Exercise: One-dimensional dynamic systems

Modelling Biological Systems, BIOS13

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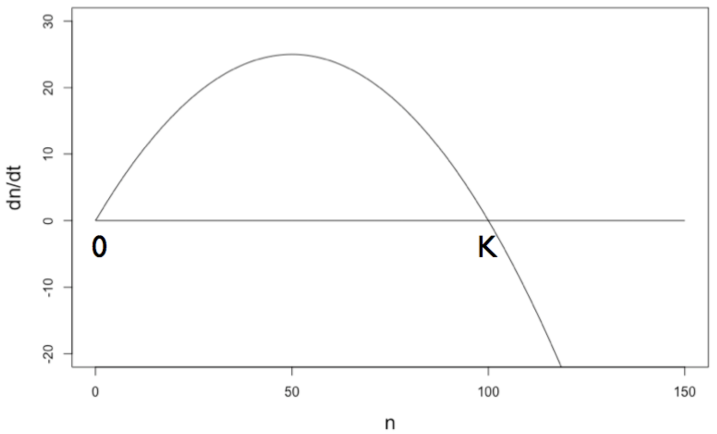
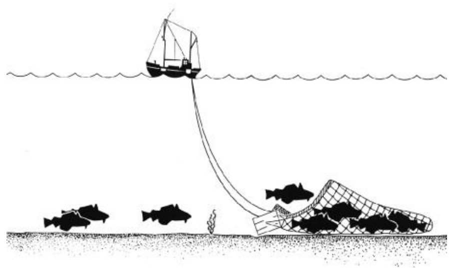
1. Consider the logistic equation that we discussed in the lecture:

$$\frac{dn}{dt} = r_0 n(1 - n/K)$$



It describes the growth of a single population with some density dependence, i.e. there is an upper bound to how large the population can become.

Now let's assume the population is harvested somehow (fishing, hunting, picking, …). The harvest is of a '*constant effort*' type, which means the number of harvested individuals per time unit is proportional to population size. In other words, the number of harvested individuals per time unit can be written *hn*, where *h* is a parameter representing the harvesting effort (the number of hours per season, or the number of fishing vessels, or something like that).



1a) Write a modified growth equation that takes harvesting into account

1b) Find the new equilibrium

1c) At what values of h does the population go extinct?

1d) Confirm that the new equilibrium is stable, if it is positive.

1e) The *yield* per time unit is simply hn. What is the yield at equilibrium?

1f) At what value of h is the yield maximized?

1g) How is the return to equilibrium affected by h?

2. Let's try running some simulations in R

2a) As a first exercise, plot the growth function as dn/dt vs. n (see figure in exercise 1).

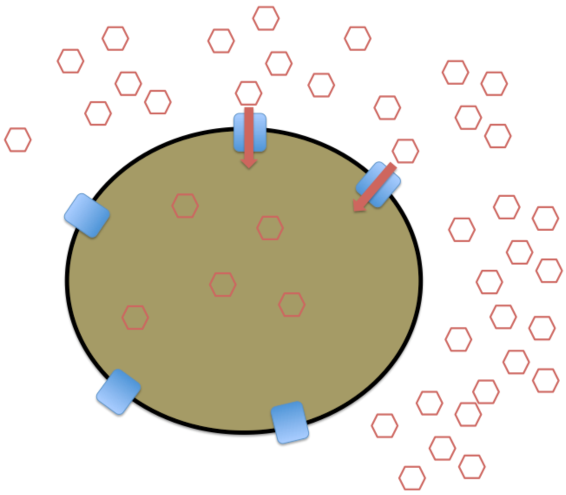
2b) Write an R-script that solves the differential equation from the previous exercise. Use the `ode` function of the `deSolve` package. See the lecture notes for an example. Make your script plot the result of the ode solver.

2c) Confirm, through your simulations, that the equilibrium you calculated in 1b) actually is an equilibrium, and that it is stable. In other words, confirm that a population with higher or lower density will approach this equilibrium over time.

2c) Confirm, again with simulations, that a higher *h* also gives slower return to equilibrium after a disturbance. Can you think of any biological implications of this?

If you have the time:

3. Nutrient uptake of a cell



Cells consume nutrients, such as sugars, but the nutrients first have to pass through the cell membrane from the surrounding fluid. The transport of nutrients through the cell membrane depends on the surrounding nutrient concentration, but also on the number of receptors (transport proteins) in the membrane. It can be shown that the nutrient concentration c in the surrounding fluid follows (approximately)

$$\frac{dc}{dt} = -\frac{K_{max}c}{k_n + c}n$$

where n is the density of cells (per unit volume), $K_{\max}$ is the maximal uptake rate for a single cell and k_n is a half-saturation constant. Both $K_{\max}$ and k_n depend on the number of receptors per cell and their efficiency. The equation above is sometimes called *Michaelis-Menten kinetics* (Edelstein-Keshet, 1988, *Mathematical Models in Biology*).

3a) Can you see that the maximal uptake per cell approaches K_{max} as c goes to infinity?

3b) Confirm that $c = 0$ is an equilibrium, and that it is stable

3b) Write an R script that solves the differential equation above and plots the result

Now assume we have chemostat conditions, i.e. that there is a constant inflow I of nutrients and an outflow μc proportional to the current concentration.

3c) Write the new equation for the nutrient dynamics!

3d) Adjust the R-script according to the new model.

3e) Is there a new equilibrium? Is it stable? (Use either simulations in R or your own calculations for this)