1-1.

(1) $f_x = 2x + y + z$. $f_y = 2y + x + z$. $f_z = 3z + y + x$

$f_{xx} = 2$. $f_{yy} = 2$. $f_{zz} = 3$. $f_{xy} = 1$. $f_{xz} = 1$ $f_{yx} = 1$ $f_{yz} = 1$ $f_{zx} = 1$. $f_{zy} = 1$.

$\Rightarrow \quad A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

$D_1 = 2 > 0$. $D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4-1 = 3 > 0$. $D_3 = 2(6-1) - (3-1) + (1-2) = 7 > 0$.

$\therefore \quad A = PD$

(2)

$adj(A) = \begin{pmatrix} 5 & 2 & -1 \\ 2 & 5 & 1 \\ -1 & 1 & 3 \end{pmatrix}' = \begin{pmatrix} 5 & -2 & -1 \\ -2 & 5 & -1 \\ -1 & -1 & 3 \end{pmatrix}$

$A^{-1} = \frac{1}{7}\begin{pmatrix} 5 & -2 & -1 \\ -2 & 5 & -1 \\ -1 & -1 & 3 \end{pmatrix}$

(3) $\det(A) = 0 \Rightarrow A$: 비가역 행렬.

비가역행렬인 경우, 어떤 b에 대해 해 X, 또는 해 무한히 많음.

$\therefore$ 조건 (b) 성립하지 않음.

1-2.

(1) i) (a) → (b)

$Ax = b$, $x = A^{-1}b$. 해 존재, 유일해. $\therefore$ (b) 성립.

ii) (b) → (c)

$Ax = 0$ $(b = 0)$, $x = 0$. $\therefore$ (c) 성립.

iii) (c) ⇒ (a)

$Ax = 0$ 의 해, 항상 $x = 0$. 임은 null space. 의미. ⇔ A = 가역.

⇒ A^{-1} 존재 $\therefore$ (a) 성립

(2) $Ax = b$ $x = A^{-1}b$. 이를 다시 대입. $A(A^{-1}b) = (AA^{-1})b = Ib = b$. $\therefore$ 해 항상 존재.

2. $BB' = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} 2-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix} \Rightarrow \det(BB') = (2-\lambda)(1-\lambda) = 0.$ $\quad \lambda_1 = 2.\ \lambda_2 = 1.\ \sigma_1 = \sqrt{2}.\ \sigma_2 = 1.$

$BB' = UDU' \Rightarrow U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ($\because$ 대각행렬).

$B'B = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$ $\quad (1-\lambda)\{(1-\lambda)^2 - 1\} = (1-\lambda)(-2+\lambda)\lambda.$ $\quad \lambda_1 = 2,\ \lambda_2 = 1.\ \lambda_3 = 0.$

i) $\lambda_1 = 2,\ v_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

ii) $\lambda_2 = 1,\ v_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

iii) $\lambda_3 = 0,\ v_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

$\Rightarrow V = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix},\ V' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$

$\Sigma = \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$\therefore\ B = U\Sigma V'$

3-1. (1)

① $(x_1, y_1), (x_2, y_2) \in C_1 \ldots \quad x_1 + y_1 \le 1, \quad x_2 + y_2 \le 1.$

$\lambda(x_1, y_1) + (1-\lambda)(x_2, y_2) = \lambda(x_1 + y_1) + (1-\lambda)(x_2 + y_2) \le \lambda \cdot 1 + (1-\lambda) \cdot 1 = 1$

$\therefore\ x + y \le 1.$ $\quad C_1 =$ convex.

② $x_1, x_2 \in C_2 \ldots \quad \|x_1\|_1 \le 1, \quad \|x_2\|_1 \le 1.$

$\|\lambda(x_1) + (1-\lambda)x_2\|_1 \le \lambda\|x_1\|_1 + (1-\lambda)\|x_2\|_1 \le 1.$ $\quad C_2 =$ convex

③ $(x_1, y_1), (x_2, y_2) \in C_3 \ldots \quad y_1 \ge e^{x_1},\ y_2 \ge e^{x_2}.$

$\lambda y_1 + (1-\lambda) y_2 \geq \lambda e^{x_1} + (1-\lambda) e^{x_2}$

$* \quad e^{\lambda x_1 + (1-\lambda)x_2} \le \lambda e^{x_1} + (1-\lambda)e^{x_2}.$

$\therefore\ \lambda y_1 + (1-\lambda) y_2 \geq e^{\lambda x_1 + (1-\lambda)x_2}.$ $\quad C_3 =$ convex

(2)

$(x_1, t_1), (x_2, t_2) \in S \ldots \quad t_1 \geq f(x_1),\ t_2 \geq f(x_2).$

$\lambda t_1 + (1-\lambda)t_2 \geq \lambda f(x_1) + (1-\lambda)f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2)$

$\Rightarrow (\lambda x_1 + (1-\lambda)x_2,\ \lambda t_1 + (1-\lambda)t_2) \in S.$ $\quad S =$ convex

3 -2 (1-(a))

$$f(\lambda x+(1-\lambda)y) \le \lambda f(x)+(1-\lambda)f(y).$$

$$+)\ \underline{g(\lambda x+(1-\lambda)y) \le \lambda g(x)+(1-\lambda)g(y).}$$

$$(f+g)(\lambda x+(1-\lambda)y) \le \lambda(f+g)(x)+(1-\lambda)(f+g)(y)$$. $f+g$ = convex.

(1-(b))

$$f(A(\lambda x+(1-\lambda)y)+b) = f(\lambda(Ax+b)+(1-\lambda)(Ay+b)) \le \lambda f(Ax+b)+(1-\lambda)f(Ay+b)$$

$f(Ax+b)$ = convex

(2)

$$f(\lambda x+(1-\lambda)y) = \|A(\lambda x+(1-\lambda)y)-b\|^2 = \|\lambda(Ax-b)+(1-\lambda)(Ay-b)\|^2 \le \lambda f(x)+(1-\lambda)f(y).$$

$\therefore$ f is convex on R^d. $\quad\hookrightarrow\ \cdots\ \|\lambda u+(1-\lambda)v\|^2 \le \lambda\|u\|^2+(1-\lambda)\|v\|^2$

4-1

(a) $P(X=0) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$. $P(X=1) = \frac{1}{3}$.

$P(Y=0) = \frac{1}{3}$. $P(Y=1) = \frac{2}{3}$.

$\Rightarrow H(X) = \frac{2}{3}\log\frac{3}{2} + \frac{1}{3}\log 3 = \log 3 - \frac{2}{3}\log 2. = \log 3 - \frac{2}{3}$

$\Rightarrow H(Y) = \frac{1}{3}\log 3 + \frac{2}{3}\log\frac{3}{2} = \log 3 - \frac{2}{3}\log 2. = \log 3 - \frac{2}{3}$

(b) $H(X,Y) = \frac{1}{3}\log 3 + \frac{1}{3}\log 3 + \frac{1}{3}\log 3 = \log 3.$

$\Rightarrow H(X|Y) = H(X,Y) - H(Y) = \log 3 - \log 3 + \frac{2}{3}\log 2 = \frac{2}{3}$

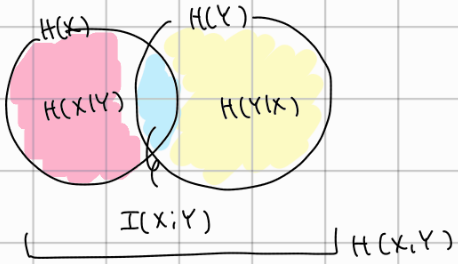
$\Rightarrow H(Y|X) = H(X,Y) - H(X) = \log 3 - \log 3 + \frac{2}{3}\log 2 = \frac{2}{3}.$

(c) $\log 3.$

(d) $H(Y) - H(Y|X) = \log 3 - \frac{2}{3} - \frac{2}{3} = \log 3 - \frac{4}{3}$

(e) $I(X;Y) = H(X) + H(Y) - H(X,Y) = 2\log 3 - \frac{4}{3} - \log 3 = \log 3 - \frac{4}{3}.$

(f)



4-2

(a) $D(p\|q) = \sum_x p(x)\log\frac{p(x)}{q(x)}$, $D(q\|p) = \sum_x q(x)\log\frac{q(x)}{p(x)}$

$D(p\|q) - D(q\|p) = \sum_x \{(p(x)+q(x))\log(p(x)) - (p(x)+q(x))\log(q(x))\} \neq 0$. always. $(\because D(p\|q), D(q\|p) \geq 0)$

$\therefore D(p\|q) \neq D(q\|p)$

(b) $\phi(E[\frac{q(x)}{p(x)}]) \leq E[\phi(\frac{q(x)}{p(x)})]$

$\Rightarrow -\log(\sum q(x)) \leq \sum p(x)\left(-\log\frac{q(x)}{p(x)}\right)$

$\Rightarrow -\log 1 = 0 \leq \sum p(x)\left(-\log\frac{q(x)}{p(x)}\right) = D(p\|q)$

$\therefore D(p\|q) \geq 0.$