CET 513 Transportation Networks and Optimization

Homework #2

Out: 10/15/2019 **Due: 10/24/2019**

Problem 1

Solve 2.C.5 Example: Cleaning up the Linear River (pg.33 of RWW) using the 2-D graphical solution method. Explicitly state 1) decision variables, 2) objective function, 3) constraints, 4) feasible region, 5) the solution procedure, and 6) the optimal solution. Is the feasible region bounded? Which procedure did you use to solve the problem? Can you use the procedure based on evaluating objective values at vertices? Why or why not? What type of solution do you get?

Problem 2

For the following LP, indicate all its extreme points in a graph and highlight those that are feasible. Is the feasible region bounded? If yes, solve this problem by evaluating the objective values at all feasible extreme points. What type of solution did you get? Highlight all solutions in the graph.

$$
\begin{aligned}
\text{Minimize } Z &= 2x_1 + x_2 \\
\text{Subject to: } 4x_1 - 12x_2 &\le -6 \\
-4x_1 + 6x_2 &\le 12 \\
4x_1 + 2x_2 &\ge 8 \\
x_1 + x_2 &\le 9 \\
4x_2 &\le 16 \\
x_1, x_2 &\ge 0
\end{aligned}
$$

Problem 3

For the following LP, convert it to the standard form first. How many slack variables did you define? Then write its dual problem and the duality theorem. Solve the primal and dual problems separately (Python or Matlab or Excel). Did you get the same solution?

$$
\begin{aligned}
\text{Minimize } Z &= 2x_1 + 3x_2 + x_3 \\
\text{Subject to: } 2x_1 + x_2 - x_3 &\ge 3 \\
x_1 + x_2 + x_3 &\ge 2 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

Problem 4:

Calculate the gradient and Hessian matrix of the following functions. Is any of them convex?

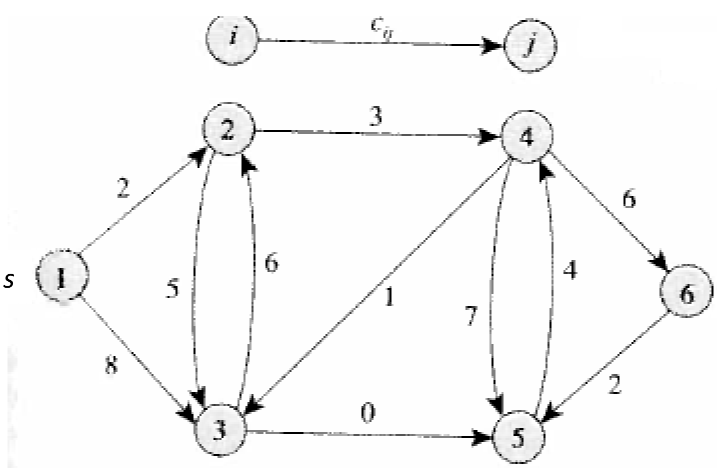
a) $f(x) = 3x_1^2 + 4x_2^2 - x_1x_2 - 5x_1 - 8x_2$

b) $f(x) = -2x_1^2 - x_2^2 - 4x_3^2 - 3x_1x_3 + 3x_2x_3 - x_1 - 2x_2 - 3x_3$

Problem 5: For this homework problem, you have two options:

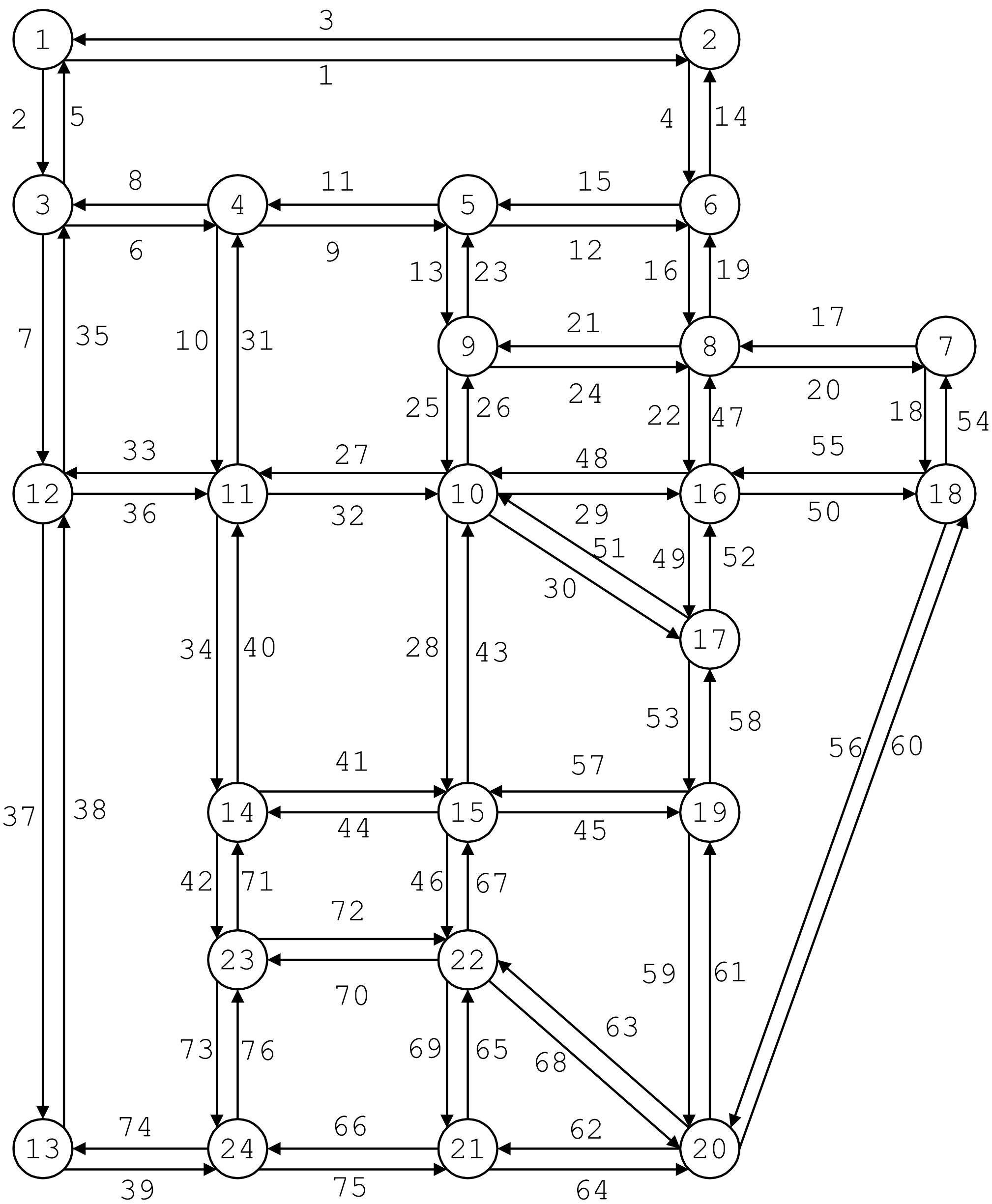
Option 1:

For the graph shown below:
(a) What are the indegree and outdegree of node 3 and node 4?
(b) List all the direct paths from node 1 to node 6.
(c) Solve the shortest path problem shown in the graph below (from node 1 to all other nodes) using the Dijkstra's algorithm. You should keep the predecessor and distance label for each node. How many distance updates did you have for this particular example?
[10 Bonus points] (d) Code the shortest path problem in Python/Matlab/Excel and solve it as an LP problem. Did you get the same solution as that by the Dijkstra's algorithm?



Option 2: *You will get 10 bonus points for working on this option*

The following graph is for the network of the City of Sioux Falls in South Dakota. Find the shortest from **Node 1** to all other nodes in the network. The network file is uploaded to Canvas. You can use "free flow time" as the cost of each link. You may do this either via the Dijkstra 's algorithm or by solving an equivalent linear program.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