
12–6.

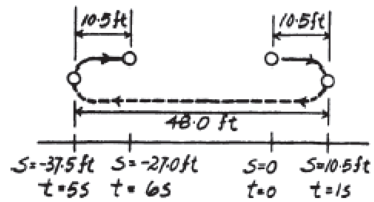
The position of a particle along a straight line is given by $s = (1.5t^3 - 13.5t^2 + 22.5t)$ ft, where t is in seconds. Determine the position of the particle when $t = 6$ s and the total distance it travels during the 6-s time interval. *Hint:* Plot the path to determine the total distance traveled.

SOLUTION

Position: The position of the particle when $t = 6$ s is

$$s|_{t=6s} = 1.5(6^3) - 13.5(6^2) + 22.5(6) = -27.0 \text{ ft} \qquad \textbf{Ans.}$$

Total Distance Traveled: The velocity of the particle can be determined by applying Eq. 12–1.



$$v = \frac{ds}{dt} = 4.50t^2 - 27.0t + 22.5$$

The times when the particle stops are

$$4.50t^2 - 27.0t + 22.5 = 0$$

$$t = 1 \text{ s} \qquad \text{and} \qquad t = 5 \text{ s}$$

The position of the particle at $t = 0$ s, 1 s and 5 s are

$$s|_{t=0\text{ s}} = 1.5(0^3) - 13.5(0^2) + 22.5(0) = 0$$

$$s|_{t=1\text{ s}} = 1.5(1^3) - 13.5(1^2) + 22.5(1) = 10.5 \text{ ft}$$

$$s|_{t=5\text{ s}} = 1.5(5^3) - 13.5(5^2) + 22.5(5) = -37.5 \text{ ft}$$

From the particle's path, the total distance is

$$s_{tot} = 10.5 + 48.0 + 10.5 = 69.0 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$s|_{t=6\text{ s}} = -27.0$ ft
$s_{tot} = 69.0$ ft

12–26.

The acceleration of a particle along a straight line is defined by $a = (2t - 9)\ \text{m/s}^2$, where t is in seconds. At $t = 0$, $s = 1$ m and $v = 10$ m/s. When $t = 9$ s, determine (a) the particle's position, (b) the total distance traveled, and (c) the velocity.

SOLUTION

$a = 2t - 9$

$$\int_{10}^{v} dv = \int_0^t (2t - 9)\, dt$$

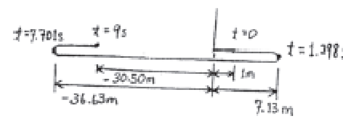
$v - 10 = t^2 - 9\,t$

$v = t^2 - 9\,t + 10$

$$\int_1^s ds = \int_0^t (t^2 - 9t + 10)\, dt$$

$$s - 1 = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t$$

$$s = \frac{1}{3}t^3 - 4.5\,t^2 + 10\,t + 1$$



Note when $v = t^2 - 9\,t + 10 = 0$:

$$t = 1.298 \text{ s and } t = 7.701 \text{ s}$$

When $t = 1.298$ s, $\quad s = 7.13$ m

When $t = 7.701$ s, $\quad s = -36.63$ m

When $t = 9$ s, $\quad s = -30.50$ m

(a) $\quad s = -30.5$ m **Ans.**

(b) $\quad s_{Tot} = (7.13 - 1) + 7.13 + 36.63 + (36.63 - 30.50)$

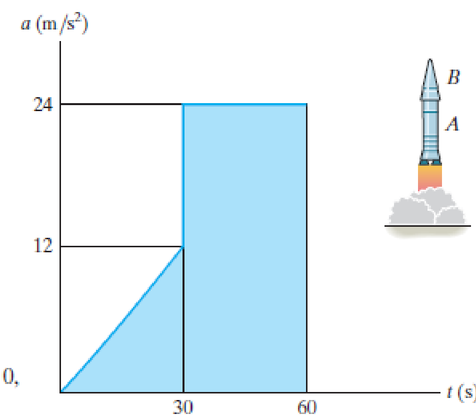
$\quad s_{Tot} = 56.0$ m **Ans.**

(c) $\quad v = 10$ m/s **Ans.**

Ans:
(a) $s = -30.5$ m
(b) $s_{\text{Tot}} = 56.0$ m
(c) $v = 10$ m/s

12–47.

A two-stage rocket is fired vertically from rest at $s = 0$ with the acceleration as shown. After 30 s the first stage, A, burns out and the second stage, B, ignites. Plot the v–t and s–t graphs which describe the motion of the second stage for $0 \le t \le 60$ s.



SOLUTION

***v–t* Graph.** The $v-t$ function can be determined by integrating $dv = a\,dt$.

For $0 \le t < 30$ s, $a = \frac{12}{30}t = \left(\frac{2}{5}t\right)$ m/s^2. Using the initial condition $v = 0$ at $t = 0$,

$$\int_0^v dv = \int_0^t \frac{2}{5}t\,dt$$

$$v = \left\{\frac{1}{5}t^2\right\} \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 30$ s,

$$v\Big|_{t=30\text{ s}} = \frac{1}{5}(30^2) = 180 \text{ m/s}$$

For $30 < t \le 60$ s, $a = 24$ m/s^2. Using the initial condition $v = 180$ m/s at $t = 30$ s,

$$\int_{180\text{ m/s}}^v dv = \int_{30\text{ s}}^t 24\,dt$$

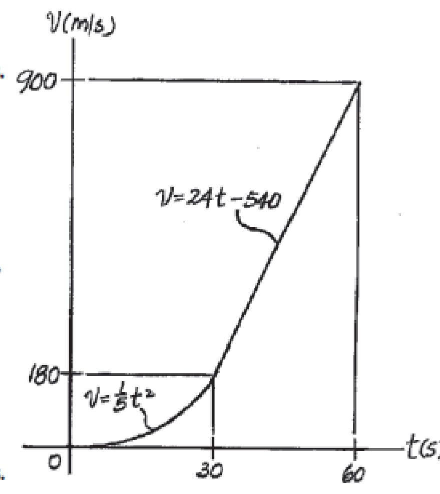
$$v - 180 = 24\,t\Big|_{30\text{ s}}^t$$

$$v = \{24t - 540\} \text{ m/s} \qquad \textbf{Ans.}$$

At $t = 60$ s,

$$v\Big|_{t=60\text{ s}} = 24(60) - 540 = 900 \text{ m/s}$$

Using these results, $v-t$ graph shown in Fig. a can be plotted.



(a)

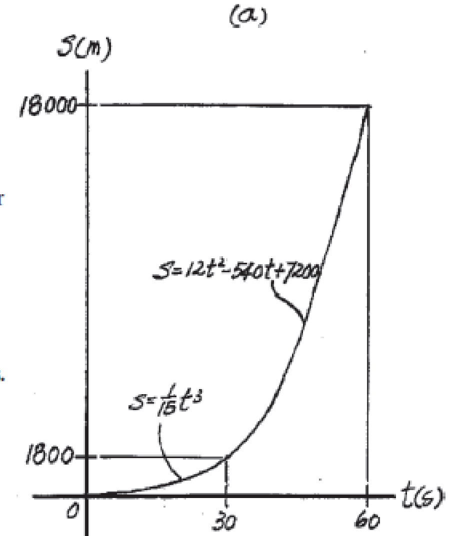
***s–t* Graph.** The $s-t$ function can be determined by integrating $ds = v\,dt$. For $0 \le t < 30$ s, the initial condition is $s = 0$ at $t = 0$.

$$\int_0^s ds = \int_0^t \frac{1}{5}t^2 dt$$

$$s = \left\{\frac{1}{15}t^3\right\} \text{ m} \qquad \textbf{Ans.}$$

At $t = 30$ s,

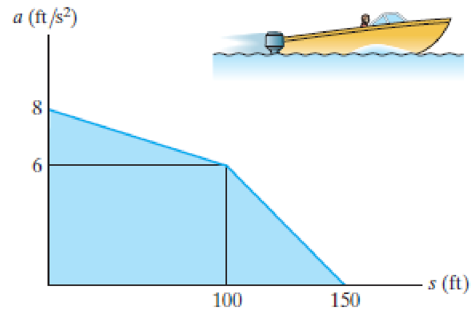
$$s\Big|_{t=30\text{ s}} = \frac{1}{15}(30^3) = 1800 \text{ m}$$



(b)

***12–56.**

Starting from rest at $s = 0$, a boat travels in a straight line with the acceleration shown by the a–s graph. Determine the boat's speed when $s = 50$ ft, 100 ft, and 150 ft.



SOLUTION

***v–s* Function.** The v–s function can be determined by integrating $v\,dv = a\,ds$. For $0 \le s < 100$ ft, $\dfrac{a-8}{s-0} = \dfrac{6-8}{100-0}$, $a = \left\{-\dfrac{1}{50}s + 8\right\}$ ft/s^2. Using the initial condition v–0 at $s = 0$,

$$\int_0^v v\,dv = \int_0^s \left(-\frac{1}{50}s + 8\right) ds$$

$$\frac{v^2}{2}\Big|_0^v = \left(-\frac{1}{100}s^2 + 8\,s\right)\Big|_0^s$$

$$\frac{v^2}{2} = 8s - \frac{1}{100}s^2$$

$$v = \left\{\sqrt{\frac{1}{50}(800\,s - s^2)}\right\} \text{ ft/s}$$

At $s = 50$ ft,

$$v|_{s=50\text{ ft}} = \sqrt{\frac{1}{50}[800\,(50) - 50^2]} = 27.39 \text{ ft/s} = 27.4 \text{ ft/s} \qquad \textbf{Ans.}$$

At $s = 100$ ft,

$$v|_{s=100\text{ ft}} = \sqrt{\frac{1}{50}[800\,(100) - 100^2]} = 37.42 \text{ ft/s} = 37.4 \text{ ft/s} \qquad \textbf{Ans.}$$

For 100 ft $< s \le 150$ ft, $\dfrac{a-0}{s-150} = \dfrac{6-0}{100-150}$; $a = \left\{-\dfrac{3}{25}s + 18\right\}$ ft/s^2. Using the initial condition $v = 37.42$ ft/s at $s = 100$ ft,

$$\int_{37.42\text{ ft/s}}^{v} v\,dv = \int_{100\text{ ft}}^{s}\left(-\frac{3}{25}s + 18\right) ds$$

$$\frac{v^2}{2}\Big|_{37.42\text{ ft/s}}^{v} = \left(-\frac{3}{50}s^2 + 18s\right)\Big|_{100\text{ ft}}^{s}$$

$$v = \left\{\frac{1}{5}\sqrt{-3s^2 + 900s - 25000}\right\} \text{ ft/s}$$

At $s = 150$ ft

$$v|_{s=150\text{ ft}} = \frac{1}{5}\sqrt{-3(150^2) + 900\,(150) - 25000} = 41.23 \text{ ft/s} = 41.2 \text{ ft/s} \qquad \textbf{Ans.}$$

Ans:
$v|_{s=50\text{ ft}} = 27.4$ ft/s
$v|_{s=100\text{ ft}} = 37.4$ ft/s
$v|_{s=150\text{ ft}} = 41.2$ ft/s

***12–72.**

The velocity of a particle is given by $\mathbf{v} = \{16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\}$ m/s, where t is in seconds. If the particle is at the origin when $t = 0$, determine the magnitude of the particle's acceleration when $t = 2$ s. Also, what is the x, y, z coordinate position of the particle at this instant?

SOLUTION

Acceleration: The acceleration expressed in Cartesian vector form can be obtained by applying Eq. 12–9.

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \{32t\mathbf{i} + 12t^2\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$$

When $t = 2$ s, $\mathbf{a} = 32(2)\mathbf{i} + 12(2^2)\mathbf{j} + 5\mathbf{k} = \{64\mathbf{i} + 48\mathbf{j} + 5\mathbf{k}\}\ \text{m/s}^2$. The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{64^2 + 48^2 + 5^2} = 80.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Position: The position expressed in Cartesian vector form can be obtained by applying Eq. 12–7.

$$d\mathbf{r} = \mathbf{v}\,dt$$

$$\int_0^{\mathbf{r}} d\mathbf{r} = \int_0^t \left(16t^2\mathbf{i} + 4t^3\mathbf{j} + (5t + 2)\mathbf{k}\right) dt$$

$$\mathbf{r} = \left[\frac{16}{3}t^3\mathbf{i} + t^4\mathbf{j} + \left(\frac{5}{2}t^2 + 2t\right)\mathbf{k}\right]\ \text{m}$$

When $t = 2$ s,

$$\mathbf{r} = \frac{16}{3}(2^3)\mathbf{i} + (2^4)\mathbf{j} + \left[\frac{5}{2}(2^2) + 2(2)\right]\mathbf{k} = \{42.7\mathbf{i} + 16.0\mathbf{j} + 14.0\mathbf{k}\}\ \text{m}.$$

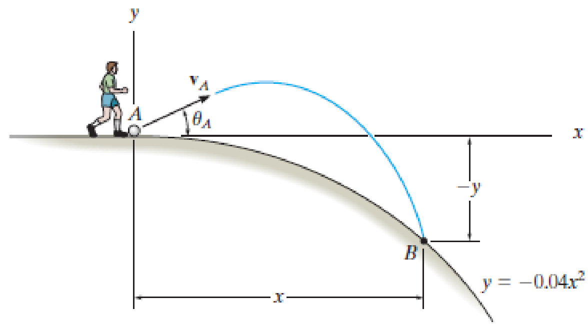
Thus, the coordinate of the particle is

$$(42.7, 16.0, 14.0)\ \text{m} \qquad \textbf{Ans.}$$

Ans:
(42.7, 16.0, 14.0) m

12–91.

The ball at A is kicked with a speed $v_A = 80$ ft/s and at an angle $\theta_A = 30°$. Determine the point $(x, -y)$ where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION

$$(v_A)_x = 80 \cos 30° = 69.28 \text{ ft/s}$$

$$(v_A)_y = 80 \sin 30° = 40 \text{ ft/s}$$

$$(\overset{+}{\rightarrow})\ s = s_0 + v_0 t$$

$$x = 0 + 69.28t \qquad \textbf{(1)}$$

$$(+\uparrow)\ s = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$-y = 0 + 40t + \frac{1}{2}(-32.2)t^2 \qquad \textbf{(2)}$$

$$y = -0.04x^2$$

From Eqs. (1) and (2):

$$-y = 0.5774x - 0.003354x^2$$

$$0.04x^2 = 0.5774x - 0.003354x^2$$

$$0.04335x^2 = 0.5774x$$

$$x = 13.3 \text{ ft} \qquad \textbf{Ans.}$$

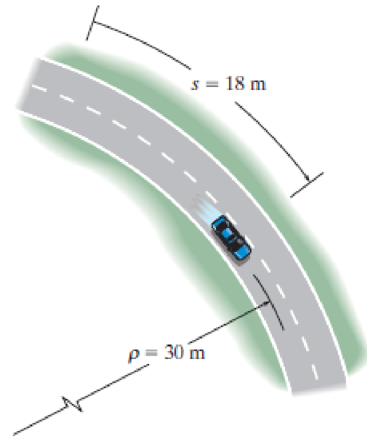
Thus

$$y = -0.04\,(13.3)^2 = -7.09 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
(13.3 ft, −7.09 ft)

*12–120.

The car travels along the circular path such that its speed is increased by $a_t = (0.5e^t)\ \text{m/s}^2$, where t is in seconds. Determine the magnitudes of its velocity and acceleration after the car has traveled $s = 18\ \text{m}$ starting from rest. Neglect the size of the car.



SOLUTION

$$\int_0^v dv = \int_0^t 0.5e^t\, dt$$

$$v = 0.5(e^t - 1)$$

$$\int_0^{18} ds = 0.5\int_0^t (e^t - 1)dt$$

$$18 = 0.5(e^t - t - 1)$$

Solving,

$$t = 3.7064\ \text{s}$$

$$v = 0.5(e^{3.7064} - 1) = 19.85\ \text{m/s} = 19.9\ \text{m/s} \qquad \textbf{Ans.}$$

$$a_t = \dot{v} = 0.5e^t|_{t=3.7064\ \text{s}} = 20.35\ \text{m/s}^2$$

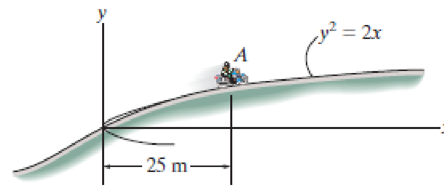
$$a_n = \frac{v^2}{\rho} = \frac{19.85^2}{30} = 13.14\ \text{m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{20.35^2 + 13.14^2} = 24.2\ \text{m/s}^2 \qquad \textbf{Ans.}$$

Ans:
$v = 19.9\ \text{m/s}$
$a = 24.2\ \text{m/s}^2$

12–134.

The motorcycle is traveling at a constant speed of 60 km/h. Determine the magnitude of its acceleration when it is at point A.



SOLUTION

Radius of Curvature:

$$y = \sqrt{2}x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{2}x^{-1/2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}\sqrt{2}x^{-3/2}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{2}\sqrt{2}x^{-1/2}\right)^2\right]^{3/2}}{\left|-\frac{1}{4}\sqrt{2}x^{-3/2}\right|}\right|_{x=25\text{ m}} = 364.21\text{ m}$$

Acceleration: The speed of the motorcycle at a is

$$v = \left(60\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 16.67\text{ m/s}$$

$$a_n = \frac{v^2}{\rho} = \frac{16.67^2}{364.21} = 0.7627\text{ m/s}^2$$

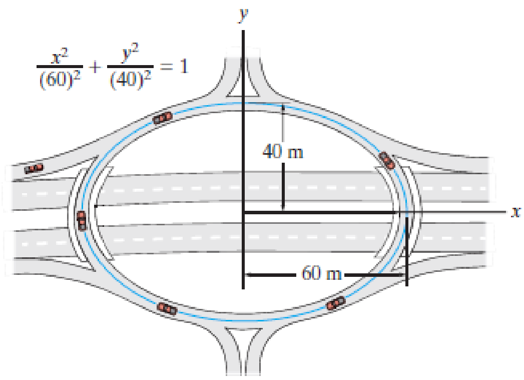
Since the motorcycle travels with a constant speed, $a_t = 0$. Thus, the magnitude of the motorcycle's acceleration at A is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0^2 + 0.7627^2} = 0.763\text{ m/s}^2$$ **Ans.**

Ans:
$a = 0.763\text{ m/s}^2$

*12–140.

Cars move around the "traffic circle" which is in the shape of an ellipse. If the speed limit is posted at 60 km/h, determine the maximum acceleration experienced by the passengers.



SOLUTION

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$b^2(2x) + a^2(2y)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{b^2x}{a^2y}$$

$$\frac{dy}{dx}y = \frac{-b^2x}{a^2}$$

$$\frac{d^2y}{dx^2}y + \left(\frac{dy}{dx}\right)^2 = \frac{-b^2}{a^2}$$

$$\frac{d^2y}{dx^2}y = \frac{-b^2}{a^2} - \left(\frac{-b^2x}{a^2y}\right)^2$$

$$\frac{d^2y}{dx^2} = \frac{-b^4}{a^2y^3}$$

$$\rho = \frac{\left[1 + \left(\frac{-b^2x}{a^2y}\right)^2\right]^{3/2}}{\left|\frac{-b^4}{a^2y^3}\right|}$$

At $x = a, y = 0$,

$$\rho = \frac{b^2}{a}$$

Then

$$a_t = 0$$

$$a_{max} = a_n = \frac{v^2}{\rho} = \frac{v^2}{\frac{b^2}{n}} = \frac{v^2a}{b^3}$$

Set $a = 60$ m, $b = 40$ m, $v = \frac{60(10^3)}{3600} = 16.67$ m/s

$$a_{max} = \frac{(16.67)^2(60)}{(40)^2} = 10.4 \text{ m/s}^2$$ **Ans.**

Ans:
$a_{max} = 10.4$ m/s^2

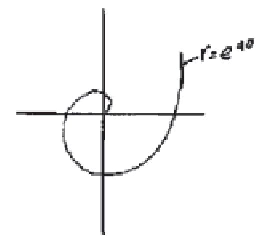
12–162.

If a particle moves along a path such that $r = (e^{at})$ m and $\theta = t$, where t is in seconds, plot the path $r = f(\theta)$, and determine the particle's radial and transverse components of velocity and acceleration.

SOLUTION

$r = e^{at} \qquad \dot{r} = ae^{at} \qquad \ddot{r} = a^2e^{et}$

$\theta = t \qquad \dot{\theta} = 1 \qquad \ddot{\theta} = 0$



$v_r = \dot{r} = ae^{at}$ **Ans.**

$v_a = r\dot{\theta} = e^{at}(1) = e^{at}$ **Ans.**

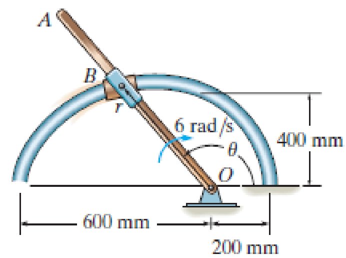
$a_r = \ddot{r} - r\dot{\theta}^2 = a^2e^{at} - e^{at}(1)^2 = e^{at}(a^2 - 1)$ **Ans.**

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{at}(0) + 2(ae^{at})(1) = 2ae^{at}$ **Ans.**

Ans:
$v_r = ae^{at}$
$v_\theta = e^{at}$
$a_r = e^{at}(a^2 - 1)$
$a_\theta = 2ae^{at}$

*12–172.

The rod OA rotates clockwise with a constant angular velocity of 6 rad/s. Two pin-connected slider blocks, located at B, move freely on OA and the curved rod whose shape is a limaçon described by the equation $r = 200(2 - \cos\theta)$ mm. Determine the speed of the slider blocks at the instant $\theta = 150°$.



SOLUTION

Velocity. Using the chain rule, the first and second time derivatives of r can be determined.

$$r = 200(2 - \cos\theta)$$

$$\dot{r} = 200(\sin\theta)\dot{\theta} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$\ddot{r} = \{200[(\cos\theta)\dot{\theta}^2 + (\sin\theta)\ddot{\theta}]\}\ \text{mm/s}^2$$

The radial and transverse components of the velocity are

$$v_r = \dot{r} = \{200(\sin\theta)\dot{\theta}\}\ \text{mm/s}$$

$$v_\theta = r\dot{\theta} = \{200(2 - \cos\theta)\dot{\theta}\}\ \text{mm/s}$$

Since $\dot{\theta}$ is in the opposite sense to that of positive θ, $\dot{\theta} = -6$ rad/s. Thus, at $\theta = 150°$,

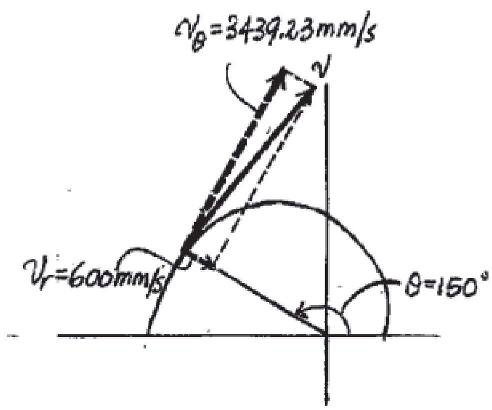
$$v_r = 200(\sin 150°)(-6) = -600\ \text{mm/s}$$

$$v_\theta = 200(2 - \cos 150°)(-6) = -3439.23\ \text{mm/s}$$

Thus, the magnitude of the velocity is

$$v = \sqrt{v_r^2 + v_\theta^2}\ \ \sqrt{(-600)^2 + (-3439.23)^2} = 3491\ \text{mm/s} = 3.49\ \text{m/s} \qquad \textbf{Ans.}$$

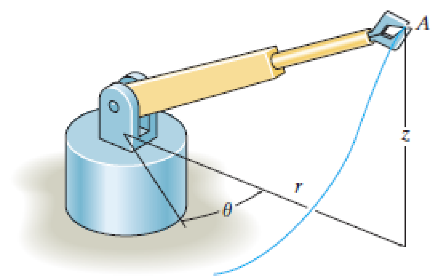
These components are shown in Fig. *a*



Ans:
$v = 3.49$ m/s

12–191.

The arm of the robot moves so that $r = 3$ ft is constant, and its grip A moves along the path $z = (3 \sin 4\theta)$ ft, where θ is in radians. If $\theta = (0.5t)$ rad, where t is in seconds, determine the magnitudes of the grip's velocity and acceleration when $t = 3$ s.



SOLUTION

$\theta = 0.5\,t \qquad r = 3 \qquad z = 3\sin 2t$

$\dot{\theta} = 0.5 \qquad \dot{r} = 0 \qquad \dot{z} = 6\cos 2t$

$\ddot{\theta} = 0 \qquad \ddot{r} = 0 \qquad \ddot{z} = -12\sin 2t$

At $t = 3$ s,

$z = -0.8382$

$\dot{z} = 5.761$

$\ddot{z} = 3.353$

$v_r = 0$

$v_\theta = 3(0.5) = 1.5$

$v_z = 5.761$

$v = \sqrt{(0)^2 + (1.5)^2 + (5.761)^2} = 5.95$ ft/s **Ans.**

$a_r = 0 - 3(0.5)^2 = -0.75$

$a_\theta = 0 + 0 = 0$

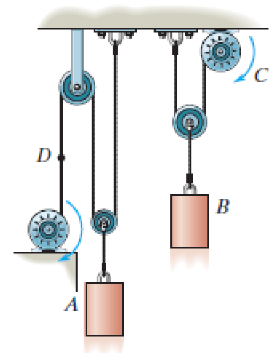
$a_z = 3.353$

$a = \sqrt{(-0.75)^2 + (0)^2 + (3.353)^2} = 3.44$ ft/s^2 **Ans.**

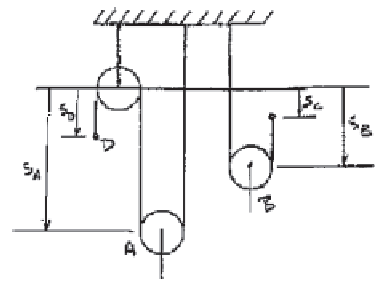
Ans:
$v = 5.95$ ft/s
$a = 3.44$ ft/s^2

*12–208.

The motor draws in the cable at C with a constant velocity of $v_C = 4$ m/s. The motor draws in the cable at D with a constant acceleration of $a_D = 8$ m/s^2. If $v_D = 0$ when $t = 0$, determine (a) the time needed for block A to rise 3 m, and (b) the relative velocity of block A with respect to block B when this occurs.



SOLUTION



(a) $a_D = 8\ \text{m/s}^2$

$v_D = 8t$

$s_D = 4t^2$

$s_D + 2s_A = l$

$\Delta s_D = -2\Delta s_A$ **(1)**

$\Delta s_A = -2t^2$

$-3 = -2t^2$

$t = 1.2247 = 1.22\ \text{s}$ **Ans.**

(b) $v_A = s_A = -4t = -4(1.2247) = -4.90\ \text{m/s} = 4.90\ \text{m/s}\uparrow$

$s_B + (s_B - s_C) = l$

$2v_B = v_C = -4$

$v_B = -2\ \text{m/s} = 2\ \text{m/s}\uparrow$

$(+\downarrow)$ $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

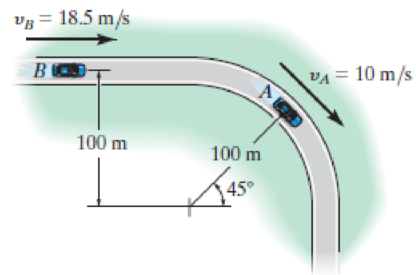
$-4.90 = -2 + v_{A/B}$

$v_{A/B} = -2.90\ \text{m/s} = 2.90\ \text{m/s}\uparrow$ **Ans.**

Ans:
$v_{A/B} = 2.90\ \text{m/s}\uparrow$

12–214.

At the instant shown, the car at A is traveling at 10 m/s around the curve while increasing its speed at 5 m/s^2. The car at B is traveling at 18.5 m/s along the straightaway and increasing its speed at 2 m/s^2. Determine the relative velocity and relative acceleration of A with respect to B at this instant.



SOLUTION

$$v_A = 10\cos 45°\mathbf{i} - 10\sin 45°\mathbf{j} = \{7.071\mathbf{i} - 7.071\mathbf{j}\}\ \text{m/s}$$

$$v_B = \{18.5\mathbf{i}\}\ \text{m/s}$$

$$v_{A/B} = v_A - v_B$$

$$= (7.071\mathbf{i} - 7.071\mathbf{j}) - 18.5\mathbf{i} = \{-11.429\mathbf{i} - 7.071\mathbf{j}\}\ \text{m/s}$$

$$v_{A/B} = \sqrt{(-11.429)^2 + (-7.071)^2} = 13.4\ \text{m/s} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{7.071}{11.429} = 31.7° \qquad \textbf{Ans.}$$

$$(a_A)_n = \frac{v_A^2}{\rho} = \frac{10^2}{100} = 1\ \text{m/s}^2 \qquad (a_A)_t = 5\ \text{m/s}^2$$

$$\mathbf{a}_A = (5\cos 45° - 1\cos 45°)\mathbf{i} + (-1\sin 45° - 5\sin 45°)\mathbf{j}$$

$$= \{2.828\mathbf{i} - 4.243\mathbf{j}\}\ \text{m/s}^2$$

$$\mathbf{a}_B = \{2\mathbf{i}\}\ \text{m/s}^2$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{u}_B$$

$$= (2.828\mathbf{i} - 4.243\mathbf{j}) - 2\mathbf{i} = \{0.828\mathbf{i} - 4.24\mathbf{j}\}\ \text{m/s}^2$$

$$a_{A/B} = \sqrt{0.828^2 + (-4.243)^2} = 4.32\ \text{m/s}^2 \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\frac{4.243}{0.828} = 79.0° \qquad \textbf{Ans.}$$

Ans:

$v_{A/B} = 13.4$ m/s

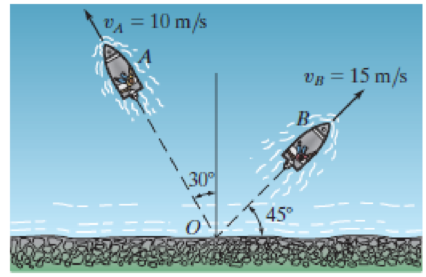
$\theta_v = 31.7°$

$a_{A/B} = 4.32$ m/s^2

$\theta_a = 79.0°$

12–223.

Two boats leave the shore at the same time and travel in the directions shown. If $v_A = 10\text{ m/s}$ and $v_B = 15\text{ m/s}$, determine the velocity of boat A with respect to boat B. How long after leaving the shore will the boats be 600 m apart?



SOLUTION

Relative Velocity. The velocity triangle shown in Fig. *a* is drawn based on the relative velocity equation $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$. Using the cosine law,

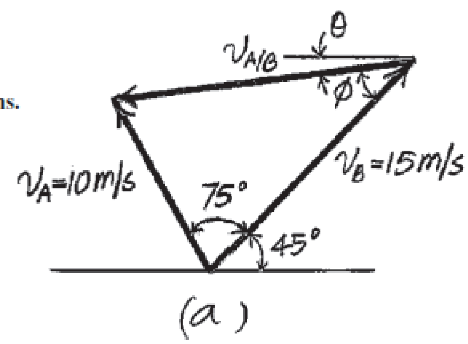
$$v_{A/B} = \sqrt{10^2 + 15^2 - 2(10)(15)\cos 75^\circ} = 15.73\text{ m/s} = 15.7\text{ m/s} \qquad \textbf{Ans.}$$

Then, the sine law gives

$$\frac{\sin\phi}{10} = \frac{\sin 75^\circ}{15.73} \qquad \phi = 37.89^\circ$$

The direction of $\mathbf{v}_{A/B}$ is defined by

$$\theta = 45^\circ - \phi = 45^\circ - 37.89^\circ = 7.11^\circ \qquad \textbf{Ans.}$$



Alternatively, we can express $\mathbf{v}_A$ and $\mathbf{v}_B$ in Cartesian vector form

$$\mathbf{v}_A = \{-10\sin 30^\circ\mathbf{i} + 10\cos 30^\circ\mathbf{j}\}\text{ m/s} = \{-5.00\mathbf{i} + 5\sqrt{3}\mathbf{j}\}\text{ m/s}$$

$$\mathbf{v}_B = \{15\cos 45^\circ\mathbf{i} + 15\sin 45^\circ\mathbf{j}\}\text{ m/s} = \{7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j}\}\text{ m/s}.$$

Applying the relative velocity equation

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$-500\mathbf{i} + 5\sqrt{3}\mathbf{j} = 7.5\sqrt{2}\mathbf{i} + 7.5\sqrt{2}\mathbf{j} + \mathbf{v}_{A/B}$$

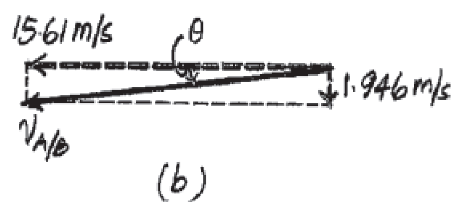
$$\mathbf{v}_{A/B} = \{-15.61\mathbf{i} - 1.946\mathbf{j}\}\text{ m/s}$$

Thus the magnitude of $\mathbf{v}_{A/B}$ is

$$v_{A/B} = \sqrt{(-15.61)^2 + (-1.946)^2} = 15.73\text{ m/s} = 15.7\text{ m/s} \qquad \textbf{Ans.}$$

And its direction is defined by angle θ, Fig. *b*,

$$\theta = \tan^{-1}\left(\frac{1.946}{15.61}\right) = 7.1088^\circ = 7.11^\circ \qquad \textbf{Ans.}$$



Here $s_{A/B} = 600$ m. Thus

$$t = \frac{s_{A/B}}{v_{A/B}} = \frac{600}{15.73} = 38.15\text{ s} = 38.1\text{ s} \qquad \textbf{Ans.}$$

Ans:
$v_{A/B} = 15.7\text{ m/s}$
$\theta = 7.11^\circ$
$t = 38.1\text{ s}$