Supplementary Material for " Optimal Ship-to-Grid Dispatch Considering Battery Thermal and Voltage Electrochemical-Thermal-Coupled Constraints"

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I. Derivations of (4a)

Substitute (1) into (2) and rearrange the resultant to yield

$$\widehat{T}_{i,j+1}^{t} = \Big(1 - \frac{h_{c,i} A_{s,i} \Delta h}{m_i C_{p,i}}\Big) T_{i,j}^{t} + \frac{\Delta h}{m_i C_{p,i}} (\epsilon H_{e,i}^{t+\Delta t} + h_{c,i} A_{s,i} T_{\text{amb}}), \quad \text{(I-1)}$$

where the variation of time-varying $H_{e,i}^{t}$ between time t and $t+\Delta t$ can be approximated by $\epsilon H_{e,i}^{t+\Delta t}$ with the constant coefficient ϵ set to 0.78. Further, substitute (1) into the corrector step in (3) to get

$$T_{i,j+1}^{t} = T_{i,j}^{t} + \frac{\Delta h}{2 m_i C_{p,i}} (2\epsilon H_{e,i}^{t+\Delta t} + h_{c,i} A_{s,i} (2T_{\text{amb}} - T_{i,j}^{t} - \widehat{T}_{i,j+1}^{t})), \quad \text{(I-2)}$$

Next, substitute (I-1) into (I-2) to get

$$T_{i,j+1}^{t} = \omega_1 T_{i,j}^{t} + \omega_2 H_{e,i}^{t+\Delta t} + \omega_3, \quad j = 0, 1, 2, ..., n-1, \quad \text{(I-3)}$$

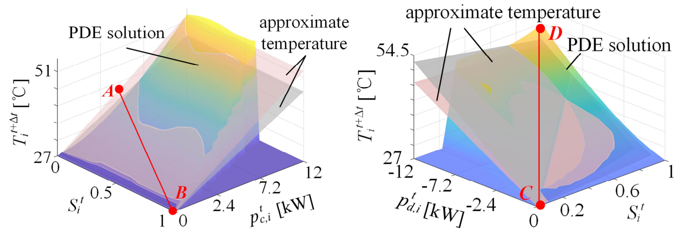
with ω_1, ω_2, and ω_3 as expressed in (4b)–(4d), respectively. The recurrence relation in (I-3) can be evaluated in closed form as

$$T_{i}^{t+\Delta t} = \omega_1^{n} T_{i}^{t} + \frac{1 - \omega_1^{n-1}}{1 - \omega_1} (\omega_2 H_{e,i}^{t+\Delta t} + \omega_3), \quad \text{(I-4)}$$

with boundary conditions $T_i^t = T_{i,0}^t$ and $T_i^{t+\Delta t} = T_{i,n}^t$.

II. Approximation Errors in Thermal Constraints

In the proposed dispatch model, errors in cell temperature variations arise from the approximation of $H_{e,i}^{t}$ in (4a). In Figs. 1(a) and 1(b), using the same parameter settings as those to produce Fig. 2, we plot variations in cell temperature $T_i^{t+\Delta t}$ with respect to S_i^t and $p_{c,i}^t$ or $p_{d,i}^t$ yielded by the PDE solver compare them to the proposed discretized temperature model in (4a). In Fig. 1(a) (Fig. 1(b)), the approximate hyperplane in grey colour corresponding to β_i (γ_i) is active below the cut line AB (CD), otherwise the approximate hyperplane in pink colour corresponding to ϕ_i (α_i) is active. Via visual examination of Fig. 1, we observe that the cell temperature approximated by (4a) is very close to the surface yielded by the PDE solver. In fact, the maximum error between the two in charging (discharging) mode is 1.2°C (1.8°C), acceptable for the optimal S2G dispatch, as demonstrated by numerical case studies.



(a) (b)

Fig. 1. Approximating battery cell temperature T_i^{t+1} during (a) charging, (b) discharging.

III. Battery Cell Parameter Values

In this study, a BESS in ES_i consists of $N_i = 41$ LiFePO_4 battery modules arranged in series configuration, each with a capacity of 306.4 kWh. We select each battery cell capacity $I_{b,i} = 314$ Ah, nominal voltage $v_{\text{flat},i} = 3.4$ V with the operating voltage ranging from 2.5 V to 3.65 V, and current rate $C_{r,i}^t \in [0, \overline{C}_{r,i}]$ and $\overline{C}_{r,i} = 1.5$. For a battery cell, $\overline{p}_i \approx 3.65 \cdot 0.314 \cdot 1.5 \approx 1.7$ kW and $\overline{v} = 3.65$ V for a battery cell. Regarding thermal parameters, each battery cell has mass $m_i = 5.529$ kg and heat capacity $C_{p,i} = 1417.2$ J/(kg · K). The forced convection air cooling with four fans is available from four side openings of a battery cell with surface area $A_{s,i} = 0.1271\,\text{m}^2$, and the heat transfer coefficient is $h_{c,i} = 5.0\,\text{W}/(\text{m}^2 \cdot \text{K})$. Other parameters for electrochemical kinetics are given in Table I.

TABLE I
Parameters for electrochemical kinetics

Parameter	Negative electrode	Separator	Positive electrode
electrode plate area (m^2)	0.163	0.163	0.163
electrode thickness (m)	$78 \cdot 10^{-6}$	$20 \cdot 10^{-6}$	$45 \cdot 10^{-6}$
Li^+ diffusion coefficient (m^2/s)	$3.9 \cdot 10^{-5}$	-	$1.8 \cdot 10^{-8}$
active electrode volume fraction (%)	0.6	-	0.6
electrolyte phase volume fraction (%)	0.3	-	0.3
max solid phase concentration (mol/m^3)	31507	-	49000
particle radius (m)	$6 \cdot 10^{-6}$	-	$5 \cdot 10^{-6}$
reaction rate efficiency (A/m^2)	$9.77 \cdot 10^{-2}$	-	$1.19 \cdot 10^{-2}$
exchange current density of side reaction (A/m^2)	10	-	10
initial electrolyte concentration (mol/m^3)	$1.25 \cdot 10^4$	$1.25 \cdot 10^4$	$1.25 \cdot 10^4$
Binder volume fraction(%)	0.1	-	0.1
Separator volume fraction(%)	-	0.4	-