

Supplementary Material for “Battery Dispatch Optimization for Electric Vehicle Aggregators: A Moving Least Squares Approach with Disjunctive Cuts ”

Chao Lei, *Student Member, IEEE*, Siqi Bu, *Senior Member, IEEE*, Qianggang Wang, *Senior Member, IEEE*, Zhouyang Ren, *Senior Member, IEEE*, Lalit Kumar Goel, *Fellow, IEEE* and Dipti Srinivasan, *Fellow, IEEE*

I. PROOF OF TIGHTENED FEASIBILITY SPACE

In terms of DCHR, the upper and lower bounds of e_i^t are denoted as $\mathcal{L}_1^{upper} = \lambda_{c,i}^t \bar{E} - \eta_{c,i} \mathbf{P}_{c,i}^t (t-1) \Delta T$ and $\mathcal{L}_1^{lower} = \lambda_{d,i}^t \underline{E} + \frac{1}{\eta_{d,i}} \mathbf{P}_{d,i}^t (t-1) \Delta T$, respectively. However, according to MIQP-based model (1a)-(1d), we express the upper and lower bounds of e_i^t by $\mathcal{L}_2^{upper} = \bar{E} - \eta_{c,i} \mathbf{P}_{c,i}^t (t-1) \Delta T$ and $\mathcal{L}_2^{lower} = \underline{E} + \frac{1}{\eta_{d,i}} \mathbf{P}_{d,i}^t (t-1) \Delta T$, respectively. Hence, we achieve the difference in algebra $\mathcal{L}_1^{upper} - \mathcal{L}_2^{upper} = (\lambda_{c,i}^t - 1) \bar{E}$. Since $\lambda_{c,i}^t \in [0, 1]$, this clearly renders $\mathcal{L}_1^{upper} \leq \mathcal{L}_2^{upper}$. Similarly, we can also achieve the lower bound subject to $\mathcal{L}_1^{lower} \geq \mathcal{L}_2^{lower}$. We have proved that DCHR has tighter relaxation lower and upper bounds than those by MIQP-based model (1a)-(1d) for $\Omega_{op,i}$. ■

II. PROOF OF THEOREM

Initially, since $\sum_{i=1}^N \mathbf{p}_{sig,i}^{t*} = \mathbf{p}_{ref}^t$, we can rewrite \mathbf{F}^* as

$$\mathbf{F}^* = \sum_{t=1}^T \left\| \sum_{i=1}^N (\mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{sig,i}^{t*}) \right\|_2 = \sum_{t=1}^T \left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2 \quad (\text{A-1})$$

where the optimal auxiliary variable $\boldsymbol{\alpha}_i^* = \mathbf{c}^T \mathbf{x}_i^{t*} - \mathbf{p}_{sig,i}^{t*}$.

Thus, our goal is to prove that the following two objective functions can get converged over $(\mathbf{x}_i^t, \mathbf{z}_i^t) \in \mathcal{X}_i, \forall i \in \mathcal{E}$, with the same optimal solution $(\mathbf{x}_i^{t*}, \mathbf{z}_i^{t*})$ for the MIQP-based model (1a)-(1d), namely

$$\operatorname{argmin}_{(\mathbf{x}_i^t, \mathbf{z}_i^t) \in \mathcal{X}_i} \sum_{t=1}^T \left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2 \iff \operatorname{argmin}_{(\mathbf{x}_i^t, \mathbf{z}_i^t) \in \mathcal{X}_i} \sum_{t=1}^T \sum_{i=1}^N \|\boldsymbol{\alpha}_i^*\|_2 \quad (\text{A-2})$$

At time t , when $\left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2$ can be minimized as a non-zero number, we suppose if there only $\exists \boldsymbol{\alpha}_N^*$ as the minimal non-zero number for the optimal solution $\mathbf{c}^T \mathbf{x}_i^{t*}$. Then, we have

$$\left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2 = \|\boldsymbol{\alpha}_1^* + \boldsymbol{\alpha}_2^* \dots + \boldsymbol{\alpha}_N^*\|_2 = \|\mathbf{0} + \mathbf{0} + \dots + \boldsymbol{\alpha}_N^*\|_2 = \|\boldsymbol{\alpha}_N^*\|_2 \quad (\text{A-3})$$

It is clear that equality (A-3) can be also equivalent to

$$\boldsymbol{\alpha}_N^* = \|\mathbf{0}\|_2 + \|\mathbf{0}\|_2 + \dots + \|\boldsymbol{\alpha}_N^*\|_2 = \sum_{i=1}^N \|\boldsymbol{\alpha}_i^*\|_2 \quad (\text{A-4})$$

Besides, if the minimal $\left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2$ is zero, then $\sum_{i=1}^N \|\boldsymbol{\alpha}_i^*\|_2 = \left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2 = 0$ naturally holds. Based on the above, we can derive (A-5) for $t \in \mathcal{T}$ by

$$\sum_{t=1}^T \left\| \sum_{i=1}^N \boldsymbol{\alpha}_i^* \right\|_2 = \sum_{t=1}^T \sum_{i=1}^N \|\boldsymbol{\alpha}_i^*\|_2 \quad (\text{A-5})$$

Thus, this equality (A-5) indicates that (A-2) can stand at the same optimal solution \mathbf{F}^* over the identical feasibility space \mathcal{X}_i , which proves this Theorem. ■