# Spatial dynamics

Modelling Biological Systems, BIOS13

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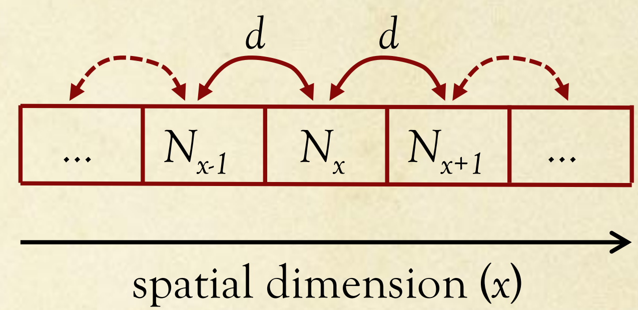
# Diffusion

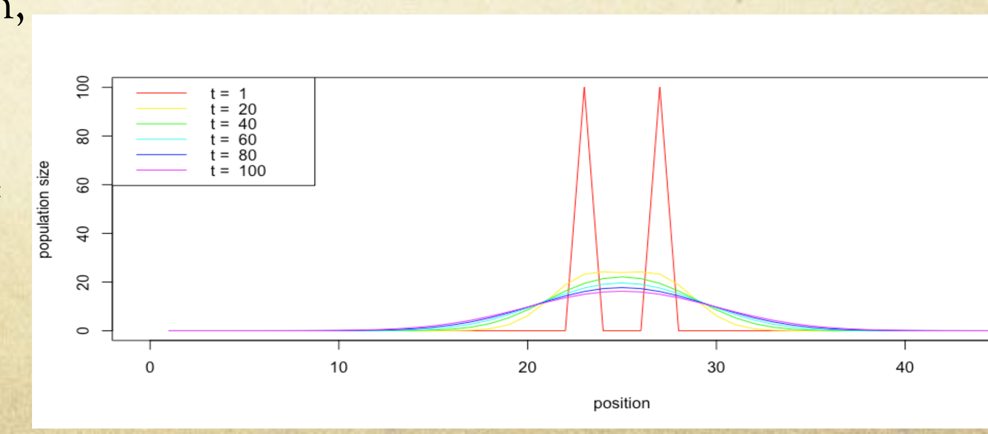
- Random movement (diffusion) of individuals (or molecules, or particles) in homogeneous space generates a Normal distribution of individuals, irrespective of the initial state.

- An important assumption is that the individuals (particles, molecules) are *many*, so many that individual fates average out. Under this assumption, the dynamics become *deterministic*. As an example, if each individual in a population of 10 has a 10% chance of moving to a neighbouring population, the actual number of moving individual can vary a lot. If the population size instead is 10.000, the number of moving individuals will always be relatively(!) close to 1000.

Model in one dimension:

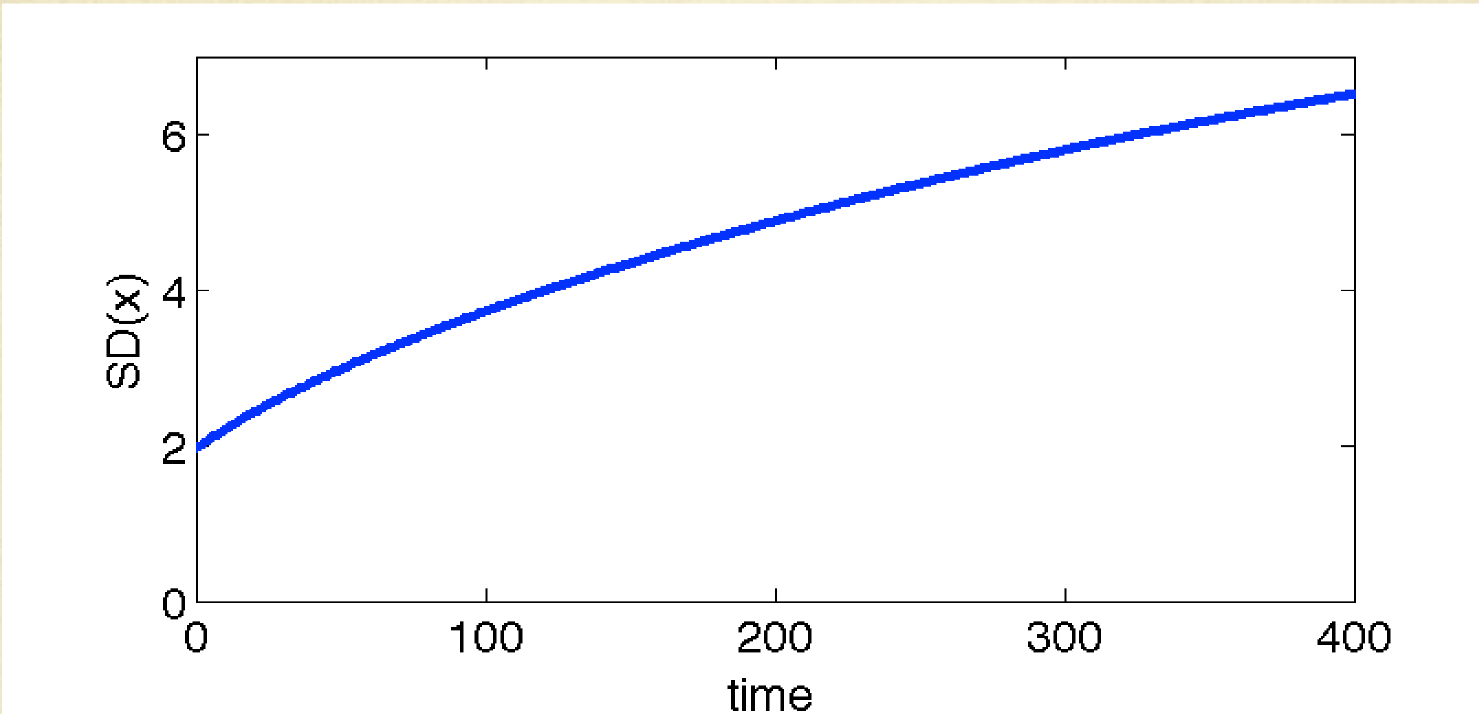
$$N_{x,t+1} = N_{x,t} - 2dN_{x,t} + dN_{x-1,t} + dN_{x+1,t}$$

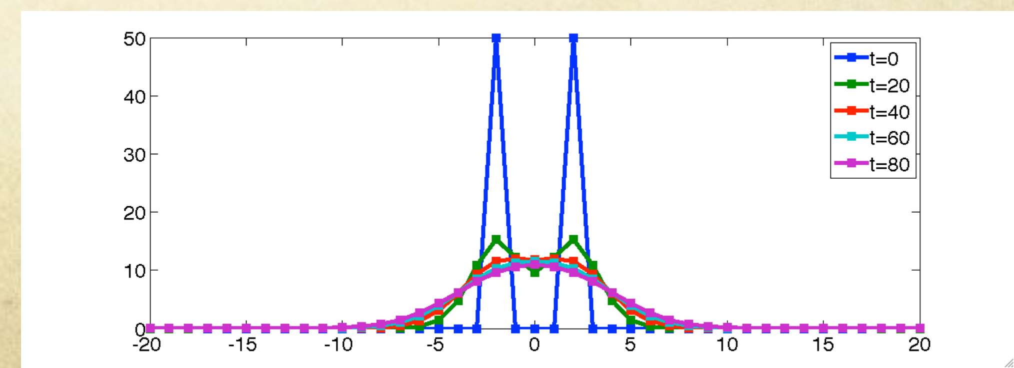


# Diffusion

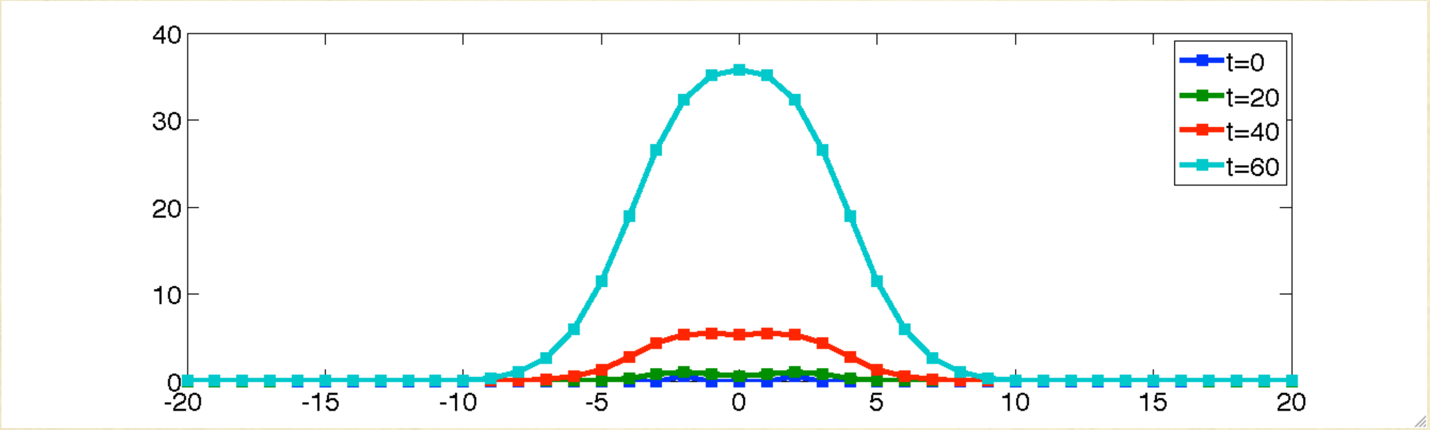
- The size of the population distribution, i.e. the *range*, increases as the square root of time:



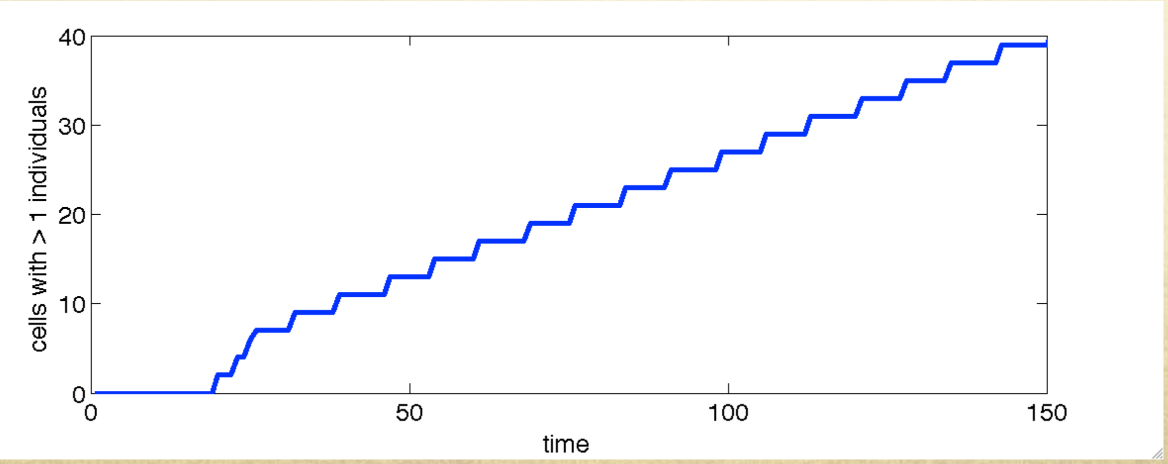



# Diffusion and growth
## (reaction-diffusion models)

- A population which disperses and at the same time grows locally can spread faster, linearly with time.
- Model in one dimension: $N_{x,t+1} = lN_{x,t} - 2dN_{x,t} + dN_{x-1,t} + dN_{x+1,t}$

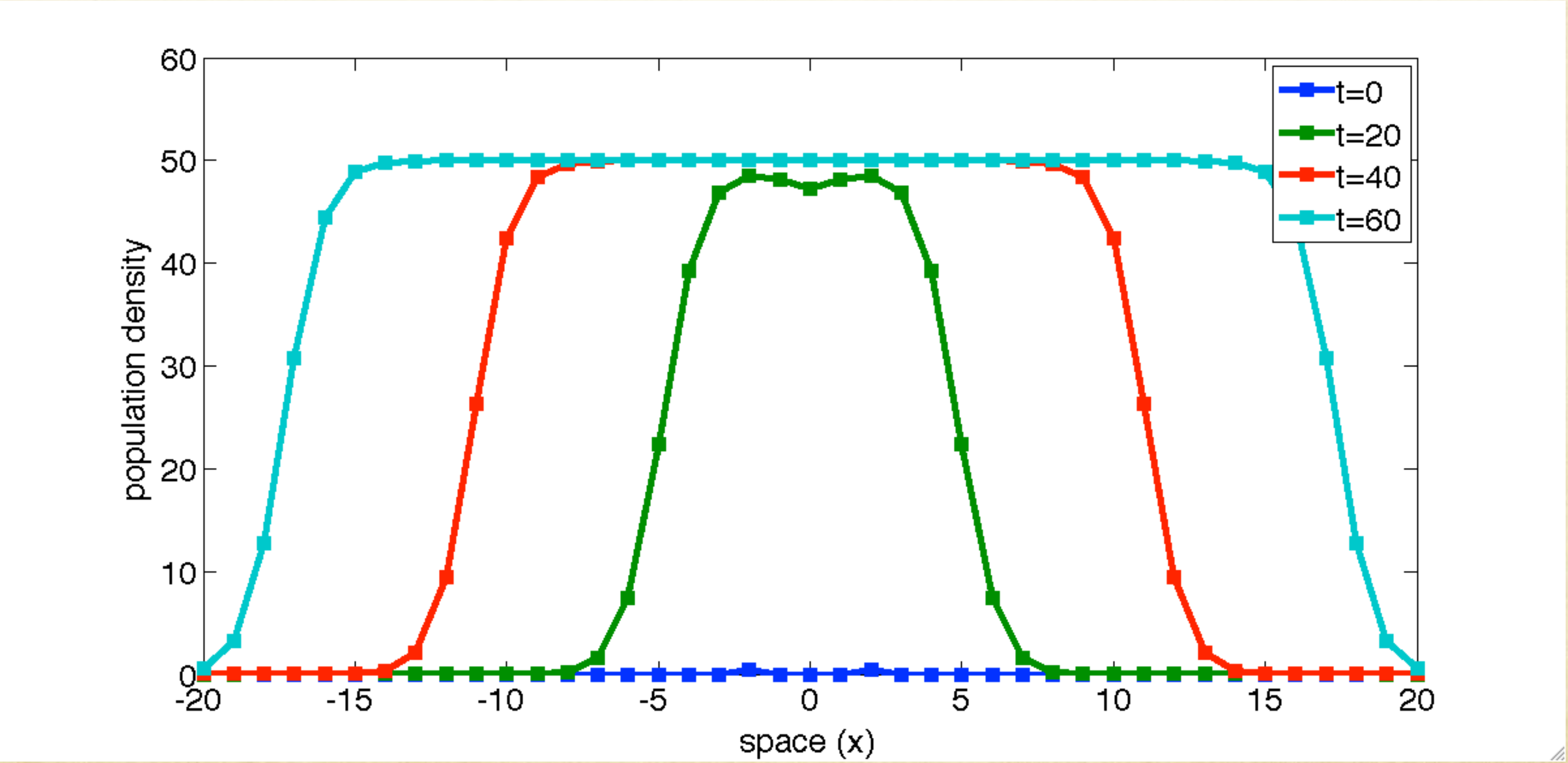



range



# Diffusion and density dependent growth

- If local population density is limited, the result is a moving invasion front, with fixed shape:



# Literature

- Fora nice introduction to spatial dynamics, have a look at "An Illustrated Guide to Theoretical Ecology" by Ted Case.
- For more in-depth analysis, check "Mathematical Models in Biology" by Leah Edelstein-Keshet.