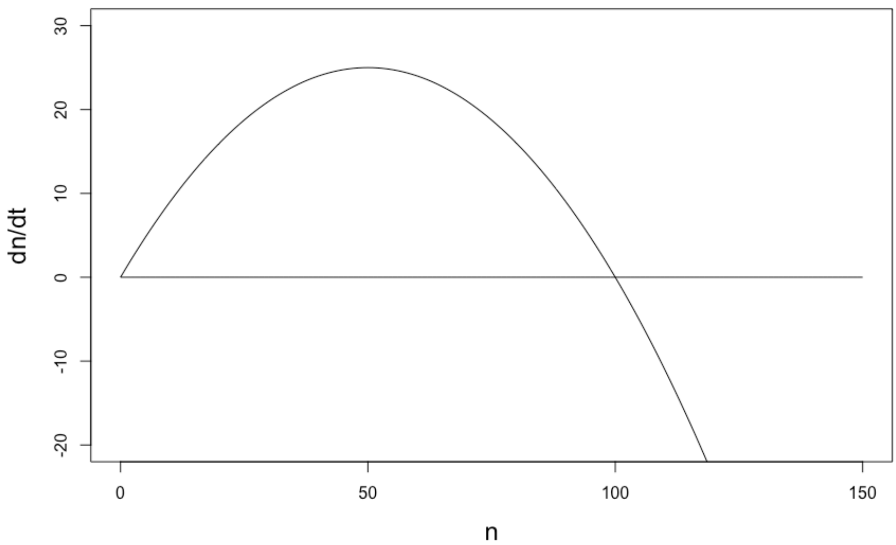
Exercise: Programming dynamic systems

In this exercise we will use the logistic equation, that was introduced in the lecture:

$$\frac{dn}{dt} = r_0 n \left(1 - \frac{n}{K}\right)$$

The idea is to (eventually) write your own numerical solver of differential equations. The `deSolve` package is forbidden in this exercise!

1. Write an R-script that plots the growth function above against population size *n*, something simular to this (add more color if you wish):



Define the values of r0 and K in the beginning of the script, such that they are easy to change if you want to. It is a good habit to define key parameters in the beginning of a program, and not embedded in the code somewhere. In this case, your script could start something like:

```
# Script plotting the logistic growth function
# Key model parameters:
r0 = 1 # intrinsic growth rate
K = 100 # carrying capacity
…
```

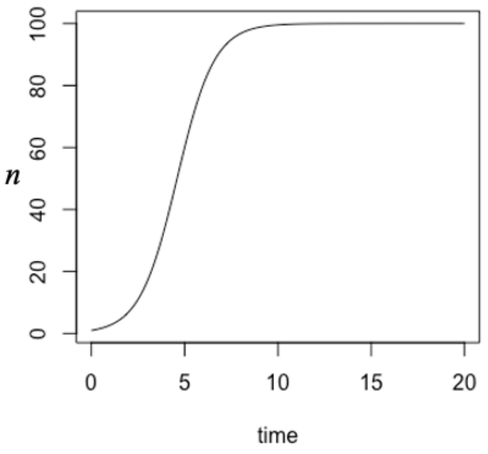
Of course, you should then actually *use* those variables in your code.

The logistic equation has an exact mathematical solution given by

$$n(t) = \frac{K}{1 + \left(\frac{K}{n(0)} - 1\right) e^{-r_0 t}}$$

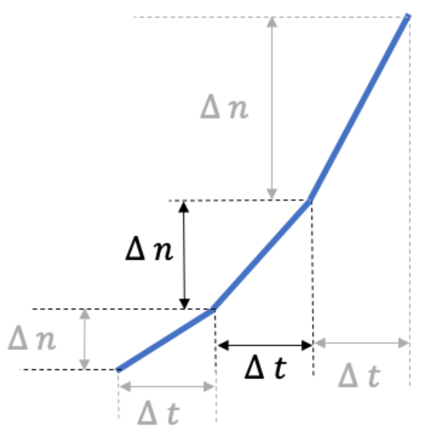
Extra: Can you see that $n \rightarrow K$ as $t \rightarrow \infty$?

2. Extend the script above to also plot the solution as given above, something like this:



Try to put the two plots side-by-side, as separate panels.
Note: We are not yet solving any differential equations, just plotting functions.

3. Make a numerical *test* of the solution. In other words, calculate $\frac{\Delta n}{\Delta t}$ for different points on the curve in exercise 2. Make a suitable plot to compare the result to the correct values given by the model ($\frac{dn}{dt} = r_0 n\left(1 - \frac{n}{K}\right)$).
The built-in R-function `diff` may come in handy. `diff(x)` calculates the difference between consequtive values of a vector `x`. Try for example `diff(c(2,3,7))` or something similar.



4. Finally, it is time to solve the logistic equation numerically. Use the procedure outlined in the lecture:
   i) Start by setting $x = x_0$, $t = 0$
   ii) Choose a small Δt
   iii) Calculate $f(t, x)$
   iv) Calculate $\Delta x = f(t, x)\Delta \mathrm{t}$
   v) Update $x \leftarrow x + \Delta x, \;\; t \leftarrow t + \Delta t$
   vi) Repeat from *iii*), until reaching final *t*.

   Does your solution match the analytical solution plotted in exercise 2?

5. (*advanced*) Re-write the program in exercise 4 to a general ode-solver, taking the function $f(t, x)$ as an input parameter. Test it with functions with known solutions, such as exponential or logistic growth. Another possible test is a function $f(t, x) = \cos{(t)}$ with solution $y = \sin(t) + y(0)$ (can you see why?).