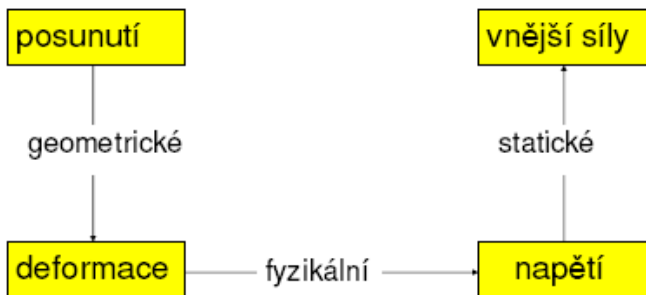


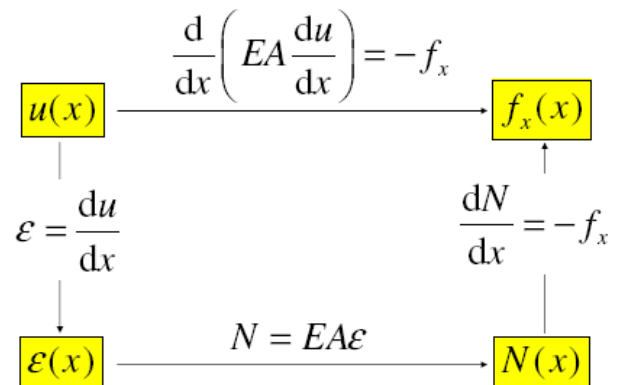
# MKP: 1D prutové prvky

## 1) Diferenciální rovnice úlohy

Základní rovnice – model kontinua



Základní rovnice  
(prut namáhaný tahem-tlakem)



## 2) Slabé řešení – variační formulace

$$\int_{\Omega} \delta u \left( \frac{d}{dx} \left( EA \frac{du}{dx} + f_x \right) \right) dx = 0$$

Integrací per partes

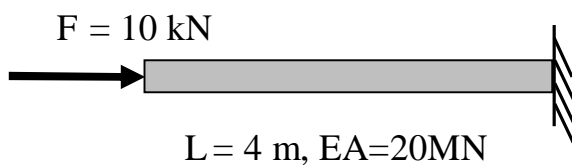
$$\int_{\Gamma} \delta u EA \frac{du}{dx} n dx - \int_{\Omega} \frac{d\delta u}{dx} EA \frac{du}{dx} dx + \int_{\Omega} \delta u f_x dx = 0$$

$$u^e(x) = \mathbf{N}^e(x) \mathbf{r}^e \rightarrow \frac{du^e}{dx} = \frac{\mathbf{N}^e(x)}{dx} \mathbf{r}^e = \mathbf{B}^e(x) \mathbf{r}^e$$

$$\delta u^e(x) = \mathbf{N}^e(x) \mathbf{w}^e \rightarrow \frac{d\delta u^e}{dx} = \frac{\mathbf{N}^e(x)}{dx} \mathbf{w}^e = \mathbf{B}^e(x)$$

$$\sum_{e=1}^n \mathbf{w}^{eT} \left\{ \underbrace{\int_{x_1^e}^{x_2^e} \mathbf{B}^{eT} EA \mathbf{B}^e dx}_{\mathbf{K}^e} \mathbf{r}^e - \underbrace{\int_{x_1^e}^{x_2^e} \mathbf{N}^{eT} f_x dx}_{\mathbf{f}_{\Omega}} - \underbrace{(\mathbf{N}^{eT} \bar{\mathbf{t}})_{\Gamma_t}}_{\mathbf{f}_{\Gamma_t}} \right\} = 0$$

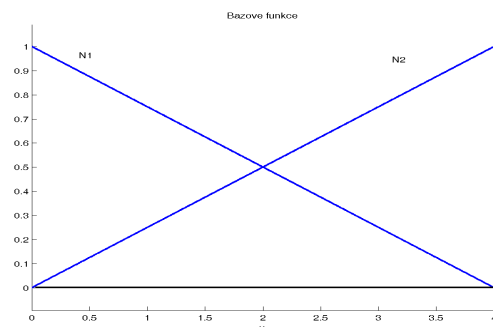
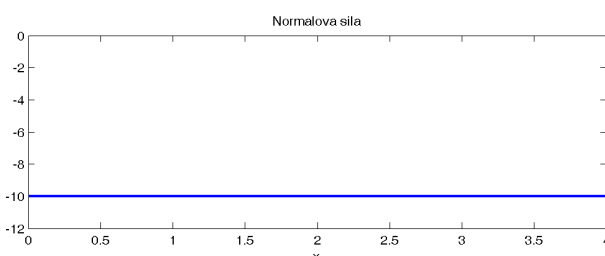
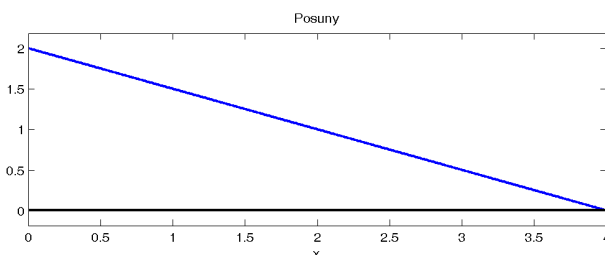
### Příklad 1



Integrací základních rovnic:

$$S = -10 \text{ kN}$$

$$u = 2 - x/2, \quad u(0) = 2 \text{ mm}, \quad u(4) = 0$$



MKP: 1 lineární prvek:

$$\mathbf{N} = [1 - 1/4 * x, \quad 1/4 * x]$$

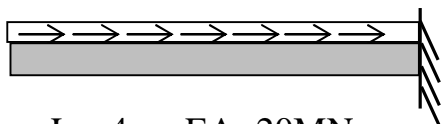
$$\mathbf{B} = d\mathbf{N}/dx = [-1/4, \quad 1/4]$$

$$\mathbf{B}^T \mathbf{D} \mathbf{B} = \begin{bmatrix} 5/4 & -5/4 \\ -5/4 & 5/4 \end{bmatrix} \quad \mathbf{K} = \int \mathbf{B}^T \mathbf{D} \mathbf{B} dx = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}$$

$$\mathbf{f} = [10, 0] \text{ kN}$$

$$\mathbf{r} = [2, 0] \text{ mm}$$

## Příklad 2 $f = 2 \text{ kN/m}$



$$L = 4 \text{ m}, EA = 20 \text{ MN}$$

### Integrací základních rovnic (Obr. 1 a 2):

$$S = -2 \cdot x \text{ kN} \quad S(4) = -8 \text{ kN}$$

$$u = 1/20 \cdot (16 - x^2), \quad u(0) = 0.8 \text{ mm}, u(4) = 0$$

### MKP: 1 lineární prvek (Obr. 3 a 4):

$$\mathbf{f} = \int \mathbf{N}^T \mathbf{f} \, dx = [4, 4] \text{ kN} \quad \mathbf{r} = [0.8, 0] \text{ mm}$$

$$S = -4 \text{ kN}$$

### MKP: 2 lineární prvky (Obr. 5, 6 a 7):

$$\mathbf{N} = [-1/2 \cdot x + 1, \quad 1/2 \cdot x] \quad \mathbf{B} = [-1/2, \quad 1/2]$$

$$\mathbf{B}^T \mathbf{D} \mathbf{B} = \begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \quad \mathbf{K}_i = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} 10 & -10 & 0 \\ -10 & 20 & -10 \\ 0 & -10 & 10 \end{bmatrix}$$

$$\mathbf{f}_i = [2, 2] \quad \mathbf{f} = [2, \quad 4, \quad 2]$$

$$\mathbf{r} = [0.8, \quad 0.6, \quad 0]$$

$$S_1 = -2 \text{ kN}, \quad S_2 = -6 \text{ kN}$$

### MKP: 1 kvadratický prvek (Obr. 8, 1 a 2):

$$\mathbf{N} = [1/8 \cdot (x-2) \cdot (x-4), -1/4 \cdot x \cdot (x-4), 1/8 \cdot x \cdot (x-2)]$$

$$\mathbf{B} = [1/4 \cdot x - 3/4, -1/2 \cdot x + 1, 1/4 \cdot x - 1/4]$$

$$\mathbf{B}^T \mathbf{D} \mathbf{B} =$$

$$[5/4 \cdot (x-3)^2, -5/2 \cdot (x-3) \cdot (x-2), 5/4 \cdot (x-3) \cdot (x-1)]$$

$$[-5/2 \cdot (x-3) \cdot (x-2), 5 \cdot (x-2)^2, -5/2 \cdot (x-2) \cdot (x-1)]$$

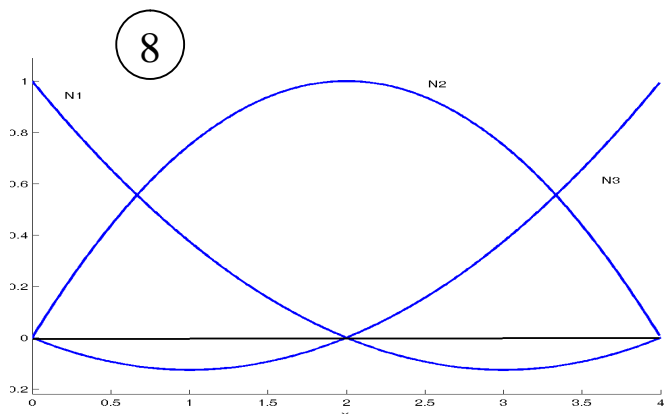
$$[5/4 \cdot (x-3) \cdot (x-1), -5/2 \cdot (x-2) \cdot (x-1), 5/4 \cdot (x-1)^2]$$

$$\mathbf{K} = \begin{bmatrix} 35/3 & -40/3 & 5/3 \\ -40/3 & 80/3 & -40/3 \\ 5/3 & -40/3 & 35/3 \end{bmatrix}$$

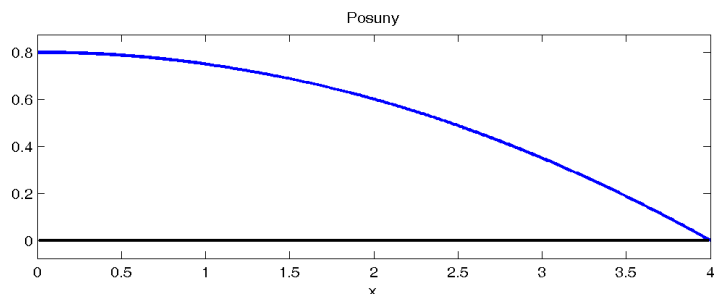
$$\mathbf{f} = [4/3, 16/3, 4/3]$$

$$\mathbf{r} = [0.8, \quad 0.6, \quad 0]$$

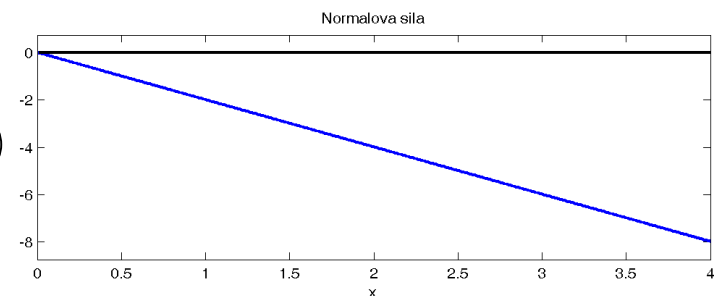
$$S = -2 \cdot x$$



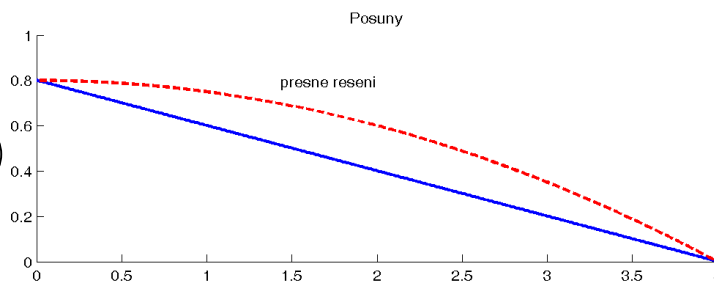
1



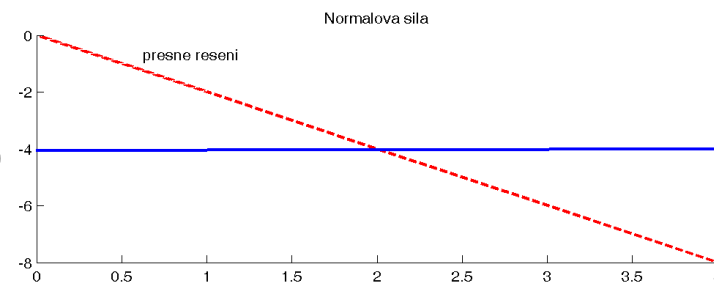
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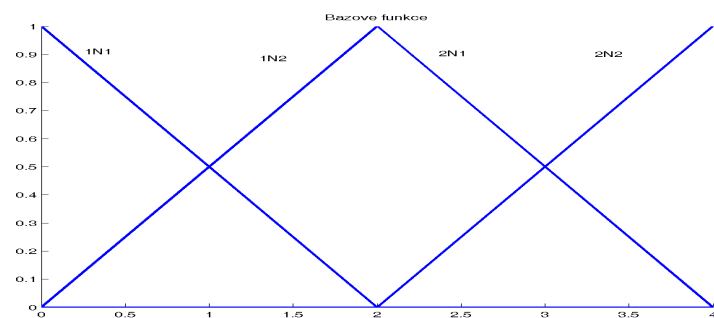
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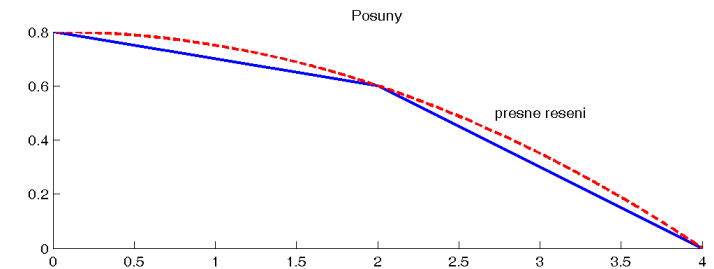
4



5



6



7

