 140L)
- 40L at 840 km: Repeat Step 3 (Cost 240L)
- C80
- F360
- T20
- F240
- T20
- F240
- T20
- F240
- D40
- T20
- B480
- T20
- B240
- T20
- B360

State of desert after this step:

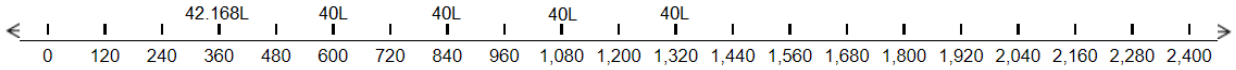


Place 2 tanks (40L) at 1,320 km: **Cost 1,240L** (Step 5)

- 40L at 360 km: Repeat Step 1 but with 40L instead of 42.168L. (Cost 100L)
- 40L at 600 km: Repeat Step 2 (Cost 140L)
- 40L at 840 km: Repeat Step 3. (Cost 240L)
- 40L at 1,080 km: Repeat Step 4. (Cost 720L)
- C80

- F360
- T20
- F240
- T20
- F240
- T20
- F240
- T20
- F240
- D40
- T20
- B480
- T20
- B240
- T20
- B240
- T20
- B360

State of desert after this step:

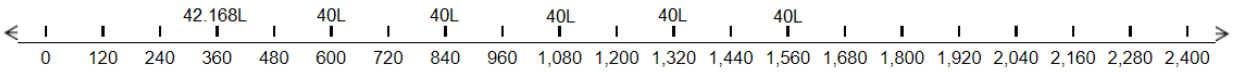


Place 2 tanks (40L) at 1560 km: **Cost 2580L** (Step 6)

- 40L at 360 km: Repeat Step 1 but with 40L instead of 42.168L. (Cost 100L)
- 40L at 600 km: Repeat Step 2 (Cost 140L)
- 40L at 840 km: Repeat Step 3. (Cost 240L)
- 40L at 1,080 km: Repeat Step 4. (Cost 720L)
- 40L at 1,320 km: Repeat Step 5. (Cost 1240L)
- C80
- F360
- T20
- F240
- T20
- F240
- T20
- F240
- T20

- F240
- T20
- F240
- D40
- T20
- B480
- T20
- B240
- T20
- B240
- T20
- B240
- T20
- B360

State of desert after this step:

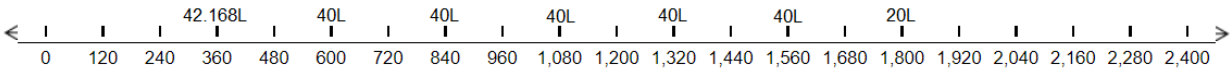


Place 1 tank (20L) at 1800 km: **Cost 2,560L** (Step 7)

- 40L at 360 km: Repeat Step 1 but with 40L instead of 42.168L. (Cost 100L)
- 40L at 600 km: Repeat Step 2 (Cost 140L)
- 40L at 840 km: Repeat Step 3. (Cost 240L)
- 40L at 1,080 km: Repeat Step 4. (Cost 720L)
- 40L at 1,320 km: Repeat Step 5. (Cost 1240L)
- C60
- F360
- T20
- F240
- T20
- F240
- T20
- F240
- T20
- F240
- T20
- F240
- T20

- F240
- D20
- T20
- B480
- T20
- B240
- T20
- B240
- T20
- B240
- T20
- B240
- T20
- B360

State of desert after this step:

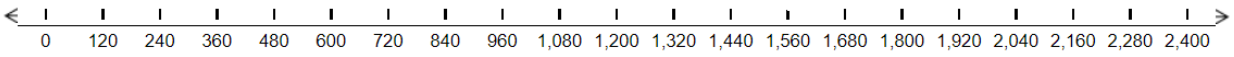


Last trip: **Cost 140L** (Step 8)

- C80
- F360
- T22.168
- F240
- T20
- F240
- T20
- F240
- T20
- F240
- T20
- F240
- T20
- F240
- T20
- F240

- T20
- F133
- B133
- T20
- B240
- T20
- B480
- T20
- B240
- T20
- B240
- T20
- B240
- T20
- B240
- T20
- B240
- T20
- B360

State of desert after this step:



Total cost to cross the desert and return: **7,722.168 Litres**.

Question 4 (One way optimal)

The vehicle can safely go across the desert optimally using **352.09 litres** of fuel. We assumed that the most efficient way to cross the desert was establishing the last base camp 140 litres or 1680 km from the end of the desert because we established previously that the further we drop cans in the desert the more fuel we had to expend to drop it there. We wanted to be 1680 km from the end of the desert because this is the closest the last drop point could be from the base camp while the vehicle is still able to make it to the end of the desert on a full tank of fuel and with four cans at the back of the trunk from that point. This also simplified the problem even more since we know from our last drop point to the end we travel 1680 kilometres or expend 140 litres of fuel the problem now become how do we travel 733 km (2413 km - 1680 km) into the desert and end up with a full tank plus four tanks worth of fuels. Another way to visualise this is instead of using distance in km to mark the journey

we instead use litres of fuel needed to travel to a certain distance e.g. instead of trying to travel 2413 km we our now trying to expend exactly 201.08 litres of fuel on a full trip into the desert. So basically the problem became how to travel 61.08 litres worth of fuel into the desert while still being on a full tank plus four cans when you reach the last drop point.

For our optimal solution, we established 2 points between the base and the end (2413 km) that we will be dropping cans of fuel off at.

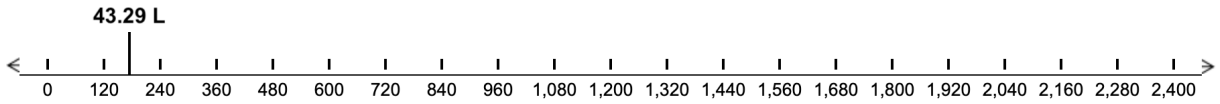
1. **P1** is established 173.2 km from the base
2. **P2** is established 733 km from the base

With aid from the format above the procedure would be described as followed:

Place 43.29 litres at 173.2 km: **Cost 28.86 litres**

- **T28.86**
- C43.29
- F173.2
- D80
- B173.2

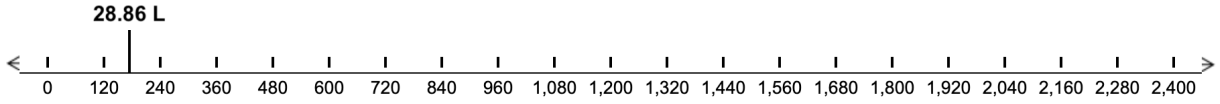
State of the desert after this step:



Place 46.67 litres at at 733 km: Cost **122.2 litres**

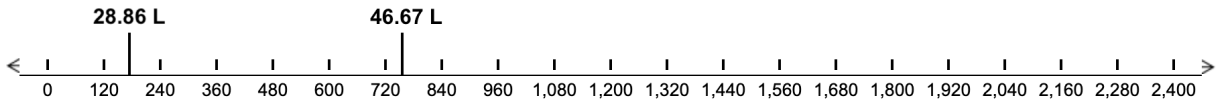
- **T60**
- C33.34
- F173.2
- **T14.43**

State of the desert after this step:



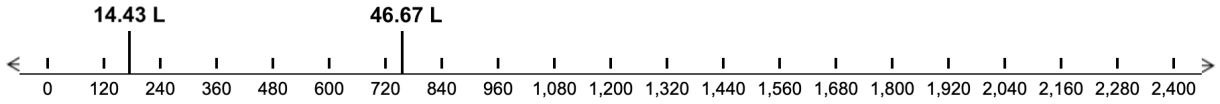
- F400.08
- **T33.34**
- F159.96
- D46.67

State of the desert after this step:



- B560.04
- **T14.43**
- B173.2

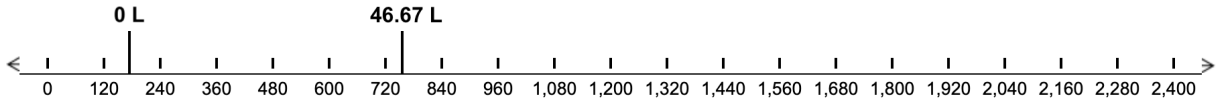
State of the desert after this step:



Last trip: Cost **201.1 L**

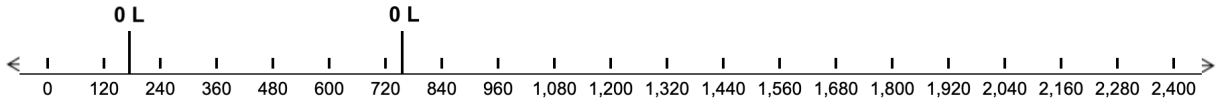
- **T60**
- C80
- F173.2
- **T14.43**

State of the desert after this step:



- F560.04
- **T46.67**

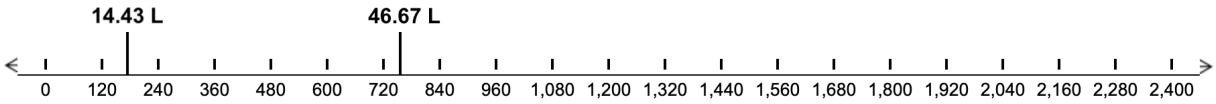
State of the desert after this step:



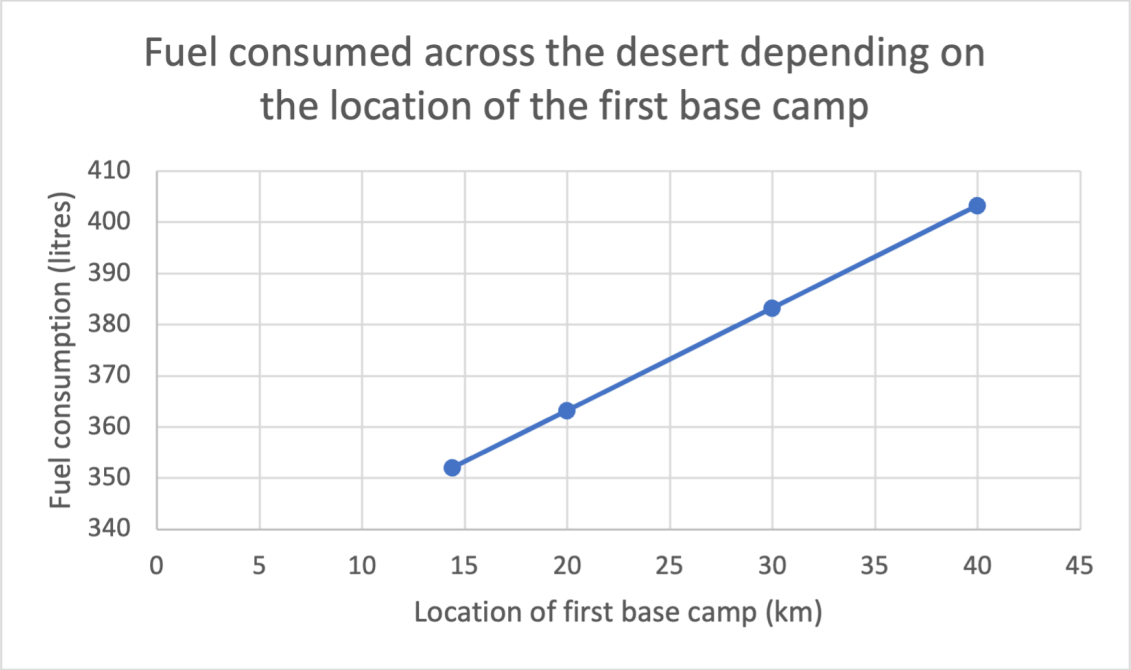
- F720
- **T60**
- F720
- **T20**
- F240

We believe that by having our last dropoff point 46.67 litres away from the first base camp, which is 14.43 litres from the start, is optimal because it is the highest distance you can travel whilst being able to drop off the same amount of fuel and travel back. This was calculated by taking one third of the 140 litres which is the max amount of fuel with a full tank and four cans in the trunk. We use

46.67 litres to travel 560.4 kilometres to travel into the desert and then drop off 46.67 litres and then use the remaining 46.67 litres to travel back to where we started. This ensures that for our final trip we can optimally fuel up first travelling 173.2 km to the first drop off point and then fueling up with 14.43 litres of fuel we left in cans there. From there then travelling 560.4 km to the last drop off point and then fueling up with the 46.67 litres of fuel we left in cans there. Therefore having a full tank plus four cans to make it all the way across the rest of the desert.



We also experimented with where we chose to put the first drop off point and found that the further away from the first base camp the more fuel was consumed as the trip between the start and the first camp was repeated the most. You can't have the first base camp closer than 173.2 km because this would mean you wouldn't be able to drop the optimal 46.67 litres of fuel at the last base camp. E.g. if you had the first camp at 10 L it would take an extra trip to fill up the second base camp with the required amount of fuel to make it to the end of the desert, since you'd only be able to drop off 40 litres at the end while spending 100 litres to get there and back. You would still be required to drop off another 10 litres to be able to make sure you can fill up to full at the second base camp.



Question 5 (There and back optimal)

Description

The vehicle can safely go across the desert and back optimally using **1645.4 litres** of fuel. For our optimal solution, we established 5 points between the base and the end (2413 km) that we will be dropping cans of fuel off at.

1. **P1** - is established 360km from the base
2. **P2** - is established 553.2km from the base
3. **P3** - is established 913.2km from the base
4. **P4** - is established 1153.2km from the base
5. **P5** - is established 1573.2km from the base

The purpose of these fuel drop off points is so we can utilise the fuel at these points to fill the vehicle back up to full - this is to allow the vehicle to continue onto the next point for further fuel drop off.

To begin our overall journey, we undertake five trips to **P1** and back. This is so we have a total of 400L of fuel (20 cans) stored at **P1**.

After these five trips, we begin our journey to leave cans at **P2**. Whenever we pass through **P2**, we utilise the fuel that was dropped off to fill the vehicles tank back up to full. From here we travel further to drop off 80L of fuel at **P2.** Now on the first trip back to base from **P2**, we will have 20L of leftover fuel meaning we just have to fuel up 10L from the **P1** stash (bringing the total down to 360L), which gives us enough fuel to get back to the base.

Now for the 2nd run to **P2** and back**,** the same process is repeated except we fill up 30L rather than 10L to ensure we get back.

For our trip to **P3** and back which is done twice, we stop at each point along the way there and back to fill the vehicle tank back up to full; allowing us to drop off 80L of fuel at **P3**. After having done this trip twice, we now have *80L* at **P3**, *96.6L* at **P2**, *180L* at **P1**.

Next up we then do a trip to **P4** (once) where we repeat the same process of filling up along the way there and back at each point and dropping off *80L* of fuel at **P4**.

This then brings us to our trip to **P5** where we dump *70L* and return back to base (as previously stated, filling up our vehicle tank along the way there and back at each point). After this trip, there is now *60L* at **P1**, *32.2L* at **P2**, *60L* at **P3**, *40L* at **P4** and *70L* at **P5** (the fuel amount at each point is double the amount required to travel one way between each of the points).

Finally, this takes us to the final trip where we travel all the way to the end (fueling up at each point, using up exactly half of what is provided at each point to ensure there is enough left on the way back) and then

travelling all the way back, fueling up with the final remaining amounts at each point.. Once returned at the base, there is no fuel left out in the desert.

The Algorithm

Trip to P1 and back (360km from base) x5

- C80
- F360
- D80
- B360

Trip to p2 and back (553.2km from base) - first run

- C80
- F360
- T30
- F193.2
- D80
- B193.2
- T10 (10L top up + 20L existing in tank = 30L required for P1 -> start)
- B360

Trip to P2 and back (553.2km from base) - second run

- C80
- F360
- T30
- F193.2
- D80
- B193.2
- T30
- B360

Trip to P3 and back (913.2km from base) x2

- C80
- F360
- T30
- F193.2
- T16.1
- F360
- D80
- B360
- T16.1
- B193.2

- T30
- B360)

Trip to P4 and back (1153.2km from base)

- C80
- F360
- T30
- F193.2
- T16.1
- F360
- T30
- F240
- D80
- B240
- T10 (the 10L + the 20L in our tank = 30L for P3->P2)
- B360
- T16.1
- B193.2
- T30
- B360 (p4)

Trip to P5 and back(1573.2km from base)

- C80
- F360
- T30
- F193.2
- T16.1
- F360
- T30
- F240
- T20
- F420
- D70
- B420
- T20
- B240
- T30
- B360
- T16.1
- B193.2
- T30
- B360

Trip to end and back (2413km from base)

- C80
- F360
- T30
- F193.2
- T16.1
- F360
- T30
- F240
- T20
- F420
- T35
- F840
- B840
- T35
- B420
- T20
- B240
- T30
- B360
- T16.1
- B193.2
- T30
- B360