Penning trap

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1 Numerical analysis

The kinematics of a charged particle in a Penning Trap is described by the system of 2nd order non-linear ordinary differential equation(ODE) below:

$$\frac{d^2}{dt^2}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{qm}{B}\frac{d}{dt}\begin{bmatrix} y \\ x \\ 0 \end{bmatrix} + \frac{qV_0}{2mz_0}\begin{bmatrix} x \\ y \\ -2z \end{bmatrix}$$

where V_0 is the electric potential difference, B is the magnetic field strength, m is the mass of the charged particle, q is the charge, z_0 is the minimum axial distance of the trap.
Define axial frequency as $w_z = \frac{qV_0}{2mz_0}$ and radial frequency as $w_c = \frac{qm}{B}$. Reduce the 2nd order ODE to a system of 1st order ODE, where $r = x(t) + iy(t)$, $s = x'(t) + iy'(t)$ and $a = z'(t)$

$$\frac{d}{dt}\begin{bmatrix} r \\ s \\ z \\ a \end{bmatrix} = \begin{bmatrix} s \\ \frac{1}{2}w_z r - iw_c s \\ a \\ -w_z^2 z \end{bmatrix}$$

Since our ODE is now first order, we can solve it by using the build-in function in Matlab ode45. This function solves the ODE using Runge-Kutta 4 method, which has higher accuracy than Euler method.

2 Parameter

In this simulation, the trajectory of an electron in a Penning trap is simulated. Below shows the parameter used in our simulation:

$$V_0 = 5.3 \times q \times 10^3$$

,

$$B = M\sqrt{\frac{2mV_0}{qz_0^2}}$$

$$z_0 = 0.001$$

$$\rho_0 = \sqrt{2} z_0$$

M is a user-defined scaling factor.Note that to ensure the charged particle is contained, $B > \sqrt{\frac{2mV_0}{qz_0^2}}$. Thus, M must always be greater than 1 to observe the motion of charged particle in Penning trap.

3 Result

The trajectory of the charged particle in 3D when M=2 is shown in Figure 1. Figure 2 shows the trajectory of the charged particle in XY plane when M=2. Figure 3 shows the position of the particle along each axis against time.Figure 4 shows the velocity against time while Figure 5 shows the acceleration of the particle against time.

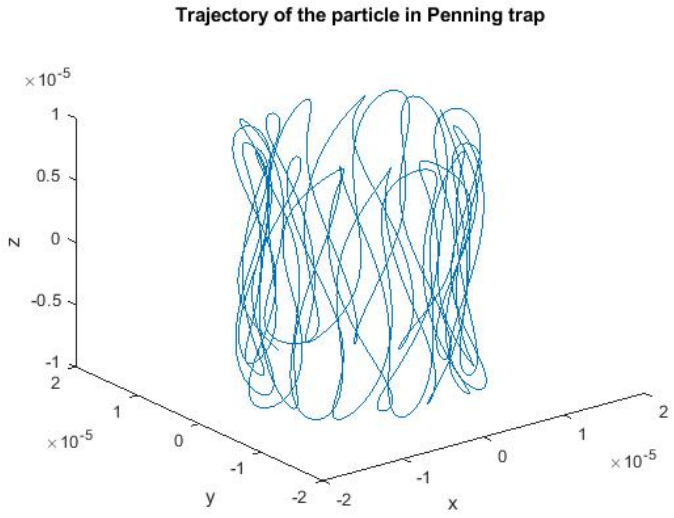

Figure 1: Trajectory of the charged particle

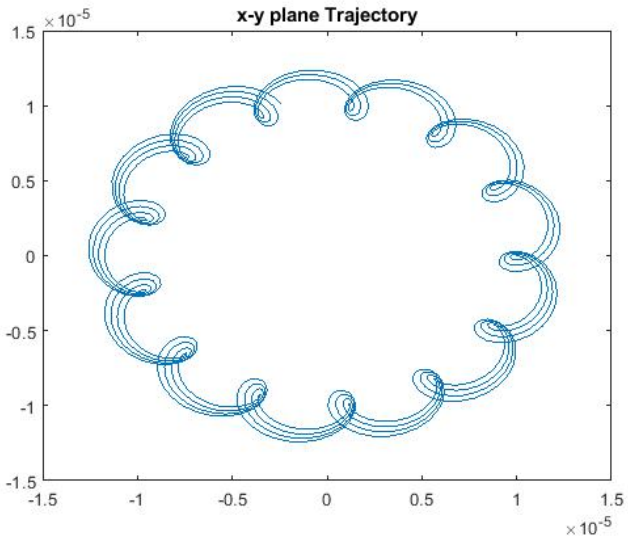

Figure 2: XY plane trajectory

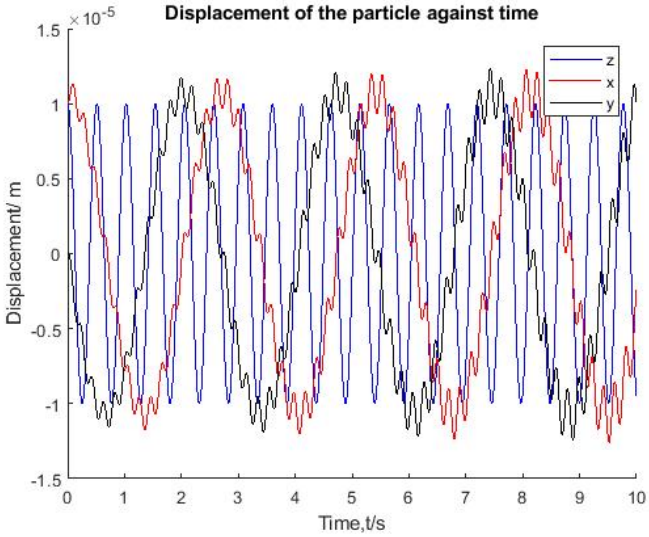

Figure 3: Position of the charged particle against time

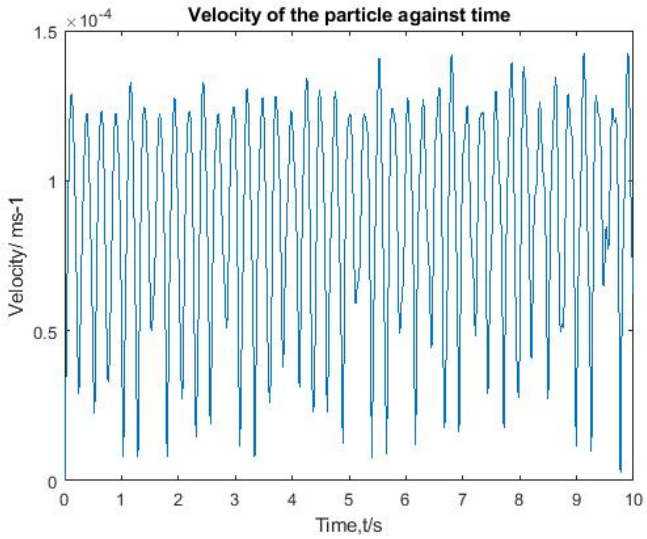

Figure 4: Velocity against time

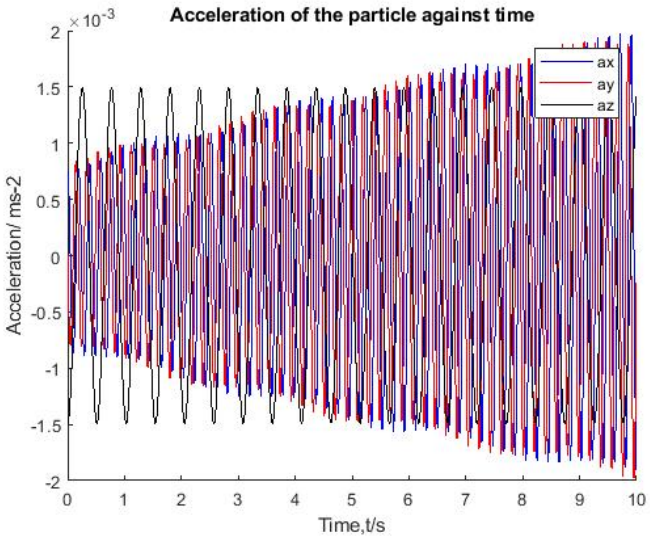

Figure 5: Acceleration against time