

CET 513 Transportation Networks and Optimization

Homework #3

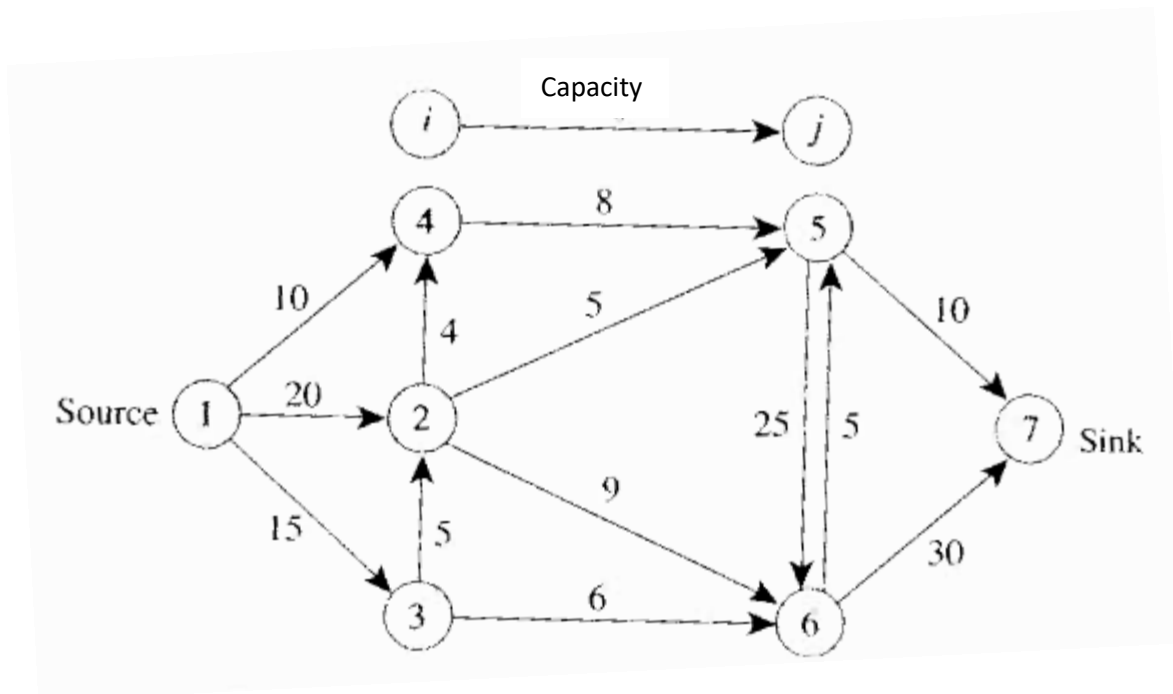
Out: 10/29/2019

Due: 11/12/2019

Problem 1

For the graph below, answer the following questions:

- (a) Solve the maximum flow problem in Python from the source to the sink.
- (b) What is the minimum cut of the graph? What is the total capacity of the links in the minimum cut?
- (c) Assume there are 10 units of product from node 1 and 5 units of product from node 2, which will be transported to node 5 (3), node 6 (7), and node 7 (5). Numbers in the parenthesis indicate the units of product needed at each destination node. Solve the minimum cost problem in Python. *Hint: please assign random costs to each link, which should follow a uniform distribution in the range of [1, 10].*



Problem 2

Perform two iterations of the Steepest Descent Method to solve the following quadratic optimization problem:

$$\min z(x_1, x_2) = 4(x_1 - 10)^2 + (x_2 - 4)^2$$

Start at $x^{(0)} = (0, 0)$

Problem 3

For the following trash management problem:

- 1) Formulate it as an optimization problem and specify its decision variables, constraints, and objective function;
- 2) Solve the problem in Python or Matlab.

New York City has 10 trash districts and is trying to determine which of the districts should be sites for trash processing plants. It costs \$1000 (annually) to haul one ton of trash one mile. The central location of each district, the number of tons of trash produced per year by each district, the annual fixed cost (in millions of dollars) of operating a trash processing facility in that district, and the variable cost (in dollars per ton of trash) for processing trash in that district are shown below:

District	X coord	Y coord	Trash (tons)	Fixed Cost (\$million)	Variable Cost (\$ / ton)
1	4	3	49	2	310
2	2	5	874	1	40
3	10	8	555	1	51
4	2	8	352	1	341
5	5	3	381	3	131
6	4	5	428	2	182
7	10	5	985	1	20
8	5	1	105	2	40
9	5	8	258	4	177
10	1	7	210	2	75

For example, district 3 is located at coordinates (10,8). This district produces 555 tons of trash per year. If we were to operate a processing plant in district 3, we would incur a fixed cost of \$1,000,000, plus \$51 for each ton of trash processed. District 3's own trash could be processed at this plant. If district 3's plant is operated, trash from other districts could be shipped to district 3 for processing, at added cost for transportation. For example, trash from district 2 would incur a shipping cost of \$1000 for each of its 874 tons of trash for each of the approximately 8.54 miles separating districts 2 and 3. (Of course, it would also incur trash processing costs at district 3.) Each plant can handle at most 1500 tons of trash. Each district must send all its trash to a single site. What trash processing plants should be used, and to which plants should the trash from each district be shipped?

Problem 4

Solve the following problem using the Bisection or Golden method for $0 \leq x \leq 3$. Also solve the problem using Python. Did you get the same solution?

$$\min 65x^6 + 71x^5 - 322x^4 - 401x^3$$