12. Dadas dos matrices de Givens de $\mathbb{R}^{2\times 2}$, G_1 y G_2, con ángulos θ y ω respectivamente, calcular e interpretar geométricamente G_1^2, G_1G_2 y $G_1^tG_1$. Pista: recordar las relaciones trigonométricas:

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$G_1^2 = \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \operatorname{sen}^2\theta & 2\operatorname{sen}\theta\cos\theta \\ -2\operatorname{sen}\theta\cos\theta & \cos^2\theta - \operatorname{sen}^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \operatorname{sen} 2\theta \\ -\operatorname{sen} 2\theta & \cos 2\theta \end{bmatrix}$$

G_1^2 rota el doble que G_1

$$G_1G_2 = \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\omega & \operatorname{sen}\omega \\ -\operatorname{sen}\omega & \cos\omega \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta+\omega & \operatorname{sen}\theta+\omega \\ -\operatorname{sen}\theta+\omega & \cos\theta+\omega \end{bmatrix}$$

G_1G_2 aplica las 2 rotaciones (suma $\theta+\omega$)

$$G_1^TG_1 = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \operatorname{sen}^2\theta & 0 \\ 0 & \cos^2\theta + \operatorname{sen}^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

G_1^T rota en sentido antihorario.
$G_1^TG_1$ rota $-\theta+\theta=0$ (no rota nada).

Para G_1, determinar el ángulo θ tal que

$$G_1 \begin{pmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$$

$$\begin{bmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} \sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix} = \begin{bmatrix} * & * \\ 0 & * \end{bmatrix}$$

$$-\operatorname{sen}\theta \cdot \sqrt{3} + \cos\theta = 0 \iff \cos\theta = \sqrt{3}\operatorname{sen}\theta$$

$$\iff \theta = 30^\circ = \frac{\pi}{6}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sqrt{3}\operatorname{sen} 30^\circ = \sqrt{3} \cdot \frac{1}{2} = \frac{\sqrt{3}}{2}$$

Trigonometric Table

α	0°	30°	45°	60°	90°	120°	180°	270°	360°
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	-1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	0	1
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0	∞	0
$\cot\alpha$	∞	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	∞	0	∞
$\sec\alpha$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	∞	-2	-1	∞	1
$\operatorname{Cosec}\alpha$	∞	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	∞	-1	∞