Supplementary Material for "Battery Dispatch Optimization for Electric Vehicle Aggregators:A Decomposition-Coordination-based Least Squares Approach with Disjunctive Cuts "

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I. Proof of Tightened Feasibility Space

In the B&B procedure by DCHR, the upper and lower bounds of $\boldsymbol{e}_i^{t+1}$ are denoted as $\mathcal{L}_1^{upper}=\boldsymbol{e}_{0,i}+\eta_{c,i}\overline{P}(t-1+\lambda_{c,i}^t)\Delta\boldsymbol{T}$ and $\mathcal{L}_1^{lower}=\boldsymbol{e}_{0,i}-\frac{1}{\eta_{d,i}}\overline{P}(t-1+\lambda_{d,i}^t)\Delta\boldsymbol{T}$, respectively. However, according to the BDO model (1a)-(1d), we express the upper and lower bounds of $\boldsymbol{e}_i^{t+1}$ by $\mathcal{L}_2^{upper}=\boldsymbol{e}_{0,i}+\eta_{c,i}\overline{P}(t)\Delta\boldsymbol{T}$ and $\mathcal{L}_2^{lower}=\boldsymbol{e}_{0,i}-\frac{1}{\eta_{d,i}}\overline{P}(t)\Delta\boldsymbol{T}$, respectively. Hence, we achieve the difference in algebra $\mathcal{L}_1^{upper}-\mathcal{L}_2^{upperr}=(\boldsymbol{\lambda}_{c,i}^t-1)\eta_{c,i}\overline{P}\Delta\boldsymbol{T}$. Since $\lambda_{c,i}^t\in\{0,1\}$, this clearly renders $\mathcal{L}_1^{upper}\leqslant\mathcal{L}_2^{upper}$. Similarly, we can also achieve the lower bound satisfying $\mathcal{L}_1^{lower}\geqslant\mathcal{L}_2^{lower}$ due to $\lambda_{d,i}^t\in\{0,1\}$. We have proved that DCHR has tighter relaxation lower and upper bounds than those by the BDO model (1a)-(1d) for $\boldsymbol{\Omega}_{op,i}$. ∎

II. Proof of Theorem

Initially, since $\sum\limits_{i=1}^{N}\boldsymbol{p}_{sig,i}^{t*}=\boldsymbol{p}_{ref}^t$, we can rewrite $\boldsymbol{F}^*$ as

$$\boldsymbol{F}^*=\sum_{t=1}^{T}\left\|\sum_{i=1}^{N}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}_{sig,i}^{t*})\right\|_2=\sum_{t=1}^{T}\left\|\sum_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2 \quad \text{(A-1)}$$

where the optimal auxiliary variable $\boldsymbol{\alpha}_i^*=\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}_{sig,i}^{t*}$.

Thus, our goal is to prove that the following two objective functions can get converged over $(\boldsymbol{x}_i^t,\boldsymbol{z}_i^t)\in\mathcal{X}_i$, $\forall i\in\mathcal{E}$, with the same optimal solution $(\boldsymbol{x}_i^{t*},\boldsymbol{z}_i^{t*})$ for the MIQP-based model (1a)-(1d), namely

$$\underset{(\boldsymbol{x}_i^t,\boldsymbol{z}_i^t)\in\mathcal{X}_i}{\operatorname{argmin}}\sum_{t=1}^{T}\left\|\sum_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2 \Longleftrightarrow \underset{(\boldsymbol{x}_i^t,\boldsymbol{z}_i^t)\in\mathcal{X}_i}{\operatorname{argmin}}\sum_{t=1}^{T}\sum_{i=1}^{N}\|\boldsymbol{\alpha}_i^*\|_2 \quad \text{(A-2)}$$

At time t, when $\left\|\sum\limits_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2$ can be minimized as a non-zero number, we suppose if there only $\exists\boldsymbol{\alpha}^*$ as the non-zero number for the optimal solution $\boldsymbol{c}^T\boldsymbol{x}_i^{t*}$. Then, we have

$$\left\|\sum_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2=\|\boldsymbol{\alpha}_1^*+\boldsymbol{\alpha}_2^*...+\boldsymbol{\alpha}_N^*\|_2=\|\boldsymbol{\alpha}^*+\boldsymbol{\alpha}^*+...+\boldsymbol{\alpha}^*\|_2=N\|\boldsymbol{\alpha}^*\|_2 \quad \text{(A-3)}$$

It is clear that equality (A-3) can be also equivalent to

$$N\|\boldsymbol{\alpha}^*\|_2=\|\boldsymbol{\alpha}^*\|_2+\|\boldsymbol{\alpha}^*\|_2+...+\|\boldsymbol{\alpha}^*\|_2=\sum_{i=1}^{N}\|\boldsymbol{\alpha}_i^*\|_2 \quad \text{(A-4)}$$

Besides, if $\boldsymbol{\alpha}^*$=0, then the $\left\|\sum\limits_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2$ can reach zero. This indicates that $\sum\limits_{i=1}^{N}\|\boldsymbol{\alpha}_i^*\|_2=\left\|\sum\limits_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2=0$ naturally holds. Based on the above, we can derive (A-5) for $t\in\mathcal{T}$ by

$$\sum_{t=1}^{T}\left\|\sum_{i=1}^{N}\boldsymbol{\alpha}_i^*\right\|_2=\sum_{t=1}^{T}\sum_{i=1}^{N}\|\boldsymbol{\alpha}_i^*\|_2 \quad \text{(A-5)}$$

Thus, this equality (A-5) indicates that (A-2) can stand at the same optimal solution F^* over the same feasibility space $\mathcal{X}_i$, which proves this Theorem. ∎