

Ejercicio 1

Para cada uno de los siguientes pares de tipos `S` y `T` indicar si están en relación de subtipado, es decir, si `S ≤ T` o `T ≤ S`, o no están relacionados.

a)

`S = !int.end
T = !float.end`

`int ≤ float`

`end ≤ end`

`!float.end ≤ !int.end`

`[S-End]`

`[S-OutS]`

`T ≤ S`

b)

`S = ?int.end
T = ?float.end`

`int ≤ float`

`end ≤ end`

`?int.end ≤ ?float.end`

`[S-End]`

`[S-InS]`

`S ≤ T`

c)

`S = !(?int.end).end
T = !(?float.end).end`

`inciso b)`

`?int.end ≤ ?float.end`

`end ≤ end`

`!(?float.end).end ≤ !(?int.end).end`

`[S-End]`

`[S-OutS]`

`T ≤ S`

d)

`S = !(!int.end).end
T = !(!float.end).end`

`inciso a)`

`!float.end ≤ !int.end`

`end ≤ end`

`[S-End]`

----- [S-OutS]
!(int.end).end ≤ !(float.end).end

$S \leq T$

e)

$S = ?(?int.end).end$
 $T = ?(?float.end).end$

inciso b)
----- [S-End]
?int.end ≤ ?float.end end ≤ end
----- [S-InS]
?(?int.end).end ≤ ?(?float.end).end

$S \leq T$

f)

$S = ?(!int.end).end$
 $T = ?(!float.end).end$

inciso a)
----- [S-End]
!float.end ≤ !int.end end ≤ end
----- [S-InS]
?(!float.end).end ≤ ?(!int.end).end

$T \leq S$

g)

$S = \otimes[l1:!int.end, l2:end]$
 $T = \otimes[l1:!int.end]$

----- [S-End]
int ≤ int end ≤ end
----- [S-OutS]
!int.end ≤ !int.end
----- [S-Choice]
 $\otimes[l1:!int.end, l2:end] \leq \otimes[l1:!int.end]$

$S \leq T$

h)

$S = \otimes[l1:!int.end]$
 $T = \otimes[l1:!float.end]$

```

----- [S-End]
int ≤ float    end ≤ end
----- [S-OutS]
!float.end ≤ !int.end
----- [S-Choice]
@[l1:!float.end] ≤ @[l1:!int.end]

```

T ≤ S

i)

```

S = @[l1:?int.end]
T = @[l1:?float.end]

```

```

----- [S-End]
int ≤ float    end ≤ end
----- [S-InS]
?int.end ≤ ?float.end
----- [S-Choice]
@[l1:?int.end] ≤ @[l1:?float.end]

```

S ≤ T

j)

```

S = &[l1:!int.end]
T = &[l1:!float.end]

```

```

----- [S-End]
int ≤ float    end ≤ end
----- [S-OutS]
!float.end ≤ !int.end
----- [S-Branch]
&[l1:!float.end] ≤ &[l1:!int.end]

```

T ≤ S

k)

```

S = &[l1:?int.end]
T = &[l1:?float.end]

```

```

----- [S-End]
int ≤ float    end ≤ end
----- [S-InS]
?int.end ≤ ?float.end
----- [S-Branch]
&[l1:?int.end] ≤ &[l1:?float.end]

```

S ≤ T

Ejercicio 2

Asumir que $S = !\text{nat.end}$ y $T = !\text{float.end}$ y que $\Gamma \vdash -1:\text{int}$ y $\Gamma \vdash 5,0:\text{float}$.

Indicar si los siguientes términos están bien tipados.

a)

```

      X      ----- [T-Nil]
 $\emptyset \vdash -1:\text{int} \quad \text{int} \leq \text{nat} \quad x+:\text{end} \vdash 0$ 
----- [T-Out]
 $x+:\text{!nat.end} \vdash x+!(-1).0$ 
 $x-:?\text{nat.end} \vdash x-(y:\text{int}).0$ 
----- [T-Par]
 $x+:\text{!nat.end}, x-:?\text{nat.end} \vdash x+!(-1).0 \mid x-(y:\text{int}).0$ 
----- [T-Res]
 $\emptyset \vdash (\text{vx:S})(x+!(-1).0 \mid x-(y:\text{int}).0)$ 
```

No está bien tipado porque no vale $\text{int} \leq \text{nat}$.

b)

```

      X      ----- [T-Nil]      X      ----- [T-Nil]
 $\emptyset \vdash -1:\text{int} \quad \text{int} \leq \text{float} \quad x+:\text{end} \vdash 0$ 
 $\text{float} \leq \text{int} \quad x-:\text{end}, y:\text{int} \vdash 0$ 
----- [T-Out]      ----- [T-In]
 $x+:\text{!float.end} \vdash x+!(-1).0$ 
 $x-:?\text{float.end} \vdash x-(y:\text{int}).0$ 
----- [T-Par]
 $x+:\text{!float.end}, x-:?\text{float.end} \vdash x+!(-1).0 \mid x-(y:\text{int}).0$ 
----- [T-Res]
 $\emptyset \vdash (\text{vx:T})(x+!(-1).0 \mid x-(y:\text{int}).0)$ 
```

No está bien tipado porque no vale $\text{float} \leq \text{int}$.

c)

```

      X      ----- [T-Nil]
 $\emptyset \vdash 5,0:\text{float} \quad \text{float} \leq \text{nat} \quad x+:\text{end} \vdash 0$ 
----- [T-Out]
 $x+:\text{!nat.end} \vdash x+!(5,0).0$ 
 $x-:?\text{nat.end} \vdash x-(y:\text{int}).0$ 
----- [T-Par]
 $x+:\text{!nat.end}, x-:?\text{nat.end} \vdash x+!(5,0).0 \mid x-(y:\text{int}).0$ 
----- [T-Res]
 $\emptyset \vdash (\text{vx:S})(x+!(5,0).0 \mid x-(y:\text{int}).0)$ 
```

No está bien tipado porque no vale $\text{float} \leq \text{nat}$.

d)

```

      X      ----- [T-Nil]      X      ----- [T-Nil]
 $\emptyset \vdash 5,0:\text{float} \quad \text{float} \leq \text{float} \quad x+:\text{end} \vdash 0$ 
 $\text{float} \leq \text{int} \quad x-:\text{end}, y:\text{int} \vdash 0$ 
----- [T-Out]      ----- [T-In]
 $x+:\text{!float.end} \vdash x+!(5,0).0$ 
 $x-:?\text{float.end} \vdash x-(y:\text{int}).0$ 
----- [T-Par]
 $x+:\text{!float.end}, x-:?\text{float.end} \vdash x+!(5,0).0 \mid x-(y:\text{int}).0$ 
```

$$\frac{}{\varnothing \vdash (vx:T)(x+!(5,0).0 \mid x-?(y:int).0)} \quad [\text{T-Res}]$$

No está bien tipado porque no vale $\text{float} \leq \text{int}$.

e)

$$\varnothing \vdash (vx:S)(x+!(-1).0 \mid x-?(y:float).0)$$

No está bien tipado por la misma razón que el inciso a).

f)

$$\frac{\begin{array}{c} X \\ \{l1, l2\} \subseteq \{l1\} \\ \hline \end{array} \quad \frac{}{\varnothing \vdash (vx:\&[l1:end, l2:end])(x+\triangleright[l1:0] \mid x-\triangleleft l2.0)} \quad [\text{T-Res}]}{\frac{\frac{\frac{}{x+:\&[l1:end, l2:end] \vdash x+\triangleright[l1:0]}{\varnothing \vdash (vx:\&[l1:end, l2:end])(x+\triangleright[l1:0] \mid x-\triangleleft l2.0)} \quad [\text{T-Par}]}{\varnothing \vdash (vx:\&[l1:end, l2:end])(x+\triangleright[l1:0] \mid x-\triangleleft l2.0)} \quad [\text{T-Res}]}$$

No está bien tipado porque el proceso de la izquierda no ofrece todas las etiquetas definidas en el canal $x+$.

g)

$$\varnothing \vdash (vx:\&[l1:end, l2:end])(x+\triangleright[l1:0] \mid x-\triangleleft l1.0)$$

No está bien tipado por la misma razón que el inciso f).

h)

$$\frac{\frac{\frac{}{\{l1\} \subseteq \{l1, l2\}}{\varnothing \vdash (vx:\&[l1:end])(x+\triangleright[l1:0, l2:0] \mid x-\triangleleft l1.0)} \quad [\text{T-Res}]}{\varnothing \vdash (vx:\&[l1:end])(x+\triangleright[l1:0, l2:0] \mid x-\triangleleft l1.0)} \quad [\text{T-Par}]}$$

Bien tipado.

i)

$$\frac{\frac{\frac{}{\{l1\} \subseteq \{l1, l2\}}{\varnothing \vdash (vx:\&[l1:end])(x+\triangleright[l1:0, l2:0] \mid x-\triangleleft l2.0)} \quad [\text{T-Res}]}{\varnothing \vdash (vx:\&[l1:end])(x+\triangleright[l1:0, l2:0] \mid x-\triangleleft l2.0)} \quad [\text{T-Par}]}$$

No está bien tipado porque el proceso de la derecha selecciona la etiqueta `l2` que no es una etiqueta disponible en el canal `x-`.

Ejercicio 3

Mostrar si los siguientes pares de tipos infinitos están en relación de subtipado.

Para probar que `S` y `T` están en relación de subtipado: `S ≤ T`, tenemos que encontrar un "type simulation" `R` tal que `(S,T) ∈ R`.

a)

```
S = μX.!int.!float.X
T = μX.μY.!int.X
```

```
R = {(S,T)}
```

```
(S,T) ∈ R
unfold(S) = !int.!float.S
unfold(T) = unfold(μY.!int.T) = !int.T
⇒ (!float.S,T) ∈ R, (int,int) ∈ R
```

```
R = {(S,T), (int,int), (!float.S,T)}
```

```
(int,int) ∈ R
⇒ int < int
```

```
(!float.S,T) ∈ R
unfold(T) = !int.T
⇒ (S,T) ∈ R, (int,float) ∈ R
```

```
R = {(S,T), (int,int), (!float.S,T), (int,float)}
```

```
(int,float) ∈ R
⇒ int < float
```

```
∴ (S,T) ∈ R ⇒ S ≤ T
```

b)

```
S = μX.!(?float.end).!(?int.end).X
T = μX.!(?float.end).X
```

```
R = {(T,S)}
```

```
(T,S) ∈ R
unfold(T) = !(?float.end).T
unfold(S) = !(?float.end).!(?int.end).S
⇒ (?float.end,?float.end) ∈ R, (T,!(?int.end).S) ∈ R
```

```
R = {(T,S), (?float.end,?float.end), (T,!(?int.end).S)}
```

```
(?float.end,?float.end) ∈ R
⇒ (float,float) ∈ R, (end,end) ∈ R
```

```
R = {(T,S), (?float.end,?float.end), (T,!(?int.end).S), (float,float), (end,end)}
```

$(T, !(?int.end).S) \in R$
 $unfold(T) = !(?float.end).T$
 $\Rightarrow (?int.end, ?float.end) \in R, (T, S) \in R$

$R = \{(T, S), (?float.end, ?float.end), (T, !(?int.end).S), (float, float), (end, end), (?int.end, ?float.end)\}$

$(float, float) \in R$
 $\Rightarrow float < float$

$(end, end) \in R$
 $\Rightarrow end < end$

$(?int.end, ?float.end) \in R$
 $\Rightarrow (int, float) \in R, (end, end) \in R$

$R = \{(T, S), (?float.end, ?float.end), (T, !(?int.end).S), (float, float), (end, end), (?int.end, ?float.end), (int, float)\}$

$(int, float) \in R$
 $\Rightarrow int < float$

$\therefore (T, S) \in R \Rightarrow T \leq S$

c)

$S = \mu X. !(?float.end). !(?int.end). X$
 $T = \mu X. !(?int.end). X$

$R = \{(S, T)\}$

$(S, T) \in R$
 $unfold(S) = !(?float.end). !(?int.end). S$
 $unfold(T) = !(?int.end). T$
 $\Rightarrow (?int.end, ?float.end) \in R, (!(?int.end). S, T) \in R$

$R = \{(S, T), (?int.end, ?float.end), (!(?int.end). S, T)\}$

$(?int.end, ?float.end) \in R$
 $\Rightarrow (int, float) \in R, (end, end) \in R$

$R = \{(S, T), (?int.end, ?float.end), (!(?int.end). S, T), (int, float), (end, end)\}$

$(!(?int.end). S, T) \in R$
 $unfold(T) = !(?int.end). T$
 $\Rightarrow (?int.end, ?int.end) \in R, (S, T) \in R$

$R = \{(S, T), (?int.end, ?float.end), (!(?int.end). S, T), (int, float), (end, end), (?int.end, ?int.end)\}$

$(int, float) \in R$
 $\Rightarrow int < float$

$(end, end) \in R$
 $\Rightarrow end < end$

```
(?int.end,?int.end) ∈ R
⇒ (int,int) ∈ R, (end,end) ∈ R
```

```
R = {(S,T), (?int.end,?float.end), (!(?int.end).S,T), (int,float), (end,end), (?int.end,?int.end), (int,int)}
```

```
(int,int) ∈ R
⇒ int < int
```

```
∴ (S,T) ∈ R ⇒ S ≤ T
```

Ejercicio 4

Para los siguientes términos, indicar si están bien tipados. En caso afirmativo, mostrar sus posibles reducciones.

a)

```
{
  let s = create () in
  let a = fork (λx. close (send x true)) (fst s) in
  let b = receive (snd s) in
  close (snd b)
}

→ [r-create]

(va){
  let s = (c+, c-) in
  let a = fork (λx. close (send x true)) (fst s) in
  let b = receive (snd s) in
  close (snd b)
}

→ [r-new][r-thread][r-let]

(va){
  let a = fork (λx. close (send x true)) (fst (c+, c-)) in
  let b = receive (snd (c+, c-)) in
  close (snd b)
}

→ [r-new][r-fork]

(va)((
  let a = () in
  let b = receive (snd (c+, c-)) in
  close (snd b)
) || {
  (λx. close (send x true)) (fst (c+, c-))
})

→ [r-new][r-par][r-thread][r-let]

(va)((
  let b = receive (snd (c+, c-)) in
  close (snd b)
```



```

) || {
  (λx. close (send x true)) (fst (c+, c-))
})

```

→ [r-new][r-par][r-thread][r-snd]

```

(va)((
  let b = receive c- in
  close (snd b)
) || {
  (λx. close (send x true)) (fst (c+, c-))
})

```

→ [r-new][r-par][r-thread][r-fst]

```

(va)((
  let b = receive c- in
  close (snd b)
) || {
  (λx. close (send x true)) c+
})

```

→ [r-new][r-par][r-thread][r-beta]

```

(va)(( let b = receive c- in close (snd b) ) || { close (send c+ true) })

```

→ [r-new][r-struct][r-comm]

```

(va)(( let b = (true, c-) in close (snd b) ) || { close c+ })

```

→ [r-new][r-par][r-thread][r-let]

```

(va)(( close (snd (true, c-)) ) || { close c+ })

```

→ [r-new][r-par][r-thread][r-snd]

```

(va)(( close c- ) || { close c+ })

```

→ [r-new][r-close]

```

(va)((()) || {()}) ≡ {()}

```