Example 1.1.2

Find the equation of the straight line L through the points (2, -1, 6) and (-4, -1, 2).

$(6, 0, 4)$

$$\begin{cases} 6t = x-2 \\ 0 = y+1 \\ 4t = z-6 \end{cases}$$

$$\boxed{\begin{cases} x = 6t+2 \quad (-\infty < t < \infty) \\ y = -1 \\ z = 4t+6 \end{cases}}$$

1.2 The Dot Product of Vectors

Definition 1.2.1 Dot product

$$Let\ F = \langle a_1, b_1, c_1 \rangle \ and \ G = \langle a_2, b_2, c_2 \rangle$$

The dot product of F with G is the scalar $F \bullet G$ *defined by*

벡터 2개

$F \bullet G \Rightarrow$ 스칼라

$$F \bullet G = a_1 a_2 + b_1 b_2 + c_1 c_2$$

스칼라

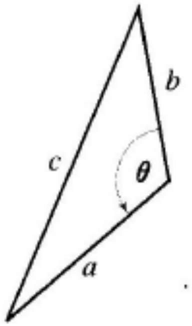
Theorem 1.2.1

Let F, G *and* H *be vectors, and let* α *be a scalar, then*

1. $F \bullet G = G \bullet F$
2. $(F + G) \bullet H = F \bullet H + G \bullet H$
3. $\alpha(F \bullet G) = (\alpha F) \bullet G = F \bullet (\alpha G)$
4. $F \bullet F = \|F\|^2$ (크기의 제곱)
5. $F \bullet F = 0$ *if and only if* $F = \langle 0, 0, 0 \rangle$

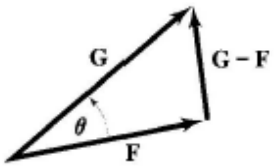
proof) $F \cdot G = a_1 a_2 + b_1 b_2 + c_1 c_2$
$= a_2 a_1 + b_2 b_1 + c_2 c_1$
$= G \cdot F$
Q.E.D.

Cosine of the angle between two vectors:



* 기준 벡터 있어야
direction 알 수 있다....
(θ도 알수있고...)

● from the law of cosines, $a^2 + b^2 - 2ab\cos\theta = c^2$



● apply this law, $\|F\|^2 + \|G\|^2 - 2\|F\|\|G\|\cos\theta = \|G - F\|^2$

$F = \langle a_1, b_1, c_1 \rangle$
$G = \langle a_2, b_2, c_2 \rangle$

$2\|F\|\|G\|\cos\theta = \cancel{\|F\|^2 + \|G\|^2} - \|G-F\|^2$

$\hookrightarrow \langle a_2 - a_1, b_2 - b_1, c_2 - c_1 \rangle$

$\cos\theta = \frac{\cancel{2}(a_1a_2 + b_1b_2 + c_1c_2)}{\cancel{2}\|F\|\|G\|} = \cancel{\|F\|^2\|G\|^2} + 2a_1a_2 + 2b_1b_2 + 2c_1c_2$

→this equation yields, $\boxed{\cos\theta = \frac{F \bullet G}{\|F\|\|G\|}}$

Definition 1.2.2 Orthogonality (Perpendicularity)

Midterm Q.1.3.

Two vectors are orthogonal (perpendicular) if and only if their dot product is zero.

$\theta = 90^\circ \to \cos 90^\circ = 0 \longrightarrow F \cdot G = 0$

Example 1.2.1

F= G=

Determine whether <-4, 1, 2> and <2, 0, 4> are orthogonal.

$\Rightarrow$ dot product $= 0$

$F \cdot G = -8 + 0 + 8 = 0$ 이므로

$\Rightarrow$ Orthogonal //

Example 1.2.2

Determine whether <6, -1, -2> and <3, 1, 4> are orthogonal.

$\langle 6, -1, -2 \rangle \cdot \langle 3, 1, 4 \rangle = 18 - 1 - 8$

$= 9 \neq 0$

$\Rightarrow$ none orthogonal //

Example 1.2.3

Determine whether the line L_1 and L_2 are perpendicular, where

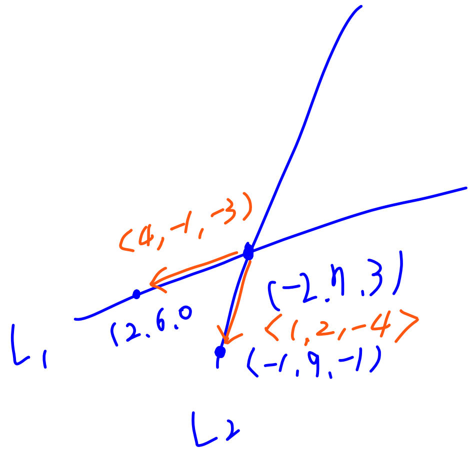
$$L_1: \begin{cases} x = 2 - 4t \\ y = 6 + t \\ z = 3t, \qquad -\infty < t < \infty \end{cases} \qquad L_2: \begin{cases} x = -2 + p \\ y = 7 + 2p \\ z = 3 - 4p, \qquad -\infty < p < \infty \end{cases}$$

$t=1$, $\begin{cases} x = -2 \\ y = 7 \\ z = 3 \end{cases}$ $\qquad$ $p=0$, $\begin{cases} x = -2 \\ y = 7 \\ z = 3 \end{cases}$

$(-2, 7, 3)$ $\qquad$ $(-2, 7, 3)$

$t=0$, $(2, 6, 0)$ $\qquad$ $p=1$, $(-1, 9, -1)$

$\langle 4, -1, -3 \rangle \cdot \langle 1, 2, -4 \rangle$



$= 4 - 2 + 12 = 14 \neq 0$

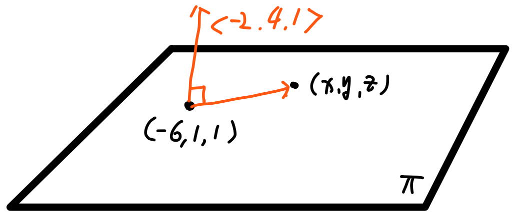
$\therefore$ none perpendicular

Example 1.2.4

Find the equation of a plane Π passing through (-6, 1, 1) and perpendicular to <-2, 4, 1>

Let (x, y, z) be any point on π.

$\langle x+6, y-1, z-1 \rangle$



$\langle -2, 4, 1 \rangle \cdot \langle x+6, y-1, z-1 \rangle = 0$

$-2x - 12 + 4y - 4 + z - 1 = 0$

$-2x + 4y + z = 17 \Rightarrow$ plane π