a)

$$\dfrac{}{\vdash 0 : \sigma}\ ax_0 \qquad \dfrac{\vdash 0 : \sigma}{\vdash succ(0) : \sigma}\ succ \qquad \sigma = Nat$$

b)

$$\dfrac{}{\vdash 0 : Nat}\ ax_0 \qquad \dfrac{\vdash 0 : Nat}{\vdash succ(0) : Nat}\ succ \qquad \dfrac{\vdash succ(0) : Nat}{\vdash isZero(succ(0)) : \sigma}\ isZero \qquad \sigma = Bool$$

c)

$$\dfrac{\dfrac{}{\vdash true : Bool}\ ax_T \quad \dfrac{}{\vdash False : Bool}\ ax_F \quad \dfrac{}{\vdash False : Bool}\ ax_F}{\vdash \text{if true then False else False} : Bool}\ if \qquad \dfrac{}{\vdash 0 : \sigma}\ ax_0 \qquad \dfrac{\vdash 0 : \sigma}{\vdash succ(0) : \sigma}\ succ$$

$$\dfrac{\vdash \text{if true then False else False} : Bool \qquad \vdash 0 : \sigma \qquad \vdash succ(0) : \sigma}{\vdash \text{if (if true then False else False) then } 0 \text{ else } succ(0) : \sigma}\ if$$

$\sigma = Nat$