Tipos Habitados

Sean σ, τ, ρ tipos cualquiera.

a) $\sigma \to \tau \to \sigma$

$M := \lambda x:\sigma.\ \lambda y:\tau.\ x$ const

$$
\dfrac{\dfrac{\dfrac{}{x:\sigma,\ y:\tau \vdash x : \sigma}\,ax_v}{x:\sigma \vdash \lambda y:\tau.x : \tau \to \sigma}\,\to_i}{\vdash \lambda x:\sigma.\ \lambda y:\tau.\ x : \sigma \to \tau \to \sigma}\,\to_i
$$

b) $\Delta := (\sigma \to \tau \to \rho) \to (\sigma \to \tau) \to \sigma \to \rho$

$M := \lambda f:(\sigma \to \tau \to \rho).\ \lambda g:(\sigma \to \tau).\ \lambda x:\sigma.\ (f x)(g x)$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash f : \sigma \to \tau \to \rho}\,ax_v \quad \dfrac{}{\Gamma \vdash x : \sigma}\,ax_v}{\Gamma \vdash f x : \tau \to \rho}\,\to_e \quad \dfrac{\dfrac{}{\Gamma \vdash g : \sigma \to \tau}\,ax_v \quad \dfrac{}{\Gamma \vdash x : \sigma}\,ax_v}{\Gamma \vdash g x : \tau}\,\to_e}{\Gamma = f : \sigma \to \tau \to \rho,\ g : \sigma \to \tau,\ x:\sigma \vdash (f x)(g x) : \rho}\,\to_e}{f : \sigma \to \tau \to \rho,\ g : \sigma \to \tau \vdash \lambda x:\sigma.\ (f x)(g x) : \sigma \to \rho}\,\to_i}{f : \sigma \to \tau \to \rho \vdash \lambda g:(\sigma \to \tau).\ \lambda x:\sigma.\ (f x)(g x) : (\sigma \to \tau) \to \sigma \to \rho}\,\to_i}{\vdash \lambda f:(\sigma \to \tau \to \rho).\ \lambda g:(\sigma \to \tau).\ \lambda x:\sigma.\ (f x)(g x) : \Delta}\,\to_i}
$$

c) $(\sigma\to\tau\to\rho)\to\tau\to\sigma\to\rho$

$M := \lambda f:(\sigma\to\tau\to\rho).\ \lambda y:\tau.\ \lambda x:\sigma.\ f\,x\,y$ flip

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash f:\sigma\to\tau\to\rho}\,ax_v\quad \dfrac{}{\Gamma\vdash x:\sigma}\,ax_v}{\Gamma\vdash f\,x:\tau\to\rho}\,{\to_e}\quad \dfrac{}{\Gamma\vdash y:\tau}\,ax_v}{\Gamma = f:\sigma\to\tau\to\rho,\ y:\tau,\ x:\sigma\ \vdash\ f\,x\,y:\rho}\,{\to_e}}{f:\sigma\to\tau\to\rho,\ y:\tau\ \vdash\ \lambda x:\sigma.\,f\,x\,y:\sigma\to\rho}\,{\to_i}}{f:\sigma\to\tau\to\rho\ \vdash\ \lambda y:\tau.\lambda x:\sigma.\,f\,x\,y:\tau\to\sigma\to\rho}\,{\to_i}}{\vdash\ \lambda f:(\sigma\to\tau\to\rho).\ \lambda y:\tau.\ \lambda x:\sigma.\ f\,x\,y:(\sigma\to\tau\to\rho)\to\tau\to\sigma\to\rho}\,{\to_i}$$

d) $(\tau\to\rho)\to(\sigma\to\tau)\to\sigma\to\rho$

$M := \lambda f:(\tau\to\rho).\ \lambda g:(\sigma\to\tau).\ \lambda x:\sigma.\ f(g\,x)$ (.) composición

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash f:\tau\to\rho}\,ax_v\quad \dfrac{\dfrac{}{\Gamma\vdash g:\sigma\to\rho}\,ax_v\quad \dfrac{}{\Gamma\vdash x:\sigma}\,ax_v}{\Gamma\vdash g\,x:\rho}\,{\to_e}}{\Gamma = f:\tau\to\rho,\ g:\sigma\to\tau,\ x:\sigma\ \vdash\ f(g\,x):\rho}\,{\to_e}}{f:\tau\to\rho,\ g:\sigma\to\tau\ \vdash\ \lambda x:\sigma.\,f(g\,x):\sigma\to\rho}\,{\to_i}}{f:\tau\to\rho\ \vdash\ \lambda g:(\sigma\to\tau).\,\lambda x:\sigma.\,f(g\,x):(\sigma\to\tau)\to\sigma\to\rho}\,{\to_i}}{\vdash\ \lambda f:(\tau\to\rho).\ \lambda g:(\sigma\to\tau).\ \lambda x:\sigma.\ f(g\,x):(\tau\to\rho)\to(\sigma\to\tau)\to\sigma\to\rho}\,{\to_i}$$