

Práctica 2

Lógica Digital

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Organización del Computador 1

| Integrante | LU | Correo electrónico |
|---------------------|--------|-----------------------|
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1. Ejercicio 1

Calculando las tablas de verdad podemos ver la equivalencia de las fórmulas booleanas.

1.a.

$$p = (p.q) + (p.\overline{q})$$

| p | q | (p.q) | + | $(p.\overline{q})$ |
|---|---|-------|---|--------------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 |

1.b.

$$x.z = (x+y).(x+\overline{y}).(\overline{x}+z)$$

| x | y | z | x.z | (x+y) | | $(x + \overline{y})$ | | $(\overline{x}+z)$ |
|---|---|---|-----|-------|---|----------------------|---|--------------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

2. Ejercicio 2

Resolviendo mediante propiedades llegamos a 2 fórmulas que a priori no parecen ser equivalentes. Notar en la última línea que a la izquierda tenemos $\overline{y}.\overline{z}$ mientras que a la derecha tenemos $\overline{y}.\overline{z}$.

$$x \oplus (y.z) = (x \oplus y).(x \oplus z)$$

$$\overline{x}.y.z + x.\overline{y.z} = (\overline{x}.y + x.\overline{y}).(\overline{x}.z + x.\overline{z})$$

$$\overline{x}.y.z + x.\overline{y.\overline{z}} = (\overline{x}.y + x.\overline{y}).\overline{x}.z + (\overline{x}.y + x.\overline{y}).x.\overline{z}$$

$$\overline{x}.y.z + x.\overline{y.\overline{z}} = \overline{x}.y.\overline{x}.z + x.\overline{y}.\overline{x}.z + \overline{x}.y.x.\overline{z} + x.\overline{y}.x.\overline{z}$$

$$\overline{x}.y.z + x.\overline{y.z} = \overline{x}.y.z + x.\overline{y}.\overline{z}$$

Calculamos la tabla de verdad para verificar.

| \boldsymbol{x} | y | z | x | \oplus | (y.z) | $(x \oplus y)$ | | $(x \oplus z)$ | |
|------------------|---|---|---|----------|-------|----------------|---|----------------|---|
| 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | |
| 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | × |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | × |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | |

Conclusión: la propiedad planteada es falsa.

3. Ejercicio 3

3.a.

Verdadero, con el operador NAND $(p|q=\overline{p.q})$ podemos representar todas las funciones booleanas: AND, OR, NOT.

Recordemos la tabla de verdad del NAND.

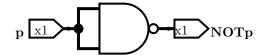
| p | q | p.q | $p q = \overline{p.q}$ |
|---|---|-----|------------------------|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

3.a.1. NOT

Utilizando la misma entrada 2 veces en un NAND podemos obtener un NOT.

$$p|p=\overline{p}$$

| p | p p | \overline{p} |
|---|-----|----------------|
| 0 | 1 | 1 |
| 1 | 0 | 0 |

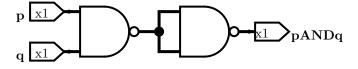


3.a.2. AND

Utilizando el NOT ya construido, podemos encadenarlo a la salida de un NAND para cancelar su negación y así obtener el resultado original del AND.

$$(p|q)|(p|q) = p.q$$

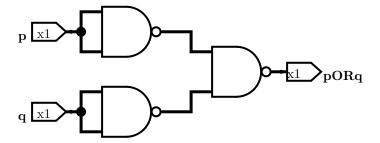
| p | q | p q | p (p q) (p q) | p.q |
|---|---|-----|---------------|-----|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |



3.a.3. OR

$$(p|p)|(q|q) = \overline{p}|\overline{q} = \overline{\overline{p}.\overline{q}} = \overline{\overline{p}} + \overline{\overline{q}} = p + q$$

| p | q | (p p) | | (q q) | p+q |
|---|---|-------|---|-------|-----|
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |



3.b.

Verdadero, con el operador NOR $(p\downarrow q=\overline{p+q})$ podemos representar todas las funciones booleanas: AND, OR, NOT.

Recordemos la tabla de verdad del NOR.

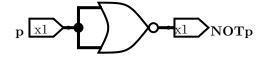
| p | q | p+q | $p \downarrow q = \overline{p+q}$ |
|---|---|-----|-----------------------------------|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

Observemos que los circuitos para armar las funciones booleanas utilizando solo la compuerta NOR son análogos a los utilizandos con la compuerta NAND.

3.b.1. NOT

$$p\downarrow p=\overline{p}$$

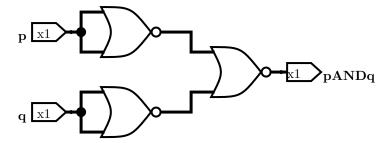
| p | $p \downarrow p$ | \overline{p} |
|---|------------------|----------------|
| 0 | 1 | 1 |
| 1 | 0 | 0 |



3.b.2. AND

$$(p\downarrow p)\downarrow (q\downarrow q)=\overline{p}\downarrow \overline{q}=\overline{\overline{p}+\overline{q}}=\overline{\overline{p}}.\overline{\overline{q}}=p.q$$

| p | q | $(p \downarrow p)$ | | $(q \downarrow q)$ | p.q |
|---|---|--------------------|---------|--------------------|-----|
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 |



3.b.3. OR

$$(p\downarrow q)\downarrow (p\downarrow q)=(\overline{p+q})\downarrow (\overline{p+q})=\overline{\overline{p+q}}=p+q$$

| p | q | $p \downarrow q$ | $(p\downarrow q)\downarrow (p\downarrow q)$ | p+q |
|---|---|------------------|---------------------------------------------|-----|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |

