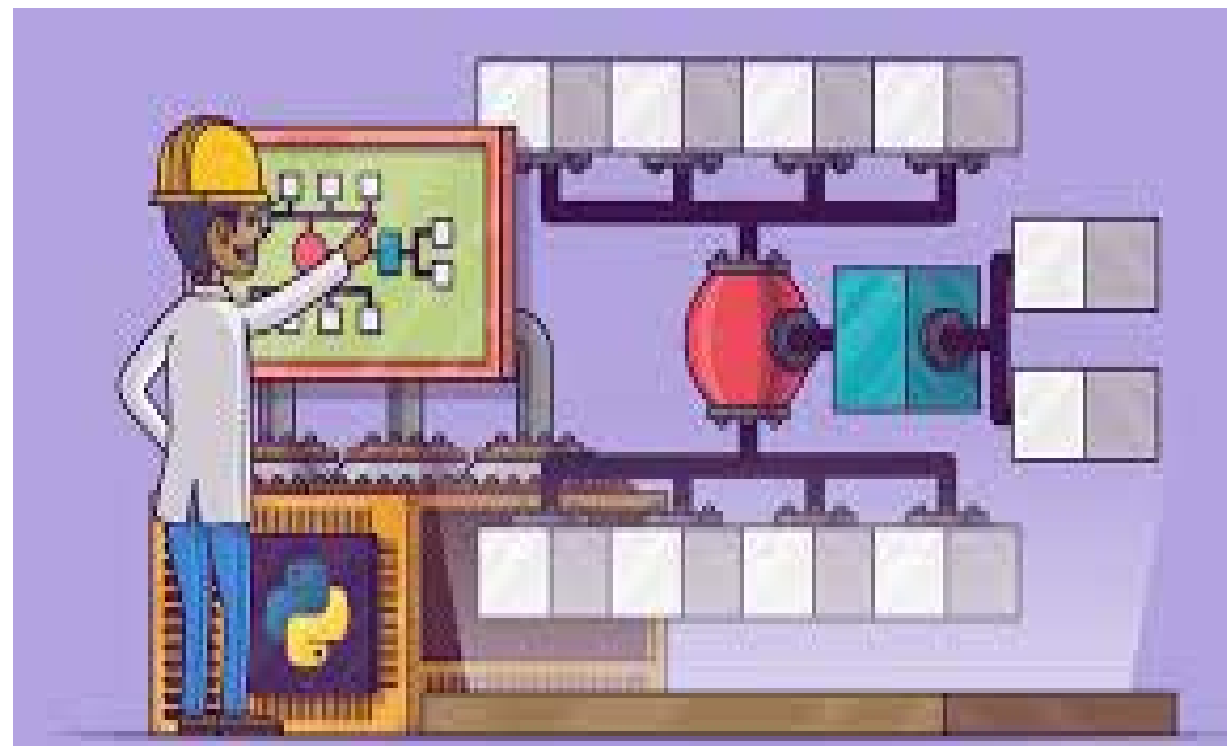




ساختمان داده ها

مدرس:
سمانه حسینی سمنانی

دانشگاه صنعتی اصفهان - دانشکده برق و
کامپیوتر





درخت ها

- مفاهیم اولیه
- پیمایش درخت
- درخت دودویی معادل
- پیاده سازی درخت
- درخت جستجوی دودویی
- درخت عبارت
- Heap tree (هرم بیشینه)



درخت ها

Red-black tree •

AVL tree •

B-Trees •



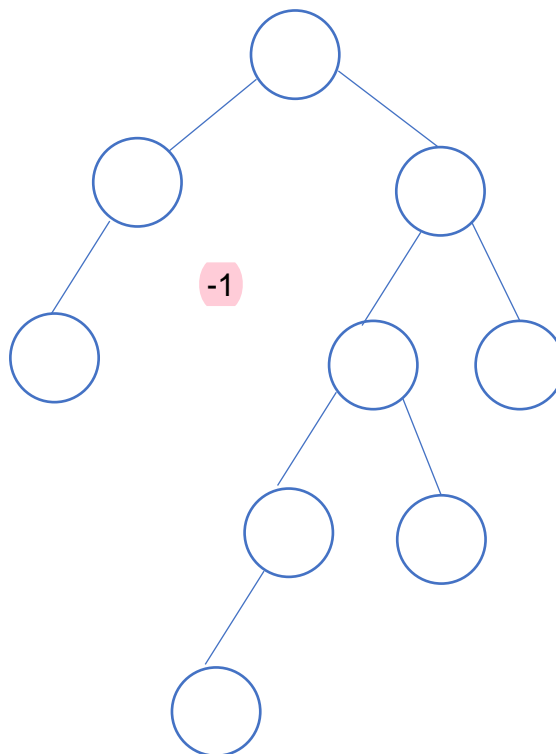
AVL tree

- An AVL tree is a binary search tree that is height balanced:
- for each node x , the heights of the left and right subtrees of x differ by at most 1.
- Landis and Adelson-Velskey in 1962
- To implement an AVL tree, we maintain an extra attribute in each node: $x.h$ is the height of node x .
- Prove that an AVL tree with n nodes has height $O(\log n)$.
- We try to keep $\text{root}.h = \log(n)$ during insertion/deletion. Using rotation



AVL tree

Is it AVL?





AVL tree height

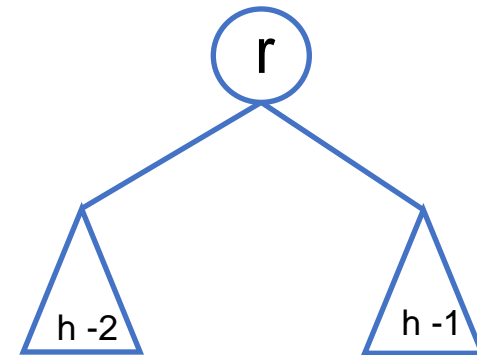
- Prove that an AVL tree with n nodes has height $O(\lg n)$.

$$M(h) \geq M(h-1) + M(h-2)$$

• حداقل تعداد گره ها در یک AVL با ارتفاع h : $M(h)$

$$M(h) \leq n$$

- Fibonacci recursive: $F(n) = F(n-1) + F(n-2)$
- $T(n) = O\left(\left(\frac{1+\sqrt{5}}{2}\right)^n\right)$
- $M(h) \geq c^h$
- $h = O(\log n)$





Height in AVL tree

- حداکثر اختلاف ارتفاع در درخت AVL

1

- حداکثر اختلاف ارتفاع در درخت Red-black

$\log n$



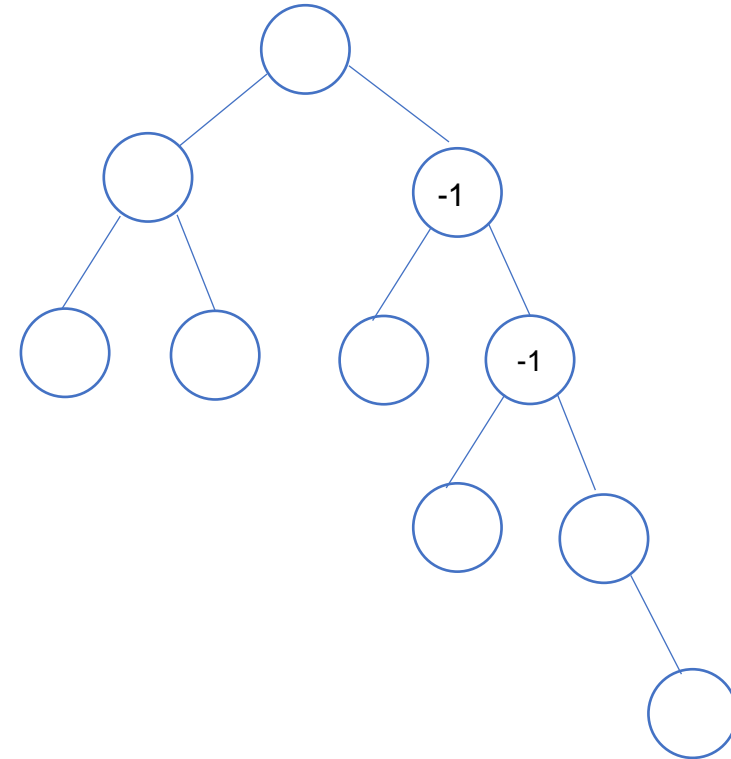
Insert/delete in AVL tree

- we first place/remove a node into the appropriate place in binary search tree order.
- the tree might no longer be height balanced.
- the heights of the left and right children of some node might differ by 2.
- Solve the problem using appropriate rotation.



Insert/delete in AVL tree

- Each node T has a Balance Factor $BF(T) = h_L - h_R$
- $BF(T) = -1$ or 0 or 1 in a AVL tree
- By insertion/deletion a node, BF's will be updated
- if $BF(T) = 2$ or $BF(T) = -2$ we need a rotation



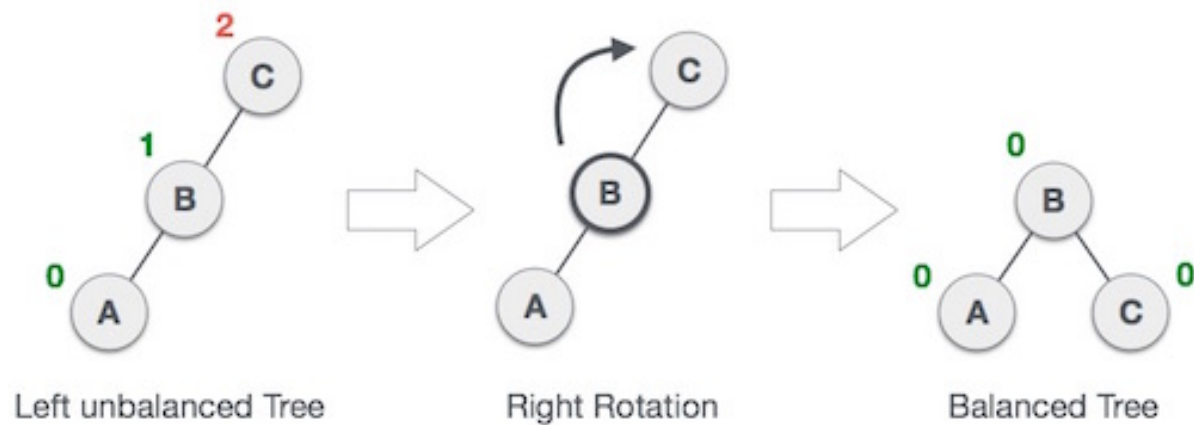


Rotation in AVL tree

- Right Rotation (R)
- Left Rotation (L)
- Left Right Rotation (LR)
- Right Left Rotation (RL)

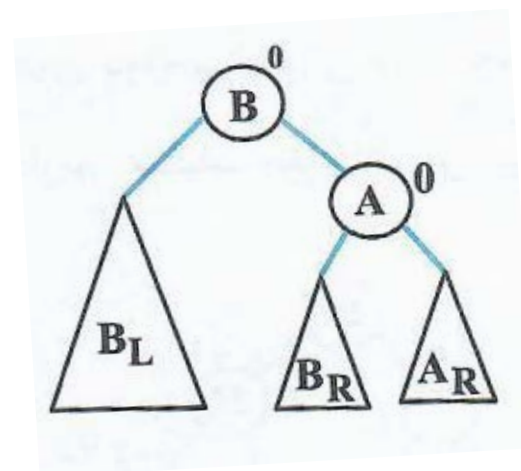
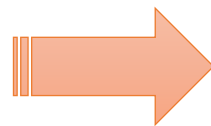
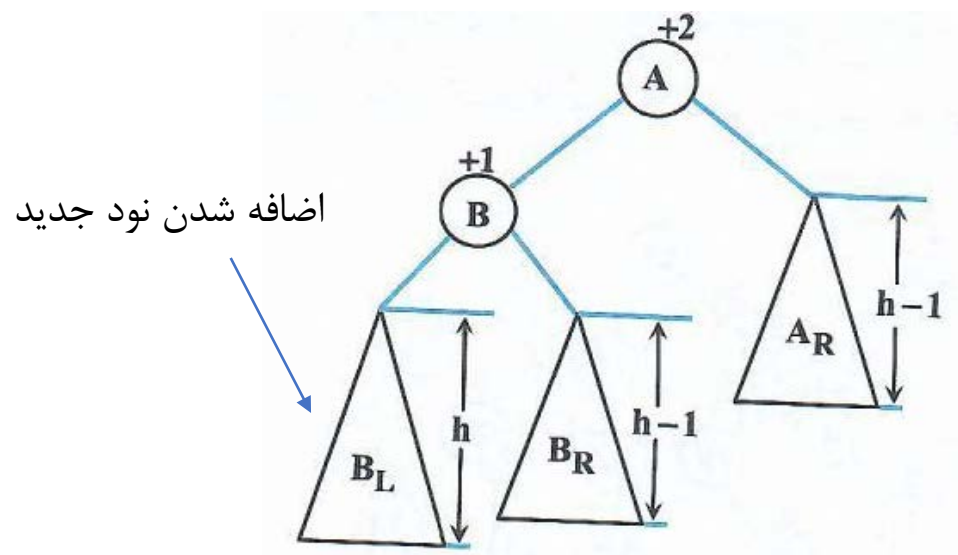


R Rotation





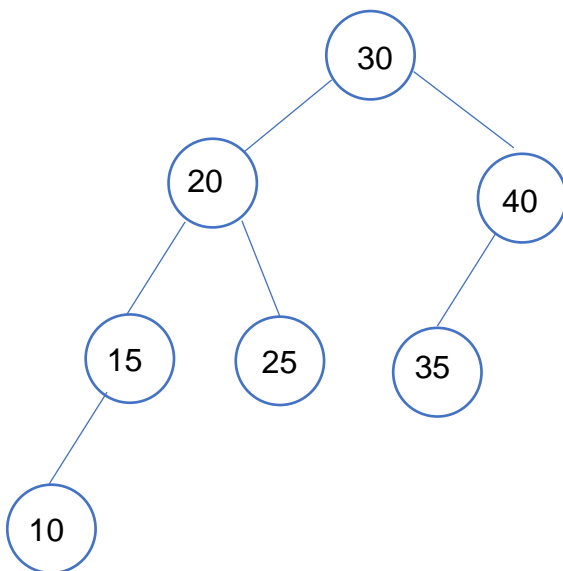
R Rotation





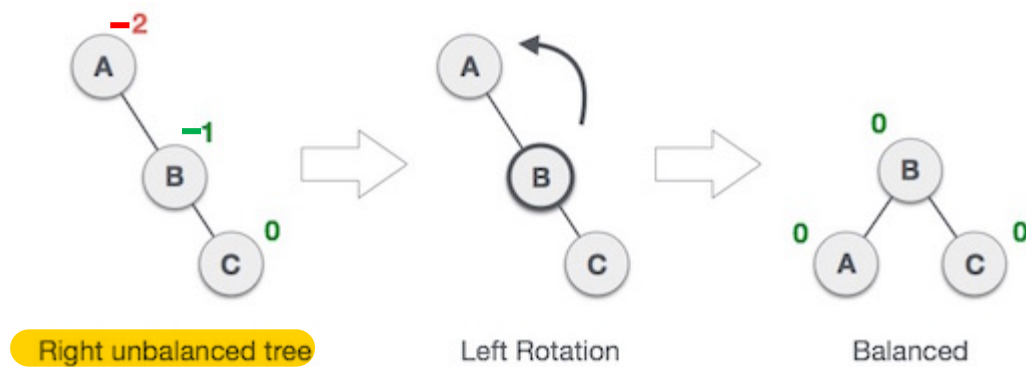
R Rotation-Example

- پیمایش inorder بعد از اضافه شدن نود ۵



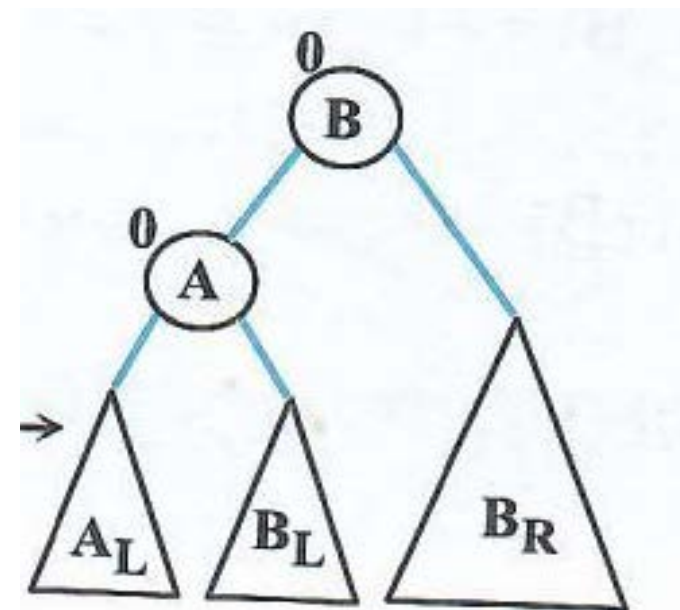
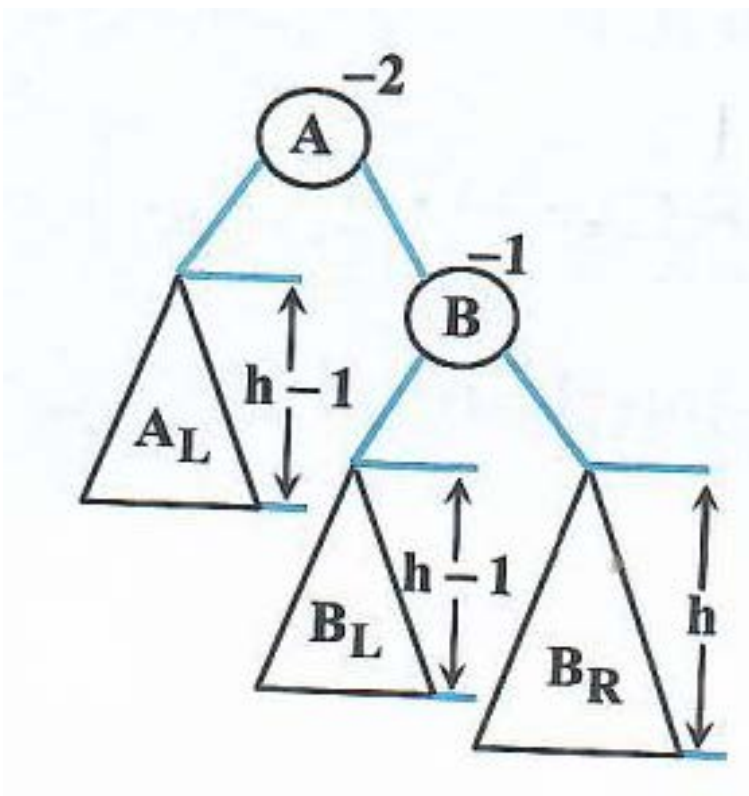


L Rotation



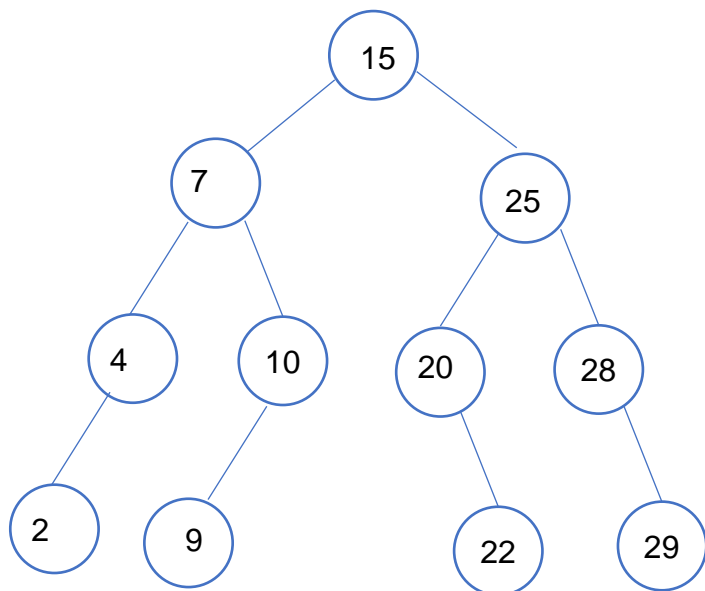


L Rotation





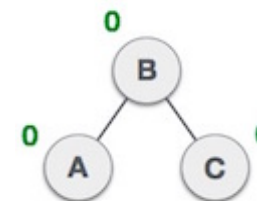
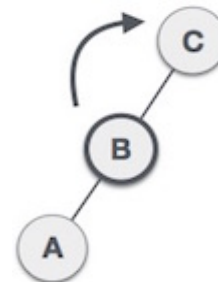
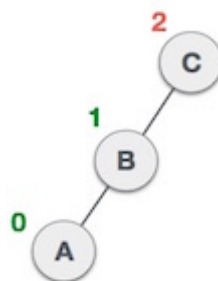
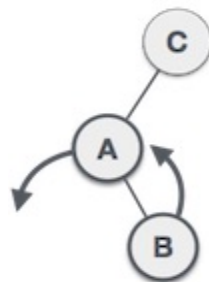
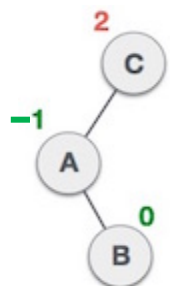
L Rotation- Example



- پیمایش postorder بعد از اضافه شدن نود 30

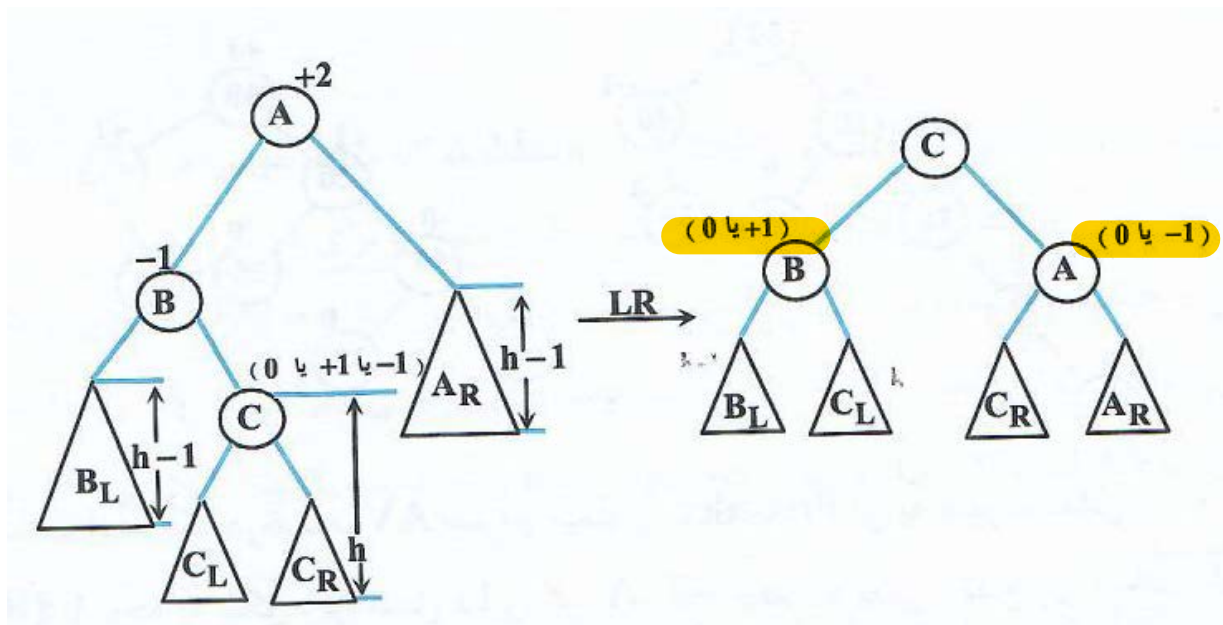


LR Rotation



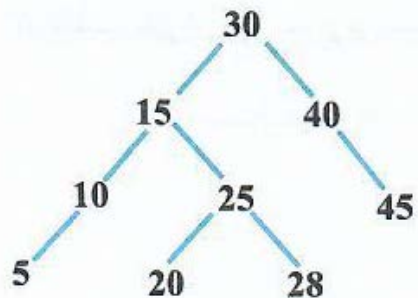


LR Rotation

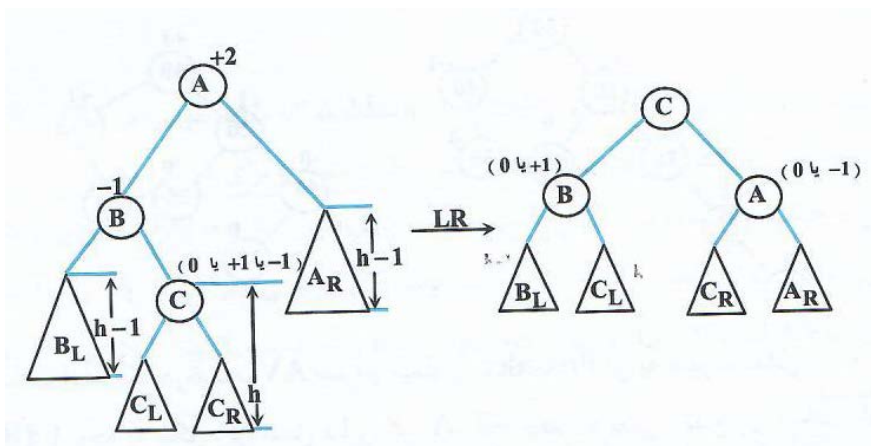




LR Rotation- Example

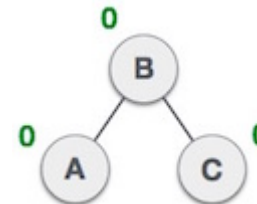
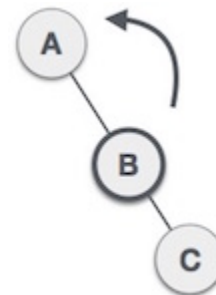
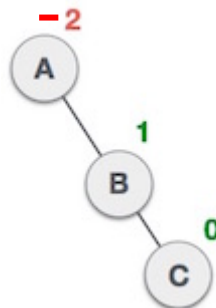
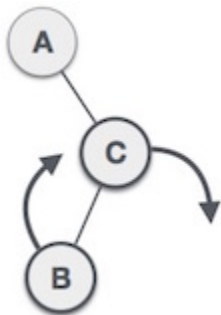
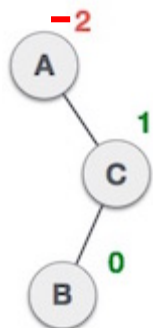


- پیمایش postorder بعد از اضافه شدن نود 29



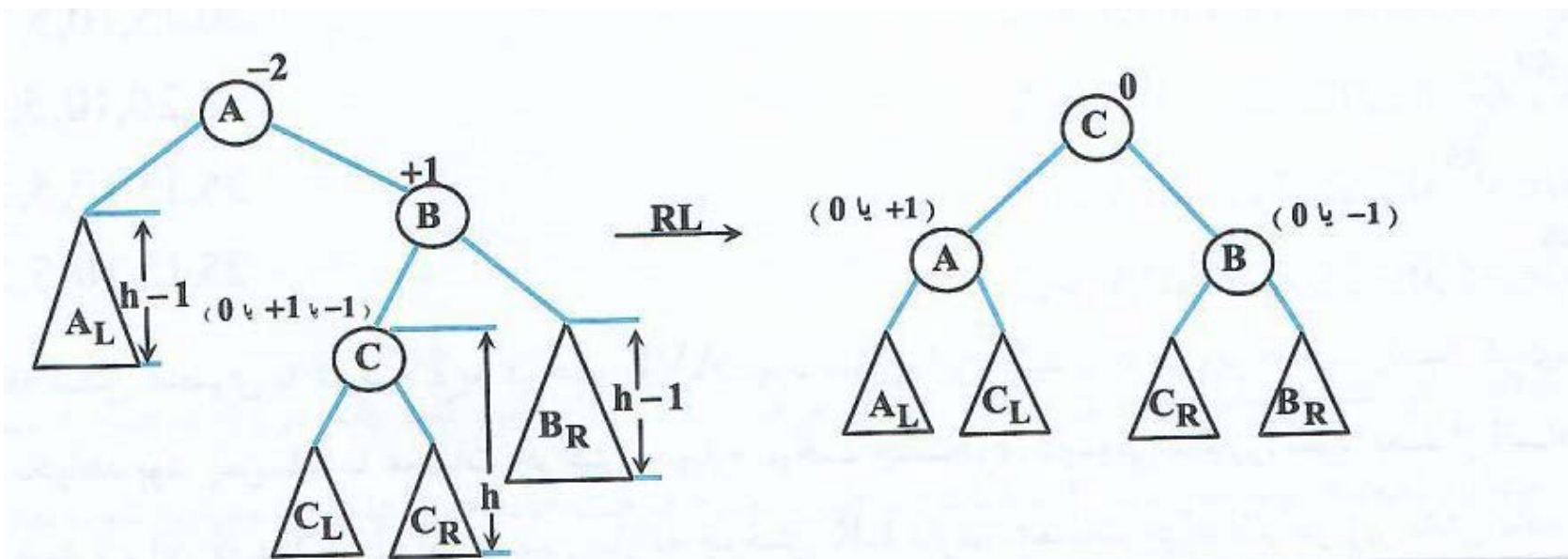


RL Rotation



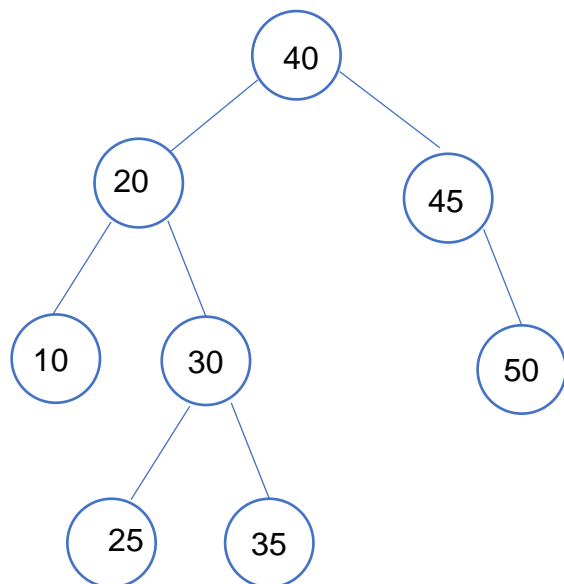


RL Rotation





RL Rotation-Example



- پیمایش preorder بعد از اضافه شدن نود 27