# Hypothesis Testing

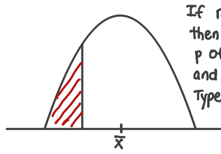
Suppose some $H_0$ with $H_a$ or $H_1$ being the inverse
(null hypothesis)

$H_0$ kept

| | | N | Y |
|---|---|---|---|
| $H_0$ Valid | N | ✓ | Type 2 Error |
| | Y | Type 1 Error | Y |

**Type 1 Error ($\alpha$)**
- $H_0$ is rejected but should hold

**Type 2 Error ($\beta$)**
- $H_0$ is kept but doesn't hold




If red is rejection, then red area is p of Type 1 Error and rest is P of Type 2 error

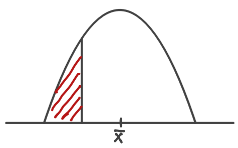
Given $\mu$, $\alpha$, and some $\bar{x}$ (either a proportion or values)

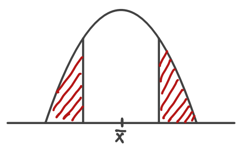
1) Determine tails (One or Two)
   - If one tail, find left or right

2) Determine if using t or z
   - $n > 30$, use z
   - $n \leq 30$, use t

3) Calculate t or z value of $\alpha$ and the other mean
   - Reject if: $\alpha < P(z)$
     $\alpha < P(t)$

$$|z_{\frac{\alpha}{2}}| < z$$
$$|t_{\frac{\alpha}{2}}| < t$$




If z value is in red, REJECT

8 juices sugar levels
11.48, 11.45, 11.48, 11.47, 11.48, 11.5, 11.42, 11.49

$H_0: \mu = 11.5$ $H_1: \mu \neq 11.5$ $\alpha = 0.05$

2 tail, not proportion, use t values

No $\bar{x}$ or s

1) $\bar{x} = \frac{11.48 \times 3 + 11.45 + 11.47 + 11.5 + 11.42 + 11.49}{8}$

$= 11.47125$

$s^2 = \frac{\sum_{n=1}^{\infty} (x_i)^2 - n\bar{x}^2}{n-1}$

$= \frac{1052.7211 - 8 \cdot (11.47125)^2}{7}$

$= 6.41 \times 10^{-4}$

$s = 0.0253\ldots$

2) $\alpha = 0.05$ $n = 8$
$\frac{\alpha}{2} = 0.025$ $n-1 = 7$

$t_{0.025,7} = \pm 2.365$

if $t < -2.365$ or $t > 2.365$, reject

3)

$T_0 = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

$= \frac{11.47125 - 11.5}{\frac{0.0253}{\sqrt{8}}}$

$= -3.2116$

$\doteq -3.21$

$\therefore$ Reject $H_0$
b/c $-3.21 < -2.365$

A type 1 error may occur

Using $df = 7$, $p = 3.21$
Between 2 tails 0.01 & 0.02
(Must be normal

## Difference in Means

- when given an 8 output

Given Following Data

| | n | mean | sd | ... |
|---|---|---|---|---|
| G1 | | | | |
| G2 | | | | |

ONLY n, mean, sd matter

$\alpha$ is given too

1) Determine if variances are equal or unequal
   - Question STATES this

2) Perform t test on two means ($t_v$ or $t_{n_1+n_2-2}$)

3) Determine confidence interval (called a, b)
   - If $t < a$ or $t > b$, REJECT
   - $\alpha/2$ only if 2 sided

| | n | mean | sd |
|---|---|---|---|
| 1 | 12 | 0.8153 | 0.0409 |
| 2 | 10 | 0.82524 | 0.01794 |

$\alpha = 0.05$

One tail (left)

variances unequal

$B_2$ has higher coefficient than $B_1$

1) T test

$$t_v = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^2}{n_1}\right) + \left(\frac{s_2^2}{n_2}\right)}}$$

$$= \frac{0.8153 - 0.82524}{\sqrt{\frac{0.0409^2}{12} + \frac{0.01794^2}{10}}}$$

$= -0.7588$

$\doteq -0.76$

2) Confidence interval

a) Find true variance

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$= \frac{\left(\frac{0.0409^2}{12} + \frac{0.01794^2}{10}\right)^2}{\frac{\left(\frac{0.0409^2}{12}\right)^2}{11} + \frac{\left(\frac{0.01794^2}{10}\right)^2}{9}}$$

$= 15.646$

$= 15$ (round down)

b) $\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2, v} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

We only want $t_{\alpha/2, v}$

No $\frac{\alpha}{2}$ b/c left tail

$t_{0.05,15} = -1.753$

Do not reject b/c $-0.76 > -1.753$

P for $-0.76$: Between 0.2 and 0.25

95% Two side interval $\alpha = 0.05$

$\bar{x}_1 - \bar{x}_2 \pm t_{\frac{\alpha}{2}, 15} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$\bar{x}_1 - \bar{x}_2 = 0.8153 - 0.82524$

$= -9.94 \times 10^{-3}$

$\doteq -0.00994$

$t_{0.025, 15} \cdot \sqrt{\frac{0.0409^2}{12} + \frac{0.01794^2}{10}}$

$= 2.131 \cdot \sqrt{1.7158 \cdot 10^{-4}}$

$= 0.02791\ldots$

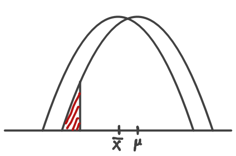
$-0.00994 \pm 0.02791\ldots$

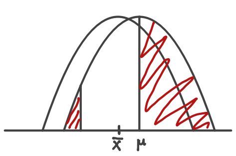
$(-0.03785, 0.0179)$

$\doteq (-0.0379, 0.0179)$

## Type 2 Error Questions ($\beta$)

- When given true mean/proportion of something




- Calculate new rejection zones with original z values

1) DO NOT COPY Z VALUES
2) Determine confidence interval of original
3) Using interval, convert to z values of new true mean
4) Calculate

## Power

- Associated with type 2 errors
- Power $= 1 - \beta$

Library reorganizes, but if proportion of unsorted books $< 0.02$, then do not reorganize

use $\alpha = 0.05$, one tail so NO $\frac{0.05}{2}$

True proportion is 0.01 left tail $n = 1000$

Inventory taken but not needed to

1) Find confidence interval

$\hat{p} \pm z_{0.05} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.02 - 1.64 \sqrt{\frac{0.02 \cdot 0.98}{1000}}$ (only left end)

$(0.0127\ldots, \infty)$

2) Normalize

$P\left(z < \frac{0.0127\ldots - 0.01}{\sqrt{\frac{0.01 \cdot 0.99}{1000}}}\right)$ $1 - 0.80785 = 0.19215$

$= P(z < 0.8706)$

$\doteq P(z < 0.87)$

$= 0.80785$

# Single Variable Least Squares

- Determines relationship existence
- Equation of line: $\hat{y} = \hat{\beta}_1 x + \hat{\beta}_0 \bar{x}$

where $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$ $\quad \hat{\beta}_0 = \bar{y} - \hat{\beta}_1$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}$$

- Residuals are calculated by $(y_i - \hat{y}_i)$
  1) After getting $y = \beta_0 + \beta_1 x$, use the x to find $\hat{y}_i$
  2) Use $y_i$ from original table

- Table given looks as such if information given:

Coefficients:

| | Estimate | Std.Error | t value | Pr(>\|t\|) |
|---|---|---|---|---|
| Intercept | $\beta_0$ | X | X | X |
| OTHER VAR | $\beta_1$ | X | $t_{n-2}$ | |

$$t_{n-2} = \frac{\hat{\beta}_1}{\sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}} \qquad \hat{\sigma}^2 = \frac{SSE}{n-2}$$

Given other values such as means of residuals

Pr(>|t|) is calculated with t value

- If p value $<0.05$, reject $H_0$ and state strong correlation

| x | 0 | 5 | 8 | 11 | 12 |
|---|---|---|---|---|---|
| y | 8 | 6 | 2 | 4 | 2 |

Determine regression line

1) $\bar{x}$ & $\bar{y}$

$\bar{x} = 7.2 \quad \bar{y} = 4.4$

2) $S_{xx} = \sum_{i=1}^{\infty} x_i^2 - n\bar{x}^2$

$= 0^2 + 5^2 + 8^2 + 11^2 + 12^2 - 5(7.2)^2$

$= 94.8$

3) $S_{xy} = \sum_{i=1}^{\infty} x_i y_i - n\bar{x}\bar{y}$

$= 0 \cdot 8 + 5 \cdot 6 + 8 \cdot 2 + 11 \cdot 4 + 12 \cdot 2 - 5 \cdot 7.2 \cdot 4.4$

$= -44.4$

4) $\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}} = \frac{-44.4}{94.8} = -0.4684\ldots$

$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$
$= 4.4 + 0.4684 \cdot 7.2$
$= 7.77215$
$= 7.7722$

$y = 7.7722 - 0.4684x$

Residuals

(0,8): $\hat{y} = 7.7722$
$(8 - 7.7722)$
$= 0.228$

(5,6): $\hat{y} = 7.7722 - 0.4684(5) = 5.4302$
$(6 - 5.4302)$
$= 0.570$

(8,2): $\hat{y} = 7.7722 - 0.4684(8) = 4.025$
$(2 - 4.025)$
$= -2.025$

(11,4): $\hat{y} = 7.7722 - 0.4684(11) = 2.6198$
$(4 - 2.6198)$
$= 1.380$

(12,2) $\hat{y} = 7.7722 - 0.4684(12)$
$= 2.1514$
$(2 - 2.1514)$
$= -0.151$

Residual Sum: $0.228^2 + 1.38^2 + 0.57^2 + 2.025^2 + 0.151^2$
$= 6.4051$ (Depends on rounding)

# Fisher's LSD Test

- Compares 2 means to see if they are similar or not

1) Calculate the absolute difference between the two means (a)

2) Calculate the LSD for two means (b)
   - If a > b, then significantly different

| Exercise | n | mean | sd |
|---|---|---|---|
| 0 | 38 | 135.16 | 13.04 |
| 1 | 38 | 129.42 | 12.12 |
| 2 | 13 | 125.23 | 9.71 |
| 3 | 11 | 122.09 | 11.09 |

Find pairs that are significantly different $\alpha = 0.05$

1) $t_{\alpha/2, N-a} = t_{0.025, 96}$
$\doteq t_{0.025, 80}$
$= 1.990$

MSE = 146.749

| $E_1$ | $E_2$ | $\lvert \bar{x}_1 - \bar{x}_2 \rvert$ | LSD | Difference |
|---|---|---|---|---|
| 0 | 1 | 5.74 | 5.53 | ✓ |
| 0 | 2 | 9.93 | 7.75 | ✓ |
| 0 | 3 | 13.07 | 8.83 | ✓ |
| 1 | 2 | 4.19 | 7.75 | X |
| 1 | 3 | 7.33 | 8.83 | X |
| 2 | 3 | 3.14 | 9.86 | X |

2) Calculate different pairs:

# ANOVA Table

- Breaks down components of variation in data
- Multiple groups with means, standard deviations, and sample size
- Distributed on table as follows, where N is the sum of all the sample, a is the # of groups

Determines if the means are similar/same

$H_0: \mu_1 = \mu_2 = \cdots$

$H_1: \mu_i \neq \mu_j$ for some (i,j) pair

| | n | SS | MS | F | Pvalue | Fcrit |
|---|---|---|---|---|---|---|
| Treatments | a-1 | $SS_{Treatments}$ | $\frac{SS_{Treatments}}{a-1}$ | $\frac{MS_{Treatments}}{MS_E}$ | From F | From P |
| Error | N-a | $SS_E$ | $\frac{SS_E}{N-a}$ | | | |
| Total | N-1 | $SS_T$ | | | | |

- A value needed would be the true mean $(\bar{y}_{\cdot\cdot})$, which is calculated by adding all values divided by # of values (n)

| Exercise | n | mean | sd |
|---|---|---|---|
| 0 | 38 | 135.16 | 13.04 |
| 1 | 38 | 129.42 | 12.12 |
| 2 | 13 | 125.23 | 9.71 |
| 3 | 11 | 122.09 | 11.09 |

1) True mean

$\ddot{y} = \frac{38 \cdot 135.16 + 38 \cdot 129.42 + 13 \cdot 125.23 + 11 \cdot 122.09}{100}$

$\ddot{y} = \frac{13025.02}{100}$

$\ddot{y} = 130.2502$

a = 4
N = 100

2) SS (sum of squares)

$SS_{Treatments} = \sum_{i=1}^{a} n_i(\bar{y}_i - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{\infty} \frac{\bar{y}_{i\cdot}}{n_i} - \frac{y_{\cdot\cdot}^2}{N}$

$= 38(135.16 - 130.2502)^2 + 38(129.42 - 130.2502)^2 + 13(125.23 - 130.2502)^2 + 11(122.09 - 130.2502)^2$

$= 2002.3328$

$SS_E = \sum_{i=1}^{a} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{\cdot\cdot})^2 = \sum_{i=1}^{a} (n_i - 1)s_i^2$

$= 37 \cdot 13.04^2 + 37 \cdot 12.12^2 + 12 \cdot 9.71^2 + 10 \cdot 11.09^2$

$= 14087.9222$

$SS_T = 2002.3328 + 14087.9222$
$= 16090.255$

3) MS

$MS_{Treatments} = \frac{2002.3328}{3}$
$= 667.444\ldots$

$MS_{Error} = \frac{14087.9222}{96}$
$= 146.749$

4)

$F = \frac{667.44}{146.749}$
$= 4.59$

| | n | SS | MS | F |
|---|---|---|---|---|
| Treatments | 3 | 2002.3328 | 667.44 | 4.59 |
| Error | 96 | 14087.9222 | 146.749 | |
| Total | 99 | 16090.255 | | |