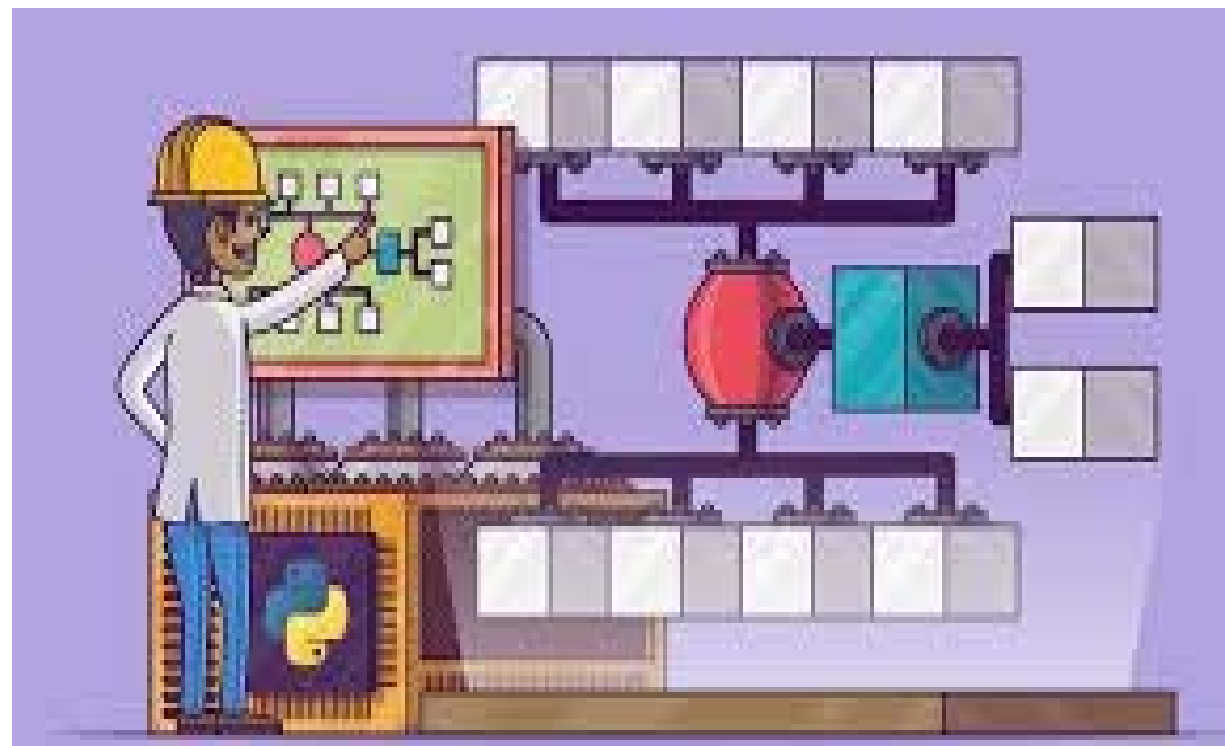




# ساختمان داده ها

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کامپیوتر





# Evaluating Postfix Expressions

- Evaluation process
- Make a single left-to-right scan of the expression.
- Place the operands on a stack until an operator is found.
- Remove, from the stack, the correct numbers of operands for the operator, perform the operation, and place the result back on the stack.



## Evaluating Postfix Expressions

```
void Eval(Expression e)
{// Evaluate the postfix expression e. It is assumed that the last token (a token
// is either an operator, operand, or '#' in e is '#'. A function NextToken is
// used to get the next token from e. The function uses the stack stack
    Stack<Token> stack;    //initialize stack
    for (Token x = NextToken (e) ; x != '#'; x = NextToken (e))
        if (x is an operand) stack.Push(x) // add to stack
        else // operator
            remove the correct number of operands for operator x from stack;
            perform the operation x and store the result (if any) onto the stack;
        }
    }
```



# Infix to Postfix

(1) Fully parenthesize expression

$a / b - c + d * e - a * c \rightarrow$

$(((((a / b) - c) + (d * e)) - a * c))$

(2) All operators replace their corresponding right parentheses.

$(((((a / b) - c) + (d * e)) - a * c))$

(3) Delete all parentheses.

$ab/c-de^*+ac^*-$



## Infix to Postfix

Infix Expression:  $A + (B * C - (D / E ^ F) * G) * H$ , where  $^$  is an exponential operator.

عملگر با اولویت پایین تر یا مساوی نمی تواند روی عملگر اولویت بالا در پشته قرار گیرد.





Symbol	Scanned	STACK	Postfix Expression	Description
1.		(		Start
2.	A	(	A	
3.	+	(+	A	
4.	(	(+(	A	
5.	B	(+(	AB	
6.	*	(+(*	AB	
7.	C	(+(*	ABC	
8.	-	(+(-	ABC*	'*' is at higher precedence than '-'
9.	(	(+(-(	ABC*	
10.	D	(+(-(	ABC*D	
11.	/	(+(-(/	ABC*D	
12.	E	(+(-(/	ABC*DE	
13.	^	(+(-(/^	ABC*DE	
14.	F	(+(-(/^	ABC*DEF	
15.	)	(+(-	ABC*DEF^/	Pop from top on Stack, that's why '^' Come first
16.	*	(+(-*	ABC*DEF^/	
17.	G	(+(-*	ABC*DEF^/G	
18.	)	(+	ABC*DEF^/G*-	Pop from top on Stack, that's why '^' Come first
19.	*	(+*	ABC*DEF^/G*-	
20.	H	(+*	ABC*DEF^/G*-H	
21.	)	Empty	ABC*DEF^/G*-H*+	END



# Rules

1. Push "(" onto Stack, and add ")" to the end of X.
2. Scan X from left to right and repeat Step 3 to 6 for each element of X until the Stack is empty.
3. If an operand is encountered, add it to Y.
4. If a left parenthesis is encountered, push it onto Stack.
5. If an operator is encountered ,then:
  1. Repeatedly pop from Stack and add to Y each operator (on the top of Stack) which has the same precedence as or higher precedence than operator.
  2. Add operator to Stack.[End of If]
6. If a right parenthesis is encountered ,then:
  1. Repeatedly pop from Stack and add to Y each operator (on the top of Stack) until a left parenthesis is encountered.
  2. Remove the left Parenthesis.[End of If]  
[End of If]
7. END.



# Rules

isp: in-stack priority: اولویت در پشته

icp: in-coming priority: اولویت در عبارت

- The left parenthesis is placed in the stack whenever it is found in the expression, but it is unstacked only when its matching right parenthesis is found.
- We remove an operator from the stack only if its isp is greater than or equal to the icp of the new operator.

عملگر با اولویت پایین تر یا مساوی نمی تواند روی عملگر اولویت بالا در پشته قرار گیرد.





# Infix to Postfix Function

```
void Postfix (Expression e)
{ // Output the postfix from of the infix expression e. NextToken
  // is as in function Eval (Program 3.18). It is assumed that
  // the last token in e is '#'. Also. '#' is used at the bottom of the stack
  Stack<Token>stack; // initialize stack
  stack.Push('#');
  for (Token x = NextToken(e); x != '#'; x = NextToken(e))
  {
    if (x is an operand) cout<<x;
    else if (x == '(')
    { // unstack until '('
      for (; stack.Top () != '('; stack.Pop())
        cout << stack.Top ();
      stack.Pop (); // unstack '('
    }
    else { // x is an operator
      for (; isp (stack.Top()) <= icp (x); stack.Pop())
        cout << stack.Top();
      stack.Push(x);
    }
  }

  // end of expression; empty the stack
  for (; !stack.IsEmpty(); cout << stack.Top(), stack.Pop());
  cout << endl;
}
```



## Analysis of postfix()

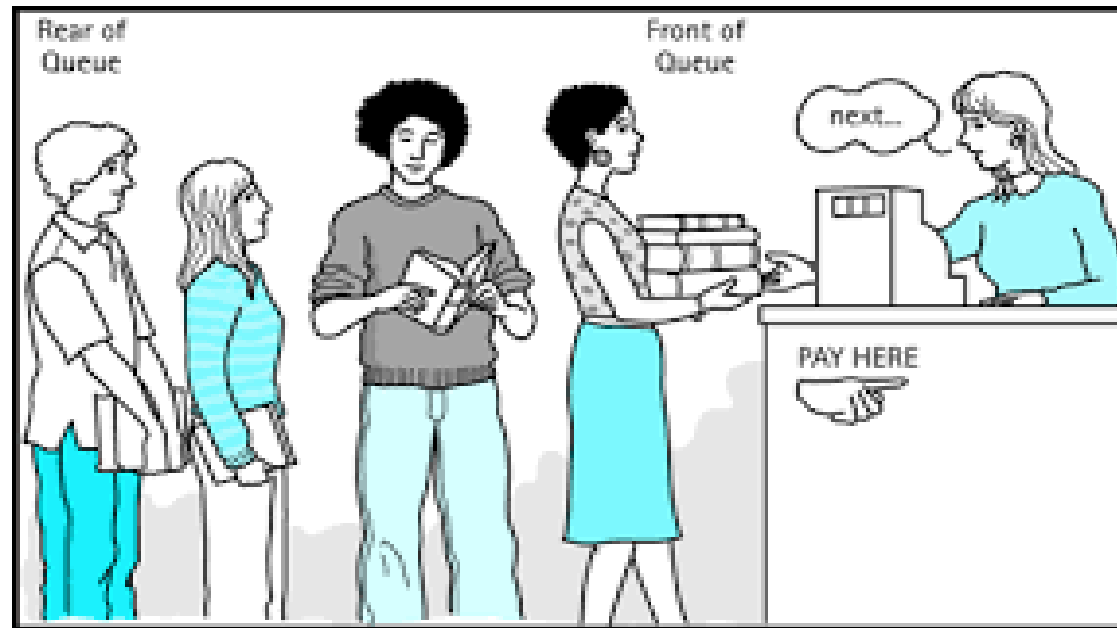
- Let  $n$  be the number of tokens in the expression.
- $\theta(n)$  time is spent extracting tokens and outputting them.
- for operands:  $\theta(1)$
- for operators:  $\theta(2)$
- Time is spent in stacking and unstacking.
- The total time spent here is  $\theta(n)$  as the number of tokens that get stacked and unstacked is linear in  $n$ .

So, the complexity of function postfix() is  $\theta(n)$ .



# Queue ADT

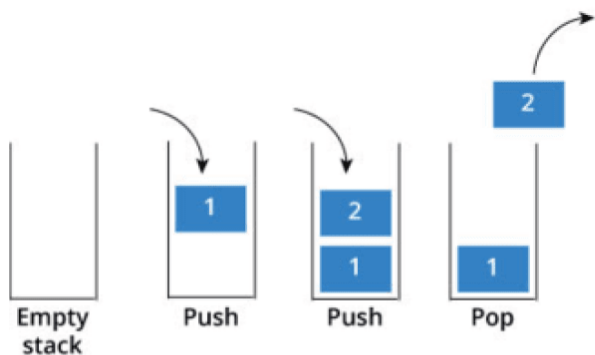
- **Queue:** an ordered list in which all insertions take place at one end and all deletions take place at the opposite end.
- First-in, last-out: **FIFO**



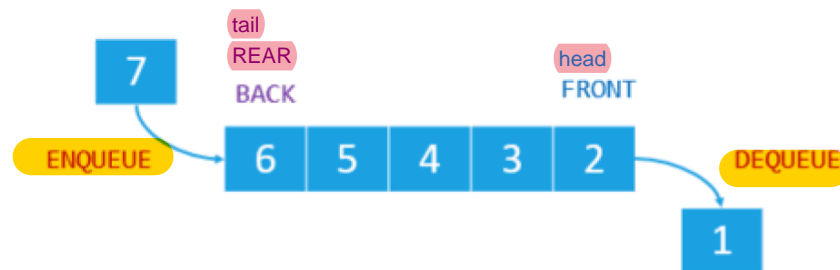


# Queue ADT

- **Queue:** an ordered list in which all insertions take place at one end and all deletions take place at the opposite end.
- First-in, last-out: **FIFO**



Stack

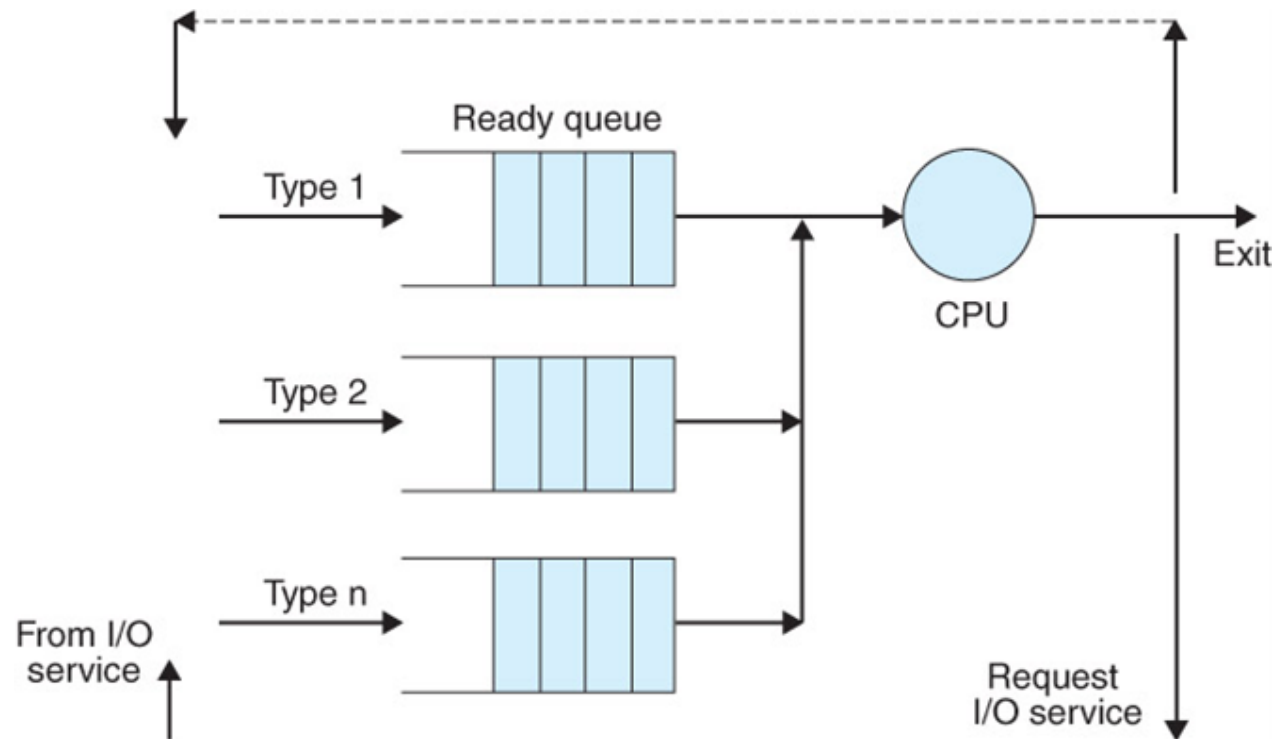


Queue



# Queue Applications

- **Queue:** an ordered list in which all insertions take place at one end and all deletions take place at the opposite end.
- First-in, last-out: **FIFO**





# Queue ADT

```
template <class T>
class Queue
{ // A finite ordered list with zero or more elements.
public:
    Queue(int queueCapacity = 10);
    // Create an empty queue whose initial capacity is queueCapacity

    bool IsEmpty() const;
    // If number of elements in the queue is 0, return true else return false.

    T& Front() const;
    // Return the element at the front of the queue.

    T& Rear() const;
    // Return the element at the rear of the queue.

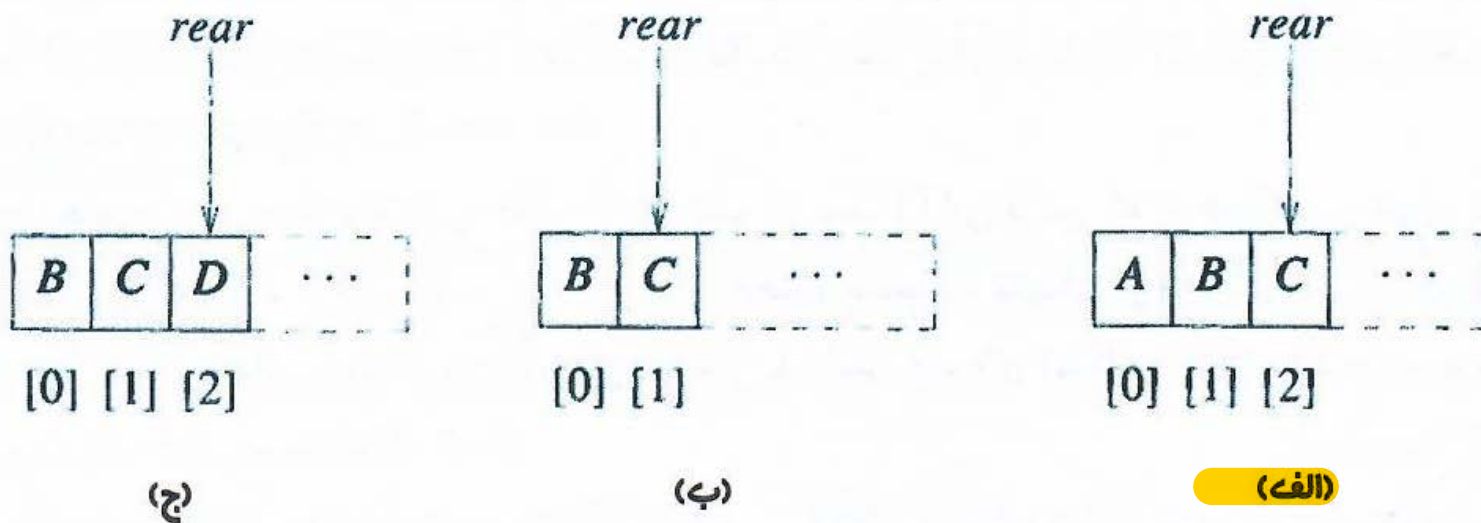
    void Push (const T& item);
    // Insert item at the rear of the queue.

    void Pop();
    // Delete the front element of the queue.
};
```



# Queue implementation using array

- پیاده سازی صف تنها با متغیر  $rear$



- هزینه درج:  $\theta(1)$

- بعد از حذف یک عنصر باید عناصر را به چپ شیف्ट دهیم.

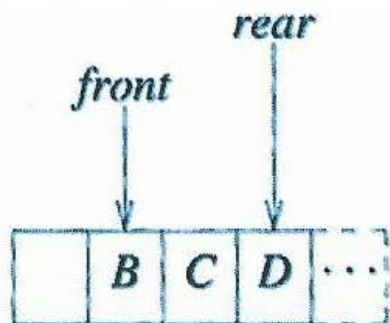
- برای صفی با  $n$  عنصر، هزینه حذف:  $\theta(n)$

- به استثنای زمان مربوط به تغییر اندازه مورد نیاز آرایه

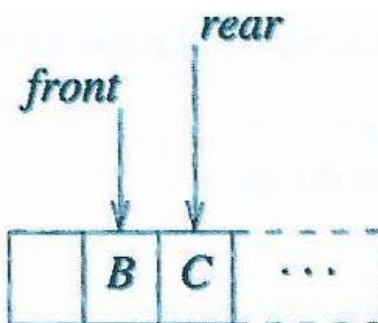


# Queue implementation using array

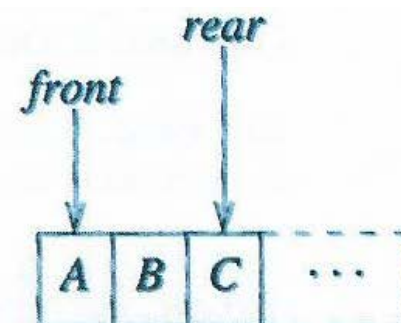
- پیاده سازی صف با دو متغیر **front** و **rear**



(الف)



(ب)



(ج)



- هزینه درج:  $\theta(1)$

- هزینه حذف:  $\theta(1)$

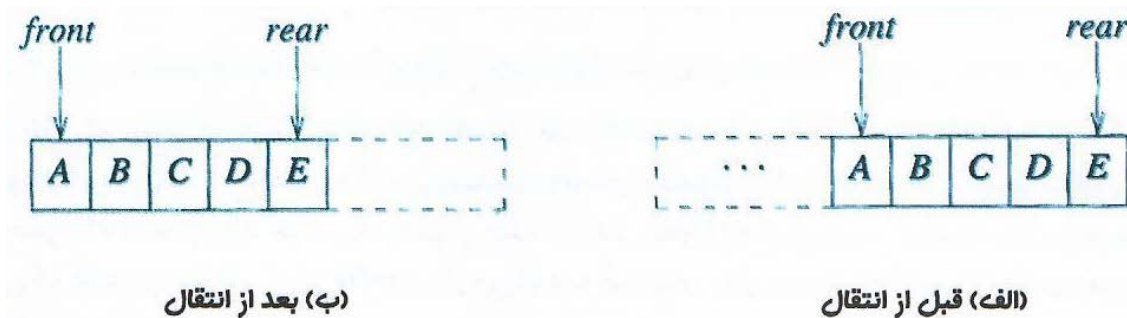
- به استثنای زمان مربوط به تغییر اندازه مورد نیاز آرایه





# صف حلقوی

• مشکل این روش:

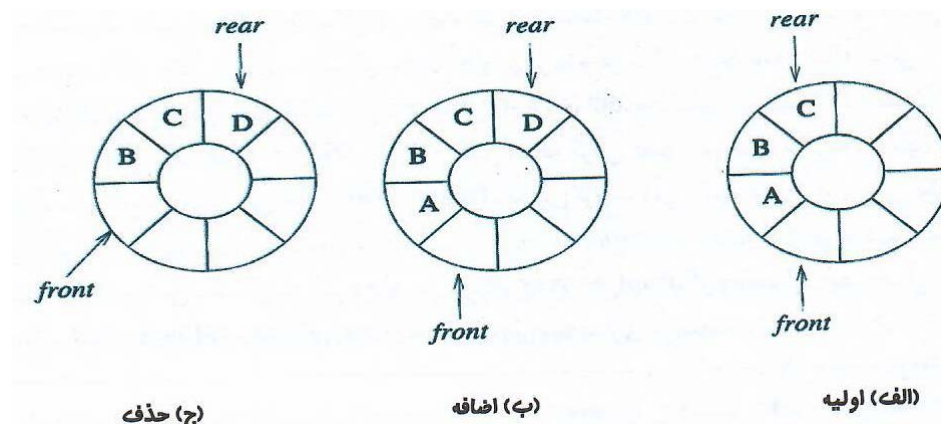


$$\text{rear} = \text{capacity} - 1$$

```
if (rear == capacity - 1) rear = 0;  
else rear++;
```

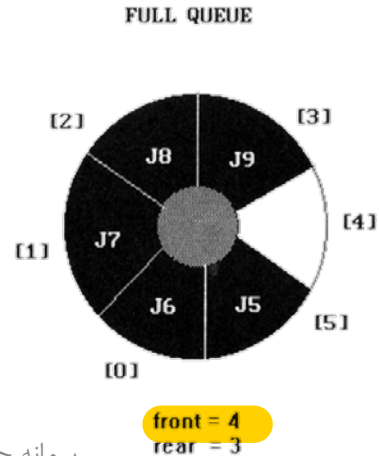
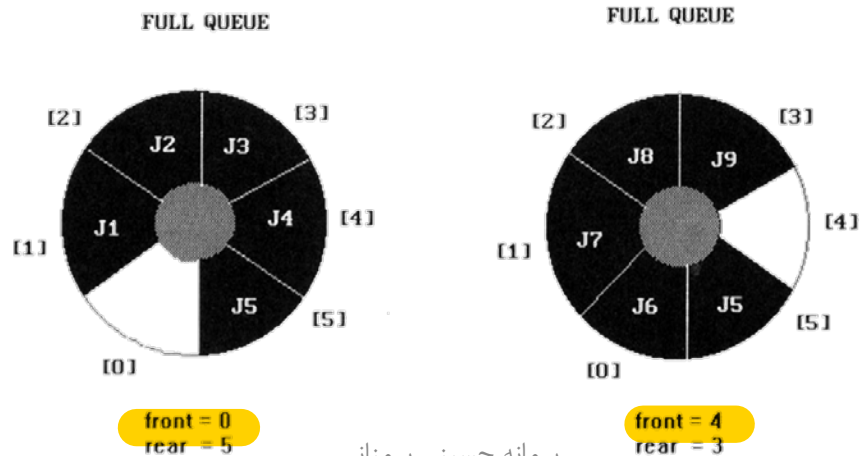
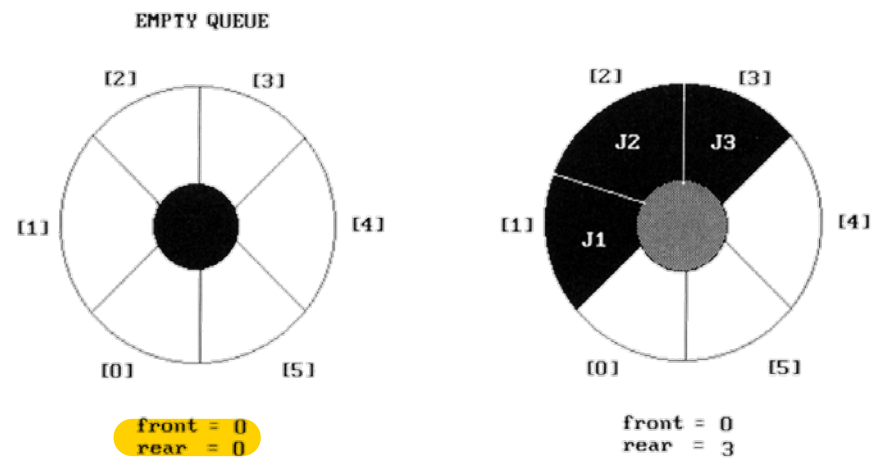
یا

$$(\text{rear} + 1) \% \text{capacity}$$





# صف حلقوی

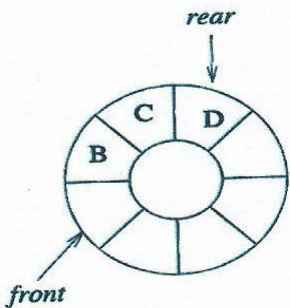




# Queue implementation using array

```
private:
    T *queue;      // array for queue elements
    int front,     // one counterclockwise from front
        rear,     // array position of rear element
        capacity; // capacity of queue array

template <class T>
Queue<T>::Queue (int queueCapacity) : capacity (queueCapacity)
{
    if (capacity < 1) throw "Queue capacity must be > 0";
    queue = new T [capacity];
    front = rear = 0;
}
```



چالش پر یا خالی بودن صف

پر یا خالی بودن صف را چگونه تشخیص دهیم؟ برای هر دو حالت  $front = rear$

راه حل برای حل مشکل:

- ۱- ذخیره تعداد عناصر موجود در صف
- ۲- اجازه ندهیم صف پر شود. همیشه یک عنصر صف را خالی نگه داریم
- ۳- نگه داشتن آخرین عمل



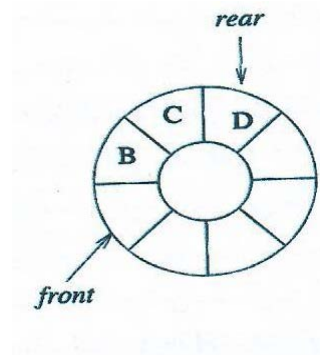
# Queue ADT

```
template <class T>
inline bool Queue<T>::IsEmpty() {return front == rear;}

template <class T>
inline T& Queue<T>::Front()
{
    if (IsEmpty ()) throw "Queue is empty. No front element";
    return queue [(front + 1) % capacity];
}

template <class T>
inline T& Queue<T>::Rear()
{
    if (IsEmpty()) throw "Queue is empty. No rear element";
    return queue [rear];
}
```

rear == front



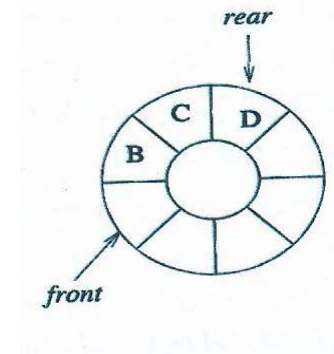


# Push in the Queue

```
template <class T>
void Queue<T>::Push(const& x)
// Add x at rear of queue.
if ((rear + 1) % capacity == front)
{ // queue full, double capacity
  // code to double queue capacity comes here
}

rear = (rear + 1) % capacity;
queue[rear] = x;
}
```

**rear + 1 == front**



front = 0  
rear = 7  
 $(rear+1) \% capacity =$   
 $8 \% 8 = 0$   
Front = 0 ->  
**queue is full**