Representing Graphs in Python

- We will use dictionaries to represent graphs. The keys will be vertices, and the values will be a list that contains the neighbors of that vertex. For example, the graph $G = (V, E)$ with $V = \{A, B, C\}$ and $E = \{\{A, B\}, \{A, C\}, \{B, C\}\}$ is represented with the dictionary: {"A" : ["B", "C"], "B" : ["A", "C"], "C" : ["A", "B"]}.

- We will use dictionaries to represent weighted graphs. The keys will be the vertices, and the values will be a list, whose entries are lists that contain a neighbor and the weight of the edge between the vertices. For example, the weighted graph $G = (V, E)$ with $V = \{A, B, C\}$ and $E = \{\{A, B\}, \{A, C\}, \{B, C\}\}$ where $\{A, B\}$ is weighted with 1, $\{A, C\}$ is weighted with 2, and $\{B, C\}$ is weighted with 2, is represented with the dictionary: {"A" : [["B", 1], ["C", 2]], "B" : [["A", 1], ["C", 2]], "C" : [["A", 2], ["B", 2]]}.

  NOTE an edge-coloring is the same as a weighted graph, so we will represent an edge-coloring in the same way! (the above example is an edge-coloring, but NOT a proper edge-coloring).

- We will use dictionaries to represent vertex-colorings. The keys will be the vertices, and the values will be the color on that vertex. For example, given the graph $G = (V, E)$ with $V = \{A, B, C\}$ and $E = \{\{A, B\}, \{A, C\}, \{B, C\}\}$, a vertex-coloring of G where A is colored with 1, B is colored with 2, and C is colored with 2 is represented using the dictionary: {"A" : 1, "B" : 2, "C" : 2}. (note that this is a vertex-coloring, but NOT a proper vertex-coloring).

- We will use dictionaries to represent rooted trees. The keys will be the vertices, and the values will be the CHILDREN of that vertex. For example, the rooted tree with root A, that has children B and C, where B has D as a child, is represented using the dictionary: {"A" : ["B", "C"], "B" : ["D"], "C" : [], "D" : []}.

- We will use dictionaries to represent directed graphs. The keys will be the vertices, and the values will be the vertices that are reached by the key vertex from a single arc. For example, the digraph with vertices $\{A, B, C, D\}$ and arcs $(A, B), (A, C), (B, D), (C, A)$, and (D, D), is represented using the dictionary: {"A" : ["B", "C"], "B" : ["D"], "C" : ["A"], "D" : ["D"]}.

- We will use a list to represent relations. Each element of the list will be a list of length 2 that correspond to the ordered pairs of the relation. For example, the relation with ground set $X = \{A, B, C, D\}$ and relation $R = \{(A, B), (A, C), (B, D), (C, A), (D, D)\}$, is represented by the list: [["A", "B"], ["A", "C"], ["B", "D"], ["C", "A"], ["D", "D"]].