NONLINEAR FINITE ELEMENT METHODS

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Material assembled from lecture notes for the course **Nonlinear Finite Elements Methods**, offered since 1987 to date at the Aerospace Engineering Sciences Department of the University of Colorado at Boulder.

Preface

This textbook presents an Introduction to the computer-based simulation of nonlinear structures by the Finite Element Method (FEM). It assembles the still "unconverged" lecture notes of **Nonlinear Finite Element Methods** or NFEM. This is an advanced graduate course offered in the Aerospace Engineering Sciences of the University of Colorado at Boulder.

NFEM was first taught on the Spring Semester 1986 and has been repeated every two or three years. Unlike the **Introduction to Finite Element Methods** (IFEM), NFEM is not a core course. It is typicall taken by second year graduate students that are interested in the topic of nonlinear simulation of mechanical systems.

Prerequisites for the course are an introductory course in finite elements such as IFEM, graduate-level calculus, linear algebra, knowledge of structural mechanics at the Mechanics of Materials level, and ability to program in a higher level language such as *Matlab* or *Mathematica*.

The course originally used Fortran 77 as computer implementation language. This has been gradually changed to *Mathematica* since 1995. Unlike IFEM the changeover is not yet complete since the course has been offered only twice since.

Book Objectives

(To be completed)

Book Organization

(To be completed)

Exercises

Each Chapter is followed by a list of homework exercises that pose problems of varying difficulty. Each exercise is labeled by a tag of the form

`[type:rating]`

The type is indicated by letters A, C, D or N for exercises to be answered primarily by analytical work, computer programming, descriptive narration, and numerical calculations, respectively. Some exercises involve a combination of these traits, in which case a combination of letters separated by + is used, e.g., A+N. For some problems heavy analytical work may be helped by the use of a computer-algebra system, in which case the type is identified as A/C.

The rating is a number between 5 and 50 that estimates the degree of difficulty of an Exercise, in the following "logarithmic" scale:

5 A simple question that can be answered in seconds, or is already answered in the text if the student has read and understood the material.

10 A straightforward question that can be answered in minutes.

15 A relatively simple question that requires some thinking, and may take on the order of half to one hour to answer.

20 Either a problem of moderate difficulty, or a straightforward one requiring lengthy computations or some programming, normally taking one to six hours of work.

25 A scaled up version of the above, estimated to require six hours to one day of work.

30 A problem of moderate difficulty that normally requires on the order of one or two days of work. Arriving at the answer may involve a combination of techniques, some background or reference material, or lenghty but straightforward programming.

40 A difficult problem that may be solvable only by gifted and well prepared individual students, or a team. Difficulties may be due to the need of correct formulation, advanced mathematics, or high level programming. With the proper preparation, background and tools these problems may be solved in hours or days, while remaining inaccessible to unprepared or average students.

50 A research problem, worthy of publication if solved.

Most Exercises have a rating of 15 or 20. Assigning three or four per week puts a load of roughly 5-10 hours of solution work, plus the time needed to prepare the answer material. Assignments of difficulty 25 or 30 are better handled by groups, or given in take-home exams. Assignments of difficulty over 30 are never assigned in the course, but provided as a challenge for an elite group.

Occasionally an Exercise has two or more distinct but related parts identified as items. In that case a rating may be given for each item. For example: [A/C:15+20]. This does not mean that the exercise as a whole has a difficulty of 35, because the scale is roughly logarithmic; the numbers simply rate the expected effort per item.

Selecting Course Material

(To be completed)

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