# Pairwise-Sequence Alignment BIFX-550

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## Agenda

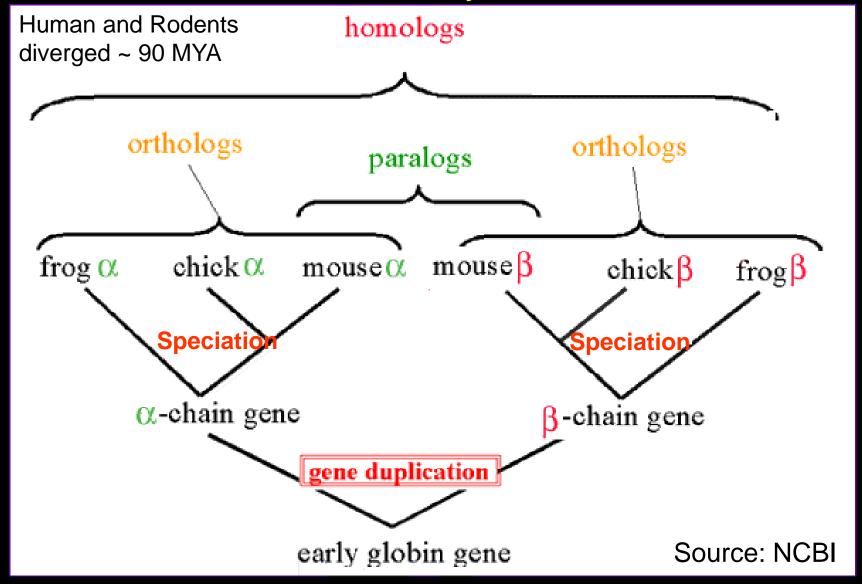
- Please make sure you have created your galaxy account; also check to make you can login
- Homology
  - Orthologs, Paralogs, Xenologs
- Scoring Matrices
  - PAM, BLOSUM
- Dynamic Programming
  - Global and Local Alignment
- Pairwise Alignment of DNA/Protein using NCBI Server

- Relatedness (homology) among proteins/DNAs
  - Common function?

- Homology (common ancestor)
  - When two sequences (proteins/genes) are highly similar, they might be <u>homologous</u>
  - Converse is not true (lack of similarity != No Homology)

– What is homology?

## Source from NCBI but modified by Dr. S. Ravichandran



## Homology

- Homology: implies evolutionary relationship
- Common ancestor
- Not measured in degrees
- Means either 2 genes/sequences are related or not
- Publications
  - Walter Fitch and Eugene Koonin

Query: h-HBB Subjct: h-Mb

# Example of Sequence Alignment

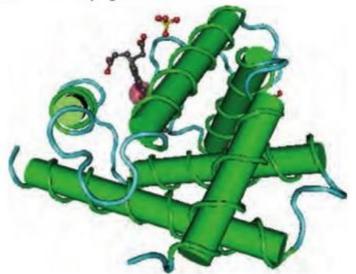
Query	4	LTPEEKSAVTALWGKVNVDEVGGEALGRLLVVYPWTQRFFESFGDLSTPDAVMGNPKV	61
NP_005359	3	.SDG.WQL.LNVEA.IPGH.Q.V.IFKGH.E.LEK.DK.KH.KSE.EMKASEDL	62
~ 1	62 63	KAHGKKVLGAFSDGLAHLDNLKGTFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHFGK .KATT.LGGI.KKKGHHEAEIKP.AQS.AT.HKIPVKYLEFISECIIQQSKHPG	121 122
Query	122	EFTPPVQAAYQKVVAGVANALAHKY 146	
NP 005359	123	D.GADA.G.MN.ALELFRKDM.SN. 147	

Score	Expect	Method	Identities	<b>Positives</b>	Gaps
43.1 bits(94)	1e-09	Compositional matrix adjust.	37/145(26%)	43/145(29%)	2/145(1%)

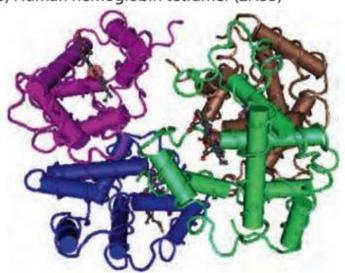
??? Gap Similar Identical Score

#### **Homologous proteins example**

(a) Human myoglobin (3RGK)



(b) Human hemoglobin tetramer (2H35)



#### **PLEASE DO NOT DISTRIBUTE-Copyright figure**

(c) Human beta globin (subunit of 2H35)

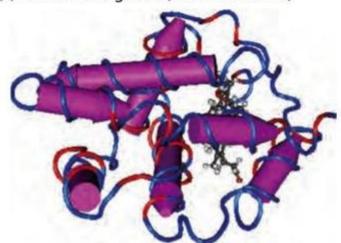
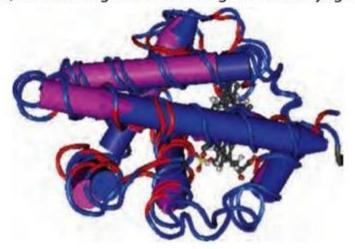


Figure 3.1 Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner

(d) Pairwise alignment of beta globin and myoglobin



Very limited sequence similarity

# How to find out whether two proteins are related?

Sequence Alignment

Comparing 3D is one way
Not all 3D information is available
So, Sequence Relationship is the common approach

## What sequences to use for alignments?

- Why protein (not DNA?) sequences?
  - Protein aa: 20 letters
  - DNA/RNA nt: 4 letters

```
CCU
CCC
CCA
CCG
```

- More information (?) in the protein sequence
  - Models using proteins can look back (identify ancestors) 1BYA (*Prof. Pearson several papers*); DNA (600 MYA)
    - glutathione transferases

# How to identify true(?) hits?



Query	303	SDVICQSEPDDSFPSSGSVSLYEVERCQQLSATILTDHQYLERTPLCAILKQKAPQQ	359
		+CQSE +DSF + S LYEVERCQQLSATILTDHQYLE+TPLCAILKQ APQQ	
Sbjct	301	FGGVCQSEQEDSFSNISSSGSVSLYEVERCQQLSATILTDHQYLEKTPLCAILKQNAPQQ	360

Query: human POT1

Sbjct: Bos mutus POT1 (Wild Yak)

Note only part of the sequence alignments are shown

#### Query: human POT1

#### conserved hypothetical protein [Trichinella spiralis]

Sequence ID: ref|XP 003378812.1|Length: 382Number of Matches: 1

Trichinella spiralis is a nematode parasite, occurring in rodents, pigs, horses, bears, and humans, and is responsible for the disease trichinosis.

```
Range 1: 238 to 341GenPeptGraphicsNext MatchPrevious Match
Alignment statistics for match #1
                                   Identities
                                               Positives
56.6 bits(135) 2e-05
                       Compositional matrix adjust. 34/111(31%)
                                                           57/111 (51%)
                                                                       9/111(8%)
Ouerv
              SFLLKVWDGTR--TPFPSWRVLIODLVLEGDLSHIHRLONLTIDILVYDNHVHVARSLKV
                                                                                        104
               ++T+VWDG+
                                                                                        295
        238
              GWILRVWDGSSPATSFKLDSVNIDGFTADEELSL--KAENFAADVFLYDEHCTVAKALKP
                                                                             155
Query
        105
              GSFLRIYSLHTKLQSMNSENQTMLSLEFHLHGGTSYGRGIRVLPESNSDVD
                                             +F +H G SYGR ++++
Sbict
        296 GDFVILYNLHLYYPYGGRSN-----CQFTMHSGNSYGRRVQLISADDELVN
```

?????

### Can't we manually align sequences?

- Works when
  - We have closely related sequences
  - Smaller number of sequences

```
Query 4 LTPEEKSAVTALWGKVNVD--EVGGEALGRLLVVYPWTQRFFESFGDLSTPDAVMGNPKV 61

NP 005359 3 SDC WOL LNV...EA.IPGH.Q.V.I..FKGH.E.LEK.DK.KH.KSE.EMKASEDL 62

Query 62 KAHGKKVLGAFSDGLAHLDNLKGTFATLSELHCDKLHVDPENFRLLGNVLVCVLAHHFGK 121

NP 005359 63 K. AT IT LGGI KKKGHHEAEIKP AOS.AT HKIPWKYLEFISECIIO OSKHPG 122

We have to align query against a DB (modest Size)

Query 122 EFTPPVQAAYQKVVAGVANALAHKY 146

NP 0053190 129 D.GADA.G.MN.ALELFRKDM.SN. 147
```

- Usual situation
- We need algorithms for alignment/evaluation
  - Math/Statistics

## Goal of Sequence Alignment

- To extract the information whether the two sequences have similar aa/nt in proper order and to access whether they are homologous
- Gaps indicate what?
  - To capture the evolution of the sequence
  - In that process allowing for Insertions, Deletions and substitutions
- To access the alignments, we need to score each residue alignment
  - Identical, similar or gap (creation & extension penalty)
  - %identity/similarity etc.

# Simple Match/Mismatch Scoring Matrix

Simple scoring matrix

Match: +2; Mismatch: -3

25% Probability of each NT occurrence

Note
A → A +2
T → T +2
(Not the same for amino acids)
Equal probability so scores are same

	A	T	G	С
Α	+2	-3	-3	-3
Т	-3	+2	-3	-3
G	-3	-3	+2	-3
С	-3	-3	-3	+2

Same points for mismatches: -3

Negative Score

Common same substitution; A  $\rightarrow$  A: 5 Less common; W  $\rightarrow$  13

**PAM70** 

**YW:7** 

**WW:13** 

	A	R	N	D	С	Õ	E	G	Н	I	L	K	M	F	P	S	T	W	Y	V	В	Z	X
A	5	-4	-2	-1	-4	-2	-1	0	-4	-2	-4	-4	-3	-6	0	1	1	-9	-5	-1	-1	-1	-2
R	-4	8	-3	-6	-5	0	-5	-6	0	-3	-6	2	-2	-7	-2	-1	-4	0	-7	-5	-4	-2	-3
N	-2	-3	6	3	-7	-1	0	-1	1	-3	-5	0	-5	-6	-3	1	0	-6	-3	-5	5	-1	-2
D	-1	-6	3	6	-9	0	3	-1	-1	-5	-8	-2	-7	-10	-4	-1	-2	-10	-7	-5	5	2	-3
С	-4	-5	-7	-9	9	-9	-9	-6	-5	-4	-10	-9	-9	-8	-5	-1	-5	-11	-2	-4	-8	-9	-6
Q	-2	0	-1	0	-9	7	2	-4	2	-5	-3	-1	-2	-9	-1	-3	-3	-8	-8	-4	-1	5	-2
E	-1	-5	0	3	-9	2	6	-2	-2	-4	-6	-2	-4	-9	-3	-2	-3	-11	-6	-4	2	5	-3
G	0	-6	-1	-1	-6	-4	-2	6	-6	-6	-7	-5	-6	-7	-3	0	-3	-10	-9	-3	-1	-3	-3
Н	-4	0	1	-1	-5	2	-2	-6	8	-6	-4	-3	-6	-4	-2	-3	-4	-5	-1	-4	0	1	-3
I	-2	-3	-3	-5	-4	-5	-4	-6	-6	7	1	-4	1	0	-5	-4	-1	-9	-4	3	-4	-4	-3
L	-4	-6	-5	-8	-10	-3	-6	-7	-4	1	6	-5	2	-1	-5	-6	-4	-4	-4	0	-6	-4	-4
K	-4	2	0	-2	-9	-1	-2	-5	-3	-4	-5	6	0	-9	-4	-2	-1	-7	-7	-6	-1	-2	-3
М	-3	-2	-5	-7	-9	-2	-4	-6	-6	1	2	0	10	-2	-5	-3	-2	-8	-7	0	-6	-3	-3
F	-6	-7	-6	-10	-8	-9	-9	-7	-4	0	-1	-9	-2	8	-7	-4	-6	-2	4	-5	-7	-9	-5
P	0	-2	-3	-4	-5	-1	-3	-3	-2	-5	-5	-4	-5	-7	7	0	-2	-9	-9	-3	-4	-2	-3
S	1	-1	1	-1	-1	-3	-2	0	-3	-4	-6	-2	-3	-4	0	5	2	-3	-5	-3	0	-2	-1
Т	1	-4	0	-2	-5	-3	-3	-3	-4	-1	-4	-1	-2	-6	-2	2	6	-8	-4	-1	-1	-3	-2
W	-9	0	-6	-10	-11	-8	-11	-10	-5	-9	-4	-7	-8	-2	-9	-3	-8	13	-3	-10	-7	-10	-7
Y	-5	-7	-3	-7	-2	-8	-6	-9	-1	-4	-4	-7	-7	4	-9	-5	-4	-3	9	-5	-4	-7	-5
V	-1	-5	-5	-5	-4	-4	-4	-3	-4	3	0	-6	0	-5	-3	-3	-1	-10	-5	6	-5	-4	-2
В	-1	-4	5	5	-8	-1	2	-1	0	-4	-6	-1	-6	-7	-4	0	-1	-7	-4	-5	5	1	-2
Z	-1	-2	-1	2	-9	5	5	-3	1	-4	-4	-2	-3	-9	-2	-2	-3	-10	-7	-4	1	5	-3
X	-2	-3	-2	-3	-6	-2	-3	-3	-3	-3	-4	-3	-3	-5	-3	-1	-2	-7	-5	-2	-2	-3	-3

http://www.sbcs.qmul.ac.uk/iupac/AminoAcid/A2021.html#AA212

#### Probability Matrix

### PAM1

PAM1	Α	R	N	D	С	Q	E	G	Н	ı	L	K	М	F	Р	S	Т	W	Υ	V
Α	0.9890	0.0004	0.0003	0.0004	0.0003	0.0004	0.0008	0.0011	0.0001	0.0002	0.0005	0.0005	0.0003	0.0001	0.0006	0.0023	0.0011	0.0000	0.0001	0.0015
R	0.0005	0.9907	0.0004	0.0002	0.0001	0.0011	0.0005	0.0004	0.0004	0.0001	0.0004	0.0033	0.0001	0.0000	0.0002	0.0005	0.0005	0.0001	0.0002	0.0002
N	0.0005	0.0005	0.9888	0.0021	0.0001	0.0007	0.0006	0.0009	0.0007	0.0002	0.0002	0.0013	0.0001	0.0001	0.0002	0.0017	0.0011	0.0000	0.0002	0.0001
D	0.0006	0.0002	0.0018	0.9905	0.0000	0.0005	0.0030	0.0007	0.0003	0.0000	0.0000	0.0005	0.0000	0.0000	0.0002	0.0008	0.0006	0.0000	0.0001	0.0000
С	0.0012	0.0002	0.0002	0.0000	0.9946	0.0001	0.0000	0.0002	0.0001	0.0002	0.0003	0.0000	0.0002	0.0003	0.0000	0.0009	0.0004	0.0001	0.0003	0.0008
Q	0.0009	0.0016	0.0008	0.0007	0.0000	0.9856	0.0028	0.0004	0.0009	0.0002	0.0008	0.0022	0.0004	0.0001	0.0005	0.0009	0.0008	0.0001	0.0001	0.0003
Е	0.0011	0.0004	0.0005	0.0028	0.0000	0.0018	0.9890	0.0003	0.0003	0.0001	0.0002	0.0015	0.0001	0.0000	0.0003	0.0007	0.0005	0.0000	0.0001	0.0004
G	0.0012	0.0003	0.0006	0.0005	0.0001	0.0002	0.0002	0.9952	0.0001	0.0000	0.0001	0.0002	0.0000	0.0000	0.0001	0.0008	0.0002	0.0000	0.0000	0.0001
Н	0.0005	0.0008	0.0013	0.0006	0.0001	0.0014	0.0007	0.0003	0.9895	0.0002	0.0003	0.0008	0.0002	0.0004	0.0002	0.0006	0.0007	0.0001	0.0013	0.0002
1	0.0002	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000	0.0001	0.9878	0.0035	0.0002	0.0010	0.0005	0.0001	0.0001	0.0006	0.0000	0.0001	0.0051
L	0.0005	0.0002	0.0001	0.0000	0.0001	0.0003	0.0001	0.0001	0.0001	0.0022	0.9919	0.0002	0.0012	0.0010	0.0002	0.0002	0.0002	0.0001	0.0002	0.0014
K	0.0006	0.0030	0.0010	0.0005	0.0000	0.0014	0.0015	0.0003	0.0003	0.0002	0.0003	0.9883	0.0002	0.0000	0.0003	0.0007	0.0009	0.0000	0.0001	0.0003
М	0.0009	0.0002	0.0001	0.0000	0.0001	0.0006	0.0003	0.0001	0.0002	0.0026	0.0048	0.0005	0.9859	0.0009	0.0000	0.0004	0.0007	0.0001	0.0002	0.0012
F	0.0002	0.0000	0.0001	0.0000	0.0001	0.0001	0.0000	0.0000	0.0002	0.0007	0.0022	0.0001	0.0005	0.9923	0.0001	0.0001	0.0002	0.0003	0.0022	0.0005
Р	0.0010	0.0003	0.0002	0.0003	0.0000	0.0004	0.0004	0.0002	0.0001	0.0001	0.0004	0.0004	0.0000	0.0000	0.9943	0.0008	0.0007	0.0000	0.0001	0.0002
S	0.0029	0.0005	0.0013	0.0007	0.0003	0.0005	0.0007	0.0010	0.0002	0.0001	0.0003	0.0006	0.0002	0.0001	0.0006	0.9862	0.0033	0.0000	0.0002	0.0003
Т	0.0014	0.0005	0.0008	0.0005	0.0001	0.0005	0.0005	0.0002	0.0002	0.0005	0.0004	0.0009	0.0003	0.0001	0.0005	0.0032	0.9879	0.0000	0.0001	0.0014
W	0.0001	0.0004	0.0001	0.0000	0.0001	0.0001	0.0001	0.0002	0.0001	0.0002	0.0005	0.0001	0.0001	0.0010	0.0000	0.0002	0.0001	0.9956	0.0010	0.0001
Υ	0.0002	0.0003	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0009	0.0002	0.0005	0.0002	0.0001	0.0028	0.0001	0.0004	0.0002	0.0004	0.9924	0.0004
V	0.0017	0.0002	0.0001	0.0000	0.0002	0.0002	0.0003	0.0001	0.0001	0.0042	0.0019	0.0002	0.0004	0.0003	0.0002	0.0002	0.0012	0.0000	0.0002	0.9884

#### Probability Matrix

## **PAM250**

PAM250	Α	R	N	D	С	Q	E	G	Н	1	L	K	М	F	Р	S	Т	W	Υ	٧
Α	0.1350	0.0460	0.0425	0.0501	0.0212	0.0359	0.0580	0.0827	0.0194	0.0480	0.0717	0.0535	0.0195	0.0239	0.0484	0.0774	0.0707	0.0057	0.0195	0.0708
R	0.0677	0.1583	0.0485	0.0496	0.0114	0.0530	0.0645	0.0583	0.0271	0.0330	0.0566	0.1097	0.0154	0.0193	0.0367	0.0580	0.0587	0.0090	0.0215	0.0437
N	0.0733	0.0568	0.1092	0.0881	0.0124	0.0442	0.0722	0.0801	0.0310	0.0304	0.0473	0.0713	0.0138	0.0202	0.0366	0.0741	0.0687	0.0057	0.0235	0.0413
D	0.0733	0.0493	0.0747	0.1593	0.0091	0.0468	0.1095	0.0752	0.0259	0.0239	0.0374	0.0662	0.0114	0.0143	0.0385	0.0671	0.0616	0.0040	0.0173	0.0352
С	0.0884	0.0323	0.0299	0.0260	0.2660	0.0215	0.0293	0.0466	0.0173	0.0446	0.0673	0.0311	0.0185	0.0340	0.0225	0.0619	0.0544	0.0103	0.0293	0.0689
Q	0.0748	0.0751	0.0534	0.0666	0.0107	0.0705	0.0863	0.0585	0.0309	0.0375	0.0653	0.0840	0.0183	0.0224	0.0434	0.0625	0.0621	0.0070	0.0220	0.0486
E	0.0781	0.0589	0.0563	0.1007	0.0094	0.0558	0.1334	0.0608	0.0256	0.0314	0.0492	0.0774	0.0144	0.0165	0.0410	0.0636	0.0605	0.0048	0.0175	0.0446
G	0.0884	0.0424	0.0496	0.0549	0.0119	0.0300	0.0483	0.3387	0.0170	0.0208	0.0339	0.0454	0.0103	0.0124	0.0318	0.0657	0.0478	0.0052	0.0130	0.0326
н	0.0647	0.0616	0.0601	0.0591	0.0138	0.0496	0.0635	0.0530	0.0946	0.0353	0.0612	0.0667	0.0170	0.0399	0.0353	0.0578	0.0580	0.0108	0.0539	0.0440
1	0.0649	0.0304	0.0239	0.0221	0.0145	0.0243	0.0315	0.0264	0.0143	0.1460	0.1779	0.0358	0.0403	0.0510	0.0251	0.0394	0.0542	0.0086	0.0276	0.1415
L	0.0597	0.0320	0.0228	0.0213	0.0134	0.0261	0.0305	0.0264	0.0153	0.1094	0.2390	0.0359	0.0435	0.0649	0.0271	0.0368	0.0462	0.0110	0.0327	0.1060
К	0.0714	0.0995	0.0552	0.0604	0.0100	0.0538	0.0768	0.0567	0.0267	0.0354	0.0576	0.1240	0.0165	0.0191	0.0396	0.0616	0.0635	0.0059	0.0200	0.0464
М	0.0671	0.0361	0.0275	0.0268	0.0153	0.0303	0.0370	0.0332	0.0175	0.1027	0.1798	0.0425	0.0608	0.0583	0.0259	0.0439	0.0535	0.0104	0.0311	0.1004
F	0.0461	0.0253	0.0225	0.0188	0.0157	0.0208	0.0237	0.0224	0.0231	0.0726	0.1501	0.0276	0.0326	0.2041	0.0191	0.0315	0.0372	0.0301	0.1054	0.0712
P	0.0834	0.0429	0.0366	0.0453	0.0093	0.0359	0.0525	0.0512	0.0182	0.0320	0.0561	0.0510	0.0130	0.0171	0.2614	0.0656	0.0632	0.0041	0.0160	0.0454
S	0.1006	0.0512	0.0558	0.0597	0.0193	0.0390	0.0614	0.0799	0.0225	0.0379	0.0575	0.0600	0.0166	0.0213	0.0495	0.0997	0.0862	0.0062	0.0213	0.0545
Т	0.0899	0.0507	0.0507	0.0536	0.0166	0.0379	0.0572	0.0569	0.0221	0.0510	0.0707	0.0605	0.0198	0.0246	0.0467	0.0844	0.1101	0.0058	0.0211	0.0698
w	0.0342	0.0366	0.0197	0.0164	0.0149	0.0201	0.0215	0.0291	0.0194	0.0381	0.0793	0.0264	0.0181	0.0937	0.0144	0.0285	0.0274	0.3398	0.0843	0.0382
Υ	0.0468	0.0351	0.0326	0.0284	0.0169	0.0253	0.0312	0.0292	0.0387	0.0489	0.0942	0.0360	0.0217	0.1312	0.0222	0.0393	0.0397	0.0336	0.1955	0.0535
V	0.0803	0.0337	0.0271	0.0273	0.0187	0.0264	0.0376	0.0346	0.0149	0.1185	0.1443	0.0394	0.0330	0.0419	0.0299	0.0475	0.0622	0.0072	0.0253	0.1501

# How to extract GONNET matrices using R?

```
#install.packages("TKF")
library(TKF)
data("GONNET")
PAM1 <- PAMn(GONNET,1)
round(PAM1[,1:20],3)
PAM250 <- PAMn(GONNET, 250)
round(PAM250[,1:20],3)
```

GONNET is an extension of PAM matrices

### **Substitution Matrices**

To create an alignment and to identify homologous sequences, we need a **score**. What score should we assign?

## Model/Probability Model

- We need Probability to get through this part
- Why?
  - Aligning two sequences; What is the prob. of this alignment compared to other alignments?
  - Random sequence model or Null Model (base model to compare with anything)
    - $X_1...X_q$ ;
    - Null prob  $q_{x1}q_{x2}...q_{xn} = \prod_{i=1}^{n} q_{x_i}$

## Dayhoff Matrix in 7 Steps

- 1978
- Step 1 of 7:
  - What mutations are accepted in closely related sequences
    - Model Accepted Point Mutation
      - Easier name: Point Accepted Mutation (PAM)
  - Collected closely (85% or >) sequences
    - Ungapped MSA
  - Used phylogenetic trees rather than comparing two sequences directly

### Step 1 of 7

#### PAUP was used for Phylogenetic Analysis

#### Sequence Alignment of Human globins/myoglobins

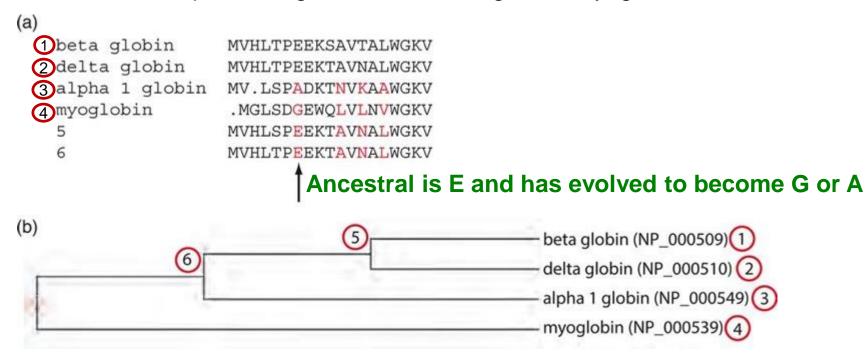
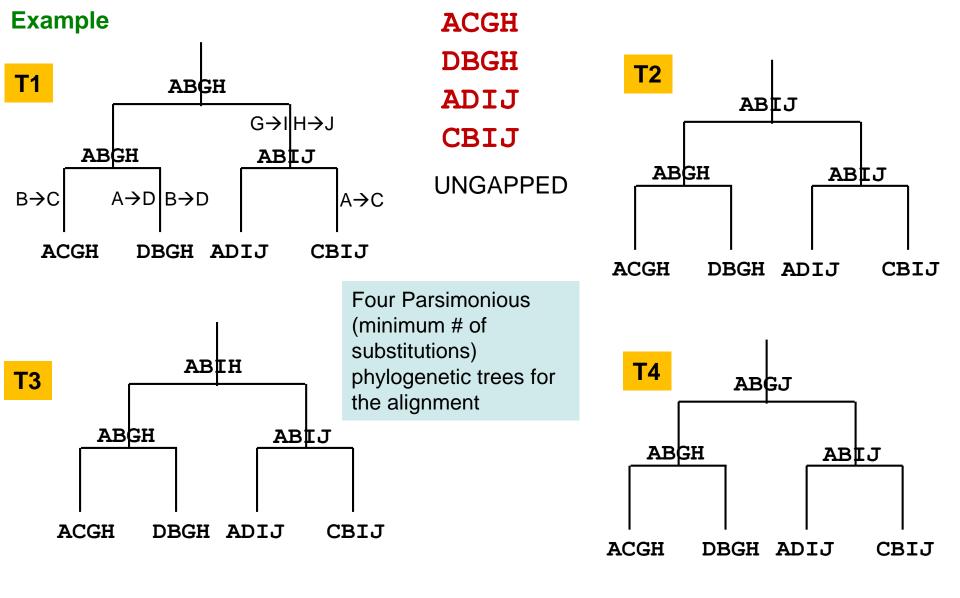


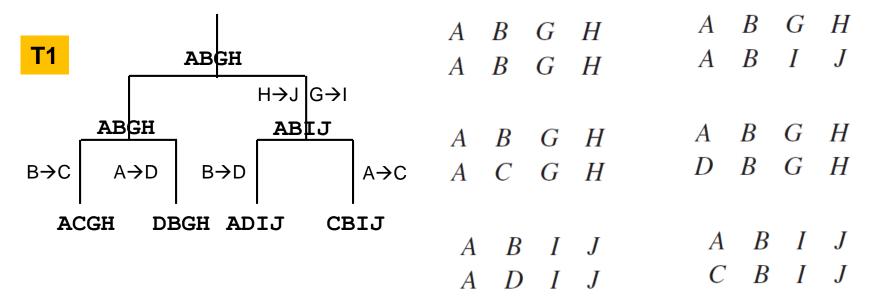
Figure 3.7 Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner



Ref: Didier Gonze, Borodovsky & Ekisheva (2007)

## Alignments -> Trees

- Ungapped alignments → Trees (T1-T4)
- Each tree produces 6 alignments



# Matrix of A(i,j) APM counts

	A	В	С	D	G	Н		J
Α		0	4	4	0	0	0	0
В	0		4	4	0	0	0	0
C	4	4		0	0	0	0	0
D	4	4	0		0	0	0	0
G	0	0	0	0		0	4	0
Н	0	0	0	0	0		0	4
1	0	0	0	0	4	0		0
J	0	0	0	0	0	4	0	
Total	8	8	8	8	4	4	4	4

## Step1-Outcome

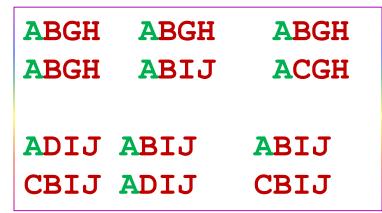
- What amino acid substitutions are likely and which ones are unlikely
  - C and W had shown to be sparsely substituted
  - N, S are commonly substituted from Dayhoff's data

- Relative mutability, m<sub>j</sub> (given the short evolutionary period)
- Note the mutation rate is different for diff.
   AA

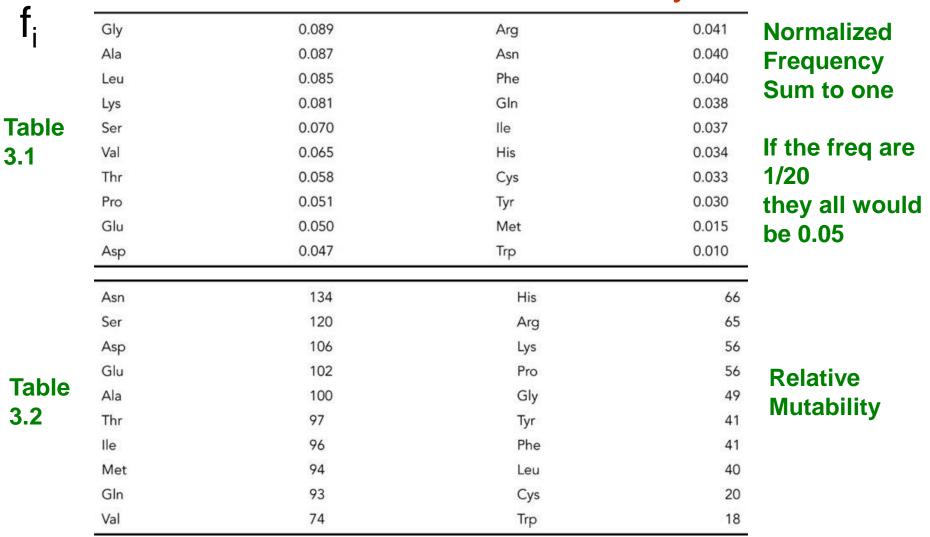
AA	Α	В		Н	G	J	C	D
Changes	8	8	4	4	4	4	8	8
Freq. of occurrence	40	40	24	24	24	24	8	8
Relative mutability m <sub>j</sub>	0.2	0.2	0.167	0.167	0.167	0.167	1	1

$$m_j = \frac{\text{number of changes of j}}{\text{number of occurances of j}}$$

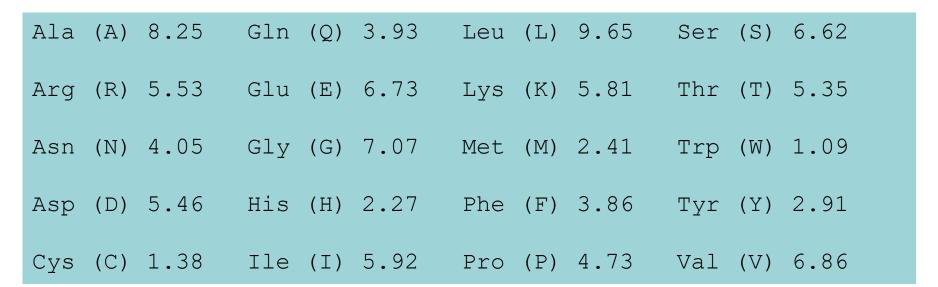
Number of times the residue occurs in the alignment (gleaned from the tree)

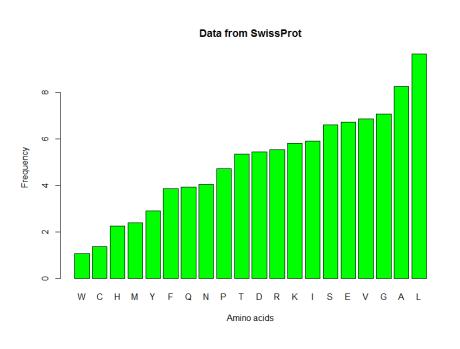


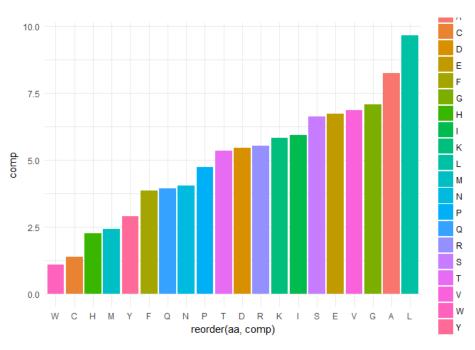
# Steps 2 and 3 (of 7): Frequency of Occurrence & Relative Mutability



S. Ravichandran, Ph.D







S. Ravichandran, Ph.D

# Effective frequencies (f<sub>i</sub>) for more than 1 block

```
Amino acid Gly Ala Leu Lys Ser Val Thr Frequency f 0.089 0.087 0.085 0.081 0.070 0.065 0.058

Amino acid Pro Glu Asp Arg Asn Phe Gln Frequency f 0.051 0.050 0.047 0.041 0.040 0.040 0.038

Amino acid Ile His Cys Tyr Met Trp Frequency f 0.037 0.034 0.033 0.030 0.015 0.010
```

Effective frequency of the 20 amino acids determined for the original alignment data(70 blocks) (Dayhoff et al., 1978)

$$f_j = k \sum_i q_j^{(b)} N^{(b)}$$

b: blocks

q<sub>j</sub><sup>(b)</sup> is the observed frequency of amino acid j in block b N<sup>(b)</sup> is the number of substitutions in a tree built for b

K is chosen such that sum of  $f_i = 1$ 

## Relative Mutability

- Different for different AAs
  - W and C are less mutable
    - Why?
  - N,S,D,E are more mutable
    - Why

O $O$ $O$ $O$ $O$ $O$ $O$ $O$ $O$ $O$	OH NH <sub>2</sub>
N⊓ <sub>2</sub>	

AA	m <sub>i</sub>	AA	m <sub>i</sub>
N	134	Н	66
S	120	R	65
D	106	K	56
E	102	Р	56
Α	100	G	49
Т	97	Υ	41
I	96	F	41
M	94	L	40
N	93	C	20
V	74	W	18

Alanine had been arbitrarily set to 100 (Dayhoff, 1978)

$$f(j) = \frac{n(j)}{N}$$

Frequency of jth amino acid

Entry, A(i,j) will contain the # of times j is mutated to i

$$m(j) = rac{\sum_{i=1, i 
eq j}^{20} A(i, j)}{n(j)}$$
 Mutability of jth amino acid A(i,j) is the count of j  $ightarrow$  i

$$\frac{1}{Nf(j)} = \frac{1}{n(j)} = \frac{m(j)}{\sum_{i=1, i \neq j}^{20} A(i,j)} \qquad \text{The above equation can be rewritten as}$$

Note the above calculation ignores self mutation (A  $\rightarrow$  A etc.)

$$f(j) = \frac{n(j)}{N}$$
 
$$\frac{1}{Nf(j)} = \frac{1}{n(j)} = \frac{m(j)}{\sum_{i=1, i \neq j}^{20} A(i, j)}$$
 
$$Goal is to compute probability matrix$$

M(i,j) is the probability of the aa in the column j having been substituted by an aa in row i over an evolutionary distance. Note this only includes non-diagonal entries

$$M(i,j) = \lambda A(i,j) \frac{m(j)}{\sum_{i=1, i \neq j}^{20} A(i,j)} = \frac{\lambda A(i,j)}{Nf(j)}$$

λ is a constant

Equation only computes the non-diagonal and the diagonal entry is just one minus of that quantity

Step 4 (of 7): Mutation <u>Probability</u> Matrix (M<sub>ij</sub>) for a certain evolutionary distance (ex 1 PAM)

						-				••••				1011			<u> </u>		<u> </u>	<u> </u>	
£8 - 5											Original a	mino acid								W	
		A	R	N	D	Cvs	Q Gln	E Glu	Gly	H His	I lie	L	K	M Met	F Phe	P	S	Thr	W	Tyr	Val
-		Ala	Arg	Asn	Asp	_			_	_	-	Leu	Lys	1000		Pro	Ser		Trp		
	Λ.	98.7	0.0	0.1	0.1	0.0	0.1	0.2	0.2	0.0	0.1	0.0	0.0	0.1	0.0	0.2	0.4	0.3	0.0	0.0	0.2
	R	0.0	99.1	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.2	0.0	0.0	0.0	0.1	0.0	0.1	0.0	0.0
1 3	N	0.0	0.0	98.2	0.4	0.0	0.0	0.1	0.1	0.2	0.0	0.0	0.1	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0
	D	0.1	0.0	0.4	98.6	0.0	0.1	0.5	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
3	C	0.0	0.0	0.0	0.0	99.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
	Q	0.0	0.1	0.0	0.1	0.0	98.8	0.3	0.0	0.2	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
-	Е	0.1	0.0	0.1	0.6	0.0	0.4	98.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	G	0.2	0.0	0.1	0.1	0.0	0.0	0.1	99.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.1
- 8	Н	0.0	0.1	0.2	0.0	0.0	0.2	0.0	0.0	99.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1,5	1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	98.7	0.1	0.0	0.2	0.1	0.0	0.0	0.1	0.0	0.0	0.3
5	L	0,0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.2	99.5	0.0	0.5	0.1	0.0	0.0	0.0	0.0	0,0	0.2
5	K	0.0	0.4	0.3	0.1	0.0	0.1	0.1	0.0	0.0	0.0	0.0	99.3	0.2	0.0	0.0	0.1	0.1	0.0	0.0	0.0
8	M	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0	98.7	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26.	F	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	99.5	0.0	0.0	0.0	0.0	0.3	0.0
	P	0.1	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	99.3	0.1	0.0	0.0	0.0	0.0
1 8	S	0.3	0.1	0.3	0.1	0.1	0.0	0.1	0.2	0.0	0.0	0.0	0.1	0.0	0.0	0.2	98.4	0.4	0.1	0.0	0.0
	T	0.2	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.1	0.1	0.0	0.1	0.3	98.7	0.0	0.0	0.1
3	W	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	99.8	0.0	0.0
	Y	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	99.5	0.0
	V	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.6	0.1	0.0	0.2	0.0	0.0	0.0	0.1	0.0	0.0	99.0



100% or 1 (probability values)

Fig 3.9 from the Pevsner Book III Edition

Each entry (i,j) shows the probability of an amino acid (j; columns) to be replaced by another amino acid (i, row) over an evolutionary distance of 1 PAM

What is 1 PAM? 1% of amino acids have changed in the sequences from which this data is derived (Note the time of evolution could be different for different sequences)

$$M(j,j) = 1 - \sum_{i=1,i 
eq j}^{20} M(i,j)$$
  $M(j,j) = 1 - \lambda m(j)$ 

λ is the same constant(can be derived with little algebra; not showing the steps)

#### Please visit,

https://en.wikipedia.org/wiki/Point\_accepted\_mutation for a nice introduction

Entries of off-diagonal mutation probability matrix

$$f(j) M(i,j) = f(i) M(j,i) = (\lambda/N) A(j,i) = (\lambda/N) A(i,j)$$

### Other PAM matrices

- What is PAM1 depend on
  - Sequence alignments that are closer
  - Also depend on the sequences that are considered
- Let us consider the extreme case of PAM0
  - Only diagonal

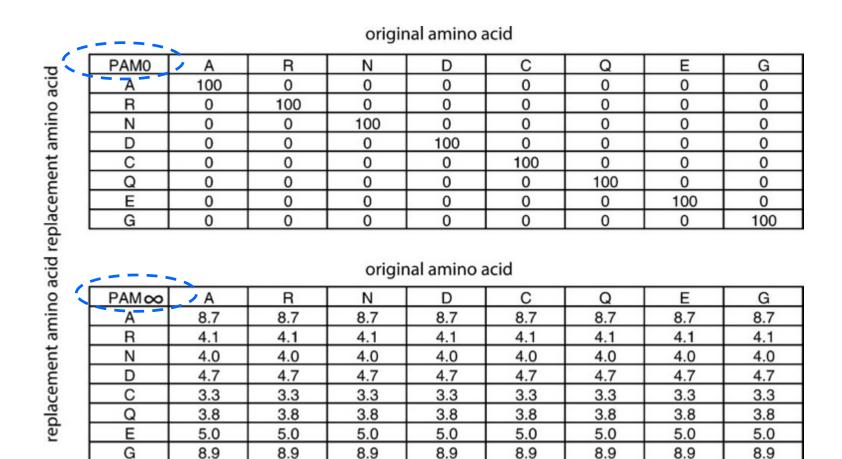


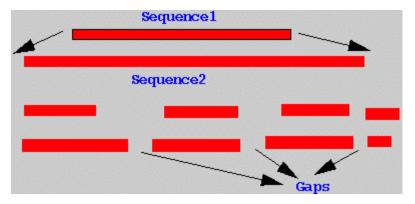
Fig 3.12 from the Pevsner Book III Edition PLEASE DO NOT DISTRIBUTE-Copyright figure

So far, we have a probability matrix, but we want to score alignments (ie how is this different from random alignments)

To get a scoring matrix, we need to convert the probability matrix into odds matrix

# Types of Alignments

- Global Alignment
  - Sequence alignment over the whole range
- Local Alignment
  - Identifying local regions (islands) by introducing gaps



# Final steps (6 and 7)

M<sub>ii</sub> is the probability that the original aa (j) will be substituted by (i)

$$R_{ij}=rac{M(i,j)f(j)}{f(i)f(j)}=rac{M(i,j)}{f(i)}$$
 R $_{ij}$  is the relatedness odds ratio

Table 3.1 provides Normalized freq. (f<sub>i</sub>)s

Think back on conditional prob.: note fis are normalized frequency

Probability of an authentic alignment =  $\frac{P(\text{aligned} \mid \text{authentic})}{P(\text{aligned} \mid \text{random})}$ 

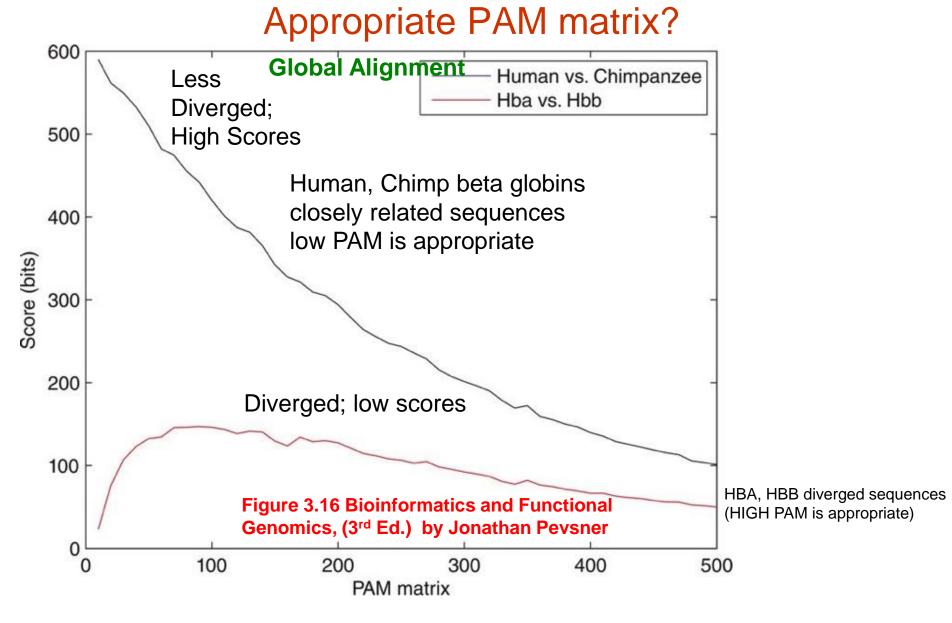
Log odds score 
$$S_{ij} = 10 imes log_{10} [rac{M(i,j)}{f(i)}]$$

Unlike M<sub>ii</sub>, S<sub>ii</sub> are symmetric.

### **PAM250**

A	2																			
R	-2	6																		
N	0	0	2				Fig	<b>j 3.</b> ′	14 f	ron	n th	e P	evs	sne	r B	ook		Edi	tior	1
D	0	-1	2	4																
C	-2	-4	-4	-5	12		3													
Q	0	1	1	2	-5	4														
E	0	-1	1	3	-5	2	4													
G	1	-3	0	1	-3	-1	0	5												
Н	-1	2	2	1	-3	3	1	-2	6											
I	-1	-2	-2	-2	-2	-2	-2	-3	-2											
L	-2	-3	-3	-4	-6	-2	-3	-4	-2	-2	6									
K	-1	3	1	0	-5	1	0	-2	0	-2	-3	5								
M	-1	0	-2	-3	-5	-1	-2	-3	-2	2	4	0	6	3 0 0						
F	-3	-4	-3	-6	-4	-5	-5	-5	-2	1	2	-5	0	9						
P	1	0	0	-1	-3	0	-1	0	0	-2	-3	-1	-2	-5	6					
S	1	0	1	0	0	-1	0	1	-1	-1	-3	0	-2	-3	1	2				
T	1	-1	0	0	-2	-1	0	0	-1	0	-2	0	-1	-3	0	1	3			
W	-6	2	-4	-7	-8	-5	-7	-7	-3	-5	-2	-3	-4	0	-6	-2	-5	17		e e
Y	-3	-4	-2	-4	0	-4	-4	-5	0	-1	-1	-4	-2	7	-5	-3	-3	0	10	
V	0	-2	-2	-2	-2	-2	-2	-1	-2	4	2	-2	2	-1	-1	-1	0	-6	-2	4
	A	R	N	D	C	Q	E	G	Н	I	L	K	M	F	P	S	T	W	Y	V

A	7															
R	-10	_	7			$P^{\Delta}$	\M1	$\cap$								
N	-7	_	9	1		1 /	VIVI I	U								
D	-6	-17	-1	8	1											
C	-10	-11	-17	-21	10	1				C	ha	rtor	0)//	+رياح	ion	any dictance
Q	-7	-4	- 7	-6	-20	9				S		ICI	GV	Jiui	IUI	ary distance
E	-5	-15	-5	0	-20	-1	8									
G	-4	_	-6	-6	-13	-10	-7	7								PAM10 PAM250
н	-11	_	-2	-7	-10	-2	- 9	-13	10							
I	-8		-8	-11	-9	-11	-8	-17	-13	9		10				$A \rightarrow A \qquad 7 \qquad 2$
L	-9		_	-19	-21	-8	-13	-14	-9	-4	7	-	1			
K	-10			-8	-20	-6	-7	-10	-10	-9	-11	7	10	1		identical pairs high score in 10 (close)
F	-8		-15	-17	-20	-7 -19	-10	-12	-17	-3 -5	-2	-4	12	9	1	1 0 ,
P	-4	- Contractor Contractor	-12	-12	-11	-6	-9	-10	-7	-12	-10	-10	-11	-13	8	i i
S	-3	_	-2	-7	-6	-8	-7	-4	-9	-10	-12	-7	-8	-9	-4	
T	-3	_	-5	-8	-11	-9	-9	-10	-11	-5	-10	-6	-7	-12	-7	
W	-2		-11	-21	-22	-19	-23	-21	-10	-20	-9	-18	-19	-7	-20	
Y	-11	-14	-7	-17	-7	-18	-11	-20	-6	-9	-10	-12	-17	-1	-20	
v	-5	_	-12	-11	-9	-10	-10	-9	-9	-1	-5	-13	-4	-12	- 9	The state of the s
8	A	R	N	D	C	Q	E	G	н	I	L	K	М	P	P	S T W Y V
A	2															
R	-2	6														
N	0		2													
D	0		2 4	ĺ		_	. A R	10 E	^							
C	-2		4 -5	12		۲	ΆΙν	1250	J							
Q	0		1 2	-5	4											
E	0		1 3	-5		4							- I			
G	1	_	0 1	_		0 5	1			_ar	ger	ev	olu	llon	ary	/ distance
H	-1		2 1	-3		1 -2					_				_	
I	-1		2 -2	-2	-2 -	2 -3		5								
L	-2		3 -4	-6	-2 -	2 -3 3 -4			6							PAM10 PAM250
K	-1		1 0	-5		0 -2			-3 5	1						
M	-1	-	2 -3	-		2 -3		2	4 0	_	1					$D \rightarrow R$ -17 -1 mismatch
F	-3		3 -6		-5 -	5 -5		1	2 -5		9					
P	1		0 -1	-3		$\frac{3}{1}$ $\frac{-3}{0}$			-3 -1		-5	6				penalty is higher in PAM10
S					0   -	. 0	_		_				2			
	-		_		_	0 1	-1	-1	-3   11	- )	- 1					compared to DANAGEO
The second division in which the second	1	0	1 0	0	-1	$\begin{array}{c c} 0 & 1 \\ 0 & 0 \end{array}$	_		-3 0		-3	0		3		compared to PAM250
T	1	0 -1	1 0 0 0	-2	-1 -1	0 0	-1	0	-2 0	-1	-3	0	1	3 5 17	1	compared to PAM250
T W	1 1 -6	0 -1 -2 -	1 0 0 0 4 -7	0 -2 -8	-1 -1 -5 -	0 0 7 -7	-1 -3	0 -5	-2 0 -2 -3	-1 -4	-3 0	0 -6	1 -2 -	5 17		compared to PAM250
T W Y	1 1 -6 -3	0 -1 2 - -4 -	1 0 0 0 4 -7 2 -4	-2 -8 0	-1 -1 -5 - -4 -	0 0 7 -7 4 -5	-1 -3 0	0 -5 -1	-2 0 -2 -3 -1 -4	-1 -4 -2	-3 0 7	0 -6 -5	1 -2 - -3 -	5 17 3 0	10	
T W	1 1 -6	0 -1 -2 -	1 0 0 0 4 -7 2 -4 2 -2	0 -2 -8 0 -2	-1 -1 -5 - -4 -	0 0 7 -7 4 -5 2 -1	-1 -3 0	0 -5	-2 0 -2 -3	-1 -4 -2	-3 0	0 -6 -5 -1	1 -2 - -3 -	5 17	10	compared to PAM250



S. Ravichandran, Ph.D

#### **BLOcks** of amino acid **SUbstitution Matrices**

#### More on this in BLAST class

#### BLOSUM

- Based on BLOCKS database
- Henikoff and Henikoff, Karlin and Altschul, Others
- Considered local MSA of distantly related proteins
- Scoring scheme similar to PAM
  - Log-odds ratio using base 2 log

$$S_{ij} = 2 * \log_2 \left\lceil \frac{M_{ij}}{f_i} \right\rceil$$

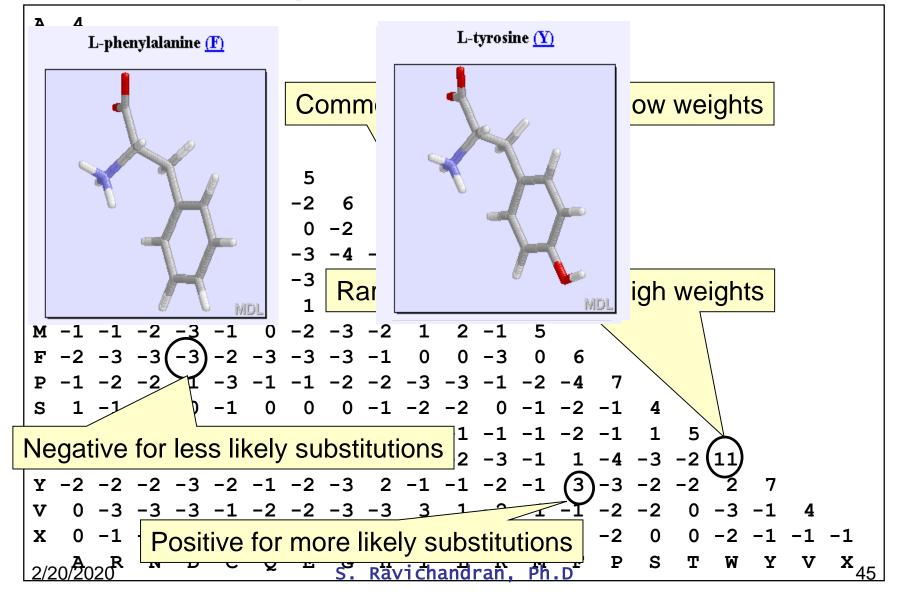
$$s(a,b) = \frac{1}{\lambda} \log(\frac{p_{ab}}{f_a f_b})$$

General form for scoring matrices according to Dr. Altschul

### **BLOSUM Summary**

- BLOSUM62 is "standard" (better performances than PAM)
- Nature Biotechnology: <a href="http://www.nature.com/nbt/journal/v22/n8/abs/nbt0804-1035.html">http://www.nature.com/nbt/journal/v22/n8/abs/nbt0804-1035.html</a>

# Example of BLOSUM62



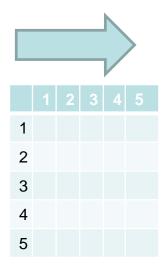
#### **BLOSUM80**

- Using sequences that share no more than 80% identity
- Sequences that are more than 80% identity are clustered and represented by a single sequence
- Why Clustering?
  - Reduces overrepresentation and bias
- BLOSUMn
  - Lower "n"s will help us identify more distantly related sequences
  - Higher "n"s will help us identify less diverged sequences

#### **BLOSUM80**

\* \* \* \*

TGNQEEYGNTSSDSSDEDY
KKLEKEEEDGISQESSEEE
KKLEKEEEDGISQESSEEE
KKLEKEEEDGISQESSEEE
KPAQEETEETSSQESAEED
KKPAQETEETSSQESAEED



TGNQEEYGNTSSDSSDEDY

KKLEKEEEDGISQESSEEE KKLEKEEEDGISQESSEEE KKLEKEEEDGISQESSEEE

KPAQEETEETSSQESAEED KKPAQETEETSSQESAEED

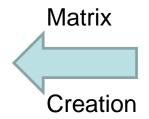


TGNQEEYGNTSSDSSDEDY

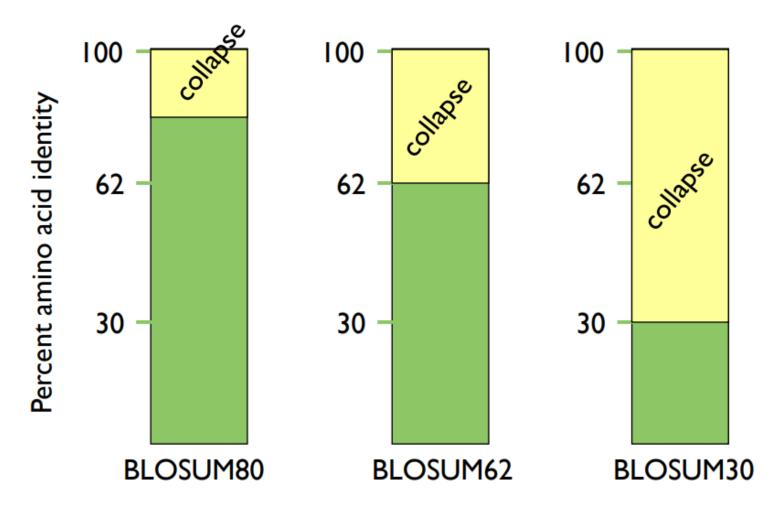
KKLEKEEEDGISQESSEEE

KPAQEETEETSSQESAEED

**BLOSUM80** 



#### **BLOSUM Matrices**



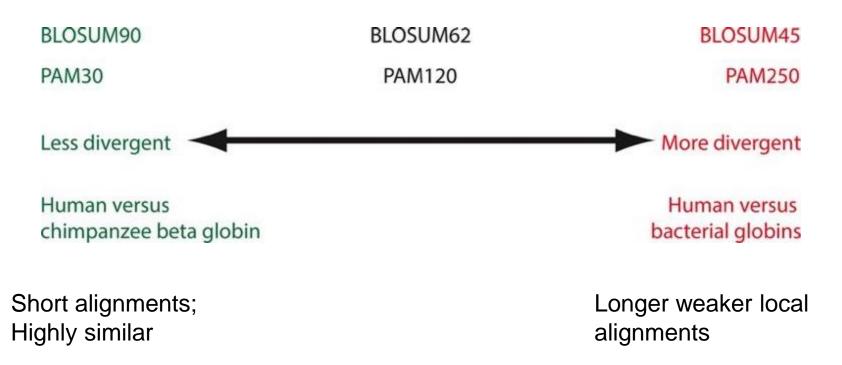
#### Proc. Natl. Acad. Sci. USA Vol. 89, pp. 10915-10919, November 1992 Biochemistry

clustering percentage in which sequence segments that are identical for at least that percentage of amino acids are grouped together. For example, if the percentage is set at 80%, and sequence segment A is identical to sequence segment B at ≥80% of their aligned positions, then A and B are clustered and their contributions are averaged in calculating pair frequencies. If C is identical to either A or B at ≥80% of aligned positions, it is also clustered with them and the contributions of A, B, and C are averaged, even though C might not be identical to both A and B at  $\geq$ 80% of aligned positions. In the above example, if 8 of the 9 sequences with A residues in the 9A-1S column are clustered, then the contribution of this column to the frequency table is equivalent to that of a 2A-1S column, which contributes 2 AS pairs. A consequence of clustering is that the contribution of closely related segments to the frequency table is reduced (or eliminated when an entire block is clustered, since this is equivalent to a single sequence in which no substitutions appear). For example, clustering at 62% reduces the number

Paper available from Class Reference folder

# **Matrix Comparison**

Figure 3.17 Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner PLEASE DO NOT DISTRIBUTE-Copyright figure



#### PAM & BLOSUM Matrices

- PAM (Dayhoff et al 1988)
  - PAM1 is the matrix obtained by comparing sequences differ by no more than 1%
  - Higher PAMX are extrapolated from PAM1)
  - PAM250: Observed difference(80%) Evolutionary distance (250)
- Limitation: Matrices are derived from alignments of sequence that are 85% identity
  - Difficult to use in Twilight Zone

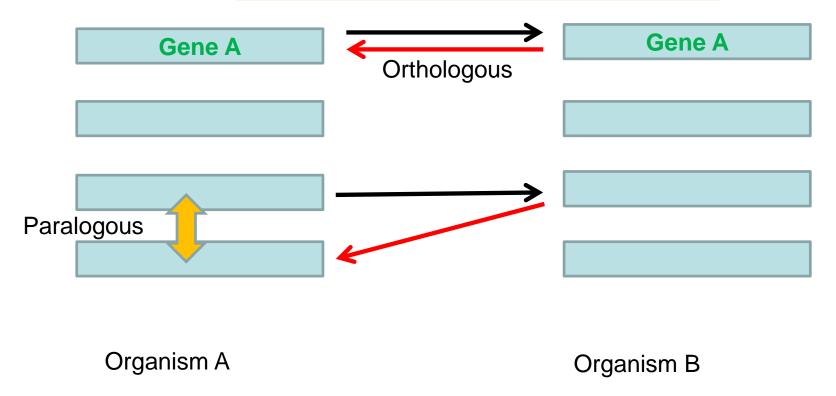
$$PAM_2 = PAM_1 * PAM_1 = (PAM_1)^2$$
  
 $PAM_{250} = (PAM_1)^{250}$ 

- Blosum (BLOcks SUbstitution Matrix)
  Heinkoff & Heinkoff (1992)
  - Derived from BLOCKS database
  - Distant relationships explained better than PAM
  - Blosum62 obtained from sequence BLOCKS clustered at >=62% identity
  - BlosumX are observed from actual alignments not extrapolated
  - Sequences are very similar use higher Blosum (low PAM)

# We introduced the concept of homology, here let us learn how to identify the homologous sequences

# Reciprocal Best Hits Concept to Deduce Homology

**BLAST** or any other alignment software



# Questions for pondering

- PAM40
  - Highly related proteins
- BLOSUM80
  - Not ideal for scoring highly related sequences
  - Why?
  - Matrix is built based on sequences that share upto 80% identity

#### What matrix for what?

BLOSUMx	Good for	% Similarity
x = 90	Short and highly similar sequences	70-90
x = 80	Often good for identifying family members	50-60
x = 62	Most effective for a variety of range of similar sequences; default in NCBI BLAST	30-40
x = 30	For diverged; weak long alignments	<30

Based on Dr. Andy Baxevanis lectures

# Divergence and Twilight zone

Take two sequences (100 aa in length), fix one and introduce mutations into the other.

Plot % identity vs PAM

PAM250 250 hits/100 aligned aa

Hit: a change in aa that occurs by mutation

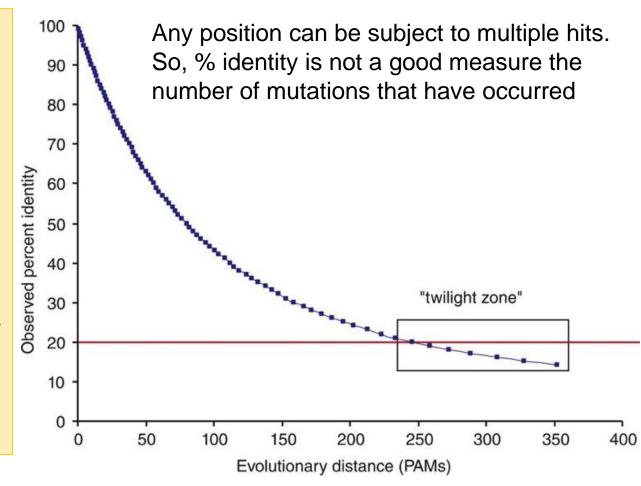


Figure 3.19 Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner PLEASE DO NOT DISTRIBUTE-Copyright figure

# Observed differences and evolutionary distance

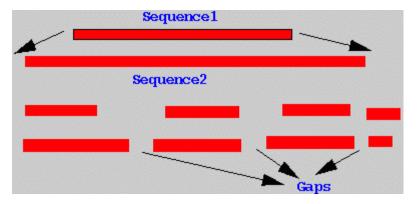
Observed differences in 100 residues	Evolutionary distance in PAMs	
1	1.0	
5	5.1	
10	10.7	
15	16.6	
20	23.1	
25	30.2	
30	38.0	
35	47	
40	56	
45	67	
50	80	
55	94	
60	112	
65	133	
70	159	
75	195	
80	246	

Table 3.3 from Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner PLEASE DO NOT DISTRIBUTE-Copyright figure

# Having solved the scoring, let us tackle how to do the alignment?

# Types of Alignments

- Global Alignment
  - Sequence alignment over the whole range
- Local Alignment
  - Identifying local regions (islands) by introducing gaps



#### Brute Force

- Creating all possible subsets to identify best alignment
- Seq A: Length M
- Seq B: Length N
- roughly 2<sup>(M+N)</sup> total comparisons
- Not an ideal method

#### Dot-matrix

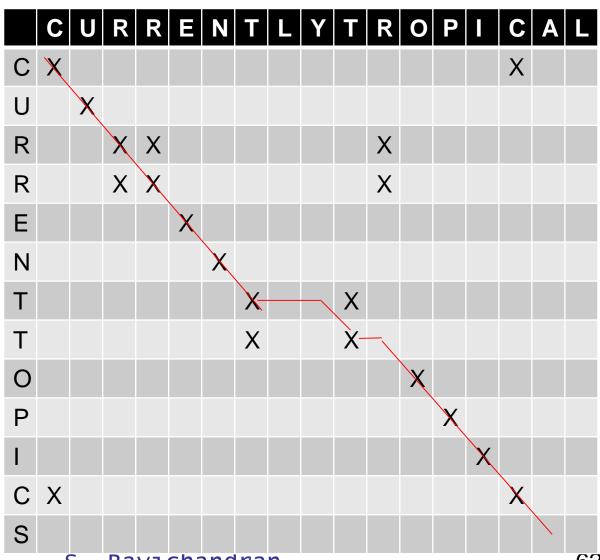
- Identifying all possible matches between two sequences
- Sequence#1: CURRENTLYTROPICAL
- Sequence#2: CURRENTTOPICS

http://www.srmuniv.ac.in/sites/default/files/files/5(6).pdf

#### Dot-matrix

Connect dots across the diagonal using either horizontal (x) or vertical lines (y)

CURRENTLYTROPICAL CURRENT-T-OPICS



2/20/2020

. Ravichandran, Ph.D

#### – Dot-matrix:

- Pros: Easy to understand and useful to identify repeats, palindrome etc
- Cons: Time consuming if more than one pairwise alignment have to be carried out

#### Dynamic Programming

 Compares each character in such a way to maximize the number of matches (identical or similar)

- Dynamic Programming (DP)
  - Global Dynamic Programming
  - Local Dynamic Programming
- DP Algorithms
  - Needleman and Wunsch (Global)
  - Smith Waterman (Local)

#### Global DP

- Generates an alignment for 2 sequences that <u>maximizes</u> the <u>matches</u> and <u>minimizes</u> the # of gaps
- End-to-end alignment
- Linear gap penalty Substitution/Mismatch
  - Ex just one value, -5
- Best when sequences are similar

# Global Alignment Reference

Needleman, S.B. and Wunsch, C.D. A general method applicable to the search for similarities in the amino acid sequence of two proteins. J Mol Biol. 48(3):443-53(1970).

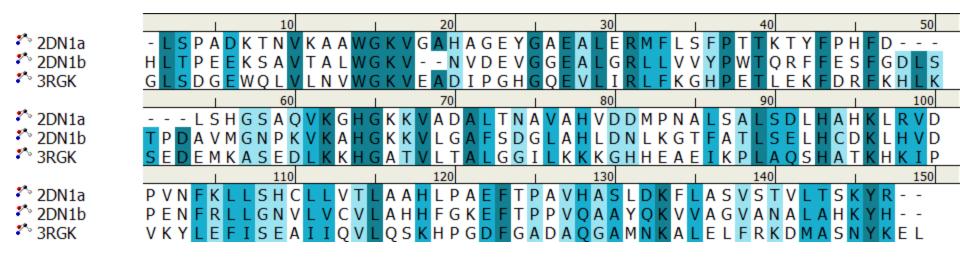
#### Local DP

Smith, T.F. and Waterman, M.S. Identification of common molecular subsequences. J Mol Biol. 147(1):195-7 (1981)

In many cases, we are inherently looking for local alignments. This is true given the fact that most proteins are modular. BLAST default matrix is BLOSUM62

# Sequence comparison of Homologous sequences

Globin family; 2DN1a/2DN1b hemoglobin-alpha/beta; 3RGK is Myoglobin



Sequence similarity 37.6%; Identity: 17.4%

#### Local DP

- Alignment that maximizes regions of similarity
- Not necessarily end-to-end
- Uses affine gap penalty
  - https://en.Wikipedia.org/wiki/Gap\_penalty
- Often uses, a one time gap penalty for each stretch of gaps plus a gap extension penalty as a function of length of gap

MM: MisMatch Sub: Substitution

# Dynamic Programming

No Gap penalty

- CGGGGGAACT
  CGGGGGGATCT 10\*8 5 = 75
- Scoring System: Match +8; sub/MM = -5
- Linear gap penalty
  - -M: +8; Sub/MM: -5; Gap = -3

C---GGGAACT
CGGGGGGGATCT

- Affine Gap penalty:
  - -M=+8; Sub/MM = -5; Gap Open = -3, Gap Ext = -1

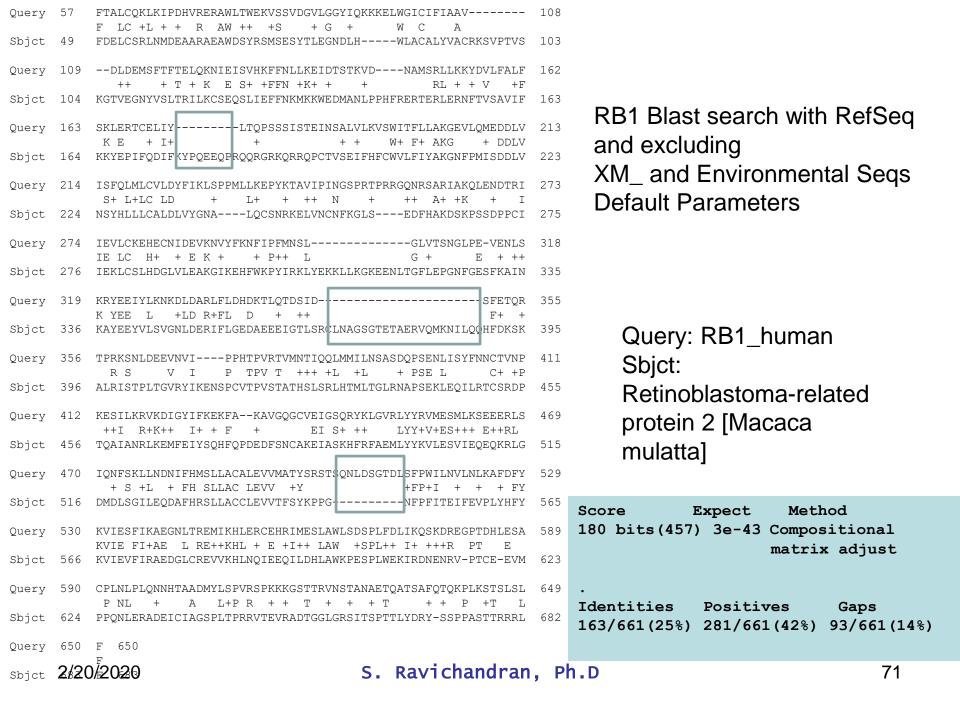
```
Deduction for a gap = G + Ln (Note G > L)
```

G: Gap Opening Penalty

L: gap-extension penalty

n: Length of a gap

C---GGGAACT
CGGGGGGGATCT



# Scoring matrix that gives a score for aligning two characters (total score taking into account INDELs)

How to generate the alignments? Algorithm??

## Dynamic Programming (DP)

- Richard Bellman, 1950s by mathematician
  - RAND Corp. on optimal decision processes
    - Funding for the project came from Navy/Military
- A name that he choose to hide his project from the then US Secretary of Defense Charles Wilson,
  - a man not friendly to either basic or mathematics research.

### Dynamic Algorithm

- Goal: To find an optimal alignment that maximizes the score of two sequences
- Can't we manually align them?
- The possible alignments are
- different alignments for two sequences of length N
  - N=100; 10<sup>58</sup> Sequence alignments!.

$$\frac{2^{2N}}{\sqrt{2\pi N}}$$

- Notation
- Two sequences x and y.
  - Length M and N respectively
- x<sub>i</sub> is the i th residue in x
- y<sub>j</sub> is the j th residue in y
- Scoring Matrix
  - Linear Gap penalty  $\gamma = -6$ 
    - Penalty increases with # of gaps

```
A G C T
A +5 -2 -2 -2
G -2 +5 -2 -2
C -2 -2 +5 -2
T -2 -2 -5
```

## What is an optimal alignment?

Recursive definition of alignment

Seq. x of length M Seq. y of length N

- How can an alignment end?
- 3 possibilities

x<sub>M</sub> Y<sub>N</sub>

$$y_{N}^{--}$$

$$\mathbf{x}_{\mathsf{M}}^{--}$$
 $\mathbf{y}_{\mathsf{N}}$ 

Note  $x_M$  indicates one residue



 The optimal alignment is the one with the highest score from the above 3 cases

## What is an optimal alignment? How can an alignment end?

Three possibilities

Seq. x of length M = 18Seq. y of length N = 8

```
CAGCACTTGGATTCTCGG
CAGC----G-T----GG
```



$$y_N^-$$

x<sub>M</sub> is aligned to a gap and Y<sub>N</sub> had already appeared earlier in the sequence alignment

- Let us look at the scoring schemes for the previous three cases
- Bigger alignments are made up of optimal sub-alignments
- You can do this recursively

 S(i,j) is the alignment score of the sequence prefix, x<sub>1</sub>...x<sub>i</sub> with y<sub>1</sub>...y<sub>i</sub>

•  $S(M,N) = S(x_{M},y_{N}) + S(M-1, N-1)$ 

$$\begin{array}{ccc} \mathbf{x}_{\mathtt{M}} & & \mathbf{1} \dots \mathbf{x}_{\mathtt{M}-1} \\ \mathbf{y}_{\mathtt{N}} & & \mathbf{1} \dots \mathbf{y}_{\mathtt{N}-1} \end{array}$$

•  $S(M-1,N) = \gamma + S(M-1,N)$ 

Y Gap penalty

•  $S(N-1,M) = \gamma + S(N-1,M)$ 

- To calculate
- S(M,N)
  - □ S(M-1,N-1), S(M,N-1), S(M-1,N)
- In turn for S(M-1,N-1), we need
  - $\square$  S(M-2,N-2),S(M-1,N-2),S(M-2,N-1)
- S(M,N-1)
  - $\square$  S(M-1,N-2), S(M,N-2), S(M-1,N-1)
- S(M-1,N)
  - $\square$  S(M-2,N-1),S(M-1,N-1),S(M-2,N)

We keep going back to building smaller and smaller pieces until we reach S(0,0)

$$A + 5 - 2 - 2 - 2$$

$$G -2 +5 -2 -2$$

$$C -2 -2 +5 -2$$

$$T -2 -2 -2 +5$$

## Recursive Definition of all scores of S(i,j)

$$S(i, j) = \max \begin{cases} S(i-1, j-1) + \sigma(x_i, y_j) \\ S(i, j) + \gamma \\ S(i, j-1) + \gamma \end{cases} \quad \gamma = -\epsilon$$

Once the matrix is filled-in. We can start at the bottom cell and ask, how we could have gotten here?

No gap with gap alignment is allowed

Possibility of more than one best alignments

### Build a matrix of dimensions m+1 by n+1

First we fill in the boundary conditions S(0,0) = 0, Fill First Row/Column

+5 Match; -2 Mis-Match and -6 for INDELs

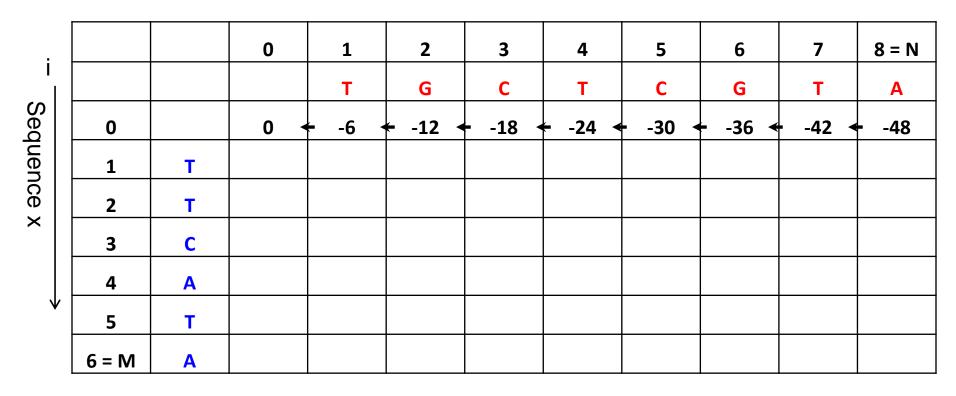
•			0	1	2	3	4	5	6	7	8 = N
Sequence x				Т	G	С	Т	С	G	Т	Α
	0										
	1	Т									
	2	Т									
	3	С									
	4	A									
	5	Т									
	6 = M	A									

Based on Eddy, S. Talk and papers

#### **TGCTCGTA**

\_\_\_\_\_

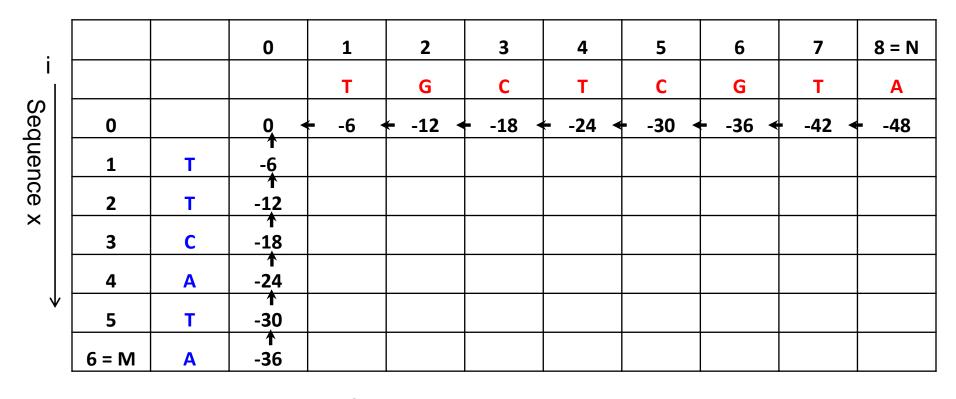
### +5 Match; -2 Mis-Match and -6 for INDELs



Based on Eddy, S. Talk and papers

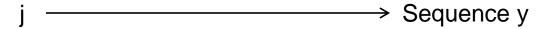
\_\_\_\_\_

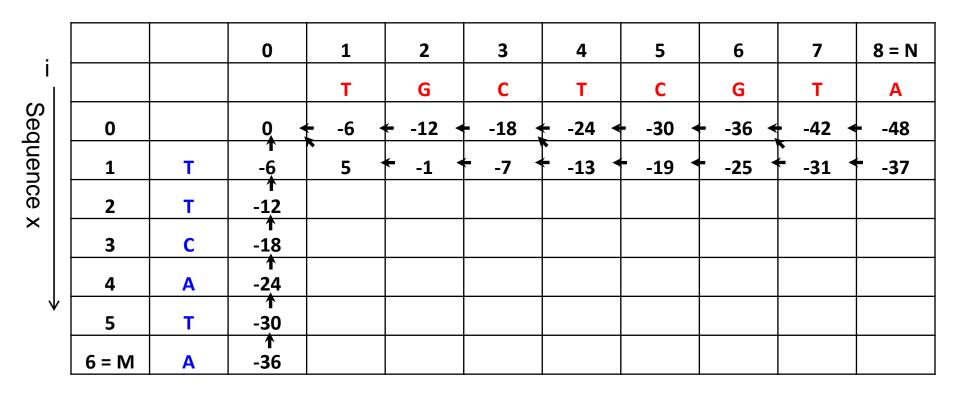
+5 Match; -2 Mis-Match and -6 for INDELs



Based on Eddy, S. Talk and papers

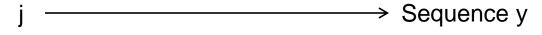
### +5 Match; -2 Mis-Match and -6 for INDELs

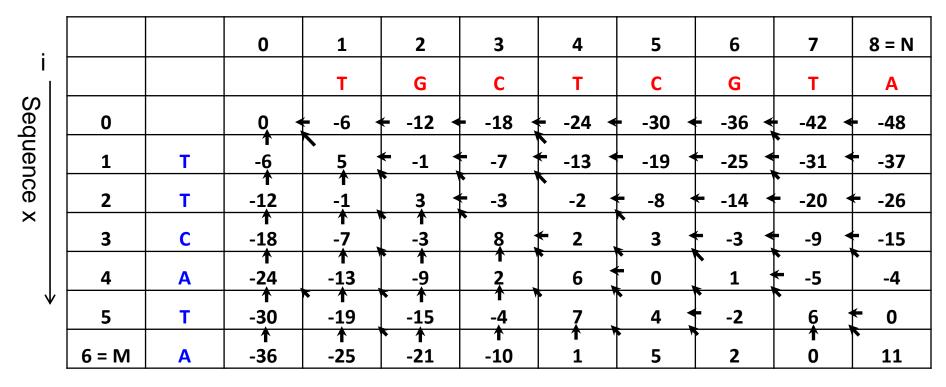




Based on Eddy, S. Talk and papers

### +5 Match; -2 Mis-Match and -6 for INDELs





Based on Eddy, S. Talk and papers

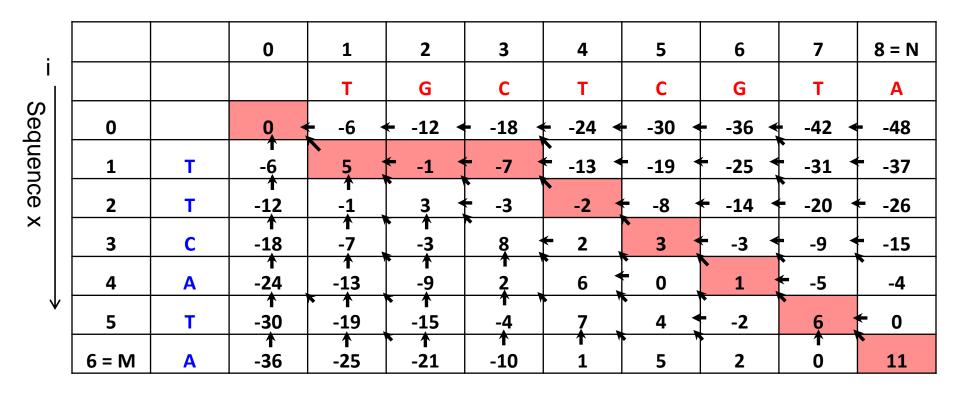
#### **TGCTCGTA**

T--TCATA

56655255 = 11

+5 Match; -2 Mis-Match and -6 for INDELs

j → Sequence y



Based on Eddy, S. Talk and papers

## Global Alignment

- Scoring Scheme:
  - Match: 1; MisMatch = 0; Gap Penalty = 0

- Guaranteed to give you the optimal alignment
  - Biologically meaningful or not is not the algorithm's problem
  - Scoring scheme should address that question
- Dyn. Prog. Algorithm can also align two random sequences (<u>Example that we worked on today</u>)
  - Statistical theory should be able to address this question using the alignment scores for this point

## Local Alignment Algorithm and Extensions

- Similar algorithm as we discussed today
  - Some differences (m x n rather than (m+1) x (n+1)

Query length: m

DB Length: N

- What is being used today?
  - A modified version
  - Why?
    - Time
    - Most problems
      - Query search DB
      - Align two sequences of length m and n is roughly m\*n
      - Or query (length m) against a DB (N) is m \* N
    - Big-oh notation (O(mn) for Needleman-Wunch)

## Types of pair-wise alignments

- Last one in this category
- Word methods
  - Heuristic; Will not produce the best alignment always but very fast; commonly used
  - More on this in future classes

## Beyond Simple DP

- FASTA
  - William Pearson (Univ. of Virginia)
- BLAST
  - Next class

# How are we doing? Comparing to Gold(??) standard

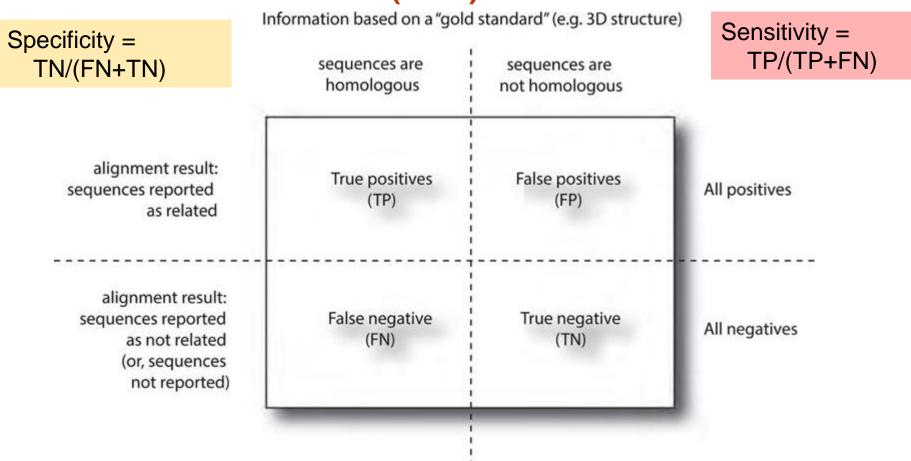


Figure 3.26 from Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner PLEASE DO NOT DISTRIBUTE-Copyright figure

### Central Limit Theorem (CLT)

"Central Limit Theorem states that the distribution of the sum (or average) of a <u>large number of</u> <u>independent, identically distributed variables</u> will be <u>approximately</u> normal, regardless of the underlying distribution."

http://www.math.uah.edu/stat/sample/CLT.html

Sample Mean Distribution will become increasingly close to a normal distribution as the sample size increases, regardless of the population distribution

Simple Random Sample: draws uniformly at random without replacement from the population

## The Central Limit Theorem (CLT)

 $X_1 = [150, 140, 130, 121.5, 141.9]$ 

Sample size:

n

Sample means: (center)

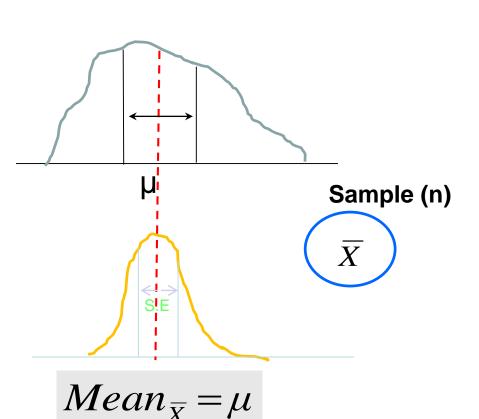
$$\overline{X}_1, \overline{X}_2, \overline{X}_3...$$

? Distribution of

They have a mean of

• Have SE  $\frac{\text{Sd}}{\sqrt{n}}$ 

$$n \to \infty$$
 Sampling  $\to$  Normal approaches



### Parameter vs Statistic

### Parameter

 Numerical descriptive measure, random, one that describes the population

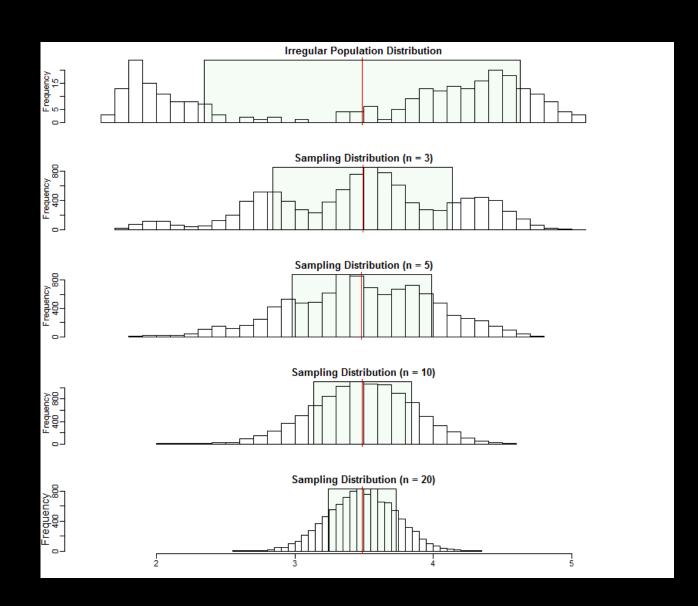
### Statistic

 Numerical descriptive measure, random, one that describes the sample

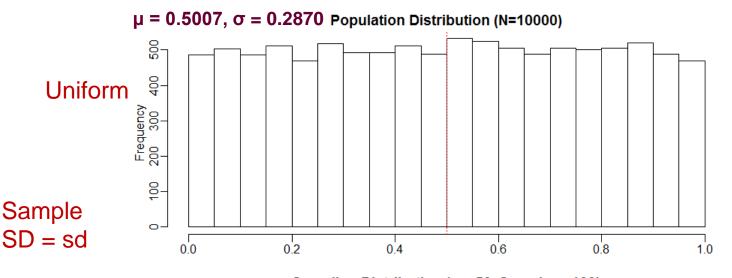
## Central Limit Theorem in Figures

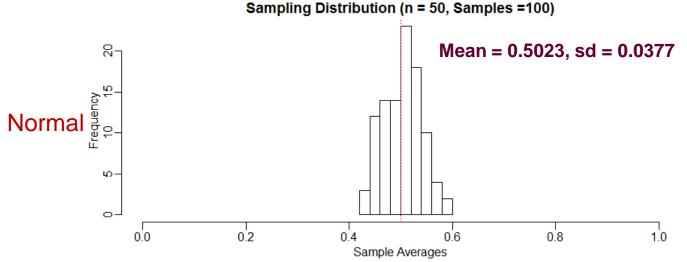
Number of Samples = 10,000

1SD Window shown



# Sampling Distribution & Population Distribution Shape





$$SE = \frac{\text{sd}}{\sqrt{n}}$$

0.00534

# Hypothesis Testing & US Judicial System

"that it is better [one hundred] guilty Persons should escape than that one innocent Person should suffer."

	Person			
Jury	Innocent	Guilty		
Not Guilty				
Guilty				

# Hypothesis Testing & US Judicial System

	Population			
US	$\mu = \mu_0$	μ ≠ μ <sub>0</sub>		
Fail to Reject				
Reject				

# Hypothesis Testing & US Judicial System

	Population			
US	$\mu = \mu_0$	μ ≠ μ <sub>0</sub>		
Fail to Reject		Type-II		
Reject	Type-I			

# Hypothesis Testing & US Judicial System

	Population			
US	$\mu = \mu_0$	μ ≠ μ <sub>0</sub>		
Fail to Reject		Type-II = β		
Reject	Type-I = $\alpha$			

 $1 - \beta = Power$ Goal is to make  $\alpha$  and  $\beta$  smaller

## Hypothesis testing by example

$$-H_0: \mu_0 = 15.5 \text{ mm Hg} \quad \text{Sample} \quad \text{Population}$$

$$-H_A: \mu_0 \neq 15.5 \text{ mm Hg} \quad \text{X} = 16.5 \text{ mm Hg} \quad \mu_0 = 15.5 \text{ mm Hg}$$

n = 49

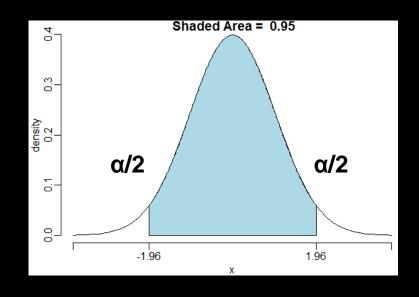
 $\sigma$  = 2.6 mm Hg

- Establish what value(s) we will accept to be different from the NULL distribution ( $\alpha$  level = 0.05)
- Assuming CLT, we can perform an one sample **Z**-test

## Hypothesis testing by example

Alpha values lead us to Critical Values

Values that represent Z values that correspond to  $\alpha/2 = \pm 2.5\%$  are called critical values ( $\pm$  1.96)



### P-value Definition

Given a H<sub>0</sub>, H<sub>A</sub> and a Test Statistic T, the p-value can be defined as

"the probability, computed assuming that  $H_0$  is true, that the test statistic would take a value as extreme or more extreme than that actually observed"

Moore, D.S. (2007) The Basic Practice of Statistics

### Statistical Significance

- Hypothesis Testing
  - Null Hypothesis (H<sub>0</sub>):
    - Two sequences are not related
  - Alternate Hypothesis (H<sub>A</sub>)
    - Yes, they are evolutionarily related
  - Cutoff (usually 0.5 but can be different)
- Generate sample random alignments of the same composition as one of query sequences (One approach)

When we align two proteins, we get a score. We often use Hypothesis Testing to ascertain whether this could have resulted by chance

### Statistical Significance

Sample mean and deviation

$$Z = \frac{x - \mu}{s}$$

- Beta globin to myoglobin
  - Scramble myoglobin 1000 times
  - Compare the random sequence with beta globin
  - Compare the real score with the distribution (Gaussian?)
- How are we doing (real score) compared to the random sample alignments
- Hypothesis testing
- P-value

## What if the distribution is not Gaussian or bell-shaped?

- Global (not Local alignment)
  - Not Gaussian
  - So, normal z-scores will be wrong
  - Refer to publications in the book
- Local alignment
  - Distribution is normal
- Normally Probability is not used, but a related value called E (more in this next class)

## Bonferroni correction for multiple comparison

- We usually compare query to a DB. So, there is a chance of identifying an accidental high scoring alignment(s)
- For multiple comparisons, people often use Bonferroni correction
  - Use stringent cut-off
  - Cut-off/# of searches =  $0.05/(10^6) \sim 10^{-8}$

## Information Theory based approach by D. Altschul

- How to identify real from random alignments
- H = Relative Entropy (expected Substitution Score/residue)
- $q_{ij}$  are target frequency  $H = \sum_{i,j} q_{i,j} s_{i,j} = \sum_{i,j} q_{i,j} \log_2 \frac{q_{ij}}{p_i p_j}$
- P<sub>i</sub> or P<sub>j</sub> are background frequencies
- PAM250: H = 0.36 bits
- PAM10: H = 3.43 bits

H: Information Content of the target and background distributions for a particular scoring matrix

$$H = \sum_{i,j} q_{i,j} s_{i,j} = \sum_{i,j} q_{i,j} \log_2 \frac{q_{ij}}{p_i p_j}$$
 H is the sum of all  $q_{ij}$  and  $S_{ij}$ 

- Altschul estimated 30 bits of information are required to identify an authentic alignment (i.e. DB space = 2^30 = 1B)
- This means you need the DB size to be
   1B to raise above the background noise.
- If you know this you can calculate what alignment length I should have to get meaningful results

## H: Information Content of the target and background distributions for a particular scoring matrix

$$H = \sum_{i,j} q_{i,j} s_{i,j} = \sum_{i,j} q_{i,j} \log_2 \frac{q_{ij}}{p_i p_j} \qquad \text{H is the sum of all } \mathbf{q}_{ij} \text{ and } \mathbf{S}_{ij}$$

• PAM10, H = 3.43, you need an alignment with at least 9 residues

$$9*3.43 = 30.87 \sim 31$$

 PAM250, H = 0.36, at least 83 aa residues are needed to distinguish an authentic alignment

$$83 * 0.36 = 29.88 \sim 30$$

Rel entropy = -H + I = -H + 4.3

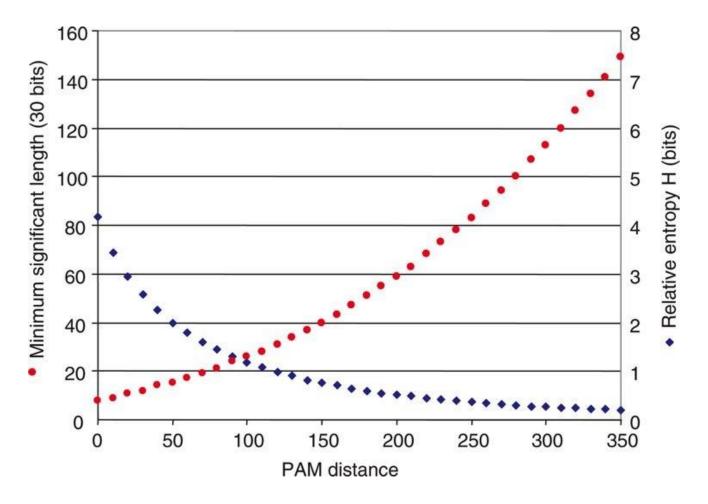


Figure 3.26 from Bioinformatics and Functional Genomics, (3<sup>rd</sup> Ed.) by Jonathan Pevsner PLEASE DO NOT DISTRIBUTE-Copyright figure

## Typos

Page number 87/88

pam250 <- pam^250

Matrix multiplication

## Computer Lab

Problems/Computer Lab

-3-2, 3-4, 3-6, 3-7

### **Thanks**

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