Thermodynamic Point of View for Urban Transportation Network

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Abstract

To explore the nature of the transportation system, this paper shows the connections between the transportation system and the thermodynamic system by regarding the vehicles as the energy. In particular, it is demonstrated that transportation systems can have a similar notion of entropy to measure the system disorder. With these correspondent notions, the principles of thermodynamics are also introduced into the transportation context. Specially, the second law of thermodynamics leads to a dissipativeness law in transportation context which can make the transportation system tend to reduce its disorder and hence become better organized. Furthermore, the approach of Shannon's entropy is also applied to present the concept of the transportation information, which reflects the system contributions to the transportation functionality. With all these efforts, this paper provides a novel thermodynamic point of view in traffic engineering.

Keywords: urban transportation system, thermodynamic system, entropy, dissipativeness theory, Shannon, information

1. Introduction

Urban transportation is an essential part of the modern cities. Due to the rapid increase of traffic demands in recent decades, the congestion problem emerges more frequently, which leads to serious economic and environment

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issues. To solve it, there have been many efforts made to study and understand the phenomenons in the urban transportation system from various approaches.

The most widely used approach is the traffic flow theory (Gartner et al., 2005). This approach describes the behaviors of the transportation system by considering the vehicle movements as flows. Indeed, the general impression of the vehicle movements in modern cities is really similar with the liquid flows. The popular traffic signal control strategies, like TRANSYT (Robertson, 1969; Hale, 2005), SCOOT (Bretherton et al., 1982), TUC (Diakaki et al., 2002), etc, are all apply this approach to model the transportation system. Some concepts of traffic flows are also used in this paper. Beyond the flow analogy, the kinematic wave theory has been also applied to study the relationship between the traffic density and the traffic flow rate, which led the Cell Transmission Model (CTM) (Daganzo, 1995; Flötteröd & Rohde, 2011). This model can accurately describe the behaviors of traffic flows. Hence, the simulation based on CTM is more approximate with the real system than other well-known queue models (Almasri & Friedrich, 2005). This model has been well-applied in the simulation (Su et al., 2013), the observer (Canudas de Wit et al., 2012) and the traffic signal control (Pohlmann & Friedrich, 2010). Another widely used model for the transportation system is the Petri-Net (PN) (Dotoli & Fanti, 2006; Ng et al., 2013), which works well with logic controllers.

However, most of these existing works only focused on the superficial transportation behaviors and tried to adapt some existing models to them. They failed to show the nature of the transportation phenomenons from more fundamental perspectives.

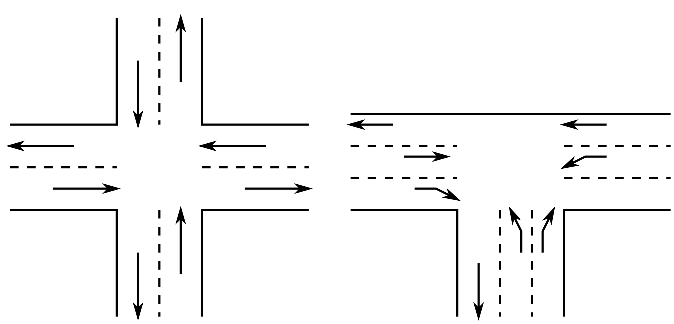
In this context, the objective of this chapter is to explore the connections between the transportation system and more general physical systems in order to show its physical nature. Indeed, the urban transportation system consists of the traffic lanes and the vehicles which can enter the system, leave the system or travel from one lane to another. This whole picture reminds us that the transportation system has many common characters with the large-scale open thermodynamic system, which consists of connected subsystems (matters) and the energy that can be transferred into the system, out of the system or from one subsystem to another. Since all physical systems are thermodynamic systems fundamentally, this chapter tries to introduce thermodynamic notions into transportation context in order to discover the nature of the transportation system and to provide better references for the traffic signal control. In particular, it is demonstrated that the transportation system can have a similar entropy notion to measure its disorder. Furthermore, based on the correspondences of notions, the principles corresponding to the laws of thermodynamics are presented as well in the following development. These efforts construct a novel thermodynamic point of view in the traffic engineering.

2. Basic Concepts and Conservation of Vehicles

2.1. Introduction to Transportation System

The two fundamental elements of the urban transportation system are the traffic streets and the vehicles. Basically, the connected traffic streets provide a network within cities so that the vehicles can travel from their departure points to their destinations. This is the basic image of the transportation functionality.

In details, a street is a linear traffic area connecting two separate points with a fixed length. When more than two streets meet in a single point, there will be a crossing area among these streets. This crossing area and the connected streets compose an intersection (Papageorgiou et al., 2003). For example, two common types of intersections are shown in Figure 1(a) and Figure 1(b), which connects 4 and 3 streets respectively.



(a) Four streets (b) Three streets

Figure 1: Examples of intersections

Based on the directions of vehicles within the street, it is further divided into several traffic lanes. For example, the 4 streets in the intersection illustrated by Figure 1(a) are all split into two lanes corresponding to two opposite directions. On the other hand, in Figure 1(b), the streets all consist of three lanes (Note that in this T-intersection, the lanes are split based on both the current and the potential directions of vehicles). A street may correspond to more than one traffic signals, but a lane can only correspond to one. Hence, the traffic lanes are more appropriate to be considered as the units of the traffic areas instead of the streets. Precisely, for each lane, the appropriate state variable is obviously the number of the vehicles within it, which, in this paper, is called the queue length of the traffic lane.

Now, observe that to accomplish the mission of transportation, a vehicle need enter the transportation network from its origin point, travel from one lane to another until leaving the system at its destination. This is the fundamental functionality of the transportation system. Clearly, the dynamic of the queue lengths is reflected by the movements of vehicles between different lanes or between the lanes and the outside. All these movements can be considered as traffic flows (Gartner et al., 2005).

Consider a transportation network including n traffic lanes, $n > 0$. Figure 2 illustrates all traffic flows related with the lane i, $i \in \{1, \cdots, n\}$. In details, x_i is the queue length of lane i; the flow r_i denotes the vehicles entering the lane i from the outside; the flow $\sigma_{i,j}$ (resp, $\sigma_{j,i}$), $j \neq i$, $j \in \{1, \cdots, n\}$, represents the vehicles travel from the lane j (resp, i) queue to the lane i (resp, j); the flow d_i is the output flow denoting the vehicles which leave the transportation system from the lane i.

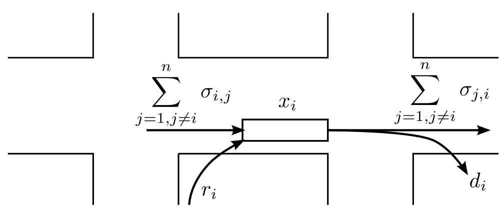


Figure 2: The traffic flows related with the lane i

To describe the dynamic of the transportation system from discrete-time approach, it is easy to know that the change of the queue length x_i during the interval k, $k \in \mathbb{N}$, is given by

$$\Delta x_i(k) = l_i(k) + r_i(k) - d_i(k) \tag{1}$$

where l_i is the sum of all exchange flows related with lane i, which means

$$l_i(k) = \sum_{j=1, j\neq i}^{n} (\sigma_{i,j}(k) - \sigma_{j,i}(k))$$

Equivalently, in vector form, the transportation system can be represented by the following state-space difference equation

$$\boldsymbol{x}(k+1) = \boldsymbol{x}(k) + \boldsymbol{l}(k) + \boldsymbol{r}(k) - \boldsymbol{d}(k), \quad \forall k \in \mathbb{N} \tag{2}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^T$, $\boldsymbol{l} = [l_1, \cdots, l_n]^T$, $\boldsymbol{r} = [r_1, \cdots, r_n]^T$, $\boldsymbol{d} = [d_1, \cdots, d_n]^T$, and $x_i(k)$ is the queue length of lane i at the beginning of the interval k.

It is important to note that without any specific assumption, (2) is a general model, which suits any kind of transportation system in any circumstance. Based on this general model, the sequel will explore the nature of transportation system from thermodynamic point of view.

2.2. Introduction to Thermodynamic System

The fundamental concept for analyzing the large-scale thermodynamic systems is the concept of energy. Indeed, assume that a matter with an unique temperature is called a subsystem, the thermodynamic system consists of a bunch of subsystems, which can store certain quantities of energy and are connected so that the energy can be transferred among them. Apparently, the state variable of a subsystem is the energy quantity stored inside it and the state changes when the energy flows occur between this subsystem and the other ones or the outside. In this paper, the work done by either the system or the environment is not concerned, hence the energy flows have only the form of heat that is driven by the temperature differences.

Based on these understandings, Haddad et al. (2005) presented a discrete-time model for the large scale open thermodynamic system. Consider the thermodynamic system including n subsystems. As shown in Figure 3, the ith subsystem is denoted by Ψ_i, $i \in \{1, \cdots, n\}$, and its stored energy is denoted by E_i. Let $E_i^* > 0$ be the thermal capacity of Ψ_i, then the temperature of Ψ_i is given by

$$T_i = E_i / E_i^* \tag{3}$$

In Figure 3, the energy flow supplied by the outside to ith subsystem is denoted by r_i^e. The flow $\sigma_{i,j}^e$ (resp, $\sigma_{j,i}^e$), $i \neq j$, $i, j \in \{1, \cdots, n\}$, represents the movement of energy from the jth (resp, ith) subsystem to the ith (resp,

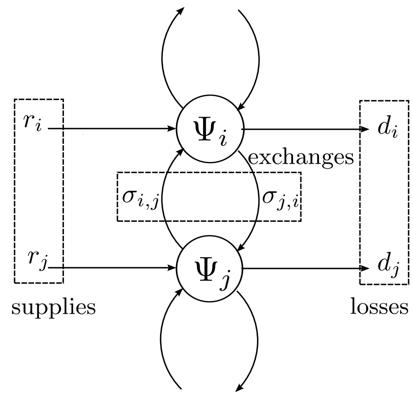


Figure 3: Thermodynamic system

jth) subsystem. The flow d_i^e, $i \in \{1, \cdots, n\}$, represents the energy loss from the ith subsystem. Combining the input, exchange and output energy flows yields the dynamic of the energy E_i stored by Ψ_i

$$\Delta E_i(k) = \sum_{j=1, j\neq i}^{n} (\sigma_{i,j}^e(k) - \sigma_{j,i}^e(k)) + r_i^e(k) - d_i^e(k) \tag{4}$$

which equivalently infers the state-space model of the thermodynamic system

$$\boldsymbol{e}(k+1) = \boldsymbol{e}(k) + \boldsymbol{l}^e(k) + \boldsymbol{r}^e(k) - \boldsymbol{d}^e(k) \tag{5}$$

where $\boldsymbol{e}(k) = [E_1(k), \cdots, E_n(k)]^T$ is the vector of the energy stored by all subsystems at the beginning of kth interval, $\boldsymbol{r}^e = [r_1^e, \cdots, r_n^e]^T$, $\boldsymbol{d}^e = [d_1^e, \cdots, d_n^e]^T$, and $\boldsymbol{l}^e = [l_1^e, \cdots, l_n^e]^T$ represents all exchange flows such that

$$l_i^e = \sum_{j=1, j\neq i}^{n} (\sigma_{i,j}^e - \sigma_{j,i}^e), \ i \in \{1, \cdots, n\}$$

2.3. Correspondences of Basic Concepts

Now, compare the transportation model (2) and the thermodynamic model (5), it is clearly observed that these two systems have many common aspects. In particular, the vehicles perform very similarly as the energy does in the thermodynamic system. Hence, the vehicles can be considered as the transportation "energy". Based on this fundamental correspondence, the thermodynamic concepts can be introduced into the transportation context.

For each traffic lane, it can contain certain number of vehicles. Meanwhile, the vehicles move between the connected lanes or between a lane and the outside. Clearly, a lane is like a subsystem in thermodynamic context which stores energy and exchanges energy with its surroundings. Hence, the traffic lane is the corresponding notion of the thermodynamic subsystem and the queue lengths x_i correspond to the energy stored in subsystems E_i, $i \in \{1, \cdots, n\}$.

The temperature is also an important thermodynamic notion. For finding its correspondence in transportation context, the corresponding notion of the thermal capacity should be firstly found. Since the lengths of traffic lanes are fixed, hence there exists the maximal capacity for each lane to contain vehicles. Let x_i^*, $i \in \{1, \cdots, n\}$, be the capacity of the lane i. It is not difficult to observe that the appropriate corresponding notion of the thermal capacity E_i^* is the lane capacity x_i^*. With this notion, we define the occupancy of a lane as the proportion of the queue length to the capacity, which is given by

$$f_i = \frac{x_i}{x_i^*}, \quad i \in \{1, \cdots, n\} \tag{6}$$

Compared with (3), the occupancies f_i obviously correspond to the temperatures T_i, $i \in \{1, \cdots, n\}$.

Table 1 summarizes the correspondences of the basic concepts between thermodynamic and transportation systems. Now, with these correspondences, the transportation system is analogized to the thermodynamic system, which provides us the opportunity to introduce thermodynamic principles into transportation context as well.

Table 1: Correspondence of basic concepts

Thermodynamic Concepts	Transportation Concepts
energy	vehicle
subsystem	traffic lane
energy stored in subsystems	queue lengths
thermal capacity	lane capacity
temperature	occupancy

2.4. Conservation of Vehicle

The first law of thermodynamics is the conservation of energy (Cengel & Boles, 2001), which means that the energy can not be created or destroyed, it can only change forms or be transferred. Specially, the change of the energy stored in any thermodynamic system equals exactly the amount of the net energy exchange with its surroundings.

Correspondingly, the conservation of vehicle holds in any transportation system, which means that the change of the vehicles within a transportation system equals exactly the amount of the net traffic flows between the system and the outside. To show this principle more clearly, the following theorem is presented and proved based on the general transportation model (2).

Theorem 1. *For the transportation system* (2)*, define*

$$U \triangleq \boldsymbol{\epsilon}^T \boldsymbol{x} \tag{7}$$

as the total number of the vehicles within the transportation network, and define

$$Q \triangleq \boldsymbol{\epsilon}^T (\boldsymbol{r} - \boldsymbol{d}) \tag{8}$$

as the number of the net vehicles moving from the outside into the transportation network, where $\boldsymbol{\epsilon} \triangleq \{1, \cdots, 1\}$*. In any circumstance, the following statement holds* $\forall k \in \mathbb{N}$

$$\Delta U(k) = Q(k) \tag{9}$$

where $\Delta U(k) = U(k+1) - U(k)$*.*

Proof. From (2), we have

$$\begin{aligned} U(k+1) &= \boldsymbol{\epsilon}^T \boldsymbol{x}(k+1) \\ &= \boldsymbol{\epsilon}^T \boldsymbol{x}(k) + \boldsymbol{\epsilon}^T \boldsymbol{l}(k) + \boldsymbol{\epsilon}^T (\boldsymbol{r}(k) - \boldsymbol{d}(k)) \\ &= U(k) + \boldsymbol{\epsilon}^T \boldsymbol{l}(k) + Q(k) \end{aligned}$$

Hence, (9) is equivalent to

$$\boldsymbol{\epsilon}^T \boldsymbol{l}(k) = 0, \quad \forall k \in \mathbb{N}$$

Now, because

$$l_i(k) = \sum_{j=1, j \neq i}^{n} (\sigma_{i,j}(k) - \sigma_{j,i}(k))$$

it infers that

$$
\begin{aligned}
\boldsymbol{\epsilon}^T \boldsymbol{l}(k) &= \sum_{i=1}^{n} l_i(k) \\
&= \sum_{i=1}^{n} \Big(\sum_{j=1, j\neq i}^{n} \sigma_{i,j}(k) - \sum_{j=1, j\neq i}^{n} \sigma_{j,i}(k) \Big) \\
&= \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \sigma_{i,j}(k) - \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \sigma_{j,i}(k) \\
&= \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \sigma_{i,j}(k) - \sum_{j=1}^{n} \sum_{i=1, i\neq j}^{n} \sigma_{j,i}(k) \\
&= 0
\end{aligned}
$$

which completes the proof. □

This theorem indicates that the total vehicles in the transportation system depends on only the traffic flows between the system and the outside, the exchange flows between different lanes can not change the total number of the vehicles within the network.

Furthermore,(9) can infer that

$$
U(k_2) = U(k_1) + \sum_{k=k_1}^{k_2-1} Q(k) \tag{10}
$$

for any $k_1, k_2 \in \mathbb{N}$, $k_2 > k_1$. Hence, according to (Willems, 1972a), the conversation of vehicle also implies that the transportation system (2) is lossless with respect to the storage function (7) and the supply function (8).

3. Transportation Entropy and Dissipativeness

3.1. Second Law of Thermodynamics

In the thermodynamic system, the energy movements obey not just the conservation principle. Beyond that, more importantly, the directions and the quantities of the energy movements must follow some specific rules. For example, without heating equipment, a hot beverage, like a cup of hot tea, will definitely become cooler in a cold room, the energy can not move from cold air to the hot beverage. These phenomenons are consistent with the second law of thermodynamics (Clausius statement, Clausius (1867)):

> Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.

Importantly, this principle is usually connected with a special concept: **entropy**. The second law of thermodynamics can be also described by the way how the entropy of thermodynamic system changes. of In particular, Clausius proposed that the increase of the system entropy due to the input energy Q from the environment is Q/T^a, where T^a is the absolute temperature at the sport where the energy transmission happens (Clausius, 1867). However, for any thermodynamic system, the actual increase of entropy is bigger than or equals to the one supplied by the environment. In other words, any thermodynamic system is creating entropy itself. This phenomenon can be presented by the following inequality

$$\Delta\psi \geq \int \frac{dQ}{T^a} \tag{11}$$

or its discrete-time version (Haddad et al., 2005)

$$\psi(k+1) \geq \psi(k) + \sum_{i=0}^{n} \frac{Q_i(k)}{T_i^a(k+1)}, \forall k \in \mathbb{N} \tag{12}$$

where ψ is the entropy of the thermodynamic system, $Q_i(k)$ is the net energy that ith subsystem absorbs from the outside during the interval k, and $T_i^a(k+1)$ is the absolute temperature of ith subsystem at the end of interval k, $i\{1, \cdots, n\}$. (11) and (12) are also known as Clausius inequality.

The Clausius inequality indicates that in any circumstance, any thermodynamic system is creating entropy so that the increase of its entropy is always beyond the one supplied by its surroundings. The Clausius inequality is consistent with the second law of thermodynamics. Furthermore, since th entropy is opposite to the capacity of the thermodynamic system to do useful work, it is also regarded as the measure of the system disorder (Bal'makov, 2001). Precisely, bigger entropy indicates that the system is worse organized. Hence, the Clausius inequality also implies that the thermodynamic system always tends to become worse organized.

For the discrete-time thermodynamic model (5), based on the Clausius inequality (12), Haddad et al. (2005) presented the formula of its entropy as follows

$$\psi(\boldsymbol{e}) \triangleq \boldsymbol{e}^{*T} \ln(a\boldsymbol{\epsilon} + \boldsymbol{P}_e \boldsymbol{e}) - \boldsymbol{\epsilon}^T \boldsymbol{e}^* \ln a \tag{13}$$

where $\psi(\boldsymbol{e})$ is the entropy, $\boldsymbol{e}^* = [E_1^*, \cdots, E_n^*]^T$, a is a positive scalar which represents the difference between the absolute temperature and the temperature defined in (3) so that $T_i^a = a + T_i$, $i \in \{1, \cdots, n\}$, and $\boldsymbol{P}_e$ is a diagonal matrix with diagonal elements $1/E_i^*$.

Note that this formula is identical with the Boltzmann entropy expression for statistical thermodynamics. Indeed, for any subsystem without phase transition, the entropies in different absolute temperatures ($T1$, $T2$) satisfy the following relationship (Cengel & Boles, 2001)

$$\psi_i(T1) - \psi_i(T2) = e_i^* \ln \frac{T1}{T2}, \quad \forall i \in \{1, \cdots, n\} \tag{14}$$

It is known that the absolute temperature of ith subsystem equals $a + \frac{e_i}{e_i^*}$. Specially, when the subsystem stores no energy ($e_i = 0$), its absolute temperature becomes a. Hence, (14) infers that

$$\begin{aligned}\psi_i(e_i) &= \psi_i(0) + e_i^* \ln \frac{a + \frac{e_i}{e_i^*}}{a} \\ &= \psi_i(0) + e_i^* \ln(a + \frac{e_i}{e_i^*}) - e_i^* \ln a\end{aligned} \tag{15}$$

Now, according to the third law of thermodynamics, the entropy approaches zero when the stored energy approaches zero, which means

$$\psi_i(0) = \lim_{e_i \to 0} \psi_i(e_i) = 0 \tag{16}$$

Therefore, we have

$$\psi_i(e_i) = e_i^* \ln(a + \frac{e_i}{e_i^*}) - e_i^* \ln a \tag{17}$$

By summarizing all subsystems, the entropy of the whole system is given by

$$\psi(\boldsymbol{e}) = \sum_{i=0}^{n} e_i^* \ln(a + \frac{e_i}{e_i^*}) - e_i^* \ln a \tag{18}$$

which can infer the Haddad's formula of entropy (13).

Furthermore, since the entropy (13) is not easy to apply in control issues, Haddad et al. (2005) also introduced a dual notion to entropy, called ectropy, to measure the system order. Correspondingly, the dual inequality to (12) is given by

$$\phi(k+1) - \phi(k) \leq \sum_{i=0}^{n} Q_i(k)T_i(k+1), \forall k \in \mathbb{N} \tag{19}$$

where ϕ is the ectropy of the thermodynamic system, $Q_i(k)T_i(k+1)$ is the input ectropy of ith subsystem supplied by the environment with the net input energy $Q_i(k)$, and $T_i(k+1)$ is the temperature of ith subsystem at the end of interval k. This inequality is also called anti-Clausius inequality, which, like Clausius inequality, is consistent with the second law of thermodynamics.

Anti-Clausius inequality indicates that in any circumstance, any thermodynamic system is destroying its ectropy so that the increase of the ectropy is always beneath the one supplied from its surroundings. Opposite with the entropy, the ectropy measures the capacity of the system to do useful work, and bigger ectropy corresponds to better organization of the system.

For the discrete-time thermodynamic model (5), based on the anti-Clausius inequality (19), Haddad et al. (2005) presented the formula of the ectropy as follows

$$\phi(\boldsymbol{e}) \triangleq \frac{1}{2}\boldsymbol{e}^T \boldsymbol{P}_e \boldsymbol{e} \tag{20}$$

Apparently, the entropy (13) and the ectropy (20) are both the function of the energy distribution within the thermodynamic system. Different with the energy, the entropy and the ectropy are both non-conservative notions. Specially in the isolated system, the sum of energy always keeps constant, but the entropy tends to increase while the ectropy tends to decrease. It is important to note that the ectropy (20) has the form of Lyapunov candidate function, which is more useful in control issues than the entropy.

3.2. Transportation Entropy

The above section has analogized the transportation system to the thermodynamic system, which encourages us to introduce entropy concept into transportation context to be the evaluation of the system performance.

The problem is that there is no natural principle in transportation context that corresponds to the second law of thermodynamics. As a matter of fact, the directions of vehicles are determined by the drivers, there is no such rule that the vehicles only move from more crowded lanes to less crowded ones. Hence, we can not introduce transportation entropy based on certain principles as it is defined in the thermodynamic context. Nevertheless, based on the correspondences of certain concepts, such as energy and temperature, we can still simply introduce the formulas in the thermodynamic system to present the transportation entropy as the measure of the system disorder.

To achieve this idea, the significations of orders in these two systems are firstly compared, and we find that these two significations are opposite. Indeed, in thermodynamic context, higher order means higher capacity to do useful work, which is related with bigger temperature differences between subsystems. For example, in a thermodynamic system including two subsystems, if more energy concentrates on one single subsystem to generate bigger temperature difference, the quantity of the potential energy movements is bigger, which means that the system can generate more useful work. In this case, the system is regarded as better organized and has higher order. But on the converse, the bigger differences between the occupancies correspond to lower order in transportation context. Indeed, for a general transportation network, if more vehicles concentrate in only a few lanes, it has bigger probability to generate congestions in these lanes and the areas in other lanes are wasted. In this case, the system is regarded as worse organized and has lower order.

Moreover, the energy input to the thermodynamic system brings more capacity to do useful work, which increases the order. However, in transportation context, the vehicle input brings more potential opportunity to generate congestions, which decrease the order. For an isolated thermodynamic system without energy input, the disorder (entropy) can only increase until it reaches equilibrium. On the converse, if, at a certain moment, the transportation system has no vehicle input, the system will keep dissipating present vehicles and consequently become better organized.

In conclusion, the order significations of these two systems are opposite, which means the disorder in the transportation system corresponds to the order in the thermodynamic system. In other words, the transportation entropy (resp, ectropy) should correspond to the thermodynamic ectropy (resp, entropy). Therefore, according to the thermodynamic ectropy and entropy formulas (20), (13), the **transportation entropy and ectropy** are defined as

$$\psi(\boldsymbol{x}) \triangleq \frac{1}{2}\boldsymbol{x}^T\boldsymbol{P}\boldsymbol{x} \tag{21}$$

$$\phi(\boldsymbol{x}) \triangleq \boldsymbol{x}^{*T}\ln(a\boldsymbol{\epsilon} + \boldsymbol{P}\boldsymbol{x}) - \boldsymbol{\epsilon}^T\boldsymbol{x}^*\ln a \tag{22}$$

where $\psi(\boldsymbol{x})$ is transportation entropy, $\phi(\boldsymbol{x})$ is transportation ectropy, a is a positive scalar, $\boldsymbol{P}$ is the diagonal matrix with the diagonal elements $1/x_i^*$, and $\boldsymbol{x}^* = [x_1^*, \cdots, x_n^*]^T$. Since the Haddad's formula of entropy is identical with the Boltzmann's one, the definitions of the transportation entropy and ectropy can also be considered as consistent with Boltzmann's approach.

It is important to note that the entropy (21) is a Lyapunov candidate function, which is much more useful in control issues than the ectropy (22). Hence, it will be the major emphasis of the sequel.

Now, this transportation entropy is supposed to measure the disorder of the transportation network. To show its signification more precisely, we consider the transportation network as a service provider and consider the vehicles traveling within it as its customers. Hence, the order of transportation network should be the sum of the qualities of the services provided to all vehicles, and the disorder is, of course, the opposite. Clearly, the services that the vehicles obtain from the urban transportation system are the traffic routes, and the qualities are evaluated by how fast the vehicles can pass these routes. Figure 4 shows the flow-density diagram based on kinetic wave model, where ρ_m is the jam density, ν is the speed and ν_m is the free-flow speed (Ukkusuri et al., 2010). It is observed that the traffic flow and the speed can be considered as functions of the traffic density. In details, the bigger traffic density generates the smaller speed and hence prolongs the delay. Indeed, despite the habits of drivers, any vehicle need spend more time to pass more crowded lanes. This indicates that the traffic densities are opposite to the qualities of services provided by the transportation network and they are therefore correspond to the system disorder.

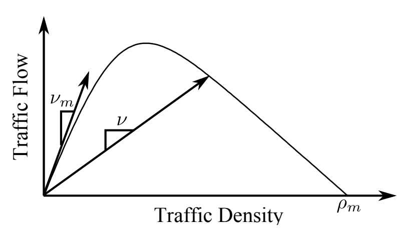


Figure 4: Density-flow chart

Now, let $I(k)$ be the sum of the traffic densities that all vehicles meet at the instant k, we have

$$I(k) = \sum_{j=1}^{N(k)} \rho_{(j)}(k) \tag{23}$$

where $N(k) \in \mathbb{N}$ is the number of vehicles inside the transportation network at the instant k, $\rho_{(j)}(k)$ is the traffic density that the vehicle j meets at time point k, $j \in \{1, \cdots, N(k)\}$.

In order to show the relationship between $I(k)$ and the entropy (21), we assume the following statement always holds.

Assumption 1. *In each lane, the vehicles are uniformly distributed.*

This assumption is reasonable because the lanes are usually the minimal measurable units in transportation network. Indeed, there have been many methods (e.g., inductive loop detectors) to measure the number of vehicles inside a traffic lane, but it is still hard to obtain the specific vehicle distribution within a lane. Hence, due to this lack of measurement, the simple solution is to assume the uniform distribution of vehicles within a lane.

Assumption 1 implies that for the vehicles in the same lane, the traffic densities are the same. Let $\rho_i(k)$ denote the traffic density of the lane i at time point k, $i \in \{1, \cdots, n\}$. Hence, (23) becomes

$$I(k) = \sum_{i=1}^{n} x_i(k)\rho_i(k)$$

Furthermore, since $\rho_i(k) = x_i(k)/h_i$ where h_i is the length of the lane i, $i \in \{1, \cdots, n\}$, the sum of the traffic densities of all vehicles is finally restated as

$$I(k) = \sum_{i=1}^{n} \frac{x_i(k)^2}{h_i} = \boldsymbol{x}(k)^T \boldsymbol{P}_h \boldsymbol{x}(k) \tag{24}$$

where $\boldsymbol{P}_h$ is the diagonal matrix with the diagonal elements $1/h_i$.

For each lane, its maximal capacity to contain the vehicles is proportional to its length, that means

$$h_i \propto x_i^* \tag{25}$$

Hence, compare the entropy (21) and the equation (24), we have the following conclusion

$$I(k) \propto \psi(\boldsymbol{x}(k)) \tag{26}$$

which means that the transportation entropy is proportional to the sum of the traffic densities of all vehicles. Consequently, it is implied that the transportation entropy (21) measures the disorder of the transportation network, which is an appropriate evaluation of the system performance. It is also important to note that the entropy (21) is a Lyapunov candidate function, which is very useful in control issues.

3.3. Dissipativity of Entropy

Though the second law of thermodynamics can not be applied in transportation context naturally, we still wonder whether it can be achieved in certain conditions. The major motive is that the transportation disorder corresponds to the thermodynamic order. If the transportation system has similar principle with the second law of thermodynamics, it will have the tendency to decrease its disorder and become better organized. More precisely, if the direction of the net traffic flow between any two transportation areas is from the more crowded one to the less crowded one as the energy does in the thermodynamic system, all congestions within the transportation network will tend to be evacuated.

Now, consider the anti-Clausius inequality (19) and its meaning in thermodynamic context, this idea is clearly to achieve the phenomenon that the increase of the transportation entropy is always beneath the one supplied by the outside. In details, the corresponding version of anti-Clausius inequality should be presented and verified.

As described above, in transportation context, the input vehicles bring disorder to the system. This supplied disorder depends on not only the number of the input vehicles, but also the distribution of them. The vehicles which enter the more crowded lanes bring more disorder than the ones with same quantity which enter the less crowded lanes. Since the occupancies represent how the lanes are crowed, it is natural to measure the supplied entropy with respect to the input flows and the occupancy factors. Let $\boldsymbol{f} = [f_1, \cdots, f_n]^T$ be the vector of all occupancies. Note that because $\boldsymbol{f}(k)$ corresponds to the beginning of the interval k while $\boldsymbol{f}(k+1)$ corresponds to the end of it, the supplied entropy in the interval k should correspond to $\boldsymbol{f}(k+1)$ rather than $\boldsymbol{f}(k)$. So, we define

$$S(k) = \boldsymbol{f}(k+1)^T \boldsymbol{r}(k)$$

as the supplied entropy in the interval k. Furthermore, since $\boldsymbol{f} = \boldsymbol{P}_x \boldsymbol{x}$ in view of (6), the supplied entropy is restated as

$$S(k) = \boldsymbol{x}(k+1)^T \boldsymbol{P}\boldsymbol{r}(k) \tag{27}$$

Now, similar with the anti-Clausius inequality (19), we propose its corresponding version in transportation context as follows

$$\psi(\boldsymbol{x}(k+1)) - \psi(\boldsymbol{x}(k)) \leq S(k), \quad \forall k \in \mathbb{N} \tag{28}$$

If this inequality is verified, the transportation system has the tendency to decrease its disorder and become better organized.

Furthermore, according to the Willems (1972b) and Hill & Moylan (1980), (28) also implies the dissipativeness with the entropy (21) as the storage function and with (27) as the supply function. In this case, (28) is called dissipation inequality. Figure 5 illustrates this dissipativeness of system disorder. Indeed, the input flows bring disorder to the transportation system to make it worse organized. At the same time, the appropriate traffic signal control dissipates the traffic so that the system stores only a part of this input disorder.

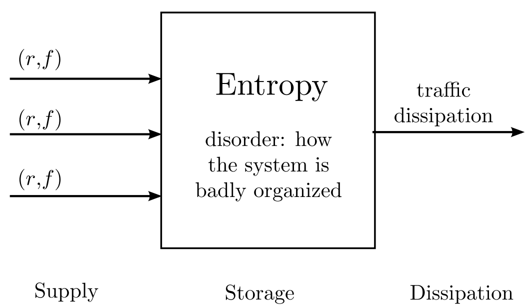


Figure 5: Dissipativity of transportation entropy

This dissipativeness corresponds to the second law of thermodynamics. According to the Clausius statement, it is implied that if this dissipativeness is verified, the direction of the net traffic exchange between two linked traffic areas is always from the more occupied one to the less occupied one. In other words, in any spot of the transportation network, the system tends to evacuate the vehicles instead of assembling them, which leads to the prevention of congestions.

This dissipativeness does not exist naturally. But, it is known that the traffic signals have the ability to control the traffic flows. Therefore, it is possible that under certain control schemes, the traffic signals can verify the dissipation inequality (28). This inequality can be considered as one of objectives for traffic signal control.

4. Nominal Situation and Equilibrium

In the researches about thermodynamic systems, the closed system, which has no exchange with the environment, is one of the most studied subjects. Indeed, the closed thermodynamic system has special features. For example, in any closed thermodynamic system, the ectropy (resp, entropy) keeps decreasing (resp, increasing) until reaching the equilibrium state when all connected subsystems have the same temperature. The equilibrium also corresponds to the minimal possible ectropy and maximal possible entropy. Since the ectropy is a Lyapunov candidate function, the closed thermodynamic system is Lyapunov stable (Haddad et al., 2005).

Unfortunately, the transportation system is always open. In fact, the openness is required by the functionality of the transportation. There is no such thing as closed transportation system. However, there exists nominal situation (Diakaki et al., 2002) in the transportation network which has some common features with the closed thermodynamic system. The nominal situation means that with certain traffic demands and certain traffic signal settings, the input vehicles equals the output vehicles in any cycle for each traffic lane, in other words, the queue lengths of all traffic lanes keep constant from cycle to cycle. In this paper, these certain traffic demands are called nominal inputs from the outside, denoted by

$$\boldsymbol{r}^N = \{r_1^N, \cdots, r_n^N\}$$

Obviously, in the nominal situation, the net exchange between the transportation system and the outside equals zero in each cycle, that means

$$Q(k) \equiv 0, \quad \forall k \in \mathbb{N} \tag{29}$$

Since the transportation system is lossless with the storage function U and the supply function Q, (29) infers that the total amount of traffic U within the whole system keeps constant in the nominal situation. This is very similar with the closed thermodynamic system which has zero exchange of energy with the environment. Hence, we propose that the nominal situation of the transportation system can be analogized to the closed status of the thermodynamic system.

Note that de Oliveira & Camponogara (2010) have presented some effective procedures to estimate the nominal parameters for the general transportation system, hence it is reasonable to assume the availability of the nominal situation in general.

Now, we define

$$\boldsymbol{\omega}(k) \triangleq \boldsymbol{r}(k) - \boldsymbol{r}^N, \quad \forall k \in \mathbb{N} \tag{30}$$

as the vector of the disturbances of the traffic demands. Since the nominal transportation system corresponds to the closed thermodynamic system, the disturbances $\boldsymbol{\omega}$ are more appropriate than the input traffic flows $\boldsymbol{r}$ to correspond to the input energy in the thermodynamic system.

Hence, let

$$S_\omega(k) = \boldsymbol{x}(k+1)^T \boldsymbol{P}\boldsymbol{\omega}(k) \tag{31}$$

be the new supply function with respect to $\boldsymbol{\omega}$. Consequently, the dissipativeness presented in previous section can be modified to the one with the entropy (21) as the storage function and with (31) as the supply function. Note that this new dissipativeness is the sufficient condition of the previous one, which is stated in the following theorem.

Theorem 2. *If the transportation system described by the difference equation* (2) *is dissipative with respect to the storage function* (21) *and the supply function* (31)*, then the system is also dissipative with respect to the same storage function and the supply function* (27)*.*

Proof. The dissipation inequality related with the storage function (21) and the supply function (31) is

$$\psi(\boldsymbol{x}(k+1)) - \psi(\boldsymbol{x}(k)) \leq S_\omega(k) \tag{32}$$

Since $\boldsymbol{r}(k) = \boldsymbol{q}(k)c$ and $\boldsymbol{\omega}(k) = (\boldsymbol{q}(k) - \boldsymbol{q}^N)c$, it follows $\boldsymbol{\omega}(k) = \boldsymbol{r}(k) - \boldsymbol{r}^N$. Hence,

$$\begin{aligned} S_\omega(k) &= \boldsymbol{x}(k+1)^T \boldsymbol{P}(\boldsymbol{r}(k) - \boldsymbol{r}^N(k)) \\ &= \boldsymbol{x}(k+1)^T \boldsymbol{P}\boldsymbol{r}(k) - \boldsymbol{x}(k+1)^T \boldsymbol{P}\boldsymbol{r}^N(k) \end{aligned}$$

Since the nominal inputs and all queue lengths are non-negative, this infers

$$S_\omega(k) \leq \boldsymbol{x}(k+1)^T \boldsymbol{P}\boldsymbol{r}(k) = S(k)$$

So, the conclusion follows. □

Now, the second law of thermodynamics corresponds to this new dissipativeness.

Finally, we close this section by introducing the concept of thermal equilibrium into the transportation context. For a closed thermodynamic system, if a pair of connected subsystems have different temperatures, the energy transmission will emerge between them so that their temperatures tend to approximate. After enough time, all connected subsystems will have the same temperature so that no energy movement can emerge any more, in other words, the system loses all its capacity to do useful work in this moment. This particular state is called thermal equilibrium state (Cengel & Boles, 2001), which also corresponds to the maximal entropy and minimal ectropy in the closed thermodynamic system.

Correspondingly, we present the concept of transportation equilibrium to denote the state when the occupancies of all traffic lanes are the same, which implies that the vehicles are uniformly distributed within the whole transportation area in the transportation equilibrium. Mathematically, the transportation equilibrium means

$$\frac{x_1}{x_1^*} = \cdots = \frac{x_n}{x_n^*} = \alpha \tag{33}$$

where

$$\alpha = \frac{\boldsymbol{\epsilon}^T \boldsymbol{x}}{\boldsymbol{\epsilon}^T \boldsymbol{x}^*} \in [0, 1]$$

The relationship between the transportation equilibrium and the transportation entropy is stated in the following theorem.

Theorem 3. *Assume that the total amount of vehicles N is given, the transportation entropy* (21) *is minimized when the system reaches the equilibrium state* (33).

Proof. To minimize the entropy, consider the problem

$$\begin{aligned} \min \psi(\boldsymbol{x}) &= \frac{1}{2}\boldsymbol{x}^T \boldsymbol{P} \boldsymbol{x} \\ s.t. \sum_{i=1}^{n} x_i &= N \end{aligned} \tag{34}$$

It is easy to know that $\psi(\boldsymbol{x})$ is a convex function. By applying the method of Lagrange multipliers to solve the problem (34), we have the equivalent unconstrained problem:

$$\min J(\boldsymbol{x}, \beta) = \frac{1}{2}\boldsymbol{x}^T \boldsymbol{P}\boldsymbol{x} + \beta(\sum_{i=1}^{n} x_i - N)$$

whose solution can be obtained by solving the equations

$$\frac{\partial J}{\partial x_i} = \frac{x_i}{x_i^*} + \beta = 0, i \in \{1, \cdots, n\} \tag{35}$$

$$\frac{\partial J}{\partial \beta} = \sum_{i=1}^{n} x_i - N = 0 \tag{36}$$

Now, (35) can easily infer that

$$\frac{x_1}{x_1^*} = \cdots = \frac{x_n}{x_n^*} = -\beta$$

Apparently, β can only equal to $-\alpha$, which completes the proof. □

Transportation equilibrium also implies that no particular concentration of vehicles exists in the system and hence the possibility to emerge congestion is extremely low. The transportation equilibrium is actually the ideal state for the transportation system and the ideal objective shared by all dynamic traffic signal control strategies.

5. Approach of Shannon's Entropy

The above studies analogized the transportation system to thermodynamic system by regarding the vehicles as the energy. Beside this approach, it is noticed that the researches of thermodynamics have been expanded to the domains other than pure physical systems. In particular, Shannon has connected the entropy with the concept of information (Shannon, 1948). This work encourages us to explore the nature of the transportation system from the information perspective, which is the emphasis of this section.

5.1. Information of Transportation System

As a very important expansion of the statistical thermodynamics, Shannon's entropy measures the information gained by measuring certain random variable (Shannon, 1948). In particular, for a random variable Y with possible states $\{y_i\}$, if the probability distribution of Y over its possible states is $\{p_i\}$, then the information gained by measuring Y can be given by the relative Shannon's entropy as follows

$$H(Y) = -\sum p_i \log_2 p_i \tag{37}$$

Note that by using 2 as the base of logarithm, the unit of the information (37) is bit.

In transportation context, the most important information is the occupancy status of the traffic areas, which can infer the degree of crowdedness. Since the Shannon's entropy measures the information gained by necessary measurement, it means the Shannon's entropy represents the unknown information or in other words the unpredictability of the information. In the transportation system, if the vehicles move more fast, the uncertainty of the occupancy of certain traffic areas will be bigger, which makes more difficult to predict the crowdedness of the system. Therefore, if the transportation system functions more smooth, the information measured by the Shannon's entropy will be greater.

Now, to measure this information, consider a single spot of lane i firstly, $\forall i \in \{1, \cdots, n\}$. Suppose that the queue length x_i and the lane capacity x_i^* are both known but there is no direct measurement on this specific spot. It can be inferred that the possibility that this spot is occupied is $\frac{x_i}{x_i^*}$, and the possibility of the otherwise case is $(1 - \frac{x_i}{x_i^*})$. Hence, the information gained by measuring this spot is

$$-[\frac{x_i}{x_i^*} \log_2 \frac{x_i}{x_i^*} + (1 - \frac{x_i}{x_i^*}) \log_2 (1 - \frac{x_i}{x_i^*})]$$

For the whole lane i, the information of all spots' occupancies can be given by

$$H_i(x_i) = -h_i[\frac{x_i}{x_i^*} \log_2 \frac{x_i}{x_i^*} + (1 - \frac{x_i}{x_i^*}) \log_2 (1 - \frac{x_i}{x_i^*})] \tag{38}$$

where h_i is the length of lane i. Since h_i is proportional to the capacity x_i^*, (38) can be simplified to

$$\begin{aligned} H_i(x_i) &= -x_i^*[\frac{x_i}{x_i^*} \log_2 \frac{x_i}{x_i^*} + (1 - \frac{x_i}{x_i^*}) \log_2 (1 - \frac{x_i}{x_i^*})] \\ &= -[x_i \log_2 \frac{x_i}{x_i^*} + (x_i^* - x_i) \log_2 (1 - \frac{x_i}{x_i^*})] \end{aligned} \tag{39}$$

By summarizing all lanes, we have the information of the entire system as follows

$$
\begin{aligned}
H(\boldsymbol{x}) &= \sum_{i=1}^{n} H_i(x_i) \\
&= -\sum_{i=1}^{n} [x_i \log_2 \frac{x_i}{x_i^*} + (x_i^* - x_i) \log_2 (1 - \frac{x_i}{x_i^*})]
\end{aligned} \tag{40}
$$

Furthermore, (40) can be restated as the vector expression:

$$
H(\boldsymbol{x}) = -\boldsymbol{x}^T \log_2 \boldsymbol{P}\boldsymbol{x} - (\boldsymbol{x}^* - \boldsymbol{x})^T \log_2 (\boldsymbol{\epsilon} - \boldsymbol{P}\boldsymbol{x}) \tag{41}
$$

We define this expression as the **transportation information**. It represents the uncertainty or unpredictability of the system crowdedness by knowing only the queue lengths and the lane capacities.

5.2. Signification

The transportation information (41) comes from the consideration of the occupancies of traffic areas. Precisely, it measures the uncertainty of the occupancies. If the occupancy of a traffic spot is more uncertain, it is more difficult to predict whether there is a vehicle in this spot without direct observation, which means this spot has higher liquidity. In other words, the greater information (41) implies the higher liquidity of the transportation system. It is clear that the liquidity corresponds to the transportation functionality. Hence, the transportation information is related with the transportation functionality of the system. The sequel will discuss this relationship in details.

Firstly, consider a single lane i, $i \in \{1, \cdots, n\}$, its information H_i due to all possible queue length x_i is illustrated in Figure 6. Observe that when the lane is empty ($x_i = 0$), the information equals zero ($H_i = 0$). This is because any spot within this lane is certainly not occupied by a vehicle. This lane is extremely clean in this case. However, since no vehicle travels inside, no transportation behavior appears in this lane. In other words, the system contributes nothing to the transportation functionality with zero usage.

On the other hand, when the lane is totally occupied ($x_i = x_i^*$), since the occupancy of any spot is also certain in this case, the information H_i also equals zero. The usage of the system is extremely high in this case. But,

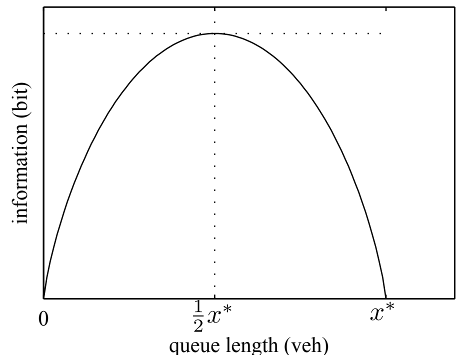


Figure 6: Information vs queue length for a lane

because there is no space to move, the transportation behavior can not happen either. Hence, with complete crowdedness, the system also contributes nothing to the transportation functionality.

Now, we can summarize that the urban transportation functions only under these two conditions:

C1 there are vehicles;

C2 there are free spaces to allow the vehicles to move.

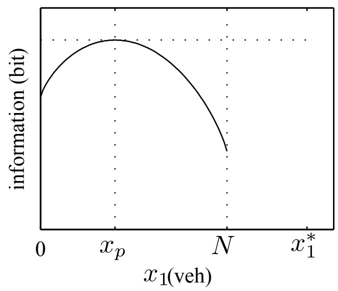
Therefore, only when $0 < x_i < x_i^*$, these two conditions are both verified so that this lane can contribute to the transportation functionality.

Observe that only when $0 < x_i < x_i^*$, the information H_i becomes positive because the occupancies are uncertain in this case. Specially, H_i maximizes when the lane is half-occupied ($x_i = x_i^*/2$). For a lane, its information H_i reflects the balance between the low usage and the high crowdedness. Hence, it can be inferred that for a lane, its information happens to correspond to its contribution to the transportation functionality.

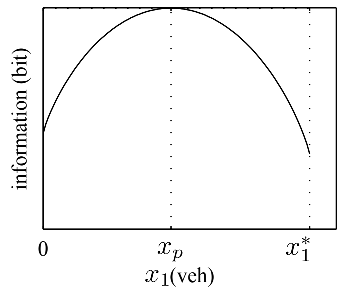
Then, we consider the relationship between the transportation information and the distribution of vehicles over different lanes. A simple example is a system including two lanes. Define x_1, x_2 are the queue lengths and x_1^*, x_2^* are the capacities. Suppose $x_1^* < x_2^*$ and the sum $N = x_1 + x_2$ is fixed. Figure 7 shows the information due to x_1 in three cases: $N \leq x_1^* \leq x_2^*$, $x_1^* \leq N \leq x_2^*$ and $x_1^* \leq x_2^* \leq N$. The information maximizes in certain point x_p, which is given by in all three cases

$$x_p = \frac{N}{x_1^* + x_2^*} x_1^*$$

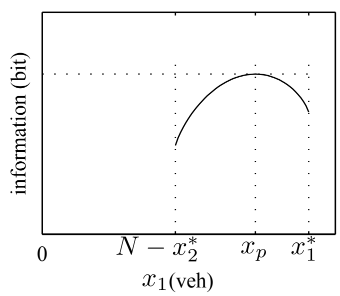
Clearly, the information reflects the balance between these two lanes.



(a) $N \leq x_1^* \leq x_2^*$



(b) $x_1^* \leq N \leq x_2^*$



(c) $x_1^* \leq x_2^* \leq N$

Figure 7: Information of two lanes with fixed total vehicles

Furthermore, the following theorem indicates that for any large-scale transportation network, the information maximizes when the vehicles are distributed uniformly.

Theorem 4. *For any transportation network including n lanes, assume that the total amount of vehicles N is given, the transportation information (41) is maximized when the vehicles are distributed uniformly.*

Proof. To maximize the information, consider the problem

$$\max H(\boldsymbol{x}) = -\boldsymbol{x}^T \log_2 \boldsymbol{P}\boldsymbol{x} - (\boldsymbol{x}^* - \boldsymbol{x})^T \log_2(\boldsymbol{\epsilon} - \boldsymbol{P}\boldsymbol{x}) \tag{42}$$
$$s.t. \sum_{i=1}^{n} x_i = N$$

Since

$$\frac{\partial^2 H}{\partial x_i \partial x_j} = 0,\ \forall i, j \in \{1, \cdots, n\}, i \neq j$$
$$\frac{\partial^2 H}{\partial x_i^2} = -\frac{1}{\ln 2}\left(\frac{1}{x_i} + \frac{1}{x_i^* - x_i}\right) < 0,\ \forall i \in \{1, \cdots, n\}$$

we know that $H(\boldsymbol{x})$ is a concave function. By applying the method of Lagrange multipliers to solve the problem (42), we have the equivalent unconstrained problem:

$$\max J(\boldsymbol{x}, \beta) = -\boldsymbol{x}^T \log_2 \boldsymbol{P}\boldsymbol{x} - (\boldsymbol{x}^* - \boldsymbol{x})^T \log_2(\boldsymbol{\epsilon} - \boldsymbol{P}\boldsymbol{x}) + \beta\left(\sum_{i=1}^{n} x_i - N\right)$$

whose solution can be obtained by solving the equations

$$\frac{\partial J}{\partial x_i} = -\log_2 \frac{x_i}{x_i^*} + \log_2\left(1 - \frac{x_i}{x_i^*}\right) + \beta = 0, \forall i \in \{1, \cdots, n\} \tag{43}$$
$$\frac{\partial J}{\partial \beta} = \sum_{i=1}^{n} x_i - N = 0 \tag{44}$$

Now, (43) can easily infer that

$$\frac{x_1}{x_1^*} = \cdots = \frac{x_n}{x_n^*} = \frac{1}{1 + 2^{-\beta}}$$

Apparently, this implies the uniform distribution of vehicles, which completes the proof. □

The uniform distribution of vehicles implies that all traffic areas are equally used, there is no waste and no overuse. This state leads to the highest overall liquidity of the transportation system.

Now, by combining the analysis of single lane and network, we can see that the transportation information reflects the balance between low usage and high crowdedness and the balance between different traffic areas. Hence, it can be further concluded that the transportation information (41) measures the contribution of the transportation system to the transportation functionality.

5.3. Comparison Between Entropy and Information

From the approaches of classical thermodynamics and expanded Shannon information, this paper has presented the entropy (21) and the information (41) for the transportation system. We close this section by comparing these two novel concepts.

From different bases, the significations of these two concepts are very different. By considering the vehicles as the energy, the transportation entropy is presented based on the parallelism between the transportation system and the thermodynamic system. It is the sum of traffic densities of all vehicles, which is opposite to the service quality supplied by the system to the vehicles. Obviously, the entropy focuses on the performances of the vehicles and it is desired to be reduced.

But on the other hand, the transportation information measures the system contribution to the transportation functionality. It comes from the consideration of the occupancy states of all traffic areas. Different from the entropy, the transportation information focuses on the performance of the transportation system and it is desired to be increased.

This difference appears in their relationships with the queue lengths. The entropy (21) is a monotonic function of queue lengths. It is obvious that every vehicle hopes to share the traffic resources with as less vehicles as possible. But, the transportation information (41) approaches zero when the number of vehicles approaches zero because transportation functionality needs the vehicles and too few of them will lead to big waste of traffic infrastructures. More precisely, the considerations based on the entropy desire that the vehicles are reduced as many as possible, while the maximization of the information desires that the lanes are half-occupied.

Apparently, the transportation entropy is more appropriate to be the objective of traffic signal control. The aim of the traffic signals focuses more on the vehicles rather than the transportation functionality of the whole system. On the other hand, the transportation information is a good evaluation of the transportation system itself. It reflects the balance between the vehicles (users) and the traffic areas (resources). In our opinion, the information (41) is also more useful in the domains, such as the transportation infrastructure design, the urban planning, etc.

At last, we note that these two notions have comme features to consider the distribution law of vehicles. No matter to minimize the entropy or to maximize the information, the vehicles are both desired to be distributed uniformly, which implies that the uniform distribution of vehicles is beneficial to both the vehicles and the transportation system.

6. Conclusion

This paper has presented a novel thermodynamic approach for studying the transportation system. The comparison between the thermodynamic system and the transportation system showed that they are similar by regarding the vehicles as the energy and regarding the traffic lanes as the thermodynamic subsystems. With this similarity, the transportation system has been analogized to the thermodynamic system. In particular, it has been demonstrated that the transportation system can have a similar notion of the entropy to measure the system disorder. With this notion, this paper presented the dissipativeness of the transportation system to correspond to the second law of thermodynamics. This dissipativeness makes the system tend to decrease the disorder and hence become better organized. Though it is not verified naturally, it can be used as the objective and the evaluation of the traffic signal control. Furthermore, this paper proposed that the nominal situation is appropriate to correspond to the closed status of the thermodynamic system. The equilibrium state was also introduced into the transportation context to represent the uniform distribution of the vehicles. Beyond the classical thermodynamic approach, this paper also applied the Shannon's point of view to present the transportation information, which measures the system contribution to the transportation functionality.

With all these efforts, this paper has constructed a novel thermodynamic point of view in traffic engineering. This work is a completely new attempt to discover the nature of the transportation system. Based on the contributions of this paper, other thermodynamic results can be further applied into the transportation context.

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