

Supplementary Material for “Differentially Private Distribution Network Reconfiguration”

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I. NON-PRIVATE DNR MODEL

The DNs are considered as a connected undirected tree $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where \mathcal{N} is the set of nodes and \mathcal{E} is the set of branches. Suppose the general mathematical formulation of a non-private DNR model in mixed-integer quadratic programming (MIQP) form can be expressed with respect to operational variables \mathbf{x}^l including active/reactive power flow variables and reactive power compensation variables, continuous parent-child relationship variables β^l and switch status indicator variables \mathbf{u}^l [1].

$$\text{Non-Private DNR : } \min_{\mathbf{x}^l, \beta^l \in \mathbb{R}, \mathbf{u}^l \in \mathbb{Z}} f_1 = (\mathbf{x}^l)^T \mathbf{c} \mathbf{x}^l \quad (\text{A-1a})$$

$$\text{s.t. } \mathcal{X} := \left\{ (\mathbf{x}^l, \beta^l, \mathbf{u}^l) \left| \begin{array}{l} \mathbf{A} \mathbf{x}^l = \mathbf{d}, \quad \begin{bmatrix} \mathbf{G}_v \\ \mathbf{G}_u \end{bmatrix} \begin{bmatrix} \mathbf{x}^l \\ \mathbf{u}^l \end{bmatrix} \leq \begin{bmatrix} \mathbf{b}_v \\ \mathbf{b}_u \end{bmatrix}, \\ \mathbf{G}_{cr} \mathbf{x}^l \leq \mathbf{b}_{cr}, \quad \mathbf{K} \begin{bmatrix} \beta^l \\ \mathbf{u}^l \end{bmatrix} = \mathbf{h} \end{array} \right. \right\} \quad (\text{A-1b})$$

where \mathcal{X} refers to the non-empty feasibility space and f_1 denotes the approximate system power loss with a fixed diagonal matrix \mathbf{c} . The inequality in (A-1b) represents the voltage security constraints, physical ranges of reactive power compensation capacitors and topology-linked branch capacity constraints, which are marked by the subscripts v , cr and u for \mathbf{G} and \mathbf{b} . The first and second equality denotes the system-wide load balance of DNs and radiality constraints.

II. MIQP-BASED DNR MODEL

Suppose that an arbitrary branch $l := (m, n), l \in \mathcal{E}$ is between nodes (m, n) and the root node is assumed as 0. The simplified DNR model is based on the linearized DistFlow equations, which is cast as a MIQP problem. The set of optimization variables involves a set of operational variables $\mathbf{x}^l := [\mathbf{P}^l, \mathbf{Q}^l, \mathbf{Q}^{cr}]^T$, switch status indicator variables $\mathbf{u}^l \in \mathbb{Z}^{|\mathcal{E}|}$ and continuous parent-child relationship variables $\beta^l \in \mathbb{R}^{2|\mathcal{E}|}$. In this vein, $\mathbf{P}^l \in \mathbb{R}^{|\mathcal{E}|}$ and $\mathbf{Q}^l \in \mathbb{R}^{|\mathcal{E}|}$ refer to the vectors of sending-end active and reactive power flows. $\mathbf{Q}^{cr} \in \mathbb{R}^{n_{cr}}$ is the vector of nodal reactive power compensation and n_{cr} is the number of capacitors. β^l are constructed with spanning tree constraints. For convenience, we express the MIQP-based DNR model in following form:

$$\min_{\mathbf{x}^l \in \mathbb{R}, \mathbf{u}^l \in \mathbb{Z}} \begin{bmatrix} \mathbf{P}^l \\ \mathbf{Q}^l \\ \mathbf{Q}^{cr} \end{bmatrix}^T \begin{bmatrix} \mathbf{D}_r & \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} \\ \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{D}_r & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} \\ \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times n_{cr}} \end{bmatrix} \begin{bmatrix} \mathbf{P}^l \\ \mathbf{Q}^l \\ \mathbf{Q}^{cr} \end{bmatrix} \quad (\text{A-2a})$$

$$\text{s.t. } \begin{bmatrix} \mathbf{A}_G^T & \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} \\ \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{A}_G^T & \mathbf{A}_{cr} \end{bmatrix} \begin{bmatrix} \mathbf{P}^l \\ \mathbf{Q}^l \\ \mathbf{Q}^{cr} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^g + \mathbf{P}^d \\ \mathbf{Q}^g + \mathbf{Q}^d \end{bmatrix} \quad (\text{A-2b})$$

$$\begin{bmatrix} 2\mathbf{D}_r & 2\mathbf{D}_x & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} & M \\ -2\mathbf{D}_r & -2\mathbf{D}_x & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} & -M \\ \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \text{diag}(\mathbb{1}_N) & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} \\ \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & -\text{diag}(\mathbb{1}_N) & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} \end{bmatrix} \begin{bmatrix} \mathbf{P}^l \\ \mathbf{Q}^l \\ \mathbf{Q}^{cr} \\ \mathbf{u}^l \end{bmatrix} \leq \begin{bmatrix} \Delta \bar{v} + M \\ -\Delta \underline{v} + M \\ \bar{Q}_{cr} \\ -\underline{Q}_{cr} \end{bmatrix} \quad (\text{A-2c})$$

$$-u^l \Gamma \leq \mathbf{P}^l, \mathbf{Q}^l \leq u^l \Gamma \quad (\text{A-2d})$$

$$\beta_{mn}^l + \beta_{nm}^l = u_{mn}^l, \quad \beta_{mn}^l = 0, \text{ if } m = 0 \quad (\text{A-2e})$$

$$\sum_{n:(m,n) \in \mathcal{E}} \beta_{mn}^l = 1, \quad \forall m \in \mathcal{N} \setminus 0 \quad (\text{A-2f})$$

$$0 \leq \beta_{mn}^l \leq 1, \quad \forall l \in \mathcal{E} \quad (\text{A-2g})$$

where (A-2a) states the quadratic active power loss of DNs under the assumption of flat voltage profiles for all nodes. $\mathbb{1}_N$ is a $|\mathcal{E}| \times 1$ vector with all ones. M refers to the big positive number and Γ denotes the branch capacity. $\mathbf{P}^g, \mathbf{Q}^g$ and $\mathbf{P}^d, \mathbf{Q}^d$ indicate the vectors of given nodal active and reactive power injections and active and reactive loads at nodes. \mathbf{A}_G is a $|\mathcal{E}|$ by $|\mathcal{N}|$ branch-node incidence matrix and \mathbf{A}_{cr} is a diagonal matrix whose i -th diagonal element is equal to 1 if node i has the reactive compensation capacitors; otherwise it is zero. \mathbf{D}_r and \mathbf{D}_x indicate the diagonal matrices whose diagonal elements are the resistance and reactance vectors, respectively. $\underline{Q}_{cr}, \bar{Q}_{cr}, \Delta \underline{v}$ and $\Delta \bar{v}$ represent the boundaries of \mathbf{Q}^{cr} and squared voltage profile deviation, respectively.

Therefore, we can summarize $\mathbf{c}, \mathbf{A}, \mathbf{G}_v, \mathbf{G}_{cr}, \mathbf{d}, \mathbf{b}_v$ and \mathbf{b}_{cr} in (A-1b) as

$$\mathbf{c} = \begin{bmatrix} \mathbf{D}_r & \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} \\ \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{D}_r & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} \\ \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times n_{cr}} \end{bmatrix} \quad (\text{A-3a})$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_G^T & \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} \\ \mathbf{0}_{|\mathcal{E}| \times |\mathcal{E}|} & \mathbf{A}_G^T & \mathbf{A}_{cr} \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} \mathbf{P}^g + \mathbf{P}^d \\ \mathbf{Q}^g + \mathbf{Q}^d \end{bmatrix} \quad (\text{A-3b})$$

$$\mathbf{G}_v = \begin{bmatrix} 2\mathbf{D}_r & 2\mathbf{D}_x & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} & M \\ -2\mathbf{D}_r & -2\mathbf{D}_x & \mathbf{0}_{|\mathcal{E}| \times n_{cr}} & -M \end{bmatrix} \quad (\text{A-3c})$$

$$\mathbf{G}_{cr} = \begin{bmatrix} \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \text{diag}(\mathbb{1}_N) & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} \\ \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} & -\text{diag}(\mathbb{1}_N) & \mathbf{0}_{n_{cr} \times |\mathcal{E}|} \end{bmatrix} \quad (\text{A-3d})$$

$$\mathbf{b}_v = \begin{bmatrix} \Delta \bar{v} + M \\ -\Delta \underline{v} + M \end{bmatrix}, \quad \mathbf{b}_{cr} = \begin{bmatrix} \bar{Q}_{cr} \\ -\underline{Q}_{cr} \end{bmatrix} \quad (\text{A-3e})$$

where \mathbf{K} and \mathbf{h} can be rewritten from (A-2e)-(A-2g) and \mathbf{G}_u and \mathbf{b}_u are inessential for (A-2d) since (A-2d) is already enforced by (3a).

REFERENCES

- [1] J. A. Taylor and F. S. Hover, “Convex models of distribution system reconfiguration,” *IEEE Trans. on Power Syst.*, vol. 27, no. 3, pp. 1407–1413, 2012.