Supplementary Material for "An ADMM-based Battery Dispatch Optimization Approach for Electric Vehicle Aggregators with Input-Discriminative Local Differential Privacy Budgets"

Chao Lei, *Member, IEEE*, Qifeng Li, *Senior Member, IEEE*, Siqi Bu, *Senior Member, IEEE*, and Qianggang Wang, *Senior Member, IEEE*

I. Reference Power Signal Vector

The given reference power signal vector $\boldsymbol{p}_{ref}^t$ for the case with $N_E = 10,000$ EVs over a 24-hour scheduling horizon is provided in Fig. 1.

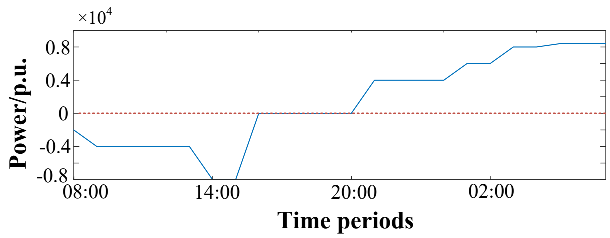

Fig. 1. Given reference power signal vector $\boldsymbol{p}_{ref}^t$ over a 24-hour scheduling horizon.

II. Parameters of Existing DP-ADMM Algorithm

According to the existing DP-ADMM algorithm in [14], the parameter $\alpha(l)$ is defined by

$$\alpha(l) = \frac{\epsilon \cdot (1+\beta)^{\frac{l-2}{4}}\left[(1+\beta)^{\frac{1}{4}} - 1\right]}{H \cdot (1+\beta)^{\frac{K-1}{4}} - 1}, \quad l = 2, ..., K \tag{II-1}$$

where $\beta = 2\tau\rho/(\rho^2 + \tau L)$, $\tau = 1$, $\rho = 5$, $L = 2$, $\epsilon = 0.01$, and $H = 1$.

III. Proof of Theorem 1

There exists an optimal BDO solution $\boldsymbol{x}_i^{t*}$ with the minimum objective function value $\boldsymbol{F}^*$ for the C-BDO model (1a)-(1d). We incorporate $\boldsymbol{p}_{sig,i}^{t*} \in \mathbb{R}$ such that $\sum\limits_{i=1}^{N_E} \boldsymbol{p}_{sig,i}^{t*} = \boldsymbol{p}_{ref}^t$. Reformulate $\|\cdot\|_2$ in the objective function (1a) by

$$\left\|\sum_{i=1}^{N_E} \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{ref}^t\right\|_2 = \left\|\sum_{i=1}^{N_E}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*})\right\|_2 \tag{III-2}$$

Then, we achieve optimality by

$$\sum_{t=1}^{N_T}\left\|\sum_{i=1}^{N_E}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*})\right\|_2 = \sum_{t=1}^{N_T}\left\|\sum_{i=1}^{N_E}\boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{ref}^t\right\|_2 = \boldsymbol{F}^* \tag{III-3}$$

This proves that $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal BDO solution with the minimum objective function value $\boldsymbol{F}^*$.

Due to the triangle inequality of norms, i.e., $|a+b| \leqslant |a| + |b|$ for any real-valued numbers a and b, we obtain that $\boldsymbol{F}^* \leqslant \boldsymbol{F} \leqslant \boldsymbol{G}$ naturally holds. This suggests that $\boldsymbol{F}^*$ can be the lower bound of $\boldsymbol{G}$ over $\mathcal{X}$. In other words, $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal solution iff the minimum function value $\boldsymbol{G}^* = \boldsymbol{F}^*$ is achieved, which demonstrates $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*})$ can also be the optimal BDO solution for minimizing the objective function $\boldsymbol{G}$ over the feasibility space $\mathcal{X}$. This proves Theorem 1. ■

IV. Proof of Theorem 2

The objective is to prove that the optimal charge-discharge solution $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds for EV$_i$. It is clear that the following (IV-4) stands.

$$\boldsymbol{s}_i^{t*} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t - \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t. \tag{IV-4}$$

We denote $\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} = \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. Then, we have

$$\boldsymbol{s}_i^{t*} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t \tag{IV-5}$$

As $\boldsymbol{s}_i^{t\dagger} \geqslant \boldsymbol{s}_i^{t*}$ and $\boldsymbol{y}_i^{t\dagger} = \boldsymbol{s}_i^{t\dagger}$ achieved at optimality, minimizing the objective function $\boldsymbol{g}_i$ ensures $\boldsymbol{s}_i^{t\dagger} = \boldsymbol{s}_i^{t*}$, where other two terms in $\boldsymbol{g}_i$ can be held at convergence, i.e.,

$$\boldsymbol{\mu}^t(\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} - \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t) = \boldsymbol{\mu}^t\boldsymbol{p}_{sig,i}^{t*} \tag{IV-6}$$

and

$$\frac{\boldsymbol{\phi}}{2}\Big\|\sum_{j=1,j\neq i}^{N_E}\widetilde{\boldsymbol{p}}_{sig,j}^{t\dagger} + \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} - \boldsymbol{p}_{ref}^t\Big\|_2^2 = \frac{\boldsymbol{\phi}}{2}\Big\|\sum_{j=1,j\neq i}^{N_E}\boldsymbol{p}_{sig,j}^{t*} + \boldsymbol{p}_{sig,i}^{t*} - \boldsymbol{p}_{ref}^t\Big\|_2^2 \approx 0 \tag{IV-7}$$

Thus, we rewrite $\boldsymbol{s}_i^{t\dagger} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. This suggests that $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds with obfuscated $\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} = \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$ for the privacy-preserving BDO model. ■

V. Proof of Theorem 3

We prove that $\mathcal{M}$ is $r_{\varepsilon,i}^t$-ID-LDP with respect to the local dataset of sub-problem i during all iterations. Let the query output answer for the sub-problem i be $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}_{sig,i}^t$, and we alternatively rewrite (V-8) in the definition of $r_{\varepsilon,i}^t$-LDP for the sub-problem i:

$$\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^t] \leqslant \mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}]e^{r_{\varepsilon,i}^t}, \tag{V-8}$$

for two $r_{\varepsilon,i}^t$-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$ in time period t. For convenience, the query output $\mathcal{O}_i^t$ of EV$_i$ for $\forall t \in \mathcal{T}$ with a random noise $\boldsymbol{\xi}_i^t$ can be written as $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}_{sig,i}^t = \boldsymbol{p}_{sig,i}^t + \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t$ where $\forall t \in \mathcal{T}$.

Therefore, the ratio of probabilities on two ε_i-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$ can be bounded by

$$\begin{aligned}
&\frac{\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^t]}{\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}]} = \frac{\mathbb{P}\left[\boldsymbol{p}_{sig,i}^t + \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t|\boldsymbol{x}_i^t\right]}{\mathbb{P}\left[\boldsymbol{p}_{sig,i}^{t'} + \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}\right]}\\
&= \frac{\mathbb{P}\left[\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t|\boldsymbol{x}_i^t\right]}{\mathbb{P}\left[\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}|\boldsymbol{x}_i^{t'}\right]} \overset{(i)}{=} \frac{\boldsymbol{\alpha}_i^t\exp\{\frac{r_{\varepsilon,i}^t\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t\|_1}{\Delta_{\rho,i}}\}}{\boldsymbol{\alpha}_i^t\exp\{\frac{r_{\varepsilon,i}^t\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\}}\\
&= \exp\left(\frac{r_{\varepsilon,i}^t\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t\|_1 - r_{\varepsilon,i}^t\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right)\\
&\overset{(ii)}{\leqslant} \exp\left(\frac{r_{\varepsilon,i}^t\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \overset{(iii)}{\leqslant} \exp\left(\frac{r_{\varepsilon,i}^t\Delta_{\rho,i}}{\Delta_{\rho,i}}\right) = e^{r_{\varepsilon,i}^t},
\end{aligned} \tag{V-9}$$

where (i) comes from the definition of the probability density function of the Gaussian distribution. In (ii) step, it is followed by the inequailty of norms, i.e., $|a| - |b| \leqslant |a - b|$ for any real-valued numbers a and b. For the (iii) step, $\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1$ denotes the ℓ_1-sensitivity on ρ-indistinguishable output datasets $\boldsymbol{p}_{sig,i}^t$ and $\boldsymbol{p}_{sig,i}^{t'}$ subject to $\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1 \leqslant \Delta_{\rho,i}$.

Accordingly, it is clear that (V-8) holds based on (V-9), which proves this Theorem. ■