

PROPORTIONS To solve problems that relate to ratios, you can use a proportion. A **proportion** is a statement of equality of two ratios.

Key Concept

Proportion

- **Words** A proportion is an equation stating that two ratios are equal.
- **Symbols** $\frac{a}{b} = \frac{c}{d}$
- **Example** $\frac{2}{3} = \frac{6}{9}$

Consider the following proportion.

$$\frac{a}{b} = \frac{c}{d}$$

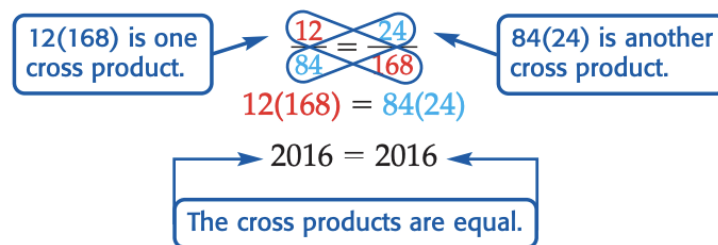
$$\frac{a}{\cancel{b}^1} \cdot \frac{1}{\cancel{d}} = \frac{c}{\cancel{d}^1} \cdot \frac{1}{\cancel{b}}$$

Multiply each side by bd to eliminate the fractions.

$$ad = cb$$

Simplify.

The products ad and cb are called the **cross products** of a proportion. Every proportion has two cross products.



Cross products can be used to determine whether two ratios form a proportion.

Key Concept

Property of Proportions

- **Words** The cross products of a proportion are equal.
- **Symbols** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$.

Example 1 Identify Proportions

Determine whether each pair of ratios forms a proportion.

a. $\frac{1}{3}, \frac{3}{9}$

$$\frac{1}{3} \stackrel{?}{=} \frac{3}{9} \quad \text{Write a proportion.}$$

$$1 \cdot 9 \stackrel{?}{=} 3 \cdot 3 \quad \text{Cross products}$$
$$9 = 9 \quad \text{Simplify.}$$

$$\text{So, } \frac{1}{3} = \frac{3}{9}.$$

b. $\frac{1.2}{4.0}, \frac{2}{5}$

$$\frac{1.2}{4.0} \stackrel{?}{=} \frac{2}{5} \quad \text{Write a proportion.}$$

$$1.2 \cdot 5 \stackrel{?}{=} 4.0 \cdot 2 \quad \text{Cross products}$$
$$6 \neq 8 \quad \text{Simplify.}$$

$$\text{So, } \frac{1.2}{4.0} \neq \frac{2}{5}.$$

Example 2 Solve Proportions

Solve each proportion.

a. $\frac{a}{25} = \frac{52}{100}$

$$\frac{a}{25} = \frac{52}{100}$$

$$a \cdot 100 = 25 \cdot 52 \quad \text{Cross products}$$

$$100a = 1300 \quad \text{Multiply.}$$

$$\frac{100a}{100} = \frac{1300}{100} \quad \text{Divide.}$$

$$a = 13$$

The solution is 13.

b. $\frac{12.5}{m} = \frac{15}{7.5}$

$$\frac{12.5}{m} = \frac{15}{7.5}$$

$$12.5 \cdot 7.5 = m \cdot 15 \quad \text{Cross products}$$

$$93.75 = 15m \quad \text{Multiply.}$$

$$\frac{93.75}{15} = \frac{15m}{15} \quad \text{Divide.}$$

$$6.25 = m$$

The solution is 6.25.

USE PROPORTIONS TO SOLVE REAL-WORLD PROBLEMS When you solve a problem using a proportion, be sure to compare the quantities in the same order.

Example 3 *Use a Proportion to Solve a Problem*

FOOD Refer to the recipe at the beginning of the lesson. How much soda should be used if 16 ounces of each type of juice are used?

Explore You know how much soda to use for 12 ounces of each type of juice. You need to find how much soda to use for 16 ounces of each type of juice.

Plan Write and solve a proportion using ratios that compare juice to soda. Let s represent the amount of soda to use in the new recipe.

Solve

$$\frac{\text{juice in original recipe}}{\text{soda in original recipe}} = \frac{\text{juice in new recipe}}{\text{soda in new recipe}}$$

$$\frac{12}{40} = \frac{16}{s} \quad \text{Write a proportion.}$$

$$12 \cdot s = 40 \cdot 16 \quad \text{Cross products}$$

$$12s = 640 \quad \text{Multiply.}$$

$$\frac{12s}{12} = \frac{640}{12} \quad \text{Divide.}$$

$$s = 53\frac{1}{3} \quad \text{Simplify.}$$

$53\frac{1}{3}$ ounces of soda should be used.

Explore Check the cross products. Since $12 \cdot 53\frac{1}{3} = 640$ and $40 \cdot 16 = 640$, the answer is correct.

Example 4 *Convert Measurements*

• ATTRACTIONS Louisville, Kentucky, is home to the world's largest baseball glove. The glove is 4 feet high, 10 feet long, 9 feet wide, and weighs 15 tons. Find the height of the glove in centimeters if $1 \text{ ft} = 30.48 \text{ cm}$.

Let x represent the height in centimeters.

$$\begin{array}{lcl} \text{customary measurement} \rightarrow & 1 \text{ ft} & \leftarrow \text{customary measurement} \\ \text{metric measurement} \rightarrow & \frac{1 \text{ ft}}{30.48 \text{ cm}} = \frac{4 \text{ ft}}{x \text{ cm}} & \leftarrow \text{metric measurement} \end{array}$$

$$1 \cdot x = 30.48 \cdot 4 \quad \text{Cross products}$$

$$x = 121.92 \quad \text{Simplify.}$$

The height of the glove is 121.92 centimeters.

Determine whether each pair of ratios forms a proportion.

3. $\frac{1}{4}, \frac{4}{16}$ $1 \times 16 \stackrel{?}{=} 4 \times 4$
 $16 = 16$ $\frac{1}{4} = \frac{4}{16}$

4. $\frac{2.1}{3.5}, \frac{3}{7}$ $2.1 \times 7 \stackrel{?}{=} 3.5 \times 3$
 $14.7 \neq 10.5$ $\frac{2.1}{3.5} \neq \frac{3}{7}$

ALGEBRA Solve each proportion.

5. $\frac{k}{35} = \frac{3}{7}$ $k \times 7 = 3 \times 35$
 $k7 = 105$
 $105/7 = 15$ $k = 15$

6. $\frac{3}{t} = \frac{18}{24}$ $3 \times 24 = 18 \times t$
 $72 = 18t$
 $72/18 = 4$ $t = 4$

7. $\frac{10}{8.4} = \frac{5}{m}$ $10 \times m = 8.4 \times 5$
 $10m = 42$
 $42/10 = 4.2$ $m = 4.2$

Determine whether each pair of ratios forms a proportion.

9. $\frac{2}{3}, \frac{8}{12}$ $2 \times 12 \stackrel{?}{=} 3 \times 8$
 $24 = 24$ $\frac{2}{3} = \frac{8}{12}$

10. $\frac{4}{2}, \frac{16}{5}$ $4 \times 5 \stackrel{?}{=} 2 \times 16$
 $20 \neq 32$ $\frac{4}{2} \neq \frac{16}{5}$

11. $\frac{1.5}{5.0}, \frac{3}{9}$ $1.5 \times 9 \stackrel{?}{=} 5.0 \times 3$
 $13.5 \neq 15$ $\frac{1.5}{5.0} \neq \frac{3}{9}$

12. $\frac{18}{2.4}, \frac{15}{2}$ $18 \times 2 \stackrel{?}{=} 2.4 \times 15$
 $36 = 36$ $\frac{18}{2.4} = \frac{15}{2}$

13. $\frac{3.4}{1.6}, \frac{5.1}{2.4}$ $3.4 \times 2.4 \stackrel{?}{=} 1.6 \times 5.1$
 $8.16 = 8.16$
 $\frac{3.4}{1.6} = \frac{5.1}{2.4}$

14. $\frac{5.3}{15.9}, \frac{2.7}{8.1}$ $2.7 \times 15.9 \stackrel{?}{=} 5.3 \times 8.1$
 $42.93 = 42.93$ $\frac{5.3}{15.9} = \frac{2.7}{8.1}$

ALGEBRA Solve each proportion.

15. $\frac{p}{6} = \frac{24}{36}$ $24 \times 6 = p36$
 $144 = p36$
 $144/36 = p$ $p = 4$

16. $\frac{w}{11} = \frac{14}{22}$ $w22 = 14 \times 11$
 $w22 = 154$
 $w = 7$

17. $\frac{4}{10} = \frac{8}{a}$ $4a = 8 \times 10$
 $4a = 80$
 $a = 20$

18. $\frac{18}{12} = \frac{24}{q}$ $18q = 24 \times 12$
 $18q = 288$
 $q = 16$

19. $\frac{5}{h} = \frac{10}{30}$ $5 \times 30 = 10h$
 $150 = 10h$
 $h = 15$

20. $\frac{51}{z} = \frac{17}{7}$ $51 \times 7 = 17z$
 $357 = 17z$
 $z = 21$

21. $\frac{7}{45} = \frac{x}{9}$ $7 \times 9 = 45x$
 $63 = 45x$
 $x = 1.4$

22. $\frac{2}{15} = \frac{c}{72}$ $2 \times 72 = 15c$
 $144 = 15c$
 $c = 9.6$

23. $\frac{7}{5} = \frac{10.5}{b}$ $7b = 10.5 \times 5$
 $7b = 52.5$
 $b = 7.5$

24. $\frac{16}{7} = \frac{4.8}{h}$ $16h = 4.8 \times 7$
 $16h = 33.6$
 $h = 2.1$

25. $\frac{2}{9.4} = \frac{0.2}{v}$ $2v = 0.2 \times 9.4$
 $2v = 1.88$
 $v = 0.94$

26. $\frac{9}{7.2} = \frac{3.5}{k}$ $9k = 3.5 \times 7.2$
 $9k = 25.2$
 $k = 2.8$

27. $\frac{a}{0.28} = \frac{4}{1.4}$ $a1.4 = 0.28 \times 4$
 $1.4a = 1.12$
 $a = 0.8$

28. $\frac{3}{14} = \frac{15}{m-3}$ $3(m-3) = 15 \times 14$
 $3m-9 = 210$
 $3m = 219$
 $m = 73$

29. $\frac{16}{x+5} = \frac{4}{5}$ $16 \times 5 = 4(x+5)$
 $80 = 4x + 20$
 $60 = 4x$
 $x = 15$

Determine whether each pair of ratios forms a proportion.

9. $\frac{2}{3}, \frac{8}{12}$ $2 \times 12 \stackrel{?}{=} 8 \times 3$
 $24 = 24$ $\boxed{\frac{2}{3} = \frac{8}{12}}$
10. $\frac{4}{2}, \frac{16}{5}$ $4 \times 5 \stackrel{?}{=} 16 \times 2$
 $20 \neq 32$ $\boxed{\frac{4}{2} \neq \frac{16}{5}}$
11. $\frac{1.5}{5.0}, \frac{3}{9}$ $1.5 \times 9 \stackrel{?}{=} 3 \times 5.0$
 $13.5 \neq 15$ $\boxed{\frac{1.5}{5.0} \neq \frac{3}{9}}$
12. $\frac{18}{2.4}, \frac{15}{2}$ $18 \times 2 \stackrel{?}{=} 15 \times 2.4$
 $36 = 36$ $\boxed{\frac{18}{2.4} = \frac{15}{2}}$
13. $\frac{3.4}{1.6}, \frac{5.1}{2.4}$ $3.4 \times 2.4 \stackrel{?}{=} 5.1 \times 1.6$
 $8.16 \neq 8.16$ $\boxed{\frac{3.4}{1.6} \neq \frac{5.1}{2.4}}$
14. $\frac{5.3}{15.9}, \frac{2.7}{8.1}$ $5.3 \times 8.1 \stackrel{?}{=} 15.9 \times 2.7$
 $42.93 = 42.93$ $\boxed{\frac{5.3}{15.9} = \frac{2.7}{8.1}}$

ALGEBRA Solve each proportion.

15. $\frac{p}{6} = \frac{24}{36}$ $36p = 144$
 $\boxed{p = 48}$
16. $\frac{w}{11} = \frac{14}{22}$ $w22 = 154$
17. $\frac{4}{10} = \frac{8}{a}$
18. $\frac{18}{12} = \frac{24}{q}$ $18q = 288$
19. $\frac{5}{h} = \frac{10}{30}$ $150 = 10h$
20. $\frac{51}{z} = \frac{17}{7}$
21. $\frac{7}{45} = \frac{x}{9}$ $63 = 45x$
22. $\frac{2}{15} = \frac{c}{72}$ $144 = 15c$
23. $\frac{7}{5} = \frac{10.5}{b}$
24. $\frac{16}{7} = \frac{4.8}{h}$ $16h = 33.6$
25. $\frac{2}{9.4} = \frac{0.2}{v}$ $1.88 = 2v$
26. $\frac{9}{7.2} = \frac{3.5}{k}$
27. $\frac{a}{0.28} = \frac{4}{1.4}$ $1.4a = 1.12$
28. $\frac{3}{14} = \frac{15}{m-3}$ $3(m-3) = 210$
 $3m-9 = 210$
29. $\frac{16}{x+5} = \frac{4}{5}$