Supplementary Material for "An ADMM-based Battery Dispatch Optimization Approach for Electric Vehicle Aggregators with Input-Discriminative Local Differential Privacy Budgets"

Chao Lei, *Member, IEEE*, Qifeng Li, *Senior Member, IEEE*, and Siqi Bu, *Senior Member, IEEE*

I. Proof of Theorem 1

There exists an optimal BDO solution $\boldsymbol{x}_i^{t*}$ with the minimum objective function value $\boldsymbol{F}^*$ for the C-BDO model (1a)-(1d). We incorporate $\boldsymbol{p}_{sig,i}^{t*} \in \mathbb{R}$ such that $\sum_{i=1}^{N_E} \boldsymbol{p}_{sig,i}^{t*} = \boldsymbol{p}_{ref}^t$. Reformulate $\|\cdot\|_2$ in the objective function (1a) by

$$\left\| \sum_{i=1}^{N_E} \boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{ref}^t \right\|_2 = \left\| \sum_{i=1}^{N_E} (\boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*}) \right\|_2 \tag{I-1}$$

Then, we achieve optimality by

$$\sum_{t=1}^{N_T} \left\| \sum_{i=1}^{N_E} (\boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*}) \right\|_2 = \sum_{t=1}^{N_T} \left\| \sum_{i=1}^{N_E} \boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{ref}^t \right\|_2 = \boldsymbol{F}^* \tag{I-2}$$

This proves that $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal BDO solution with the minimum objective function value $\boldsymbol{F}^*$.

Due to the triangle inequality of norms, i.e., $|a+b| \leqslant |a| + |b|$ for any real-valued numbers a and b, we obtain that $\boldsymbol{F}^* \leqslant \boldsymbol{F} \leqslant \boldsymbol{G}$ naturally holds. This suggests that $\boldsymbol{F}^*$ can be the lower bound of $\boldsymbol{G}$ over $\mathcal{X}$. In other words, $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal solution iff the minimum function value $\boldsymbol{G}^* = \boldsymbol{F}^*$ is achieved, which demonstrates $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*})$ can also be the optimal BDO solution for minimizing the objective function $\boldsymbol{G}$ over the feasibility space $\mathcal{X}$. This proves Theorem 1. ■

II. Proof of Theorem 2

First, we prove that the optimal charge-discharge solution $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds for EV$_i$. It is clear that $\boldsymbol{s}_i^{t*} = \boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger} \boldsymbol{\xi}_i^t - \boldsymbol{\alpha}_i^{t\dagger} \boldsymbol{\xi}_i^t$ stands. We denote $\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} = \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger} \boldsymbol{\xi}_i^t$. Then, we have $\boldsymbol{s}_i^{t*} = \boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} + \boldsymbol{\alpha}_i^{t\dagger} \boldsymbol{\xi}_i^t$. As $\boldsymbol{s}_i^{t\dagger} \geqslant \boldsymbol{s}_i^{t*}$ and $\boldsymbol{y}_i^{t\dagger} = \boldsymbol{s}_i^{t\dagger}$ achieved at optimality, minimizing the objective function $\boldsymbol{g}_i$ ensures $\boldsymbol{s}_i^{t\dagger} = \boldsymbol{s}_i^{t*}$, where other two terms in $\boldsymbol{g}_i$ can be held, i.e., $\boldsymbol{\mu}^t (\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} - \boldsymbol{\alpha}_i^t \boldsymbol{\xi}_i^t) = \boldsymbol{\mu}^t \boldsymbol{p}_{sig,i}^{t*}$ and $\frac{\boldsymbol{\phi}}{2} || \sum_{j=1, j\neq i}^{N_E} \widetilde{\boldsymbol{p}}_{sig,j}^{t\dagger} + \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} - \boldsymbol{p}_{ref}^t ||_2^2 = \frac{\boldsymbol{\phi}}{2} || \sum_{j=1, j\neq i}^{N_E} \boldsymbol{p}_{sig,j}^{t*} + \boldsymbol{p}_{sig,i}^{t*} - \boldsymbol{p}_{ref}^t ||_2^2 \approx 0$ at convergence. Thus, we rewrite $\boldsymbol{s}_i^{t\dagger} = \boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} + \boldsymbol{\alpha}_i^{t\dagger} \boldsymbol{\xi}_i^t$. This suggests that $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds with obfuscated $\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} = \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger} \boldsymbol{\xi}_i^t$ for the privacy-preserving BDO model. ■

III. Proof of Theorem 3

We prove that $\mathcal{M}$ is $r_{\varepsilon,i}^t$-ID-LDP with respect to the local dataset of sub-problem i during all iterations. Let the query output answer for the sub-problem i be $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}_{sig,i}^t$, and we alternatively rewrite (III-3) in the definition of $r_{\varepsilon,i}^t$-LDP for the sub-problem i:

$$\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \boldsymbol{x}_i^t] \leqslant \mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \boldsymbol{x}_i^{t'}] e^{r_{\varepsilon,i}^t}, \tag{III-3}$$

for two $r_{\varepsilon,i}^t$-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$ in time period t. For convenience, the query output $\mathcal{O}_i^t$ of EV$_i$ for $\forall t \in \mathcal{T}$ with a random noise $\boldsymbol{\xi}_i^t$ can be written as $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}_{sig,i}^t = \boldsymbol{p}_{sig,i}^t + \boldsymbol{\alpha}_i^t \boldsymbol{\xi}_i^t$ where $\forall t \in \mathcal{T}$.

Therefore, the ratio of probabilities on two ε_i-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$ can be bounded by

$$\begin{aligned}
&\frac{\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \boldsymbol{x}_i^t]}{\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \boldsymbol{x}_i^{t'}]} = \frac{\mathbb{P}\left[\boldsymbol{p}_{sig,i}^t + \boldsymbol{\alpha}_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t | \boldsymbol{x}_i^t\right]}{\mathbb{P}\left[\boldsymbol{p}_{sig,i}^{t'} + \boldsymbol{\alpha}_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t | \boldsymbol{x}_i^{t'}\right]} \\
&= \frac{\mathbb{P}\left[\boldsymbol{\alpha}_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t | \boldsymbol{x}_i^t\right]}{\mathbb{P}\left[\boldsymbol{\alpha}_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'} | \boldsymbol{x}_i^{t'}\right]} \overset{(i)}{=} \frac{\boldsymbol{\alpha}_i^t \exp\{\frac{r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t\|_1}{\Delta_{\rho,i}}\}}{\boldsymbol{\alpha}_i^t \exp\{\frac{r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\}} \\
&= \exp\left(\frac{r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t\|_1 - r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \\
&\overset{(ii)}{\leqslant} \exp\left(\frac{r_{\varepsilon,i}^t \|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \overset{(iii)}{\leqslant} \exp\left(\frac{r_{\varepsilon,i}^t \Delta_{\rho,i}}{\Delta_{\rho,i}}\right) = e^{r_{\varepsilon,i}^t},
\end{aligned} \tag{III-4}$$

where (i) comes from the definition of the probability density function of the Gaussian distribution. In (ii) step, it is followed by the inequailty of norms, i.e., $|a| - |b| \leqslant |a - b|$ for any real-valued numbers a and b. For the (iii) step, $\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1$ denotes the ℓ_1-sensitivity on ρ-indistinguishable output datasets $\boldsymbol{p}_{sig,i}^t$ and $\boldsymbol{p}_{sig,i}^{t'}$ subject to $\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1 \leqslant \Delta_{\rho,i}$.

Accordingly, it is clear that (III-3) holds based on (III-4), which proves this Theorem. ■