15기 김정민

1-(1)

$f(x,y,z) = x^2+y^2+1.5z^2+xy+yz+zx$

$f_{xx}=2,\ f_{yy}=2,\ f_{zz}=2\ /\ f_{xy}=1,\ f_{xz}=1,\ f_{yz}=1$ $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$

$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ ① $=2$

② $=4-1=3$

③ $=\det(A)=7$ $\therefore$ A는 PD

1-(2)

$A^{-1} = \begin{pmatrix} \frac{5}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{5}{7} & -\frac{1}{7} \\ -\frac{1}{7} & -\frac{1}{7} & \frac{3}{7} \end{pmatrix}$

1-(3)

$xy \to kxy$면 A'은 $\begin{pmatrix} 2 & k & 1 \\ k & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ $\det(A') = (2-k)(3k+4) = 0$

$\to k = 2$ or $-\frac{4}{3}$

i) $\det(A')=0$이면 A'은 비가역 → 성립X

ii) $\det(A') \neq 0$이면 A는 가역 → 모든 b에 대해 해 존재&유일 ∴ b 성립X

2.

$B=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $BB^T = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ $B^TB = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\lambda = 2, 1, 0$

$V = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$ $V^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \end{pmatrix}$$

3-(1)-1

$C_1: \{\lambda x_1 + (1-\lambda)x_2\} + \{\lambda y_1 + (1-\lambda)y_2\} = \lambda(x_1+y_1) + (1-\lambda)(x_2+y_2) \le \lambda + (1-\lambda) = 1$

C_2 ????

$C_3: \lambda y_1 + (1-\lambda)y_2 \ge \lambda e^{x_1} + (1-\lambda)e^{x_2} \ge e^{\lambda x_1 + (1-\lambda)x_2}$. ∴ C_1, C_3 convex C_2는 모르겠음.

3-(1)-2

$\lambda t_1 + (1-\lambda)t_2 \ge \lambda f(x_1) + (1-\lambda) f(x_2) \ge f(\lambda x_1 + (1-\lambda)x_2)$ ∴ S convex.

3-(2)

(a) $(f+g)(\lambda x + (1-\lambda)y) = f(\lambda x + (1-\lambda)y) + g(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y) + \lambda g(x) + (1-\lambda) g(y)$

$= \lambda(f+g)(x) + (1-\lambda)(f+g)(y)$ ∴ convex

(b) $f(A(\lambda x + (1-\lambda)y) + b) = f(\lambda(Ax+b) + (1-\lambda)(Ay+b)) \le \lambda f(Ax+b) + (1-\lambda) f(Ay+b)$

∴ convex

3-2-2

$u = Ax - b$, $v = Ay - b$ 라 하자

then $f(\lambda x + (1-\lambda)y) = \|A(\lambda x + (1-\lambda)y) - b\|^2 = \|\lambda u + (1-\lambda)v\|^2$

$\|\lambda u + (1-\lambda)v\|^2 = \lambda\|u\|^2 + (1-\lambda)\|v\|^2 - \lambda(1-\lambda)\|u-v\|^2 \le \lambda\|u\|^2 + (1-\lambda)\|v\|^2$

$f(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y)$ 이므로 convex.

4.(1)

X \ Y	1	2
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

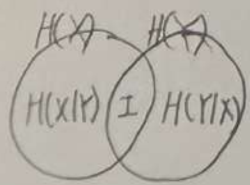
$P(X=0) = \frac{2}{3}$, $P(X=1) = \frac{1}{3}$

$P(Y=0) = \frac{1}{3}$, $P(Y=1) = \frac{2}{3}$

(a) $H(X) = H(Y) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}$

(b) $H(X|Y) = P(Y=0)\cdot 0 + P(Y=1)\cdot 1 = \frac{2}{3}$, 대칭이므로 $H(Y|X) = \frac{2}{3}$

(c) $H(X,Y) = H(Y) + H(X|Y) = \left(\log_2 3 - \frac{2}{3}\right) + \frac{2}{3} = \log_2 3$.



(d) $H(Y) - H(Y|X) = \log_2 3 - \frac{2}{3} - \frac{2}{3} = \log_2 3 - \frac{4}{3}$.

(e) $I(X;Y) = H(Y) - H(Y|X) = \log_2 3 - \frac{4}{3}$

(f)

4-(2). a

$p = (0.9, 0.1)$ $q = (0.5, 0.5)$

4-(2) b

이산형에서 $D(p\|q) = \sum p_x \log\frac{p_x}{q_x} = -\sum p_x \log\frac{q_x}{p_x}$

$z_x = q_x/p_x$ 라 두면 log는 concave 이므로

$\sum p_x \log z_x \le \log\left(\sum p_x z_x\right)$.

$-\sum p_x \log z_x \ge -\log\left(\sum p_x z_x\right)$ 인데,

$\sum p_x z_x = 1$ 이므로

$D(p\|q) \ge -\log 1 = 0$ $\therefore D(p\|q) \ge 0$ //.