■ How to obtain A_R ?

Definition 2.2.2 Elementary row operations

1. Interchange of two rows
2. Multiplication of a row by a nonzero scalar 0이아닌 스칼라를 열에 곱하기
3. Addition of a scalar multiple of one row to **another row**

(the one row remains the same, i.e., unchanged.)

Note: Recall ***Example 1.5.7***

- We solved: $\alpha_1 \langle 1,0 \rangle + \alpha_2 \langle 0,2 \rangle + \alpha_3 \langle 1,2 \rangle = \langle 0,0 \rangle$

➔ In a matrix form: $$\mathrm{AX} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \alpha_3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note: Recall ***Example 1.5.7***

- We solved:

$$\alpha_1 \langle 1,0 \rangle + \alpha_2 \langle 0,2 \rangle + \alpha_3 \langle 1,2 \rangle = \langle 0,0 \rangle$$

$$\langle \alpha_1 + \alpha_3, 2\alpha_2 + 2\alpha_3 \rangle = \langle 0,0 \rangle$$

$$\begin{cases} \alpha_1 + \alpha_3 = 0 \\ 2\alpha_2 + 2\alpha_3 = 0 \end{cases}$$

➔ In a matrix form:

$$\text{AX} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \alpha_3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha_1 + \alpha_3 \\ 2\alpha_2 + 2\alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Note: **1. Interchange of two rows** in **Definition 2.2.2 Elementary row operations**

$$\alpha_1\langle 1,0\rangle+\alpha_2\langle 0,2\rangle+\alpha_3\langle 1,2\rangle=\langle 0,0\rangle$$

$$\langle \alpha_1+\alpha_3, 2\alpha_2+2\alpha_3\rangle=\langle 0,0\rangle$$

- We solved:

$$\begin{cases}\alpha_1+\alpha_3=0\\ 2\alpha_2+2\alpha_3=0\end{cases}\rightarrow\begin{cases}2\alpha_2+2\alpha_3=0\\ \alpha_1+\alpha_3=0\end{cases}$$

➔ In a matrix form:

$$AX=\begin{bmatrix}1&0&1\\0&2&2\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}=\alpha_1\begin{bmatrix}1\\0\end{bmatrix}+\alpha_2\begin{bmatrix}0\\2\end{bmatrix}+\alpha_3\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

교환 가능

$$\begin{bmatrix}\alpha_1+\alpha_3\\2\alpha_2+2\alpha_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\rightarrow\begin{bmatrix}2\alpha_2+2\alpha_3\\\alpha_1+\alpha_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

0이아닌 스칼라를 열에 곱하기

Note: **2. Multiplication of a row by a nonzero scalar** in **Definition 2.2.2 Elementary row operations**

$$\alpha_1\langle 1,0\rangle+\alpha_2\langle 0,2\rangle+\alpha_3\langle 1,2\rangle=\langle 0,0\rangle$$

$$\langle \alpha_1+\alpha_3, 2\alpha_2+2\alpha_3\rangle=\langle 0,0\rangle$$

- We solved:

$$\begin{cases}\alpha_1+\alpha_3=0\\ 2\alpha_2+2\alpha_3=0\end{cases}\rightarrow\begin{cases}\underline{2\times\alpha_1+2\times\alpha_3=2\times 0}\\ 2\alpha_2+2\alpha_3=0\end{cases}$$

➔ In a matrix form:

$$\text{AX}=\begin{bmatrix}1&0&1\\0&2&2\end{bmatrix}\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{bmatrix}=\alpha_1\begin{bmatrix}1\\0\end{bmatrix}+\alpha_2\begin{bmatrix}0\\2\end{bmatrix}+\alpha_3\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}\alpha_1+\alpha_3\\ 2\alpha_2+2\alpha_3\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\rightarrow\begin{bmatrix}2\times\alpha_1+2\times\alpha_3\\ 2\alpha_2+2\alpha_3\end{bmatrix}=\begin{bmatrix}2\times 0\\ 0\end{bmatrix}$$

Note: **3. Addition of a scalar multiple of one row to another row**

한 행의 상수배를 다른 행에 더하기

(the one row remains the same, i.e., unchanged.)

변화 없이 그대로 남아있음

in **Definition 2.2.2 Elementary row operations**

- We solved:

더하기

$$\begin{cases} \alpha_1 + \alpha_3 = 0 \\ 2\alpha_2 + 2\alpha_3 = 0 \end{cases} \rightarrow \begin{cases} 2\alpha_1 + 2\alpha_3 = 0 \\ 2\alpha_2 + 2\alpha_3 = 0 \end{cases} \rightarrow \begin{cases} 2\alpha_1 + 2\alpha_3 = 0 \\ (-1)\times 2\alpha_2 + (-1)\times 2\alpha_3 = (-1)\times 0 \end{cases}$$

$$\rightarrow \begin{cases} 2\alpha_1 - 2\alpha_2 = 0 \\ 2\alpha_2 + 2\alpha_3 = 0 \end{cases}$$

→ 원래 그대로

➔ In a matrix form:

$$\begin{bmatrix} \alpha_1 + \alpha_3 \\ 2\alpha_2 + 2\alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2\alpha_1 + 2\alpha_3 \\ 2\alpha_2 + 2\alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2\alpha_1 + 2\alpha_3 \\ (-1)\times 2\alpha_2 + (-1)\times 2\alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ (-1)\times 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 2\alpha_1 - 2\alpha_2 \\ 2\alpha_2 + 2\alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

2.3 Solution of Homogeneous Systems of Linear Equations

Definition 2.3.1 r = rank(A)

→ $r = \text{rank}(A)$ = the number of leading entries in A_R

leading entry의 갯수

Note: $\text{rank}(A) = \text{rank}(A_R)$

Theorem 2.3.1

→ $\text{rank}(A_R) = \dim(R(A_R))$

차원의 갯수

Theorem 2.3.2

→ $\text{rank}(A) = \dim(R(A))$

Note: Recall ***Example 1.5.3***

row space, colum space
2가지를 만들 수 있음!!

- 2 x 3 matrix: $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$

➔ $\underbrace{\text{3-dimensional row vectors (} 1 \times 3 \text{ matrices): } [1 \quad 0 \quad 1]}_{3개}$ and $[0 \quad 2 \quad 2]$

$$R(A) = \{\alpha_1 [1 \quad 0 \quad 1] + \alpha_2 [0 \quad 2 \quad 2] \mid \alpha_1 \text{ and } \alpha_2 \text{ are scalars.}\}$$

Find A_R for a given $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$.

★ basis (일종의 뼈대)
: 벡터를 생성할 때, 최소한으로
필요한 것들의 집합

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \xrightarrow{\frac{1}{2}\times(\text{2nd row})} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = A_R$$

Note: $\text{rank}(A) = \text{rank}(A_R) = 2$

Note: $\dim(\, R(A_R)\,) = 2$

$\Rightarrow$ $B(R(A_R)) = \{[1 \quad 0 \quad 1], [0 \quad 1 \quad 1]\}$

R(A_R) 생성시,
최소한으로 필요한 것들

i) $\alpha_1[1\ 0\ 1] + \alpha_2[0\ 1\ 1] = [0\ 0\ 0]$
$[\alpha_1\ 0\ \alpha_1] + [0\ \alpha_2\ \alpha_2] = [0\ 0\ 0]$
$[\alpha_1\ \ \alpha_2\ \ \alpha_1+\alpha_2] = [0\ 0\ 0]$

$\begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0 \\ \alpha_1 + \alpha_2 = 0 \end{cases}$ $\therefore \begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0 \end{cases}$ only the solution

$\therefore$ $B(R(A_R))$ is linearly independent

ii) $\alpha_1[1\ 0\ 1] + \alpha_3[0\ 1\ 1]$
$= \beta_1[1\ 0\ 1] + \beta_2[0\ 1\ 1]$
Where $\alpha_1 = \beta_1$ and $\alpha_3 = \beta_2$

$\therefore$ $B(R(A_R)) = P(R(A_R))$

$$R(A) = \{\alpha_1[1 \quad 0 \quad 1] + \alpha_2[0 \quad 2 \quad 2] \mid \alpha_1 \text{ and } \alpha_2 \text{ are scalars.}\}$$
$$= \{\alpha_1[1 \quad 0 \quad 1] + 2\alpha_2[0 \quad 1 \quad 1] \mid \alpha_1 \text{ and } 2\alpha_2 \text{ are scalars.}\} = R(A_R)$$
$$= \{\alpha_1[1 \quad 0 \quad 1] + \alpha_3[0 \quad 1 \quad 1] \mid \alpha_1 \text{ and } \alpha_3 \text{ are scalars}\}$$

★ ***Note:*** Elementary row operations do not change R(A), i.e., R(A) = R(A_R).

Therefore, dim(R(A)) = dim(R(A_R)).