candidate (3of) Final Q.3

Example 2.3.3 Find the general solution of the system

$$AX = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2 change

non-leading entry

corresponding column

leading entry

corresponding column

$$\begin{array}{cccc} & \downarrow & \downarrow & \downarrow \\ A_R X = & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

표현가능

$$\begin{bmatrix} \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

answer

$$\therefore X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ 0 \\ 0 \end{bmatrix} = \boxed{\alpha_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

Express leading entry corresponding variables

in terms of non-leading entry corresponding variables

$AX=O_{n1}$

R^3
m=3

(nxm)(mx1)=(nx1)

2x3 3x1 2x1

R(A) N(A) C(A)

R^2
n=2

leading entry

A

a linear transformation

leading entry corresponding column $C(A)$

※ $R(A) = \text{Rank } A = \text{leading } 1\text{의 수} = C(A)$

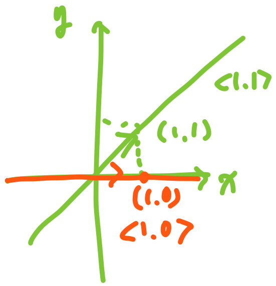
※ $\dim(C(A)) = \dim(R(A))$

non-leading entry corresponding column

Candidate (4 of 5) Final Q.3

Example 2.3.4 Find the general solution of the system

$$AX = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow \quad A_R X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



$C(A) = \{\underbrace{\alpha_1\begin{bmatrix}1\\1\end{bmatrix} + \alpha_2\begin{bmatrix}0\\0\end{bmatrix} + \alpha_3\begin{bmatrix}0\\0\end{bmatrix}}_{\alpha_1\begin{bmatrix}1\\1\end{bmatrix}} \mid \alpha_1, \alpha_2, \alpha_3 \text{ are scalars}\}$

$C(A_R) = \{\underbrace{\alpha_1\begin{bmatrix}1\\0\end{bmatrix} + \alpha_2\begin{bmatrix}0\\0\end{bmatrix} + \alpha_3\begin{bmatrix}0\\0\end{bmatrix}}_{\alpha_1\begin{bmatrix}1\\0\end{bmatrix}} \mid \alpha_1, \alpha_2, \alpha_3 \text{ are scalars}\}$

$\Rightarrow C(A) \neq C(A_R)$

$[\alpha_1] = [0]$ $\quad \alpha_2 = \alpha_2$, $\alpha_3 = \alpha_3$

Solution $\quad N(A)$

$$\therefore X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \boxed{\alpha_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

Express leading entry corresponding variables

in terms of non-leading entry corresponding variables

Candidate (5 of 5) Final Q.3

Example 2.3.5 Find the general solution of the system

$$AX = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \rightarrow$$

leading entry corresponding column

non-leading entry corresponding column

$$\begin{array}{cccc} & \downarrow & \downarrow & \downarrow \\ A_R X = & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} & & \end{array} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

로 표현 가능

$$[\alpha_1] = [\overset{-\alpha_3}{\cancel{\alpha_3}}]$$

solution

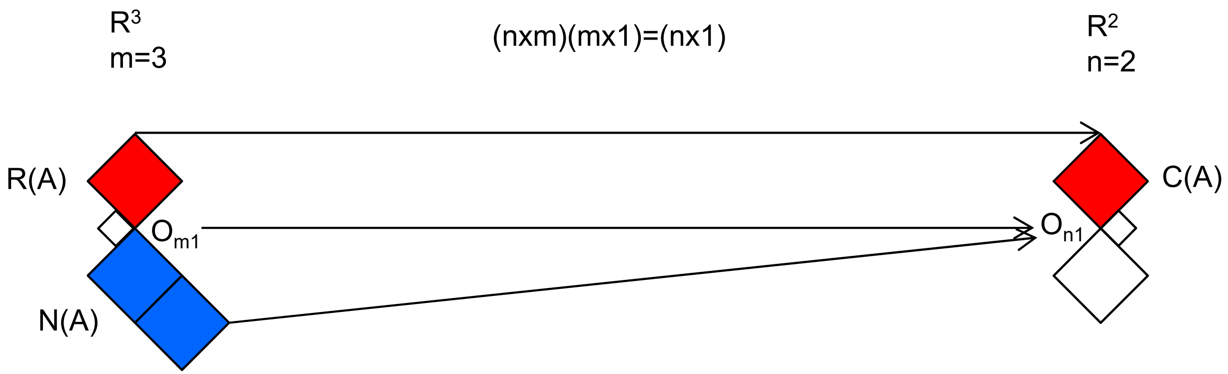
$$\therefore X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} \overset{-\alpha_3}{\cancel{\alpha_3}} \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \boxed{\alpha_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}$$

Express leading entry corresponding variables

in terms of non-leading entry corresponding variables

$AX=B$

$(n \times m)(m \times 1)=(n \times 1)$



2.4 Solution of Nonhomogeneous Systems of Linear Equations

Definition 2.4.1 Nonhomogeneous Systems of Linear Equations

➔The system $AX = B$ is nonhomogeneous if $B \neq O_{n1}$.

B가 non-zero 벡터면,

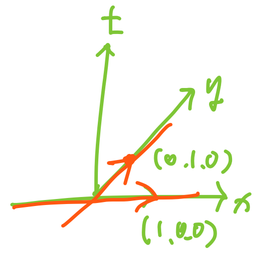
Definition 2.4.2 Orthogonal Subspaces

직교

➔Two subspaces S_1 and S_2 of the same space R^n are orthogonal if every vector s_1 in S_1 is orthogonal to every vector s_2 in S_2 (i.e., $s_1 \cdot s_2 = 0$ for all s_1 and s_2), and denoted by $S_1 \perp S_2$.

Note: $\dim(S_1) + \dim(S_2) \leq n$

$1 + 1 = 2 \leq 3$



$S_1 = \{\alpha_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mid \alpha_1$ is a scalars$\}$

$\dim(S_1) = 1$

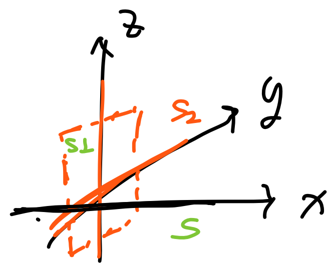
$S_2 = \{\alpha_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \mid \alpha_1$ is a scalar$\}$ in $\mathbb{R}3$

$\dim(S_2) = 1$

Definition 2.4.3 Orthogonal Complements

➔Given a subspace S of R^n, the space of all the vectors orthogonal to S is called the orthogonal complement of S, and denoted by $S^\perp$.

$S^\perp = \{\alpha_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mid \alpha_1, \alpha_2$ are scalars$\}$



Note: $\dim(S) + \dim(S^\perp) = n$

Note: Recall ***Example 2.3.1***

$$AX = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \quad \therefore X = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \alpha_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$R(A) = \{\alpha_1[1 \quad 0 \quad 1] + \alpha_2[0 \quad 2 \quad 2] \mid \alpha_1 \text{ and } \alpha_2 \text{ are scalars.}\}$$

$$N(A) = \left\{ \alpha_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \middle| \alpha_3 \text{ is a scalar.} \right\}$$

$$cf.) \quad AX = \begin{bmatrix} \alpha_3[1 \quad 0 \quad 1][-1 \quad -1 \quad 1]^T \\ \alpha_3[0 \quad 2 \quad 2][-1 \quad -1 \quad 1]^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R \bullet N = 0$, where $R \in R(A)$ and $N \in N(A)$,

becuase $R \bullet N$

$$= (\alpha_1[1 \quad 0 \quad 1] + \alpha_2[0 \quad 2 \quad 2]) \left(\alpha_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} \right)$$

$$= \alpha_1\alpha_3[1 \quad 0 \quad 1] \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + \alpha_2\alpha_3[0 \quad 2 \quad 2] \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$$= 0$$

i.e., generally, $N(A)$ is solutions for $AX = O_{n1}$.

Therefore, $R(A) \perp N(A)$ in R^m

Also, by **Fundamental Theorem of Linear Algebra, Part I** (ii) rank(A) + dim(N(A)) = m

Fundamental Theorem of Linear Algebra, Part II

$$\boxed{\boxed{R(A)} = N(A)^{\perp}}$$

Final Q.1.4

in R^m