Supplementary Material for "Optimal Electric Ship-to-Grid Dispatch Considering Electrochemical-Thermal-Coupled Battery Cell Thermal and Voltage Constraints"

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I. Derivations of (4a)

According to $\widehat{T}^t_{i,j+1} = T^t_{i,j} + \Delta h \frac{\mathrm{d}T^t_{i,j}}{\mathrm{d}t}$, we obtain $\widehat{T}^t_{i,j+1}$ by rearranging,

$$\widehat{T}^t_{i,j+1} = (1 - \frac{h_{c,i}A_{s,i}\Delta h}{m_i C_{p,i}})T^t_{i,j} + \frac{\Delta h(\epsilon H^{t+\Delta t}_{e,i} + h_{c,i}A_{s,i}T_{\mathrm{amb}})}{m_i C_{p,i}}, \tag{I-1}$$

where the variation of time-varying $H^t_{e,i}$ between $[t, t+\Delta t]$ in (A-1) can be replaced by $\epsilon H^{t+\Delta t}_{e,i}$ and the constant coefficient ϵ is set to 0.78.

Furthermore, we calculate next corrector step by,

$$T^t_{i,j+1} = T^t_{i,j} + \frac{\Delta h}{2}\Big(\frac{\mathrm{d}T^t_{i,j}}{\mathrm{d}t}\Big|_{T^t_{i,j}=T^t_{i,j}} + \frac{\mathrm{d}T^t_{i,j}}{\mathrm{d}t}\Big|_{T^t_{i,j}=\widehat{T}^t_{i,j+1}} \Big) = T^t_{i,j} + \frac{\Delta h}{2m_i C_{p,i}}[2\epsilon H^{t+\Delta t}_{e,i} + h_{c,i}A_{s,i}(2T_{\mathrm{amb}} - T^t_{i,j} - \widehat{T}^t_{i,j+1})], \tag{I-2}$$

Substituting $\widehat{T}^t_{i,j+1}$ into (I-2), we obtain

$$T^t_{i,j+1} = \omega_1 T^t_{i,j} + \omega_2 H^{t+\Delta t}_{e,i} + \omega_3, \quad j = 0, 1, 2, ..., n-1, \tag{I-3}$$

$$\begin{cases} \omega_1 = 1 - \frac{h_{c,i}A_{s,i}\Delta h}{2m_i C_{p,i}}(2 - \frac{h_{c,i}A_{s,i}\Delta h}{m_i C_{p,i}}), \\ \omega_2 = \frac{\epsilon \Delta h}{m_i C_{p,i}} - \frac{\epsilon h_{c,i}A_{s,i}\Delta h^2}{2m_i^2 C_{p,i}^2}, \\ \omega_3 = \frac{T_{\mathrm{amb}} h_{c,i}A_{s,i}\Delta h}{m_i C_{p,i}} - \frac{T_{\mathrm{amb}} h_{c,i}^2 A_{s,i}^2 \Delta h^2}{2m_i^2 C_{p,i}^2}. \end{cases} \tag{I-4}$$

Thus, it is evident that (I-3) represents a recurrence sequence, and we rearrange the recurrence sequence by

$$T^{t+\Delta t}_i = \omega_1^n T^t_i + \frac{1 - \omega_1^{n-1}}{1 - \omega_1}(\omega_2 H^{t+\Delta t}_{e,i} + \omega_3), \tag{I-5}$$

where $T^{t+\Delta t}_i = T^t_{i,n}$ and $T^t_i = T^t_{i,0}$ at time intervals $t+1$ and t, respectively.

II. Error Discussion for Thermal Constraints

The cell temperature error comes from the approximation of of $H^t_{e,i}$ in (2a). We plot the cell temperatures $T^{t+\Delta t}_i$ with respect to S^t_i and $p^t_{c,i}$ or $p^t_{d,i}$ from the PDE solver and our proposed discretized temperature equation (2a), where the cell temperature in the end of time period t is $T^t_i = 27$°C over $\Delta t = 15$min in Fig.1(a) and (b). In Fig.1(a), the approximate cell temperature $T^{t+\Delta t}_i$ is active on the grey plane when $(S^t_i, p^t_{c,i})$ is below the cut lines AB and CD, otherwise it is on the light red plane. For Fig.1(b), the approximate cell temperature $T^{t+\Delta t}_i$ is the intersection of two planes. It is clear that the approximate cell temperatures $T^{t+\Delta t}_i$ by (2a) is very close to the cell temperature surface by the PDE solver in two sub-figures. The maximum errors between cell temperatures by the PDE solver and the approximate temperatures in charging and discharging modes are less than 1.2°C and 1.8°C, which can be acceptable for the S2G dispatch problem.

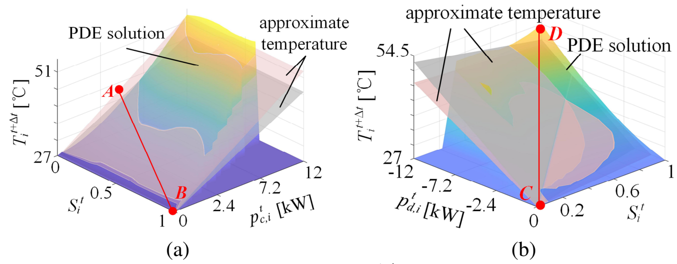

Fig. 1. Estimation of cell temperature T^{t+1}_i: (a) charge; (b) discharge.

III. Parameters for Heat Estimation and Electrochemical Kinetics

In this study, the BESS in the i-th ES consists of $N_i = 41$ $\mathrm{LiFePO_4}$ battery modules arranged in series configuration, each with a capacity of 306.4 kWh. We select each battery cell capacity $I_{b,i} = 314$ Ah, nominal voltage $v_{\mathrm{flat},i} = 3.4$ V with the operating voltage ranging from 2.5 V to 3.65 V, and current rate $C^t_{r,i} \in [0, \overline{C}_{r,i}]$ and $\overline{C}_{r,i} = 1.5$. For a battery cell, $\overline{p}_i \approx 3.65 \cdot 0.314 \cdot 1.5 \approx 1.7$ kW and $\overline{v} = 3.65$ V for a battery cell. Regarding thermal parameters, each battery cell has mass $m_i = 2162.2$ kg and heat capacity $C_{p,i} = 1417.2\,\mathrm{J/(kg \cdot K)}$. The forced convection air cooling with four fans is available from four side openings of a battery cell with surface area $A_{\mathrm{s},i} = 0.1271\,\mathrm{m}^2$, and the heat transfer coefficient is $h_{c,i} = 5.0\,\mathrm{W/(m^2 \cdot K)}$. Other parameters for electrochemical kinetics are given in Tab.I (See next page).

TABLE I
PARAMETERS FOR ELECTROCHEMICAL KINETICS

Parameter	Negative electrode	Separator	Positive electrode
electrode plate area (m^2)	0.163	0.163	0.163
electrode thickness (m)	$78 \cdot 10^{-6}$	$20 \cdot 10^{-6}$	$45 \cdot 10^{-6}$
Li^+ diffusion coefficient (m^2/s)	$3.9 \cdot 10^{-5}$	-	$1.8 \cdot 10^{-8}$
active electrode volume fraction (%)	0.6	-	0.6
electrolyte phase volume fraction (%)	0.3	-	0.3
max solid phase concentration (mol/m^3)	31507	-	49000
particle radius (m)	$6 \cdot 10^{-6}$	-	$5 \cdot 10^{-6}$
reaction rate efficiency (A/m^2)	$9.77 \cdot 10^{-2}$	-	$1.19 \cdot 10^{-2}$
exchange current density of side reaction (A/m^2)	10	-	10
initial electrolyte concentration (mol/m^3)	$1.25 \cdot 10^4$	$1.25 \cdot 10^4$	$1.25 \cdot 10^4$
Binder volume fraction(%)	0.1	-	0.1
Separator volume fraction(%)	-	0.4	-