

Supplementary Material for “An ADMM-based Battery Dispatch Optimization Approach for Electric Vehicle Aggregators with Input-Discriminative Local Differential Privacy Budgets”

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I. REFERENCE POWER SIGNAL VECTOR

The given reference power signal vector \mathbf{p}_{ref}^t over a 24-hour scheduling horizon is provided in Fig. 1.

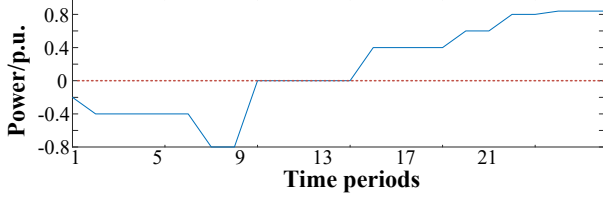


Fig. 1. Given reference power signal vector \mathbf{p}_{ref}^t over a 24-hour scheduling horizon.

II. PROOF OF THEOREM 1

There exists an optimal BDO solution \mathbf{x}_i^{t*} with the minimum objective function value \mathbf{F}^* for the C-BDO model (1a)-(1d). We incorporate $\mathbf{p}_{sig,i}^{t*} \in \mathbb{R}$ such that $\sum_{i=1}^{N_E} \mathbf{p}_{sig,i}^{t*} = \mathbf{p}_{ref}^t$. Reformulate $\|\cdot\|_2$ in the objective function (1a) by

$$\left\| \sum_{i=1}^{N_E} \mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{ref}^t \right\|_2 = \left\| \sum_{i=1}^{N_E} (\mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{sig,i}^{t*}) \right\|_2 \quad (\text{II-1})$$

Then, we achieve optimality by

$$\sum_{t=1}^{N_T} \left\| \sum_{i=1}^{N_E} (\mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{sig,i}^{t*}) \right\|_2 = \sum_{t=1}^{N_T} \left\| \sum_{i=1}^{N_E} \mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{ref}^t \right\|_2 = \mathbf{F}^* \quad (\text{II-2})$$

This proves that $(\mathbf{x}_i^{t*}, \mathbf{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal BDO solution with the minimum objective function value \mathbf{F}^* .

Due to the triangle inequality of norms, i.e., $|a + b| \leq |a| + |b|$ for any real-valued numbers a and b , we obtain that $\mathbf{F}^* \leq \mathbf{F} \leq \mathbf{G}$ naturally holds. This suggests that \mathbf{F}^* can be the lower bound of \mathbf{G} over \mathcal{X} . In other words, $(\mathbf{x}_i^{t*}, \mathbf{p}_{sig,i}^{t*}) \in \mathcal{X}$ can be the optimal solution iff the minimum function value $\mathbf{G}^* = \mathbf{F}^*$ is achieved, which demonstrates $(\mathbf{x}_i^{t*}, \mathbf{p}_{sig,i}^{t*})$ can also be the optimal BDO solution for minimizing the objective function \mathbf{G} over the feasibility space \mathcal{X} . This proves Theorem 1. ■

III. PROOF OF THEOREM 2

The objective is to prove that the optimal charge-discharge solution $\mathbf{x}_i^{t\dagger} = \mathbf{x}_i^{t*}$ holds for EV_i. It is clear that the following (III-3) stands.

$$\mathbf{s}_i^{t*} = \mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{sig,i}^{t*} + \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t - \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t. \quad (\text{III-3})$$

We denote $\tilde{\mathbf{p}}_{sig,i}^{t\dagger} = \mathbf{p}_{sig,i}^{t*} + \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t$. Then, we have

$$\mathbf{s}_i^{t*} = \mathbf{c}^T \mathbf{x}_i^{t*} - \tilde{\mathbf{p}}_{sig,i}^{t\dagger} + \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t \quad (\text{III-4})$$

As $\mathbf{s}_i^{t\dagger} \geq \mathbf{s}_i^{t*}$ and $\mathbf{y}_i^{t\dagger} = \mathbf{s}_i^{t\dagger}$ achieved at optimality, minimizing the objective function \mathbf{g}_i ensures $\mathbf{s}_i^{t\dagger} = \mathbf{s}_i^{t*}$, where other two terms in \mathbf{g}_i can be held at convergence, i.e.,

$$\boldsymbol{\mu}^t (\tilde{\mathbf{p}}_{sig,i}^{t\dagger} - \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t) = \boldsymbol{\mu}^t \mathbf{p}_{sig,i}^{t*} \quad (\text{III-5})$$

and

$$\frac{\phi}{2} \left\| \sum_{j=1, j \neq i}^{N_E} \tilde{\mathbf{p}}_{sig,j}^{t\dagger} - \tilde{\mathbf{p}}_{sig,i}^{t\dagger} - \mathbf{p}_{ref}^t \right\|_2^2 = \frac{\phi}{2} \left\| \sum_{j=1, j \neq i}^{N_E} \mathbf{p}_{sig,j}^{t*} + \mathbf{p}_{sig,i}^{t*} - \mathbf{p}_{ref}^t \right\|_2^2 \approx 0 \quad (\text{III-6})$$

Thus, we rewrite $\mathbf{s}_i^{t\dagger} = \mathbf{c}^T \mathbf{x}_i^{t*} - \tilde{\mathbf{p}}_{sig,i}^{t\dagger} + \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t$. This suggests that $\mathbf{x}_i^{t\dagger} = \mathbf{x}_i^{t*}$ holds with obfuscated $\tilde{\mathbf{p}}_{sig,i}^{t\dagger} = \mathbf{p}_{sig,i}^{t*} + \alpha_i^{t\dagger} \boldsymbol{\xi}_i^t$ for the privacy-preserving BDO model. ■

IV. PROOF OF THEOREM 3

We prove that \mathcal{M} is $r_{\varepsilon,i}^t$ -ID-LDP with respect to the local dataset of sub-problem i during all iterations. Let the query output answer for the sub-problem i be $\mathcal{O}_i^t = \tilde{\mathbf{p}}_{sig,i}^t$, and we alternatively rewrite (IV-7) in the definition of $r_{\varepsilon,i}^t$ -LDP for the sub-problem i :

$$\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \mathbf{x}_i^t] \leq \mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \mathbf{x}_i^{t'}] e^{r_{\varepsilon,i}^t}, \quad (\text{IV-7})$$

for two $r_{\varepsilon,i}^t$ -indistinguishable charge-discharge solutions \mathbf{x}_i^t and $\mathbf{x}_i^{t'}$ in time period t . For convenience, the query output \mathcal{O}_i^t of EV_i for $\forall t \in \mathcal{T}$ with a random noise $\boldsymbol{\xi}_i^t$ can be written as $\mathcal{O}_i^t = \tilde{\mathbf{p}}_{sig,i}^t = \mathbf{p}_{sig,i}^t + \alpha_i^t \boldsymbol{\xi}_i^t$ where $\forall t \in \mathcal{T}$.

Therefore, the ratio of probabilities on two ε_i -indistinguishable charge-discharge solutions \mathbf{x}_i^t and $\mathbf{x}_i^{t'}$ can be bounded by

$$\begin{aligned} \frac{\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \mathbf{x}_i^t]}{\mathbb{P}[\mathcal{M}_i = \mathcal{O}_i^t | \mathbf{x}_i^{t'}]} &= \frac{\mathbb{P}[\mathbf{p}_{sig,i}^t + \alpha_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t | \mathbf{x}_i^t]}{\mathbb{P}[\mathbf{p}_{sig,i}^{t'} + \alpha_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t | \mathbf{x}_i^{t'}]} \\ &= \frac{\mathbb{P}[\alpha_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \mathbf{p}_{sig,i}^t | \mathbf{x}_i^t]}{\mathbb{P}[\alpha_i^t \boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \mathbf{p}_{sig,i}^{t'} | \mathbf{x}_i^{t'}]} \stackrel{(i)}{=} \frac{\alpha_i^t \exp\left\{\frac{r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \mathbf{p}_{sig,i}^t\|_1}{\Delta_{\rho,i}}\right\}}{\alpha_i^t \exp\left\{\frac{r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \mathbf{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right\}} \\ &= \exp\left(\frac{r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \mathbf{p}_{sig,i}^t\|_1 - r_{\varepsilon,i}^t \|\mathcal{O}_i^t - \mathbf{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \\ &\stackrel{(ii)}{\leq} \exp\left(\frac{r_{\varepsilon,i}^t \|\mathbf{p}_{sig,i}^t - \mathbf{p}_{sig,i}^{t'}\|_1}{\Delta_{\rho,i}}\right) \stackrel{(iii)}{\leq} \exp\left(\frac{r_{\varepsilon,i}^t \Delta_{\rho,i}}{\Delta_{\rho,i}}\right) = e^{r_{\varepsilon,i}^t}, \end{aligned} \quad (\text{IV-8})$$

where (i) comes from the definition of the probability density function of the Gaussian distribution. In (ii) step, it is followed by the inequality of norms, i.e., $|a| - |b| \leq |a - b|$ for any real-valued numbers a and b . For the (iii) step, $\|\mathbf{p}_{sig,i}^t - \mathbf{p}_{sig,i}^{t'}\|_1$ denotes the ℓ_1 -sensitivity on ρ -indistinguishable output datasets $\mathbf{p}_{sig,i}^t$ and $\mathbf{p}_{sig,i}^{t'}$ subject to $\|\mathbf{p}_{sig,i}^t - \mathbf{p}_{sig,i}^{t'}\|_1 \leq \Delta_{\rho,i}$.

Accordingly, it is clear that (IV-7) holds based on (IV-8), which proves this Theorem. ■