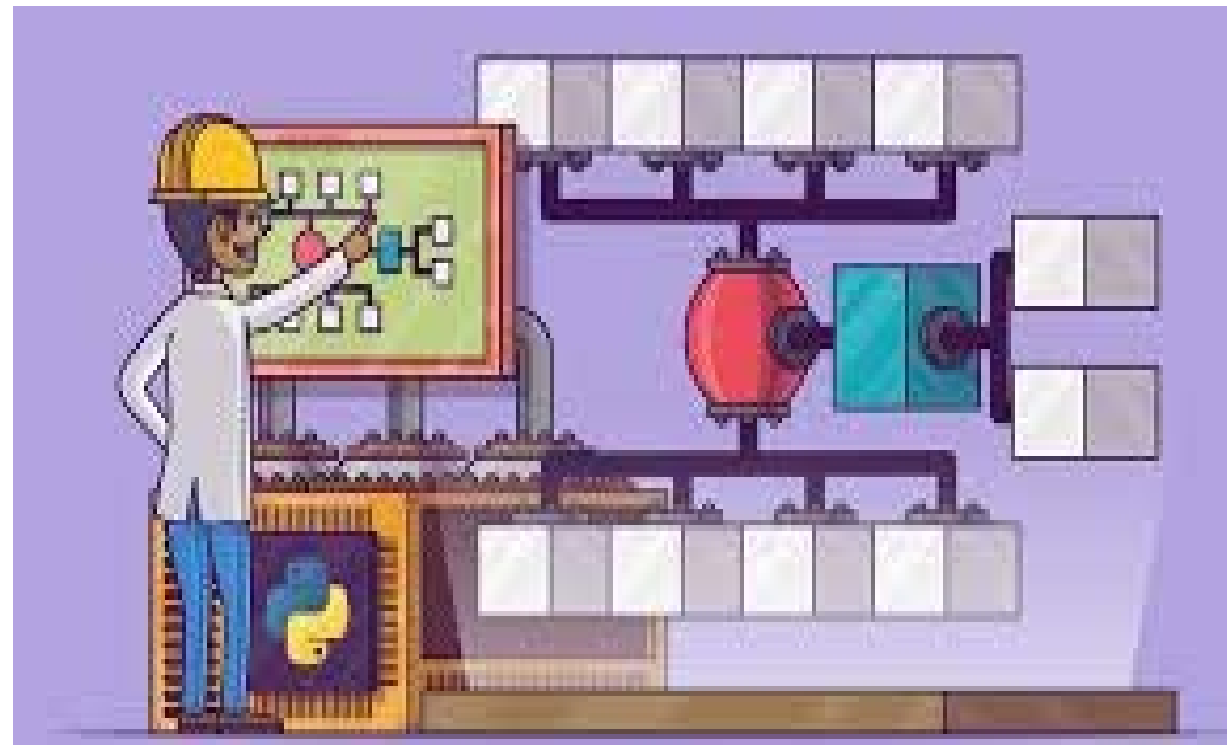




# ساختمان داده ها

مدرس:  
سمانه حسینی سمنانی

دانشگاه صنعتی اصفهان - دانشکده برق و  
کامپیوتر





# درخت ها

- مفاهیم اولیه
- پیمایش درخت
- درخت دودویی معادل
- پیاده سازی درخت
- درخت جستجوی دودویی
- درخت عبارت
- Heap tree (هرم بیشینه)



# درخت ها

Red-black tree •

AVL tree •

→ B-Trees •



# Deleting a key from a B-tree

- Had to ensure that a node didn't get too big due to insertion
- Ensure that a node doesn't get too small during deletion
- Except that the root is allowed to have fewer than the minimum number  $t - 1$  of keys



# Deleting a key from a B-tree

1. If the key  $k$  is in node  $x$  and  $x$  is a leaf, delete the key  $k$  from  $x$ .
2. If the key  $k$  is in node  $x$  and  $x$  is an internal node, do the following:
  - a. If the child  $y$  that precedes  $k$  in node  $x$  has at least  $t$  keys, then find the predecessor  $k'$  of  $k$  in the subtree rooted at  $y$ . Recursively delete  $k'$ , and replace  $k$  by  $k'$  in  $x$ . (We can find  $k'$  and delete it in a single downward pass.)



# Deleting a key from a B-tree

- b.** If  $y$  has fewer than  $t$  keys, then, symmetrically, examine the child  $z$  that follows  $k$  in node  $x$ . If  $z$  has at least  $t$  keys, then find the successor  $k'$  of  $k$  in the subtree rooted at  $z$ . Recursively delete  $k'$ , and replace  $k$  by  $k'$  in  $x$ . (We can find  $k'$  and delete it in a single downward pass.)
- c.** Otherwise, if both  $y$  and  $z$  have only  $t - 1$  keys, merge  $k$  and all of  $z$  into  $y$ , so that  $x$  loses both  $k$  and the pointer to  $z$ , and  $y$  now contains  $2t - 1$  keys. Then free  $z$  and recursively delete  $k$  from  $y$ .



# Deleting a key from a B-tree

3. If the key  $k$  is not present in internal node  $x$ , determine the root  $x.c_i$  of the appropriate subtree that must contain  $k$ , if  $k$  is in the tree at all. If  $x.c_i$  has only  $t - 1$  keys, execute step 3a or 3b as necessary to guarantee that we descend to a node containing at least  $t$  keys. Then finish by recursing on the appropriate child of  $x$ .



# Deleting a key from a B-tree

- a. If  $x.c_i$  has only  $t - 1$  keys but has an immediate sibling with at least  $t$  keys, give  $x.c_i$  an extra key by moving a key from  $x$  down into  $x.c_i$ , moving a key from  $x.c_i$ 's immediate left or right sibling up into  $x$ , and moving the appropriate child pointer from the sibling into  $x.c_i$ .
- b. If  $x.c_i$  and both of  $x.c_i$ 's immediate siblings have  $t - 1$  keys, merge  $x.c_i$  with one sibling, which involves moving a key from  $x$  down into the new merged node to become the median key for that node.

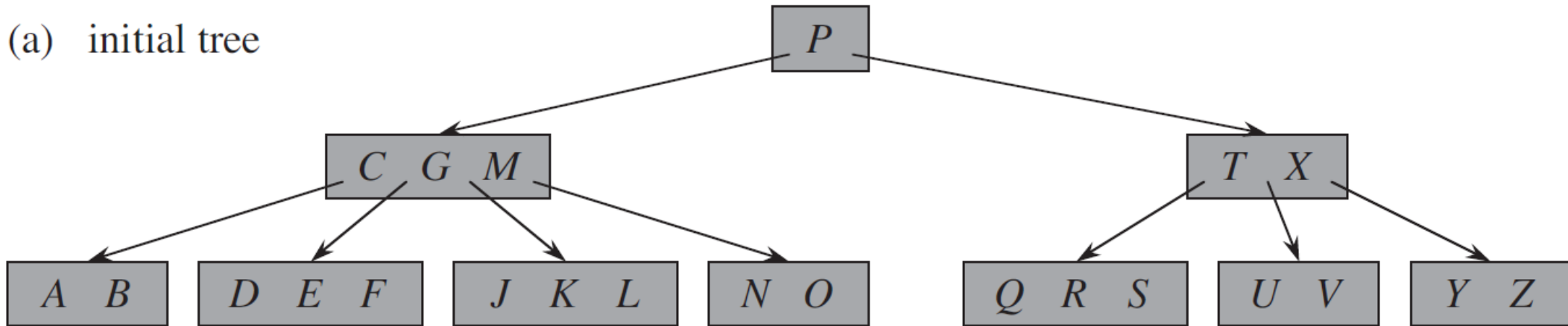




# Deleting a key from a B-tree

The minimum degree for this B-tree is  $t = 3$ , so a node (other than the root) cannot have fewer than 2 keys.

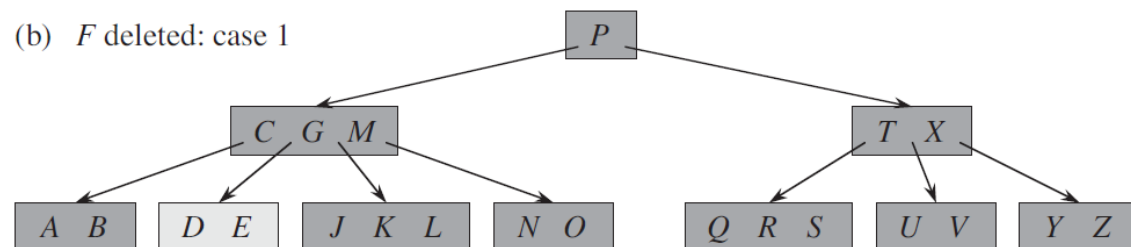
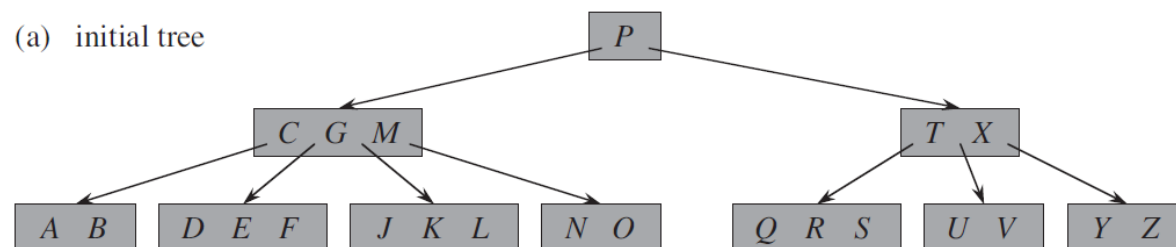
(a) initial tree





# Deleting a key from a B-tree

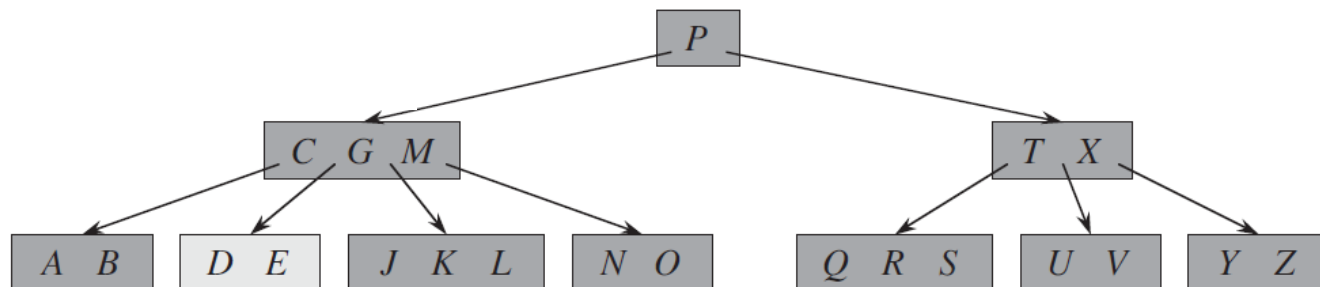
The minimum degree for this B-tree is  $t = 3$ , so a node (other than the root) cannot have fewer than 2 keys.



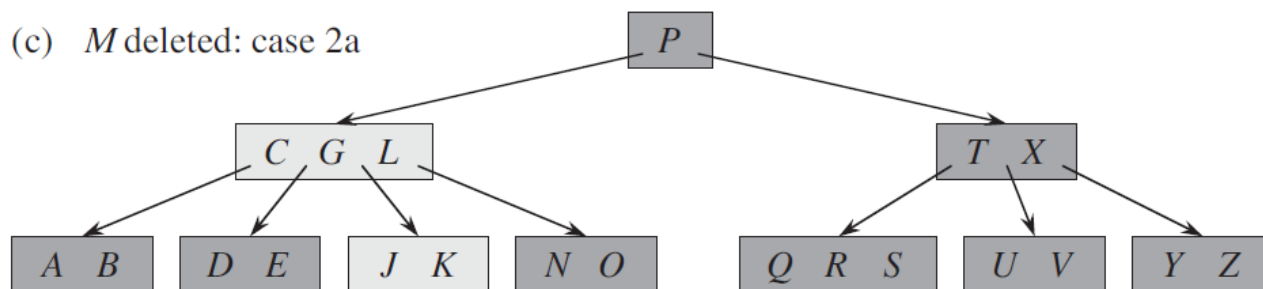
1. If the key  $k$  is in node  $x$  and  $x$  is a leaf, delete the key  $k$  from  $x$ .



# Deleting a key from a B-tree



(c) *M* deleted: case 2a



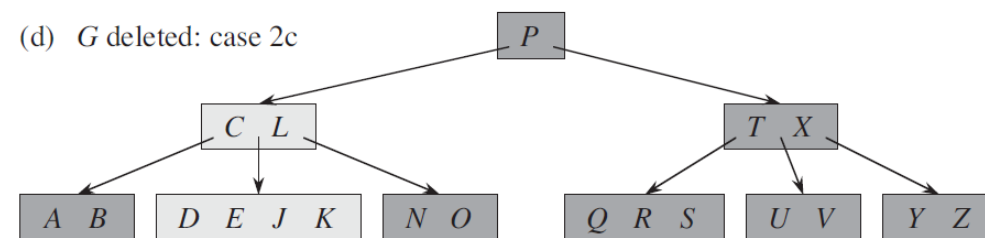
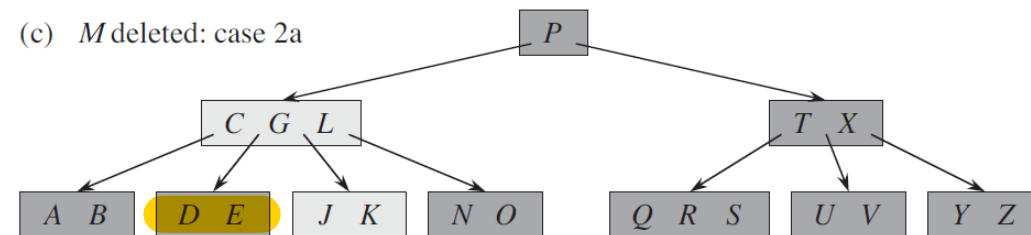
2. If the key  $k$  is in node  $x$  and  $x$  is an internal node, do the following:
  - a. If the child  $y$  that precedes  $k$  in node  $x$  has at least  $t$  keys, then find the predecessor  $k'$  of  $k$  in the subtree rooted at  $y$ . Recursively delete  $k'$ , and replace  $k$  by  $k'$  in  $x$ . (We can find  $k'$  and delete it in a single downward pass.)



# Deleting a key from a B-tree

2. If the key  $k$  is in node  $x$  and  $x$  is an internal node, do the following:

- If the child  $y$  that precedes  $k$  in node  $x$  has at least  $t$  keys, then find the predecessor  $k'$  of  $k$  in the subtree rooted at  $y$ . Recursively delete  $k'$ , and replace  $k$  by  $k'$  in  $x$ . (We can find  $k'$  and delete it in a single downward pass.)
- If  $y$  has fewer than  $t$  keys, then, symmetrically, examine the child  $z$  that follows  $k$  in node  $x$ . If  $z$  has at least  $t$  keys, then find the successor  $k'$  of  $k$  in the subtree rooted at  $z$ . Recursively delete  $k'$ , and replace  $k$  by  $k'$  in  $x$ . (We can find  $k'$  and delete it in a single downward pass.)
- Otherwise, if both  $y$  and  $z$  have only  $t - 1$  keys, merge  $k$  and all of  $z$  into  $y$ , so that  $x$  loses both  $k$  and the pointer to  $z$ , and  $y$  now contains  $2t - 1$  keys. Then free  $z$  and recursively delete  $k$  from  $y$ .

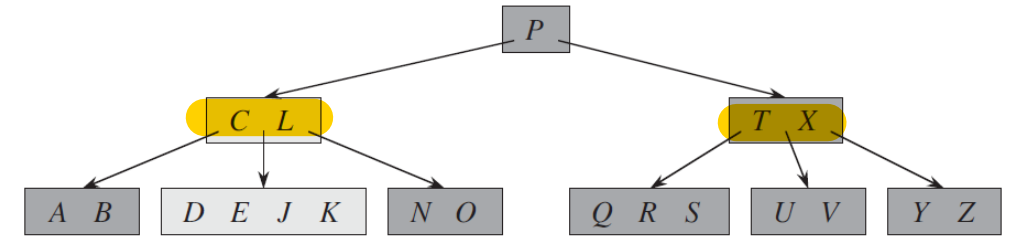




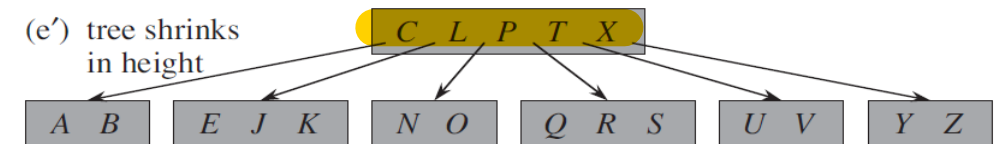
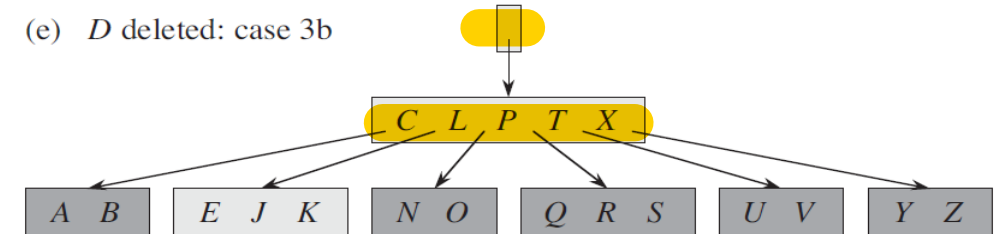
# Deleting a key from a B-tree

3. If the key  $k$  is not present in internal node  $x$ , determine the root  $x.c_i$  of the appropriate subtree that must contain  $k$ , if  $k$  is in the tree at all. If  $x.c_i$  has only  $t - 1$  keys, execute step 3a or 3b as necessary to guarantee that we descend to a node containing at least  $t$  keys. Then finish by recursing on the appropriate child of  $x$ .

- If  $x.c_i$  has only  $t - 1$  keys but has an immediate sibling with at least  $t$  keys, give  $x.c_i$  an extra key by moving a key from  $x$  down into  $x.c_i$ , moving a key from  $x.c_i$ 's immediate left or right sibling up into  $x$ , and moving the appropriate child pointer from the sibling into  $x.c_i$ .
- If  $x.c_i$  and both of  $x.c_i$ 's immediate siblings have  $t - 1$  keys, merge  $x.c_i$  with one sibling, which involves moving a key from  $x$  down into the new merged node to become the median key for that node.



(e)  $D$  deleted: case 3b



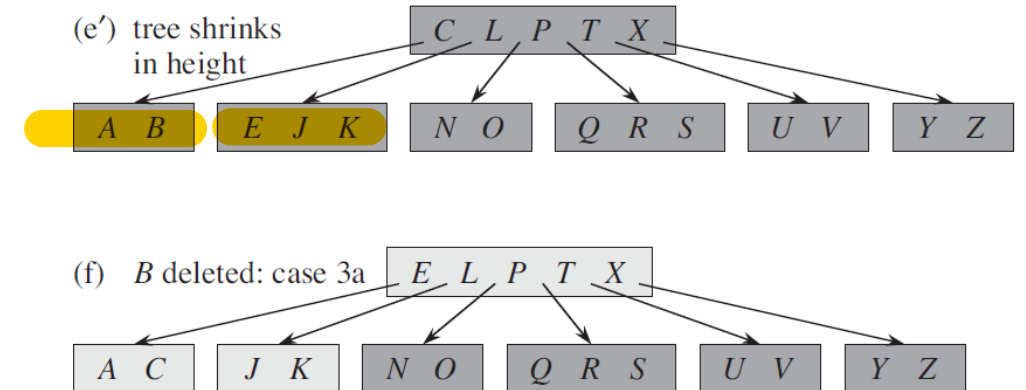
Although this procedure seems complicated, it involves only  $O(h)$  disk operations for a B-tree of height  $h$



# Deleting a key from a B-tree

3. If the key  $k$  is not present in internal node  $x$ , determine the root  $x.c_i$  of the appropriate subtree that must contain  $k$ , if  $k$  is in the tree at all. If  $x.c_i$  has only  $t - 1$  keys, execute step 3a or 3b as necessary to guarantee that we descend to a node containing at least  $t$  keys. Then finish by recursing on the appropriate child of  $x$ .

- If  $x.c_i$  has only  $t - 1$  keys but has an immediate sibling with at least  $t$  keys, give  $x.c_i$  an extra key by moving a key from  $x$  down into  $x.c_i$ , moving a key from  $x.c_i$ 's immediate left or right sibling up into  $x$ , and moving the appropriate child pointer from the sibling into  $x.c_i$ .
- If  $x.c_i$  and both of  $x.c_i$ 's immediate siblings have  $t - 1$  keys, merge  $x.c_i$  with one sibling, which involves moving a key from  $x$  down into the new merged node to become the median key for that node.



Although this procedure seems complicated, it involves only  $O(h)$  disk operations for a B-tree of height  $h$