

Answer: Running time is $O(n)$

Explanation: There is a single loop that runs $n-5$ times. Each time the loop runs it executes instruction in the loop header and 1 instruction in the body of the loop. The total number of instructions is $2 * (n - 5) + 1$ (for the last loop check) $= 2n - 9 = O(n)$. (also OK: $\Theta(n)$).

```
1)
num=0;
for (i = n; i >= 0; i--)
    num--;
```

Answer: Running time is $O(n)$

Explanation: after the 1st instruction, there is a single loop that runs $(n+1)$ times; each time the loop runs it executes the instruction in the loop header and 1 instruction in the body of the loop. The total number of instructions is $2*(n+1) + 1$ (for the last loop check) $+ 1 = 2n + 4 = O(n)$.

```
2)
num=0;
for (i = 0; i <= n * n; i=i+2)
    num=num+2;
```

Answer: Running time is $O(n^2)$.

Explanation: after the 1st instruction, there is a single loop that runs $(n^2)/2 + 1$ (including last loop check);

therefore $2[(n^2)/2] + 1 + 1 = n^2 + 2 = O(n^2)$.

0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36

n=2	4
n=3	6
n=4	10
n=5	14
n=6	20

```
3)
num=0;
for (i = 1; i <= n; i=i*2)
    num++;
```

Answer: $O(\log N)$

Explanation: Since the number of iterations decreases by half, loop has $\log N + 1$ complexity (inclusive of last loop check); therefore $2(\log N) + 1 + 1 = 2\log N + 2 = O(\log N)$.

```
4)
num=1;
for (i = 0; i < n; i++)
    for (j = 0; j <= i; j++)
        num = num * 2
```

Answer: $O(n^2)$

Explanation: total loop iteration pattern follows the triangular number sequence which is $n(n+1)/2$. Thus total time complexity is $n(n+1)/2 + n$ (number of inner loop last checks) + 1 (last outer loop check) = $O(n^2)$.

Eg let $n = 4$

I value	0	1	2	3	4
Outer iterations	1st	2nd	3rd	4th	5th
Inner iterations	1	2	3	4	5

Total 10 inner instructions for $n = 4$

```
*5)
p=10;
num=0;
plimit=100,000;
```

thus $O(n^2) = (n + 1) + (n^2 - np + 1)$

```
for (i = p; i<=plimit; i++)          ((10^5) - p + 1)
    for (j = 1; j<=i; j++)          ((10^5) - 9) n
        num = num + 1;
```

Answer: $O(n^2)$

Explanation: let $plimit = n$; I first find a pattern for the number of inner iterations generated per outer iteration and I found that the inner iterations has the following pattern: 10,21,33,46 . I then find the primary and secondary difference and used a general quadratic formula $an^2 + bn + c$; I then sub 3 values $n = 10, 11, 12$ to the equations to get 3 equations to solve for the variables a, b and c . The result is $\frac{1}{2}n^2 + \frac{1}{2}n - 45$, which is the algorithm for finding the number of iterations for a n number of outer iterations.

Hence $\frac{1}{2} * n^2 + \frac{1}{2} * n - 45 + n - p$ instructions occur, $O(n^2)$

```
6)
num=0;
```

```
for (i = n*n; i>=0; i=i/2)          log(n^2)
    for (j = 1; j<=n; j++)          n
        num = num + i;
```

Answer: $O(n \log(n^2))$

Explanation: Since outer loop iterations $\log(n^2)$ due to the initial I being n^2 but every iteration decreases by $i/2$, it shows a log function; inner loop then iterates n times for each iteration of the outer loop.

Thus the total iteration is $n * \log(n^2) + n$ (last checks of inner loop) + 1 (last check of outer loop) + $\log(n^2)$ (total outer loop iterations = $O(n * \log(n^2))$).

7)

```
num=0;
for (i = 0; i<n*n-1; i++)           n^2 - 1 times
    for (j = 0; j< i * i; j++)      I^2 times
        num = 2*num;
```

Answer: $O(n^3)$

Explanation: as complexity of loop is $[n*(n+1)*(2n+1)]/6$. We know this because for every iteration, the inner loop has to iterate squared the index of outer iteration which is similar to the sum of squares number sequence.

8)

```
num=0;
for (i = 0; i<= n*n; i++)
    num++;
for (i = 1; i<=n; i=i*2)
    for (j=n; j>= 1; j=j/2)
        num++;
```

Ans: $O(n^2)$

Explanation: 1st loop is $n^2 + 1 + 1$ (last check); 2nd loop is $\log(n)*\log(n) + \log(n)$ because outer of 2nd loop increases in a multiple of 2 while inner of 2nd loop decreases by a division of 2.

Since n^2 is upper bound of $(\log(n) * \log(n))$, complexity of this algorithm is $O(n^2)$.

9*)

```
for (i = 0; i < n; i++) {           n times
    smallest = i;
    for (j = i+1; j <= n; j++) {
        if (a[j] < a[smallest])
            smallest = j;
    }
    swap(a, i, smallest); // has three instructions
}
```

Ans: $O(n^2)$

Explanation:

for loop follows the triangular number sequence thus has complexity of $n(n+1)/2 * 2$ (assume have to swap elements for every iteration); swap has $3n$ instructions in total.

Thus total complexity is $O(n^2)$.

10)

```
num = 0;
i = 0;
```

```
while (i<n) {
    j = 0;
    while (j<100) {
        //constant time operations
    }
}
```

```
        j++;  
    }  
    i++;  
}
```

Ans: $O(n)$

Explanation: since while loop is $(i < n)$ and $I = 0$, outer loop iterates n times;

Inner while loop iterates 100 times for every outer loop iteration, thus complexity of inner loop is $100 * n$.

Total complexity = $100 * n + n$ (inner + last inner loop check) + $n + 1$ (outer + outer last loop check) = $O(n)$.