

17. Sea $U = I - 2uu^T$ un reflector ortogonal. Sea x tal que $x = v + w$ con v múltiplo de u y w ortogonal a u . Mostrar que $Ux = -v + w$. Interpretar geométicamente en \mathbb{R}^n .

$$\begin{aligned}
 Ux &= (I - 2uu^T)(v + w) \\
 &= v + w - 2uu^T(v + w) \\
 &= v + w - 2uu^Tv - 2uu^Tw \\
 &= v + w - 2uu^Tv \quad \downarrow \\
 &= v + w - 2v \\
 &= -v + w
 \end{aligned}$$

$$\begin{aligned}
 v &= \alpha u \\
 uu^Tv &= uu^T\alpha u \\
 &= \alpha uu^Tu \\
 &= \alpha u \|u\|_2^2 \quad \|u\|_2^2 = 1 \\
 &= \alpha u \\
 &= v
 \end{aligned}$$

