Supplementary Material for "Interconnected Active Distribution Networks: A Differentially Private Reconfiguration Approach "

Chao Lei, *Student Member, IEEE* and Siqi Bu, *Senoir Member, IEEE*

I. MIQP-BASED DNR MODEL

The interconnected active distribution networks (ADNs) are considered as a connected undirected tree $\mathcal{G}=(\mathcal{N},\mathcal{E})$, where $\mathcal{N}$ is the set of nodes and $\mathcal{E}$ is the set of branches. Suppose that an arbitrary branch $l:=(m,n),\forall l\in\mathcal{E}$ is between nodes (m,n) and S denotes the set of root nodes regarded as multiple substations [1]. The simplified distribution network reconfiguration (DNR) model is based on the linearized DistFlow equations, which is cast as a mixed-integer quadratic programming (MIQP) problem [2]. The set of optimization variables involves a set of operational variables $\boldsymbol{x}^l:=[\boldsymbol{P}^l,\boldsymbol{Q}^l,\boldsymbol{Q}^{cr}]^T$, switch status indicator variables $\boldsymbol{u}^l\in\mathbb{Z}^{|\mathcal{E}|}$ and continuous parent-child relationship variables $\boldsymbol{\beta}^l\in\mathbb{R}^{2|\mathcal{E}|}$. In this vein, $\boldsymbol{P}^l\in\mathbb{R}^{|\mathcal{E}|}$ and $\boldsymbol{Q}^l\in\mathbb{R}^{|\mathcal{E}|}$ refer to the vectors of sending-end active and reactive power flows. $\boldsymbol{Q}^{cr}\in\mathbb{R}^{|n_{cr}|}$ is the vector of nodal reactive power compensation and n_{cr} is the number of capacitors. $\boldsymbol{\beta}^l$ are constructed with spanning tree constraints with multiple sources. For convenience, we express the MIQP-based DNR model in following form:

$$\min_{\boldsymbol{x}^l,\boldsymbol{\beta}^l\in\mathbb{R},\boldsymbol{u}^l\in\mathbb{Z}} f=\begin{bmatrix}\boldsymbol{P}^l\\\boldsymbol{Q}^l\\\boldsymbol{Q}^{cr}\end{bmatrix}^T\begin{bmatrix}\boldsymbol{D}_r & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}}\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{D}_r & \mathbf{0}_{|\mathcal{E}|\times n_{cr}}\\\mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times n_{cr}}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}^l\\\boldsymbol{Q}^l\\\boldsymbol{Q}^{cr}\end{bmatrix} \tag{A-1a}$$

$$\text{s.t.}\begin{bmatrix}\boldsymbol{A}_{\mathcal{G}}^T & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}}\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{A}_{\mathcal{G}}^T & \boldsymbol{A}_{cr}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}^l\\\boldsymbol{Q}^l\\\boldsymbol{Q}^{cr}\end{bmatrix}=\begin{bmatrix}-\boldsymbol{P}^g+\boldsymbol{P}^d\\-\boldsymbol{Q}^g+\boldsymbol{Q}^d\end{bmatrix} \tag{A-1b}$$

$$\begin{bmatrix}2\boldsymbol{D}_r & 2\boldsymbol{D}_x & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & M\\-2\boldsymbol{D}_r & -2\boldsymbol{D}_x & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -M\\\mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times|\mathcal{E}|} & diag(\mathbb{1}_c) & \mathbf{0}_{n_{cr}\times|\mathcal{E}|}\\\mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times|\mathcal{E}|} & -diag(\mathbb{1}_c) & \mathbf{0}_{n_{cr}\times|\mathcal{E}|}\end{bmatrix}\begin{bmatrix}\boldsymbol{P}^l\\\boldsymbol{Q}^l\\\boldsymbol{Q}^{cr}\\\boldsymbol{u}^l\end{bmatrix}\leqslant\begin{bmatrix}\Delta\overline{\boldsymbol{v}}+M\\-\Delta\underline{\boldsymbol{v}}+M\\\overline{\boldsymbol{Q}}_{\boldsymbol{cr}}\\-\underline{\boldsymbol{Q}}_{\boldsymbol{cr}}\end{bmatrix} \tag{A-1c}$$

$$\begin{bmatrix}diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\\-diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & -diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\end{bmatrix}\begin{bmatrix}\boldsymbol{P}^l\\\boldsymbol{Q}^l\\\boldsymbol{Q}^{cr}\\\boldsymbol{u}^l\end{bmatrix}\leqslant\mathbf{0} \tag{A-1d}$$

$$\beta_{mn}^l+\beta_{nm}^l=u_{mn}^l,\quad \beta_{mn}^l=0,\ \text{if}\ m=S \tag{A-1e}$$

$$\sum_{n:(m,n)\in\mathcal{E}}\beta_{mn}^l=1,\ \forall m\in\mathcal{N}\backslash S \tag{A-1f}$$

$$0\leqslant\beta_{mn}^l\leqslant1,\ \forall l\in\mathcal{E} \tag{A-1g}$$

where (A-1a) states the quadratic active power loss of DNs under the assumption of flat voltage profiles for all nodes. $\mathbb{1}_c$ and $\mathbb{1}_N$ are the $n_{cr}\times1$ and $|\mathcal{E}|\times1$ vectors with all ones, respectively. M refers to the big positive number and Γ denotes the branch capacity. $\boldsymbol{P}^g,\boldsymbol{Q}^g$ and $\boldsymbol{P}^d,\boldsymbol{Q}^d$ indicate the vectors of given nodal active and reactive power injections and active and reactive loads at nodes. $\boldsymbol{A}_{\mathcal{G}}$ is a $|\mathcal{E}|$ by $|\mathcal{N}|$ branch-node incidence matrix and $\boldsymbol{A}_{cr}$ is a diagonal matrix whose i-th diagonal element is equal to 1 if node i has the reactive compensation capacitors; otherwise this entry is zero. $\boldsymbol{D}_r$ and $\boldsymbol{D}_x$ indicate the diagonal matrices whose diagonal elements are the resistance and reactance vectors, respectively. $\underline{\boldsymbol{Q}}_{\boldsymbol{cr}}$, $\overline{\boldsymbol{Q}}_{\boldsymbol{cr}}$, $\Delta\underline{\boldsymbol{v}}$ and $\Delta\overline{\boldsymbol{v}}$ represent the boundaries of $\boldsymbol{Q}^{cr}$ and squared voltage profile deviation, respectively. (A-1e)-(A-1g) represents the spanning tree constraints with multiple sources for interconnected ADNs [2].

Therefore, we can summarize $\boldsymbol{c}$, $\boldsymbol{A}$, $\boldsymbol{G}_v$, $\boldsymbol{G}_{cr}$, $\boldsymbol{d}$, $\boldsymbol{b}_v$ and $\boldsymbol{b}_{cr}$ in (A-1b) as

$$\boldsymbol{c}=\begin{bmatrix}\boldsymbol{D}_r & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}}\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{D}_r & \mathbf{0}_{|\mathcal{E}|\times n_{cr}}\\\mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times n_{cr}}\end{bmatrix} \tag{A-2a}$$

$$\boldsymbol{A}=\begin{bmatrix}\boldsymbol{A}_{\mathcal{G}}^T & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}}\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \boldsymbol{A}_{\mathcal{G}}^T & \boldsymbol{A}_{cr}\end{bmatrix},\ \boldsymbol{d}=\begin{bmatrix}-\boldsymbol{P}^g+\boldsymbol{P}^d\\-\boldsymbol{Q}^g+\boldsymbol{Q}^d\end{bmatrix} \tag{A-2b}$$

$$\boldsymbol{G}_v=\begin{bmatrix}2\boldsymbol{D}_r & 2\boldsymbol{D}_x & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & M\\-2\boldsymbol{D}_r & -2\boldsymbol{D}_x & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -M\end{bmatrix} \tag{A-2c}$$

$$\boldsymbol{G}_u=\begin{bmatrix}diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\\-diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\\\mathbf{0}_{|\mathcal{E}|\times|\mathcal{E}|} & -diag(\mathbb{1}_N) & \mathbf{0}_{|\mathcal{E}|\times n_{cr}} & -\Gamma\end{bmatrix} \tag{A-2d}$$

$$\boldsymbol{G}_{cr}=\begin{bmatrix}\mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times|\mathcal{E}|} & diag(\mathbb{1}_c) & \mathbf{0}_{n_{cr}\times|\mathcal{E}|}\\\mathbf{0}_{n_{cr}\times|\mathcal{E}|} & \mathbf{0}_{n_{cr}\times|\mathcal{E}|} & -diag(\mathbb{1}_c) & \mathbf{0}_{n_{cr}\times|\mathcal{E}|}\end{bmatrix} \tag{A-2e}$$

$$\boldsymbol{b}_v=\begin{bmatrix}\Delta\overline{\boldsymbol{v}}+M\\-\Delta\underline{\boldsymbol{v}}+M\end{bmatrix},\ \boldsymbol{b}_{cr}=\begin{bmatrix}\overline{\boldsymbol{Q}}_{\boldsymbol{cr}}\\-\underline{\boldsymbol{Q}}_{\boldsymbol{cr}}\end{bmatrix} \tag{A-2f}$$

$$\boldsymbol{b}_u=\mathbf{0}_{4|\mathcal{E}|\times1} \tag{A-2g}$$

where $\boldsymbol{K}$ and $\boldsymbol{h}$ can be rewritten from (A-1e)-(A-1g).

II. NON-PRIVATE DNR MODEL

To avoid heavy notions, we express the general mathematical formulation of a non-private DNR model in the MIQP form with respect to operational variables $\boldsymbol{x}^l$ including active/reactive power flow variables and reactive power compensation variables, continuous parent-child relationship variables $\boldsymbol{\beta}^l$ and switch status indicator variables $\boldsymbol{u}^l$. This is the compact form of MIQP-based DNR model in section I.

$$\textbf{Non-Private DNR}:\ \min_{\boldsymbol{x}^l,\boldsymbol{\beta}^l\in\mathbb{R},\boldsymbol{u}^l\in\mathbb{Z}} f=(\boldsymbol{x}^l)^T\boldsymbol{c}\boldsymbol{x}^l \tag{A-3a}$$

$$\text{s.t.}\ \mathcal{X}:=\left\{(\boldsymbol{x}^l,\boldsymbol{\beta}^l,\boldsymbol{u}^l)\ \middle|\ \begin{array}{l}\boldsymbol{A}\boldsymbol{x}^l=\boldsymbol{d},\ \begin{bmatrix}\boldsymbol{G}_v\\\boldsymbol{G}_u\end{bmatrix}\begin{bmatrix}\boldsymbol{x}^l\\\boldsymbol{u}^l\end{bmatrix}\leqslant\begin{bmatrix}\boldsymbol{b}_v\\\boldsymbol{b}_u\end{bmatrix},\\\boldsymbol{G}_{cr}\boldsymbol{x}^l\leqslant\boldsymbol{b}_{cr},\ \boldsymbol{K}\begin{bmatrix}\boldsymbol{\beta}^l\\\boldsymbol{u}^l\end{bmatrix}=\boldsymbol{h}\end{array}\right\} \tag{A-3b}$$

where $\mathcal{X}$ refers to the non-empty feasibility space and f_1 denotes the approximate system power loss with a fixed diagonal matrix $\boldsymbol{c}$. The inequality in (A-3b) represents the voltage security constraints, physical ranges of reactive power compensation capacitors and topology-linked branch capacity constraints, which are marked by the subscripts v, cr and u for $\boldsymbol{G}$ and $\boldsymbol{b}$. The first and second equality denotes the system-wide load balance of ADNs and radiality constraints.

REFERENCES

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[2] J. A. Taylor and F. S. Hover, "Convex models of distribution system reconfiguration," *IEEE Trans. on Power Syst.*, vol. 27, no. 3, pp. 1407–1413, 2012.