Definition 2.3.2 Homogeneous Systems of Linear Equations

➔The system $AX = B$ is homogeneous if $B = O_{n1}$.

싹다 제로 !!

- Note: O_{n1} is the n x 1 zero matrix, i.e., $O_{n1} = \left.\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}\right\} n \;\; zeros$,

  alternatively called an n-dimensional zero vector $\underbrace{\langle 0, 0, \cdots, 0 \rangle}_{n \;\; componets}$.

Note: Elementary row operations do not change N(A), i.e., N(A) = N(A_R). (By inspection)

Therefore, dim(N(A)) = dim(N(A_R)).

Note: Recall ***Example 1.5.7***

- We solved:

$$\alpha_1 \langle 1,0 \rangle + \alpha_2 \langle 0,2 \rangle + \alpha_3 \langle 1,2 \rangle = \langle 0,0 \rangle$$
$$\langle \alpha_1 + \alpha_3, 2\alpha_2 + 2\alpha_3 \rangle = \langle 0,0 \rangle$$
$$\begin{cases} \alpha_1 + \alpha_3 = 0 \\ 2\alpha_2 + 2\alpha_3 = 0 \end{cases}$$

leading entry corresponding column

non-leading entry corresponding column

Candidate (1of) Final Q.3

Example 2.3.1 Find the general solution of the system

$$AX = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow \quad A_R X = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 + \alpha_3 \\ \alpha_2 + \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} -\alpha_3 \\ -\alpha_3 \end{bmatrix}$$

Answer

$$\therefore X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} -\alpha_3 \\ -\alpha_3 \\ \alpha_3 \end{bmatrix} = \boxed{\alpha_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}}$$

Express leading entry corresponding variables in terms of non-leading entry corresponding variables

Note: r = rank(A) = dim(R(A))

* r+dim(N(A)) =m

열벡터의 m..

Then, dim(N(A)) = m - rank(A) = m - r =1

N(A)의 vector의 개수?....

3   2

This is **Fundamental Theorem of Linear Algebra, Part I** (ii)

Note: rank(A) = dim(C(A_R))

Then, dim(C(A)) = dim(C(A_R))   although C(A) ≠ C(A_R)

- We solved: $\alpha_1\langle 1,0\rangle + \alpha_2\langle 0,2\rangle + \alpha_3\langle 1,2\rangle = \langle 0,0\rangle$

➔ In a matrix form:

$$\text{AX} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 2 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$C(A) = \{\alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 2 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 2 \end{bmatrix} \mid \alpha_1, \alpha_2\ \alpha_3$ are scarlars$\}$

$$\text{A}_\text{R}\text{X} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$C(A_R) = \{\alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 1 \end{bmatrix} \mid \alpha_1, \alpha_2\ \alpha_3$ are scalars$\}$

because elementary row operations do not change the corresponding independency among columns,

i.e., N(A) = N(A_R).

This is **Fundamental Theorem of Linear Algebra, Part I** (i)

Note: Recall ***Example 1.5.7***

$C(A_R) = \{\alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\mid \alpha_1, \alpha_2$ and α_3 are scarlars$\}$

$B(C(A_R)) = \{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\}$

(i) $\alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} \alpha_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{cases} \alpha_1 = 0 \\ \alpha_2 = 0 \end{cases}$ only the solution

$\therefore B(C(A_R))$ is learnarly independant

(ii) $\alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= (\alpha_1 + \alpha_3)\begin{bmatrix} 1 \\ 0 \end{bmatrix} + (\alpha_2 + \alpha_3)\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \beta_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

where $\beta_1 = \alpha_1 + \alpha_3$ and
$\beta_2 = \alpha_2 + \alpha_3$

$\therefore B(C(A_R)) = P(C(A_R))$

$\therefore$ By (i) and (ii)
$B(C(A_R))$ is a basis for $C(A_R)$

$N(A) = N(A_R) = \left\{ \alpha_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \middle| \ \alpha_2 \text{ is a scalar.} \right\}$

$B(N(A)) = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\alpha_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 0 \\ \alpha_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\alpha_1 = 0$ only the solution

$\therefore B(N(A))$ is linearly independent

candidate (2 of) Final Q.3

Example 2.3.2 Find the general solution of the system

$$AX = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow$$

leading entry corresponding column

non-leading entry corresponding column

$$A_R X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(ii) $\alpha_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = m_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, where $m_1 = \alpha_2$

$\therefore B(N(A)) = P(N(A))$

$\therefore$ By (i) and (ii), $B(N(A))$ is a basis for $N(A)$. Q.E.D.

$\dim(N(A)) = 1$ //

Answer

$$\therefore X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_2 \\ 0 \end{bmatrix} = \boxed{\alpha_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

Express leading entry corresponding variables

in terms of non-leading entry corresponding variables