Subject: Data: Page:

۱← می دانیم $x(t)$ حقیقی است ←

$$Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} \xleftrightarrow{FT} Re\{x(j\omega)\}$$

$$LFT\{Re\{x(j\omega)\}\} = |t|e^{-|t|}$$

$$\text{So} \rightarrow Ev\{x(t)\} = \frac{x(t)+x(-t)}{2} = |t|e^{-|t|}$$

برای $t \le 0$ ← $x(t)=0$ ← برای $t>0$ ← $x(-t)$ ← پس

$$x(t) = 2|t|e^{-|t|} \quad for \quad t \ge 0$$

$$x(t) = 2te^{-t}u(t)$$

۲← $x(3t) \xleftrightarrow{FT} \frac{1}{3}X\left(\frac{j\omega}{3}\right)$ ، $h(3t) \xleftrightarrow{FT} \frac{1}{3}H\left(\frac{j\omega}{3}\right)$

$$G(j\omega) = FT\{x(3t) * h(3t)\} = \frac{1}{9}X\left(\frac{j\omega}{3}\right)H\left(\frac{j\omega}{3}\right)$$

$$y(j\omega) = FT\{x(t) * h(t)\} = X(j\omega)H(j\omega)$$

$$\rightarrow y\left(\frac{j\omega}{3}\right) = X\left(\frac{j\omega}{3}\right)H\left(\frac{j\omega}{3}\right) \rightarrow G(j\omega) = \frac{1}{9}y\left(\frac{j\omega}{3}\right)$$

$$\rightarrow g(t) = \frac{1}{3}y(3t) \Rightarrow B=3 \text{ ، } A=\frac{1}{3}$$

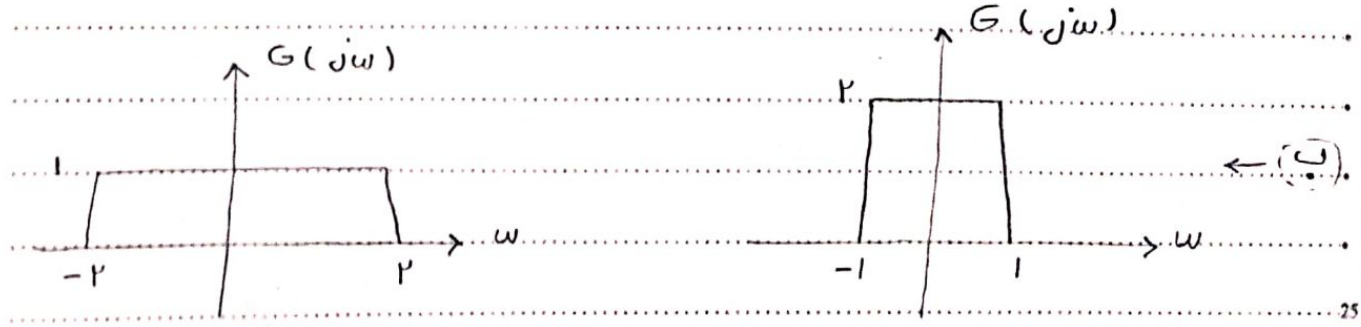
← ۳

← ۴

الف) $w(t) = \cos t \xleftrightarrow{FT} w(j\omega) = \pi [\delta(\omega-1) + \delta(\omega+1)]$

$g(t) = x(t)\cos t \xleftrightarrow{FT} G(j\omega) = \frac{1}{2\pi} [x(j\omega) * w(j\omega)]$

$\rightarrow G(j\omega) = \frac{1}{2} x(j(\omega-1)) + \frac{1}{2} x(j(\omega+1))$



← ب

$\rightarrow x(t) = \frac{2\sin}{\pi t}$

۵ ← جواب های نهایی :

الف) $x(t) = \begin{cases} e^{j2\pi t} & |t|<2 \\ 0 & \text{other} \end{cases}$

ب) $x(t) = \frac{1}{2} e^{-j\frac{\pi}{3}} \delta(t-4) + \frac{1}{2} e^{j\frac{\pi}{3}} \delta(t+4)$

ج) $x(t) = \frac{2j}{\pi} \sin t + \frac{3}{\pi} \cos(2\pi t)$

۶ ←

الف) $x_1(jw) = \pi e^{j\frac{\pi}{12}} \delta(w-4) + \pi e^{j\frac{\pi}{12}} \delta(w+4)$

$y_1(jw) = x_1(jw) H(jw) = 0 \longrightarrow y_1(t) = 0$

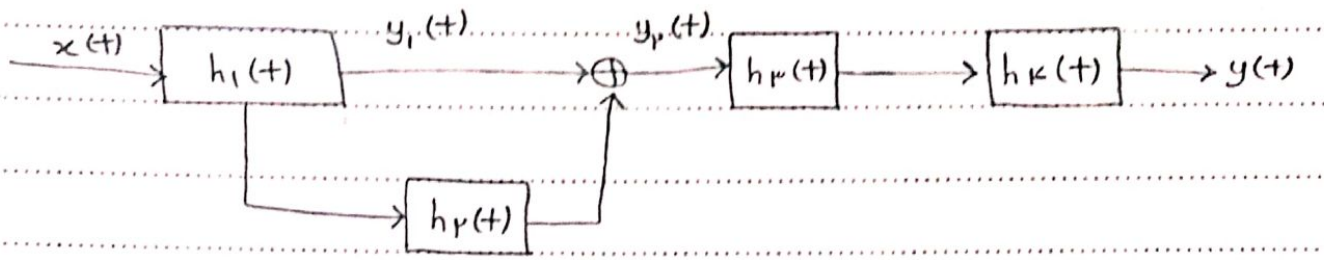
$H(jw) = \begin{cases} e^{-jw} & |w|<4 \\ 0 & \text{other} \end{cases}$

ب) $x_2(jw) = \frac{\pi}{j} \left[\sum_{k=0}^{\infty} \left(\frac{1}{2}\right) \{ \delta(w-3k) - \delta(w+3k) \} \right]$

$\longrightarrow y_2(jw) = x_2(jw) H(jw) = \frac{\pi}{j} \left[\left(\frac{1}{2}\right) \{ \delta(w-3) - \delta(w+3) \} e^{-jw} \right]$

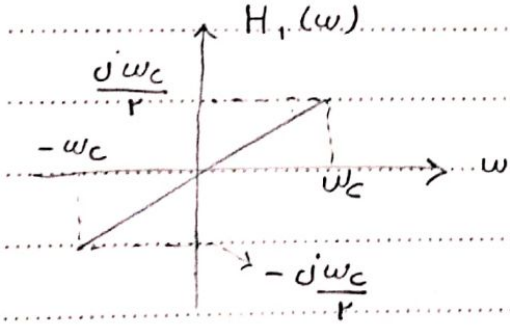
$\longrightarrow y_2(t) = \frac{1}{2} \sin(3t-1)$

← V.



$$\frac{\sin \omega_0 t}{\pi t} \longleftrightarrow \frac{1}{2} \text{rect}\left(\frac{\omega}{2\omega_c}\right)$$

$$h_1(t) = \frac{d}{dt}\left[\frac{\sin(\omega_0 t)}{\pi t}\right] \longleftrightarrow \frac{j\omega}{2} \text{rect}\left(\frac{\omega}{2\omega_c}\right)$$



$$x(t) = \sin(2\omega_c t) + \cos\left(\frac{1}{2}\omega_c t\right) \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega - 2\omega_c) - \delta(\omega + 2\omega_c)\right] + \pi\left[\delta\left(\omega - \frac{\omega_c}{2}\right) + \delta\left(\omega + \frac{\omega_c}{2}\right)\right]$$

$$y_1(\omega) = x(\omega) H_1(\omega) = \frac{j\omega_c}{2}\pi\,\delta\left(\omega - \frac{\omega_c}{2}\right) - \frac{j\omega_c}{2}\pi\,\delta\left(\omega + \frac{\omega_c}{2}\right) = \frac{-\omega_c}{2j}\pi\left[\delta\left(\omega - \frac{\omega_c}{2}\right) - \delta\left(\omega + \frac{\omega_c}{2}\right)\right] \Longrightarrow y(t) = \frac{-\omega_c}{2}\sin\left(\frac{\omega_c}{2}t\right)$$

$$y_p(w) = y_1(w)\left[1 + H_p(w)\right] = \frac{-w_c}{2j}\pi\left[\delta\left(w - \frac{w_c}{2}\right) - \delta\left(w + \frac{w_c}{2}\right)\right]\left[1 + e^{\frac{-j2\pi w}{w_c}}\right] =$$

$$\frac{-w_c}{2j}\pi\left[\underbrace{\left(1 + e^{-j\pi}\right)}_{0}\delta\left(w - \frac{w_c}{2}\right) - \underbrace{\left(1 + e^{+j\pi}\right)}_{0}\delta\left(w + \frac{w_c}{2}\right)\right]$$

$$= 0$$

$$\longrightarrow y_2(w) = 0 \longrightarrow y(w) = 0 \longrightarrow y(t) = 0$$

← ۱

الف) از ۲ طرف تبدیل فوریه می‌گیریم // $H(jw) = \frac{y(jw)}{x(jw)} = \frac{2}{-w^2 + 2jw + 8} =$

$$\frac{1}{jw+2} - \frac{1}{jw+4} \xrightarrow[\text{می‌گیریم}]{\text{عکس فوریه}} h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

ب) $x(jw) = \frac{1}{(2+jw)^2}$

$$y(jw) = x(jw)H(jw) = \frac{2}{-w^2 + 2jw + 8} \times \frac{1}{(2+jw)^2}$$

$$y(t) = \frac{1}{4}e^{-2t}u(t) - \frac{1}{2}te^{-2t}u(t) + t^2e^{-2t}u(t) - \frac{1}{4}e^{-4t}u(t)$$