15. Sea $u \in \mathbb{R}^n$ tal que $\|u\|_2 = 1$. Demostrar que la matriz $Q := I - 2uu^t$ es ortogonal y simétrica.

Simetría QVQ: $Q = Q^T$

$$Q^T = (I - 2uu^T) = I^T - 2(uu^T)^T = I - 2uu^T$$

$\therefore$ Q es simétrica.

Ortogonal QVQ: $Q^{-1} = Q^T$

$$
\begin{aligned}
QQ^T = I \quad &\iff \quad QQ = I \\
&\iff \quad (I - 2uu^T)(I - 2uu^T) = I \\
&\iff \quad I - 2uu^T - 2uu^T + 4(uu^T)(uu^T) = I \\
&\iff \quad I - 4uu^T + 4(uu^Tuu^T) = I \\
&\iff \quad I - 4uu^T + 4(u\|u\|_2^2 u^T) = I \\
&\iff \quad I - 4uu^T + 4uu^T = I \qquad \|u\|_2^2 = 1 \\
&\iff \quad I = I
\end{aligned}
$$

$\therefore$ Q es ortogonal.