Supplementary Material for "An ADMM-based Battery Dispatch Optimization Approach for Electric Vehicle Aggregators with Customized Local Differential Privacy Budgets"

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I. Proof of Theorem 1

There exists an optimal BDO solution $\boldsymbol{x}_i^{t*}$ with the minimum objective function value $\boldsymbol{F}^*$ for the non-private BDO model (1a)-(1e). Suppose $\boldsymbol{p}^{t*}_{sig,i} = \boldsymbol{c}^T\boldsymbol{x}_i^{t*}$, and then $\left\|\sum_{i=1}^{N_E}\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}^t_{ref}\right\|_2 = \left\|\sum_{i=1}^{N_E}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}^{t*}_{sig,i})\right\|_2$ such that $\sum_{i=1}^{N_E}\boldsymbol{p}^{t*}_{sig,i} = \boldsymbol{p}^t_{ref}$. Thus, $\sum_{t=1}^{N_T}\left\|\sum_{i=1}^{N_E}(\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}^{t*}_{sig,i})\right\|_2 = \sum_{t=1}^{N_T}\left\|\sum_{i=1}^{N_E}\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}^t_{ref}\right\|_2 = \boldsymbol{F}^*$. This proves that $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}^{t*}_{sig,i}) \in \mathcal{X}$ can be the optimal BDO solution with the minimum objective function value $\boldsymbol{F}^*$.

Due to the triangle inequality of norms, i.e., $|a+b| \leqslant |a|+|b|$ for any real-valued numbers a and b, we obtain that $\boldsymbol{F}^* \leqslant \boldsymbol{F} \leqslant \boldsymbol{G}$ naturally holds. This suggests that $\boldsymbol{F}^*$ can be the lower bound of $\boldsymbol{G}$ over $\mathcal{X}$. In other words, $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}^{t*}_{sig,i}) \in \mathcal{X}$ can be the optimal solution iff the minimum function value $\boldsymbol{G}^* = \boldsymbol{F}^*$ is achieved, which demonstrates $(\boldsymbol{x}_i^{t*}, \boldsymbol{p}^{t*}_{sig,i})$ can be also the optimal BDO solution for minimizing the objective function $\boldsymbol{G}$ over the feasibility space $\mathcal{X}$. This proves Theorem 1. ■

II. Proof of Theorem 2

First, we prove that the optimal charge-discharge solution $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds for EV customer i. It is clear that $\boldsymbol{d}_i^{t*}{=}\boldsymbol{c}^T\boldsymbol{x}_i^{t*}-\boldsymbol{p}^{t*}_{sig,i}{+}\boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t{-}\boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$ stands. We denote $\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,i} = \boldsymbol{p}^{t*}_{sig,i}{+}\boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. Then, we have $\boldsymbol{d}_i^{t*}{=}\boldsymbol{c}^T\boldsymbol{x}_i^{t*}{-}\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,i}{+}\boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. As $\boldsymbol{d}_i^{t\dagger} \geqslant \boldsymbol{d}_i^{t*}$ and $\boldsymbol{y}_i^{t\dagger} = \boldsymbol{d}_i^{t\dagger}$ achieved at optimality, minimizing the objective function $\boldsymbol{g}_i$ ensures $\boldsymbol{d}_i^{t\dagger} = \boldsymbol{d}_i^{t*}$, where other two terms in $\boldsymbol{g}_i$ can be held, i.e., $\boldsymbol{\mu}^t(\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,i} - \boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t) = \boldsymbol{\mu}^t\boldsymbol{p}^{t*}_{sig,i}$ and $\frac{\boldsymbol{\rho}}{2}||\sum_{j=1,j\neq i}^{N_E}\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,j}{+}\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,i}{-}\boldsymbol{p}^t_{ref}||_2^2 = \frac{\boldsymbol{\rho}}{2}||\sum_{j=1,j\neq i}^{N_E}\boldsymbol{p}^{t*}_{sig,j}{+}\boldsymbol{p}^{t*}_{sig,i}{-}\boldsymbol{p}^t_{ref}||_2^2 \approx 0$ at convergence. Thus, we rewrite $\boldsymbol{d}_i^{t\dagger}{=}\boldsymbol{c}^T\boldsymbol{x}_i^{t*}{-}\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,i}+\boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$. This suggests that $\boldsymbol{x}_i^{t\dagger} = \boldsymbol{x}_i^{t*}$ holds with obfuscated $\widetilde{\boldsymbol{p}}^{t\dagger}_{sig,i} = \boldsymbol{p}^{t*}_{sig,i}{+}\boldsymbol{\alpha}_i^{t\dagger}\boldsymbol{\xi}_i^t$ for the proposed privacy-preserving BDO model.

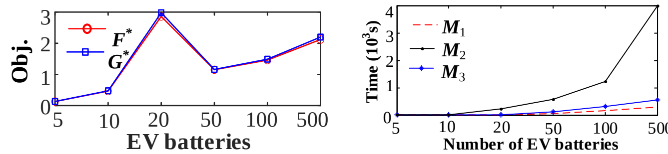
Second, we prove that $\mathcal{M}_i$ is ε_i-differentially locally private with respect to the local dataset of sub-problem i during all iterations. According to the definition of Gaussian mechanism $\mathcal{M}_i$, let the query output answer for the sub-problem i be $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}^t_{sig,i}$, and we alternatively rewrite (A-1) in the definition of ε_i-LDP for the sub-problem i [1]:

$$\mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^t] \leqslant \mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}]e^{\varepsilon_i}, \tag{A-1}$$

for two ε_i-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$. For convenience, the query output $\mathcal{O}_i^t$ of EV customer i for $\forall t \in \mathcal{T}$ with a random noise $\boldsymbol{\xi}_i^t$ can be written as $\mathcal{O}_i^t = \widetilde{\boldsymbol{p}}^t_{sig,i} = \boldsymbol{p}^t_{sig,i}{+}\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t$ where $\forall t \in \mathcal{T}$.

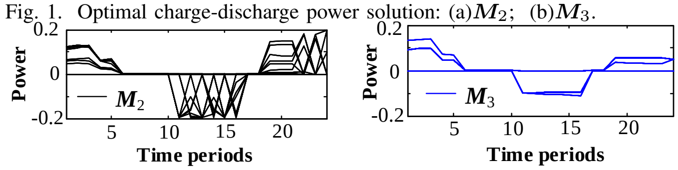
Therefore, the ratio of probabilities on two ε_i-indistinguishable charge-discharge solutions $\boldsymbol{x}_i^t$ and $\boldsymbol{x}_i^{t'}$ can be bounded by

$$\begin{aligned}
&\frac{\mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^t]}{\mathbb{P}_\xi[\mathcal{M}_i = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}]} = \frac{\mathbb{P}_\xi\left[\boldsymbol{p}^t_{sig,i}+\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t|\boldsymbol{x}_i^t\right]}{\mathbb{P}_\xi\left[\boldsymbol{p}^{t'}_{sig,i}+\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t|\boldsymbol{x}_i^{t'}\right]}\\
&=\frac{\mathbb{P}_\xi\left[\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}^t_{sig,i}|\boldsymbol{x}_i^t\right]}{\mathbb{P}_\xi\left[\boldsymbol{\alpha}_i^t\boldsymbol{\xi}_i^t = \mathcal{O}_i^t - \boldsymbol{p}^{t'}_{sig,i}|\boldsymbol{x}_i^{t'}\right]} \overset{(i)}{=} \frac{\boldsymbol{\alpha}_i^t\exp\{\frac{\varepsilon_i\|\mathcal{O}_i^t-\boldsymbol{p}^t_{sig,i}\|_1}{\Delta_\rho}\}}{\boldsymbol{\alpha}_i^t\exp\{\frac{\varepsilon_i\|\mathcal{O}_i^t-\boldsymbol{p}^{t'}_{sig,i}\|_1}{\Delta_\rho}\}}\\
&=\exp\left(\frac{\varepsilon_i\|\mathcal{O}_i^t-\boldsymbol{p}^t_{sig,i}\|_1-\varepsilon_i\|\mathcal{O}_i^t-\boldsymbol{p}^{t'}_{sig,i}\|_1}{\Delta_\rho}\right)
\end{aligned}$$



(a) (b)

Fig. 1. Optimal charge-discharge power solution: (a)$\boldsymbol{M_2}$; (b)$\boldsymbol{M_3}$.



(a) (b)

Fig. 2. Optimal charge-discharge power solution: (a)$\boldsymbol{M_2}$; (b)$\boldsymbol{M_3}$.

$$\overset{(ii)}{\leqslant} \exp\left(\frac{\varepsilon_i\|\boldsymbol{p}^t_{sig,i}-\boldsymbol{p}^{t'}_{sig,i}\|_1}{\Delta_\rho}\right) \overset{(iii)}{\leqslant} \exp\left(\frac{\varepsilon_i\Delta_\rho}{\Delta_\rho}\right) = e^{\varepsilon_i}, \tag{A-2}$$

where (i) comes from the definition of the probability density function of the Gaussian distribution. In (ii) step, it is followed by the inequailty of norms, i.e., $|a|-|b| \leqslant |a-b|$ for any real-valued numbers a and b. For the (iii) step, $\|\boldsymbol{p}^t_{sig,i}-\boldsymbol{p}^{t'}_{sig,i}\|_1$ denotes the ℓ_1-sensitivity on ρ-indistinguishable output datasets $\boldsymbol{p}^t_{sig,i}$ and $\boldsymbol{p}^{t'}_{sig,i}$ subject to $\|\boldsymbol{p}^t_{sig,i}-\boldsymbol{p}^{t'}_{sig,i}\|_1 \leqslant \Delta_\rho$.

Accordingly, it is clear that (A-1) holds based on (A-2), which proves this Theorem. ■

III. Additional Case Study for Theorem 1

The computational performance of this proposed ADMM-based BDO approach by **Theorem 1** can be verified in this section. For brevity, three methods are compared: the LP-based approach in [2] ($\boldsymbol{M_1}$), and the MIQP-based BDO model solved using commercial solver MOSEK ($\boldsymbol{M_2}$) for comparison with the proposed ADMM-based BDO method ($\boldsymbol{M_3}$). In Fig. 1(a), we plot the minimum objective function value $\boldsymbol{F}^*$ obtained from the benchmark $\boldsymbol{M_2}$ and compare it to $\boldsymbol{G}^*$ obtained from the proposed $\boldsymbol{M_3}$ for different numbers of EVs. We observe that, indeed, $\boldsymbol{G}^* \approx \boldsymbol{F}^*$ in all cases. Moreover, Fig. 1(b) reports the computation time with respect to the numbers of EV batteries. $\boldsymbol{M_1}$ incurs the least computational time because $\boldsymbol{M_1}$ is free of binary variables, but it can result in infeasible solutions featuring simultaneous charging and discharging. And $\boldsymbol{M_3}$ is superior to both $\boldsymbol{M_2}$ with respect to computational time. In fact, for the case with $N_E = 500$ EVs, $\boldsymbol{M_3}$ converges to the optimal solution in 14.00% and 18.67% of the time taken by $\boldsymbol{M_2}$. To further demonstrate the numbers of charging/discharging switches in $\boldsymbol{M_2}$ and $\boldsymbol{M_3}$, Figs. 2(a) and 2(b) present the specific optimal charge-discharge power solutions for the case with $N_E = 10$ EVs. In Fig. 2(a), $\boldsymbol{M_2}$ incurs many charging and discharging actions over 24 hours, while $\boldsymbol{M_3}$ imposes few charging and discharging actions, as shown in Fig. 2(b). This is highly beneficial as charging/discharging switches degrade the remaining useful life of EV batteries. This case study showcases that an ADMM-based BDO approach established by **Theorem 1** outperforms existing methods for solving BDO problems.

References

[1] C. Dwork, A. Roth *et al.*, "The algorithmic foundations of differential privacy," *Foundations and Trends in Theoretical Computer Science*, vol. 9, no. 3–4, pp. 211–407, 2014.