Supplementary Material for "An ADMM-based Battery Dispatch Optimization for Electric Vehicle Aggregators with Input-Discriminative Local Differential Privacy Budgets"

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I. Proof of Theorem 1

There exists an optimal battery dispatch optimization (BDO) solution $\boldsymbol{x}_i^{t*}$ with the minimum objective function value $\boldsymbol{F}^*$ for the non-private BDO model (1a)-(1e). We incorporate $\boldsymbol{p}_{sig,i}^{t*} \in \mathbb{R}$ such that $\sum\limits_{i=1}^{N_E} \boldsymbol{p}_{sig,i}^{t*} = \boldsymbol{p}_{ref}^t$. Then, $\left\|\sum\limits_{i=1}^{N_E} \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{ref}^t\right\|_2 = \left\|\sum\limits_{i=1}^{N_E}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*})\right\|_2$ can hold. Clearly, $\sum\limits_{t=1}^{N_T}\left\|\sum\limits_{i=1}^{N_E}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*})\right\|_2 = \sum\limits_{t=1}^{N_T}\left\|\sum\limits_{i=1}^{N_E}\boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{ref}^t\right\|_2 = \boldsymbol{F}^*$ can be achieved. This proves that $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal BDO solution with the minimum objective function value $\boldsymbol{F}^*$.

Due to the triangle inequality of norms, i.e., $|a+b| \leqslant |a| + |b|$ for any real-valued numbers a and b, we obtain that $\boldsymbol{F}^* \leqslant \boldsymbol{F} \leqslant \boldsymbol{G}$ naturally holds. This suggests that $\boldsymbol{F}^*$ can be the lower bound of $\boldsymbol{G}$ over $\mathcal{X}$. In other words, $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal solution iff the minimum function value $\boldsymbol{G}^* = \boldsymbol{F}^*$ is achieved, which demonstrates $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}_{sig,i}^{t*})$ can be also the optimal BDO solution for minimizing the objective function $\boldsymbol{G}$ over the feasibility space $\mathcal{X}$. This proves Theorem 1. ■

II. Proof of Theorem 2

First, we prove that the optimal charge-discharge solution $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds for the i-th EV. It is clear that $\boldsymbol{d}_i^{t*} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t - \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$ stands. We denote $\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} = \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. Then, we have $\boldsymbol{d}_i^{t*} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. As $\boldsymbol{d}_i^{t\dagger} \geqslant \boldsymbol{d}_i^{t*}$ and $\boldsymbol{y}_i^{t\dagger} = \boldsymbol{d}_i^{t\dagger}$ achieved at optimality, minimizing the objective function $\boldsymbol{g}_i$ ensures $\boldsymbol{d}_i^{t\dagger} = \boldsymbol{d}_i^{t*}$, where other two terms in $\boldsymbol{g}_i$ can be held, i.e., $\boldsymbol{\mu}^t(\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} - \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t) = \boldsymbol{\mu}^t\boldsymbol{p}_{sig,i}^{t*}$ and $\frac{\boldsymbol{\rho}}{2}||\sum\limits_{j=1,j\neq i}^{N_E}\widetilde{\boldsymbol{p}}_{sig,j}^{t\dagger} + \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} - \boldsymbol{p}_{ref}^t||_2^2 = \frac{\boldsymbol{\rho}}{2}||\sum\limits_{j=1,j\neq i}^{N_E}\boldsymbol{p}_{sig,j}^{t*} + \boldsymbol{p}_{sig,i}^{t*} - \boldsymbol{p}_{ref}^t||_2^2 \approx 0$ at convergence. Thus, we rewrite $\boldsymbol{d}_i^{t\dagger} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*} - \widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. This suggests that $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds with obfuscated $\widetilde{\boldsymbol{p}}_{sig,i}^{t\dagger} = \boldsymbol{p}_{sig,i}^{t*} + \boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$ for the proposed privacy-preserving BDO model.

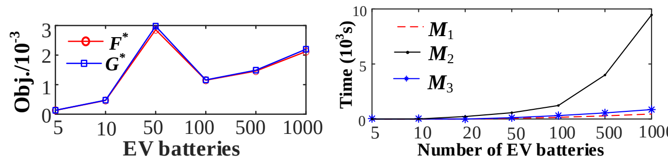
Second, we prove that $\mathcal{M}$ is $r_{\varepsilon,i}$-input-discriminative differentially locally private (ID-LDP) with respect to the local dataset of sub-problem i during all iterations. Let the query output answer for the sub-problem i be $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}_{sig,i}^t$, and we alternatively rewrite (A-1) in the definition of ε_i-LDP for the sub-problem i [1]:

$$\mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^t] \leqslant \mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}]e^{r_{\varepsilon,i}}, \tag{A-1}$$

for two $r_{\varepsilon,i}$-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$. For convenience, the query output $\mathcal{O}_i^t$ of i-th EV for $\forall t \in \mathcal{T}$ with a random noise $\boldsymbol{\xi}_i^t$ can be written as $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}_{sig,i}^t = \boldsymbol{p}_{sig,i}^t + \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t$ where $\forall t \in \mathcal{T}$.

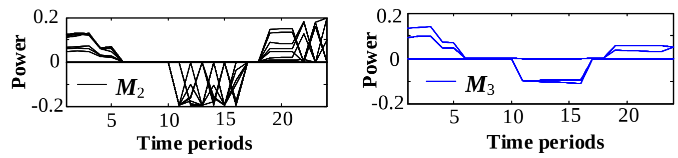
Therefore, the ratio of probabilities on two ε_i-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$ can be bounded by

$$\begin{aligned}
&\frac{\mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^t]}{\mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}]} = \frac{\mathbb{P}_\xi\left[\boldsymbol{p}_{sig,i}^t + \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t|\boldsymbol{x}_i^t\right]}{\mathbb{P}_\xi\left[\boldsymbol{p}_{sig,i}^{t'} + \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}\right]} \\
&= \frac{\mathbb{P}_\xi\left[\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t|\boldsymbol{x}_i^t\right]}{\mathbb{P}_\xi\left[\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}|\boldsymbol{x}_i^{t'}\right]} \overset{(i)}{=} \frac{\boldsymbol{\alpha}_i^t\exp\{\frac{r_{\varepsilon,i}\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t\|_1}{\Delta_{\rho,i}}\}}{\boldsymbol{\alpha}_i^t\exp\{\frac{r_{\varepsilon,i}\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\}} \\
&= \exp\left(\frac{r_{\varepsilon,i}\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^t\|_1 - r_{\varepsilon,i}\|\mathcal{O}_i^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \\
&\overset{(ii)}{\leqslant} \exp\left(\frac{r_{\varepsilon,i}\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \overset{(iii)}{\leqslant} \exp\left(\frac{r_{\varepsilon,i}\Delta_{\rho,i}}{\Delta_{\rho,i}}\right) = e^{r_{\varepsilon,i}},
\end{aligned} \tag{A-2}$$



(a) (b)

Fig. 1. (a) Optimal objective function values; (b) CPU time.



(a) (b)

Fig. 2. Optimal charge-discharge power solution: (a) $\boldsymbol{M}_2$; (b) $\boldsymbol{M}_3$.

where (i) comes from the definition of the probability density function of the Gaussian distribution. In (ii) step, it is followed by the inequailty of norms, i.e., $|a| - |b| \leqslant |a-b|$ for any real-valued numbers a and b. For the (iii) step, $\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1$ denotes the ℓ_1-sensitivity on ρ-indistinguishable output datasets $\boldsymbol{p}_{sig,i}^t$ and $\boldsymbol{p}_{sig,i}^{t'}$ subject to $\|\boldsymbol{p}_{sig,i}^t - \boldsymbol{p}_{sig,i}^{t'}\|_1 \leqslant \Delta_{\rho,i}$.

Accordingly, it is clear that (A-1) holds based on (A-2), which proves this Theorem. ■

III. Additional Case Study for Theorem 1

The computational performance of this proposed ADMM-based BDO approach by **Theorem 1** can be verified in this section. For brevity, three methods are compared: the LP-based approach in [2] ($\boldsymbol{M_1}$), and the MIQP-based BDO model solved using commercial solver MOSEK ($\boldsymbol{M_2}$) for comparison with the proposed ADMM-based BDO method ($\boldsymbol{M_3}$). In Fig. 1(a), we plot the minimum objective function value $\boldsymbol{F}^*$ obtained from the benchmark $\boldsymbol{M_2}$ and compare it to $\boldsymbol{G}^*$ obtained from the proposed $\boldsymbol{M_3}$ for the case with the different numbers of EVs, i.e, 5, 20, 20, 50, 100, 500 and 1000. We observe that, indeed, $\boldsymbol{G}^* \approx \boldsymbol{F}^*$ in all cases. Moreover, Fig. 1(b) reports the computation time with respect to the numbers of EV batteries. $\boldsymbol{M_1}$ incurs the least computational time because $\boldsymbol{M_1}$ is free of binary variables, but it can result in infeasible solutions featuring simultaneous charging and discharging. And $\boldsymbol{M_3}$ is superior to both $\boldsymbol{M_2}$ with respect to computational time. In fact, for the case with $N_E = 1000$ EVs, $\boldsymbol{M_3}$ converges to the optimal solution in 12.67% of the time taken by $\boldsymbol{M_2}$. To further demonstrate the numbers of charging/discharging switches in $\boldsymbol{M_2}$ and $\boldsymbol{M_3}$, Figs. 2(a) and 2(b) present the specific optimal charge-discharge power solutions for the case with $N_E = 10$ EVs. In Fig. 2(a), $\boldsymbol{M_2}$ incurs many charging and discharging actions over 24 hours, while $\boldsymbol{M_3}$ imposes few charging and discharging actions, as shown in Fig. 2(b). This is highly beneficial as charging/discharging switches degrade the remaining useful life of EV batteries. This case study showcases that an ADMM-based BDO approach established by **Theorem 1** outperforms existing methods for solving BDO problems in terms of computational time and switching times.

REFERENCES

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[2] D. Pozo, "Convex hull formulations for linear modeling of energy storage systems," *IEEE Trans. on Power Syst.*, vol. 38, no. 6, pp. 5934–5936, 2023.