حوری دهش ۹۸۲۱۴۱۳

۱ ←

الف) $u(t-a) = \frac{1}{۲}\left(1 + \text{sign}(t-a)\right)$

$$U(j\omega) = \int_{-\infty}^{+\infty} \frac{1}{۲}\left(1 + \text{sign}(t-a)\right) e^{-j\omega t}\, dt =$$

$$\frac{1}{۲}\left(\int_{-\infty}^{+\infty} e^{-j\omega t}\, dt + \int_{-\infty}^{+\infty} \text{sign}(t-a)\, e^{-j\omega t}\, dt\right) =$$

$$\frac{1}{۲}\left(۲\pi\,\delta(\omega) + \frac{۲e^{-j\omega a}}{j\omega}\right) = \pi\,\delta(\omega) + \frac{e^{-j\omega a}}{j\omega}$$

$$D(j\omega) = \int_{-\infty}^{+\infty} \delta(t-a)\, e^{-j\omega t}\, dt = e^{-j\omega a}$$

ب) $FT\left\{\int_{-\infty}^{t} x(\tau)\, d\tau\right\} = FT\left\{x(t) * u(t)\right\} = X(j\omega)\,U(j\omega) =$

$$\frac{X(j\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$$

ج)

$$u(t-a) = \int_{-\infty}^{t} \delta(\tau - a)\, d\tau$$

$$FT\left\{u(t-a)\right\} = \frac{D(j\omega)}{j\omega} + \pi D(0)\,\delta(\omega) = \frac{e^{-j\omega a}}{j\omega} +$$

$$\pi\,\delta(\omega)$$

۲— الف)

$$x_1[n] = \frac{\sin(\frac{n\pi}{5})}{n\pi} \xleftrightarrow{FT} X_1(e^{j\omega}) = \begin{cases} 1 & |\omega| < \frac{\pi}{5} \\ 0 & \frac{\pi}{5} \le |\omega| < \pi \end{cases}$$

$$x_2[n] = \cos\left(\frac{7\pi n}{2}\right) = \cos\left(\frac{\pi n}{2}\right) \xleftrightarrow{FT}$$

$$X_2(e^{j\omega}) = \pi\left\{\delta\left(\omega - \frac{\pi}{2}\right) + \delta\left(\omega + \frac{\pi}{2}\right)\right\} \qquad 0 \le |\omega| < \pi$$

$$x[n] = x_1[n]\,x_2[n] \longrightarrow X(e^{j\omega}) = X_2(e^{j\omega}) * X_1(e^{j\omega}) =$$

$$\begin{cases} 1 & \frac{3\pi}{10} < |\omega| < \frac{7\pi}{10} \\ 0 & \text{other} \end{cases}$$

ب)

$$x'[n] = a^{|n|}, \quad 0 < a < 1$$

$$X'(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} a^{|n|} e^{-j\omega n} = \sum_{n=-\infty}^{-1} a^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} a^{n} e^{-j\omega n}$$

$$\rightarrow X'(e^{j\omega}) = \sum_{n=1}^{+\infty} (ae^{j\omega})^n + \sum_{n=0}^{\infty} (ae^{-j\omega})^n \longrightarrow$$

$$X'(e^{j\omega}) = \frac{ae^{j\omega}}{1-ae^{j\omega}} + \frac{1}{1-ae^{-j\omega}} = \frac{1-a^2}{|1-ae^{j\omega}|}$$

$$\rightarrow X'(e^{j\omega}) = \frac{۱-a^{۲}}{(۱-a\cos\omega)^{۲}+a^{۲}\sin^{۲}\omega} = \frac{۱-a^{۲}}{۱+a^{۲}-۲a\cos\omega}$$

$$\rightarrow x[n] = (n-۳)\,x'[n]\Big|_{a=\frac{۱}{۷}} \xleftrightarrow{DTFT} \left[j\frac{dX'(e^{j\omega})}{d\omega} - ۳X'(e^{j\omega})\right]\Bigg|_{a=\frac{۱}{۷}}$$

$$= X(e^{j\omega})$$

ج) $X(e^{j\omega}) = \sum_{n=-\infty}^{-۲}\left(\frac{۱}{۳}\right)^{n} e^{-j\omega} = \sum_{n=۲}^{+\infty}\left(\frac{۱}{۳}e^{j\omega}\right)^{n} =$

$$\frac{e^{۲j\omega}}{۹} \times \frac{۱}{۱-\frac{۱}{۳}e^{j\omega}}$$

۳ ←

الف) $X(e^{j\omega}) = \frac{۱-\frac{۱}{۳}e^{-j\omega}}{-\frac{۱}{۸}(e^{-۲j\omega}+۲e^{-j\omega}-۸)} = \frac{۱-\frac{۱}{۳}e^{-j\omega}}{-\frac{۱}{۸}(e^{-j\omega}+۴)(e^{-j\omega}-۲)} =$

$$\frac{۱-\frac{۱}{۳}e^{-j\omega}}{(۱+\frac{e^{-j\omega}}{۴})(۱-\frac{e^{-j\omega}}{۲})} = \frac{A}{(۱+\frac{e^{-j\omega}}{۴})} + \frac{B}{(۱-\frac{e^{-j\omega}}{۲})} \rightarrow$$

$$A - \frac{A}{۲}e^{-j\omega} + B + \frac{B}{۴}e^{-j\omega} = ۱ - \frac{۱}{۳}e^{-j\omega}$$

$$\rightarrow \begin{cases} A + B = ۱ \\ \frac{B}{۴} - \frac{A}{۲} = \frac{-۱}{۳} \end{cases} \rightarrow B = \frac{۲}{۹} \,,\; A = \frac{۷}{۹} \rightarrow$$

$$x[n] = \frac{7}{9} \times \left(\frac{1}{4}\right)^n u[n] + \frac{2}{9} \times \left(\frac{1}{2}\right)^n u[n]$$

ب) $$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{k=-\infty}^{+\infty} (-1)^k \, \delta\left(\omega - \frac{2\pi}{5} k\right) e^{-j\omega n} \, d\omega =$$

$$\frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} (-1)^k \underbrace{\int_{-\pi}^{\pi} \delta\left(\omega - \frac{2\pi}{5} k\right) e^{j\omega n} \, d\omega}_{\to\, k = 0, \pm 1, \pm 2}$$

$$\rightarrow x[n] = \frac{1}{2\pi}\left[1 + e^{j\left(\frac{4\pi}{5}\right)n} + e^{-j\left(\frac{4\pi}{5}\right)n} - e^{j\left(\frac{2\pi}{5}\right)n} - e^{-j\left(\frac{2\pi}{5}\right)n}\right]$$

$$\rightarrow x[n] = \frac{1}{2\pi}\left[1 + 2\cos\left(\frac{4\pi}{5} n\right) - 2\cos\left(\frac{2\pi}{5} n\right)\right]$$

ج) $$X(e^{j\omega}) = \frac{1}{1 - \frac{1}{3} e^{-j\omega}} - \frac{\left(\frac{1}{3}\right)^{n_0} e^{-j\omega}}{1 - \frac{1}{3} e^{-j\omega}}$$

$$\rightarrow x[n] = \left(\frac{1}{3}\right)^n u[n] - \left(\frac{1}{3}\right)^{n_0} \left(\frac{1}{3}\right)^{n-n_0} u[n-n_0]$$

د) $$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} \, d\omega = \frac{-1}{2\pi} \int_{-\pi}^{0} 2j e^{j\omega n} \, d\omega +$$

$$\frac{1}{2\pi} \int_{0}^{\pi} 2j e^{j\omega n} \, d\omega = \frac{j}{\pi}\left[\frac{e^{jn\pi}}{jn} - \frac{1 - e^{-jn\pi}}{jn}\right] =$$

$$\frac{-4}{n\pi} \sin^2\left(\frac{n\pi}{2}\right)$$

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ب ~~(الف~~ $x[-n] \xleftrightarrow{FT} X(e^{-j\omega})$ ← ۴

$x^*[n] \xleftrightarrow{FT} X^*(e^{j\omega})$

$\rightarrow x_۲[n] = \dfrac{x^*[-n] + x[n]}{۲} \xleftrightarrow{FT} \frac{۱}{۲}\left(X(e^{j\omega}) + X^*(e^{-j\omega})\right)$

$= Re\{X(e^{j\omega})\}$

الف) $x[-n] \xleftrightarrow{FT} X(e^{-j\omega})$

$x[-n-1] \xleftrightarrow{FT} e^{j\omega n} X(e^{-j\omega})$

$X[1-n] \xleftrightarrow{FT} e^{-j\omega n} X(e^{-j\omega})$

$\rightarrow x_1[n] = x[1-n] + [-1-n] \xleftrightarrow{FT} e^{-j\omega n} X(e^{-j\omega}) +$

$e^{j\omega n} X(e^{-j\omega}) = (۲\cos\omega) X(e^{-j\omega})$

ج) $n x[n] \xleftrightarrow{FT} j\dfrac{dX(e^{j\omega})}{d\omega}$

$\rightarrow n^۲ x[n] \xleftrightarrow{FT} -\dfrac{d^۲ X(e^{j\omega})}{d\omega^۲} \rightarrow$

$x_۳[n] = (n-1)^۲ x[n] = n^۲ x[n] - ۲n x[n] + 1 \xleftrightarrow{FT}$

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$$-\frac{d^2X(e^{j\omega})}{d\omega^2} - 2j\frac{dX(e^{j\omega})}{d\omega} + X(e^{j\omega})$$

ه ←

الف) $\sum_{k=1}^{15} \sin(k\omega)$ یک سیگنال حقیقی و فرد است و طبق دوگانی تبدیل فوریه داریم ← زوج تبدیل فوریه آن یک سیگنال کاملا موهومی و فرد است.

توی این سیگنال یک شیفت زمانی بر روی زوج فوریه سیگنال بالا ایجاد می کند پس سیگنال حاصل موهومی است ولی زوج یا فرد نیست.

ب) این سیگنال از ترکیب دو سیگنال زوج و فرد ساخته شده پس فرد است و با توجه به ضرایب آن موهومی نیز است پس زوج تبدیل فوریه آن حقیقی و فرد است.

ج)

۶ ←

الف)

$$x[n] \longrightarrow y[n] \xleftrightarrow{FT} X(e^{j\omega}) \longrightarrow Y(e^{j\omega})$$

$$x[n] = a x_1[n] + b x_2[n] \xleftrightarrow{FT} X(e^{j\omega}) = a X_1(e^{j\omega}) + b X_2(e^{j\omega})$$

$$Y_1(e^{j\omega}) = 2X_1(e^{j\omega}) + e^{-j\omega} X_1(e^{j\omega}) - \frac{dX_1(e^{j\omega})}{d\omega}$$

$$Y_2(e^{j\omega}) = 2X_2(e^{j\omega}) + e^{-j\omega} X_2(e^{j\omega}) - \frac{dX_2(e^{j\omega})}{d\omega}$$

$$aY_1(e^{j\omega}) + bY_2(e^{j\omega}) = a\left[2X_1(e^{j\omega}) + e^{-j\omega} X_1(e^{j\omega}) - \frac{dX_1(e^{j\omega})}{d\omega}\right] + b\left[2X_2(e^{j\omega}) + e^{-j\omega} X_2(e^{j\omega}) - \frac{dX_2(e^{+j\omega})}{d\omega}\right]$$

$$= 2\left(aX_1(e^{j\omega}) + bX_2(e^{j\omega})\right) + e^{-j\omega}\left(aX_1(e^{j\omega}) + bX_2(e^{j\omega})\right) - \frac{d\left(aX_1(e^{j\omega}) + bX_2(e^{j\omega})\right)}{d\omega} = Y(e^{j\omega})$$

← خطی است

$$x[n] \xleftrightarrow{FT} X(e^{j\omega})$$

$$x_1[n] = x[n-n_0] \xleftrightarrow{FT} e^{-j\omega n_0} X(e^{j\omega})$$

$$Y_1(e^{j\omega}) = 2X_1(e^{j\omega}) + e^{-j\omega} X_1(e^{j\omega}) - \frac{dX_1(e^{j\omega})}{d\omega} =$$

$$2e^{-j\omega n_0} X(e^{j\omega}) + e^{-j\omega} e^{-j\omega n_0} X(e^{j\omega}) - \frac{de^{-j\omega n_0} X(e^{j\omega})}{d\omega} =$$

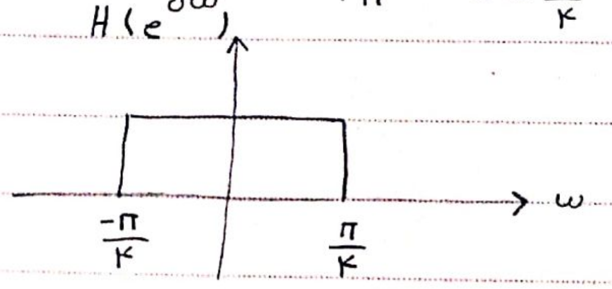
$$e^{-j\omega n_0}\left[2X(e^{j\omega}) + e^{-j\omega} X(e^{j\omega}) - \frac{dX(e^{j\omega})}{d\omega} + j e^{-j\omega} X(e^{j\omega})\right]$$

نه تغییرناپذیر با زمان نیست

ب) $x[n] = \delta[n] \xleftrightarrow{FT} X(e^{j\omega}) = 1$

$$H(e^{j\omega}) = 2 + e^{-j\omega} \xleftrightarrow{FT} h[n] = 2\delta[n] + \delta[n-1]$$

ج) $$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{\omega-\frac{\pi}{4}}^{\omega+\frac{\pi}{4}} X(e^{j\theta})\, H(e^{j(\omega-\theta)})\, d\theta$$



$$y[n] = 2x[n]\frac{\sin(\frac{n\pi}{4})}{n}$$

۷ ←

الف) $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1-\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$

$\longrightarrow$ عکس تبدیل فوریه $\longrightarrow h[n] = 3\left(\frac{1}{4}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n]$

ب) $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1-\frac{1}{2}e^{-j\omega}}{(1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{8}e^{-j\omega})} =$

$(1-\frac{1}{2}e^{-j\omega})X(e^{j\omega}) = (1-\frac{1}{3}e^{-j\omega})(1-\frac{1}{8}e^{-j\omega})Y(e^{j\omega}) \Rightarrow$

$X(e^{j\omega}) - \frac{1}{2}e^{-j\omega}X(e^{j\omega}) = Y(e^{j\omega}) - \frac{11}{24}e^{-j\omega}Y(e^{+j\omega}) +$

$\frac{1}{24}e^{-2j\omega}Y(e^{j\omega}) \longrightarrow$

$x[n] - \frac{1}{2}x[n-1] = y[n] - \frac{11}{24}y[n-1] + \frac{1}{24}y[n-2]$

←۸

$$x_1[n] = (-1)^n \left(h[n] * \left((-1)^n x[n] \right) \right) =$$

$$e^{jn\pi} \left(h[n] * \left(e^{jn\pi} x[n] \right) \right) = \left(e^{jn\pi} h[n] \right) * \left(e^{2jn\pi} x[n] \right) =$$

$$\left(e^{jn\pi} h[n] \right) * x[n]$$

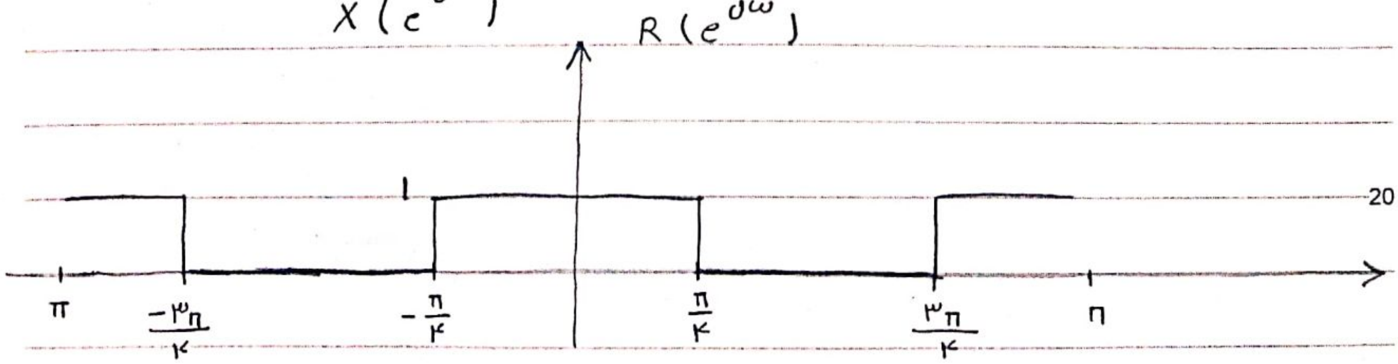
$$X_1(e^{j\omega}) = H(e^{j(\omega-\pi)}) X(e^{j\omega})$$

$$X_2(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega})$$

$$Y(e^{j\omega}) = X_1(e^{j\omega}) + X_2(e^{j\omega}) = H(e^{j(\omega-\pi)}) X(e^{j\omega}) +$$

$$H(e^{j\omega}) X(e^{j\omega}) = X(e^{j\omega}) \left(H(e^{j(\omega-\pi)}) + H(e^{j\omega}) \right)$$

$$R(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = H(e^{j(\omega-\pi)}) + H(e^{j\omega})$$



یک فیلتر bandpass است