

$$\sigma = (\forall x. \exists y. R(x, y)) \Rightarrow \exists y. \forall x. R(x, y)$$

$$\begin{aligned} \neg \sigma &= \neg((\forall x. \exists y. R(x, y)) \Rightarrow \exists y. \forall x. R(x, y)) \\ &= \neg(\neg(\forall x. \exists y. R(x, y)) \vee \exists y. \forall x. R(x, y)) \\ &= (\forall x. \exists y. R(x, y)) \wedge \neg \exists y. \forall x. R(x, y) \\ &= (\forall x. \exists y. R(x, y)) \wedge \forall y. \exists x. \neg R(x, y) \\ &= (\forall x. \exists y. R(x, y)) \wedge \forall z. \exists w. \neg R(w, z) \\ &= \forall x. \exists y. \exists w. \forall z. (R(x, y) \wedge \neg R(w, z)) \\ &= \forall x. \forall z. (R(x, f(x)) \wedge \neg R(g(x), z)) \\ &= \{ \underbrace{\{R(x, f(x))\}}_1, \underbrace{\{\neg R(g(x), z)\}}_2 \} = C \end{aligned}$$

$$\begin{aligned} 1 \text{ y } 2: \text{ mgu } \{ R(x_1, f(x_1)) \doteq R(g(x_2), z) \} \\ &= \text{mgu } \{ x_1 \doteq g(x_2), f(x_1) \doteq z \} && \text{decompose} \\ &= \text{mgu } \{ f(g(x_2)) \doteq z \} && \text{elim } \{ x_1 \doteq g(x_2) \} \\ &= \text{mgu } \{ \} && \text{swap, elim } \{ z \doteq f(g(x_2)) \} \end{aligned}$$

$$S = \{ x_1 \doteq g(x_2), z \doteq f(g(x_2)) \}$$

$$3 = S(\{\}) = \{\}$$

$$C \vdash \perp \Rightarrow \neg \sigma \vdash \perp \Rightarrow \vdash \sigma$$

$\therefore \sigma$  válida