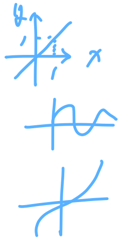
# Advanced Engineering Mathematics II

선형 대수
Linear Algebra 값이 존재

(직선 = 선형) $x$

$\cancel{\sin} x$

$\cancel{x^3}$



$x^2 = 1$ (o)

$1+1=2$ (x)

## Chapter 1

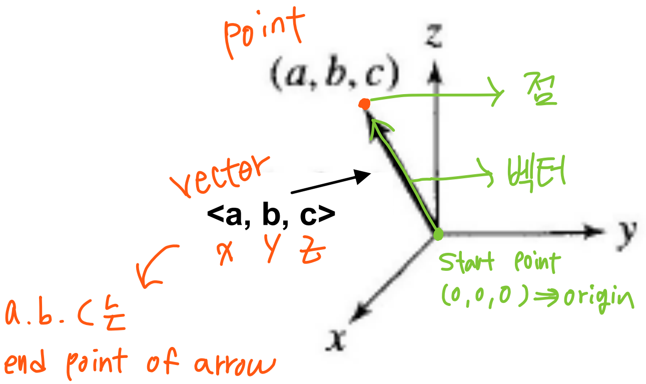
# Vector Spaces

## 1.1 The Algebra and Geometry of Vectors

- **Scalar: a real number** (실수) ex) 3, 4.7, π(3.14…) (방향 X, 크기만 갖는다)
- **Vector: a direction and a magnitude** (방향, 크기)

➔One way of efficiently conveying the magnitude and direction of a vector is to define as an ordered triple <a, b, c> of real numbers 벡터의 크기·방향을 전달하는 가장 효과적 방법(표현)

- **A geometric way of representing a vector <a, b, c>** 위치벡터




- an arrow (화살표) from the origin to the point (a, b, c)
- the direction of the arrow gives the direction of the vector (방향 다르면 다른 벡터)
- its length represents the magnitude 길이 = 크기 — 두 arrow는 길이는(크기는) 같지만 방향 다르므로 서로 다른 벡터
- in the vector <a, b, c> (x y z; end point of arrow), a is the first component, b is the second component, and c is the third component

크기

## Magnitude or Norm: the length of the arrow from the origin to the point (a, b, c)

(0,0,0)

- the norm of a vector F is denoted $\|F\|$

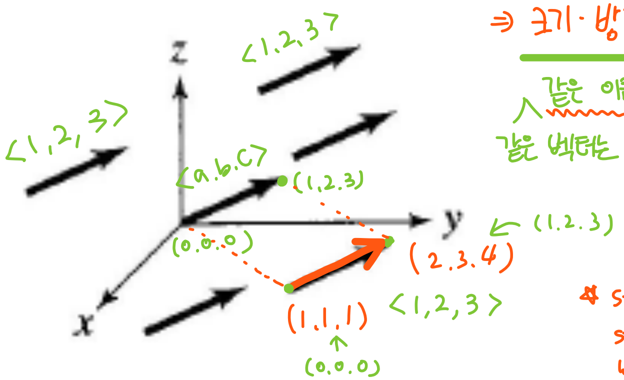
- if F= <a, b, c>, $\boxed{\|F\| = \sqrt{a^2 + b^2 + c^2}}$

## Two vectors are equal ~~/~~ if and only if ~~/~~ their respective components are equal:

각각 3가지 components가 다 같으면 같은 벡터

- $\langle a,b,c \rangle = \langle x,y,z \rangle \quad$ *if and only if* $a = x,\ b = y,\ c = z$

## All of the arrow with the same length and direction represent the same vectors




⇒ 크기·방향 같으면, 같은 벡터

같은 벡터는 같은 이름을 갖는다.

※ start point, end point 다르더라도 start point를 origin으로 설정했을 때 (위치벡터) 벡터의 이름을 그대로 사용한다. (단 같은 벡터일 경우)

→ 벡터 2개 필요

* 실수 (IR) (real number)
⇒ +, −, ×, ÷ 가능

## Definition 1.1.1 Vector sum of F and G

$$\text{let } F = \langle a_1, b_1, c_1 \rangle \text{ and } G = \langle a_2, b_2, c_2 \rangle$$

$$F + G = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$$

벡터끼리 operation
⇒ 벡터

## Definition 1.1.2 Scalar product of a vetor <a, b, c> and a scalar α

정의 (증명 불필요)
= 약속

스칼라곱 벡터 스칼라 필요

$$\alpha \langle a, b, c \rangle = \langle \alpha a, \alpha b, \alpha c \rangle$$

## Theorem 1.1.1

이론 (증명 필요)

*Let $F, G$ and $H$ be vectors, and let $\alpha$ and $\beta$ be scalars, then*

1. $F + G = G + F$
2. $(F + G) + H = F + (G + H)$
3. $F + \langle 0, 0, 0 \rangle = F$
4. $\alpha(F + G) = \alpha F + \alpha G$
5. $(\alpha\beta)F = \alpha(\beta F)$
6. $(\alpha + \beta)F = \alpha F + \beta F$

Proof) Let, $F = \langle a_1, b_1, c_1 \rangle$ $G = \langle a_2, b_2, c_2 \rangle$

$F + G = \langle a_1, b_1, c_1 \rangle + \langle a_2, b_2, c_2 \rangle = \langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle$

$= \langle a_2 + a_1, b_2 + b_1, c_2 + c_1 \rangle$

$= \langle a_2, b_2, c_2 \rangle + \langle a_1, b_1, c_1 \rangle$

$= G + F$

Q.E.D

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