

# Netzwerkkodierung in Theorie und Praxis

## *Praktische Anwendungen der Netzwerkkodierung*

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M.Sc. Juan Cabrera

Deutsche Telekom Chair of Communication Networks (ComNets)



## *Netzwerkkodierungstheorie*

Professor Dr.-Ing. Eduard Jorswieck

Dipl.-Ing. Johannes Richter

Theoretische Nachrichtentechnik



## Practical Implementations of Network Coding



Lecturer: Professor Frank Fitzek



Assistant: M.Sc. Juan Cabrera

## Overview

This course introduces the students to the challenges and approaches of the state of the art implementations of network coding. The course is taught not just through lectures, but also with hands-on exercises using the KODO software library.

The initial lectures refresh the knowledge of the students of the theoretical background of network coding, e.g., the min-cut max-flow of a network, inter-flow network coding, and intra-flow Random Linear Network Coding (RLNC). The student is then introduced to the state of the art software library KODO and the advanced implementations of network coding such as systematic, sparse, tunable sparse, sliding window, etc. The course also covers the benefits of network coding in distributed storage applications. By the end of the course, the student will be introduced to advanced applications of network coding, e.g., Coded TCP, MORE, FULCRUM.

The exercises will teach the students how to use sockets in python as well as the python bindings of the KODO software library for implementing unicast and broadcast communication applications.

## Time Schedule

Lectures: Wednesdays 9:20 – 10:50

Exercises: Thursdays (Odd weeks) 14:50 – 16:20

Show 10 entries Search:

Date	Type	Room	Topic
04.Apr.2016 16:40-18:10	L1	GÖR/0127/U	Presentation of the chair; Organisation of the course; 5G Intro; Butterfly; min cut max flow.
06.Apr.2016	L2	VM8/0E02/U	Inter Flow NC; Index Coding; Zick Zack Coding; CATWOMAN
11.Apr.2016 16:40-18:10	L3	GÖR/0127/U	Analog Inter Flow Network Coding
13.Apr.2016	L4	VM8/0E02/U	Random Linear Network Coding (Basics)
14.Apr.2016	E1	GÖR/0229/U	UDP transmissions with python sockets. Unicast and Broadcasts.
20.Apr.2016	L5	VM8/0E02/U	KODO
27.Apr.2016	L6	VM8/0E02/U	RLNC advanced (sparse, tunable)
28.Apr.2016	E2	GÖR/0229/U	

## ComNets

Deutsche Telekom Chair  
of Communication Networks

## Latest News

- February 19th, 2016  
Wirtschaftswoche & Handelsblat report on research of ComNets & 5G – Prof. Fitzek head and center of European Research: Wirtschaftswoche & Handelsblat
- January 8th, 2016  
New open position at the chair: Research Fellow
- January 7th, 2016  
Deutsche Telekom announces collaboration and sponsorship of the 'Deutsche Telekom Chair for Communication Networks' and becomes an industrial partner of the 5G Lab Germany. Press Release

## Tweets by @ComNets\_TUD

- ComNets Chair at TUD Retweeted
- C. Bettstetter @bettstetter  
Workshop on 5G mobile networks and tactile Internet hosted by @frankfitzek and team in Dresden on June 10, 2016. [ig.kn.ei.tum.de/doku.php?id=te...](http://ig.kn.ei.tum.de/doku.php?id=te...)
- ComNets Chair at TUD Retweeted
- Frank Fitzek @frankfitzek  
Keynote at NOSSDAV/MoVid: mmays2016.itc.aau.at/keynote-5g-ena... #tactileinternet #5g @5g\_lab @ComNets\_TUD
- ComNets Chair at TUD @ComNets\_TUD  
#haec #openstack working. What is better than late night programming with success killing a problem we were fighting for months #victory

- Here all information for the lecture and the exercise can be found.
- Slides
- Links
  - Steinwurf
  - Python
  - KODOMARK (google play)

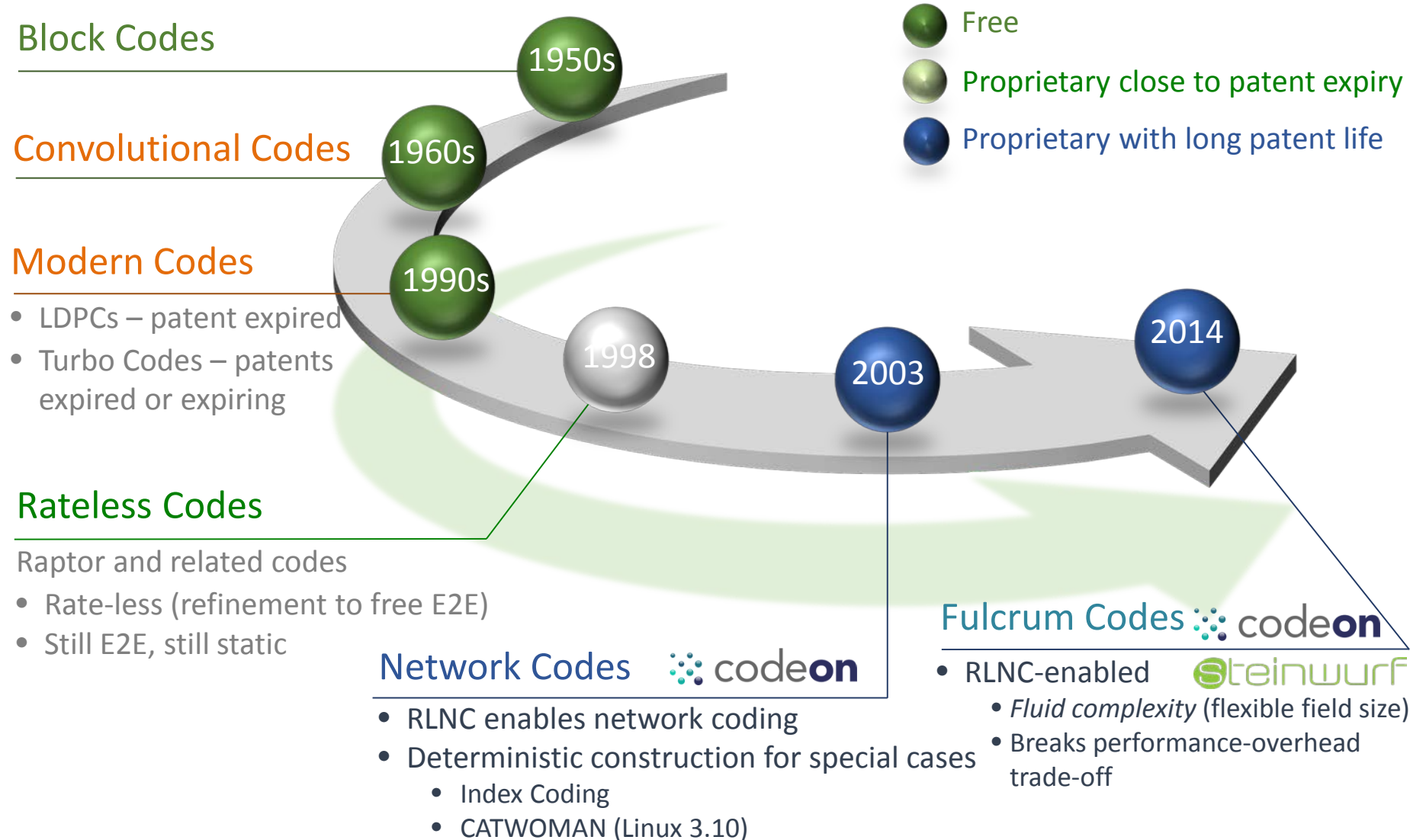
Please check every week!

# Random Linear Network Coding

Lecture 4

# Coding History

# Evolution of Coding



# The Technology: RLNC

*At the heart of many communications problems is a collectors' problem.*

## Traditional Approach

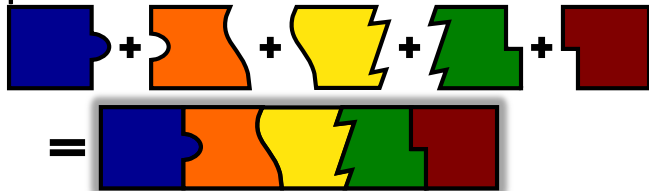
- Data broken into pieces



- k-piece data set  $\rightarrow$  k pieces



- All pieces needed

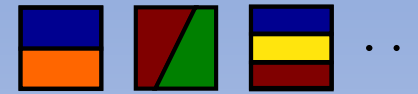


- Only these pieces will do

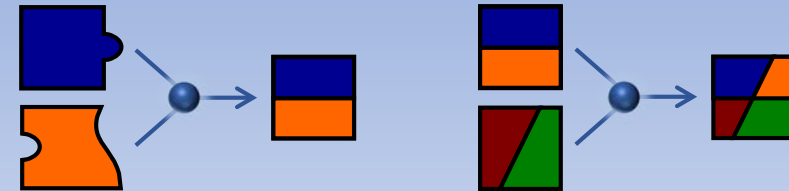


## RLNC

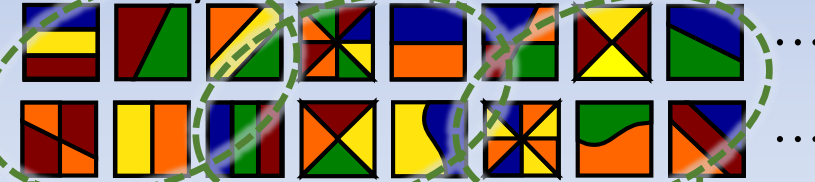
- Mixtures created from pieces



- Any node can create mixtures



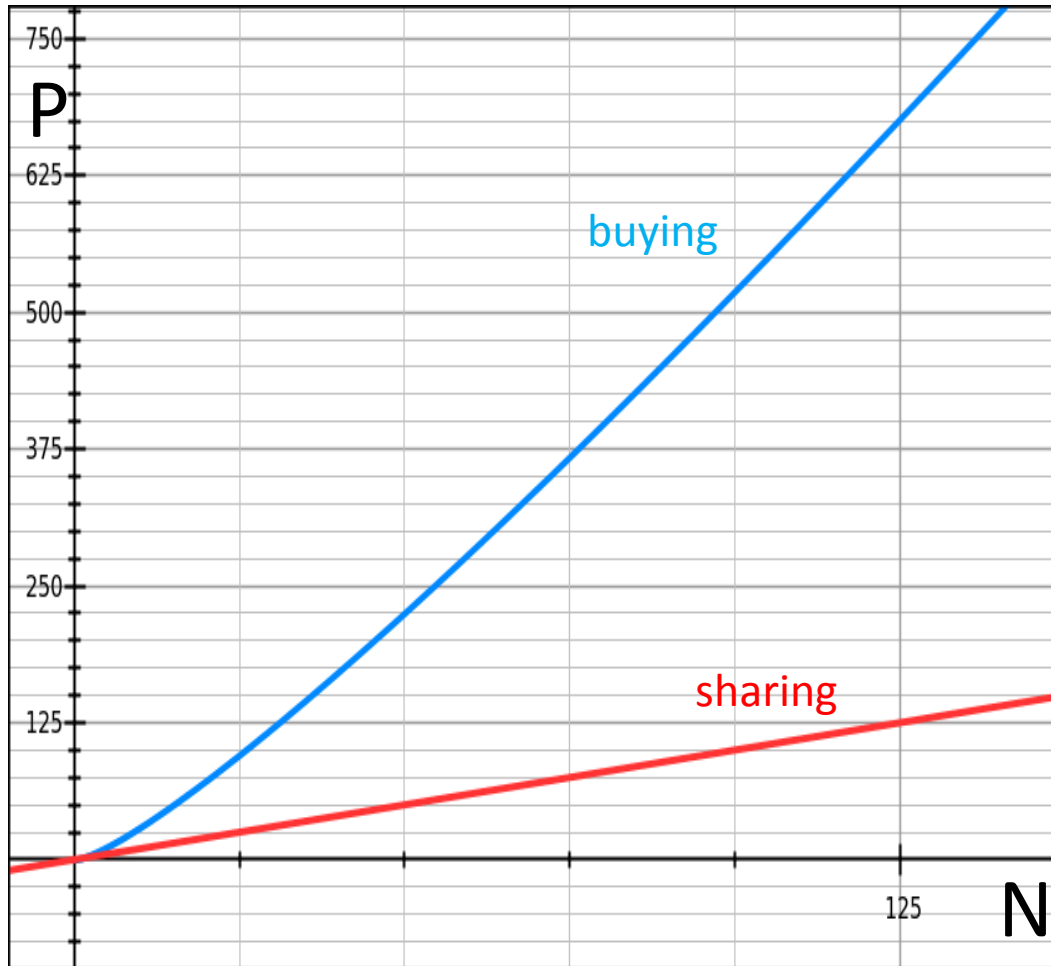
- Many mixtures possible



- Any k mixtures will do



# Collector's Problem



$$P = N * (\ln(N) + 0.577)$$

# Random Linear Network Coding

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{pmatrix} = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} & \alpha_{1,5} & \alpha_{1,6} \\ \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,4} & \alpha_{2,5} & \alpha_{2,6} \\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & \alpha_{3,4} & \alpha_{3,5} & \alpha_{3,6} \\ \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} & \alpha_{4,5} & \alpha_{4,6} \\ \alpha_{5,1} & \alpha_{5,2} & \alpha_{5,3} & \alpha_{5,4} & \alpha_{5,5} & \alpha_{5,6} \\ \alpha_{6,1} & \alpha_{6,2} & \alpha_{6,3} & \alpha_{6,4} & \alpha_{6,5} & \alpha_{6,6} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix}$$

Original  
packets

Gaussian elimination  $n \times n$  matrix requires  $An^3 + Bn^2 + Cn$  operations.



# Random Linear Network Coding

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{pmatrix} = \begin{matrix} \text{coding} \\ \text{coefficients} \end{matrix} \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} & \alpha_{1,5} & \alpha_{1,6} \\ \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,4} & \alpha_{2,5} & \alpha_{2,6} \\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & \alpha_{3,4} & \alpha_{3,5} & \alpha_{3,6} \\ \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} & \alpha_{4,5} & \alpha_{4,6} \\ \alpha_{5,1} & \alpha_{5,2} & \alpha_{5,3} & \alpha_{5,4} & \alpha_{5,5} & \alpha_{5,6} \\ \alpha_{6,1} & \alpha_{6,2} & \alpha_{6,3} & \alpha_{6,4} & \alpha_{6,5} & \alpha_{6,6} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix}$$

Gaussian elimination  $n \times n$  matrix requires  $An^3 + Bn^2 + Cn$  operations.

# Random Linear Network Coding

coded  
packets

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{pmatrix} = \begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} & \alpha_{1,5} & \alpha_{1,6} \\ \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,4} & \alpha_{2,5} & \alpha_{2,6} \\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & \alpha_{3,4} & \alpha_{3,5} & \alpha_{3,6} \\ \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} & \alpha_{4,5} & \alpha_{4,6} \\ \alpha_{5,1} & \alpha_{5,2} & \alpha_{5,3} & \alpha_{5,4} & \alpha_{5,5} & \alpha_{5,6} \\ \alpha_{6,1} & \alpha_{6,2} & \alpha_{6,3} & \alpha_{6,4} & \alpha_{6,5} & \alpha_{6,6} \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{pmatrix}$$

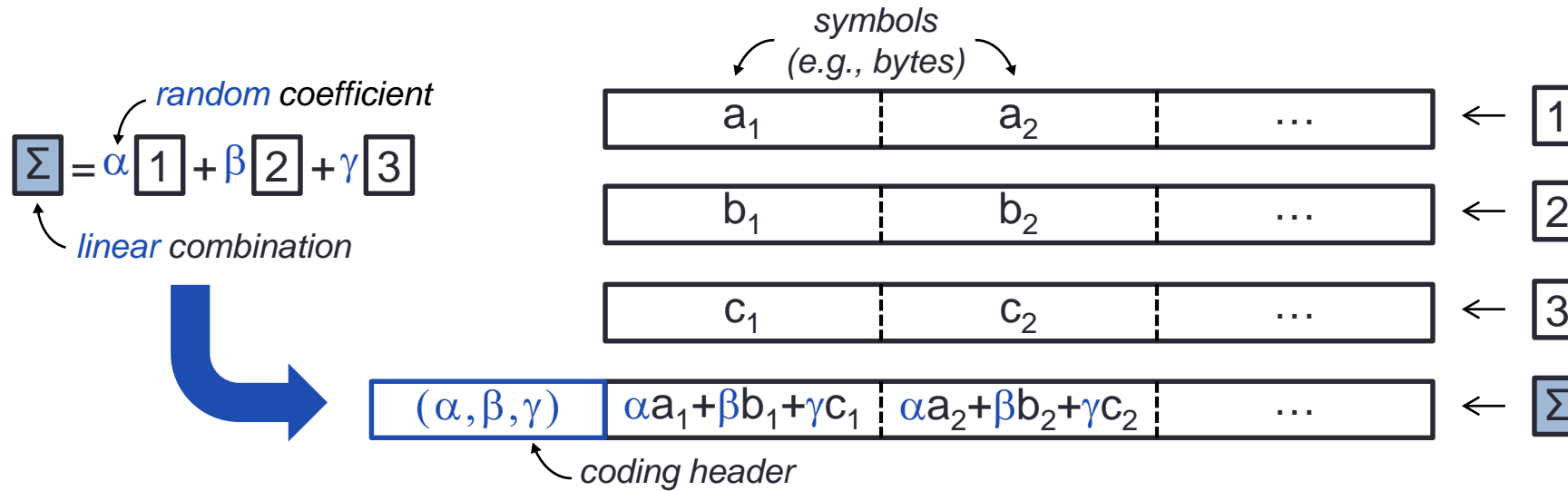
Gaussian elimination  $n \times n$  matrix requires  $An^3 + Bn^2 + Cn$  operations.

# Random Linear Network Coding

$$\begin{pmatrix} C_1 \\ \vdots \\ C_G \\ C_{G+1} \\ \vdots \\ C_K \end{pmatrix} = \begin{pmatrix} \alpha_{1,1} & \dots & \alpha_{1,G} \\ \vdots & \ddots & \vdots \\ \alpha_{G,1} & \dots & \alpha_{G,G} \\ \alpha_{G+1,1} & \dots & \alpha_{G+1,G} \\ \vdots & \ddots & \vdots \\ \alpha_{K,1} & \dots & \alpha_{K,G} \end{pmatrix} \begin{pmatrix} P_1 \\ \vdots \\ P_G \end{pmatrix}$$

Rateless code: can output any number of coded packets.  
(such as Fountain codes, but better than RS)

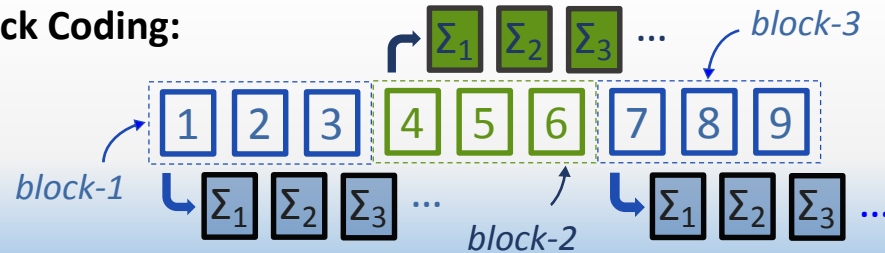
# How does RLNC work?



- Random Generation of Coefficients
- Code Embedded within Data
- No State Tracking
- New IT Paradigms

## Multiple encoding schemes

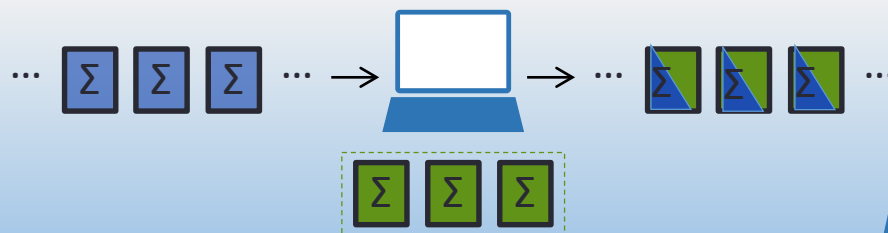
### Block Coding:



### Sliding Window Encoding:



### Multi-hop Re-encoding:



## One decoding scheme (simple equation-solving)

$$1 = a'_1 \Sigma_1 + b'_1 \Sigma_2 + g'_1 \Sigma_3$$

$$2 = a'_2 \Sigma_1 + b'_2 \Sigma_2 + g'_2 \Sigma_3$$

$$3 = 1 + b'_3 \Sigma_2 + g'_3 \Sigma_3$$

*obtained through  
Gaussian Elimination*

*Can decode using both  
encoded and un-encoded packets*

# RLNC: The Technology

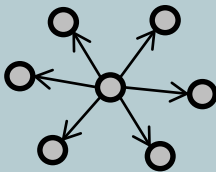
## Coding Today

(all End-to-End)

Classical



Multicast



## Coding Tomorrow with RLNC

Classical + Sliding Window Encoding



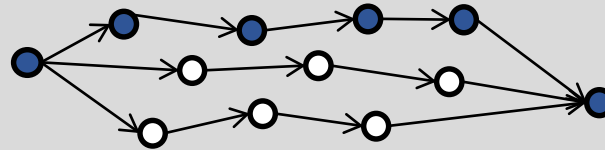
Real time video streaming,  
TCP, SDN...

Multihop



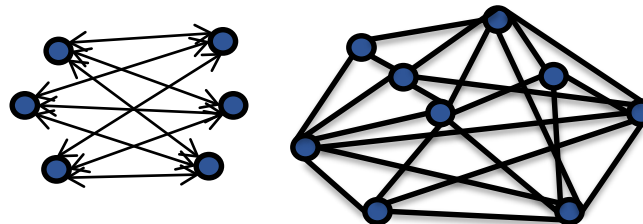
Edge caches, wireless mesh,  
reliable multicast, satellites,  
small relay topologies, SDN...

Multipath



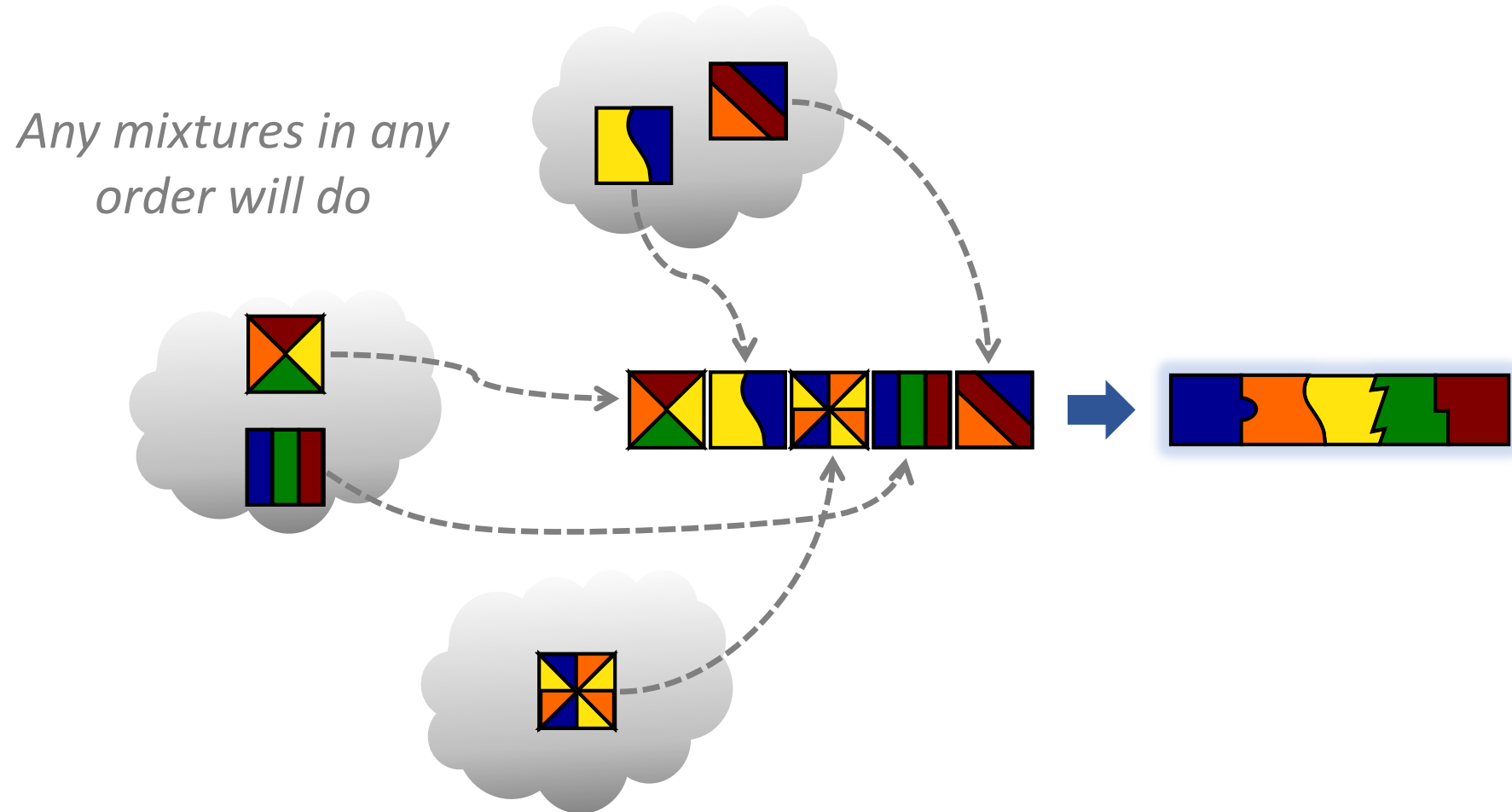
Multi-source streaming  
Multipath TCP, channel  
bundling, heterogeneous  
network combining, SDN...

Multisource – Multi-destination / Mesh



Distributed cloud, SDN,  
advanced mesh (IoT, car2car,  
M2M, smart grid) ...

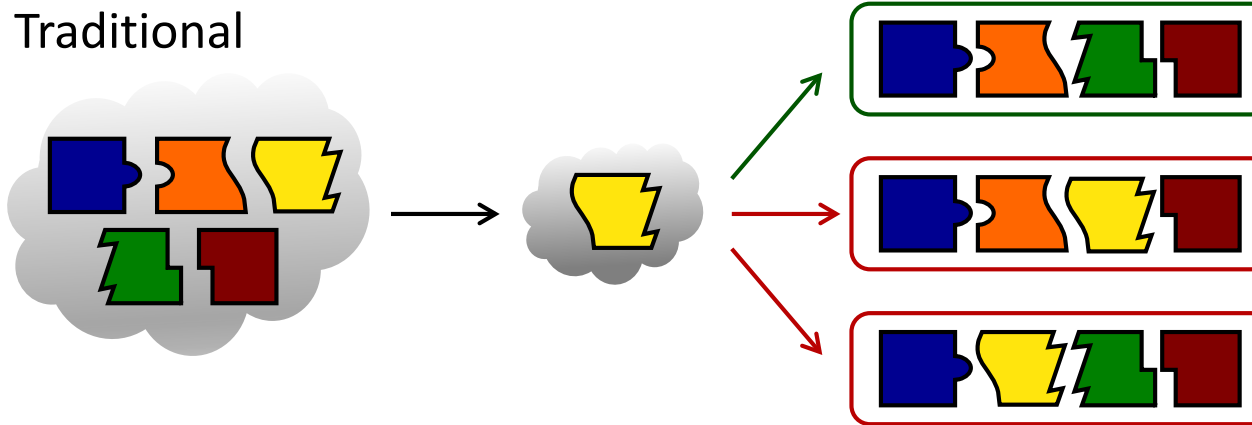
# Multipath – Multicloud



- RLNC Enables “Stateless Communications”

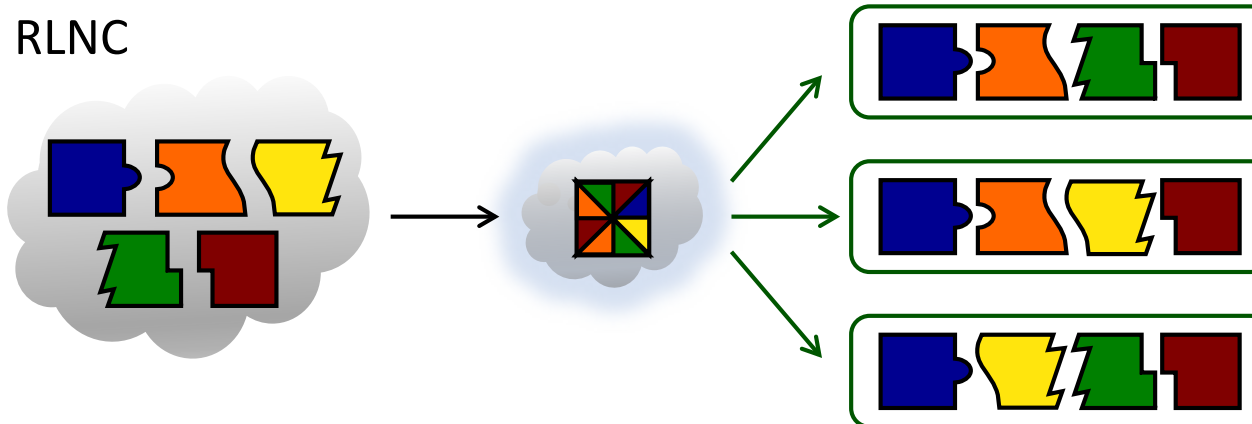
# Coded Edge Caching

Traditional



- One small cache can't satisfy everyone
- Only the device missing the piece in the edge cache can reconstruct

RLNC

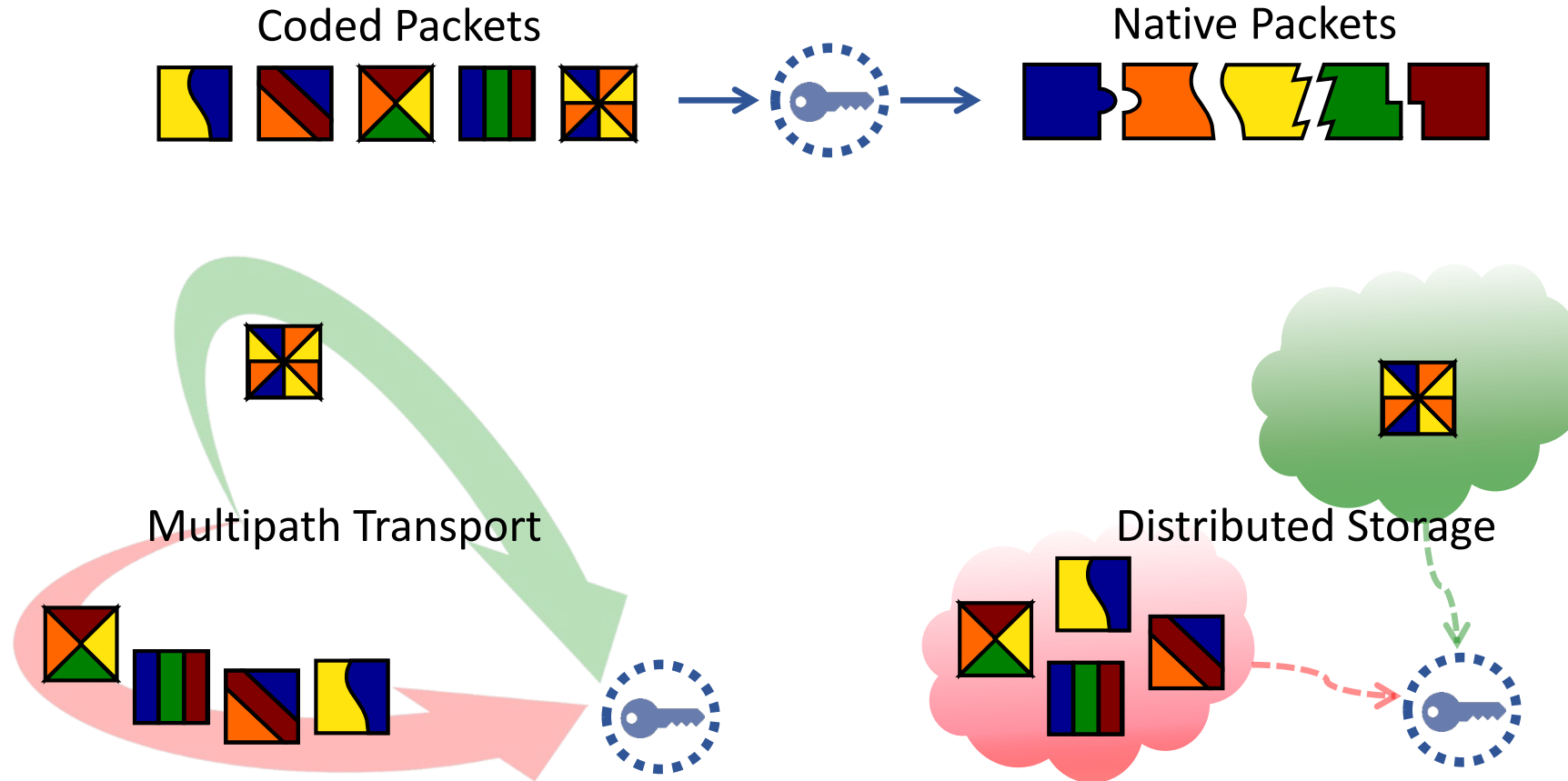


- Mixtures enable all nodes to reconstruct
- An example of RLNC's enhancement of non-coded systems

■ A Little RLNC Can Go a Long Way

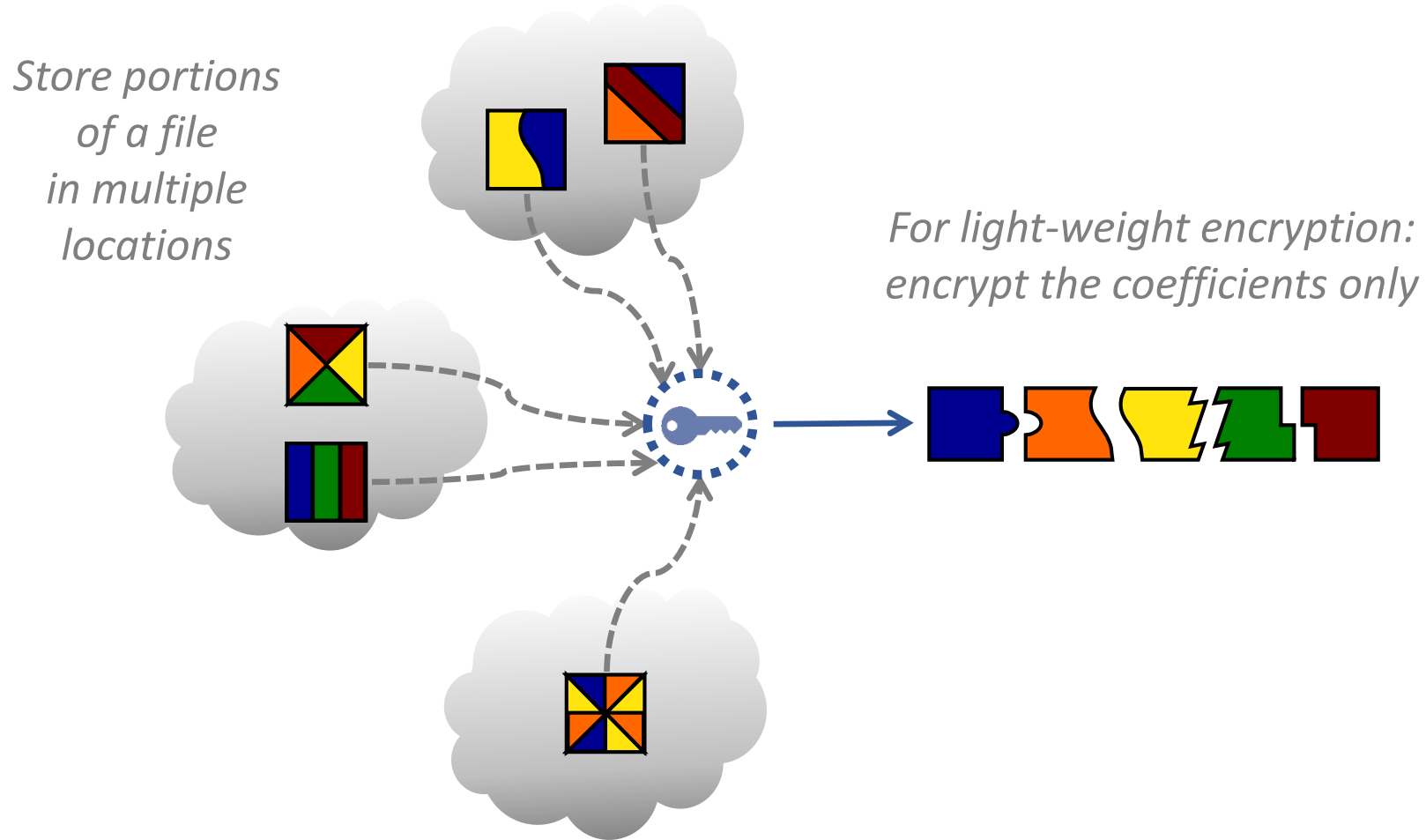


# Coding as an Additional Security Measure



- Data on a given path/cloud acts as a cypher

# RLNC Improves Cloud Security

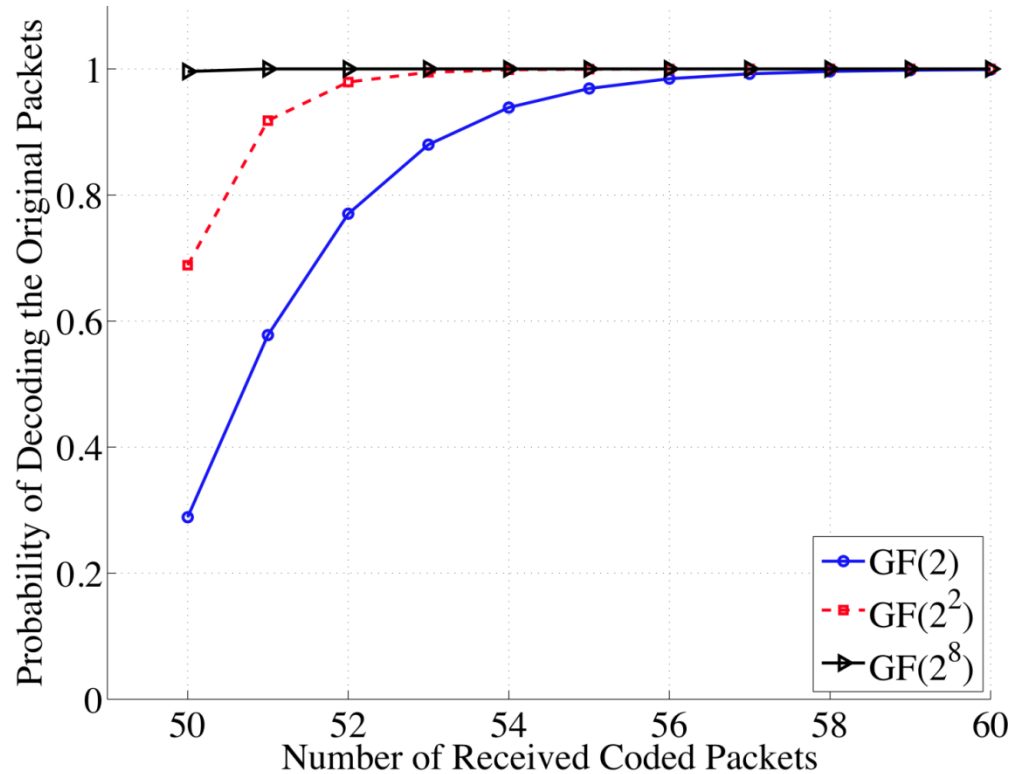


- Mixture equations "unlock" data from mixtures

# Key Parameters of RLNC

- **Generation size:** number of packets that are (currently) coded together.
- **Field size:** number of elements in the finite field
- Both have an impact on:
  - Performance
  - Complexity

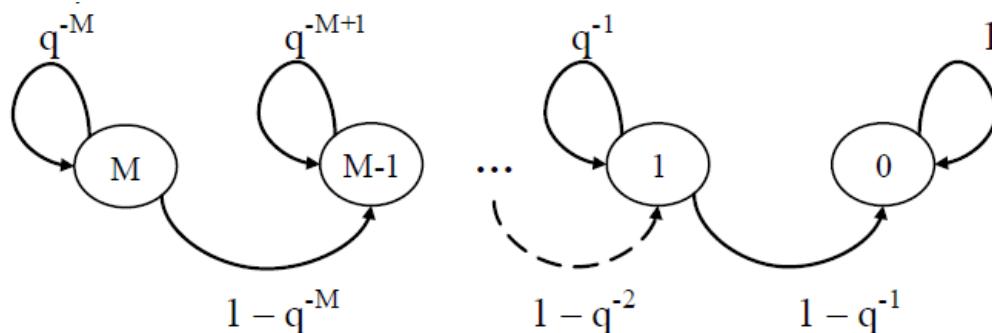
# Field and Generation Size



Small field sizes are resulting in linear dependent coded packets.

1.6 packets extra per generation in case of binary field sizes.

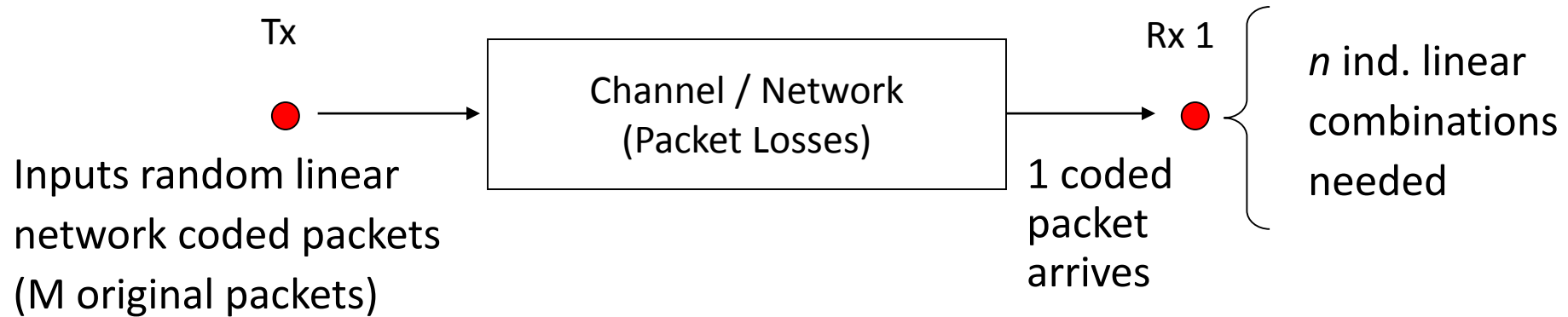
Large fields sizes (2<sup>8</sup> or higher) have nearly no linear dependency.



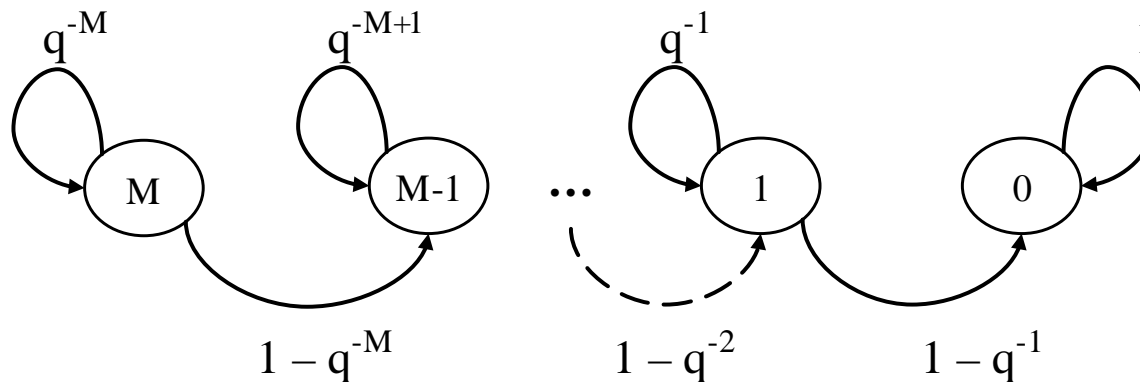
Theory is aiming for large field sizes and large generation sizes!

# Field Size Analysis

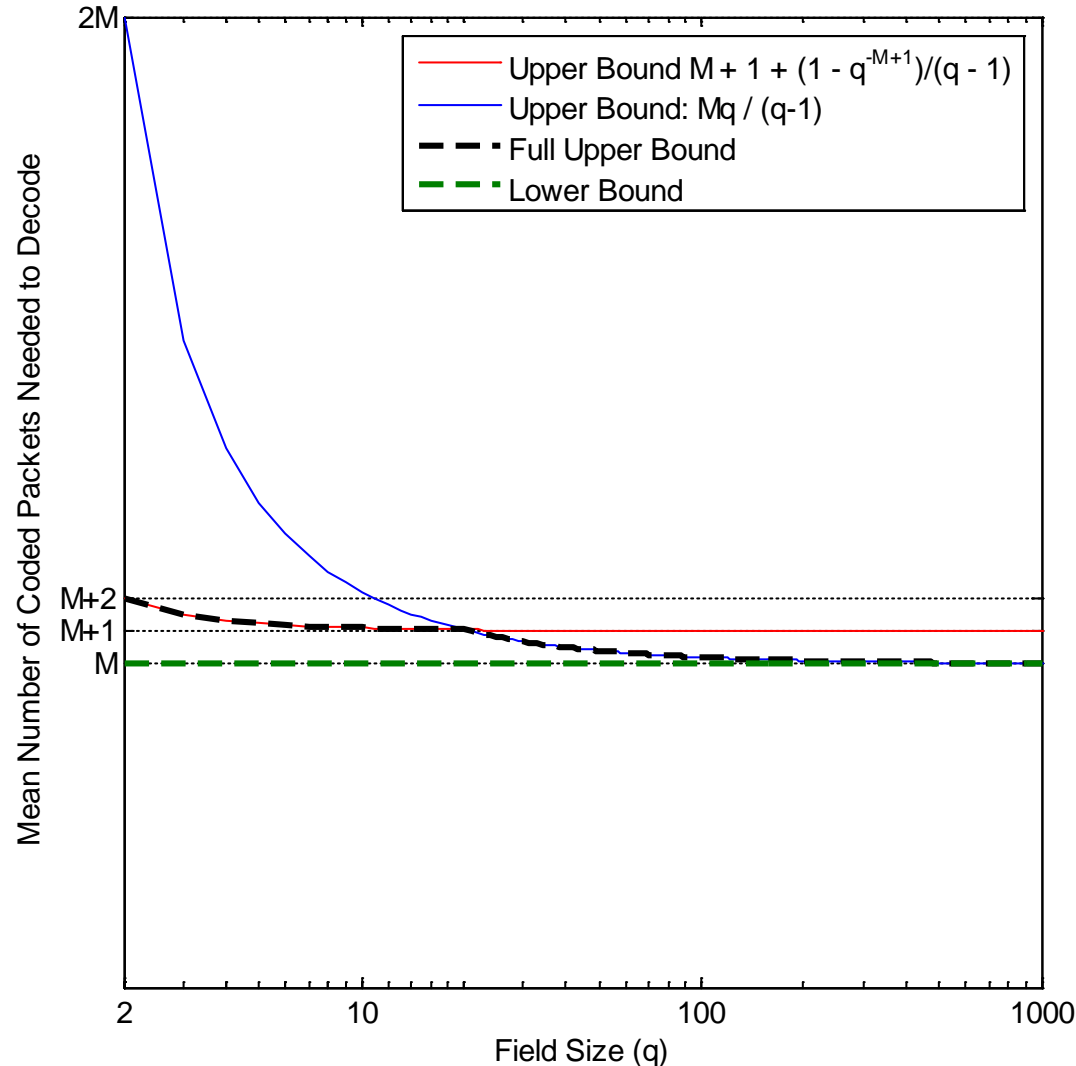
# Field Size Analysis for RLNC



–Modeled as a Markov chain



# Field Size Analysis



$$E[N_c] = \sum_{k=1}^M \frac{1}{1 - q^{-k}}$$

$$\leq \min \left( M \frac{q}{q-1}, M + 1 + \frac{1 + q^{-M+1}}{q-1} \right)$$

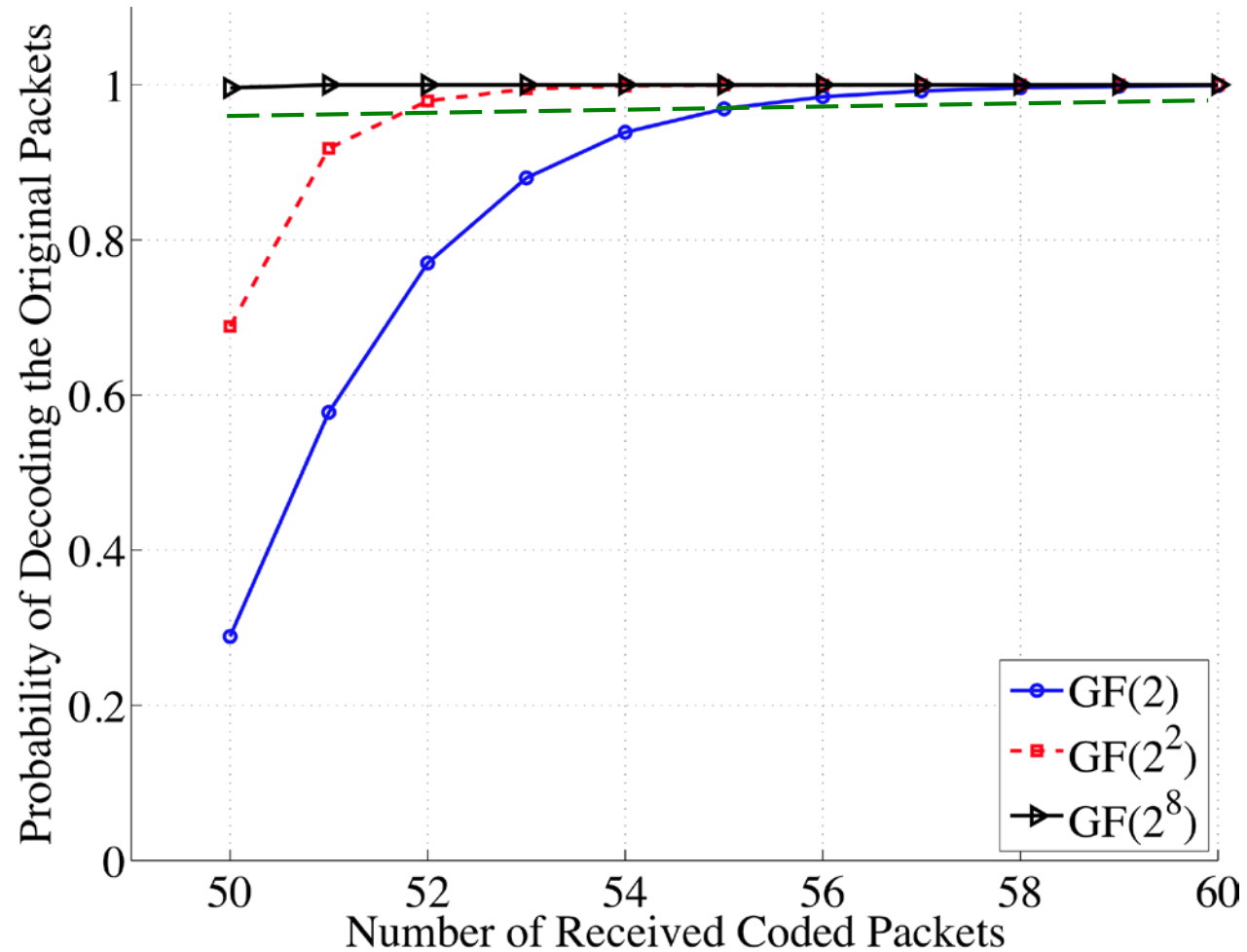
If  $M$  is large:

- Little overhead
- Small performance degradation

Later today you will simulate

- Task 7: single link, GF(2), generation size 8

# Field Size Analysis: Distribution





# DEMO

# Recoding Potential

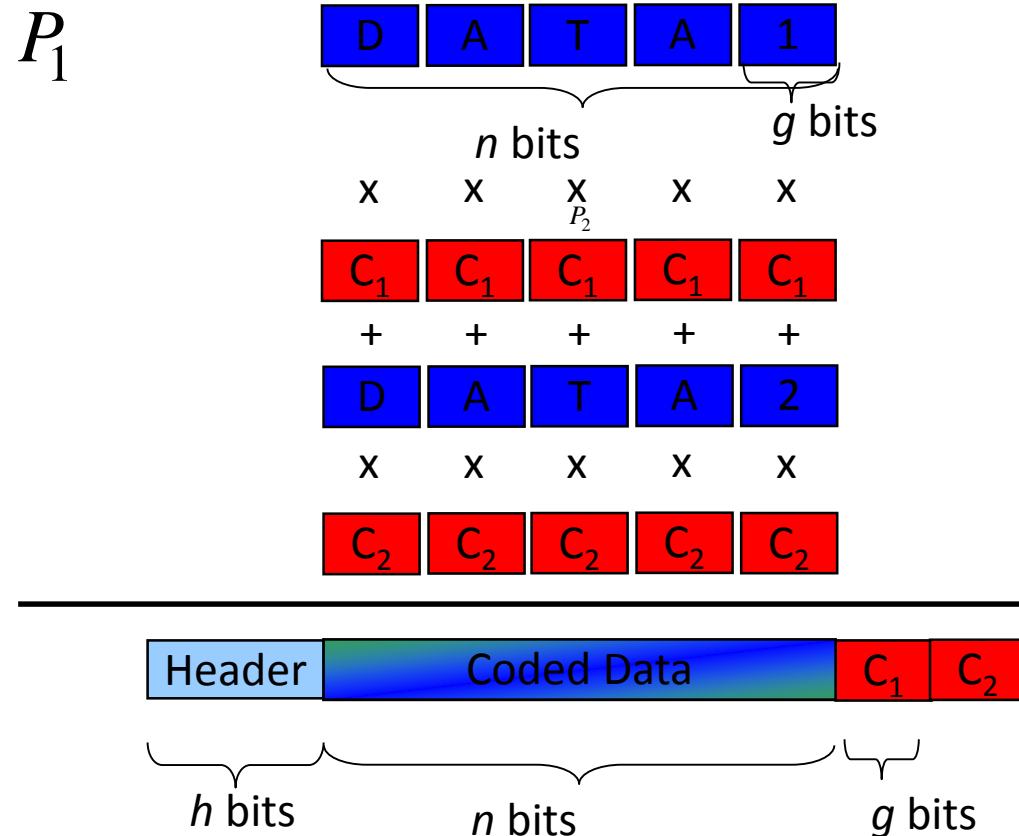
# Generating a Coded Packet

- Generating a linear network coded packet (CP)

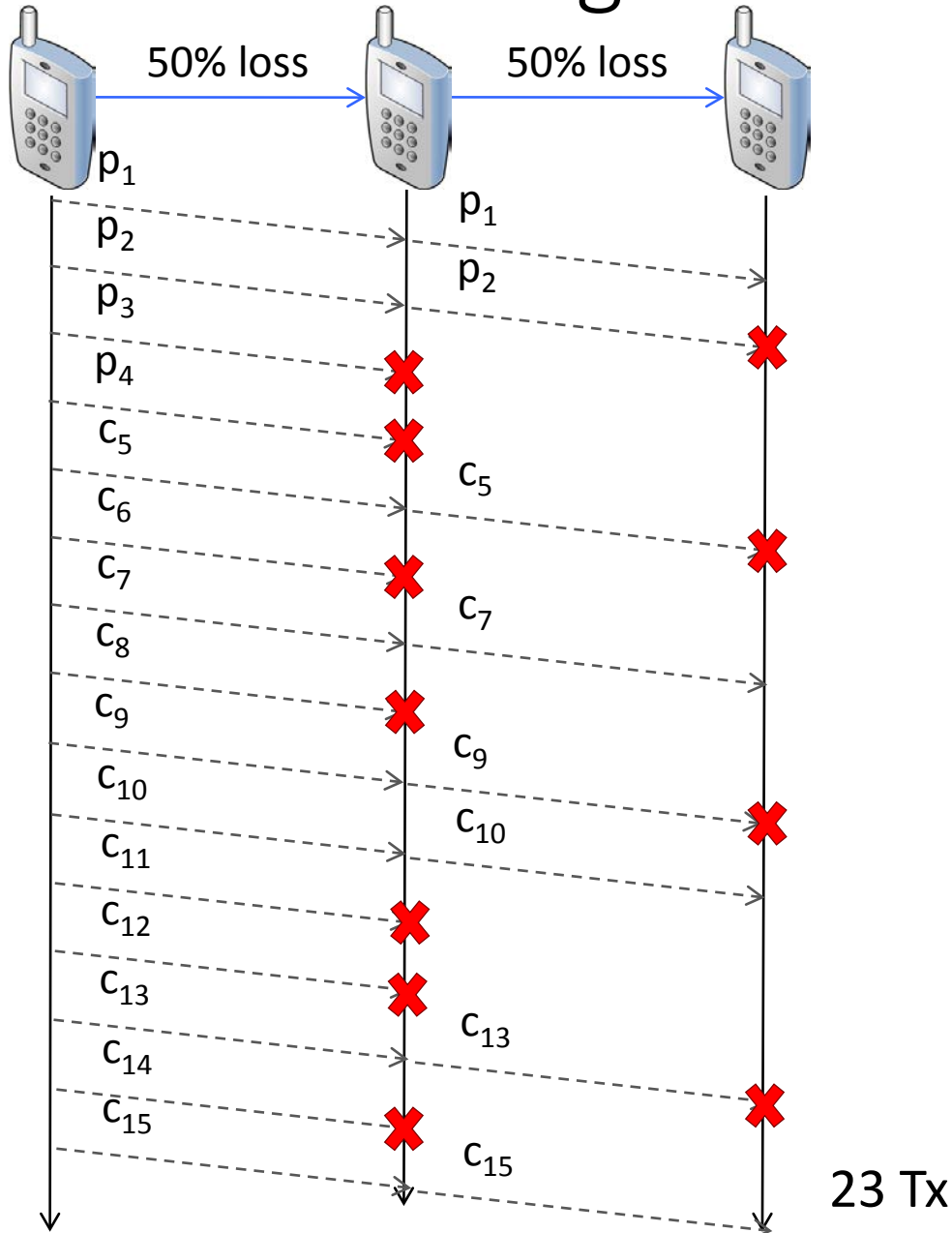
$$CP_j = \sum_i C_i P_i$$

- Operations over finite field of size.

e.g.  $g = 8$  bits,  $q = 256$



# No Recoding



- Simple operation: forward
- Structure of code is preserved
- Issues
  - Delay per batch of packets
  - Missing transmission opportunities
  - Equivalent loss probability: compounding each channel's loss

Loss of channel  $i$ :  $e_i$

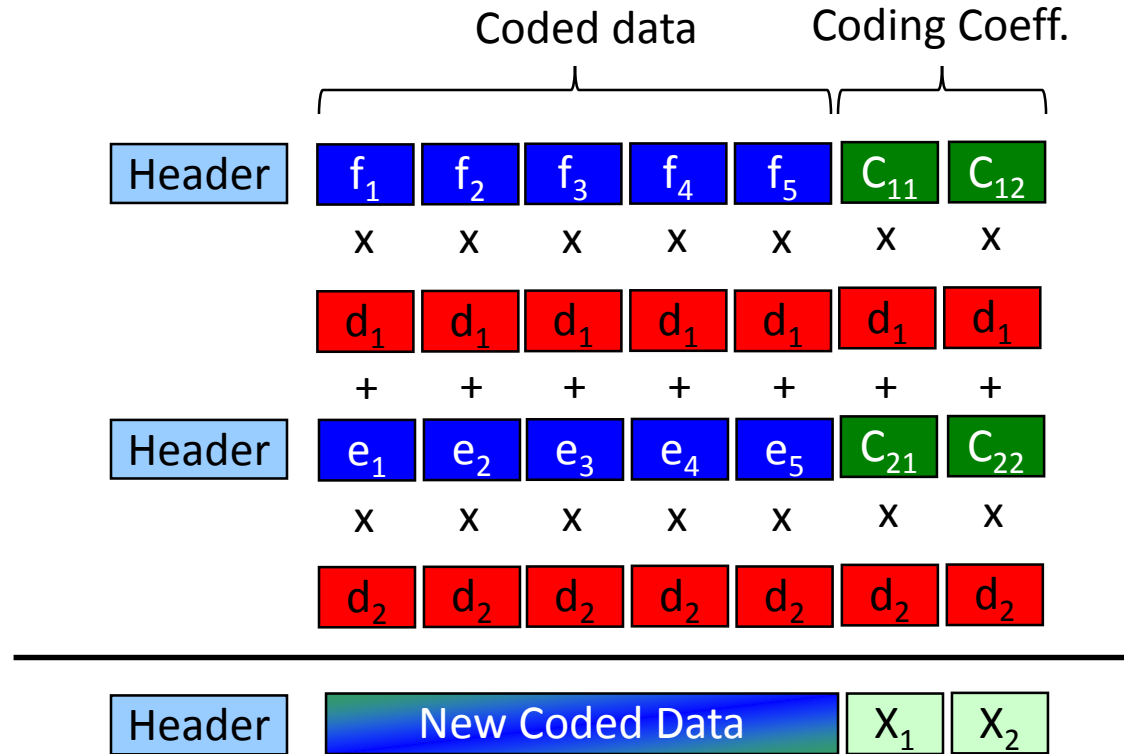
Equivalent success prob of a packet:  
 $(1-e_1) (1-e_2)$

$$c_i = \sum \alpha_{ij} p_j$$

✗ Packet loss

# Recoding Packets

- Generating a new linear network coded packet (CP)

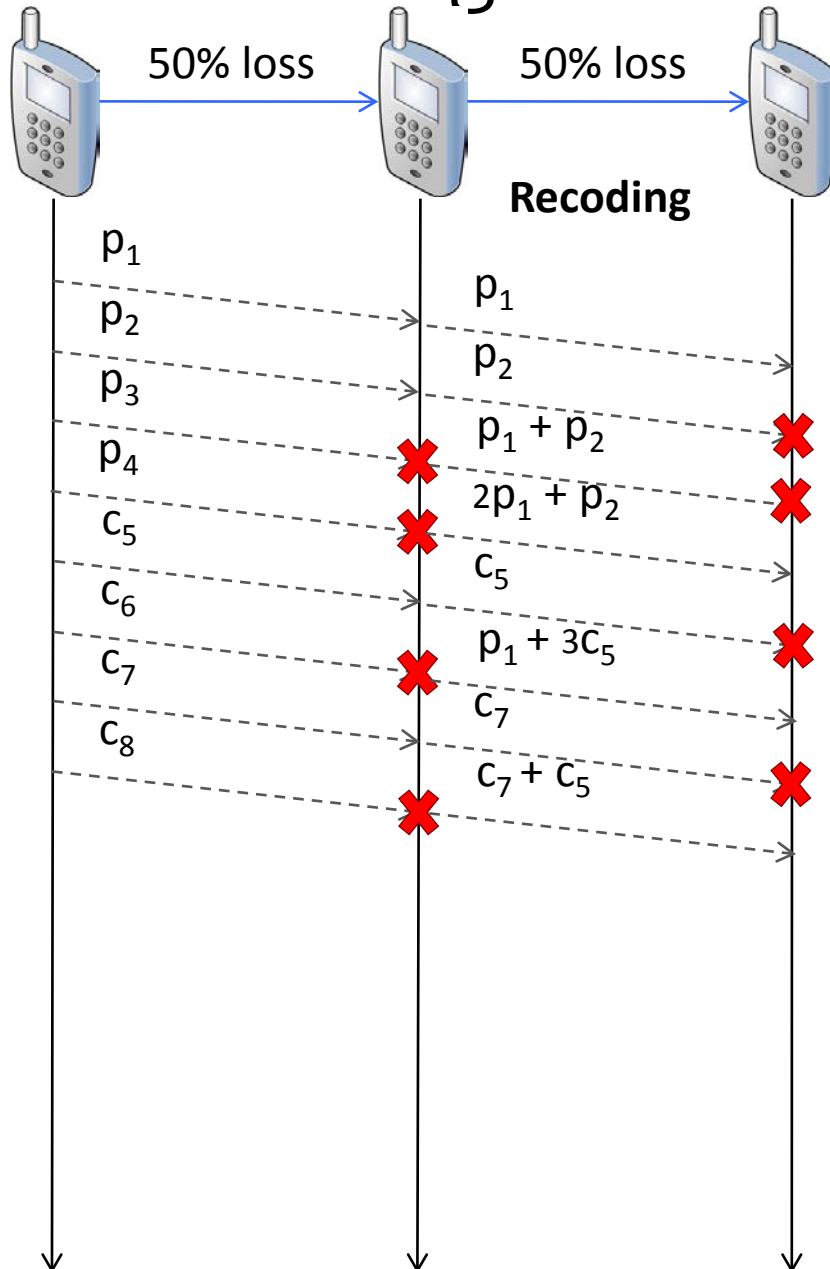


$$X_1 = d_1 C_{11} + d_2 C_{21} \quad \text{and} \quad X_2 = d_1 C_{12} + d_2 C_{22}$$

$$\text{Recall that: } f = C_{11}P_1 + C_{12}P_2 \quad \text{and} \quad e = C_{21}P_1 + C_{22}P_2$$

$$\text{Thus, } d_1f + d_2e = d_1C_{11}P_1 + d_1C_{12}P_2 + d_2C_{21}P_1 + d_2C_{22}P_2 = X_1P_1 + X_2P_2$$

# Recoding



- A bit more complex: recode
  - Equivalent to encoding in worst case
  - Not so bad
- Structure of code may be changed
  - Some exceptions
- Issues
  - If left unchecked, can have unnecessary transmissions, e.g.,  $c_8$
  - Additional processing needed

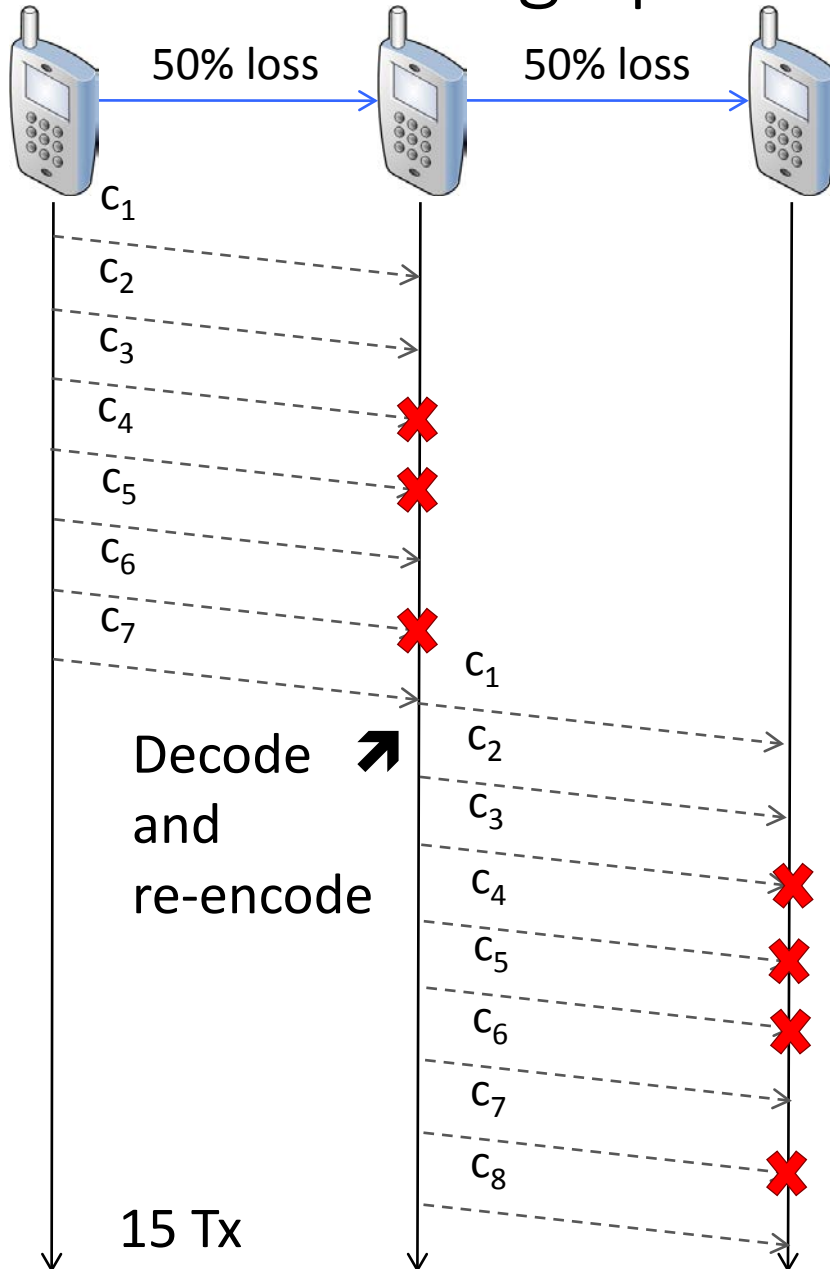
Advantage: equivalent success prob of a *linear combination*:

$$\min\{1-e_1, 1-e_2\}$$

$$c_i = \sum \alpha_{ij} p_j$$

✗ Packet loss

# Is “recoding” possible with other linear codes?



Consider Reed-Solomon, LT, etc

- Structure is not composable
  - Mixing coded packets does not produce a “valid” coded packet
  - Different structure, properties are lost
- Recoding means receiving enough coded packets, decode, and *then* re-encode
- Issues
  - Delay per batch
  - Computational effort
  - Can also have unnecessary Tx
- Success prob of a *linear combination*:  $\min\{1-e_1, 1-e_2\}$

# Software Defined Network (SDN)

## Example



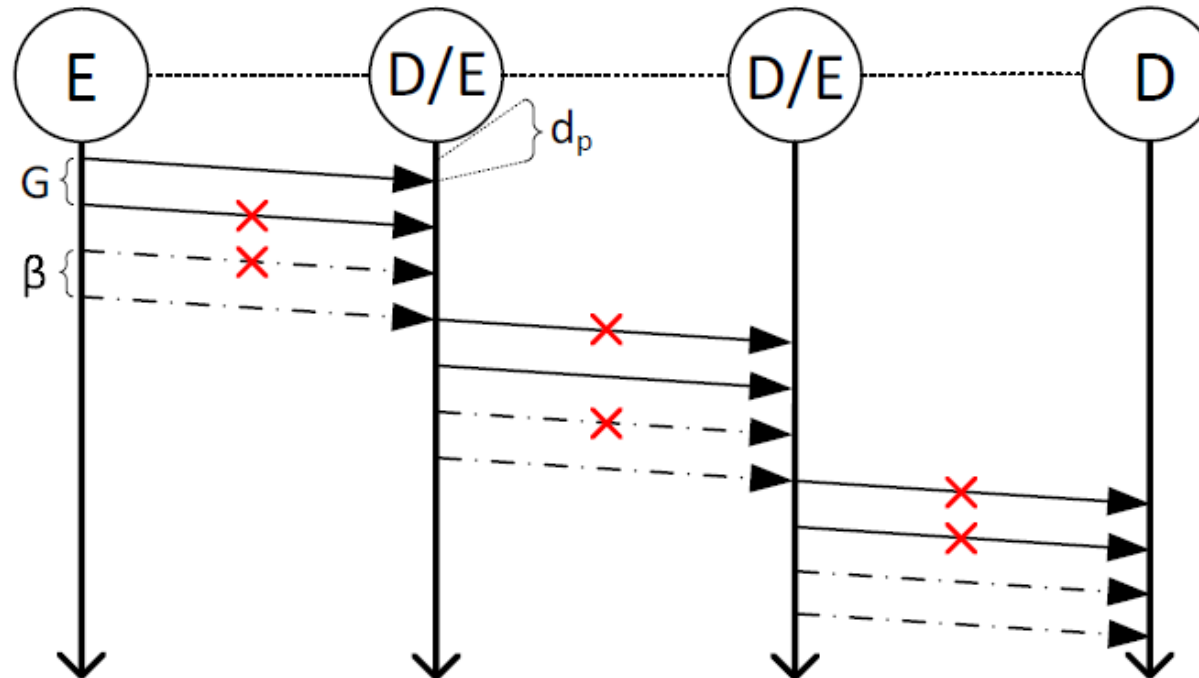


# Software Defined Networks

## ■ Hop by Hop Coding Scheme: Store and Forward

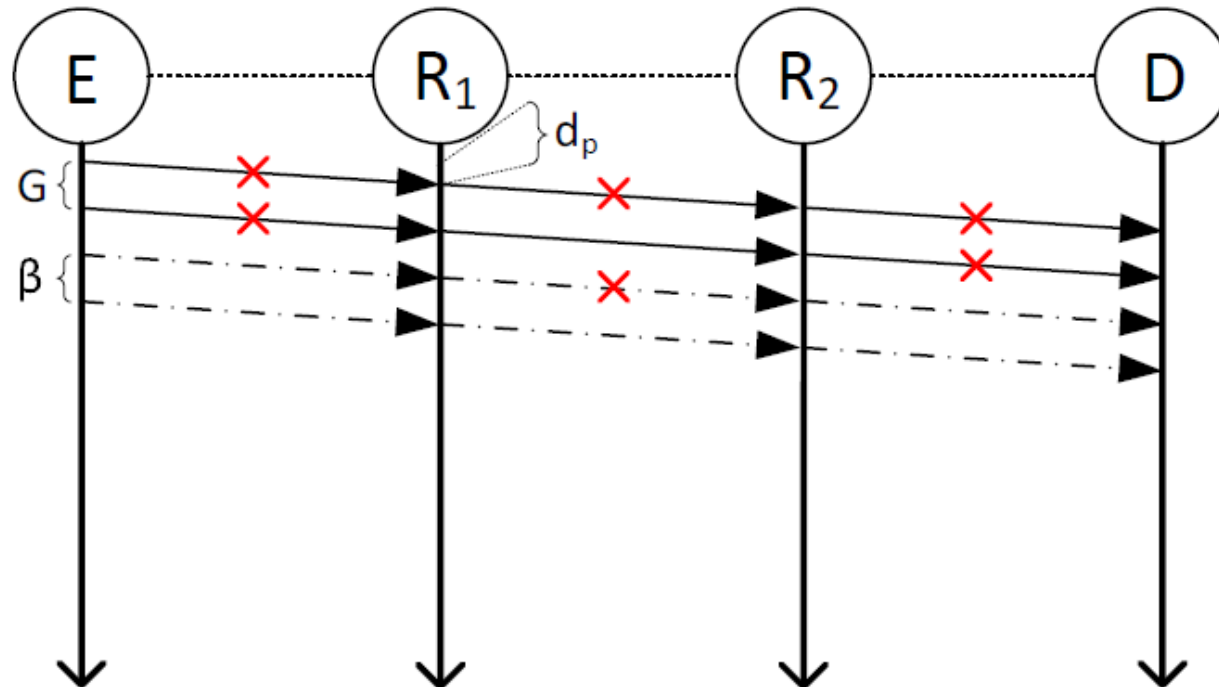
$$P_{HbH} = G \cdot H \cdot \left( \frac{1}{1 - \epsilon} \right)$$

$$D_{HbH} = G \cdot \left( \frac{1}{1 - \epsilon} \right) \cdot H \cdot d_p$$



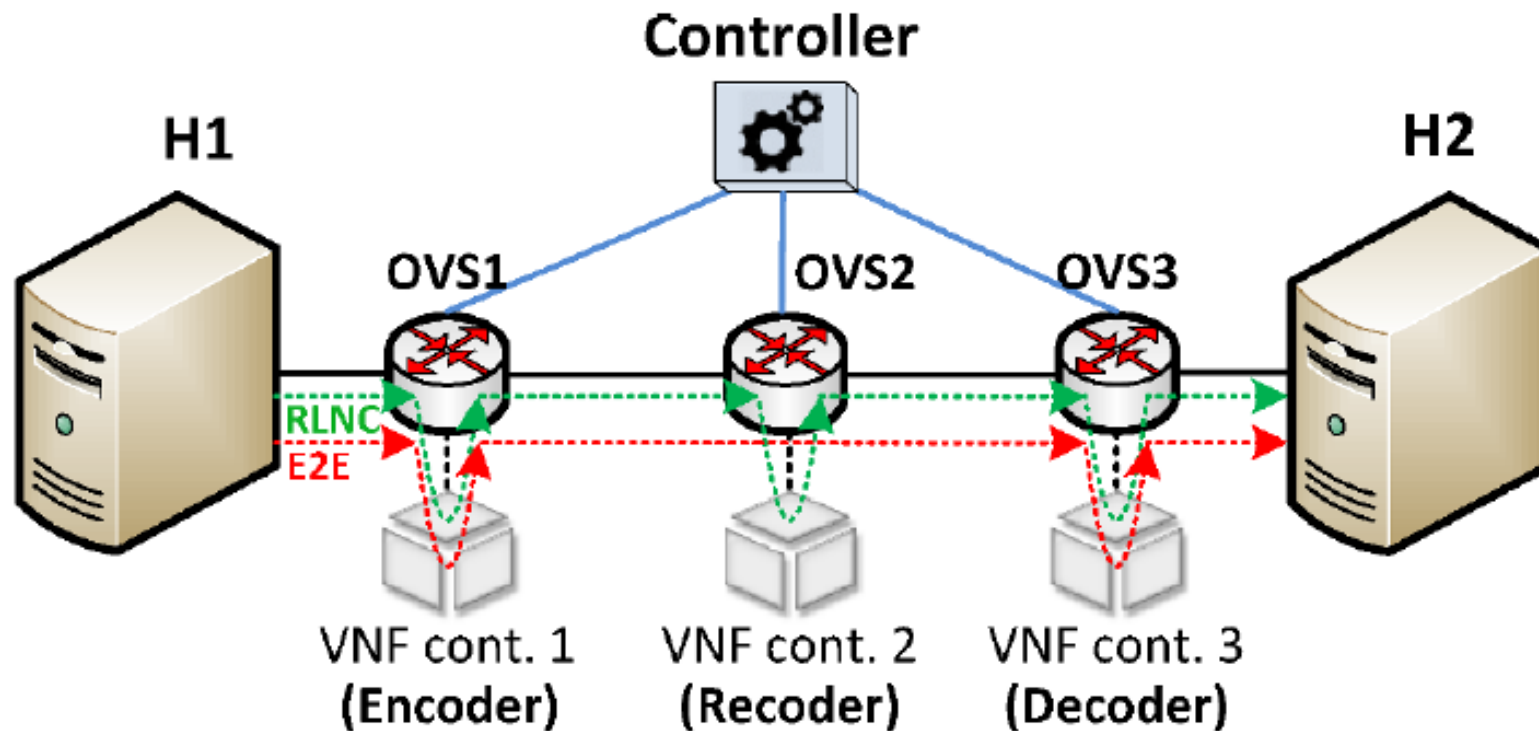
- Network Coding Scheme: Compute and Forward

$$P_{RLNC} = G \cdot H \cdot \left( \frac{1}{1 - \epsilon} \right) \quad D_{RLNC} = \left( G \cdot \left( \frac{1}{1 - \epsilon} \right) + (H - 1) \right) \cdot d_p$$

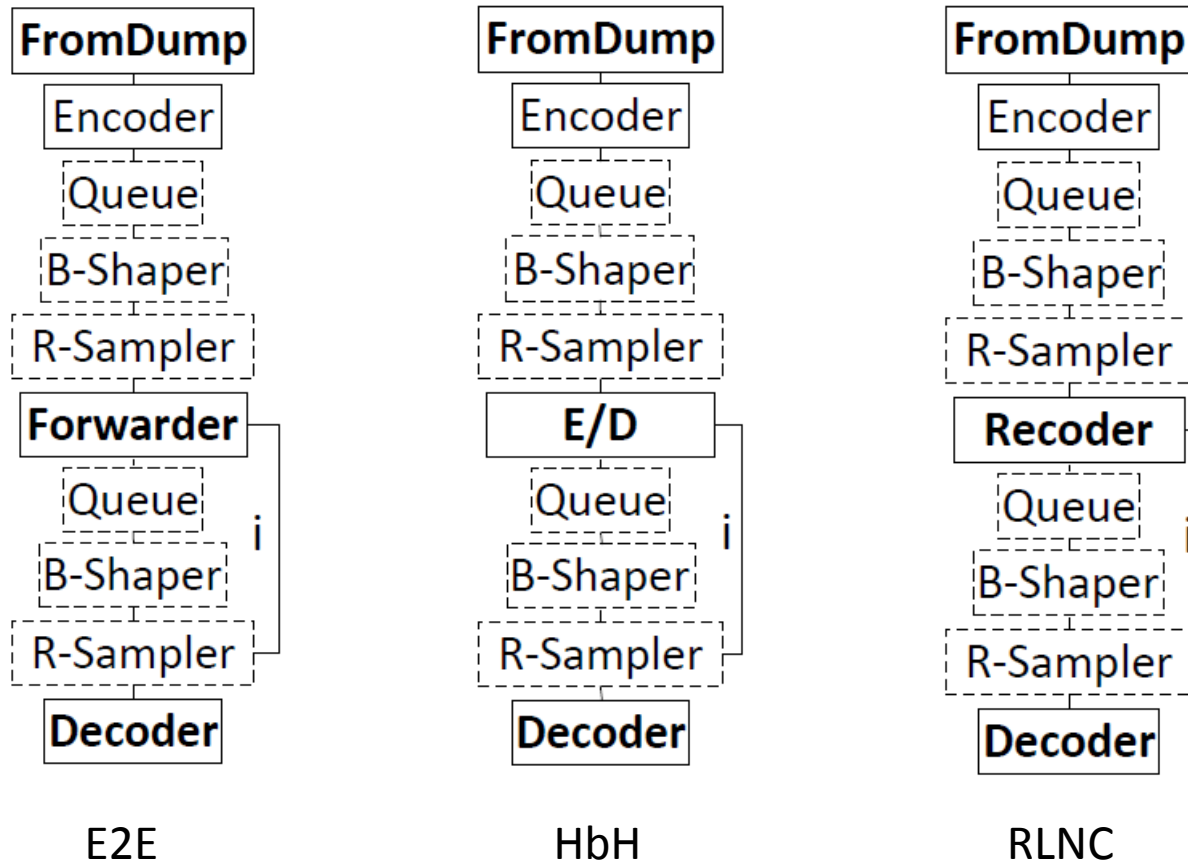


# SDN testbed

- Example with ESCAPE prototyping environment

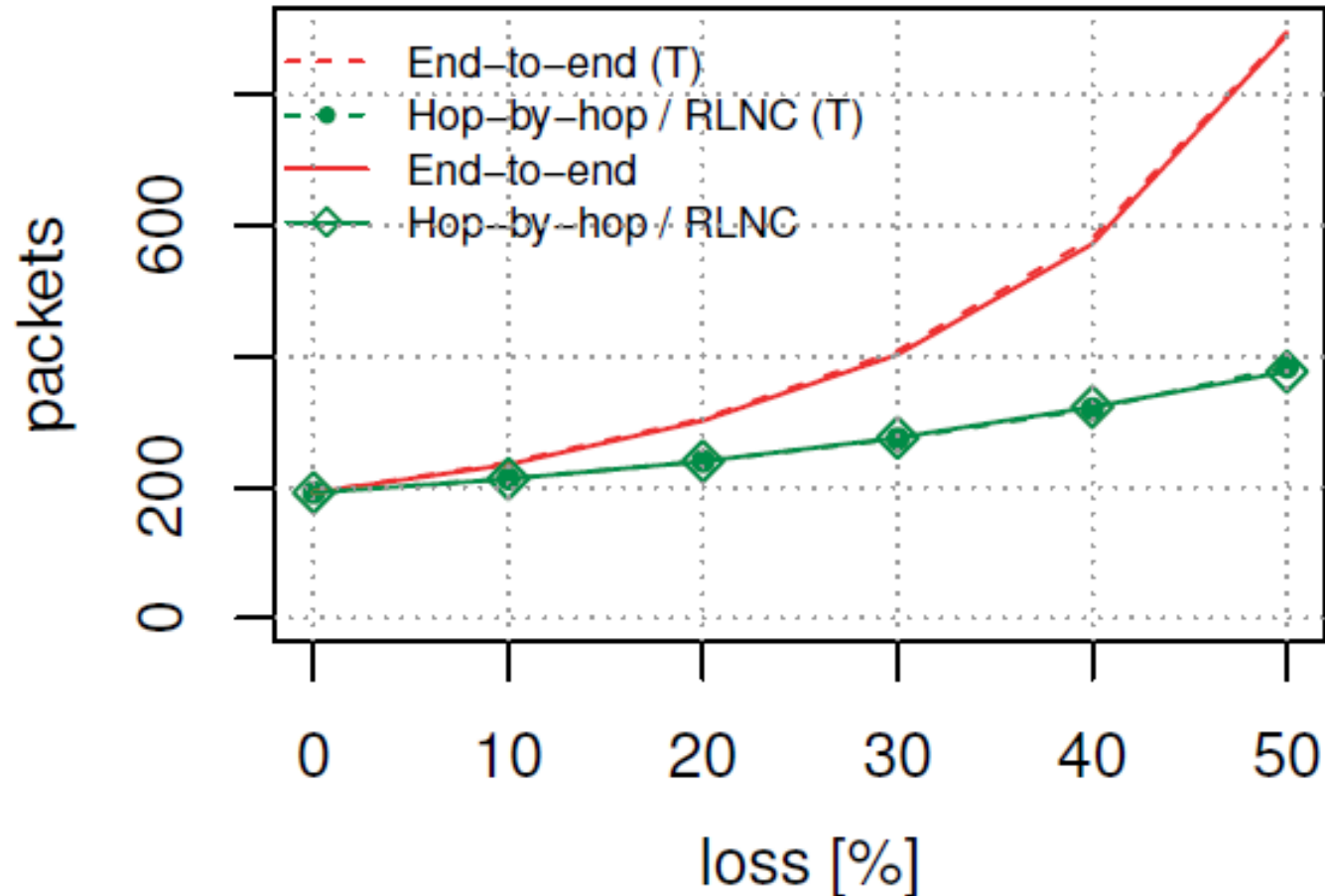


- Click configuration



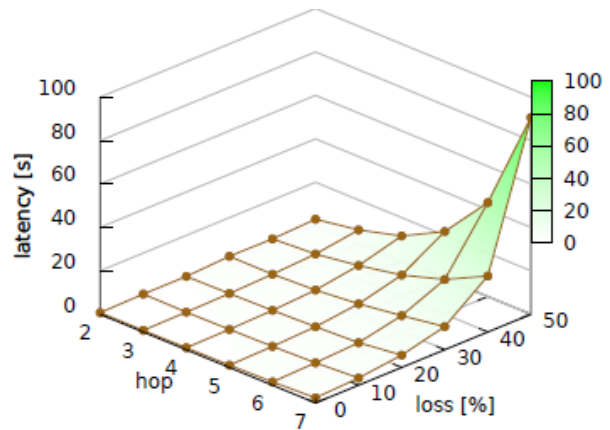
# Software Defined Networks

- Packets injected into network for the three approaches

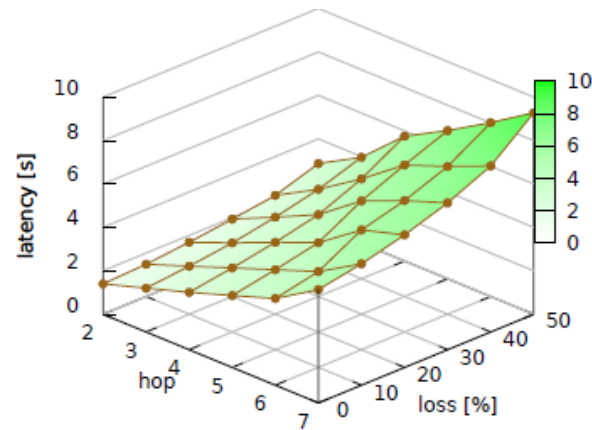


# Results

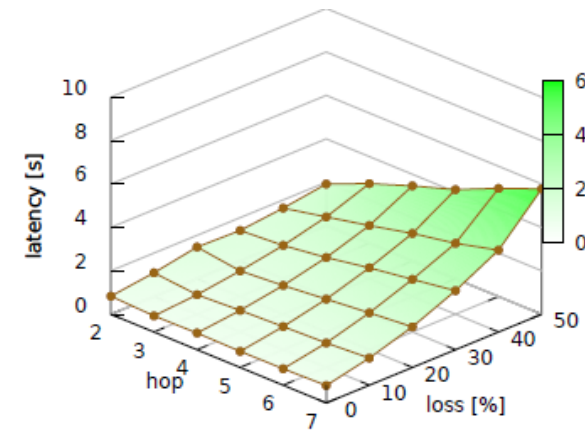
- Packets 64 – Size 250 B – Bitrate 0.25 Mb/s



end-to-end



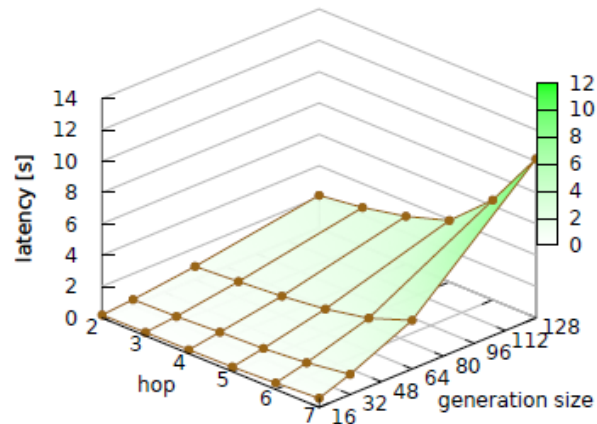
hop-by-hop



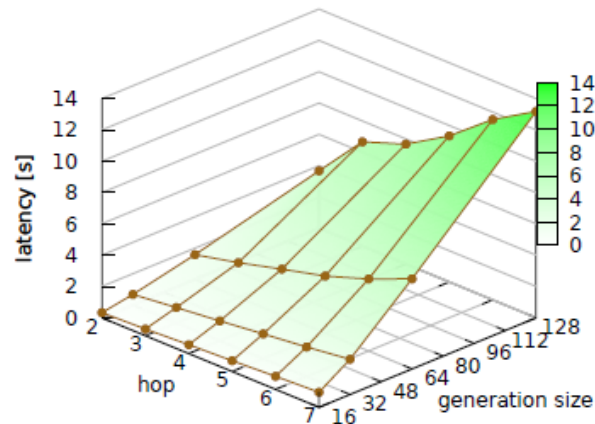
RLNC

# Results

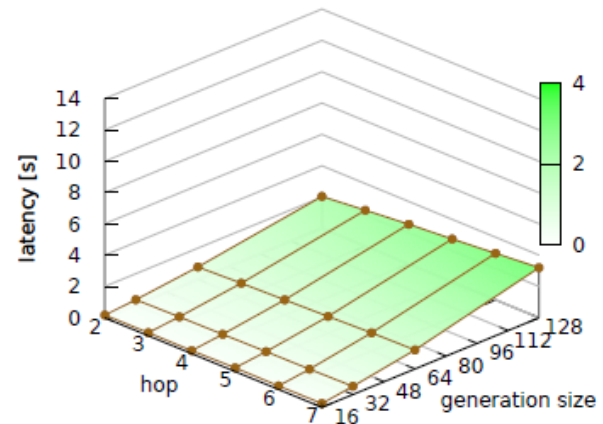
- Size 250 B – Loss 10% – Bitrate 0.25 Mb/s



end-to-end



hop-by-hop

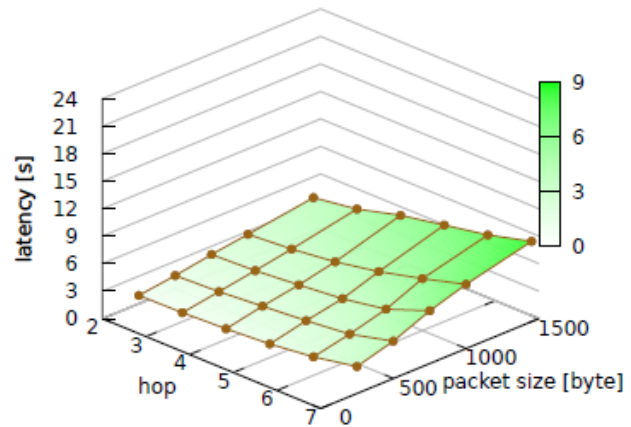


RLNC

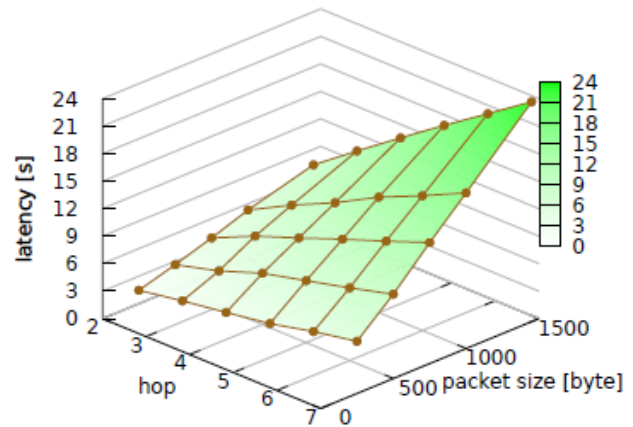


# Results

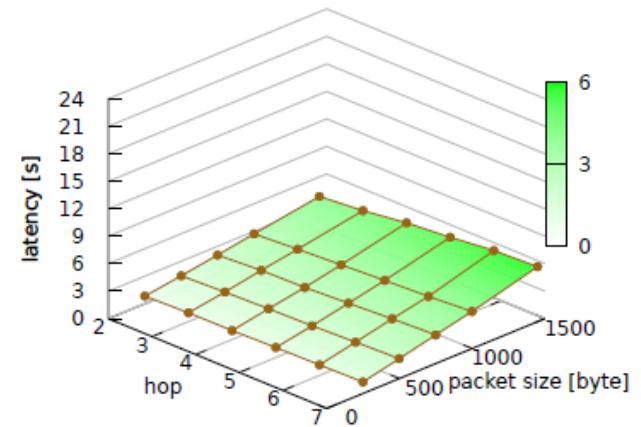
- Packets 64 – Loss 10% – Bitrate 0.25 Mb/s



end-to-end



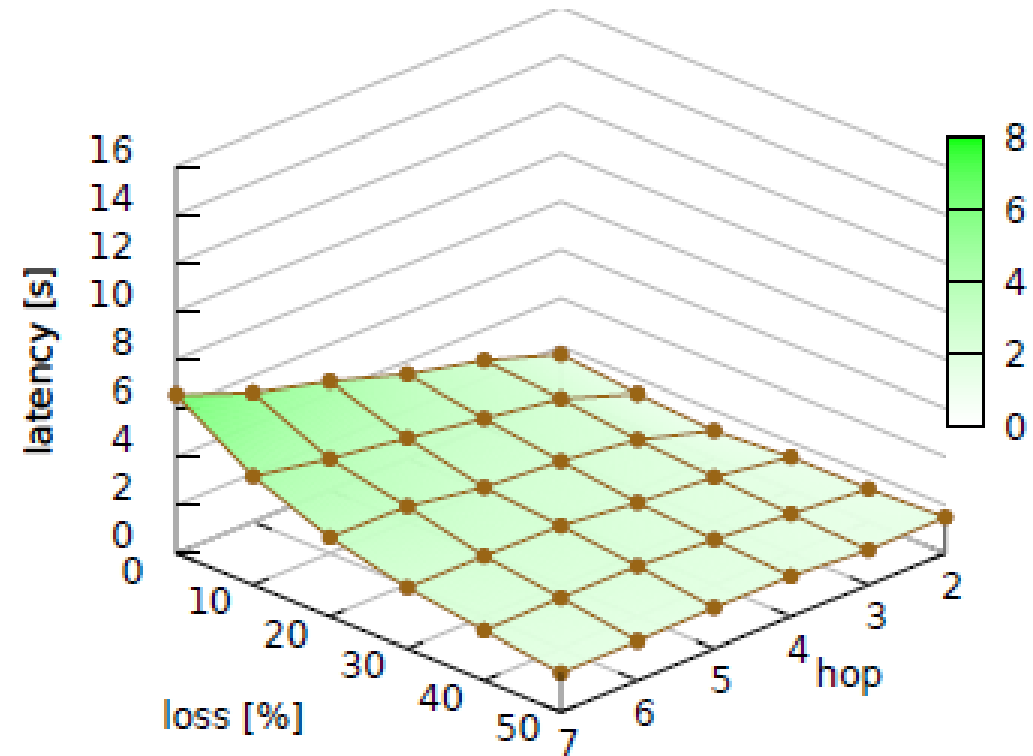
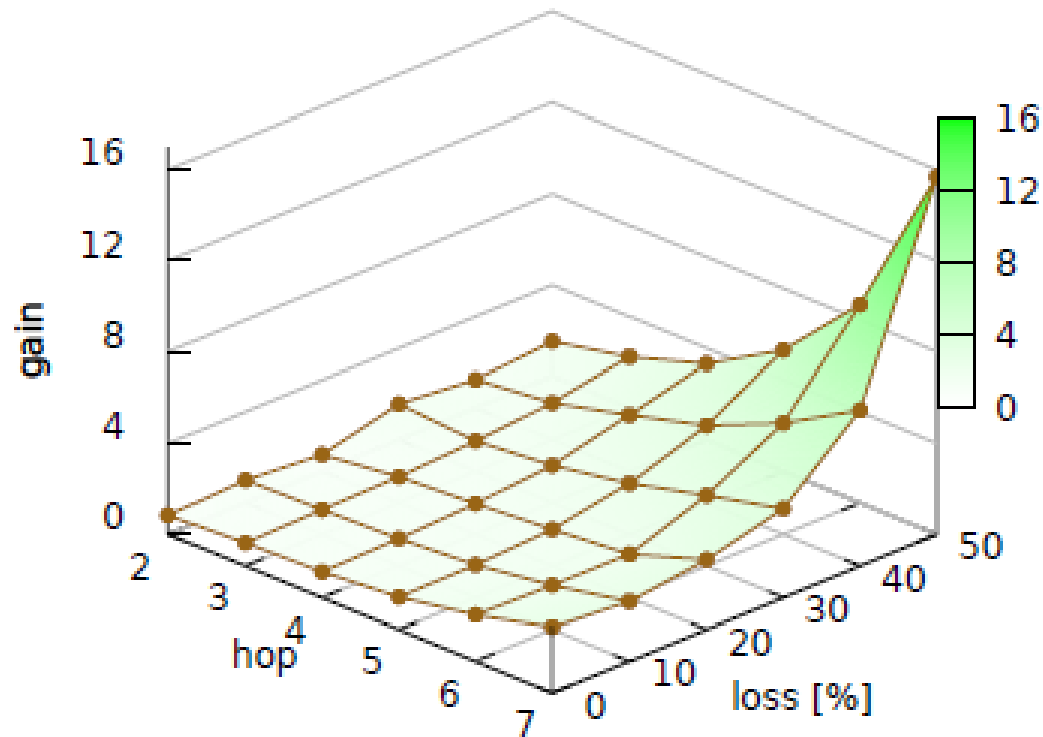
hop-by-hop



RLNC

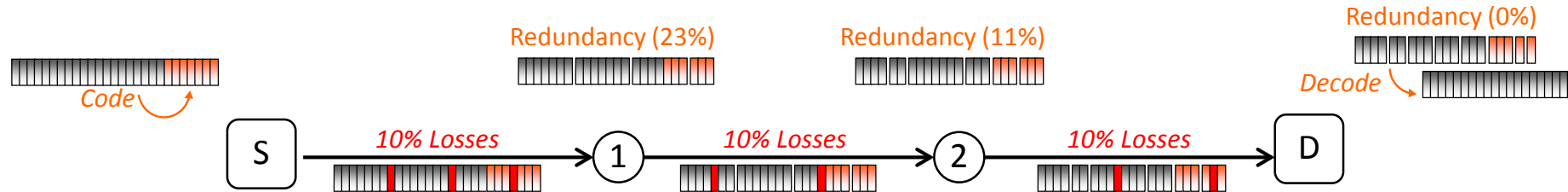
# Software Defined Networks

- Latency gain of e2e vs RLNC (left) and hbh vs RLNC(right)

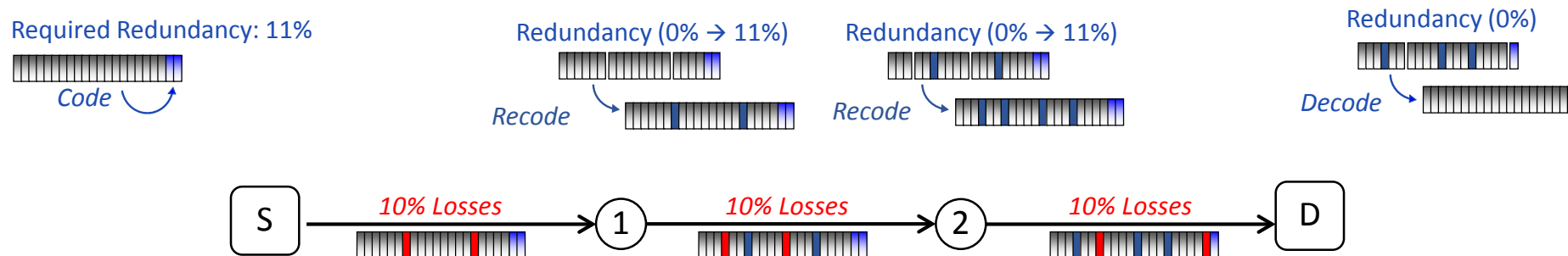


# The Recoding Advantage

**Coding End-to-End Overhead = Cumulative Losses (37%)**

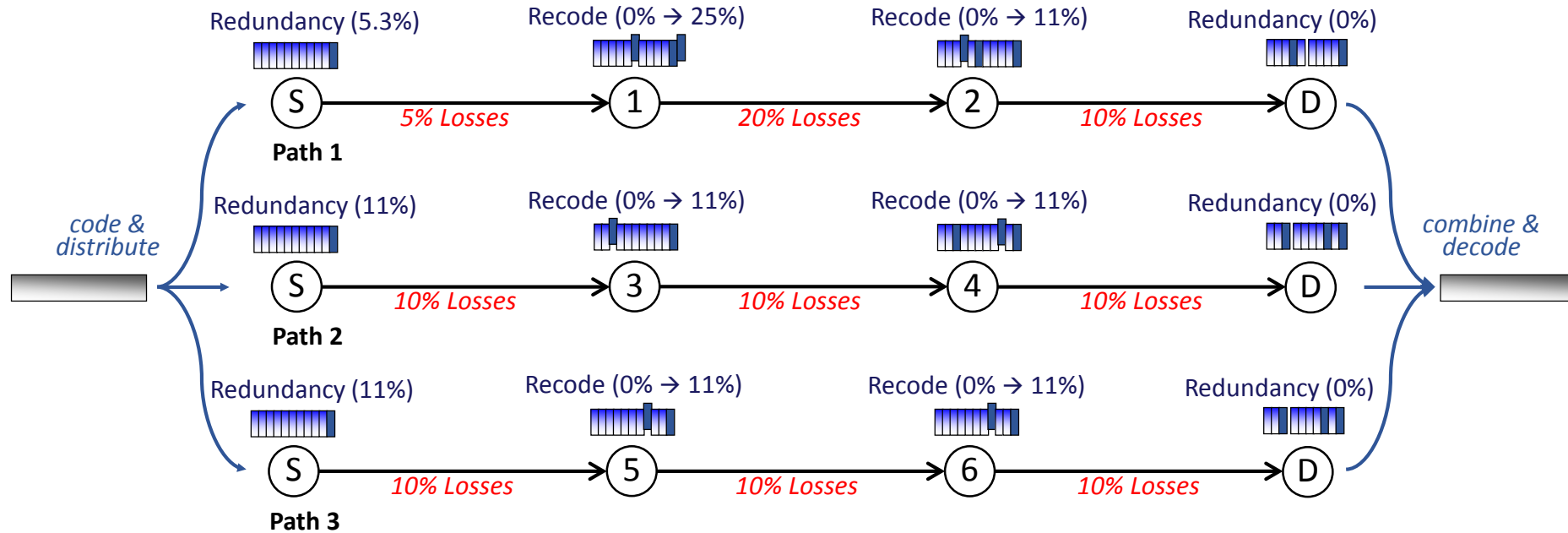


**Re-Coding Overhead = Single Worst-Case Loss (11%)**



- Optimal and Dynamic Loss Compensation

# Recoding + Multipath Advantage

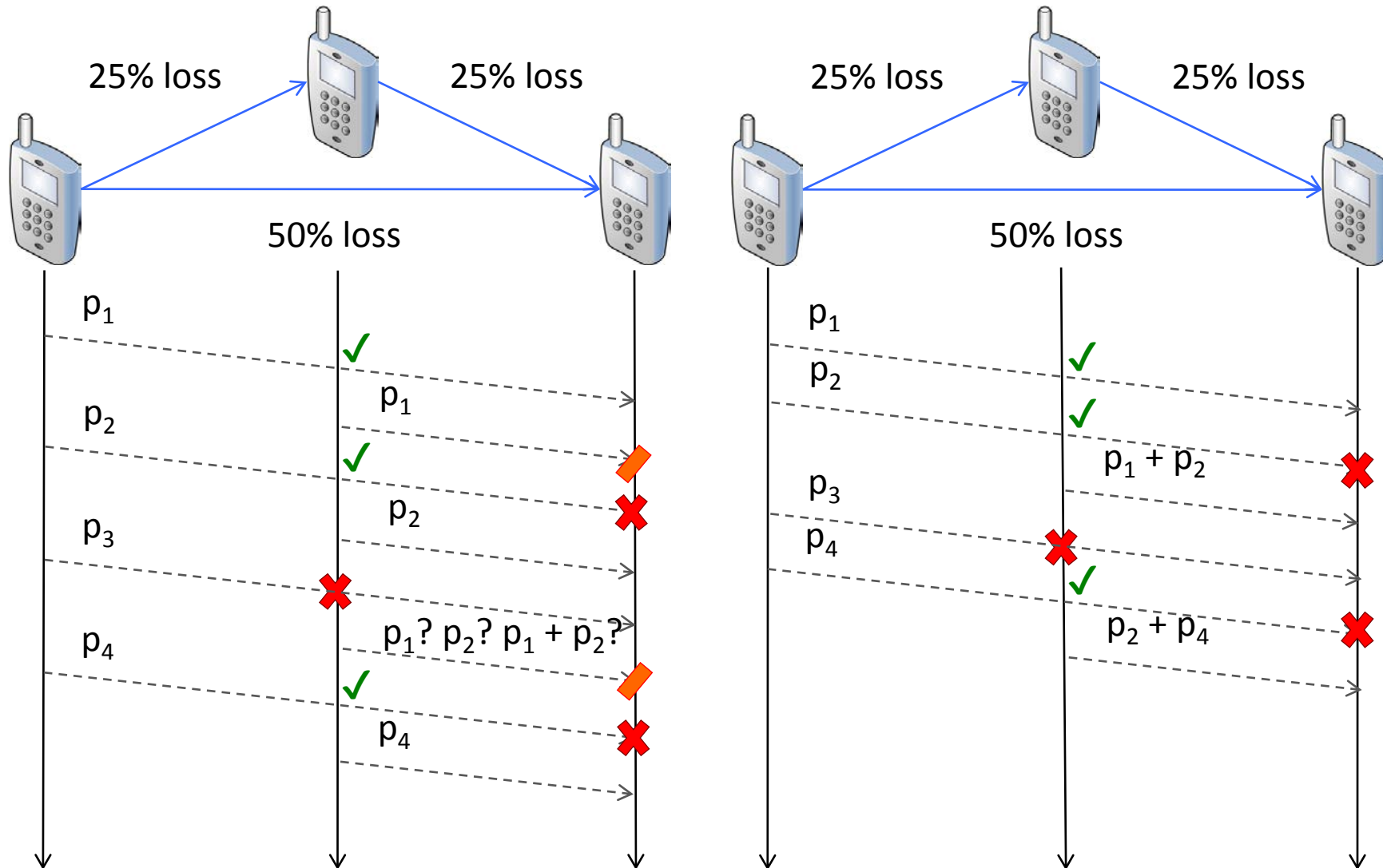


*Summing without scheduling*  
multipath

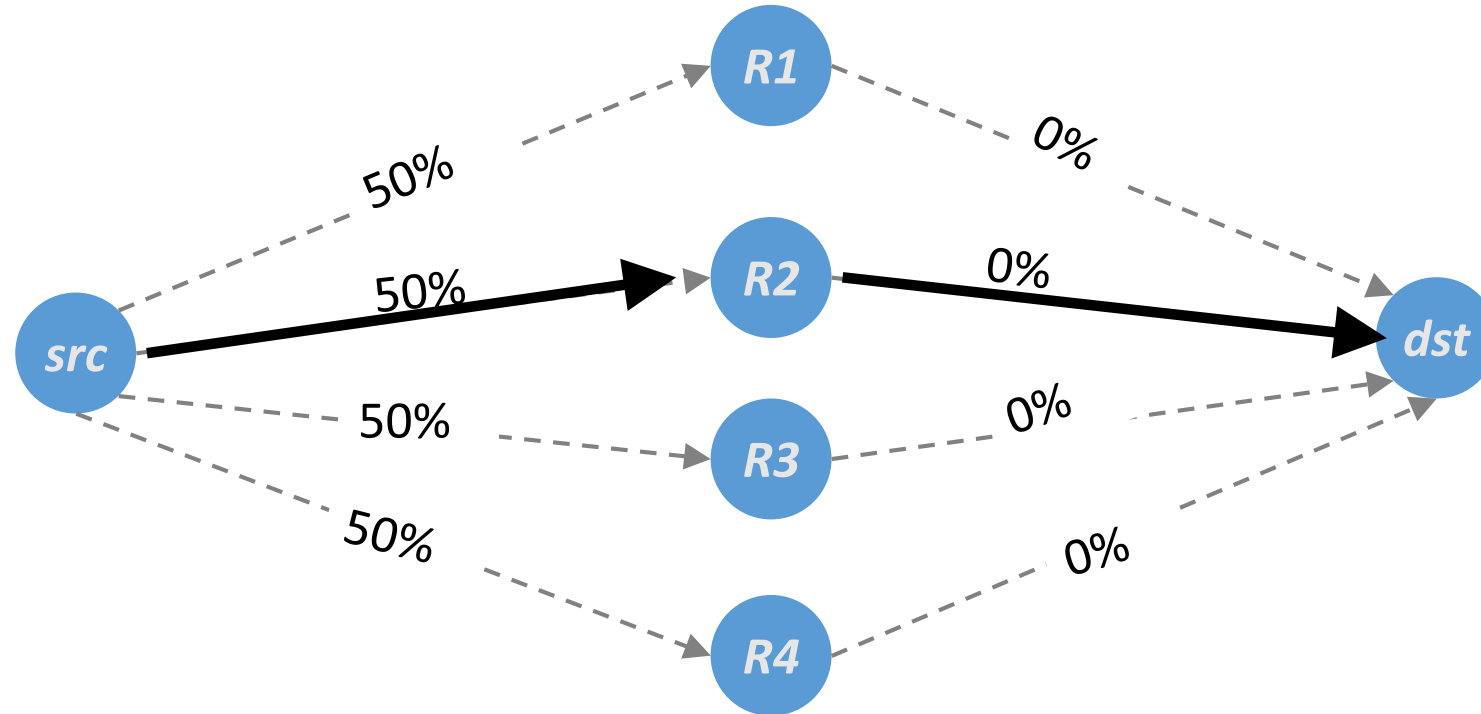
- Total Throughput =  $\sum$  (path rate – worst link)
- Inject 10Mbps at each path → Obtain 26Mbps seamlessly

## ■ Native Bandwidth Aggregation

# Other Recoding Issues

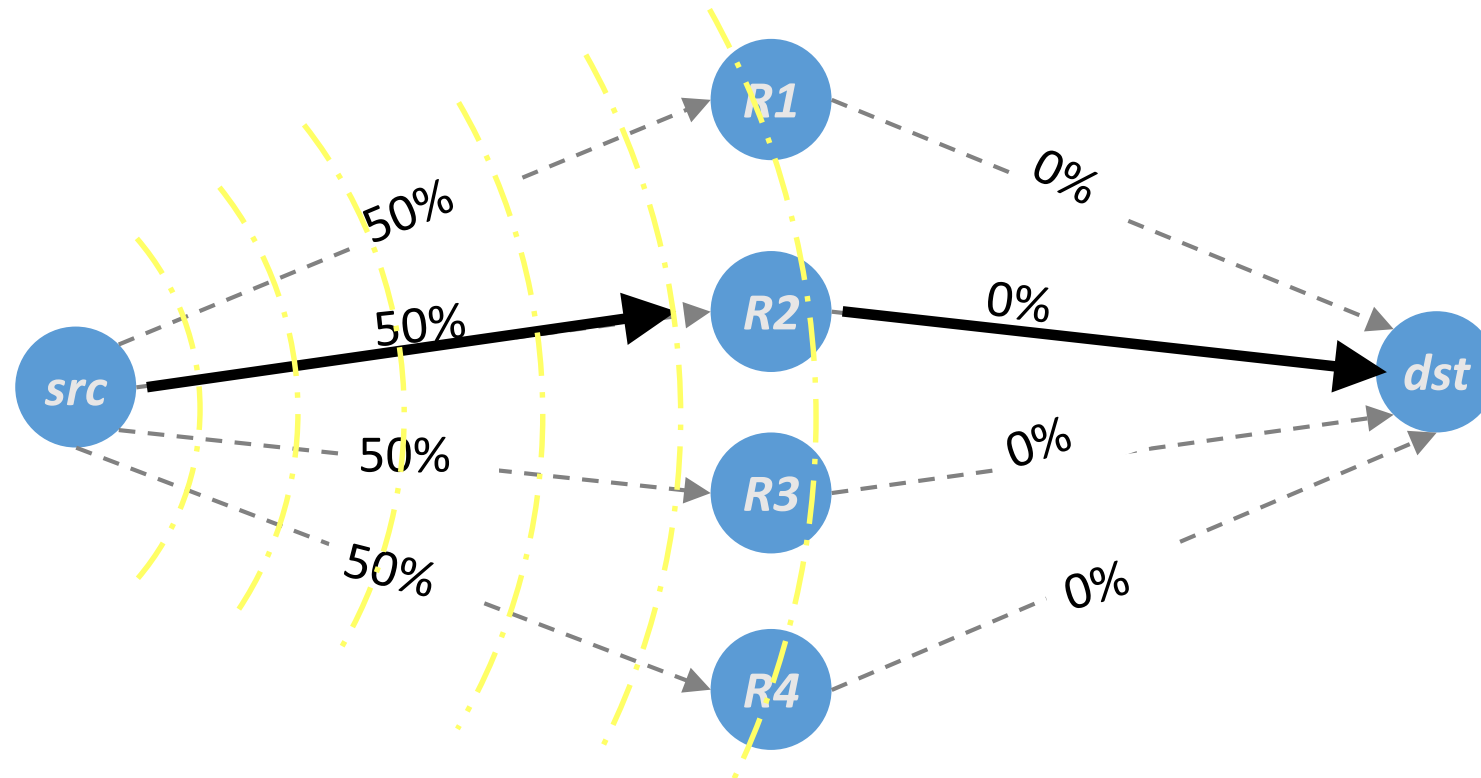


# Sender Is in a Bad Spot



- Best single path → Prob. of loss 50%

# Sender Is in a Bad Spot



- Best single path → Prob. of loss 50%
- Spatial diversity to the rescue
  - Any router forward packet → Prob. of loss  $0.54 = 6\%$

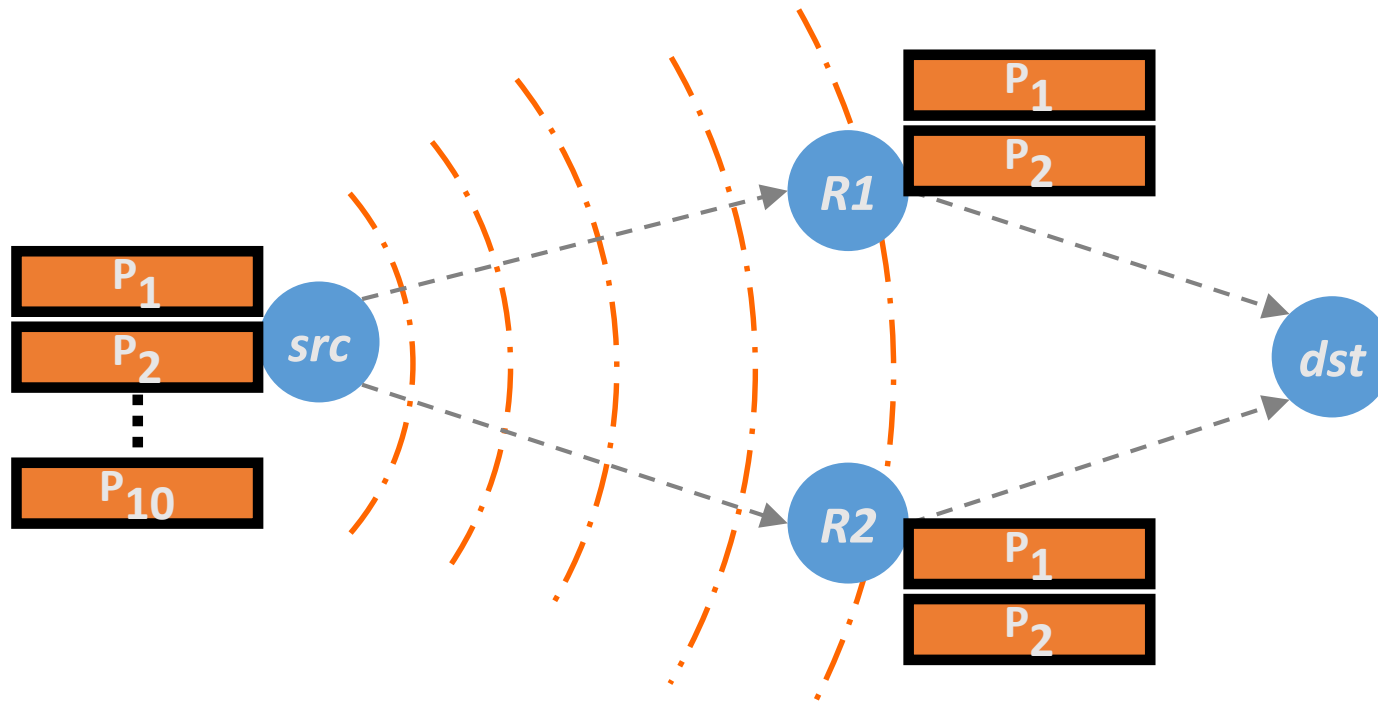
# Challenge with Using Spatial Diversity

- Overlap in received packets → Routers forward duplicates



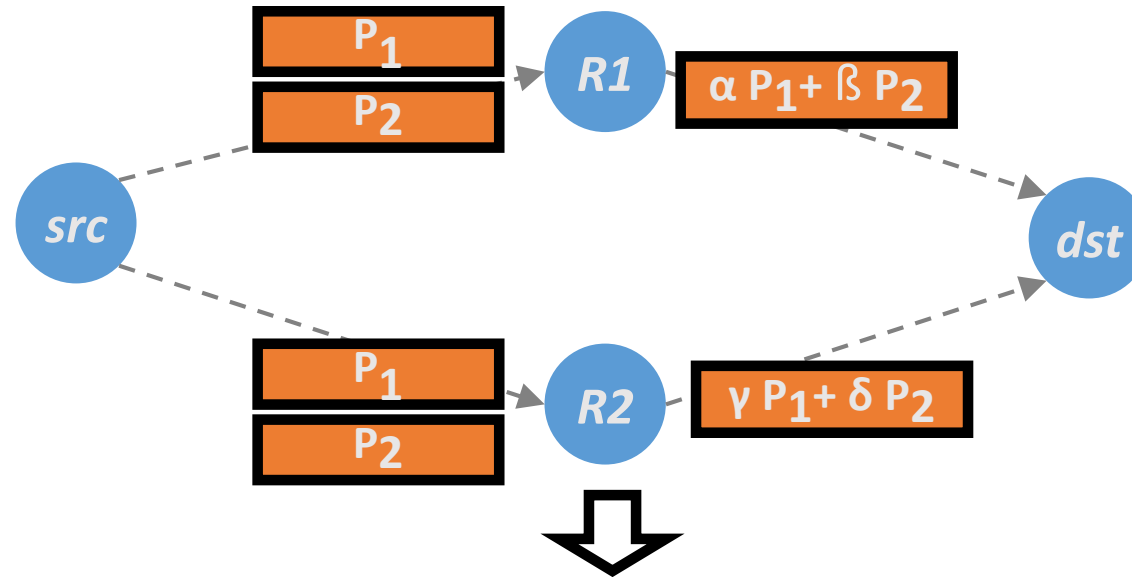
# Challenge with Using Spatial Diversity

- Overlap in received packets → Routers forward duplicates



# Random Linear Network Coding

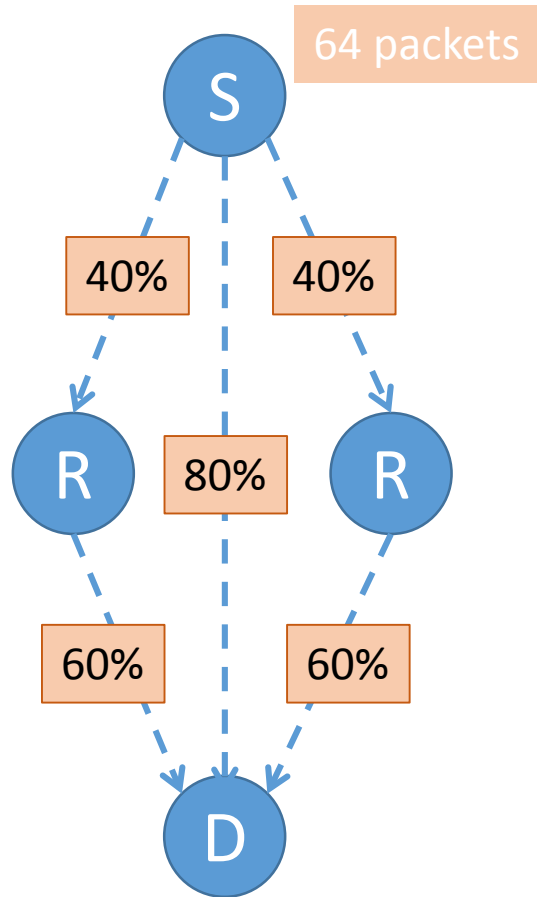
- Each router forwards random combinations of packets



Randomness prevents duplicates

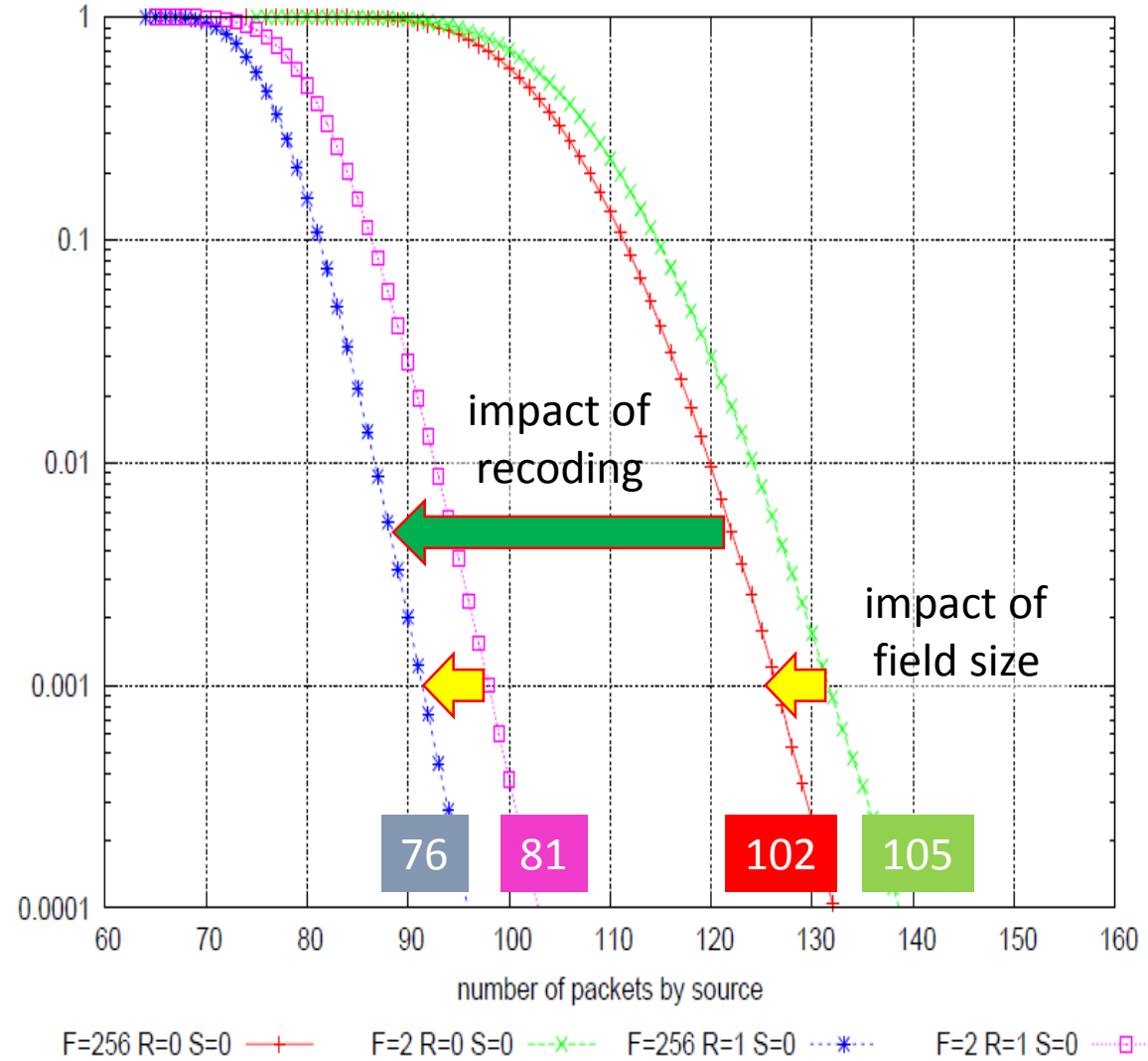
Network coding exploits spatial diversity to improve dead spots

# Impact of Recoding



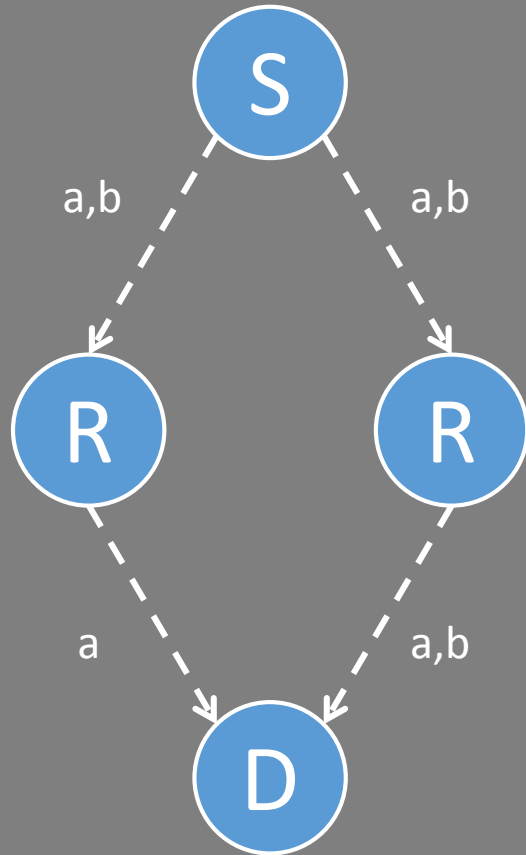
No need for signalling!

prob. D has not received all 64 after X trans.



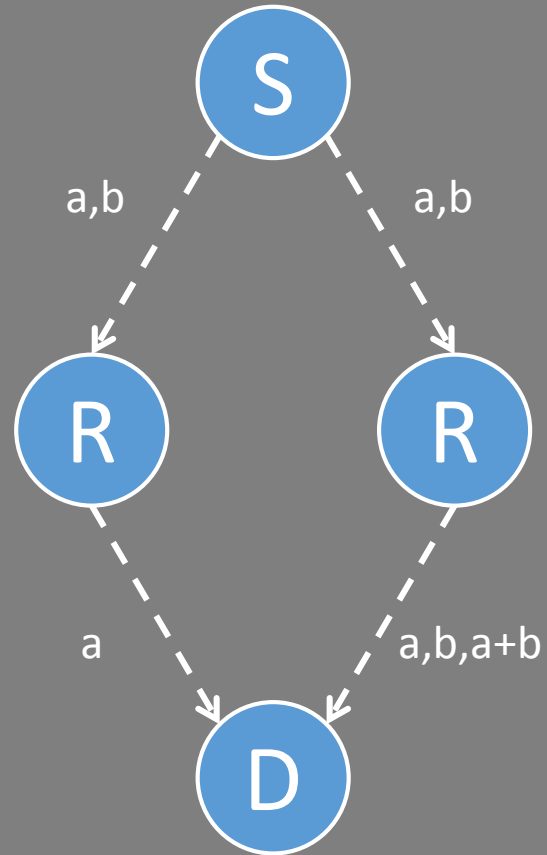
# Coding

No coding



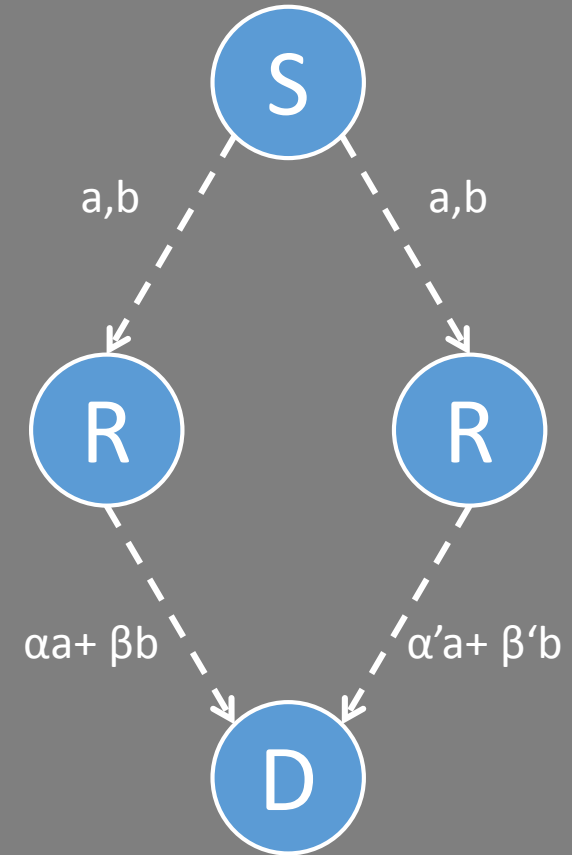
$$P_{\text{succ}} = 0.5$$

Binary coding



$$P_{\text{succ}} = 0.6667$$

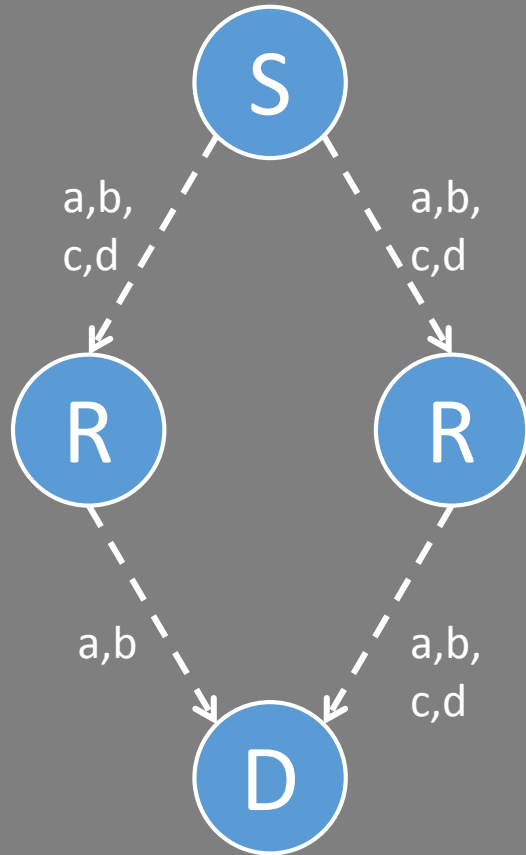
RLNC coding



$$P_{\text{succ}} = 1$$

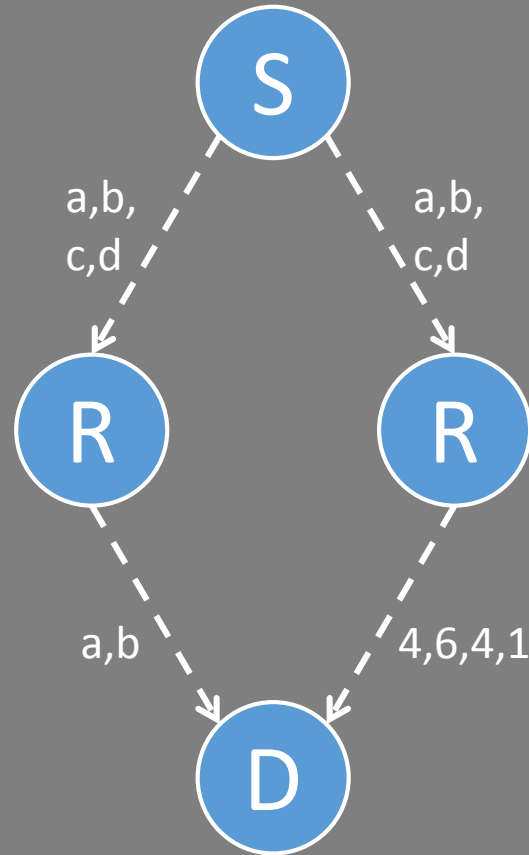
# Coding

No coding



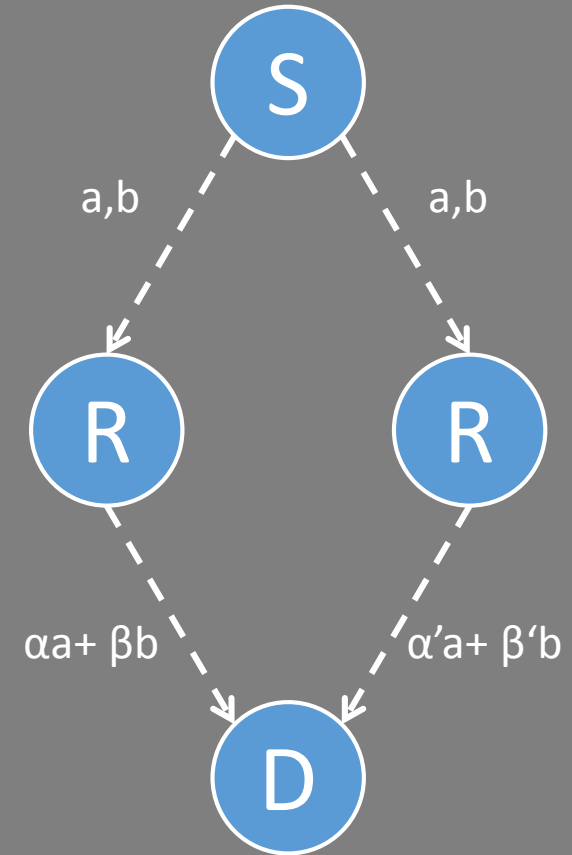
$$P_{\text{succ}} = 0.5$$

Binary coding



$$P_{\text{succ}} = 1 - 2/15$$

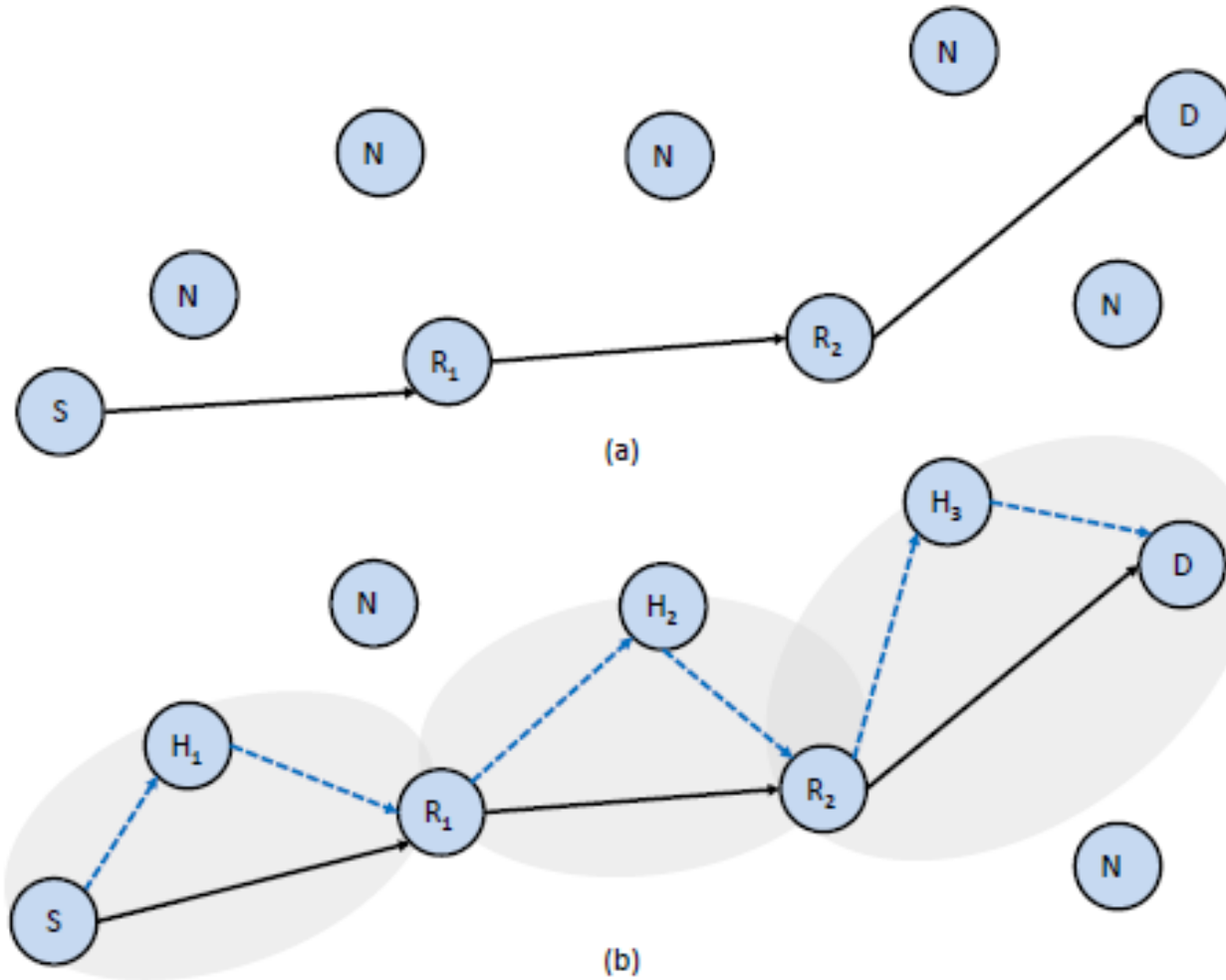
RLNC coding



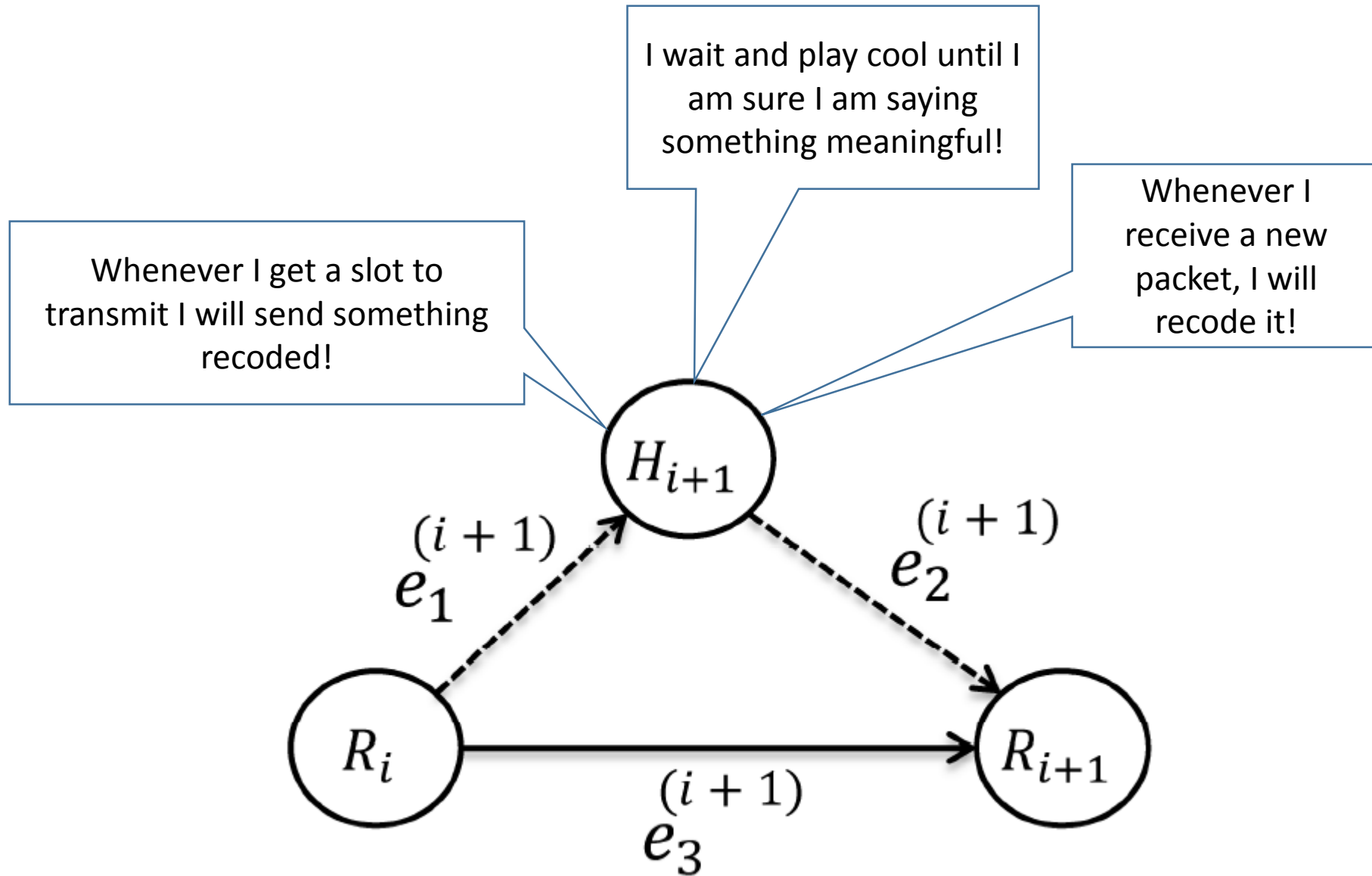
$$P_{\text{succ}} = 1$$

# Impact of the protocol design

# RLNC in real meshed

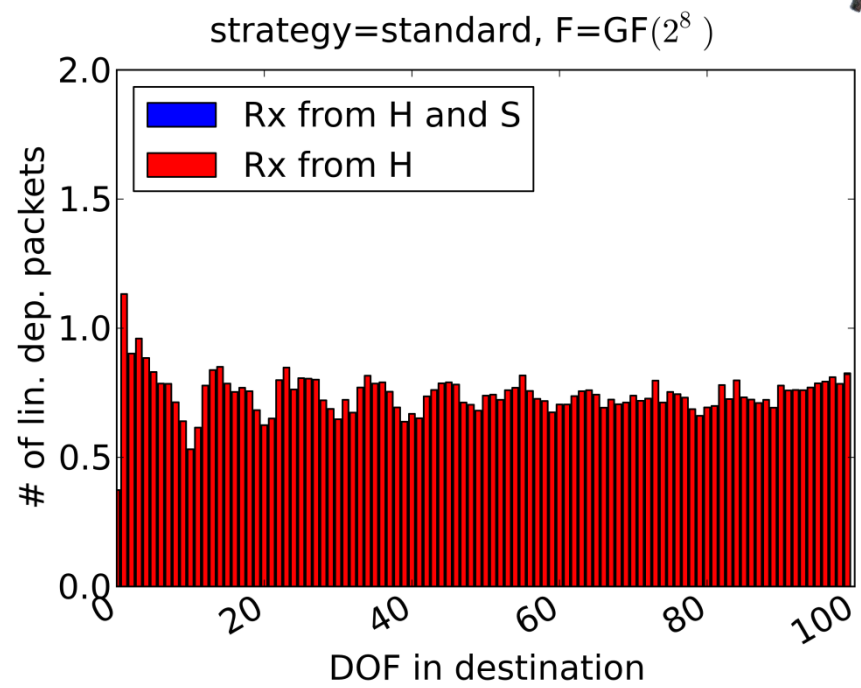
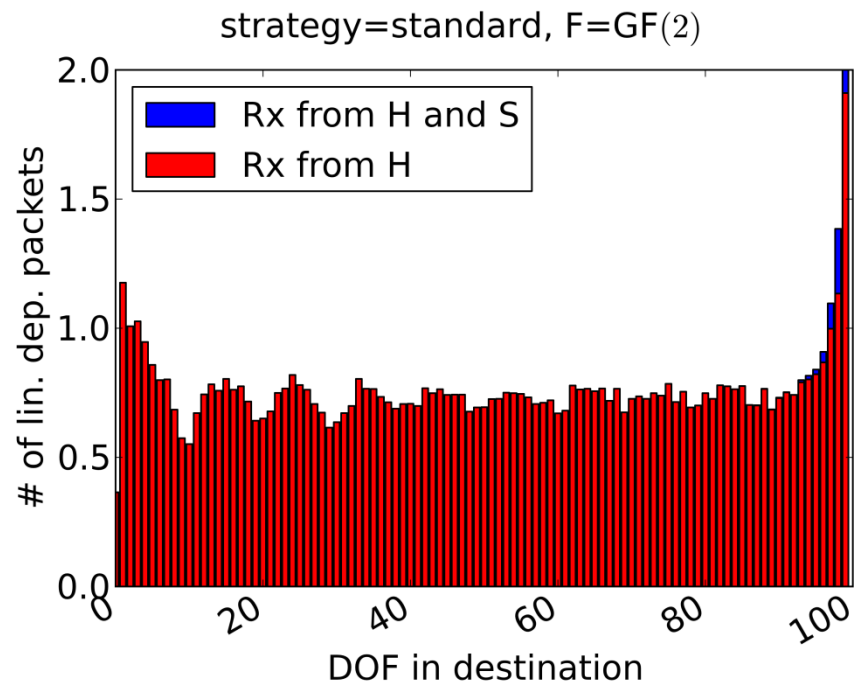
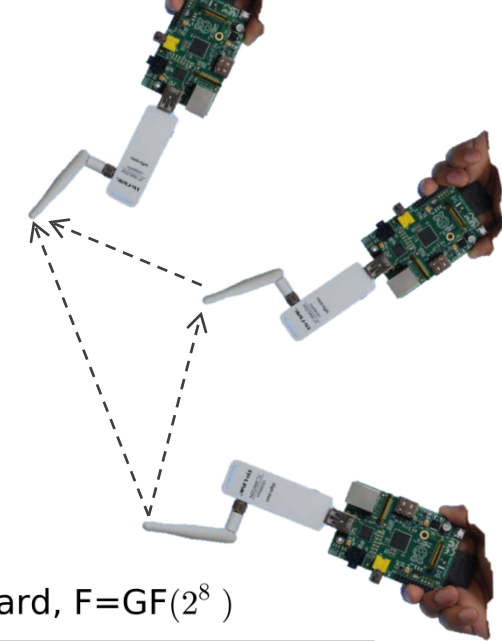
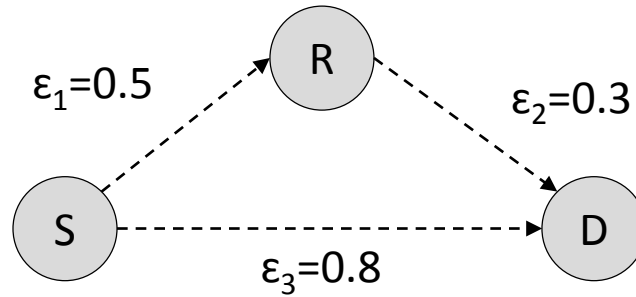


# Strategies under Investigation

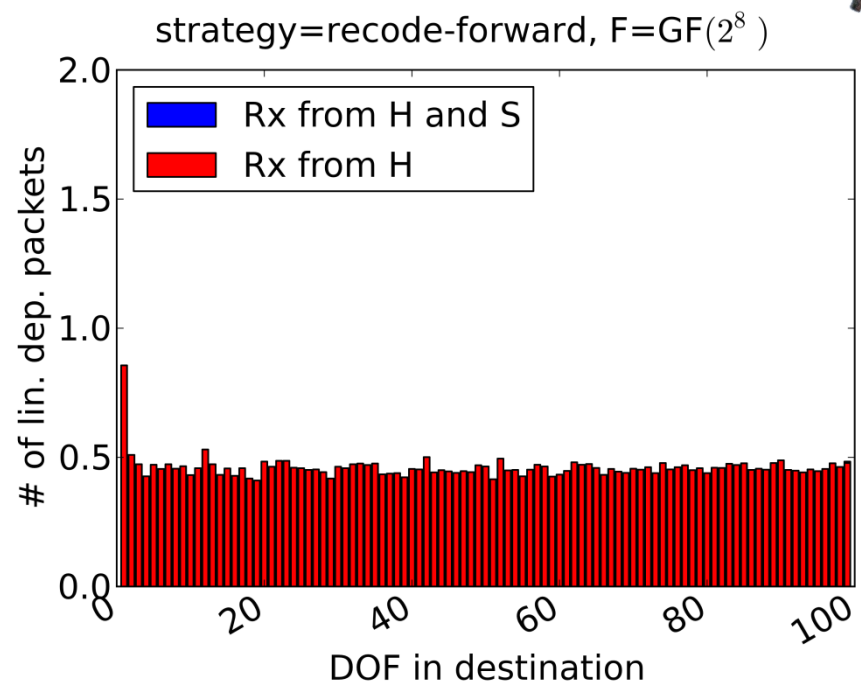
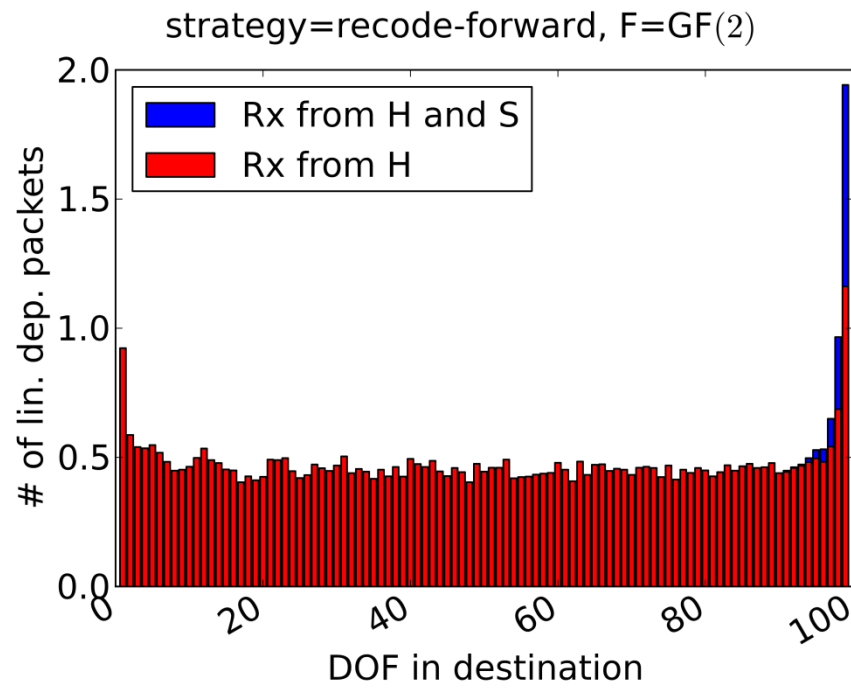
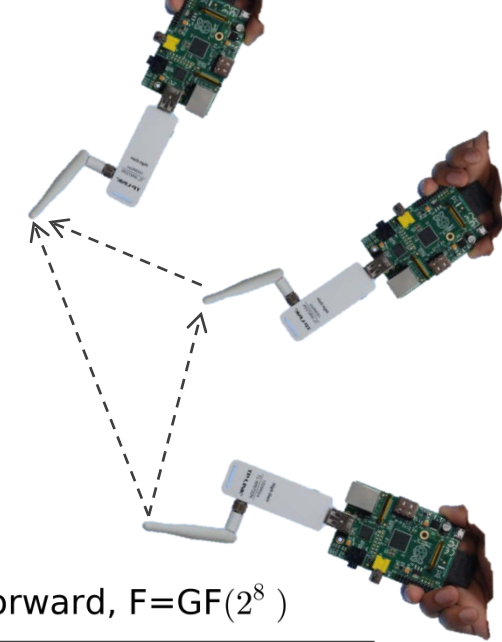
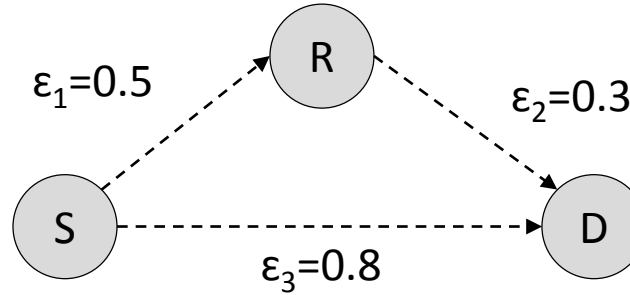




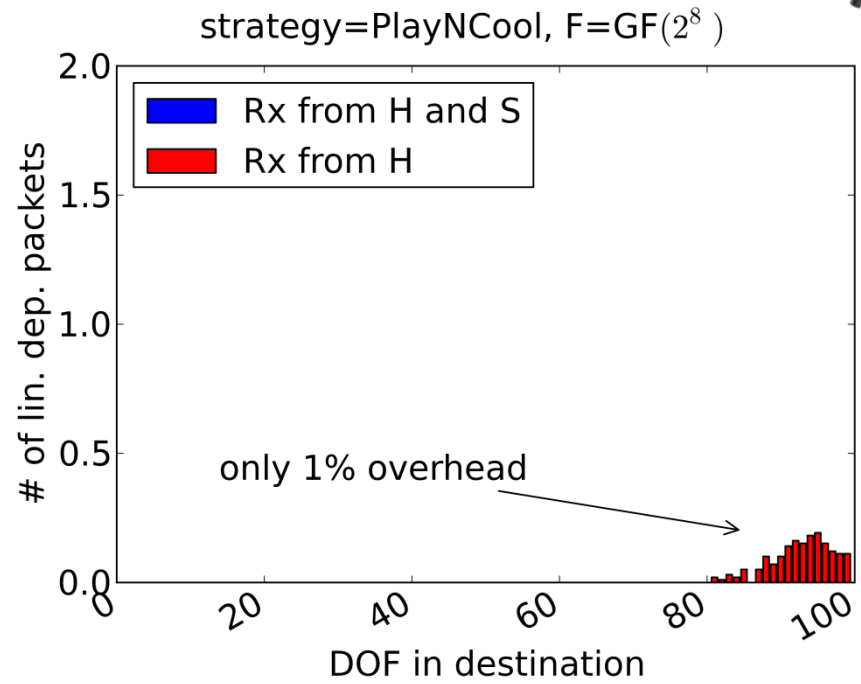
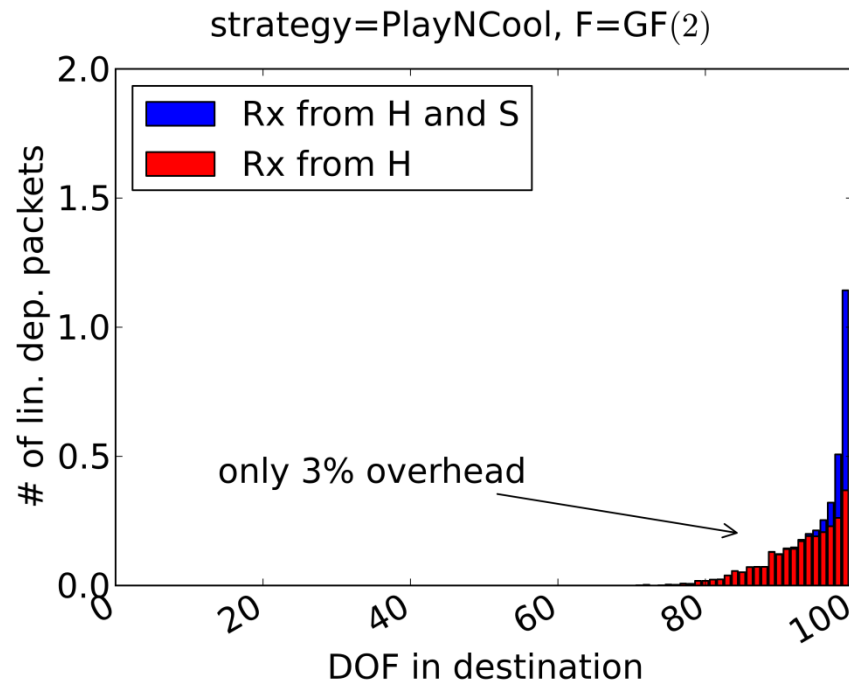
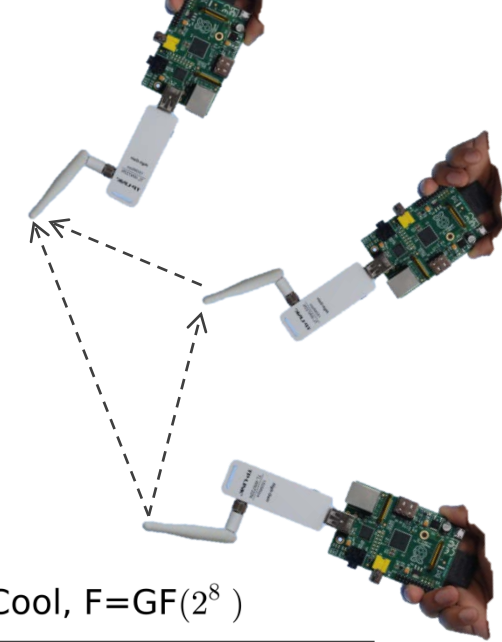
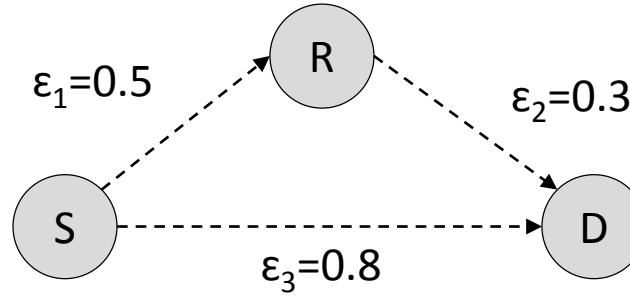
# Recode



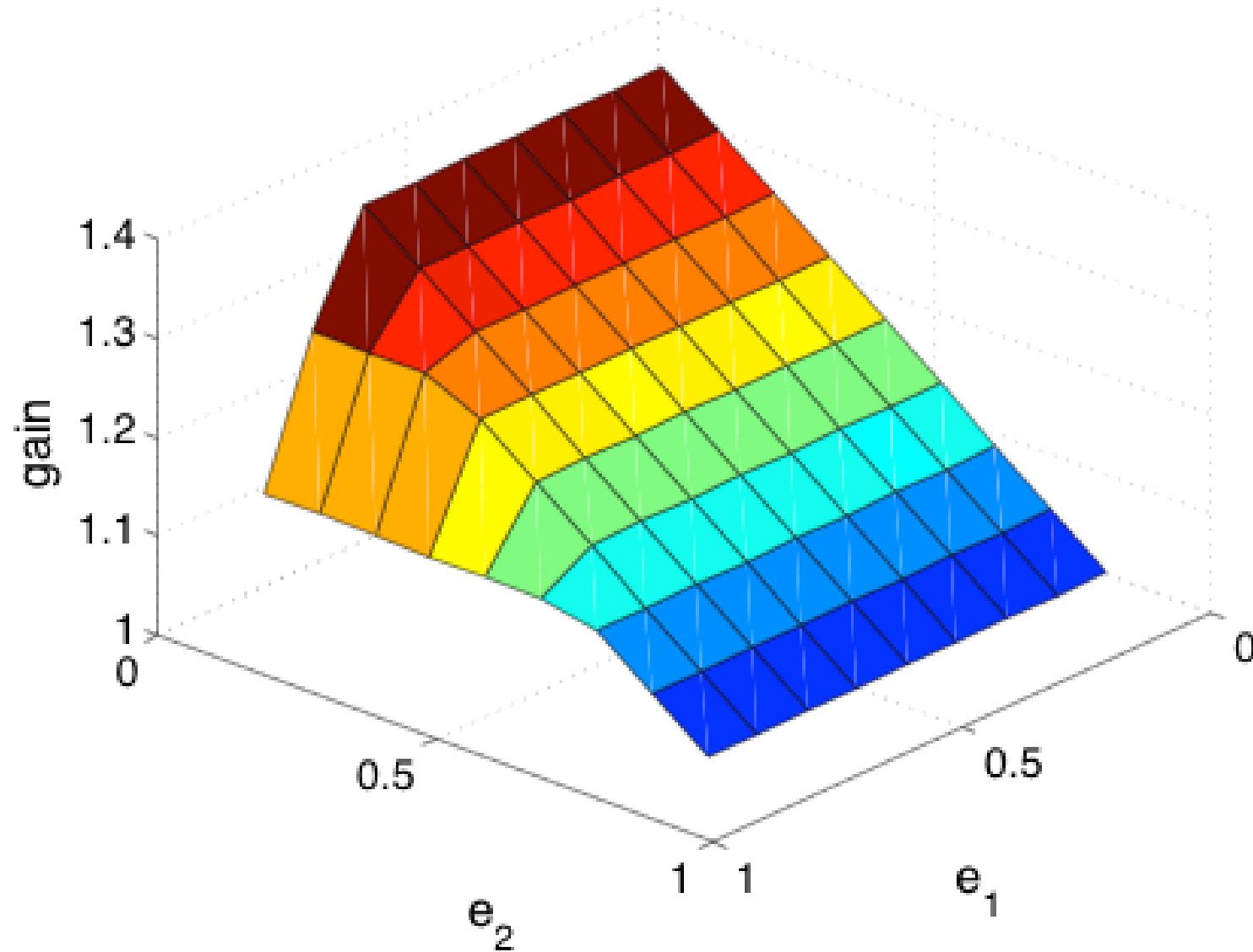
# Recode on new arrival



# PlayNCool



# Moderate Losses in the Direct Link



$e_3=0.30$ , 4 neighbors active

# High Losses in the Direct Link

