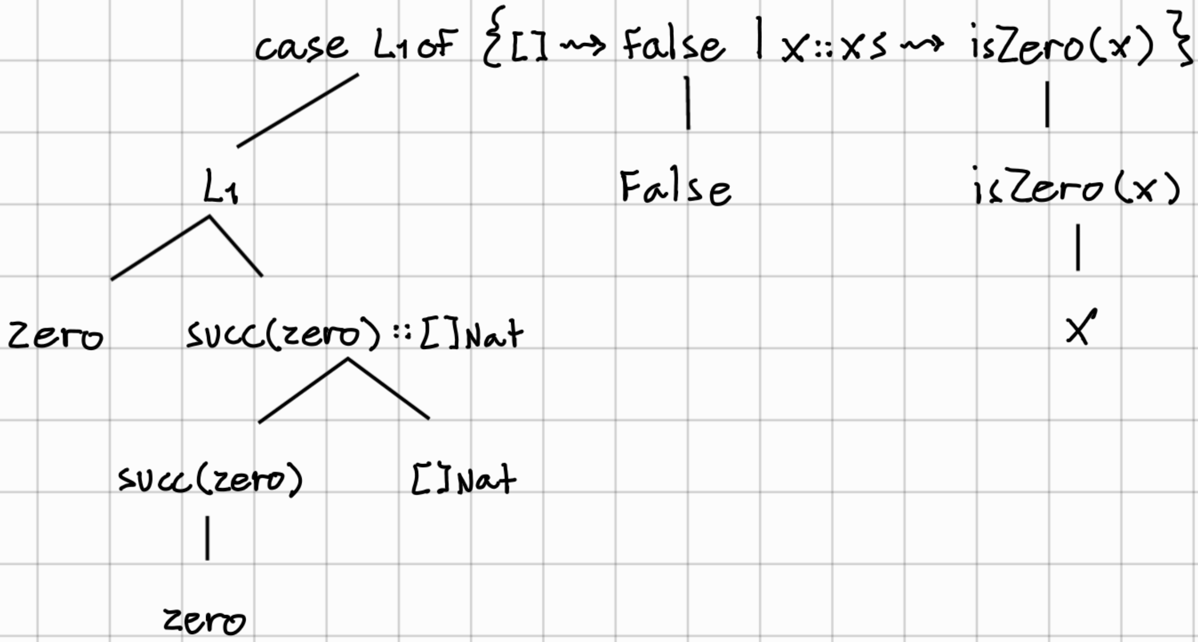
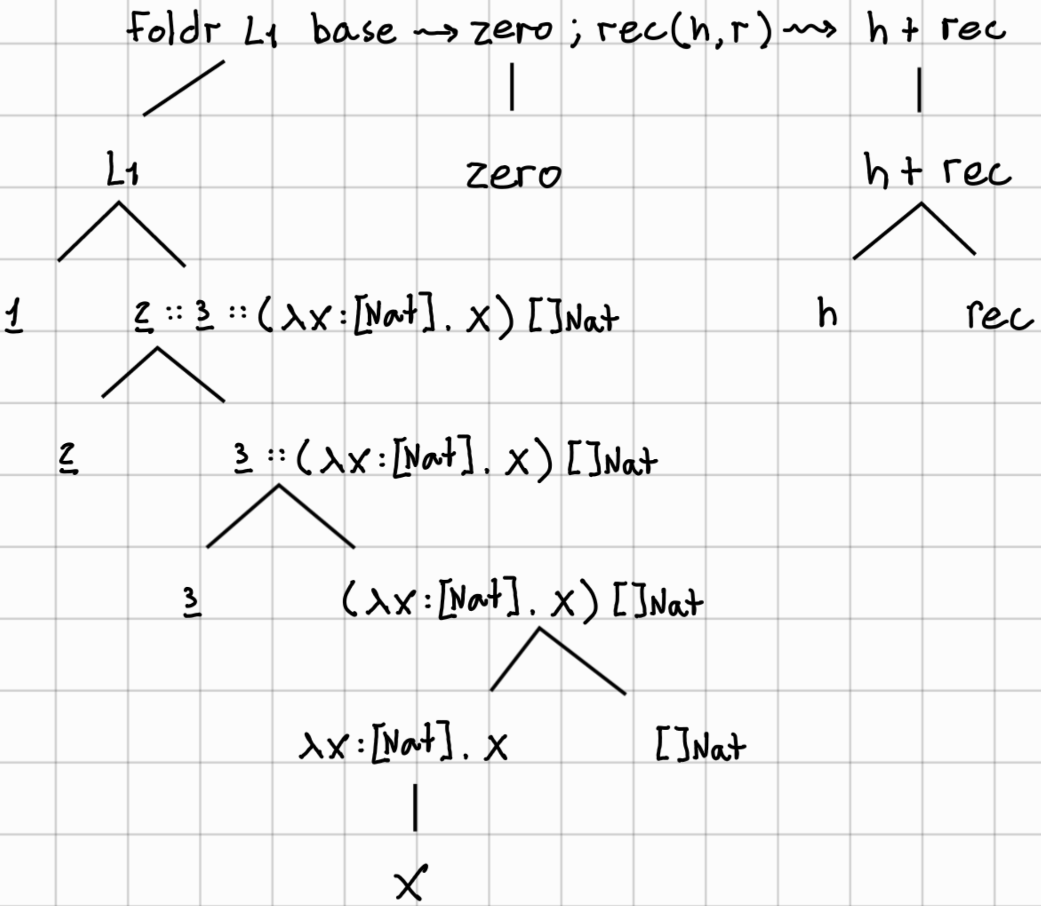
Extensión de listas

$L_1 \stackrel{def}{=} zero :: succ(zero) :: [\,]_{Nat}$



$L_2 \overset{def}{=} \underline{1} :: \underline{2} :: \underline{3} :: (\lambda x:[Nat].\ x)\ [\,]Nat$



Reglas de tipado

$$\frac{}{\Gamma \vdash [\,]_\sigma : [\sigma]}\ \text{T-[]}$$

$$\frac{\Gamma \vdash M : \sigma \qquad \Gamma \vdash N : [\sigma]}{\Gamma \vdash M :: N : [\sigma]}\ \text{T-::}$$

$$\frac{\Gamma \vdash M : [\sigma] \qquad \Gamma \vdash N : \tau \qquad \Gamma, h : \sigma, t : [\sigma] \vdash O : \tau}{\Gamma \vdash \text{case } M \text{ of } \{[\,] \rightsquigarrow N \mid h :: t \rightsquigarrow O\} : \tau}\ \text{T-Case}$$

$$\frac{\Gamma \vdash M : [\sigma] \qquad \Gamma \vdash N : \tau \qquad \Gamma, h : \sigma, r : \tau \vdash O : \tau}{\Gamma \vdash \text{foldr } M \text{ base} \rightsquigarrow N ; \text{rec}(h, r) \rightsquigarrow O : \tau}\ \text{T-Foldr}$$

$\Gamma = x : Bool,\ y : [Bool]$

$L \overset{def}{=} x :: x :: y$

$A \overset{def}{=} \text{if } y \text{ then } x \text{ else } []_{Bool}$

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash x : Bool}\,ax_v \qquad \dfrac{\dfrac{}{\Gamma \vdash x : Bool}\,ax_v \quad \dfrac{}{\Gamma \vdash y : [Bool]}\,ax_v}{\Gamma \vdash x :: y : [Bool]}\,T\text{-}::}{\Gamma \vdash L : [Bool]}\,T\text{-}:: \qquad \dfrac{}{\Gamma \vdash y : [Bool]}\,ax_v \qquad \dfrac{\dfrac{}{\Delta \vdash y : Bool}\,ax_v \quad \dfrac{}{\Delta \vdash x : [Bool]}\,ax_v \quad \dfrac{}{\Delta \vdash []_{Bool} : [Bool]}\,T\text{-}[]}{\Delta = \Gamma, y : Bool, x : [Bool] \vdash A : [Bool]}\,if}{\Gamma \vdash foldr\ L\ \ base \rightsquigarrow y\ ;\ rec(y, x) \rightsquigarrow A\ :\ [Bool]}\,T\text{-}foldr$$

$\downarrow$

En realidad no podemos mantener estos nombres de variables porque en el contexto Γ ya hay otras variables llamadas x e y. Lo correcto sería renombrar estas variables y sus ocurrencias libres en A. Si consideramos que las variables se identifican por nombre y color, entonces esto puede zafar ya que $y \neq y$, $x \neq x$.

Si no reescribir el término así:

$foldr\ L\ \ base \rightsquigarrow y\ ;\ rec(h, r) \longrightarrow A\{y := h\}\{x := r\}$

Semántica operacional

$V ::= \ldots \mid [\,]_\sigma \mid V :: V$

Reglas de reducción

$\text{case } [\,]_\sigma \text{ of } \{[\,] \rightsquigarrow N \mid h :: t \rightsquigarrow O\} \to N$ E-CV

$\text{case } V :: W \text{ of } \{[\,] \rightsquigarrow N \mid h :: t \rightsquigarrow O\} \to O\{h := V\}\{t := W\}$ E-CR

$\text{Foldr } [\,]_\sigma \text{ base} \rightsquigarrow N ;\ \text{rec}(h, r) \rightsquigarrow O \to N$ E-FV

$\text{Foldr } V :: W \text{ base} \rightsquigarrow N ;\ \text{rec}(h, r) \rightsquigarrow O$

$\quad \to O\{h := V\}\{r := \text{Foldr } W \text{ base} \rightsquigarrow N ;\ \text{rec}(h, r) \rightsquigarrow O\}$ E-FR

Si $M \to M'$

$\text{case } M \text{ of } \{[\,] \rightsquigarrow N \mid h :: t \rightsquigarrow O\}$

$\quad \to \text{case } M' \text{ of } \{[\,] \rightsquigarrow N \mid h :: t \rightsquigarrow O\}$ E-CC

$\text{Foldr } M \text{ base} \rightsquigarrow N ;\ \text{rec}(h, r) \rightsquigarrow O$

$\quad \to \text{Foldr } M' \text{ base} \rightsquigarrow N ;\ \text{rec}(h, r) \rightsquigarrow O$ E-FC

$M :: N \to M' :: N$ E-::C$_1$

$V :: M \to V :: M'$ E-::C$_2$