

Definition of Big θ :

If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$, then $f(n) = \theta(g(n))$.

$f(n) = O(g(n))$, if there are positive constants c and n_0 such that $f(n) \leq c * g(n)$ for all $n \geq n_0$.

$f(n) = \Omega(g(n))$, if there are positive constants c and n_0 such that $f(n) \geq c * g(n)$ for all $n \geq n_0$.

11. $n^2 + n/2 + 1 = \theta(n^3)$

False, no constants can be chosen since $g(n^3) \geq f * g(n^2)$ for all c .

$f(n^2) = O(g(n^3))$ because for $c = 2$, $n_0 = 2$, $g(n^3) \geq f * g(n^2)$ for all $n \geq n_0$.

$f(n^2) \neq \Omega(g(n^3))$ because for any c , $g(n^3) \geq f * g(n^2)$ for all $n \geq n_0$. Eg if $c = 5$, for all $n \geq n_0 = 5$, $g(n^3) \geq f * g(n^2)$.

12.

False, because no constants can be chosen since $g(\log 64) \geq f(1)$ for all c .

$g = O(f)$ but $f \neq O(g)$ as $\log 64 = 6 > 1$.

Likewise, $g \neq \Omega(f)$ but $f = \Omega(g)$ because $1 < 6$.

13.

False. Because no constants can be chosen since $f(n \log n) \geq g(\log n)$ for all c .

$f(n \log n) = \Omega(g)$ but $g \neq \Omega(f)$ because for any n , $n \log n \geq \log n$.

Likewise, $f(n \log n) \neq O(g)$ but $g = O(f)$.

14.

False. Because no constants can be chosen since $2^n \geq n$ for all c .

15.

True since $n^5/n^2 = n^3$. Let c be 5 and n_0 be 5. Then $5(n^5/n^2)$ always $> n^3$.

Likewise, $5(n^3)$ always $\geq (n^5/n^2)$. Therefore 15 is true.