Supplementary Material for “Differentially Private Distribution Network Reconfiguration ”

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I. Non-Private DNR Model

The DNs are considered as a connected undirected tree $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ is the set of branches. Suppose the general mathematical formulation of a non-private DNR model in mixed-integer quadratic programming (MIQP) form can be expressed with respect to operational variables $\boldsymbol{x}^l$ including active/reactive power flow variables and reactive power compensation variables, continuous parent-child relationship variables $\boldsymbol{\beta}^l$ and switch status indicator variables $\boldsymbol{u}^l$ [1].

$$\textbf{Non-Private DNR}: \quad \min_{\boldsymbol{x}^l, \boldsymbol{\beta}^l \in \mathbb{R}, \boldsymbol{u}^l \in \mathbb{Z}} \; f_1 = (\boldsymbol{x}^l)^T \boldsymbol{c} \boldsymbol{x}^l \tag{A-1a}$$

$$\text{s.t. } \mathcal{X} := \left\{ (\boldsymbol{x}^l, \boldsymbol{\beta}^l, \boldsymbol{u}^l) \left| \begin{array}{l} \boldsymbol{A}\boldsymbol{x}^l = \boldsymbol{d}, \; \begin{bmatrix} \boldsymbol{G}_v \\ \boldsymbol{G}_u \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^l \\ \boldsymbol{u}^l \end{bmatrix} \leqslant \begin{bmatrix} \boldsymbol{b}_v \\ \boldsymbol{b}_u \end{bmatrix}, \\ \boldsymbol{G}_{cr}\boldsymbol{x}^l \leqslant \boldsymbol{b}_{cr}, \; \boldsymbol{K} \begin{bmatrix} \boldsymbol{\beta}^l \\ \boldsymbol{u}^l \end{bmatrix} = \boldsymbol{h} \end{array} \right. \right\} \tag{A-1b}$$

where $\mathcal{X}$ refers to the non-empty feasibility space and f_1 denotes the approximate system power loss with a fixed diagonal matrix $\boldsymbol{c}$. The inequality in (A-1b) represents the voltage security constraints, physical ranges of reactive power compensation capacitors and topology-linked branch capacity constraints, which are marked by the subscripts v, cr and u for $\boldsymbol{G}$ and $\boldsymbol{b}$. The first and second equality denotes the system-wide load balance of DNs and radiality constraints.

II. MIQP-based DNR Model

Suppose that an arbitrary branch $l := (m, n), l \in \mathcal{E}$ is between nodes (m, n) and the root node is assumed as 0. The simplified DNR model is based on the linearized DistFlow equations, which is cast as a MIQP problem. The set of optimization variables involves a set of operational variables $\boldsymbol{x}^l := [\boldsymbol{P}^l, \boldsymbol{Q}^l, \boldsymbol{Q}^{cr}]^T$, switch status indicator variables $\boldsymbol{u}^l \in \mathbb{Z}^{|\mathcal{E}|}$ and continuous parent-child relationship variables $\boldsymbol{\beta}^l \in \mathbb{R}^{2|\mathcal{E}|}$. In this vein, $\boldsymbol{P}^l \in \mathbb{R}^{|\mathcal{E}|}$ and $\boldsymbol{Q}^l \in \mathbb{R}^{|\mathcal{E}|}$ refer to the vectors of sending-end active and reactive power flows. $\boldsymbol{Q}^{cr} \in \mathbb{R}^{|n_{cr}|}$ is the vector of nodal reactive power compensation and n_{cr} is the number of capacitors. $\boldsymbol{\beta}^l$ are constructed with spanning tree constraints. For convenience, we express the MIQP-based DNR model in following form:

$$\min_{\boldsymbol{x}^l \in \mathbb{R}, \boldsymbol{u}^l \in \mathbb{Z}} \begin{bmatrix} \boldsymbol{P}^l \\ \boldsymbol{Q}^l \\ \boldsymbol{Q}^{cr} \end{bmatrix}^T \begin{bmatrix} \boldsymbol{D}_r & \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} \\ \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{D}_r & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} \\ \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times n_{cr}} \end{bmatrix} \begin{bmatrix} \boldsymbol{P}^l \\ \boldsymbol{Q}^l \\ \boldsymbol{Q}^{cr} \end{bmatrix} \tag{A-2a}$$

$$\text{s.t.} \begin{bmatrix} \boldsymbol{A}_{\mathcal{G}}^T & \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} \\ \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{A}_{\mathcal{G}}^T & \boldsymbol{A}_{cr} \end{bmatrix} \begin{bmatrix} \boldsymbol{P}^l \\ \boldsymbol{Q}^l \\ \boldsymbol{Q}^{cr} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{P}^g + \boldsymbol{P}^d \\ -\boldsymbol{Q}^g + \boldsymbol{Q}^d \end{bmatrix} \tag{A-2b}$$

$$\begin{bmatrix} 2\boldsymbol{D}_r & 2\boldsymbol{D}_x & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} & M \\ -2\boldsymbol{D}_r & -2\boldsymbol{D}_x & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} & -M \\ \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & diag(\mathbb{1}_N) & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} \\ \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & -diag(\mathbb{1}_N) & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} \end{bmatrix} \begin{bmatrix} \boldsymbol{P}^l \\ \boldsymbol{Q}^l \\ \boldsymbol{Q}^{cr} \\ \boldsymbol{u}^l \end{bmatrix} \leqslant \begin{bmatrix} \Delta\overline{\boldsymbol{v}} + M \\ -\Delta\underline{\boldsymbol{v}} + M \\ \overline{\boldsymbol{Q}}_{\boldsymbol{cr}} \\ -\underline{\boldsymbol{Q}}_{\boldsymbol{cr}} \end{bmatrix} \tag{A-2c}$$

$$-u^l\Gamma \leqslant \boldsymbol{P}^l, \boldsymbol{Q}^l \leqslant u^l\Gamma \tag{A-2d}$$

$$\beta_{mn}^l + \beta_{nm}^l = u_{mn}^l, \quad \beta_{mn}^l = 0, \text{ if } m = 0 \tag{A-2e}$$

$$\sum_{n:(m,n)\in\mathcal{E}} \beta_{mn}^l = 1, \; \forall m \in \mathcal{N}\backslash 0 \tag{A-2f}$$

$$0 \leqslant \beta_{mn}^l \leqslant 1, \; \forall l \in \mathcal{E} \tag{A-2g}$$

where (A-2a) states the quadratic active power loss of DNs under the assumption of flat voltage profiles for all nodes. $\mathbb{1}_N$ is a $|\mathcal{E}| \times 1$ vector with all ones. M refers to the big positive number and Γ denotes the branch capacity. $\boldsymbol{P}^g, \boldsymbol{Q}^g$ and $\boldsymbol{P}^d, \boldsymbol{Q}^d$ indicate the vectors of given nodal active and reactive power injections and active and reactive loads at nodes. $\boldsymbol{A}_{\mathcal{G}}$ is a $|\mathcal{E}|$ by $|\mathcal{N}|$ branch-node incidence matrix and $\boldsymbol{A}_{cr}$ is a diagonal matrix whose i-th diagonal element is equal to 1 if node i has the reactive compensation capacitors; otherwise it is zero. $\boldsymbol{D}_r$ and $\boldsymbol{D}_x$ indicate the diagonal matrices whose diagonal elements are the resistance and reactance vectors, respectively. $\underline{\boldsymbol{Q}}_{\boldsymbol{cr}}$, $\overline{\boldsymbol{Q}}_{\boldsymbol{cr}}$, $\Delta\underline{\boldsymbol{v}}$ and $\Delta\overline{\boldsymbol{v}}$ represent the boundaries of $\boldsymbol{Q}^{cr}$ and squared voltage profile deviation, respectively.

Therefore, we can summarize $\boldsymbol{c}$, $\boldsymbol{A}$, $\boldsymbol{G}_v$, $\boldsymbol{G}_{cr}$, $\boldsymbol{d}$, $\boldsymbol{b}_v$ and $\boldsymbol{b}_{cr}$ in (A-1b) as

$$\boldsymbol{c} = \begin{bmatrix} \boldsymbol{D}_r & \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} \\ \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{D}_r & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} \\ \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times n_{cr}} \end{bmatrix} \tag{A-3a}$$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_{\mathcal{G}}^T & \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} \\ \boldsymbol{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{A}_{\mathcal{G}}^T & \boldsymbol{A}_{cr} \end{bmatrix}, \; \boldsymbol{d} = \begin{bmatrix} -\boldsymbol{P}^g + \boldsymbol{P}^d \\ -\boldsymbol{Q}^g + \boldsymbol{Q}^d \end{bmatrix} \tag{A-3b}$$

$$\boldsymbol{G}_v = \begin{bmatrix} 2\boldsymbol{D}_r & 2\boldsymbol{D}_x & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} & M \\ -2\boldsymbol{D}_r & -2\boldsymbol{D}_x & \boldsymbol{0}_{|\mathcal{E}|\times n_{cr}} & -M \end{bmatrix} \tag{A-3c}$$

$$\boldsymbol{G}_{cr} = \begin{bmatrix} \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & diag(\mathbb{1}_N) & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} \\ \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} & -diag(\mathbb{1}_N) & \boldsymbol{0}_{n_{cr}\times|\mathcal{E}|} \end{bmatrix} \tag{A-3d}$$

$$\boldsymbol{b}_v = \begin{bmatrix} \Delta\overline{\boldsymbol{v}} + M \\ -\Delta\underline{\boldsymbol{v}} + M \end{bmatrix}, \; \boldsymbol{b}_{cr} = \begin{bmatrix} \overline{\boldsymbol{Q}}_{\boldsymbol{cr}} \\ -\underline{\boldsymbol{Q}}_{\boldsymbol{cr}} \end{bmatrix} \tag{A-3e}$$

where $\boldsymbol{K}$ and $\boldsymbol{h}$ can be rewritten from (A-2e)-(A-2g) and $\boldsymbol{G}_u$ and $\boldsymbol{b}_u$ are inessential for (A-2d) since (A-2d) is already enforced by (3a).

References

[1] J. A. Taylor and F. S. Hover, “Convex models of distribution system reconfiguration,” *IEEE Trans. on Power Syst.*, vol. 27, no. 3, pp. 1407–1413, 2012.