Tipos Habitados

Sean σ, τ, ρ tipos cualquiera.

a) $\sigma \to \tau \to \sigma$

$M := \lambda x{:}\sigma.\ \lambda y{:}\tau.\ x$ const

$$\dfrac{\dfrac{}{x:\sigma,\ y:\tau \vdash x:\sigma}\ ax_V}{\dfrac{x:\sigma \vdash \lambda y{:}\tau.\ x : \tau \to \sigma}{\vdash \lambda x{:}\sigma.\ \lambda y{:}\tau.\ x : \sigma \to \tau \to \sigma}\ \to_i}\ \to_i$$

b) $\Delta := (\sigma \to \tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho$

$M := \lambda f{:}(\sigma \to \tau \to \rho).\ \lambda g{:}(\sigma \to \tau).\ \lambda x{:}\sigma.\ (fx)(gx)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : \sigma \to \tau \to \rho}\ ax_V \quad \dfrac{}{\Gamma \vdash x : \sigma}\ ax_V}{\Gamma \vdash fx : \tau \to \rho}\ \to_e \quad \dfrac{\dfrac{}{\Gamma \vdash g : \sigma \to \tau}\ ax_V \quad \dfrac{}{\Gamma \vdash x : \sigma}\ ax_V}{\Gamma \vdash gx : \tau}\ \to_e}{\Gamma = f : \sigma \to \tau \to \rho,\ g : \sigma \to \tau,\ x : \sigma \vdash (fx)(gx) : \rho}\ \to_e}{f : \sigma \to \tau \to \rho,\ g : \sigma \to \tau \vdash \lambda x{:}\sigma.\ (fx)(gx) : \sigma \to \rho}\ \to_i}{f : \sigma \to \tau \to \rho \vdash \lambda g{:}(\sigma \to \tau).\ \lambda x{:}\sigma.\ (fx)(gx) : (\sigma \to \tau) \to \sigma \to \rho}\ \to_i}{\vdash \lambda f{:}(\sigma \to \tau \to \rho).\ \lambda g{:}(\sigma \to \tau).\ \lambda x{:}\sigma.\ (fx)(gx) : \Delta}\ \to_i$$

c) $(\sigma \to \tau \to \rho) \to \tau \to \sigma \to \rho$

$M := \lambda f:(\sigma \to \tau \to \rho).\ \lambda y:\tau.\ \lambda x:\sigma.\ f\ x\ y$ flip

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : \sigma \to \tau \to \rho}\,ax_V \quad \dfrac{}{\Gamma \vdash x : \sigma}\,ax_V}{\Gamma \vdash f\ x : \tau \to \rho}\,{\to}e \quad \dfrac{}{\Gamma \vdash y : \tau}\,ax_V}{\Gamma = f : \sigma \to \tau \to \rho,\ y:\tau,\ x:\sigma \vdash f\ x\ y : \rho}\,{\to}e}{f : \sigma \to \tau \to \rho,\ y:\tau \vdash \lambda x:\sigma.\ f\ x\ y : \sigma \to \rho}\,{\to}i}{f : \sigma \to \tau \to \rho \vdash \lambda y:\tau.\ \lambda x:\sigma.\ f\ x\ y : \tau \to \sigma \to \rho}\,{\to}i$$

$$\vdash \lambda f:(\sigma \to \tau \to \rho).\ \lambda y:\tau.\ \lambda x:\sigma.\ f\ x\ y : (\sigma \to \tau \to \rho) \to \tau \to \sigma \to \rho \quad {\to}i$$

d) $(\tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho$

$M := \lambda f:(\tau \to \rho).\ \lambda g:(\sigma \to \tau).\ \lambda x:\sigma.\ f\,(g\,x)$ (.) composición

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : \tau \to \rho}\,ax_V \quad \dfrac{\dfrac{}{\Gamma \vdash g : \sigma \to \tau}\,ax_V \quad \dfrac{}{\Gamma \vdash x : \sigma}\,ax_V}{\Gamma \vdash g\ x : \tau}\,{\to}e}{\Gamma = f : \tau \to \rho,\ g : \sigma \to \tau,\ x : \sigma \vdash f\,(g\,x) : \rho}\,{\to}e}{f : \tau \to \rho,\ g : \sigma \to \tau \vdash \lambda x:\sigma.\ f\,(g\,x) : \sigma \to \rho}\,{\to}i}{f : \tau \to \rho \vdash \lambda g:(\sigma \to \tau).\ \lambda x:\sigma.\ f\,(g\,x) : (\sigma \to \tau) \to \sigma \to \rho}\,{\to}i$$

$$\vdash \lambda f:(\tau \to \rho).\ \lambda g:(\sigma \to \tau).\ \lambda x:\sigma.\ f\,(g\,x) : (\tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho \quad {\to}i$$