1.3 The Cross Product of Vectors

Definition 1.3.1 Cross product <벡터의 외적>

* dot product
F·G는 벡터
$= a_1a_2+b_1b_2+c_1c_2$

Let $F=\langle a_1,b_1,c_1\rangle$ *and* $G=\langle a_2,b_2,c_2\rangle$

The cross product of F *with* G *is the vector* $F\times G$ *defined by*

$$F\times G=\langle b_1c_2-b_2c_1,\ a_2c_1-a_1c_2,\ a_1b_2-a_2b_1\rangle$$

vector

■ Remembering the components of F x G:

- using **3 x 3** "determinant",

unit vector

$i = \langle 1,0,0\rangle$
$j = \langle 0,1,0\rangle$
$k = \langle 0,0,1\rangle$

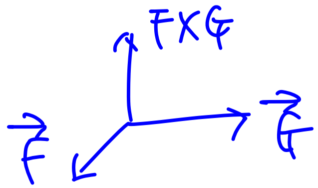
main diagram   sub diagram

$$F\times G=\begin{array}{cc} & \begin{array}{ccc} + & - & + \end{array} \\ \begin{array}{c} \\ F \\ G \end{array} & \begin{vmatrix} \mathrm{i} & \mathrm{j} & \mathrm{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}\end{array}=+\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}\mathrm{i}-\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}\mathrm{j}+\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}\mathrm{k}$$

$$=\langle b_1c_2-b_2c_1,\ -(a_1c_2-a_2c_1),\ a_1b_2-a_2b_1\rangle$$

$$=\langle b_1c_2-b_2c_1,\ -(a_1c_2-a_2c_1),\ a_1b_2-a_2b_1\rangle$$

$$=\langle b_1c_2-b_2c_1,\ a_2c_1-a_1c_2,\ a_1b_2-a_2b_1\rangle$$



Theorem 1.3.1

Let F, G and H be vectors, and let α be a scalar, then ⇒ $F \cdot G = 0$인지로 증명 가능

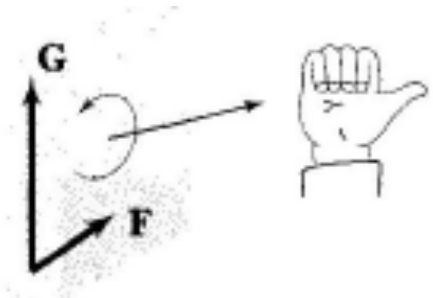
1. $F \times G$ *is orthogonal to both* F *and* G
2. 크기 $\|F \times G\| = \|F\|\|G\| \sin\theta$, *where* θ *is the angle between* F *and* G
3. *if* $F \neq \langle 0, 0, 0 \rangle$ *and* $G \neq \langle 0, 0, 0 \rangle$, *then* $F \times G = \langle 0, 0, 0 \rangle$ *if and only if* F *and* G *are parallel*
4. $F \times (G + H) = F \times G + F \times H$
5. $(\alpha F) \times G = \alpha (F \times G) = F \times (\alpha G)$
6. $F \times G = -G \times F$

$\sin\theta = 0$

평행

* $\theta = 60°$ 이면 $\sin\theta = 0$ ⟶

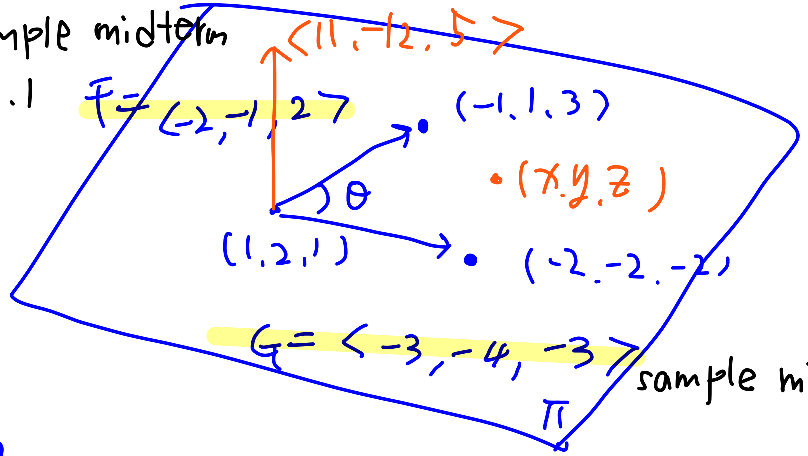
Direction of F x G:



Example 1.3.1

Find the equation of a plane Π passing through (1, 2, 1), (-1, 1, 3), and (-2, -2, -2)

sample midterm Q.3.1



sample midterm Q.3.2

sample midterm Q.3.4

$$\langle x-1, y-2, z-1\rangle \cdot \langle 11,-12,5\rangle = 0$$

$$11x-11-12y+24+5z-5=0$$

$$\Pi: 11x-12y+5z=-8$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -1 & 2 \\ -3 & -4 & -3 \end{vmatrix}$$

sample midterm Q.3.3

$$F\times G=\langle 3+8, -(6+6), 8-3\rangle$$
$$=\langle 11,-12,5\rangle$$

(1, 1, 4)

(1, 3, 3)

$$F\times G=\langle 3-12, -(3-4), (3-1)\rangle$$
$$=\langle -9,1,2\rangle$$

$$\langle -9,1,2\rangle\cdot\langle x-1, y-1, z\rangle=0$$

$$-9x+9+y-1+2z=0$$