Supplementary Material for "Battery Dispatch Optimization for Electric Vehicle Aggregators: A Moving Least Squares Approach with Disjunctive Cuts "

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I. PROOF OF TIGHTENED FEASIBILITY SPACE

In terms of DCHR, the upper and lower bounds of e_i^t are denoted as $\mathcal{L}_1^{upper} = \lambda_{c,i}^t \overline{E} - \eta_{c,i} \boldsymbol{P}_{c,i}^t(t-1)\Delta T$ and $\mathcal{L}_1^{lower} = \lambda_{d,i}^t \underline{E} + \frac{1}{\eta_{d,i}} \boldsymbol{P}_{d,i}^t(t-1)\Delta T$, respectively. However, according to MIQP-based model (1a)-(1d), we express the upper and lower bounds of e_i^t by $\mathcal{L}_2^{upper} = \overline{E} - \eta_{c,i} \boldsymbol{P}_{c,i}^t(t-1)\Delta T$ and $\mathcal{L}_2^{lower} = \underline{E} + \frac{1}{\eta_{d,i}} \boldsymbol{P}_{d,i}^t(t-1)\Delta T$, respectively. Hence, we achieve the difference in algebra $\mathcal{L}_1^{upper} - \mathcal{L}_2^{upperr} = (\lambda_{c,i}^t - 1)\overline{E}$. Since $\lambda_{c,i}^t \in [0,1]$, this clearly renders $\mathcal{L}_1^{upper} \leqslant \mathcal{L}_2^{upper}$. Similarly, we can also achieve the lower bound subject to $\mathcal{L}_1^{lower} \geqslant \mathcal{L}_2^{lower}$. We have proved that DCHR has tighter relaxation lower and upper bounds than those by MIQP-based model (1a)-(1d) for $\Omega_{op,i}$. ∎

II. PROOF OF THEOREM

Initially, since $\sum_{i=1}^{N} \boldsymbol{p}_{sig,i}^{t*} = \boldsymbol{p}_{ref}^t$, we can rewrite $\boldsymbol{F}^*$ as

$$\boldsymbol{F}^* = \sum_{t=1}^{T} \left\| \sum_{i=1}^{N} (\boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*}) \right\|_2 = \sum_{t=1}^{T} \left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2 \quad \text{(A-1)}$$

where the optimal auxiliary variable $\boldsymbol{\alpha}_i^* = \boldsymbol{c}^T \boldsymbol{x}_i^{t*} - \boldsymbol{p}_{sig,i}^{t*}$.

Thus, our goal is to prove that the following two objective functions can get converged over $(\boldsymbol{x}_i^t, \boldsymbol{z}_i^t) \in \mathcal{X}_i, \forall i \in \mathcal{E}$, with the same optimal solution $(\boldsymbol{x}_i^{t*}, \boldsymbol{z}_i^{t*})$ for the MIQP-based model (1a)-(1d), namely

$$\underset{(\boldsymbol{x}_i^t, \boldsymbol{z}_i^t) \in \mathcal{X}_i}{\arg\min} \sum_{t=1}^{T} \left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2 \iff \underset{(\boldsymbol{x}_i^t, \boldsymbol{z}_i^t) \in \mathcal{X}_i}{\arg\min} \sum_{t=1}^{T} \sum_{i=1}^{N} \|\boldsymbol{\alpha}_i^*\|_2 \quad \text{(A-2)}$$

At time t, when $\left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2$ can be minimized as a non-zero number, we suppose if there only $\exists \boldsymbol{\alpha}^*$ as the non-zero number for the optimal solution $\boldsymbol{c}^T \boldsymbol{x}_i^{t*}$. Then, we have

$$\left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2 = \|\boldsymbol{\alpha}_1^* + \boldsymbol{\alpha}_2^* ... + \boldsymbol{\alpha}_N^*\|_2 = \|\boldsymbol{\alpha}^* + \boldsymbol{\alpha}^* + ... + \boldsymbol{\alpha}^*\|_2 = N \|\boldsymbol{\alpha}^*\|_2 \quad \text{(A-3)}$$

It is clear that equality (A-3) can be also equivalent to

$$N \|\boldsymbol{\alpha}^*\|_2 = \|\boldsymbol{\alpha}^*\|_2 + \|\boldsymbol{\alpha}^*\|_2 + ... + \|\boldsymbol{\alpha}^*\|_2 = \sum_{i=1}^{N} \|\boldsymbol{\alpha}_i^*\|_2 \quad \text{(A-4)}$$

Besides, if $\boldsymbol{\alpha}^* = 0$, then the $\left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2$ can reach zero. This indicates that $\sum_{i=1}^{N} \|\boldsymbol{\alpha}_i^*\|_2 = \left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2 = 0$ naturally holds. Based on the above, we can derive (A-5) for $t \in \mathcal{T}$ by

$$\sum_{t=1}^{T} \left\| \sum_{i=1}^{N} \boldsymbol{\alpha}_i^* \right\|_2 = \sum_{t=1}^{T} \sum_{i=1}^{N} \|\boldsymbol{\alpha}_i^*\|_2 \quad \text{(A-5)}$$

Thus, this equality (A-5) indicates that (A-2) can stand at the same optimal solution F^* over the same feasibility space $\mathcal{X}_i$, which proves this Theorem. ∎