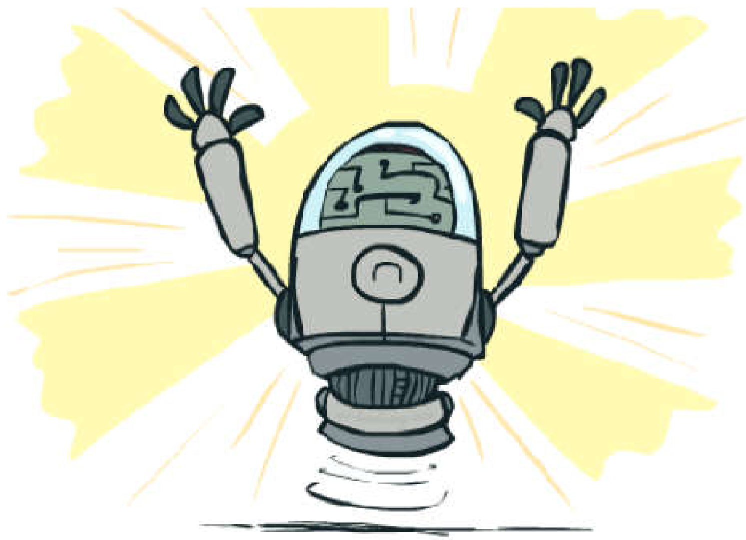
Improving Backtracking

- General-purpose ideas give huge gains in speed

- Ordering:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - [In what order should constraints be checked?]

- Filtering: Can we detect inevitable failure early?
- Structure: Can we exploit the problem structure?

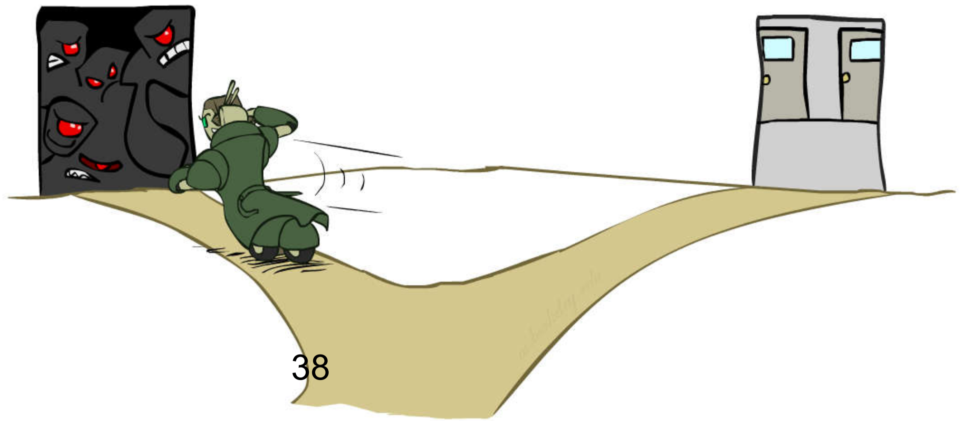


Ordering

Ordering: Minimum Remaining Values

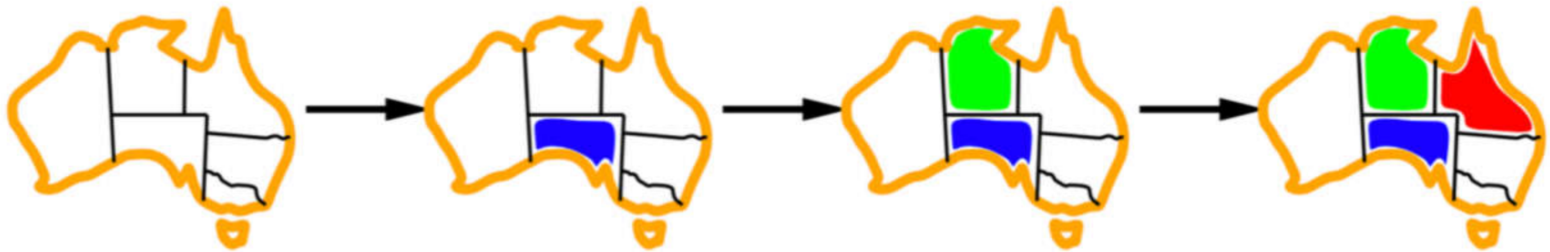
- Variable Ordering: Minimum remaining values (MRV):
  - Choose the variable with the fewest legal left values in its domain

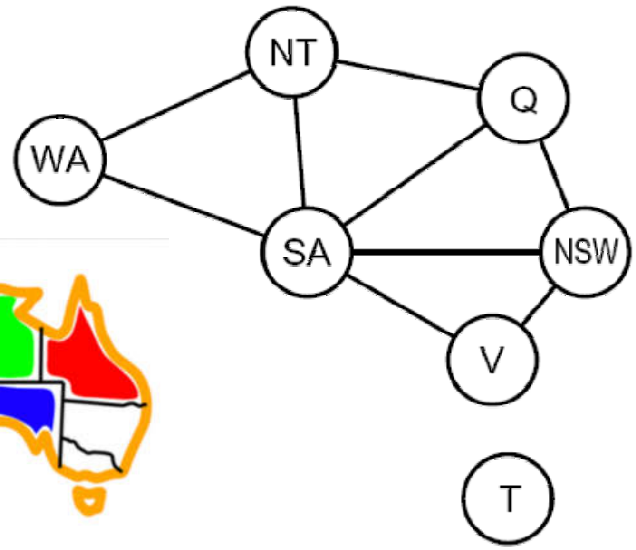
- Why min rather than max?
  - Find bad assignment quickly(fast converge)
- Also called "most constrained variable"
- "Fail-fast" ordering



Ordering: Degree heuristic

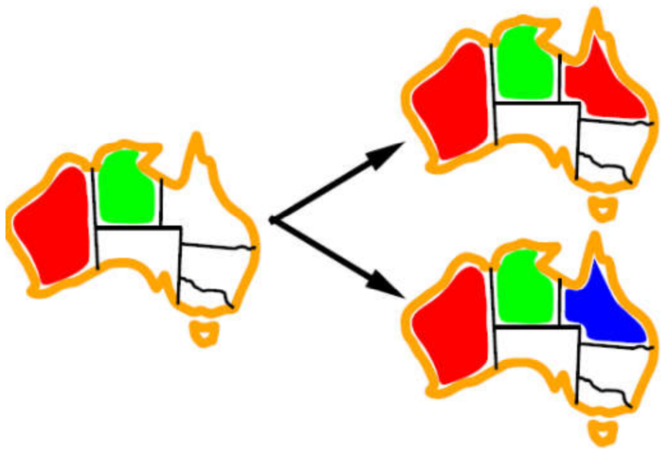
- Variable Ordering: when MRV are same
- Choose the variable with the most constraints on remaining variables





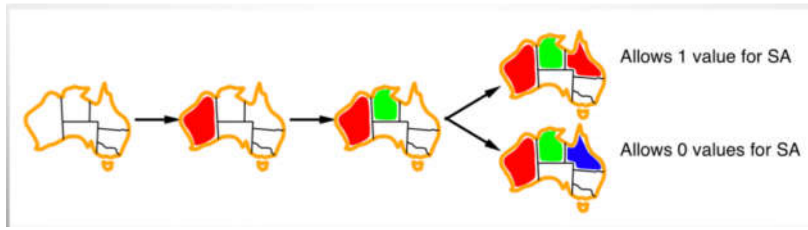
Ordering: Least Constraining Value

- Value Ordering: Least Constraining Value
  - Given a choice of variable, choose the *least constraining value*
  - I.e., the one that rules out the fewest values in the remaining variables
  - Note that it may take some computation to determine this! (E.g., rerunning filtering)



- Why least rather than most?

- Combining these ordering ideas makes 1000 queens feasible

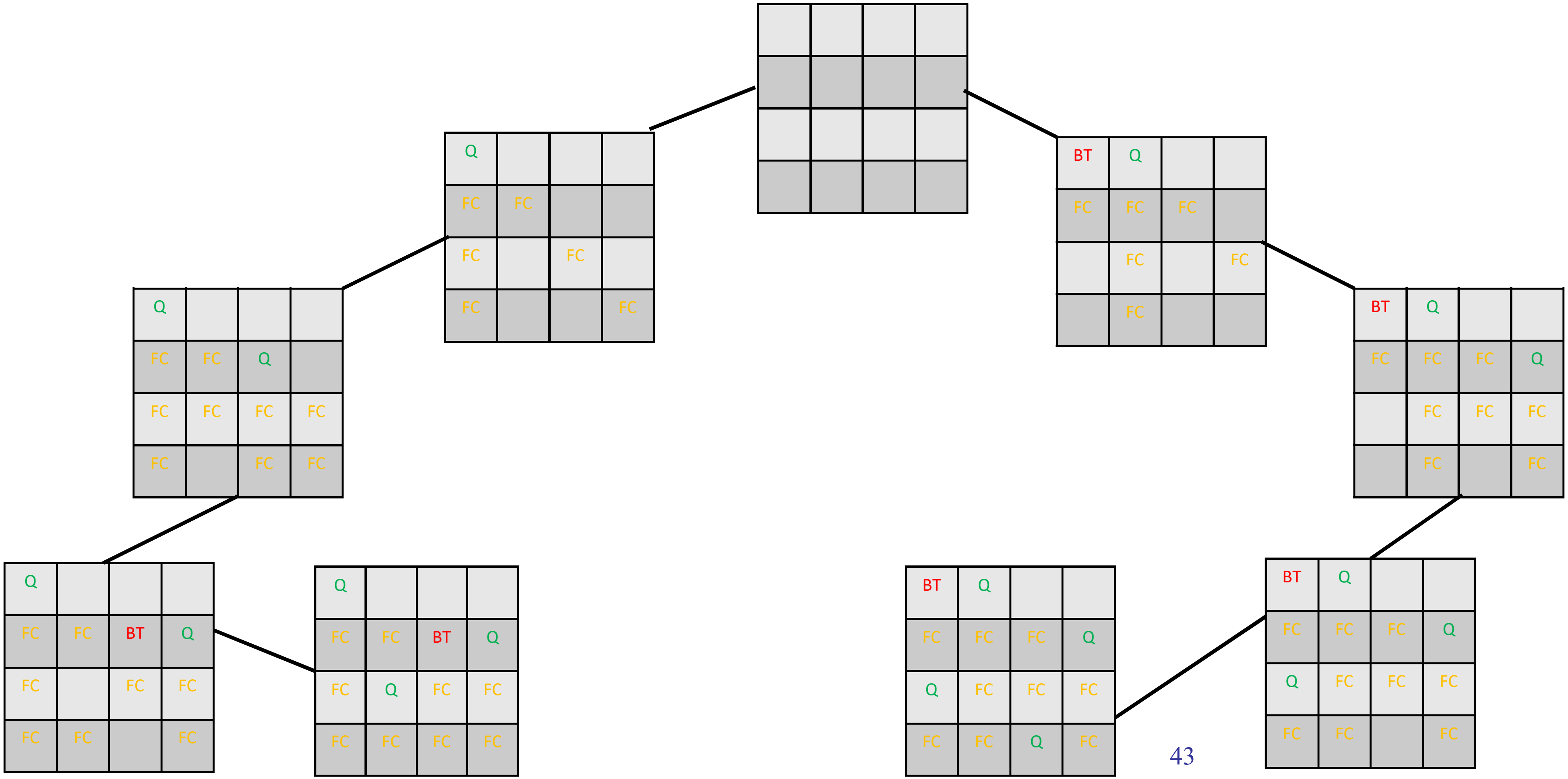


Filtering

Filtering: Forward Checking

- Idea1:
  - Filtering: Keep track of domains for unassigned variables and cross off bad options
  - Forward checking: Cross off values that violate a constraint when added to the existing assignment
  - Stop when there is no legal option for domains of a unassigned variable(save time just one step)

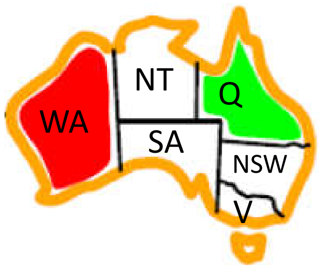
Forward Checking

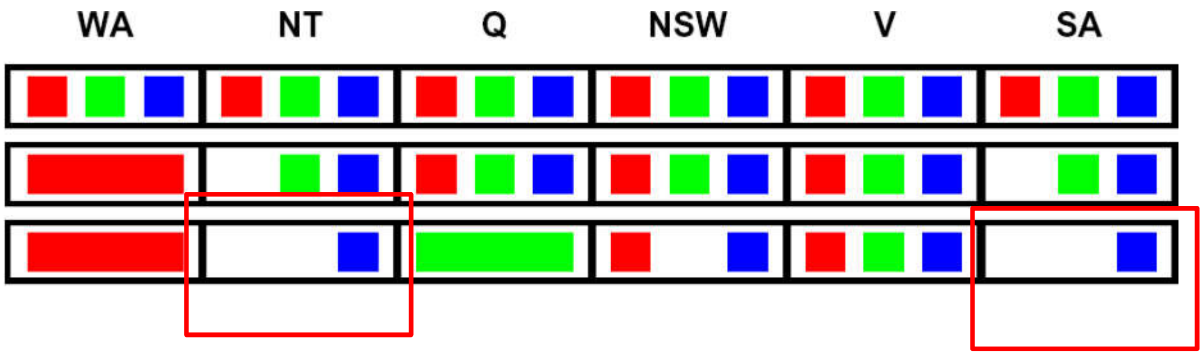


[40-417, SUT]

Filtering: Constraint Propagation

- **Forward checking** propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:
  - NT and SA cannot both be blue!
  - Why didn't we detect this yet?

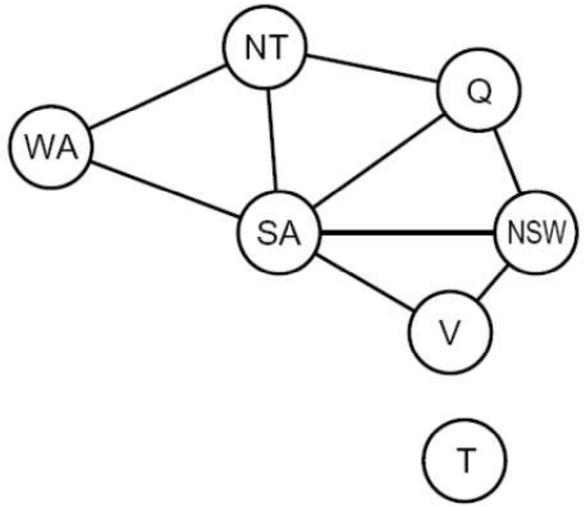




- Idea2: check constraint in next unassigned variable(save more time)
- *Constraint propagation:* reason from constraint to constraint

Consistency of A Arc

- The idea of forward checking can be generalized into
  the principle of arc consistency
- Each undirected edge of the constraint graph as two directed edges pointing in opposite directions.
- Each of these directed edges is called an arc.

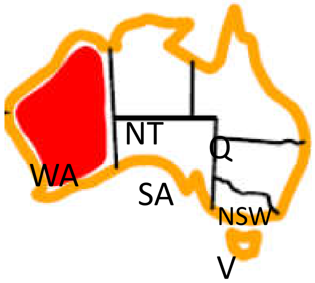


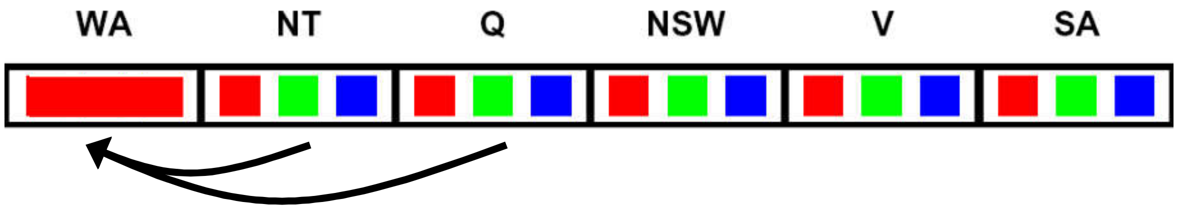
- Arc -> constraint check direction

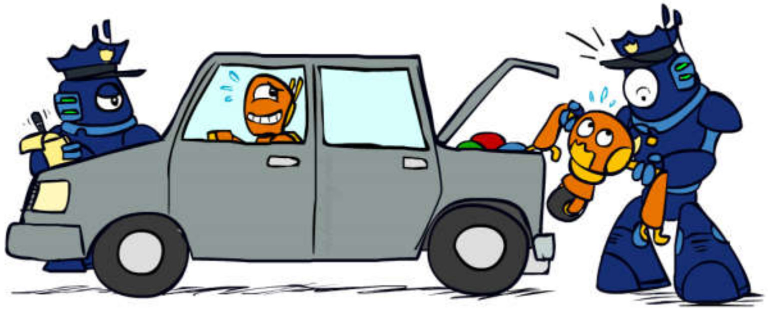
- adding all arcs between unassigned variables
  sharing a constraint to a queue Q

Consistency of A Single Arc

- An arc X $\rightarrow$ Y is consistent iff for *every* x there is *some* y which could be assigned without violating a constraint





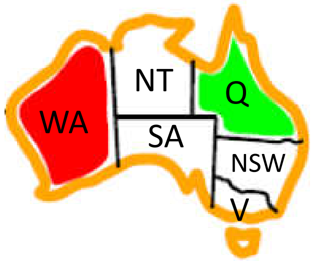


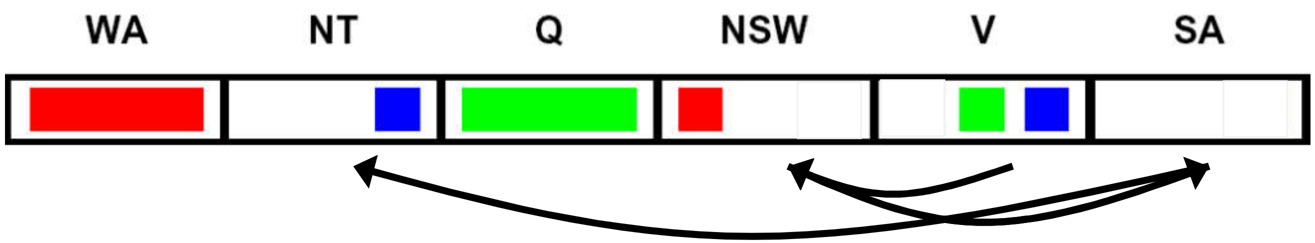
Delete from the tail!

- Forward checking: Enforcing consistency of arcs pointing to each new assignment

Arc Consistency of an Entire CSP

- A simple form of propagation makes sure all arcs are simultaneously consistent:





- Arc consistency detects failure earlier than forward checking
- Important: If X loses a value, neighbors of X need to be rechecked!
- Can be run as a preprocessor or after each assignment
- What's the downside of enforcing arc consistency?

Remember: Delete from the tail!

Enforcing Arc Consistency in a CSP

function AC-3(csp) **returns** the CSP, possibly with reduced domains
  inputs: csp, a binary CSP with variables $\{X_1,\ X_2,\ \ldots,\ X_n\}$
  local variables: $queue$, a queue of arcs, initially all the arcs in csp

  while $queue$ is not empty **do**
    $(X_i,\ X_j) \leftarrow$ REMOVE-FIRST($queue$)
    if REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **then**
      for each X_k **in** NEIGHBORS[X_i] **do**
        add $(X_k,\ X_i)$ to $queue$

---

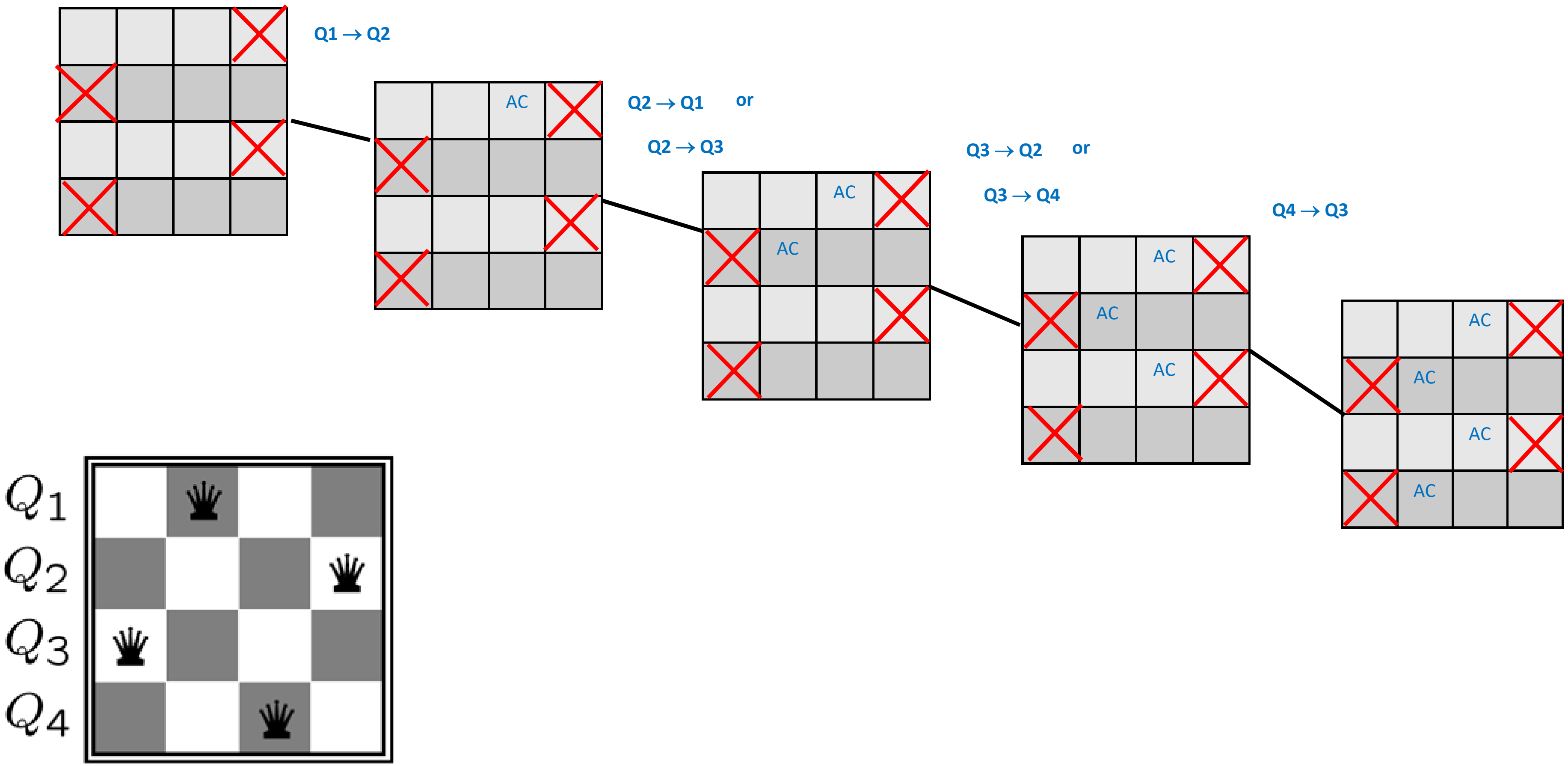
function REMOVE-INCONSISTENT-VALUES($X_i,\ X_j$) **returns** true iff succeeds
  $removed \leftarrow false$
  for each x **in** DOMAIN[X_i] **do**
    if no value y in DOMAIN[X_j] allows (x,y) to satisfy the constraint $X_i \leftrightarrow X_j$
      then delete x from DOMAIN[X_i]; $removed \leftarrow true$
  return $removed$

- Runtime: $O(n^2d^3)$, can be reduced to $O(n^2d^2)$
- … but detecting all possible future problems is NP-hard – why?

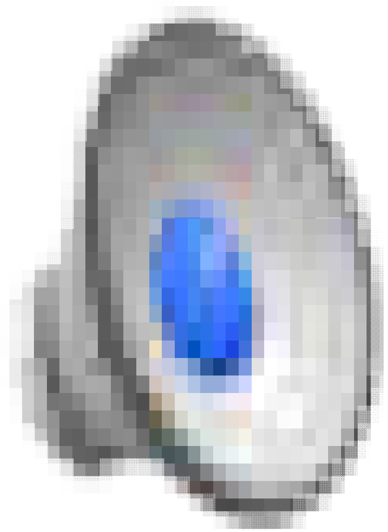
AC-3: time complexity

- Time complexity (n variables, c binary constraints, d domain size): $O(cd^3)$
  - Each arc (X_k, X_i) is inserted in the queue at most d times.
    - At most all values in domain X_i can be deleted.
  - Checking consistency of an arc: $O(d^2)$

Arc Consistency



Video of Demo Arc Consistency – CSP Applet – n Queens



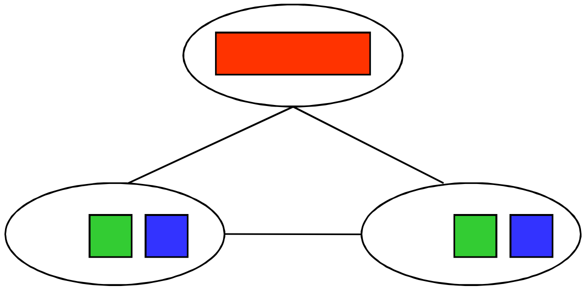
Ordering: Most-constraining-constraint

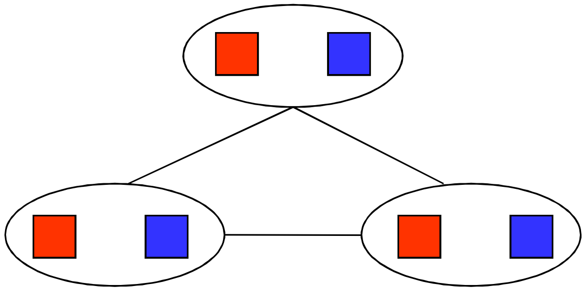
- Constraint Ordering: Most Constraining constraint
- Prefer testing constraints that are more difficult to
- Called Fail First Principle = “Fail-fast” ordering

Limitations of Arc Consistency

- After enforcing arc consistency:
  - Can have one solution left
  - Can have multiple solutions left
  - Can have no solutions left (and not know it)

- Arc consistency still runs inside a backtracking search!



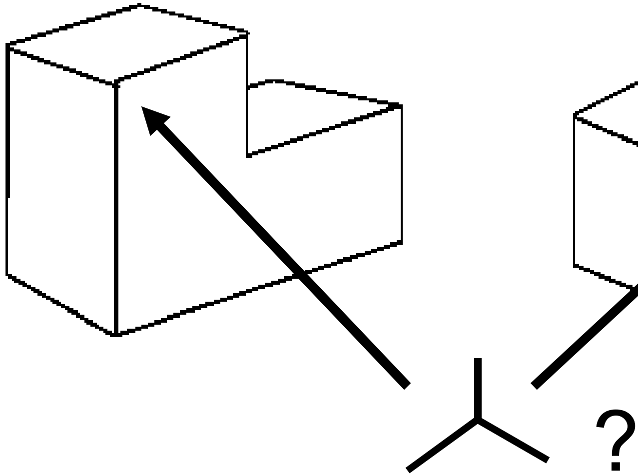


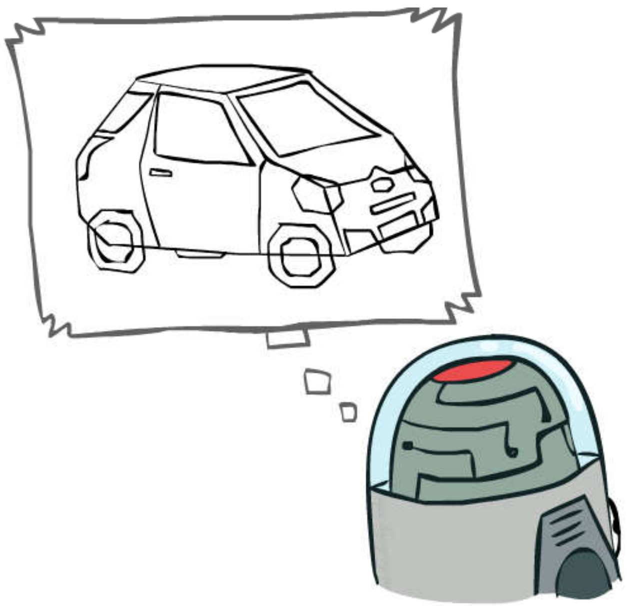
What went wrong here?

[Demo: coloring -- forward checking]
[Demo: coloring -- arc consistency]

Example: The Waltz Algorithm

- The Waltz algorithm is for interpreting line drawings of solid polyhedra as 3D objects
- An early example of an AI computation posed as a CSP





- Approach:
  - Each intersection is a variable
  - Adjacent intersections impose constraints on each other
  - Solutions are physically realizable 3D interpretations

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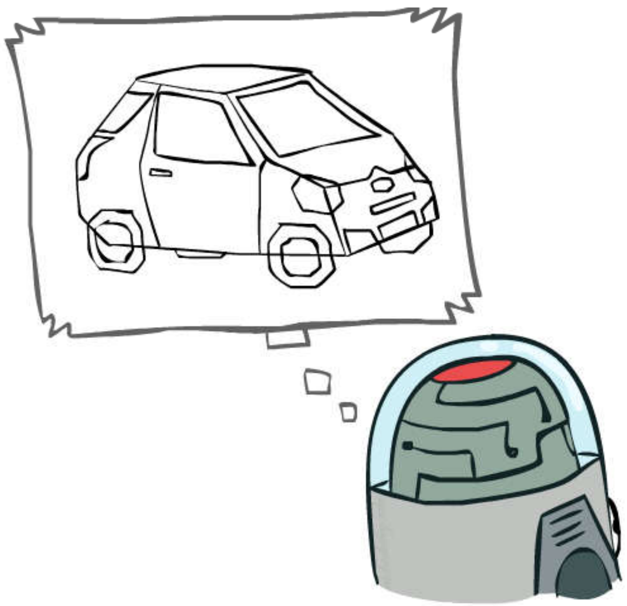

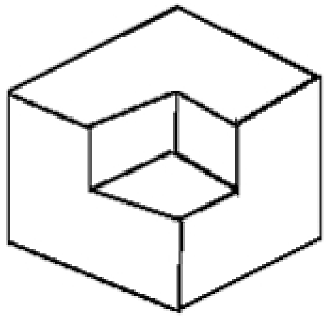

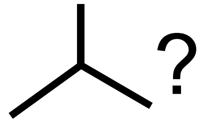

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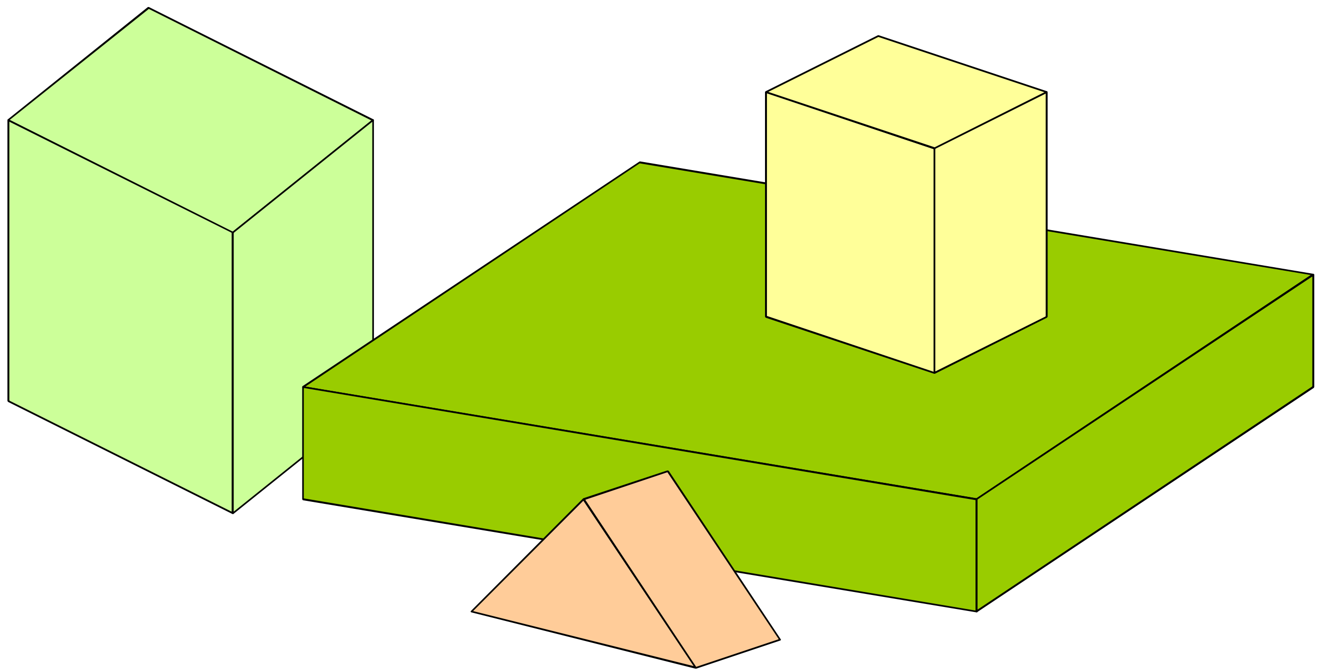
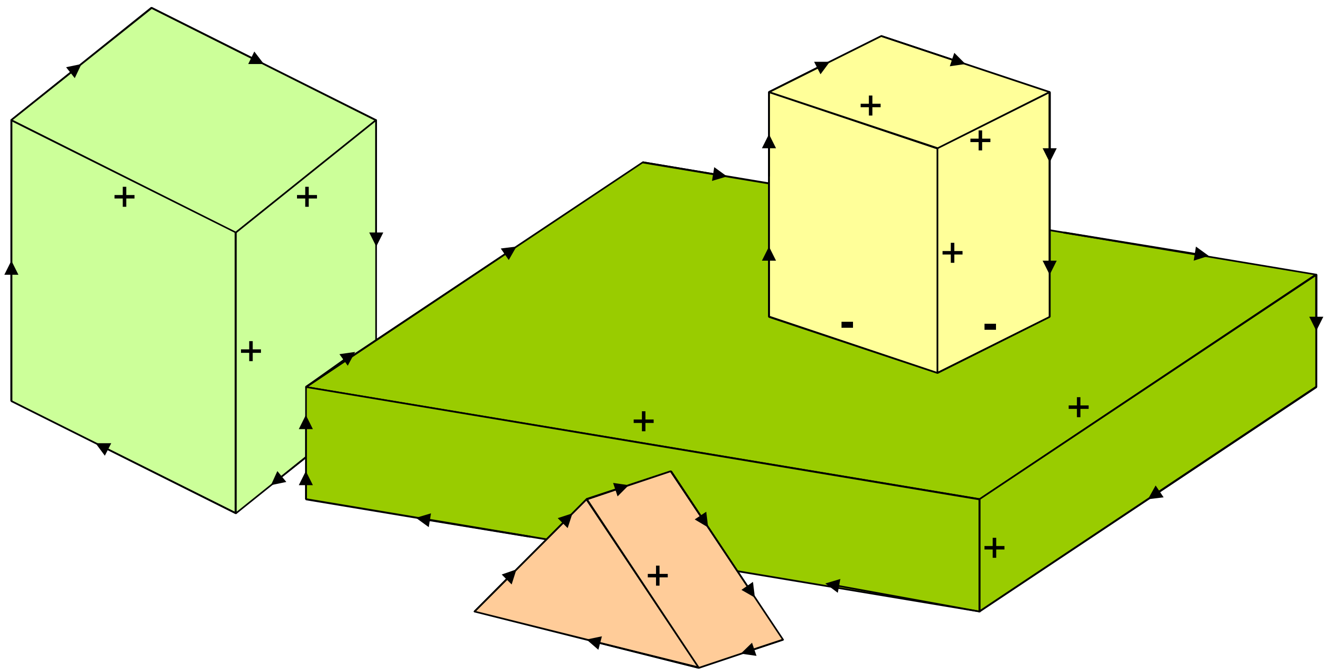
Edge Labeling in Computer Vision



Edge Labeling in Computer Vision



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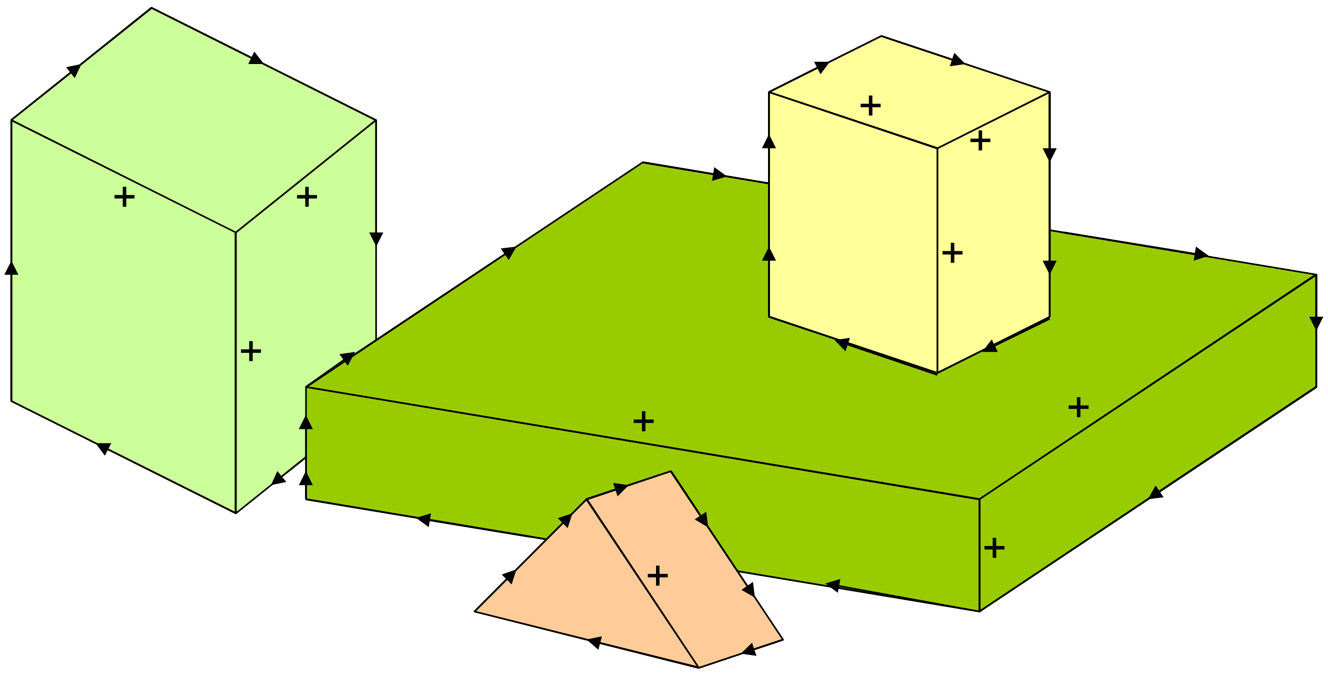
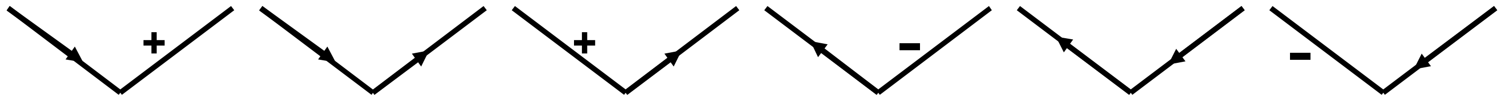


Labels of Edges

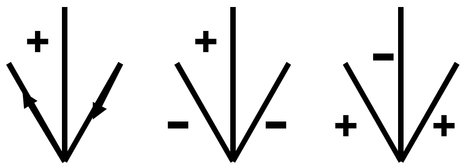
- Convex edge:
  - two surfaces intersecting at an angle greater than 180°
- Concave edge
  - two surfaces intersecting at an angle less than 180°
- + convex edge, both surfaces visible
- − concave edge, both surfaces visible
- ← convex edge, only one surface is visible (i.e., on the right side of ←)

Junction Label Sets

L-junctions

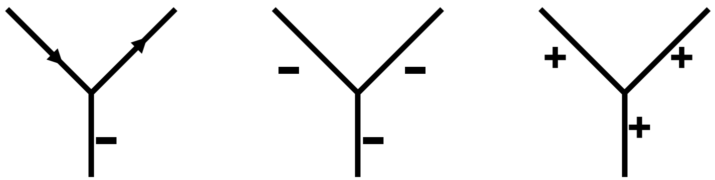


Arrow-junctions

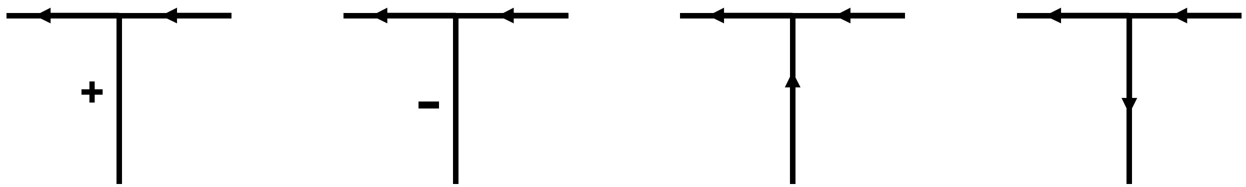


From the total of 208 ($4^2 + 4^3 + 4^3 + 4^3$) possible cases, only 16 is valid

Y-junctions



T-junctions

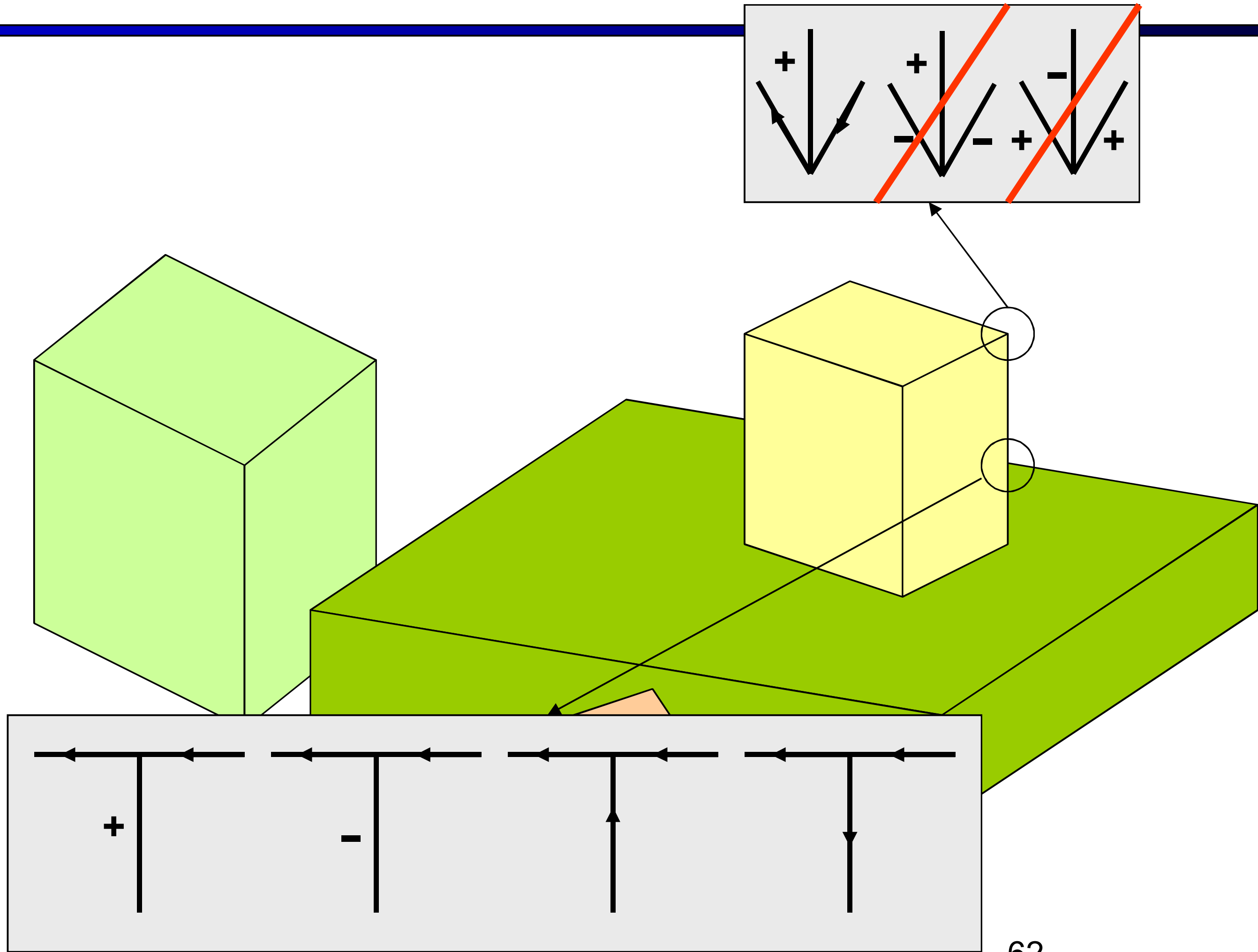


(Waltz, 1975; Mackworth, 1977)

Edge Labeling as a CSP

- A variable is associated with each junction
- The domain of a variable is the label set of the corresponding junction
- Each constraint imposes that the values given to two adjacent junctions give the same label to the joining edge

Edge labeling example



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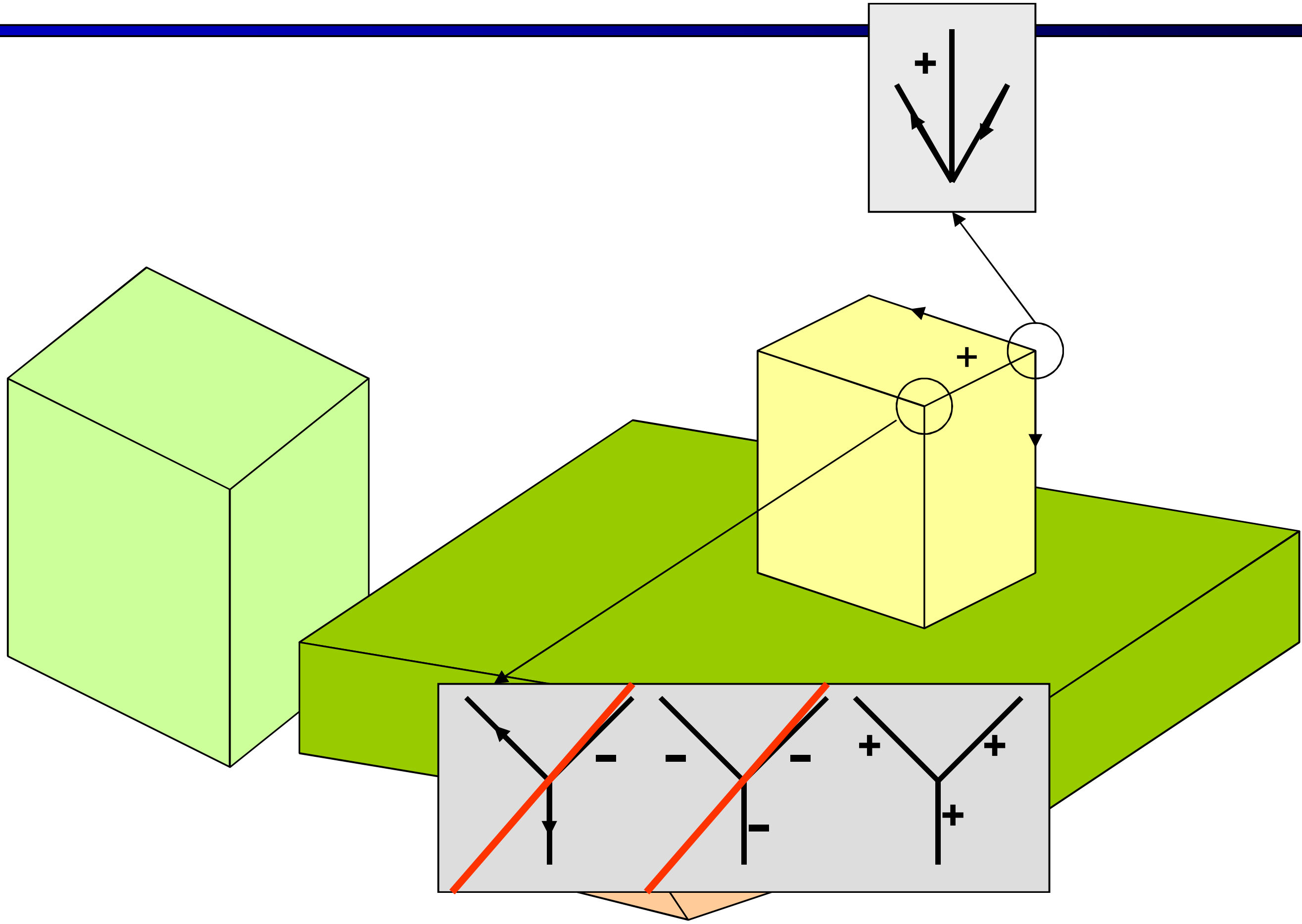

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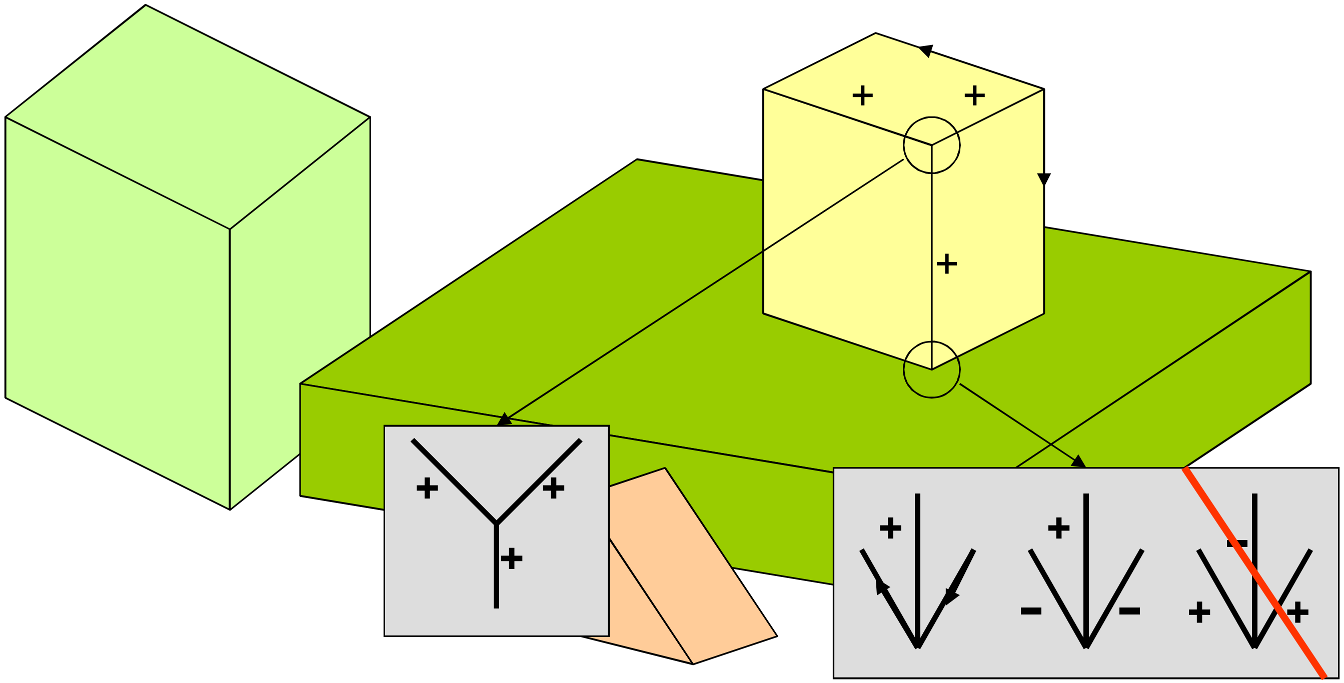

Edge labeling example