5. Sea $u_1, \ldots, u_n$ una base ortonormal de vectores de $\mathbb{R}^n$. Demostrar que para cualquier vector $x \in \mathbb{R}^n$, la coordenada de x respecto de u_k es igual a $u_k^t x$, para cualquier $k = 1, \ldots, n$.

$B = \{u_1, \cdots, u_n\}$ b.o.n. de $\mathbb{R}^n$

$X = \alpha_1 u_1 + \cdots + \alpha_n x_n$

$$\begin{aligned} u_k^T X &= u_k^T(\alpha_1 u_1 + \cdots + \alpha_n u_n) \\ &= \alpha_1 \underbrace{u_k^T u_1} + \cdots + \alpha_k \underbrace{u_k^T u_k} + \cdots + \alpha_n \underbrace{u_k^T u_n} = \alpha_k \end{aligned}$$

$u_k^T u_1 = 0$ $\qquad$ $u_k^T u_k = 1$ $\qquad$ $u_k^T u_n = 0$ $\qquad$ $u_i^T u_j = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

$\therefore \quad X = (\underset{\alpha_1}{\underset{\downarrow}{u_1^T X}}, \cdots, \underset{\alpha_n}{\underset{\downarrow}{u_n^T X}})_B$ coordenadas de x en base B