Free Lunch in the LETF Market

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Go with market

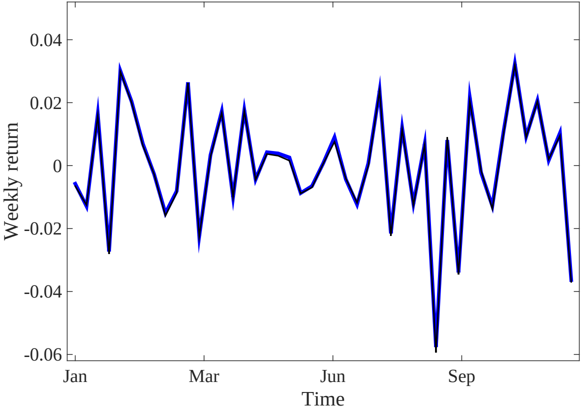


Figure 1: Weekly returns of ETF (SPY) and stock market (S&P 500) (20150101-20151112)

Leverage up/down

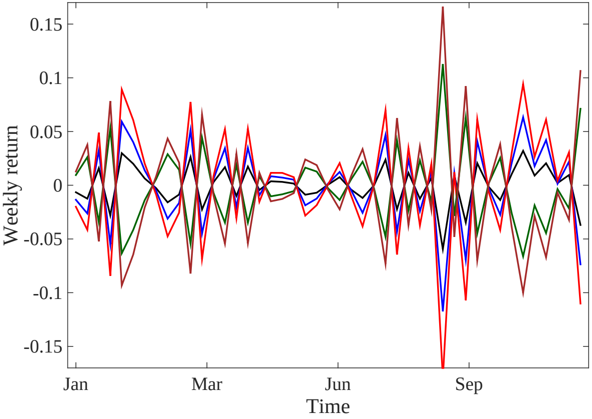


Figure 2: Weekly returns of LETFs (SSO, UPRO, SDS, SPXU) and stock market (S&P 500) (20150101-20151112)

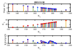
ETFs & LETFs

- Exchange-traded funds (ETFs): tracking returns on financial quantities and yielding the identical daily return,
  e.g., SPDR S&P 500 ETF (SPY) tracks the S&P 500.

- Leveraged exchange-traded funds (LETFs): promising a fixed leverage ratio β w.r.t. a given underlying asset or index,
  e.g.,

LETF	β
ProShares Ultra S&P500 (SSO)	2
ProShares UltraPro S&P 500 (UPRO)	3
ProShares UltraShort S&P500 (SDS)	-2
ProShares UltraPro Short S&P 500 (SPXU)	-3

Table 1: LETFs with different β Illustration



Implied volatility paradoxon

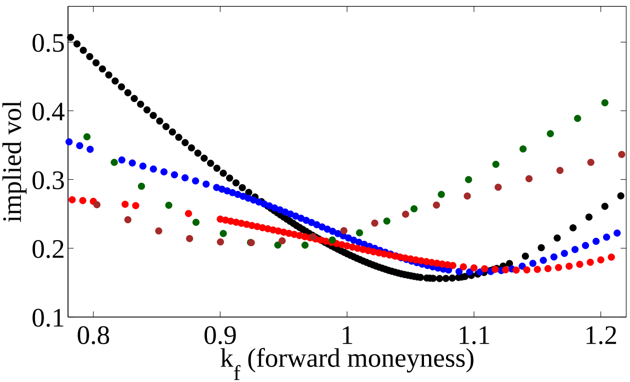


Figure 3: Implied volitility of (L)ETF options (SPY, SSO, UPRO, SDS, SPXU) with 21 days to maturity

Outline

1. Motivation ✓
2. Trading Strategy
3. Empirical Result
4. Conclusion

Price relationship of (L)ETFs

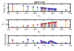
- For SPY ($\beta = 1$) and SSO ($\beta = 2$),

$$\log(L_{2T}) - \log(L_{20}) = 2\left\{\log(L_{1T}) - \log(L_{10})\right\}, \qquad (1)$$

where $t = 0$ is the date for designing the strategy and $t = T$ is the delivery date for the corresponding LETF options.

- Let $\alpha = \frac{\log(L_{20})}{\log(L_{10})}$, Equation (1) can be written as

$$L_{2T} = L_{10}^{\alpha-2} L_{1T}^{2}. \qquad (2)$$



The strategy I

1. Long 1 share SPY at $t = 0$ until $t = T$.
2. Short 1 share call option on SPY with strike K_1 at $t = 0$ until the maturity T.
3. Short $L_{10}^{2-\alpha} K_1^{-1}$ shares SSO at $t = 0$ until $t = T$.
4. Long $L_{10}^{2-\alpha} K_1^{-1}$ shares call option on SSO with strike $K_2 = L_{10}^{\alpha-2} K_1^2$ at $t = 0$ until the maturity T.

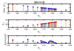
The initial profit (cost if negative) of this strategy is

$$IP_0 = -L_{10} + C_{10} + L_{10}^2 K_1^{-1} - L_{10}^{2-\alpha} K_1^{-1} C_{20}. \tag{3}$$

The portfolio value at time T

	Value at time T	
Position	$L_{1T} \le K_1$	$L_{1T} > K_1$
1.	L_{1T}	L_{1T}
2.	0	$K_1 - L_{1T}$
3.	$-L_{1T}^2 K_1^{-1}$	$-L_{1T}^2 K_1^{-1}$
4.	0	$L_{1T}^2 K_1^{-1} - K_1$
Sum	$L_{1T}\left(1 - \frac{L_{1T}}{K_1}\right) \ge 0$	0

Table 2: Portfolio value at time T of trading strategy I

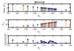


The strategy II

1. Short 1 share SPY at $t = 0$ until $t = T$.
2. Short 1 share put option on SPY with strike K_1 at $t = 0$ until the maturity T.
3. Long $L_{10}^{2-\alpha} K_1^{-1}$ shares SSO at $t = 0$ until $t = T$.
4. Long $L_{10}^{2-\alpha} K_1^{-1}$ shares put option on SSO with strike $K_2 = L_{10}^{\alpha-2} K_1^2$ at $t = 0$ until the maturity T.

The initial profit (cost if negative) of this strategy is

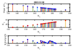
$$IP_0 = L_{10} + P_{10} - L_{10}^2 K_1^{-1} - L_{10}^{2-\alpha} K_1^{-1} P_{20}. \tag{4}$$



The portfolio value at time T

	Value at time T	
Position	$L_{1T} < K_1$	$L_{1T} \geq K_1$
1.	$-L_{1T}$	$-L_{1T}$
2.	$L_{1T} - K_1$	0
3.	$L_{1T}^2 K_1^{-1}$	$L_{1T}^2 K_1^{-1}$
4.	$K_1 - L_{1T}^2 K_1^{-1}$	0
Sum	0	$L_{1T}\left(\frac{L_{1T}}{K_1} - 1\right) \geq 0$

Table 3: Portfolio value at time T of trading strategy II

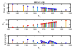


The strategy III

1. Short 1 share call option on SPY with strike K_1 at $t = 0$ until the maturity T.
2. Short 1 share put option on SPY with strike K_1 at $t = 0$ until the maturity T.
3. Long $L_{10}^{2-\alpha} K_1^{-1}$ shares call option on SSO with strike $K_2 = L_{10}^{\alpha-2} K_1^2$ at $t = 0$ until the maturity T.
4. Long $L_{10}^{2-\alpha} K_1^{-1}$ shares put option on SSO with strike $K_2 = L_{10}^{\alpha-2} K_1^2$ at $t = 0$ until the maturity T.

The initial profit (cost if negative) of this strategy is

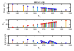
$$IP_0 = C_{10} + P_{10} - L_{10}^{2-\alpha} K_1^{-1} C_{20} - L_{10}^{2-\alpha} K_1^{-1} P_{20}. \tag{5}$$



The portfolio value at time T

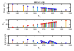
	Value at time T	
Position	$L_{1T} < K_1$	$L_{1T} \geq K_1$
1.	0	$K_1 - L_{1T}$
2.	$L_{1T} - K_1$	0
3.	0	$L_{1T}^2 K_1^{-1} - K_1$
4.	$K_1 - L_{1T}^2 K_1^{-1}$	0
Sum	$L_{1T}\left(1 - \frac{L_{1T}}{K_1}\right) > 0$	$L_{1T}\left(\frac{L_{1T}}{K_1} - 1\right) \geq 0$

Table 4: Portfolio value at time T of trading strategy III



Data

- Objects: prices of SPY, SSO and the corresponding call and put options with various strikes and the maturity date being Jan. 15, 2016
- Sample period: Jan. 1, 2015-Nov. 12, 2015
- Data source: Datastream



Arbitrage opportunities

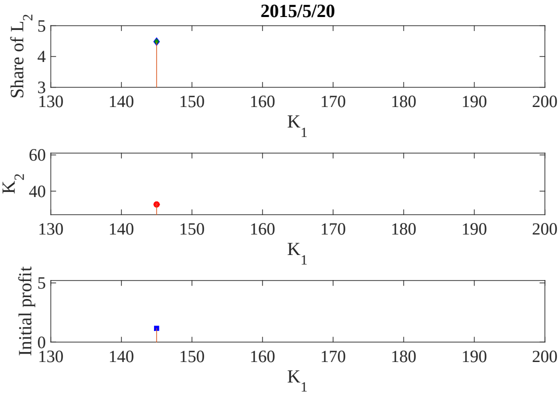
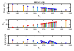


Figure 4: Arbitrage opportunities of strategy I (20150101-20151112)



Arbitrage opportunities

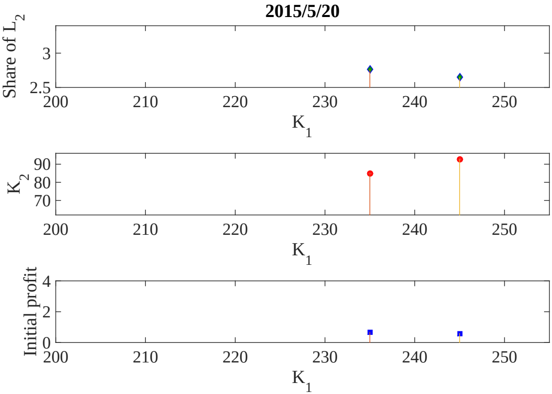
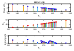
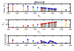


Figure 5: Arbitrage opportunities of strategy II (20150101-20151112)



Conclusion

- Implied volatility paradoxon of LETFs implies there exist arbitrage opportunities in the LETF market;
- We design some trading strategies by longing and shorting LETFs and LETF options to capture these opportunities;
- It shows our strategies can indeed bring free lunch in the LETF market.



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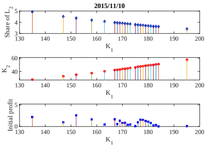

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S&P 500 and (L)ETFs Return

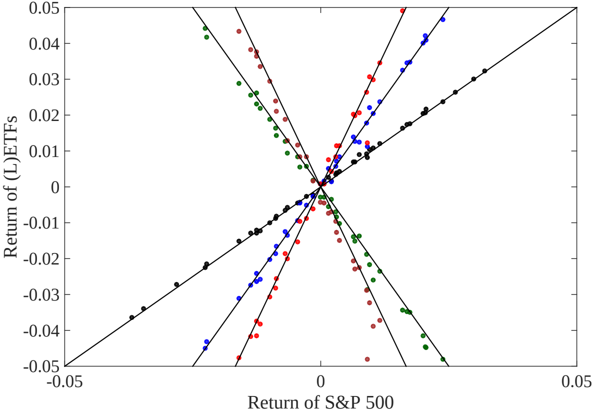

Figure 6: Weekly return relationship of S&P 500 and (L)ETFs (SPY, SSO, UPRO, SDS, SPXU) (20150101-20151112) Back