R Exercises in Linear Algebra 1

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This exercise aims to connect linear algebra concepts we learned earlier with the dynamical systems that we will learn later in the course.

In a California redwood forest, the spotted owl is the primary predator of the wood rat. Suppose that the following system models the interaction between owls and rats. Here, $o(t)$ represents the owl population at time t and $r(t)$ represents the rat population at time t:

$$\begin{cases} o(t+1) = 0.5o(t) + 0.3r(t) \\ r(t+1) = -0.2o(t) + 1.2r(t) \end{cases}$$

1. Let $\vec{x}(t) = \begin{bmatrix} o(t) \\ r(t) \end{bmatrix}$.

    a. Find a matrix A such that the above system can be written into the matrix form
    $$\vec{x}(t+1) = A\vec{x}(t)$$

2. Suppose there are currently 2000 owls and 1000 rats.

    a. How many owls and rats are there next year?
    b. How many owls and rats are there the year after next year?

3. Write a R script that can use matrix multiplication to calculate $\vec{x}(t+1) = A\vec{x}(t)$ and find the population sizes of owls and rats in the next ten years. Plot the population sizes of owls and rats in the next ten years.

Note: for matrix multiplication, use %*% instead of *