Advanced
Engineering Mathematics II

Chapter 2

Marices
and Systems of Linear Equations

2.1 Subspaces of a matrix A

Definition 2.1.1 Column spaces, C(A)

➔The subspace of R^n consisting of all linear combinations of the column vectors of an n x m matrix A is called the column space of the matrix A, namely C(A),

행렬 A의 열들의 선형 조합으로 만들 수 있는 모든 집합

Note: Recall ***Example 1.5.3***

- 2-dimensional vectors: <1,0>, <0,2> and <1,2>

➔ 2-dimensional column vectors (2 x 1 matrices): $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

조합으로 만들어 진거

➔ 2 x 3 matrix: $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$

Definition 2.1.2 Row spaces, R(A)

➔The subspace of R^m (consisting of all the linear combinations of the row vectors of an n x m matrix A) is called the row space of the matrix A, namely R(A).

Note: Recall ***Example 1.5.3***

행렬A의 행들의 선형 조합으로 만들 수 있는 모든 집합

- 2 x 3 matrix: $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}$

➔ 3-dimensional row vectors (1 x 3 matrices): $[1 \quad 0 \quad 1]$ and $[0 \quad 2 \quad 2]$

Definition 2.1.3 Null spaces, N(A)

➔The nullspace consists of all column vectors which satisfy AX = O_{n1}, namely N(A).

제로 벡터

i.e., N(A) = { X | AX=O_{n1} }

Note: Recall ***Example 1.5.7***

- We solved: $\alpha_1\langle 1,0\rangle + \alpha_2\langle 0,2\rangle + \alpha_3\langle 1,2\rangle = \langle 0,0\rangle$

➔ In a matrix form:

$$\begin{array}{cc} & \begin{array}{cc} A & X \end{array} \\ \mathrm{AX} = & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array}$$

cf.)

$$\mathrm{AX} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \alpha_1\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \alpha_2\begin{bmatrix} 0 \\ 2 \end{bmatrix} + \alpha_3\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Fundamental Theorem of Linear Algebra, Part I

(i) $\boxed{\dim(\overset{2}{R(A)}) = \boxed{\dim(\overset{2}{C(A)})}}$ Final Q.1.3

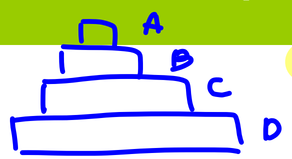
(ii) $\underbrace{\dim(R(A))}_{\text{차원}\,=\,2} + \underbrace{\dim(N(A))}_{=\,1} = m$

for an n x m matrix A

$2\times3\quad A : \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2.2 Row-reduced echelon form

감소된 행 사다리꼴



rank

■ Leading entry:

행의

첫째 원소가 nonzero entry 면 행사다리꼴!

행 사다리꼴

➔if a row has nonzero element, the leading entry of that row is its first nonzero entry,

읽음

reading from left to right

* 행이 nonzero 요소를 갖고 있으면 그 행 사다리꼴은 첫번째 nonzero 사다리꼴이다.

Definition 2.2.1 Row-reduced echelon form

➔A_R is a row-reduced echelon matrix if A_R has the following properties,

leading 1 이라 부른다.

1. The leading entry of any nonzero row is 1 전체가 0으로 구성되지 않는 행의 0을 제외한 첫 숫자는 1
2. If a_{ij} is a leading entry, all other elements of column j are zero leading 1을 포함하는 각 열은 leading 1의 위아래가 0이여야 함.
3. Each row of all zero elements lies below any row having a nonzero element
   전체가 0으로 구성된 행이 있다면 그렇지 않은 행의 아래에 위치시킨다.
4. If $a_{i_1 j_1}$ and $a_{i_2 j_2}$ are the leading entries and $i_1 < i_2$, then $j_1 < j_2$

아래행의 leading 1은 윗행의 leading 1보다 더 오른쪽에 있어야 함

연속된 leading 1이 있을때,

Typical row-reduced echelon matrix: simply called "a reduced matrix AR"

$$A_R = \left[\begin{array}{c|c|c|c|cccc|c} 1 & 0 & * & 0 & * & * & * & * & 0 \\ \hline 0 & 1 & * & 0 & * & * & * & * & 0 \\ \hline 0 & 0 & 0 & 1 & * & * & * & * & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$, where * stands for an arbitrary number

임의의

Note: AR is unique $\boxed{\text{so that}}$ it is the reduced matrix AR

Examples of AR:

$$A_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$ ⇒ Rank$(A_r) = 2$

$$A_R = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$