Tipos Comportamentales y Contratos

Práctica 4: Subtyping y lambda cálculo con sesiones

Asumiremos la siguiente relación de subtipado para tipos básicos: $\mathtt{nat} \leqslant \mathtt{int} \leqslant \mathtt{float}$

1. Para cada uno de los siguientes pares de tipos S y T indicar si están en relación de subtipado, es decir, si $S \leqslant T$ o $T \leqslant S$, o no están relacionados.

   a) $S = \mathtt{!int.end}$ y $T = \mathtt{!float.end}$.
   b) $S = \mathtt{?int.end}$ y $T = \mathtt{?float.end}$.
   c) $S = \mathtt{!(?int.end).end}$ y $T = \mathtt{!(?float.end).end}$.
   d) $S = \mathtt{!(!int.end).end}$ y $T = \mathtt{!(!float.end).end}$.
   e) $S = \mathtt{?(?int.end).end}$ y $T = \mathtt{?(?float.end).end}$.
   f) $S = \mathtt{?(!int.end).end}$ y $T = \mathtt{?(!float.end).end}$.
   g) $S = \oplus[\mathtt{l}_1 : \mathtt{!int.end}, \mathtt{l}_2 : \mathtt{end}]$ $S = \oplus[\mathtt{l}_1 : \mathtt{!int.end}]$
   h) $S = \oplus[\mathtt{l}_1 : \mathtt{!int.end}]$ $S = \oplus[\mathtt{l}_1 : \mathtt{!float.end}]$
   i) $S = \oplus[\mathtt{l}_1 : \mathtt{?int.end}]$ $S = \oplus[\mathtt{l}_1 : \mathtt{?float.end}]$
   j) $S = \&[\mathtt{l}_1 : \mathtt{!int.end}]$ $S = \&[\mathtt{l}_1 : \mathtt{!float.end}]$
   k) $S = \&[\mathtt{l}_1 : \mathtt{?int.end}]$ $S = \&[\mathtt{l}_1 : \mathtt{?float.end}]$

2. Asumir que $S = \mathtt{!nat.end}$ y $T = \mathtt{!float.end}$ y que $\Gamma \vdash -1 : \mathtt{int}$ y $\Gamma \vdash 5{,}0 : \mathtt{float}$. Indicar si los siguientes términos están bien tipados:

   a) $(\nu x{:}S)(x^{+}\mathtt{!}(-1)\mathtt{.}0 \mid x^{-}\mathtt{?}(y{:}\mathtt{int})\mathtt{.}0)$
   b) $(\nu x{:}T)(x^{+}\mathtt{!}(-1)\mathtt{.}0 \mid x^{-}\mathtt{?}(y{:}\mathtt{int})\mathtt{.}0)$
   c) $(\nu x{:}S)(x^{+}\mathtt{!}(5{,}0)\mathtt{.}0 \mid x^{-}\mathtt{?}(y{:}\mathtt{int})\mathtt{.}0)$
   d) $(\nu x{:}T)(x^{+}\mathtt{!}(5{,}0)\mathtt{.}0 \mid x^{-}\mathtt{?}(y{:}\mathtt{int})\mathtt{.}0)$
   e) $(\nu x{:}S)(x^{+}\mathtt{!}(-1)\mathtt{.}0 \mid x^{-}\mathtt{?}(y{:}\mathtt{float})\mathtt{.}0)$
   f) $(\nu x{:}\&[\mathtt{l}_1 : \mathtt{end}, \mathtt{l}_2 : \mathtt{end}])(x^{+} \triangleright [\mathtt{l}_1 : 0] \mid x^{-} \triangleleft \mathtt{l}_2\mathtt{.}0)$
   g) $(\nu x{:}\&[\mathtt{l}_1 : \mathtt{end}, \mathtt{l}_2 : \mathtt{end}])(x^{+} \triangleright [\mathtt{l}_1 : 0] \mid x^{-} \triangleleft \mathtt{l}_1\mathtt{.}0)$
   h) $(\nu x{:}\&[\mathtt{l}_1 : \mathtt{end}])(x^{+} \triangleright [\mathtt{l}_1 : 0, \mathtt{l}_2 : 0] \mid x^{-} \triangleleft \mathtt{l}_1\mathtt{.}0)$
   i) $(\nu x{:}\&[\mathtt{l}_1 : \mathtt{end}])(x^{+} \triangleright [\mathtt{l}_1 : 0, \mathtt{l}_2 : 0] \mid x^{-} \triangleleft \mathtt{l}_2\mathtt{.}0)$

3. Mostrar si los siguientes pares de tipos infinitos están en relación de subtipado.

   a) $S = \mu X\,.\,\mathtt{!int.!float.}X$ y $T = \mu X\,.\,\mu Y\,.\,\mathtt{!int.}X$
   b) $S = \mu X\,.\,\mathtt{!(?float.end).!(?int.end).}X$ y $T = \mu X\,.\,\mathtt{!(?float.end).}X$
   c) $S = \mu X\,.\,\mathtt{!(?float.end).!(?int.end).}X$ y $T = \mu X\,.\,\mathtt{!(?int.end).}X$

4. Para los siguientes términos, indicar si están bien tipado. En caso afirmativo, mostrar sus posibles reducciones.

a)
```
let s = create () in
let a = fork (λ x. close (send true x)) (fst s) in
let b = receive (snd s) in
close (snd b)
```

b)
```
let s = create () in
let a = fork (λ x. close (send true x)) (fst s) in
let b = receive (snd s) in
let b = receive (snd s) in
close (snd b)
```

c)
```
let s = create () in
let a = fork (λ x. close (send true x)) (fst s) in
let b = receive (snd s) in
let b = receive (snd b) in
close (snd b)
```