CET 513 Transportation Networks and Optimization

CET513
Homework #2

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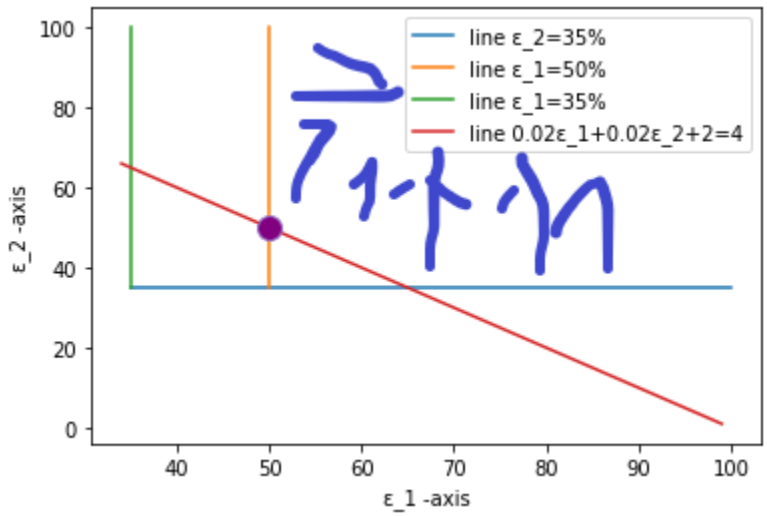
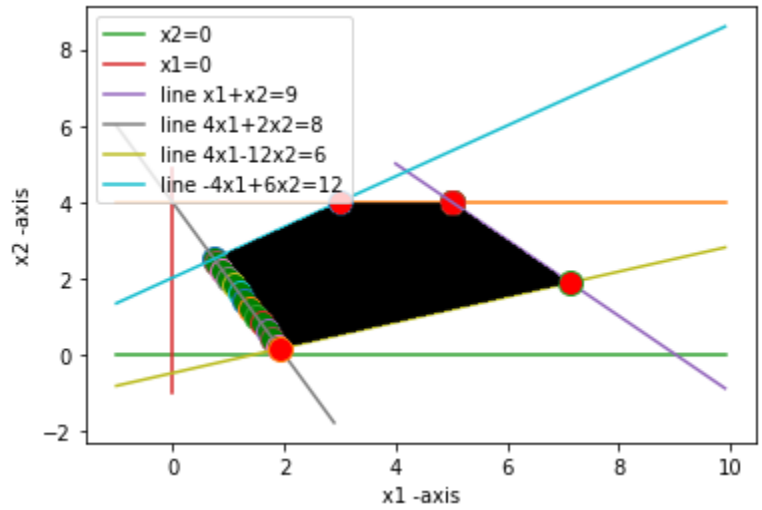
Problem 1

1) decision variables,
ε_1 ε_2 is removal efficiency in percent in 1, 2
2) objective function:
Minimize Z=20000ε_1 + 10000ε_2
3) constraints:
1.0+0.1ε_1>=6
2+0.02ε_1+0.02ε_2>=4
ε_1,ε_2>=35%
4) Feasible region is above orange line, red line , blue line. In the picture, the blue arrow shows the area occurs in the outside of three line(direction betraying from these three lines).
5) procedure :
iterate the vertices of the crossline
orange line cross red:
$\varepsilon_1 = 50, \varepsilon_2 = 50$
redline cross blue line:
$\varepsilon_1 = 65, \varepsilon_2 = 35$
And we know the point which greater will be larger in Objective function Z, so the answer must occur nearby lowest part of the feasible region. We can get the answer by iterating vertices in this case, because vertices are the lowest part of the feasible region.
After calculating, the minimize Z will occur when $\varepsilon_1 = 50, \varepsilon_2 = 50$.
6) feasible region is not bounded



Problem 2



Yes, the feasible region is bounded.

Extreme point:

x_1= 3, x_2= 4

x_1= 27/14 , x_2= 1/7

x_1= 57/8 , x_2= 15/8

x_1= 5 , x_2=4

x_1= 0.75 , x_2= 2.5

Objective value=4; while x_1, x_2 @ 4 x_1+2 x_2=8, x_1 in [0.75, 27/14], x_2 in [2.5, 1/7]

Problem 3

Standard form:

Minimize Z=2 x_1+ 3x_2+x_3

2 x_1+ x_2- x_3- s_1=3

x_1+ x_2+x_3- s_2=2

x_1,x_2,x_3, s_1, s_1 ≥ 0

convert:

variable:

y_1, y_2

Maximize Z=3 y_1+ 2y_2

$-2y_1 - y_2 - s_1 = 2$

$-y_1 - y_2 - s_2 = 3$

$y_1 - y_2 - s_3 = 1$

$y_1, y_2, s_1, s_2, s_3 \geq 0$

Yes, answer is same,

y_1, y_2=[0.33333333 1.33333333]

x_1,x_2,x_3=[1.66666667e+00 2.17603713e-14 3.33333333e-01]

objective= 3.666666666666748

Problem 4:

Calculate the gradient and Hessian matrix of the following functions. Is any of them convex?

a) Gradient
$$=\begin{bmatrix}6x_1 - x_2 - 5\\ 8x_2 - x_1 - 8\end{bmatrix}$$
Hessian
$$=\begin{bmatrix}6 & -1\\ -1 & 8\end{bmatrix}$$

b) Gradient
$$=\begin{bmatrix}-4x_1 - 3x_3 - 1\\ -2x_2 + 3x_2 -\\ 8x_3 - 3x_1 + 3x_2 - 3\end{bmatrix}$$
Hessian
$$=\begin{bmatrix}-4 & 0 & -3\\ 0 & -4 & 3\\ -3 & 3 & -8\end{bmatrix}$$

Problem 5:

Option 1:

(a) Node 3:
indegree is 3
outdegree is 2
Node 4:
indegree is 2
outdegree is 3

(b) direct paths from node 1 to node 6:
1→2→4→6
1→2→3→5→4→6
1→3→5→4→6
1→3→2→4→6

(c) the distance value change 3 times

(d) Yes, I get the same solution as that by Dikstra's algorithm
the code present of (c) and(d)
the value change! To 3 from 8.0 changes to 7.0
original path is:
1 → 3
New path is :
1 → 2 → 3
the value change! To 3 from 7.0 changes to 6.0
original path is:
1 → 2 → 3
New path is :

1 → 2 → 4 → 3
the value change! To 5 from 12.0 changes to 6.0
original path is:
1 → 2 → 4 → 5
New path is :
1 → 2 → 4 → 3 → 5
6 states expanded.
The link of each node and its cost:
1 → 2 is : 2
1 → 3 is : 8
2 → 3 is : 5
2 → 4 is : 3
4 → 3 is : 1
4 → 5 is : 7
4 → 6 is : 6
3 → 2 is : 6
3 → 5 is : 0
5 → 4 is : 4
6 → 5 is : 2
minimize cost to :1 is 0.0
1
minimize cost to :2 is 2.0
1 → 2
minimize cost to :4 is 5.0
1 → 2 → 4
minimize cost to :3 is 6.0
1 → 2 → 4 → 3
minimize cost to :5 is 6.0
1 → 2 → 4 → 3 → 5
minimize cost to :6 is 11.0
1 → 2 → 4 → 6

Option 2:

The following shows that the flow start from 1 with the min Cost

Arc	Flow / Capacity	Cost
1 -> 2	1 / 25900	6

Minimum cost: 6

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4

Minimum cost: 4

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 4	1 / 17111	4

Minimum cost: 8

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 4	1 / 17111	4
4 -> 5	1 / 17783	2

Minimum cost: 10

Arc	Flow / Capacity	Cost
1 -> 2	1 / 25900	6
2 -> 6	1 / 4958	5

Minimum cost: 11

Arc	Flow / Capacity	Cost
1 -> 2	1 / 25900	6
2 -> 6	1 / 4958	5
6 -> 8	1 / 4899	2
8 -> 7	1 / 7842	3

Minimum cost: 16

Arc	Flow / Capacity	Cost
1 -> 2	1 / 25900	6
2 -> 6	1 / 4958	5
6 -> 8	1 / 4899	2

Minimum cost: 13

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 4	1 / 17111	4
4 -> 5	1 / 17783	2
5 -> 9	1 / 10000	5

Minimum cost: 15

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 4	1 / 17111	4
4 -> 5	1 / 17783	2
5 -> 9	1 / 10000	5

9 -> 10 1 / 13916 3
Minimum cost: 18

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 4	1 / 17111	4
4 -> 11	1 / 4909	6

Minimum cost: 14

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 12	1 / 23403	4

Minimum cost: 8

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 12	1 / 23403	4
12 -> 13	1 / 25900	3

Minimum cost: 11

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 12	1 / 23403	4
11 -> 14	1 / 4877	4
12 -> 11	1 / 4909	6

Minimum cost: 18

Arc	Flow / Capacity	Cost
1 -> 3	1 / 23403	4
3 -> 4	1 / 17111	4
4 -> 11	1 / 4909	6
11 -> 14	1 / 4877	4
14 -> 15	1 / 5128	5

Minimum cost: 23

Arc	Flow / Capacity	Cost
1 -> 2	1 / 25900	6
2 -> 6	1 / 4958	5
6 -> 8	1 / 4899	2
8 -> 16	1 / 5046	5

Minimum cost: 18

```
  Arc    Flow / Capacity  Cost
1 -> 2     1  / 25900        6
2 -> 6     1  / 4958         5
6 -> 8     1  / 4899         2
8 -> 16    1  / 5046         5
16 -> 17   1  / 5230         2
Minimum cost: 20

  Arc    Flow / Capacity  Cost
1 -> 2     1  / 25900        6
2 -> 6     1  / 4958         5
6 -> 8     1  / 4899         2
7 -> 18    1  / 23403        2
8 -> 7     1  / 7842         3
Minimum cost: 18

  Arc    Flow / Capacity  Cost
1 -> 2     1  / 25900        6
2 -> 6     1  / 4958         5
6 -> 8     1  / 4899         2
8 -> 16    1  / 5046         5
16 -> 17   1  / 5230         2
17 -> 19   1  / 4824         2
Minimum cost: 22

  Arc    Flow / Capacity  Cost
1 -> 2     1  / 25900        6
2 -> 6     1  / 4958         5
6 -> 8     1  / 4899         2
7 -> 18    1  / 23403        2
8 -> 7     1  / 7842         3
18 -> 20   1  / 23403        4
Minimum cost: 22

  Arc    Flow / Capacity  Cost
1 -> 3     1  / 23403        4
3 -> 12    1  / 23403        4
12 -> 13   1  / 25900        3
13 -> 24   1  / 5091         4
24 -> 21   1  / 4885         3
Minimum cost: 18
```

```
  Arc    Flow / Capacity  Cost
1 -> 3     1  / 23403        4
3 -> 12    1  / 23403        4
12 -> 13   1  / 25900        3
13 -> 24   1  / 5091         4
21 -> 22   1  / 5230         2
24 -> 21   1  / 4885         3
Minimum cost: 20
```

```
  Arc    Flow / Capacity  Cost
1 -> 3     1  / 23403        4
3 -> 12    1  / 23403        4
12 -> 13   1  / 25900        3
13 -> 24   1  / 5091         4
24 -> 23   1  / 5079         2
Minimum cost: 17
```

```
  Arc    Flow / Capacity  Cost
1 -> 3     1  / 23403        4
3 -> 12    1  / 23403        4
12 -> 13   1  / 25900        3
13 -> 24   1  / 5091         4
Minimum cost: 15
```