Definition 1.5.4 Bases, B(V)

↱ 벡터 집합

- A basis B (for a vector space V) is a set of vectors in V (having two properties at once:)

  (1) It is linearly independent. 선형 독립

  (2) It spans the vector space. 벡터 공간을 생성

B(V)

⇒ 벡터공간 V에 대해서 2개의 특성을 갖는 벡터 집합

(벡터 조)

$1 < \dim(S) = 2 < 3$

a basis

= minimal spanning set

= maximal independent set

⇒ B(S)가 S의 basis 인지 증명!!
(1) 선형독립
(2) 벡터공간으로 span 가능한지

Example 1.5.9

$S = \{\alpha_1\langle 1,0\rangle + \alpha_2\langle 0,2\rangle + \alpha_3\langle 1,2\rangle \mid \alpha_1, \alpha_2$ and α_3 are scalars$\}$
⇒ Vector space

Let B(S)={ <1,0>, <0,2> }. Prove that B(S) is a basis for S in *Example 1.5.3.*

Final Q.2.2

(1) B(S)가 선형독립인지

(1) $\alpha_1\langle 1,0\rangle + \alpha_2\langle 0,2\rangle = \langle 0,0\rangle$

$\langle \alpha_1, 2\alpha_2\rangle = \langle 0,0\rangle$ $\boxed{\alpha_1 = 0,\ \alpha_2 = 0}$

∴ B(S) is linear independent

Final Q.2.3

(2) B(S) 가 S에 대한 벡터공간 생성 집합인지

(2) $\alpha_1\langle 1,0\rangle + \alpha_2\langle 0,2\rangle + \alpha_3\langle 1,2\rangle$ ⇒ 하나의 벡터

$= \alpha_1\langle 1,0\rangle + \alpha_2\langle 0,2\rangle$

$+ \alpha_3\langle 1,0\rangle + \alpha_3\langle 0,2\rangle$

$= \underbrace{(\alpha_1+\alpha_3)}_{\beta_1}\langle 1,0\rangle + \underbrace{(\alpha_2+\alpha_3)}_{\beta_2}\langle 0,2\rangle$

⇒ $B(S) = \{\langle 1,0\rangle, \langle 0,2\rangle\}$이므로
S가 이것로 이루어져있음

∴ $B(S) = P(S)$ → spanning set

∴ Based on (i) and (ii).
B(S) is a basis for S.

Example 1.5.10

Let B(S)={ <1,0> }. Prove that B(S) is a basis for S in *Example 1.5.4*.

$S = \{\alpha_1\langle 1,0\rangle \mid \alpha_1$ is a scalar$\}$

(i) $\alpha_1\langle 1,0\rangle = \langle 0,0\rangle$

$\boxed{\alpha_1 = 0}$ ⇒ only zero solution

∴ B(S) is linearly independant

(ii) $\alpha_1\langle 1,0\rangle = \beta_1\langle 1,0\rangle$

where $\alpha_1 = \beta_1$

* vector space S에 있는 어떤 벡터나
<1,0>의 선형결합으로 표현 가능

∴ $B(S) = P(S)$

∴ Based on (i) and (ii).
B(S) is a basis for S.

차원

Definition 1.5.5 Dimension, dim(V)

The number of vectors in a basis (for a vector space V) is the dimension of V

=> vector space V에 대한 basis 안에 있는 벡터의 수

Final Q.1.2 The number of vectors in every basis for a vector space is the same

=> True

Example 1.5.11

Find the dimension of S in *Example 1.5.3*.

2

Example 1.5.12

Find the dimension of S in *Example 1.5.4*.

1

*하지만 basis에 있는 벡터의 수는 같음!!

이것도 S의 basis 될 수 있음

$B_2(S) = \{\langle 2.0 \rangle\}$

(i) $\alpha_1 \langle 2,0 \rangle = \langle 0,0 \rangle$

$\begin{cases} 2\alpha_1 = 0 \\ 0 = 0 \end{cases}$ $\therefore \alpha_1 = 0$

↳ linearly independant

(ii) $\alpha_1 \langle 1.0 \rangle = \eta_1 \langle 2.0 \rangle$

where $\frac{\alpha_1}{2} = \eta_1$