Machete de interpretación y semántica denotacional

Intérprete con estrategia Call By Name (CBN)

$$\frac{\Gamma' \vdash M \hookrightarrow V}{\Gamma, x = \langle M, \Gamma' \rangle, \Delta \vdash x \hookrightarrow V} \; x \notin \mathsf{D}(\Delta)$$

$$\frac{}{\Gamma \vdash \lambda x.M \hookrightarrow \langle x, M, \Gamma \rangle} \qquad \frac{\Gamma \vdash M \hookrightarrow \langle x, M', \Gamma' \rangle \quad \Gamma', x = \langle N, \Gamma \rangle \vdash M' \hookrightarrow V}{\Gamma \vdash MN \hookrightarrow V}$$

$$\frac{}{\Gamma \vdash \mathsf{True} \hookrightarrow \mathsf{True}} \qquad \frac{}{\Gamma \vdash \mathsf{False} \hookrightarrow \mathsf{False}}$$

$$\frac{\Gamma \vdash M \hookrightarrow \mathsf{True} \quad \Gamma \vdash N_1 \hookrightarrow V}{\Gamma \vdash \mathsf{if}\ M\ \mathsf{then}\ N_1\ \mathsf{else}\ N_2 \hookrightarrow V} \qquad \frac{\Gamma \vdash M \hookrightarrow \mathsf{False} \quad \Gamma \vdash N_2 \hookrightarrow V}{\Gamma \vdash \mathsf{if}\ M\ \mathsf{then}\ N_1\ \mathsf{else}\ N_2 \hookrightarrow V}$$

$$\frac{\Gamma, x = \langle \mu x.M, \Gamma \rangle \vdash M \hookrightarrow V}{\Gamma \vdash \mu x.M \hookrightarrow V}$$

Intérprete con estrategia Call By Value (CBV)

$$\frac{}{\Gamma, x = V, \Delta \vdash x \hookrightarrow V}\ x \notin \mathsf{D}(\Delta) \qquad \frac{\Gamma' \vdash \mu y.M \to V}{\Gamma, x = \langle \mu y.M, \Gamma' \rangle, \Delta \vdash x \hookrightarrow V}\ x \notin \mathsf{D}(\Delta)$$

$$\frac{}{\Gamma \vdash \lambda x.M \hookrightarrow \langle x, M, \Gamma \rangle} \qquad \frac{\Gamma \vdash N \hookrightarrow W \quad \Gamma \vdash M \hookrightarrow \langle x, M', \Gamma' \rangle \quad \Gamma', x = W \vdash M' \hookrightarrow V}{\Gamma \vdash MN \hookrightarrow V}$$

$$\frac{}{\Gamma \vdash \mathsf{True} \hookrightarrow \mathsf{True}} \qquad \frac{}{\Gamma \vdash \mathsf{False} \hookrightarrow \mathsf{False}}$$

$$\frac{\Gamma \vdash M \hookrightarrow \mathsf{True} \quad \Gamma \vdash N_1 \hookrightarrow V}{\Gamma \vdash \mathsf{if}\ M\ \mathsf{then}\ N_1\ \mathsf{else}\ N_2 \hookrightarrow V} \qquad \frac{\Gamma \vdash M \hookrightarrow \mathsf{False} \quad \Gamma \vdash N_2 \hookrightarrow V}{\Gamma \vdash \mathsf{if}\ M\ \mathsf{then}\ N_1\ \mathsf{else}\ N_2 \hookrightarrow V}$$

$$\frac{\Gamma, x = \langle \mu x.M, \Gamma \rangle \vdash M \hookrightarrow V}{\Gamma \vdash \mu x.M \hookrightarrow V}$$

Extensión de los intérpretes con números naturales

CBN y CBV

$$\frac{}{\Gamma \vdash \text{zero} \hookrightarrow \text{zero}}$$

$$\frac{\Gamma \vdash M \hookrightarrow \text{zero}}{\Gamma \vdash \text{pred}(M) \hookrightarrow \text{zero}}$$

$$\frac{\Gamma \vdash M \hookrightarrow \text{zero}}{\Gamma \vdash \text{isZero}(M) \hookrightarrow \text{True}}$$

$$\frac{\Gamma \vdash M \hookrightarrow V}{\Gamma \vdash \text{succ}(M) \hookrightarrow \text{succ}(V)}$$

$$\frac{\Gamma \vdash M \hookrightarrow \text{succ}(V)}{\Gamma \vdash \text{pred}(M) \hookrightarrow V}$$

$$\frac{\Gamma \vdash M \hookrightarrow \text{succ}(V)}{\Gamma \vdash \text{isZero}(M) \hookrightarrow \text{False}}$$

Semántica denotacional del Cálculo Lambda (sin error)

$$[\![x]\!]_v = v(x)$$

$$[\![\lambda x{:}\tau.M]\!]_v = V^{[\![\tau]\!]} \mapsto [\![M]\!]_{v,x=V}$$

$$[\![MN]\!]_v = [\![M]\!]_v [\![N]\!]_v$$

$$[\![\,\mathsf{True}\,]\!]_v = \mathsf{true}$$

$$[\![\mathsf{False}]\!]_v = \mathsf{false}$$

$$[\![0]\!]_v = 0$$

$$[\![\mathsf{succ}(M)]\!]_v = [\![M]\!]_v + 1$$

$$[\![\mathsf{if}\ M\ \mathsf{then}\ N\ \mathsf{else}\ O]\!]_v = \begin{cases} [\![N]\!]_v & \text{si } [\![M]\!]_v = \mathsf{true} \\ [\![O]\!]_v & \text{si } [\![M]\!]_v = \mathsf{false} \\ \bot & \text{si } [\![M]\!]_v = \bot \end{cases}$$

$$[\![\mathsf{pred}(M)]\!]_v = \begin{cases} 0 & \text{si } [\![M]\!]_v = 0 \\ [\![M]\!]_v - 1 & \text{si no} \end{cases}$$

$$[\![\mathsf{isZero}(M)]\!]_v = \begin{cases} \mathsf{true} & \text{si } [\![M]\!]_v = 0 \\ \mathsf{false} & \text{si } [\![M]\!]_v = n \neq 0 \\ \bot & \text{si } [\![M]\!]_v = \bot \end{cases}$$

$$[\![\mu x{:}\tau.M]\!]_v = \mathsf{FIX}(V^{[\![\tau]\!]} \mapsto [\![M]\!]_{v,x=V})$$