HW#2 2018-17738 진동훈

3.3

$$x(t) = 2+\cos\left(\tfrac{2}{3}\pi t\right)+4\sin\left(\tfrac{5}{3}\pi t\right)$$

$$= 2+\tfrac{1}{2}e^{j\left(\frac{2}{3}\pi\right)t}+\tfrac{1}{2}e^{-j\left(\frac{2}{3}\pi\right)t}-2je^{j\left(\frac{5}{3}\pi\right)t}+2je^{-j\left(\frac{5}{3}\pi\right)t}$$

GCD of $\left(\frac{2}{3}\pi, \frac{5}{3}\pi\right) = \frac{1}{3}\pi$

$$\boxed{\therefore\ a_0=2,\ a_2=a_{-2}=\tfrac{1}{2},\ a_5=a^*_{-5}=-2j}$$

3.15

$$H(j\omega)=\begin{cases}1, & |\omega|\le 100\\ 0, & |\omega|>100\end{cases}$$

S is lowpass filter

$x(t) \xrightarrow{S} y(t) = x(t)$, $T=\frac{\pi}{6}$ for $x(t)$

$\therefore \omega_0 = \frac{2\pi}{T} = 12$

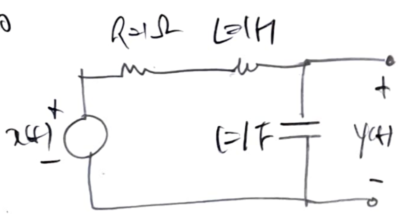
$a_k \neq 0$ for $|k|\omega_0 \le 100$

$|k| \le \frac{100}{12}$, $(k\in\mathbb{Z})$

$\therefore |k| \le 8$

$$\boxed{\therefore\ a_k \text{ is } 0 \text{ for } |k|>8}$$

3.20



(a) $x(t) = LC\frac{d^2y(t)}{dt^2}+RC\frac{dy(t)}{dt}+y(t)$

$$\boxed{\therefore\ x(t) = \frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+y(t)}$$

(b) $x(t) = e^{j\omega t}$

$$\boxed{\therefore\ H(j\omega) = \frac{1}{-\omega^2+j\omega+1}}$$

(c) $x(t)=\sin(t) = \frac{1}{2j}e^{j\left(\frac{2\pi}{2\pi}\right)t}-\frac{1}{2j}e^{-j\left(\frac{2\pi}{2\pi}\right)t}$

$\therefore a_1 = a_{-1}^* = \frac{1}{2j}$

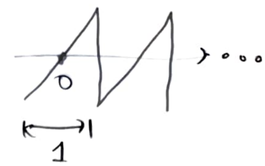
$\therefore y(t) = a_1H(j)e^{jt}-a_1H(-j)e^{-jt}$

$= \frac{1}{2j}\left(\frac{1}{j}e^{jt}+\frac{1}{j}e^{-jt}\right)$

$= -\frac{1}{2}(2\cos t) = \boxed{-\cos t}$

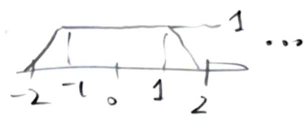
3.22 (a)

3.22-(a) $a_0 = 0$, $a_k = j\dfrac{(-1)^k}{k\pi}$, $(k \neq 0)$, $T = 1$



3.22-(b)

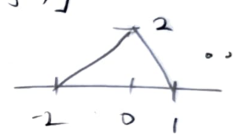
$a_0 = \dfrac{1}{2}$, $a_k = \begin{cases} 0 & (k \text{ is even}) \\ \dfrac{6}{\pi^2 k^2} \sin\left(\dfrac{\pi}{2}k\right) \sin\left(\dfrac{\pi k}{6}\right), & (k \text{ is odd}) \end{cases}$ $T = 6$



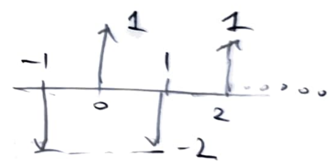
3.22-(c)

$a_0 = 1$, $a_k = j\dfrac{3}{2\pi^2 k^2}\left[e^{j\frac{2\pi}{3}k} \sin\left(\frac{2\pi}{3}k\right) + 2e^{j\frac{\pi}{3}k} \sin\left(\frac{\pi}{3}k\right)\right]$

$(k \neq 0)$, $T = 3$

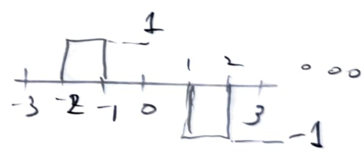


3.22-(d) $a_0 = -\dfrac{1}{2}$, $a_k = \dfrac{1}{2} - (-1)^k$, $(k \neq 0)$, $T = 2$



3.22-(e) $a_0 = 0$, $a_k = \begin{cases} 0 & (k \text{ is even}) \\ \dfrac{\cos\left(\frac{2\pi}{3}k\right) - \cos\left(\frac{\pi}{3}k\right)}{j \cdot \left(\frac{\pi}{3}k\right)} & (k \text{ is odd}) \end{cases}$

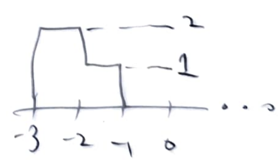
$T = 6$



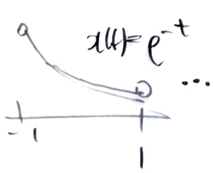
3.22-(f)

$a_0 = \dfrac{3}{4}$, $a_k = \dfrac{e^{-j\frac{\pi}{2}k} \sin\left(\frac{\pi}{2}k\right) + e^{-j\frac{\pi}{4}k} \sin\left(\frac{\pi}{4}k\right)}{k\pi}$

$T = 4$



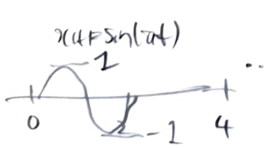
(b) $T = 2$



$$\boxed{a_k = \frac{(-1)^k}{2(1+jk\pi)}\left[e - \frac{1}{e}\right]}$$

(c)

$T = 4$



$a_k = \begin{cases} 0 & (k \text{ is even}) \\ \dfrac{4}{\pi(4-k^2)} & (k \text{ is odd}) \end{cases}$

3.26

$x(t)$ is a periodic signal with Fourier coefficients of $a_k = \begin{cases} 2 & (k=0) \\ j(\frac{1}{2})^{|k|} & (\text{otherwise}) \end{cases}$

(a) $x(t)$ is real $\Leftrightarrow x(t) = x^*(t)$

$\Leftrightarrow a_k = a_{-k}^*$ (X) ($\because \{j(\frac{1}{2})^{|k|}\}^* \neq j(\frac{1}{2})^{|k|}$)

$\boxed{\therefore x(t) \text{ is not real}}$

(b)
$x(t)$ is even $\Leftrightarrow x(t) = x(-t)$

$\Leftrightarrow a_k = a_{-k}$ (O) ($\because j(\frac{1}{2})^{|k|} = j(\frac{1}{2})^{|-k|}$)

$\boxed{\therefore x(t) \text{ is even}}$

(c)
let Fourier coefficient of $\frac{dx(t)}{dt}$, b_k

$\frac{dx(t)}{dt} \xleftrightarrow{FS} b_k = jk\frac{2\pi}{T_0}a_k$

$\therefore b_k = \begin{cases} 0 & (k=0) \\ -k(\frac{1}{2})^{|k|}(\frac{2\pi}{T_0}) & (\text{otherwise}) \end{cases}$

$b_k \neq b_{-k}$ $\boxed{\therefore \frac{dx(t)}{dt} \text{ is not even}}$

3.34 $h(t) = e^{-4|t|}$

$H(j\omega) = \int_{-\infty}^{\infty} e^{-4|t|}e^{-j\omega t}dt = \frac{1}{4+j\omega} + \frac{1}{4-j2\pi k}$

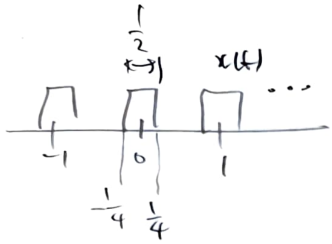
(a) $x(t) = \sum_{-\infty}^{\infty}\delta(t-n)$, $T=1$, $\omega_0 = 2\pi$, $a_k = 1$ for $\forall k$

$y(t) \xleftrightarrow{FS} b_k = a_k H(jk\omega_0) = \boxed{\frac{1}{4+j2\pi k} + \frac{1}{4-j2\pi k}}$

(b) $x(t) = \sum_{-\infty}^{\infty}(-1)^n\delta(t-n)$, $T=2$, $\omega_0 = \pi$, $a_k = \begin{cases} 0 & (k \text{ is even}) \\ 1 & (k \text{ is odd}) \end{cases}$

$y(t) \xleftrightarrow{FS} b_k = a_k H(jk\omega_0) = \boxed{\begin{cases} 0 \\ \frac{1}{4+j\pi k} + \frac{1}{4-j\pi k} \end{cases}}$

(c)

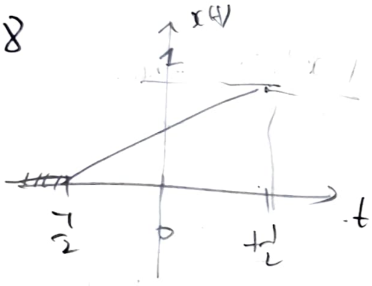


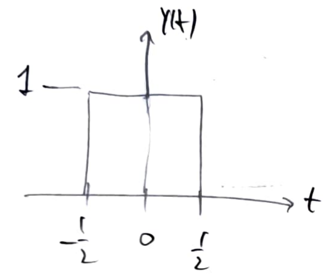
$T = 1,\ \omega_0 = 2\pi$

$$a_k = \begin{cases} \frac{1}{2} & (k=0) \\ 0 & (k \text{ is even}, k \neq 0) \\ \frac{\sin(\frac{\pi}{2}k)}{\pi k} & (k \text{ is odd}) \end{cases}$$

$$y(t) \overset{FS}{\longleftrightarrow} b_k = a_k H(jk\omega_0) = \boxed{\begin{cases} \frac{1}{4} & (k=0) \\ 0 & (k \text{ is even}, k\neq 0) \\ \frac{\sin(\frac{\pi}{2}k)}{\pi k}\left[\frac{1}{4+j2\pi k} + \frac{1}{4-j2\pi k}\right] & (k \text{ is odd}) \end{cases}}$$

4.8





(a) $x(t) = \int_{-\infty}^{t} y(t)\,dt$

$$x(t) \overset{FS}{\longleftrightarrow} X(j\omega) = \frac{1}{j\omega} Y(j\omega) + \pi\delta(\omega)$$

$$Y(j\omega) = \frac{2\sin(\frac{\omega}{2})}{\omega}$$

$$\boxed{\therefore X(j\omega) = \frac{2\sin(\frac{\omega}{2})}{j\omega^2} + \pi\delta(\omega)}$$

(b) $g(t) = x(t) - \frac{1}{2}$

$$\overset{FS}{\longleftrightarrow} G(j\omega) = X(j\omega) - \left(\frac{1}{2}\right)\cdot 2\pi\delta(\omega)$$

$$= \boxed{\frac{2\sin(\frac{\omega}{2})}{j\omega^2}}$$

4.14 $\mathcal{F}^{-1}\{(1+j\omega)X(j\omega)\} = Ae^{-2t}u(t)$

$$\therefore X(j\omega) = \frac{A}{(1+j\omega)(2+j\omega)} = A\left\{\frac{1}{1+j\omega} - \frac{1}{2+j\omega}\right\} \leftrightarrow x(t) = Ae^{-t}u(t) - Ae^{-2t}u(t)$$

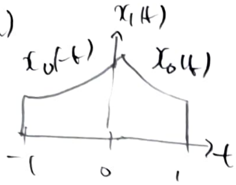
by Parseval's Relation, $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi = 2\pi \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$\boxed{\therefore \int_{-\infty}^{\infty} |x(t)|^2 dt = 1}$$

4.23

$x_0(t) = \begin{cases} e^{t} & (0 \le t \le 1) \\ 0 & (\text{otherwise}) \end{cases} \leftrightarrow X_0(j\omega) = \dfrac{1-e^{-(1+j\omega)}}{1+j\omega}$

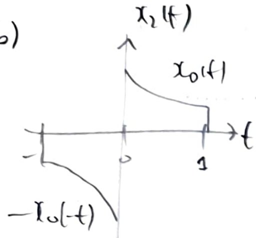
(a)



$x_1(t) = x_0(t) + x_0(-t)$

$\leftrightarrow X_1(j\omega) = X_0(j\omega) + X_0(-j\omega) = \boxed{\dfrac{2-2e^{-1}(\cos\omega + \omega\sin\omega)}{1+\omega^2}}$

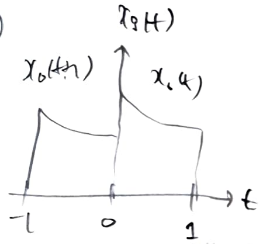
(b)



$x_2(t) = x_0(t) - x_0(-t)$

$\therefore X_2(j\omega) = X_0(j\omega) - X_0(-j\omega) = \boxed{j\left(\dfrac{-2\omega + 2e^{-1}(\sin\omega + \omega\cos\omega)}{1+\omega^2}\right)}$

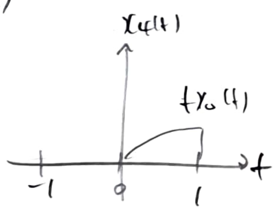
(c)



$x_3(t) = x_0(t) + x_0(t+1)$

$\therefore X_3(j\omega) = X_0(j\omega) + e^{j\omega}X_0(-j\omega) = \boxed{\dfrac{1+e^{j\omega} - e^{-1}(1+e^{-j\omega})}{1+j\omega}}$

(d)



$x_4(t) = tx_0(t)$

$\therefore X_4(j\omega) = j\dfrac{d}{d\omega}X_0(j\omega) = \boxed{\dfrac{1-e^{-1}(2e^{-j\omega} + j\omega e^{-j\omega})}{(1+j\omega)^2}}$

4.26

(a) (i) $Y(j\omega) = X(j\omega)H(j\omega) = \left[\dfrac{1}{(2+j\omega)^2}\right]\left[\dfrac{1}{4+j\omega}\right] = \dfrac{1}{4}\cdot\dfrac{1}{4+j\omega} - \dfrac{1}{4}\cdot\dfrac{1}{2+j\omega} + \dfrac{1}{2}\cdot\dfrac{1}{(2+j\omega)^2}$

$\boxed{\therefore y(t) = \frac{1}{4}e^{-4t}u(t) - \frac{1}{4}e^{-2t}u(t) + \frac{1}{2}te^{-2t}u(t)}$

(ii)

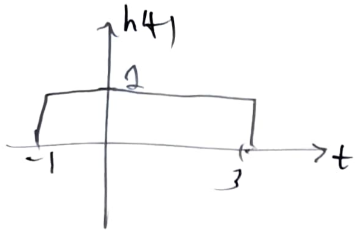
$Y(j\omega) = X(j\omega)H(j\omega) = \left[\dfrac{1}{(2+j\omega)^2}\right]\left[\dfrac{1}{(4+j\omega)^2}\right] = \dfrac{1}{4}\dfrac{1}{2+j\omega} + \dfrac{1}{4}\dfrac{1}{(2+j\omega)^2} - \dfrac{1}{4}\dfrac{1}{4+j\omega}$

$\boxed{\therefore y(t) = \frac{1}{4}e^{-2t}u(t) + \frac{1}{4}te^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t) + \frac{1}{4}te^{-4t}u(t)}$

(iii) $Y(j\omega) = X(j\omega)H(j\omega) = \left[\dfrac{1}{1+j\omega}\right]\left[\dfrac{1}{1-j\omega}\right] = \dfrac{1}{2}\dfrac{1}{1+j\omega} + \dfrac{1}{2}\dfrac{1}{1-j\omega}$

$\boxed{\therefore y(t) = \frac{1}{2}e^{-|t|}}$

(b)



$$y(t) = x(t)*h(t) = \begin{cases} 0 & (t<1) \\ 1-e^{-(t-1)} & (1<t\le 5) \\ e^{-(t-5)} - e^{-(t-1)} & (t>5) \end{cases}$$

$$\Leftrightarrow Y(j\omega) = \frac{2e^{-j3\omega}\sin(2\omega)}{\omega(1+j\omega)} = \underbrace{\left[\frac{e^{-j2\omega}}{1+j\omega}\right]}_{X(j\omega)}\underbrace{\left[\frac{e^{-j\omega}\cdot 2\sin(2\omega)}{\omega}\right]}_{H(j\omega)}$$

$\boxed{\therefore\ y(t) = x(t)*h(t) \text{ equals } H(j\omega)X(j\omega)}$

4.32

$$h(t) = \frac{\sin(4(t-1))}{\pi(t-1)}, \quad \text{let } h_1(t-1) = h(t)$$

$$h_1(t) = \frac{\sin 4t}{\pi t} \Leftrightarrow H_1(j\omega) = \begin{cases} e^{-j\omega} & |\omega|<4 \\ 0 & (\text{otherwise}) \end{cases}$$

$\downarrow$ shift the time

$$\therefore H(j\omega) = \begin{cases} e^{-j\omega} & |\omega|<4 \\ 0 & (\text{otherwise}) \end{cases}$$

(a) $X_1(j\omega) = \pi e^{j\frac{\pi}{12}}\delta(\omega-6) + \pi e^{j\frac{\pi}{12}}\delta(\omega+6)$

$Y_1(j\omega) = X_1(j\omega)H(j\omega) = 0 \Rightarrow \boxed{y_1(t) = 0}$

(b) $X_2(j\omega) = \sum_{k=0}^{\infty} \frac{\pi}{j}\cdot\left(\frac{1}{2}\right)^k \{\delta(\omega-3k) - \delta(\omega+3k)\}$

$Y_2(j\omega) = X_2(j\omega)H(j\omega) = \left(\frac{\pi}{j}\right)\left[\left(\frac{1}{2}\right)\{\delta(\omega-3)-\delta(\omega+3)\}e^{-j\omega}\right]$

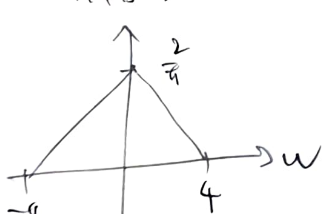
$\boxed{\therefore\ y_2(t) = \frac{1}{2}\sin(3t-1)}$

(c) $X_3(j\omega) = \begin{cases} e^{j\omega} & (|\omega|<4) \\ 0 & (\text{otherwise}) \end{cases}$

$Y_3(j\omega) = X_3(j\omega)H(j\omega) = \begin{cases} 1 & (|\omega|<4) \\ 0 & (\text{otherwise}) \end{cases}$

$\boxed{\therefore\ y_3(t) = x_3(t+1) = \frac{\sin 4t}{\pi t}}$

(d) $X_4(j\omega)$



$\therefore Y_4(j\omega) = X_4(j\omega)H(j\omega) = X_4(j\omega)e^{-j\omega}$

$\boxed{\therefore\ y_4(t) = x_4(t-1) = \left\{\frac{\sin(2(t-1))}{\pi(t-1)}\right\}^2}$

4.35

(a) $|H(j\omega)| = \dfrac{\sqrt{a^2+\omega^2}}{\sqrt{a^2+\omega^2}} = \boxed{1}$

$\measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right) - \tan^{-1}\left(\frac{\omega}{a}\right) = \boxed{-2\tan^{-1}\left(\frac{\omega}{a}\right)}$

$H(j\omega) = -1 + \dfrac{2a}{a+j\omega} \leftrightarrow \boxed{h(t) = -\delta(t) + 2a e^{-at} u(t)}$

(b) $a=1$, $|H(j\omega)| = 1$, $\measuredangle H(j\omega) = -2\tan^{-1}\omega$

$\boxed{y(t) = \cos\left(\frac{t}{\sqrt{3}} - \frac{\pi}{3}\right) - \cos\left(t - \frac{\pi}{2}\right) + \cos\left(\sqrt{3}t - \frac{2}{3}\pi\right)}$