4. Sea $Q \in \mathbb{R}^{n \times n}$ ortogonal. Probar que:

a) $det(Q) = 1$ ó -1

b) $\kappa_2\ (Q) = 1$

a)

Q ortogonal $\Rightarrow Q^{-1} = Q^T$

$$QQ^T = I \Rightarrow det(QQ^T) = det(I)$$
$$\Rightarrow det(Q) \cdot det(Q^T) = 1$$
$$\Rightarrow det(Q) \cdot det(Q) = 1$$
$$\Rightarrow det(Q)^2 = 1$$
$$\Rightarrow |det(Q)| = 1$$

$\therefore\ det(Q) = 1$ o -1

b)

$$\|Q\|_2 = \max_{x:\|x\|_2=1} \|Qx\|_2 = \max_{x:\|x\|_2=1} \sqrt{(Qx)^T Qx} = \max_{x:\|x\|_2=1} \sqrt{x^T \underbrace{Q^T Q}_{I} x}$$

$$= \max_{x:\|x\|_2=1} \sqrt{x^T x} = \max_{x:\|x\|_2=1} \|x\|_2 = 1$$

$$K_2(Q) = \underbrace{\|Q\|_2}_{1} \cdot \underbrace{\|Q^{-1}\|_2}_{1} = 1$$

$$Q^{-1} = Q^T \Rightarrow \|Q^{-1}\|_2 = \|Q^T\|_2 = \|Q\|_2 = 1$$

Solo para norma 2