1-1

(1)

헤시안행렬 $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}$

$D_1 > 0$, $D_2 = 4-1 > 0$,

$\det(A) = 13 > 0$ $\Rightarrow$ PD

(2)

$$\left(\begin{array}{ccc|ccc} 2 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 0 & 0 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} \frac{6}{13} & -\frac{3}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{6}{13} & -\frac{2}{13} \\ -\frac{2}{13} & \frac{1}{13} & \frac{4}{13} \end{pmatrix}$$

(3)

$\det(A) = 0 \Rightarrow$ 특이행렬

이때 $Ax = b$는 해가 없거나 무한히 존재

→ (b) 성립X

② $B^TB =$

$BB^T = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} \lambda^1 = 2 \\ \lambda^2 = 1 \end{pmatrix}$ $\begin{pmatrix} \sigma^1 = \sqrt{2} \\ \sigma^2 = 1 \end{pmatrix}$

$(B - \lambda I)v = 0$

① $\lambda = 2$

$\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

x자유, $y=0$

$v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

② $\lambda = 1$

$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow$ $x = 0$, $y =$ 자유

$v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$V_1 = \frac{1}{\sqrt{2}}(1, 1, 0)$

$V_2 = (0, 0, 1)$

$V_3 = \frac{1}{\sqrt{2}}(-1, 1, 0)$

$V = \begin{pmatrix} \frac{1}{\sqrt{2}}, & \frac{1}{\sqrt{2}}, & 0 \\ 0 & 0 & 1 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}$

$\therefore$

$U = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Sigma = \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$V^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$

③

convex set 정의

$x_1, x_2 \in C$, $\theta \in [0,1]$, $\theta x_1 + (1-\theta) x_2 \in C$

① C_1은 Half-space ⇒ convex

② C_2는 삼각부등식

$\|\theta x_1 + (1-\theta) x_2\|_1 \le 1$ → convex

② $g(x) = e^x > 0$ convex

$\boxed{3-2}$

1-(a) f, g convex

$f(\theta x + (1-\theta) y) \le \theta f(x) + (1-\theta) f(y)$

$g(\theta x + (1-\theta) y) \le \theta g(x) + (1-\theta) g(y)$

$(f+g)(\theta x + (1-\theta) y) \le \theta (f+g)(x) + (1-\theta)(f+g)(y)$

∴ convex

1-(b) $f(A(\theta x + (1-\theta) y) + b) \le \theta f(Ax+b) + (1-\theta) f(Ay+b)$

↳ $f(Ax+b)$ → convex

2. $f(x) = \|Ax-b\|^2$

$f(\theta x + (1-\theta) y) = \|\theta(Ax-b) + (1-\theta)(Ay-b)\|^2$

$\le \theta \|Ax-b\|^2 + (1-\theta)\|Ay-b\|^2 = \theta f(x) + (1-\theta) f(y)$

→ convex

4.

(a) $H(X) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3}$

$H(Y) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{2}{3}\log_2\frac{2}{3}$

$H(X) = H(Y) \approx 0.918$

(b)

$H(X|Y)$

. $Y=0 \rightarrow X=0$ $\quad H(X|Y=0)=0$

$Y=1 \rightarrow P(X=0 \mid Y=1) = \frac{1}{3}/\frac{2}{3} = \frac{1}{2}$ $/ P(X=1 \mid Y=1) = \frac{1}{2} \rightarrow H(X|Y=1)=1$

$P(Y=0)\times 0 + P(Y=1)\times 1 = \frac{1}{3}\times 0 + \frac{2}{3}\times 1 = \frac{2}{3}$

: $H(Y|X)$

. $X=0 \rightarrow Y=(\frac{1}{2},\frac{1}{2}) \rightarrow H(Y|X=0) = 1$ $\Big] \frac{2}{3}$

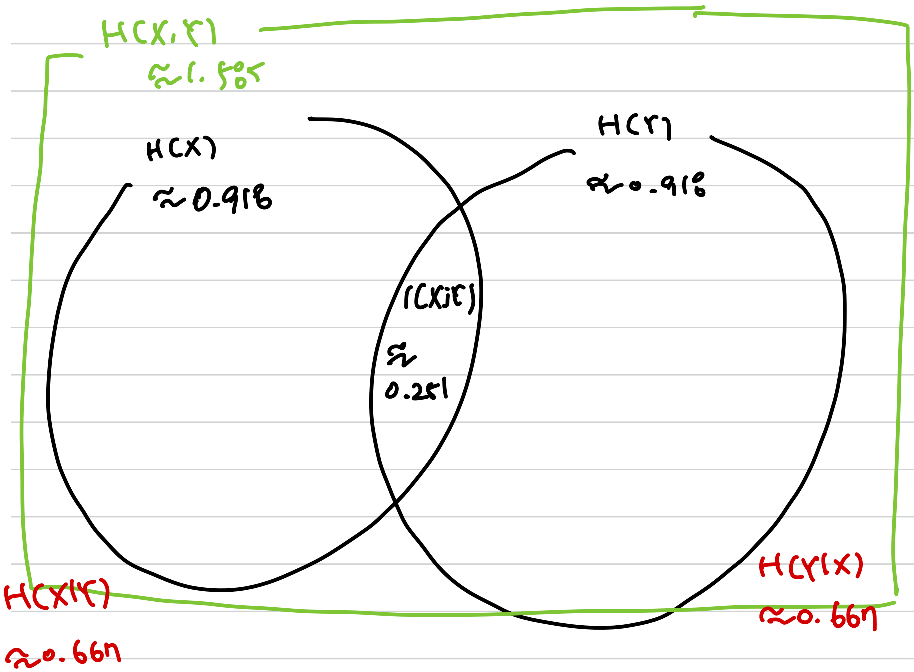
. $X=1 \rightarrow Y=1$ $\quad H(Y|X=1) = 0$

(c) $H(X,Y) = -3\times\frac{1}{3}\log_2\frac{1}{3} = \log_2 3$

(d) $0.918 - 0.667 = 0.251$

(e) $I(X;Y) = H(Y) - H(Y|X)$

$= 0.918 - 0.667 \rightarrow 0.251$


H(X,Y)
≈1.585
H(X)
≈0.918
H(Y)
≈0.918
I(X;Y)
≈
0.251
H(X|Y)
≈0.667
H(Y|X)
≈0.667