$\sigma = (\exists X. \forall Y. R(X,Y)) \Rightarrow \forall Y. \exists X. R(X,Y)$

QVQ: σ es válida. Para esto, refutemos $\neg\sigma$ con el método de resolución.

$$
\begin{aligned}
\neg\sigma &= \neg((\exists X. \forall Y. R(X,Y)) \Rightarrow \forall Y. \exists X. R(X,Y)) \\
&= \neg(\neg(\exists X. \forall Y. R(X,Y)) \vee \forall Y. \exists X. R(X,Y)) \\
&= \neg\neg(\exists X. \forall Y. R(X,Y)) \wedge \neg(\forall Y. \exists X. R(X,Y)) \\
&= (\exists X. \forall Y. R(X,Y)) \wedge \exists Y. \neg\exists X. R(X,Y) \\
&= (\exists X. \forall Y. R(X,Y)) \wedge \exists Y. \forall X. \neg R(X,Y) \\
&= (\exists X. \forall Y. R(X,Y)) \wedge \exists Z. \forall W. \neg R(W,Z) \\
&= \exists X. (\forall Y. R(X,Y) \wedge \exists Z. \forall W. \neg R(W,Z)) \\
&= \exists X. \forall Y. (R(X,Y) \wedge \exists Z. \forall W. \neg R(W,Z)) \\
&= \exists X. \forall Y. \exists Z. (R(X,Y) \wedge \forall W. \neg R(W,Z)) \\
&= \exists X. \forall Y. \exists Z. \forall W. (R(X,Y) \wedge \neg R(W,Z)) \\
&= \forall Y. \exists Z. \forall W. (R(C,Y) \wedge \neg R(W,Z)) \\
&= \forall Y. \forall W. (R(C,Y) \wedge \neg R(W,F(Y))) \\
&= \forall Y. \forall W. R(C,Y) \wedge \forall Y. \forall W. \neg R(W,F(Y)) \\
&= \{\underbrace{\{R(C,Y)\}}_{1}, \underbrace{\{\neg R(W,F(Y))\}}_{2}\} = C
\end{aligned}
$$

Por 1 y 2:

$$
\begin{aligned}
&\mathrm{mgu}\{R(C,Y_1) \doteq R(W,F(Y_2))\} \\
&= \mathrm{mgu}\{C \doteq W, Y_1 \doteq F(Y_2)\} && \text{decompose} \\
&= \mathrm{mgu}\{Y_1 \doteq F(Y_2)\} && \text{swap, elim } \{W := C\} \\
&= \mathrm{mgu}\{\} && \text{elim } \{Y_1 := F(Y_2)\} \\
&S = \{Y_1 := F(Y_2), W := C\} \\
&3 = S(\{\}) = \{\}
\end{aligned}
$$

$C \vdash \bot \Rightarrow \neg\sigma \vdash \bot \Rightarrow \vdash \sigma \qquad \therefore \sigma$ es válida