Supplementary Material for "A Disjunctive Convex Hull Relaxation Formulation for Battery Operation without Binary Variables and Charge-Discharge Complementarity Constraints"

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I. Proof of Lemma 1

Suppose that there exists $\eta_c \boldsymbol{P}_c^t \Delta \boldsymbol{T} + \frac{1}{\eta_d} \boldsymbol{P}_d^t \Delta \boldsymbol{T} > 0$ such that $\boldsymbol{\lambda}_d^t = 1$ at time t, then we have $\boldsymbol{\lambda}_c^t = 0$ and $\boldsymbol{P}_c^t = 0$ according to (1b). This induces $\frac{1}{\eta_d} \boldsymbol{P}_d^t \Delta \boldsymbol{T} < 0$, which contradicts $\eta_c \boldsymbol{P}_c^t \Delta \boldsymbol{T} + \frac{1}{\eta_d} \boldsymbol{P}_d^t \Delta \boldsymbol{T} > 0$ as the initial assumption. Thereby, $\boldsymbol{\lambda}_d^t = 1$ enables $\frac{1}{\eta_d} \boldsymbol{P}_d^t \Delta \boldsymbol{T} < 0$. Similarly, this theorem can be also proven for which $\eta_c \boldsymbol{P}_c^t \Delta \boldsymbol{T} > 0$ is consistent with $\boldsymbol{\lambda}_c^t = 1$. Besides, if this battery device is in an idle mode, then $\boldsymbol{P}_d^t = \boldsymbol{P}_c^t = 0$, which (1a) and (1b) still stand. ■

II. Proof of Tightened Feasibility Space

In terms of DCHR, the upper and lower bounds of e^t are denoted as $\mathcal{L}_1^{upper} = \boldsymbol{\lambda}_c^t \overline{E} - \eta_c \boldsymbol{P}_c^t (t-1) \Delta T$ and $\mathcal{L}_1^{lower} = \boldsymbol{\lambda}_d^t \underline{E} + \frac{1}{\eta_d} \boldsymbol{P}_d^t (t-1) \Delta T$, respectively. However, according to HCH-LP method, we express the upper and lower bounds of e^t by $\mathcal{L}_2^{upper} = \overline{E} - \eta_c \boldsymbol{P}_c^t (t-1) \Delta T$ and $\mathcal{L}_2^{lower} = \underline{E} + \frac{1}{\eta_d} \boldsymbol{P}_d^t (t-1) \Delta T$, respectively. Hence, we achieve the difference in algebra $\mathcal{L}_1^{upper} - \mathcal{L}_2^{upperr} = (\boldsymbol{\lambda}_c^t - 1) \overline{E}$. Since $\lambda_c^t \in [0,1]$, this clearly renders $\mathcal{L}_1^{upper} \leqslant \mathcal{L}_2^{upper}$. Similarly, we can also achieve the lower bound subject to $\mathcal{L}_1^{lower} \geqslant \mathcal{L}_2^{lower}$. We have proved that DCHR has tighter relaxation lower and upper bounds than those by HCH-LP for $\boldsymbol{\Omega}_{id}$. ■

III. Proof of Lemma 2

Suppose $\boldsymbol{\lambda}_c^{t*}$ and $\boldsymbol{\lambda}_d^{t*}$ are optimal solutions. Since $(\boldsymbol{\lambda}_c^{t*} + \boldsymbol{\lambda}_d^{t*})^2 = 1$ holds, we derive $2\boldsymbol{\lambda}_c^{t*} \boldsymbol{\lambda}_d^{t*} = 0$ according to $(\boldsymbol{\lambda}_c^{t*})^2 + (\boldsymbol{\lambda}_d^{t*})^2 = 1$ in this Lemma. In other words, $\boldsymbol{\lambda}_c^{t*}, \boldsymbol{\lambda}_d^{t*}$ simply equals to either 0 or 1, which proves this lemma. ■

IV. Proof of Theorem

Suppose the optimal solution for the MIQP-based BDO model is $(\boldsymbol{e}^{t*}, \boldsymbol{P}_c^{t*}, \boldsymbol{P}_d^{t*}, \boldsymbol{\lambda}_c^{t*}, \boldsymbol{\lambda}_d^{t*})$. First, we show that $(\boldsymbol{\lambda}_c^{t*}, \boldsymbol{\lambda}_d^{t*})$ can be optimal for the SOCP-based BDO model. By substituting (2) into $\widetilde{\boldsymbol{z}}^t$, we can obtain $\widetilde{\boldsymbol{z}}^t = 1 + (\boldsymbol{\lambda}_{c,0}^t)^2 - (\boldsymbol{\lambda}_{d,0}^t)^2 - 2(\boldsymbol{\lambda}_{c,0}^t - \boldsymbol{\lambda}_{d,0}^t)\boldsymbol{\lambda}_c^t$. This $\widetilde{\boldsymbol{z}}^t$ is a linear expression only with respect to $\boldsymbol{\lambda}_c^t$. If $\exists \boldsymbol{\lambda}_{c,0}^t \neq \boldsymbol{\lambda}_{d,0}^t$, the minimal $||\boldsymbol{s}^t||_2$ can be achieved at an optimal charging indicator $\boldsymbol{\lambda}_c^{t*} = 1$ if $\boldsymbol{\lambda}_{c,0}^t - \boldsymbol{\lambda}_{d,0}^t > 0$, or an optimal charging indicator $\boldsymbol{\lambda}_c^{t*} = 0$ if $\boldsymbol{\lambda}_{c,0}^t - \boldsymbol{\lambda}_{d,0}^t < 0$. It is clear that such optimal charging indicator $\boldsymbol{\lambda}_c^{t*}$=0 or 1 can satisfy indicator-related constraints (2), (8) and $(\boldsymbol{\lambda}_c^t)^2 + (\boldsymbol{\lambda}_d^t)^2 \leqslant 1$. This demonstrates that $(\boldsymbol{\lambda}_c^{t*}, \boldsymbol{\lambda}_d^{t*})$ is valid for achiaveing the smallest $||\boldsymbol{s}^t||_2$ in the objective function F of the SOCP-based BDO model.

Next, we show that $(\boldsymbol{e}^{t*}, \boldsymbol{P}_c^{t*}, \boldsymbol{P}_d^{t*})$ can be the optimal solution for the SOCP-based BDO model. Recall that $(\boldsymbol{e}^{t*}, \boldsymbol{P}_c^{t*}, \boldsymbol{P}_d^{t*})$=argmin $||\boldsymbol{P}_c^t + \boldsymbol{P}_d^t - \boldsymbol{p}_{sig}^t||_2 \ s.t. \ \boldsymbol{\Omega}_{id} \cup \boldsymbol{\Omega}_{op}$ under $(\boldsymbol{\lambda}_c^{t*}, \boldsymbol{\lambda}_d^{t*})$ for the MIQP-based BDO model. Thus, $(\boldsymbol{e}^{t*}, \boldsymbol{P}_c^{t*}, \boldsymbol{P}_d^{t*})$ can enable to achieve the smallest $||\boldsymbol{P}_c^t + \boldsymbol{P}_d^t - \boldsymbol{p}_{sig}^t||_2$ over $\mathcal{X}$ for the SOCP-based BDO model, which suggests that $(\boldsymbol{e}^{t*}, \boldsymbol{P}_c^{t*}, \boldsymbol{P}_d^{t*})$ is optimal.

Based on the above, this original MIQP-based BDO model can be theoretically equivalent to this SOCP-based BDO model with the same optimal solution $(\boldsymbol{e}^{t*}, \boldsymbol{P}_c^{t*}, \boldsymbol{P}_d^{t*}, \boldsymbol{\lambda}_c^{t*}, \boldsymbol{\lambda}_d^{t*})$, which proves this Theorem. ■