

## Session Types

### FuSe: An Ocaml implementation of binary session types<sup>1</sup>

<sup>1</sup>Padovani, L. (2017). A simple library implementation of binary sessions. Journal of Functional Programming, 27. The implementation can be downloaded from <https://github.com/boystrange/FuSe>

#### Syntax

$$t, s ::= \text{bool} \mid \text{int} \mid \dots \mid T \mid \alpha$$

$$T, S ::= \text{end} \mid !t.T \mid ?t.T \mid \oplus[l_i : T_i]_{i \in I} \mid \&[l_i : T_i]_{i \in I} \mid A \mid \bar{A}$$

- ▶ FuSe provides polymorphic session types
- ▶  $\alpha$  is a type variable
- ▶  $A$  is a session type variable
- ▶  $\bar{A}$  the dual of a session type variable

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- ▶  $[l_i : t_i]_{i \in I}$ : Variants (disjoint sums)

## Duality

$$\begin{aligned} \overline{\text{end}} &= \text{end} \\ \overline{(?t.T)} &= !t.\bar{T} \\ \overline{(!t.T)} &= ?t.\bar{T} \\ \overline{\&[l_i : T_i]_{i \in I}} &= \oplus[l_i : \bar{T}_i]_{i \in I} \\ \overline{\oplus[l_i : T_i]_{i \in I}} &= \&[l_i : \bar{T}_i]_{i \in I} \\ \overline{\bar{A}} &= A \end{aligned}$$

## Module Session

```

val send      :  $\alpha \rightarrow !\alpha.A \rightarrow A$ 
val receive   :  $?\alpha.A \rightarrow \alpha \times A$ 
val create    :  $\text{unit} \rightarrow A \times \bar{A}$ 
val close     :  $\text{end} \rightarrow \text{unit}$ 
val branch    :  $\&[1_i : A_i]_{i \in I} \rightarrow [1_i : A_i]_{i \in I}$ 
val select    :  $(A_k \rightarrow [1_i : \bar{A}_i]_{i \in I}) \rightarrow \oplus[1_i : A_i]_{i \in I} \rightarrow A_k$ 

```

## Main idea

- ▶ Session types: Products + Sums + Linearity
- ▶ Ornella Dardha, Elena Giachino, and Davide Sangiorgi. Session types revisited. PPDP'12.

## Two types

- ▶  $\emptyset$ , which is not inhabited (no constructor)
- ▶  $\langle \rho, \sigma \rangle$  for channels:
  - ▶ receiving messages of type  $\rho$
  - ▶ sending messages of type  $\sigma$ .
  - ▶  $\rho$  and  $\sigma$  instantiated with  $\emptyset$  to indicate that no message is respectively received and/or sent

## Representation of session types

## Encoding

$$\begin{aligned}
\llbracket \text{end} \rrbracket &= \langle \emptyset, \emptyset \rangle \\
\llbracket ?t.T \rrbracket &= \langle \llbracket t \rrbracket \times \llbracket T \rrbracket, \emptyset \rangle \\
\llbracket !t.T \rrbracket &= \langle \emptyset, \llbracket t \rrbracket \times \llbracket \bar{T} \rrbracket \rangle \\
\llbracket \&[1_i : T_i]_{i \in I} \rrbracket &= \langle \llbracket 1_i : \llbracket T_i \rrbracket \rrbracket_{i \in I}, \emptyset \rangle \\
\llbracket \oplus[1_i : T_i]_{i \in I} \rrbracket &= \langle \emptyset, \llbracket 1_i : \llbracket \bar{T}_i \rrbracket \rrbracket_{i \in I} \rangle \\
\llbracket A \rrbracket &= \langle \rho_A, \sigma_A \rangle \\
\llbracket \bar{A} \rrbracket &= \langle \sigma_A, \rho_A \rangle
\end{aligned}$$

## Examples

 $? \alpha.A$ 

$$\llbracket ? \alpha.A \rrbracket = \langle \alpha \times \langle \rho_A, \sigma_A \rangle, \emptyset \rangle$$

 $T = \oplus[\text{End} : \text{end}, \text{Msg} : !\alpha. ?\beta.\text{end}]$ 

$$\begin{aligned}
\llbracket T \rrbracket &= \langle \emptyset, [\text{End} : \llbracket \text{end} \rrbracket, \text{Msg} : \llbracket ?\alpha. !\beta.\text{end} \rrbracket] \rangle \\
&= \langle \emptyset, [\text{End} : \langle \emptyset, \emptyset \rangle, \text{Msg} : \langle \alpha \times \llbracket !\beta.\text{end} \rrbracket, \emptyset \rangle] \rangle \\
&= \langle \emptyset, [\text{End} : \langle \emptyset, \emptyset \rangle, \text{Msg} : \langle \alpha \times \langle \emptyset, \beta \times \llbracket \text{end} \rrbracket \rangle, \emptyset \rangle] \rangle \\
&= \langle \emptyset, [\text{End} : \langle \emptyset, \emptyset \rangle, \text{Msg} : \langle \alpha \times \langle \emptyset, \beta \times \langle \emptyset, \emptyset \rangle \rangle, \emptyset \rangle] \rangle
\end{aligned}$$

 $\bar{T} = \&[\text{End} : \text{end}, \text{Msg} : ?\alpha. !\beta.\text{end}]$ 

$$\begin{aligned}
\llbracket \bar{T} \rrbracket &= \langle [\text{End} : \llbracket \text{end} \rrbracket, \text{Msg} : \llbracket ?\alpha. !\beta.\text{end} \rrbracket], \emptyset \rangle \\
&= \langle [\text{End} : \langle \emptyset, \emptyset \rangle, \text{Msg} : \langle \alpha \times \llbracket !\beta.\text{end} \rrbracket, \emptyset \rangle], \emptyset \rangle \\
&= \langle [\text{End} : \langle \emptyset, \emptyset \rangle, \text{Msg} : \langle \alpha \times \langle \emptyset, \beta \times \llbracket \text{end} \rrbracket \rangle, \emptyset \rangle], \emptyset \rangle \\
&= \langle [\text{End} : \langle \emptyset, \emptyset \rangle, \text{Msg} : \langle \alpha \times \langle \emptyset, \beta \times \langle \emptyset, \emptyset \rangle \rangle, \emptyset \rangle], \emptyset \rangle
\end{aligned}$$

## Theorem

If  $\llbracket \mathcal{T} \rrbracket = \langle t, s \rangle$ , then  $\llbracket \bar{\mathcal{T}} \rrbracket = \langle s, t \rangle$ .

## Session

```
module Session : sig
  type 0
  type (ρ,σ) st (* OCaml syntax for ⟨ρ,σ⟩ *)
  val create : unit → (ρ,σ) st × (σ,ρ) st
  val close : (0,0) st → unit
  val send : α → (0,(α × (σ,ρ) st)) st → (ρ,σ) st
  val receive : ((α × (ρ,σ) st),0) st → α × (ρ,σ) st
  val select : ((σ,ρ) st → α) → (0,[>] as α) st → (ρ,σ) st
  val branch : ([>] as α,0) st → α
end
```

## Non linear usage of channels

```
let client ep x y =
  let _ = Session.send x ep in
  let ep = Session.send y ep in
  let result, ep = Session.receive ep in
  Session.close ep;
  result

let service ep =
  let x, ep = Session.receive ep in
  let ep = Session.send x ep in
  Session.close ep

let _ =
  let a, b = Session.create () in
  let _ = Thread.create service a in
  print_int (client b 1 2)
```

The program is well-typed

```
val client : !α. ?α. → α → α → β
val service : ?α. !β. → unit
```

Its execution raises the exception `Session.InvalidEndpoint`