Evaluar en el intérprete CBN las siguientes expresiones.

I. $(\lambda x.x)$ zero

II. $(\lambda x.\lambda x.x)\ \underline{2}\ \underline{3}$

III. $(\lambda x.\lambda y.(\lambda x.\text{if isZero}(x) \text{ then } y \text{ else } x)\ x)\ \underline{5}\ \underline{4}$

IV. $(\lambda x.(\lambda f.(\lambda y.f\ \underline{6})\ \underline{5})\ (\lambda y.\text{if isZero}(y) \text{ then } x \text{ else } y))\ \underline{4}$

I)

$$\dfrac{\dfrac{}{\vdash (\lambda x.x) \hookrightarrow \langle x, x, \emptyset\rangle} \qquad \dfrac{\dfrac{}{\vdash \text{zero} \hookrightarrow \text{zero}}}{x = \langle \text{zero}, \emptyset\rangle \vdash x \hookrightarrow \text{zero}}}{\vdash (\lambda x.x)\ \text{zero} \hookrightarrow \text{zero}}$$

II)

$$\dfrac{\dfrac{\vdash \lambda x.\lambda x.x \hookrightarrow \langle x, \lambda x.x, \emptyset\rangle \qquad \Gamma_1 : x = \langle \underline{2}, \emptyset\rangle \vdash \lambda x.x \hookrightarrow \langle x, x, \Gamma_1\rangle}{\vdash (\lambda x.\lambda x.x)\ \underline{2} \hookrightarrow \langle x, x, \Gamma_1\rangle} \qquad \dfrac{}{\vdash \underline{3} \hookrightarrow \underline{3}}}{\vdash (\lambda x.\lambda x.x)\ \underline{2}\ \underline{3} \hookrightarrow \underline{3}}$$

$$\Gamma_1, x = \langle \underline{3}, \emptyset\rangle \vdash x \hookrightarrow \underline{3}$$

III)

$(\lambda X.\lambda Y.(\lambda X.\ \text{if isZero}(x)\ \text{then } y \text{ else } x)\ X)\ \underline{5}\ \underline{4}$

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\vdash \underline{5} \hookrightarrow \underline{5}}}{X=\langle \underline{5},\emptyset\rangle,\ Y=\langle \underline{4},\emptyset\rangle \vdash X \hookrightarrow \underline{5}}}{\Gamma,\ x=\langle X,\Gamma\rangle \vdash X \hookrightarrow \underline{5}}}{\Gamma,\ x=\langle X,\Gamma\rangle \vdash \text{isZero}(x) \hookrightarrow \text{False} \quad \Gamma \vdash X \hookrightarrow \underline{5}}}{\Gamma,\ x=\langle X,\Gamma\rangle \vdash \text{if isZero}(x)\ \text{then } y \text{ else } x \hookrightarrow \underline{5}}}{\Gamma \vdash \lambda X.A \hookrightarrow \langle X, A, \Gamma\rangle \quad \Gamma,\ x=\langle X,\Gamma\rangle \vdash A \hookrightarrow \underline{5}}}{\Gamma:\ X=\langle \underline{5},\emptyset\rangle,\ Y=\langle \underline{4},\emptyset\rangle \vdash (\lambda X.A)\ X \hookrightarrow \underline{5}}$$

$$\frac{\vdash \lambda X.\lambda Y.(\lambda X.A)\ X \hookrightarrow \langle X, B, \emptyset\rangle \qquad X=\langle \underline{5},\emptyset\rangle \vdash B \hookrightarrow \langle Y, C, X=\langle \underline{5},\emptyset\rangle\rangle}{\vdash (\lambda X.\lambda Y.(\lambda X.A)\ X)\ \underline{5} \hookrightarrow \langle Y, C, X=\langle \underline{5},\emptyset\rangle\rangle}$$

$$\vdash (\lambda X.\lambda Y.(\lambda X.A)\ X)\ \underline{5}\ \underline{4} \hookrightarrow \underline{5}$$

$\overbrace{\lambda X.\lambda Y.(\lambda X.A)\ X}^{B}$

$\overbrace{\lambda X.A}^{C}$

$A = \text{if isZero}(x)\ \text{then } y \text{ else } x$

IV) $(\lambda x.(\lambda f.(\lambda y.\ f\ \underline{6})\ \underline{5})\ (\lambda y.\text{if iszero}(y)\text{ then } x \text{ else } y))\ \underline{4}$

$B' = \text{if iszero}(y)\text{ then } x \text{ else } y$

$\Delta \vdash \underline{6} \hookrightarrow \underline{6}$

$\Pi \vdash y \hookrightarrow \underline{6}$ $\qquad$ $\Delta \vdash \underline{6} \hookrightarrow \underline{6}$

$x=\langle \underline{4}, \emptyset\rangle \vdash B \hookrightarrow \langle y, B', x=\langle \underline{4}, \emptyset\rangle\rangle$ $\qquad$ $\Pi \vdash \text{iszero}(y) \hookrightarrow ff$ $\qquad$ $\Pi \vdash y \hookrightarrow \underline{6}$

$\Gamma, y=\langle \underline{5}, \Gamma\rangle \vdash f \hookrightarrow \langle y, B', x=\langle \underline{4}, \emptyset\rangle\rangle$ $\qquad$ $\Pi : x=\langle \underline{4}, \emptyset\rangle,\ y=\langle \underline{6}, \Delta\rangle \vdash B' \hookrightarrow \underline{6}$

$\Gamma \vdash \lambda y.\ f\ \underline{6} \hookrightarrow \langle y, f\underline{6}, \Gamma\rangle$ $\qquad$ $\Delta : \Gamma, y=\langle \underline{5}, \Gamma\rangle \vdash f\ \underline{6} \hookrightarrow \underline{6}$

$\Gamma : x=\langle \underline{4}, \emptyset\rangle, f=\langle B, x=\langle \underline{4}, \emptyset\rangle\rangle \vdash (\lambda y.\ f\ \underline{6})\ \underline{5} \hookrightarrow \underline{6}$

$x=\langle \underline{4}, \emptyset\rangle \vdash A \hookrightarrow \langle f, (\lambda y.\ f\ \underline{6})\ \underline{5}, x=\langle \underline{4}, \emptyset\rangle\rangle$

$\vdash (\lambda x.AB) \hookrightarrow \langle x, AB, \emptyset\rangle$ $\qquad$ $x=\langle \underline{4}, \emptyset\rangle \vdash AB \hookrightarrow \underline{6}$

$\vdash (\lambda x.\underbrace{(\lambda f.(\lambda y.\ f\ \underline{6})\ \underline{5})}_{A}\ \underbrace{(\lambda y.\text{if iszero}(y)\text{ then } x \text{ else } y)}_{B})\ \underline{4} \hookrightarrow \underline{6}$