Inferencia de Tipos
Machete

Paradigmas (de Lenguajes) de Programación

1. Algoritmo de inferencia

- $\mathbb{W}(x) \rightsquigarrow \{x : ?k\} \vdash x : ?k, \quad ?k$ incógnita fresca
- $\mathbb{W}(\mathit{0}) \rightsquigarrow \emptyset \vdash \mathit{0} : \mathit{Nat}$
- $\mathbb{W}(\mathit{true}) \rightsquigarrow \emptyset \vdash \mathit{true} : \mathit{Bool}$
- $\mathbb{W}(\mathit{false}) \rightsquigarrow \emptyset \vdash \mathit{false} : \mathit{Bool}$
- $\mathbb{W}(\mathit{succ}(U)) \rightsquigarrow S(\Gamma) \vdash S(\mathit{succ}(M)) : \mathit{Nat}$ donde
  - $\mathbb{W}(U) = \Gamma \vdash M : \tau$
  - $S = MGU\{\tau \stackrel{?}{=} \mathit{Nat}\}$
- $\mathbb{W}(\mathit{pred}(U)) \rightsquigarrow S(\Gamma) \vdash S(\mathit{pred}(M)) : \mathit{Nat}$ donde
  - $\mathbb{W}(U) = \Gamma \vdash M : \tau$
  - $S = MGU\{\tau \stackrel{?}{=} \mathit{Nat}\}$
- $\mathbb{W}(\mathit{iszero}(U)) \rightsquigarrow S(\Gamma) \vdash S(\mathit{iszero}(M)) : \mathit{Bool}$ donde
  - $\mathbb{W}(U) = \Gamma \vdash M : \tau$
  - $S = MGU\{\tau \stackrel{?}{=} \mathit{Nat}\}$
- $\mathbb{W}(\mathit{if}\ U\ \mathit{then}\ V\ \mathit{else}\ W) \rightsquigarrow S(\Gamma_1) \cup S(\Gamma_2) \cup S(\Gamma_3) \vdash S(\mathit{if}\ M\ \mathit{then}\ P\ \mathit{else}\ Q) : S(\sigma)$ donde
  - $\mathbb{W}(U) = \Gamma_1 \vdash M : \rho$
  - $\mathbb{W}(V) = \Gamma_2 \vdash P : \sigma$
  - $\mathbb{W}(W) = \Gamma_3 \vdash Q : \tau$
  - $S = MGU\{\sigma \stackrel{?}{=} \tau, \rho \stackrel{?}{=} \mathit{Bool}\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_i, x : \sigma_2 \in \Gamma_j, i, j \in \{1, 2, 3\}\}$
- $\mathbb{W}(\lambda x.U) \rightsquigarrow \Gamma' \vdash \lambda x : \tau'.M : \tau' \to \rho$ donde
  - $\mathbb{W}(U) = \Gamma \vdash M : \rho$
  - $\tau' = \begin{cases} \alpha \text{ si } x : \alpha \in \Gamma \\ ?k \text{ con } ?k \text{ variable fresca en otro caso} \end{cases}$
  - $\Gamma' = \Gamma \ominus \{x\}$
- $\mathbb{W}(U\,V) \rightsquigarrow S(\Gamma_1) \cup S(\Gamma_2) \vdash S(M\,N) : S(?k)$ donde
  - $\mathbb{W}(U) = \Gamma_1 \vdash M : \tau$
  - $\mathbb{W}(V) = \Gamma_2 \vdash N : \rho$
  - $?k$ variable fresca
  - $S = MGU\{\tau \stackrel{?}{=} \rho \to ?k\} \cup \{\sigma_1 \stackrel{?}{=} \sigma_2 \mid x : \sigma_1 \in \Gamma_1, x : \sigma_2 \in \Gamma_2\}$

2. Algoritmo de unificación (Martelli-Montanari)

2.1. Reglas

Se enuncian las reglas para constructores de tipo C en general de cualquier aridad, y en particular para los constructores de tipo de λ^b

$$\sigma, \tau ::= Nat \mid Bool \mid \sigma \to \tau$$

1. **Descomposición**

   $\{\sigma_1 \to \sigma_2 \stackrel{?}{=} \tau_1 \to \tau_2\} \cup G \mapsto \{\sigma_1 \stackrel{?}{=} \tau_1, \sigma_2 \stackrel{?}{=} \tau_2\} \cup G$

   $\{Bool \stackrel{?}{=} Bool\} \cup G \mapsto G$

   $\{Nat \stackrel{?}{=} Nat\} \cup G \mapsto G$

   Caso general

   $\{C(\sigma_1, \dots, \sigma_n) \stackrel{?}{=} C(\tau_1, \dots, \tau_n)\} \cup G \mapsto \{\sigma_1 \stackrel{?}{=} \tau_1, \dots, \sigma_n \stackrel{?}{=} \tau_n\} \cup G$

2. **Eliminación de par trivial**

   $\{?k \stackrel{?}{=} ?k\} \cup G \mapsto G$

3. **Swap**: si σ no es una variable

   $\{\sigma \stackrel{?}{=} ?k\} \cup G \mapsto \{?k \stackrel{?}{=} \sigma\} \cup G$

4. **Eliminación de variable**: si $?k \notin FV(\sigma)$

   $\{?k \stackrel{?}{=} \sigma\} \cup G \mapsto_{\{?k:=\sigma\}} G\{?k := \sigma\}$

5. **Colisión**

   $\{\sigma \stackrel{?}{=} \tau\} \cup G \mapsto \textbf{falla}$, con $(\sigma, \tau) \in T \cup T^{-1}$ donde

   $T = \{(Bool, Nat), (Nat, \sigma_1 \to \sigma_2), (Bool, \sigma_1 \to \sigma_2)\}$ y T^{-1} representa invertir cada par

   Caso general: si $C \neq C'$ son constructores de tipo diferentes

   $\{C(\dots) \stackrel{?}{=} C'(\dots)\} \cup G \mapsto \textbf{falla}$

6. **Occur check**: si $?k \neq \sigma$ y $?k \in FV(\sigma)$

   $\{?k \stackrel{?}{=} \sigma\} \cup G \mapsto \textbf{falla}$

2.2. Ejemplos

2.2.1. Secuencia exitosa

$$\begin{array}{ll}
 & \{(Nat \to ?1) \to (?1 \to ?3) \stackrel{?}{=} ?2 \to (?4 \to ?4) \to ?2\} \\
\mapsto^1 & \{Nat \to ?1 \stackrel{?}{=} ?2, ?1 \to ?3 \stackrel{?}{=} (?4 \to ?4) \to ?2\} \\
\mapsto^3 & \{?2 \stackrel{?}{=} Nat \to ?1, ?1 \to ?3 \stackrel{?}{=} (?4 \to ?4) \to ?2\} \\
\mapsto^4_{\{?2:=Nat \to ?1\}} & \{?1 \to ?3 \stackrel{?}{=} (?4 \to ?4) \to (Nat \to ?1)\} \\
\mapsto^1 & \{?1 \stackrel{?}{=} ?4 \to ?4, ?3 \stackrel{?}{=} Nat \to ?1\} \\
\mapsto^4_{\{?1:=?4 \to ?4\}} & \{?3 \stackrel{?}{=} Nat \to (?4 \to ?4)\} \\
\mapsto^4_{\{?3:=Nat \to (?4 \to ?4)\}} & \emptyset
\end{array}$$

El MGU es

$$\begin{aligned}
& \{?3 := Nat \to (?4 \to ?4)\} \circ \{?1 := ?4 \to ?4\} \circ \{?2 := Nat \to ?1\} \\
= & \{?2 := Nat \to (?4 \to ?4),\ ?1 := ?4 \to ?4,\ ?3 := Nat \to (?4 \to ?4)\}
\end{aligned}$$

2.2.2. Secuencia fallida

$$
\begin{array}{ll}
& \{?1 \to (?2 \to ?1) \stackrel{?}{=} ?2 \to ((?1 \to Nat) \to ?1)\} \\
\mapsto^1 & \{?1 \stackrel{?}{=} ?2, ?2 \to ?1 \stackrel{?}{=} (?1 \to Nat) \to ?1\} \\
\mapsto^4_{\{?1:=?2\}} & \{?2 \to ?2 \stackrel{?}{=} (?2 \to Nat) \to ?2\} \\
\mapsto^1 & \{?2 \stackrel{?}{=} ?2 \to Nat, ?2 \stackrel{?}{=} ?2\} \\
\mapsto^6 & \textbf{falla}
\end{array}
$$

2.2.3. Constructores en general

Se usan los constructores de tipos de listas,

$$\sigma ::= \ldots \mid [\sigma]$$

$$
\begin{array}{ll}
& \{(?3 \to ?4 \to ?4) \to ?4 \to [?3] \to ?4 \stackrel{?}{=} ((?1 \to ?2) \to [?1] \to [?2]) \to ?5\} \\
\mapsto^1 & \{?3 \to ?4 \to ?4 \stackrel{?}{=} (?1 \to ?2) \to [?1] \to [?2], ?4 \to [?3] \to ?4 \stackrel{?}{=} ?5\} \\
\mapsto^3 & \{?3 \to ?4 \to ?4 \stackrel{?}{=} (?1 \to ?2) \to [?1] \to [?2],\ ?5 \stackrel{?}{=} ?4 \to [?3] \to ?4\} \\
\mapsto^4_{\{?5:=?4\to[?3]\to?4\}} & \{?3 \to ?4 \to ?4 \stackrel{?}{=} (?1 \to ?2) \to [?1] \to [?2]\} \\
\mapsto^1 & \{?3 \stackrel{?}{=} ?1 \to ?2,\ ?4 \to ?4 \stackrel{?}{=} [?1] \to [?2]\} \\
\mapsto^4_{\{?3:=?1\to?2\}} & \{?4 \to ?4 \stackrel{?}{=} [?1] \to [?2]\} \\
\mapsto^1 & \{?4 \stackrel{?}{=} [?1],\ ?4 \stackrel{?}{=} [?2]\} \\
\mapsto^4_{\{?4:=[?1]\}} & \{[?1] \stackrel{?}{=} [?2]\} \\
\mapsto^1 & \{?1 \stackrel{?}{=} ?2\} \\
\mapsto^4_{\{?1:=?2\}} & \emptyset
\end{array}
$$

El MGU es

$$
\begin{aligned}
&\{?1 := ?2\} \circ \{?4 := [?1]\} \circ \{?3 := ?1 \to ?2\} \circ \{?5 := ?4 \to [?3] \to ?4\} \\
= &\{?5 := ?[?2] \to [?2 \to ?2] \to ?[?2],\ ?3 := ?2 \to ?2,\ ?4 := [?2],\ ?1 := ?2\}
\end{aligned}
$$