دانشکده برق و کامپیوتر

به نام خدا

فصل دوم: آشنایی با سلف و خازن و تحلیل مدارهای مرتبه اول RL و RC

ارائه کننده:

روحانی

آبان ۱۴۰۰

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تعاریف اولیه

خازن الکتریکی

$$i = \frac{dq}{dt} \qquad \boxed{i = C\frac{dv}{dt}} \qquad [1]$$

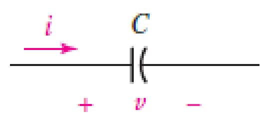

FIGURE 7.1 Electrical symbol and current-voltage conventions for a capacitor.

where v and i satisfy the conventions for a passive element, as shown in Fig. 7.1. We should bear in mind that v and i are functions of time; if needed, we can emphasize this fact by writing $v(t)$ and $i(t)$ instead. From Eq. [1], we may determine the unit of capacitance as an ampere-second per volt, or c$C = \varepsilon A/d$,olt. We will now define the ***farad***[1] (F) as one coulomb per volt, and use this as our unit of capacitance.

(a) (b) (c)

FIGURE 7.2 Several examples of commercially available capacitors. (*a*) Left to right: 270 pF ceramic, 20 μF tantalum, 15 nF polyester, 150 nF polyester. (*b*) Left: 2000 μF 40 VDC rated electrolytic, 25,000 μF 35 VDC rated electrolytic. (*c*) Clockwise from smallest: 100 μF 63 VDC rated electrolytic, 2200 μF 50 VDC rated electrolytic, 55 F 2.5 VDC rated electrolytic, and 4800 μF 50 VDC rated electrolytic. Note that generally speaking larger capacitance values require larger packages, with one notable exception above. What was the tradeoff in that case?

$$C = \varepsilon A/d,$$

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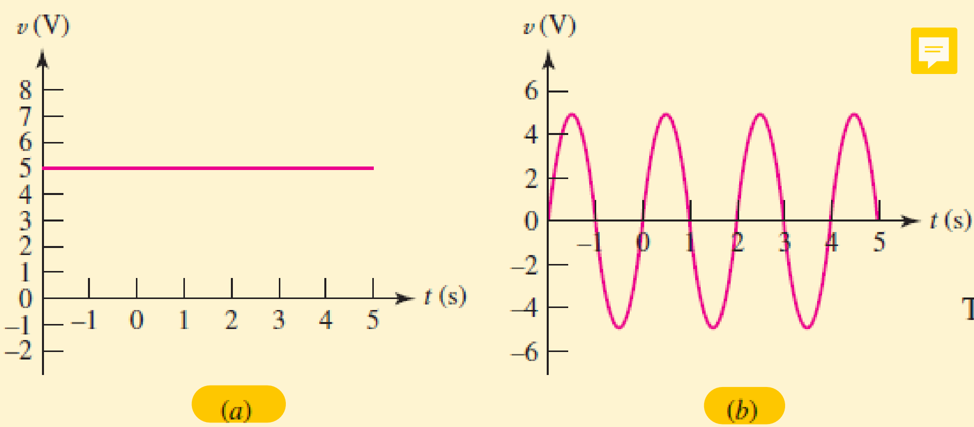
تعاریف اولیه: خازن
مثال

Determine the current i flowing through the capacitor of Fig. 7.1 for the two voltage waveforms of Fig. 7.3 if $C = 2$ F.

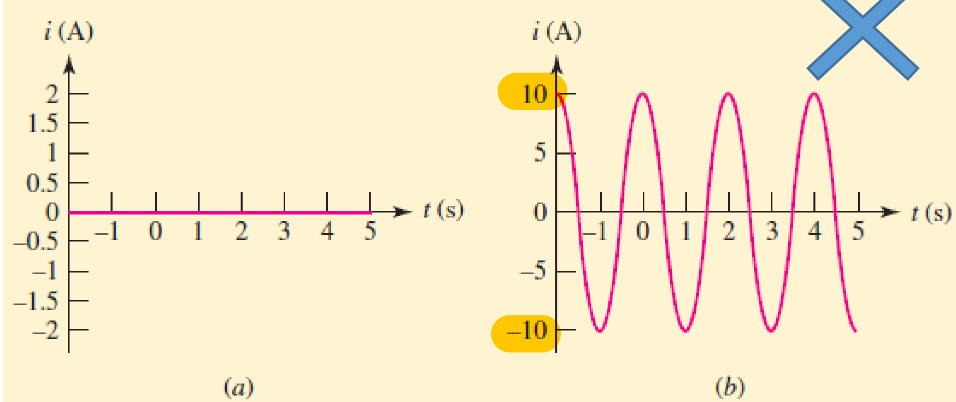


■ **FIGURE 7.3** (*a*) A dc voltage applied to the terminals of the capacitor. (*b*) A sinusoidal voltage waveform applied to the capacitor terminals.

The current i is related to the voltage v across the capacitor by Eq. [1]:

$$i = C\frac{dv}{dt}$$

For the voltage waveform depicted in Fig. 7.3*a*, $dv/dt = 0$, so $i = 0$; the result is plotted in Fig. 7.4*a*. For the case of the sinusoidal waveform of Fig. 7.3*b*, we expect a cosine current waveform to flow in response, having the same frequency and twice the magnitude (since $C = 2$ F). The result is plotted in Fig. 7.4*b*.



■ **FIGURE 7.4** (*a*) $i = 0$ as the voltage applied is dc. (*b*) The current has a cosine form in response to a sine wave voltage.

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The capacitor voltage may be expressed in terms of the current by integrating Eq. [1]. We first obtain

$$dv = \frac{1}{C} i(t)\, dt$$

and then integrate[2] between the times t_0 and t and between the corresponding voltages $v(t_0)$ and $v(t)$:

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\, dt' + v(t_0)} \qquad [2]$$

Equation [2] may also be written as an indefinite integral plus a constant of integration:

$$v(t) = \frac{1}{C}\int i\, dt + k$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i\, dt'$$

Since the integral of the current over any time interval is the corresponding charge accumulated on the capacitor plate into which the current is flowing, we may also define capacitance as

$$q(t) = Cv(t)$$

where $q(t)$ and $v(t)$ represent instantaneous values of the charge on either plate and the voltage between the plates, respectively.

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$$p = vi = Cv\frac{dv}{dt}$$

The change in energy stored in its electric field is simply

$$\int_{t_0}^{t} p\,dt' = C\int_{t_0}^{t} v\frac{dv}{dt'}\,dt' = C\int_{v(t_0)}^{v(t)} v'\,dv' = \frac{1}{2}C\left\{[v(t)]^2 - [v(t_0)]^2\right\}$$

and thus

$$w_C(t) - w_C(t_0) = \tfrac{1}{2}C\left\{[v(t)]^2 - [v(t_0)]^2\right\} \qquad [3]$$

where the stored energy is $w_C(t_0)$ in joules (J) and the voltage at t_0 is $v(t_0)$. If we select a zero-energy reference at t_0, implying that the capacitor voltage is also zero at that instant, then

$$\boxed{w_C(t) = \tfrac{1}{2}Cv^2} \qquad [4]$$

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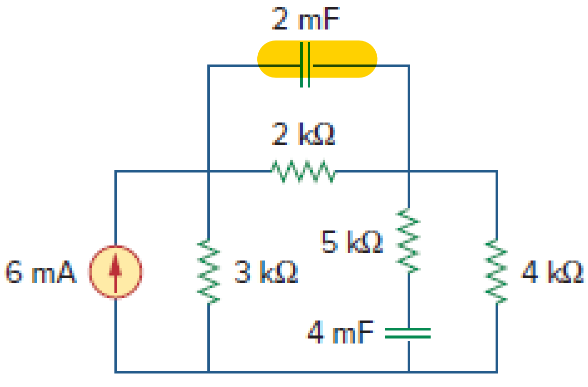
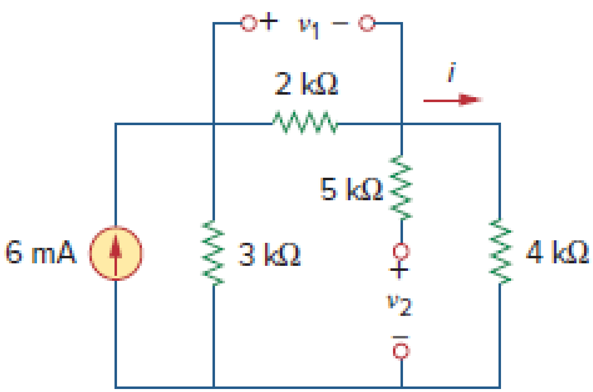
انرژی ذخیره شده میان صفحات خازن: مثال

Example 6.5

Obtain the ener gy stored in each capacitor in Fig. 6.12(a) under dc conditions.



(a)



(b)

Solution:

Under dc conditions, we replace each capacitor with an open circuit, as shown in Fig. 6.12(b). The current through the series combination of the 2-kΩ and 4-kΩ resistors is obtained by current division as

$$i = \frac{3}{3+2+4}(6\text{ mA}) = 2\text{ mA}$$

Hence, the voltages v_1 and v_2 across the capacitors are

$$v_1 = 2000i = 4\text{ V} \qquad v_2 = 4000i = 8\text{ V}$$

and the energies stored in them are

$$w_1 = \frac{1}{2}C_1v_1^2 = \frac{1}{2}(2\times 10^{-3})(4)^2 = 16\text{ mJ}$$

$$w_2 = \frac{1}{2}C_2v_2^2 = \frac{1}{2}(4\times 10^{-3})(8)^2 = 128\text{ mJ}$$

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تعاریف اولیه

سلف الکتریکی

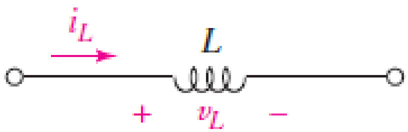
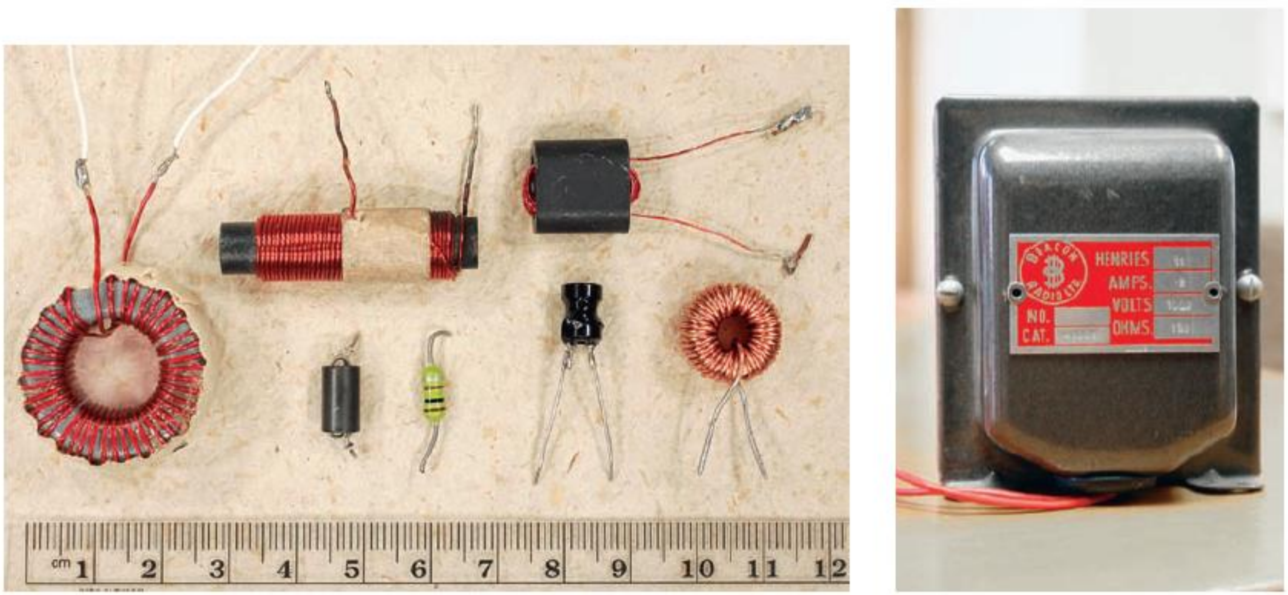


FIGURE 7.10 Electrical symbol and current-voltage conventions for an inductor.

$$v = L\frac{di}{dt}$$



(*a*) (*b*)

FIGURE 7.11 (*a*) Several different types of commercially available inductors, sometimes also referred to as "chokes." Clockwise, starting from far left: 287 μH ferrite core toroidal inductor, 266 μH ferrite core cylindrical inductor, 215 μH ferrite core inductor designed for VHF frequencies, 85 μH iron powder core toroidal inductor, 10 μH bobbin-style inductor, 100 μH axial lead inductor, and 7 μH lossy-core inductor used for RF suppression. (*b*) An 11 H inductor, measuring 10 cm (tall) $\times$ 8 cm (wide) $\times$ 8 cm (deep).

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تعاریف اولیه: سلف
مثال

Given the waveform of the current in a 3 H inductor as shown in Fig. 7.12*a*, determine the inductor voltage and sketch it.

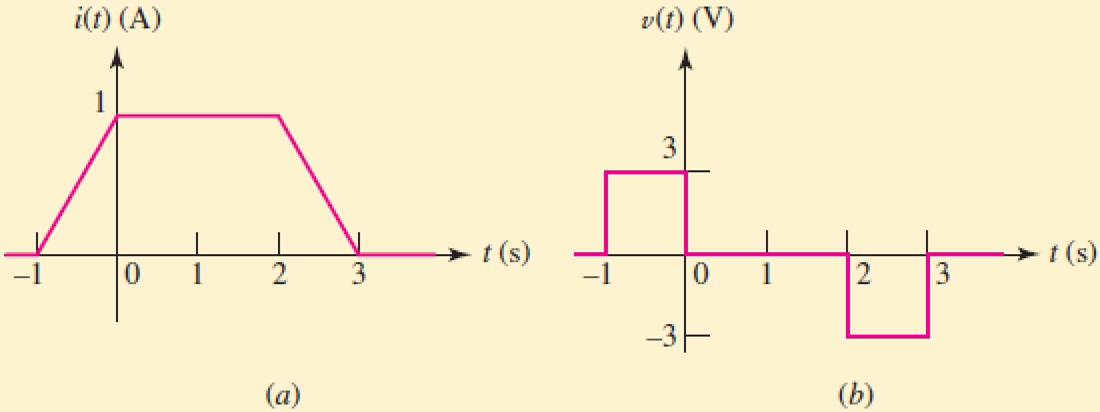

FIGURE 7.12 (*a*) The current waveform in a 3 H inductor. (*b*) The corresponding voltage waveform, $v = 3\,di/dt$.

Defining the voltage v and the current i to satisfy the passive sign convention, we may obtain v from Fig. 7.12*a* using Eq. [5]:

$$v = 3\frac{di}{dt}$$

Since the current is zero for $t < -1$ s, the voltage is zero in this interval. The current then begins to increase at the linear rate of 1 A/s, and thus a constant voltage of $L\,di/dt = 3$ V is produced. During the following 2 s interval, the current is constant and the voltage is therefore zero. The final decrease of the current results in $di/dt = -1$ A/s, yielding $v = -3$ V. For $t > 3$ s, $i(t)$ is a constant (zero), so that $v(t) = 0$ for that interval. The complete voltage waveform is sketched in Fig. 7.12*b*.

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We have defined inductance by a simple differential equation,

$$v = L\frac{di}{dt}$$

and we have been able to draw several conclusions about the characteristics of an inductor from this relationship. For example, we have found that we may consider an inductor to be a short circuit to direct current, and we have agreed that we cannot permit an inductor current to change abruptly from one value to another, because this would require that an infinite voltage and power be associated with the inductor. The simple defining equation for inductance contains still more information, however. Rewritten in a slightly different form,

$$di = \frac{1}{L} v\, dt$$

it invites integration. Let us first consider the limits to be placed on the two integrals. We desire the current i at time t, and this pair of quantities therefore provides the upper limits on the integrals appearing on the left and right sides of the equation, respectively; the lower limits may also be kept general by merely assuming that the current is $i(t_0)$ at time t_0. Thus,

$$\int_{i(t_0)}^{i(t)} di' = \frac{1}{L}\int_{t_0}^{t} v(t')\, dt'$$

which leads to the equation

$$i(t) - i(t_0) = \frac{1}{L}\int_{t_0}^{t} v\, dt'$$

$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\, dt' + i(t_0)} \qquad [6]$$

Equation [5] expresses the inductor voltage in terms of the current, whereas Eq. [6] gives the current in terms of the voltage. Other forms are

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Let us now turn our attention to power and energy. The absorbed power is given by the current-voltage product

$$p = vi = Li\,\frac{di}{dt}$$

The energy w_L accepted by the inductor is stored in the magnetic field around the coil. The change in this energy is expressed by the integral of the power over the desired time interval:

$$\int_{t_0}^{t} p\,dt' = L\int_{t_0}^{t} i\,\frac{di}{dt'}\,dt' = L\int_{i(t_0)}^{i(t)} i'\,di'$$
$$= \frac{1}{2}L\left\{[i(t)]^2 - [i(t_0)]^2\right\}$$

Thus,

$$w_L(t) - w_L(t_0) = \tfrac{1}{2}L\left\{[i(t)]^2 - [i(t_0)]^2\right\} \qquad [9]$$

where we have again assumed that the current is $i(t_0)$ at time t_0. In using the energy expression, it is customary to assume that a value of t_0 is selected at which the current is zero; it is also customary to assume that the energy is zero at this time. We then have simply

$$\boxed{w_L(t) = \tfrac{1}{2}Li^2} \qquad [10]$$

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تعاریف اولیه: سلف
انرژی ذخیره شده میان سیم پیچ های سلف: مثال

Example 6.10

Consider the circuit in Fig. 6.27(a). Under dc conditions, find: (a) i, v_C, and i_L, (b) the energy stored in the capacitor and inductor.

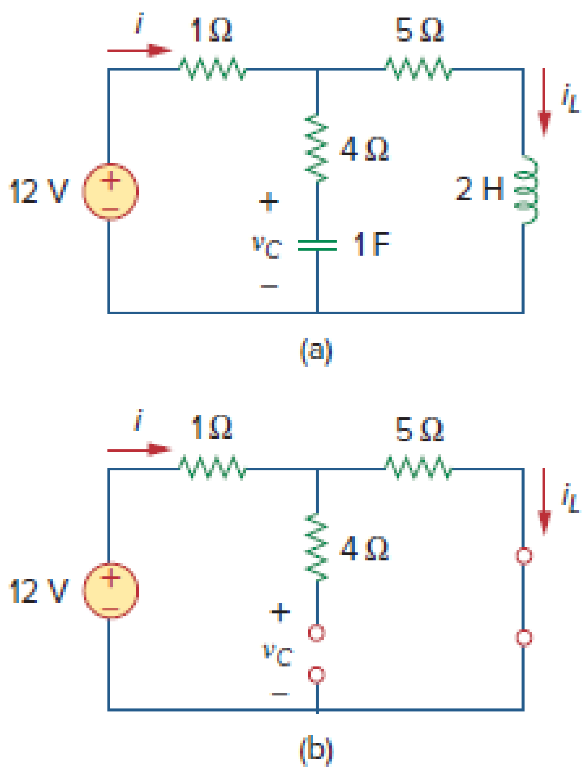

Figure 6.27
For Example 6.10.

Solution:

(a) Under dc conditions, we replace the capacitor with an open circuit and the inductor with a short circuit, as in Fig. 6.27(b). It is evident from Fig. 6.27(b) that

$$i = i_L = \frac{12}{1+5} = 2 \text{ A}$$

The voltage v_C is the same as the voltage across the 5-Ω resistor. Hence,

$$v_C = 5i = 10 \text{ V}$$

(b) The energy in the capacitor is

$$w_C = \frac{1}{2}Cv_C^2 = \frac{1}{2}(1)(10^2) = 50 \text{ J}$$

and that in the inductor is

$$w_L = \frac{1}{2}Li_L^2 = \frac{1}{2}(2)(2^2) = 4 \text{ J}$$

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اتصال سری سلف ها

$$\begin{aligned} v_s &= v_1 + v_2 + \cdots + v_N \\ &= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + \cdots + L_N \frac{di}{dt} \\ &= (L_1 + L_2 + \cdots + L_N) \frac{di}{dt} \end{aligned}$$

or, written more concisely,

$$v_s = \sum_{n=1}^{N} v_n = \sum_{n=1}^{N} L_n \frac{di}{dt} = \frac{di}{dt} \sum_{n=1}^{N} L_n$$

But for the equivalent circuit we have

$$v_s = L_{eq} \frac{di}{dt}$$

$$L_{eq} = \sum_{n=1}^{N} L_n \qquad [11]$$

and thus the equivalent inductance is

$$L_{eq} = L_1 + L_2 + \cdots + L_N$$

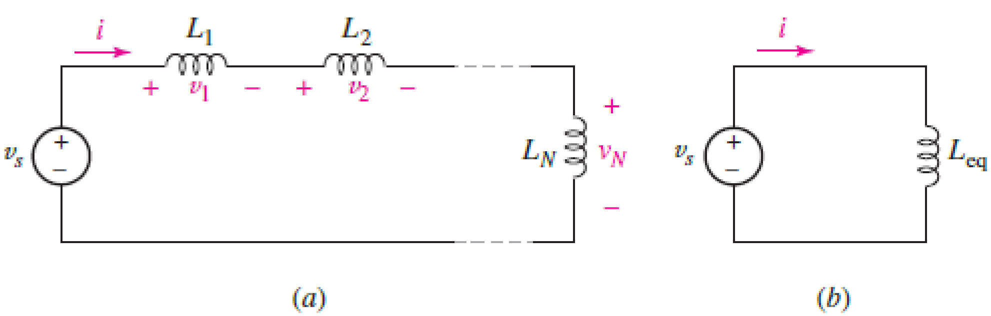

FIGURE 7.18 (*a*) A circuit containing N inductors in series. (*b*) The desired equivalent circuit, in which $L_{eq} = L_1 + L_2 + \cdots + L_N$.

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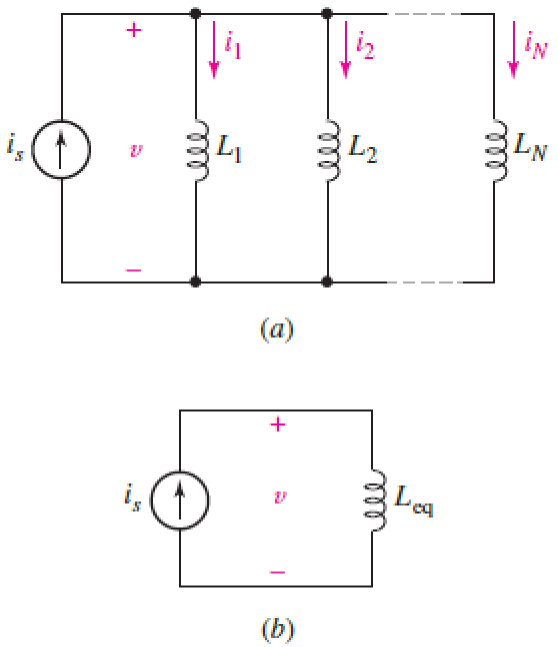

FIGURE 7.19 (*a*) The parallel combination of *N* inductors. (*b*) The equivalent circuit, where $L_{eq} = [1/L_1 + 1/L_2 + \cdots + 1/L_N]^{-1}$.

Inductors in Parallel

The combination of a number of parallel inductors is accomplished by writing the single nodal equation for the original circuit, shown in Fig. 7.19*a*,

$$i_s = \sum_{n=1}^{N} i_n = \sum_{n=1}^{N}\left[\frac{1}{L_n}\int_{t_0}^{t} v\,dt' + i_n(t_0)\right]$$
$$= \left(\sum_{n=1}^{N}\frac{1}{L_n}\right)\int_{t_0}^{t} v\,dt' + \sum_{n=1}^{N} i_n(t_0)$$

and comparing it with the result for the equivalent circuit of Fig. 7.19*b*,

$$i_s = \frac{1}{L_{eq}}\int_{t_0}^{t} v\,dt' + i_s(t_0)$$

Since Kirchhoff's current law demands that $i_s(t_0)$ be equal to the sum of the branch currents at t_0, the two integral terms must also be equal; hence,

$$L_{eq} = \frac{1}{1/L_1 + 1/L_2 + \cdots + 1/L_N} \qquad [12]$$

For the special case of two inductors in parallel,

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \qquad [13]$$

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اتصال سری خازن ها

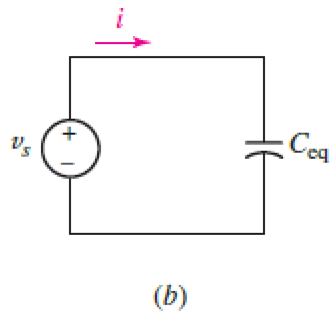

FIGURE 7.20 (*a*) A circuit containing N capacitors in series. (*b*) The desired equivalent circuit, where $C_{eq} = [1/C_1 + 1/C_2 + \cdots + 1/C_N]^{-1}$.

Capacitors in Series

In order to find a capacitor that is equivalent to N capacitors in series, we use the circuit of Fig. 7.20*a* and its equivalent in Fig. 7.20*b* to write

$$v_s = \sum_{n=1}^{N} v_n = \sum_{n=1}^{N} \left[\frac{1}{C_n} \int_{t_0}^{t} i\, dt' + v_n(t_0) \right]$$
$$= \left(\sum_{n=1}^{N} \frac{1}{C_n} \right) \int_{t_0}^{t} i\, dt' + \sum_{n=1}^{N} v_n(t_0)$$

and

$$v_s = \frac{1}{C_{eq}} \int_{t_0}^{t} i\, dt' + v_s(t_0)$$

However, Kirchhoff's voltage law establishes the equality of $v_s(t_0)$ and the sum of the capacitor voltages at t_0; thus

$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + \cdots + 1/C_N} \qquad [14]$$

and capacitors in series combine as do conductances in series, or resistors in parallel. The special case of two capacitors in series, of course, yields

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \qquad [15]$$

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Finally, the circuits of Fig. 7.21 enable us to establish the value of the capacitor which is equivalent to N parallel capacitors as

$$C_{eq} = C_1 + C_2 + \cdots + C_N \qquad [16]$$

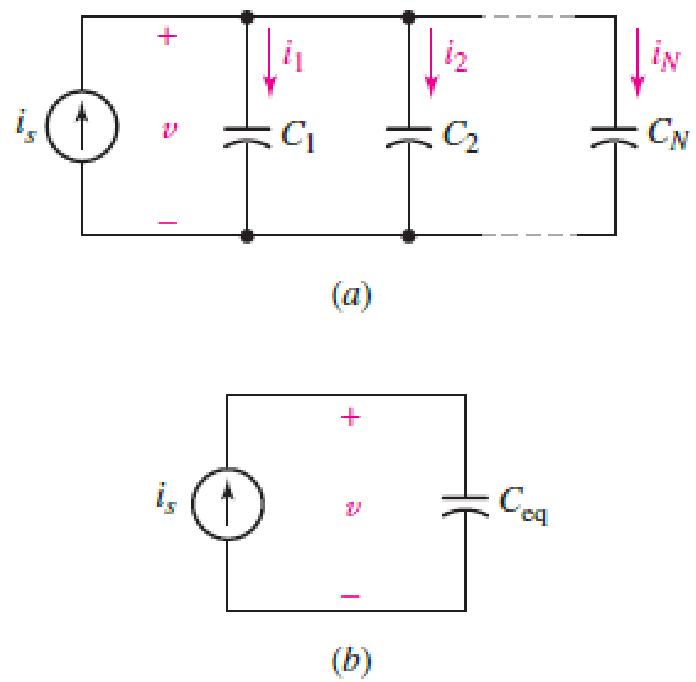


FIGURE 7.21 (*a*) The parallel combination of N capacitors. (*b*) The equivalent circuit, where $C_{eq} = C_1 + C_2 + \cdots + C_N$.

اتصالات سلف و خازن
مثال و تمرین:

Simplify the network of Fig. 7.22*a* using series-parallel combinations.

The 6 μF and 3 μF series capacitors are first combined into a 2 μF equivalent, and this capacitor is then combined with the 1 μF element with which it is in parallel to yield an equivalent capacitance of 3 μF. In addition, the 3 H and 2 H inductors are replaced by an equivalent 1.2 H inductor, which is then added to the 0.8 H element to give a total equivalent inductance of 2 H. The much simpler (and probably less expensive) equivalent network is shown in Fig. 7.22*b*.

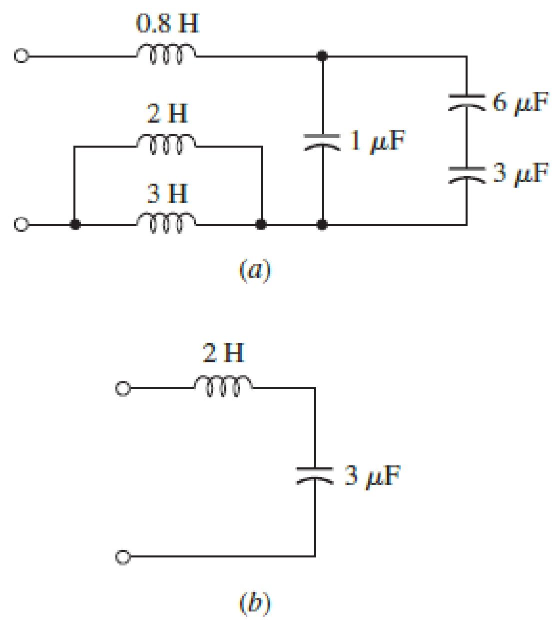


FIGURE 7.22 (*a*) A given *LC* network. (*b*) A simpler equivalent circuit.

PRACTICE

7.8 Find C_{eq} for the network of Fig. 7.23.

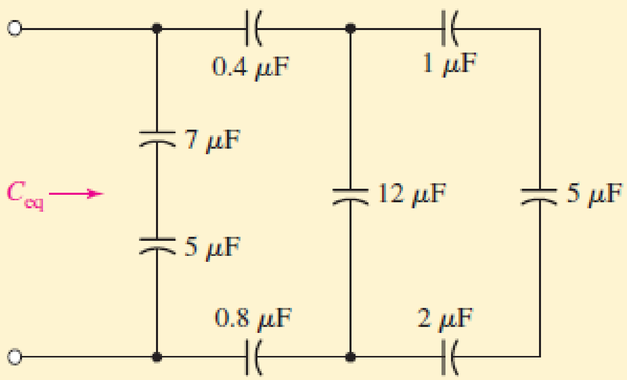
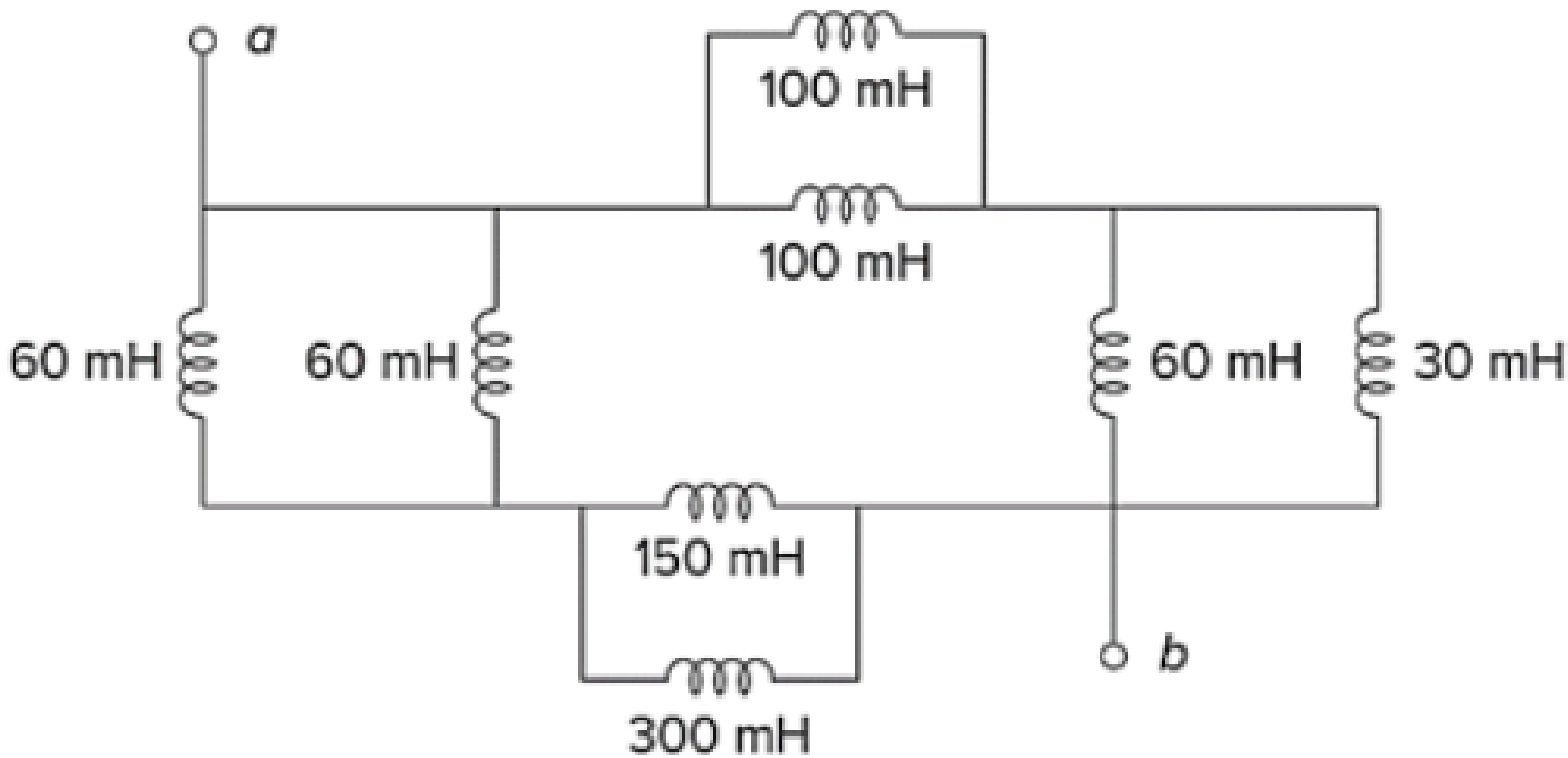


FIGURE 7.23

Ans: 3.18 μF.

اتصالات سلف و خازن: تمرین (به عهده دانشجو):

سلف معادل دیده شده از ترمینال a و b را در شکل زیر بیابید.



اتصالات سلف و خازن: **ادامه تمرین (به عهده دانشجو):**

خلاصه روابط سلف و خازن

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TABLE 6.1

Important characteristics of the basic elements.†

Relation	Resistor (R)	Capacitor (C)	Inductor (L)
v-i:	$v = iR$	$v = \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau + v(t_0)$	$v = L\frac{di}{dt}$
i-v:	$i = v/R$	$i = C\frac{dv}{dt}$	$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)\,d\tau + i(t_0)$
p or w:	$p = i^2R = \frac{v^2}{R}$	$w = \frac{1}{2}Cv^2$	$w = \frac{1}{2}Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short circuit
Circuit variable that cannot change abruptly:	Not applicable	v	i

†*Passive sign convention is assumed.*

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تمرین: (به عهده دانشجو)

For the circuit in Fig. 6.33, $i(t) = 4(2 - e^{-10t})$ mA. If $i_2(0) = -1$ mA, find: (a) $i_1(0)$; (b) $v(t)$, $v_1(t)$, and $v_2(t)$; (c) $i_1(t)$ and $i_2(t)$.

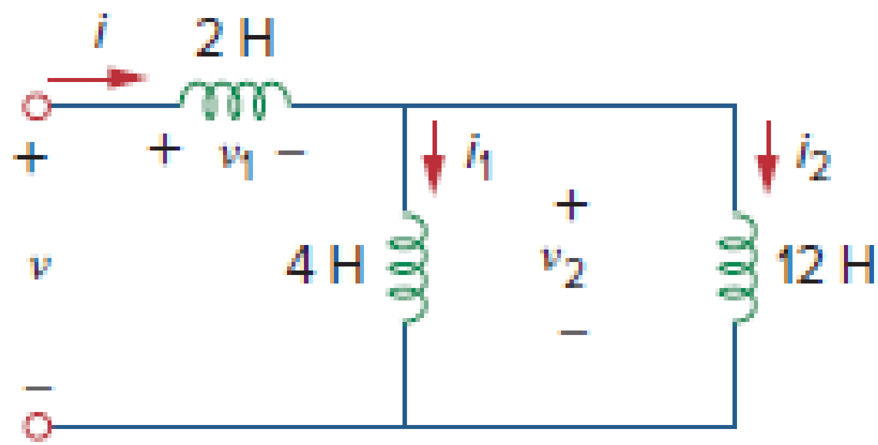

Figure 6.33
For Example 6.12.

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ادامه تمرین: (به عهده دانشجو)

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ادامه تمرین: (به عهده دانشجو)

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مدارهای مرتبه اول

پاسخ گذرای مدار RL (پاسخ ورودی صفر)

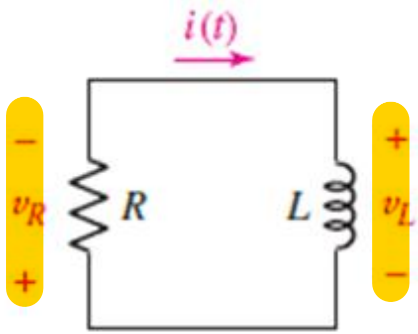


FIGURE 8.1 A series *RL* circuit for which *i*(*t*) is to be determined, subject to the initial condition that $i(0) = I_0$.

We begin our study of transient analysis by considering the simple series *RL* circuit shown in Fig. 8.1. Let us designate the time-varying current as $i(t)$; we will represent the value of $i(t)$ at $t = 0$ as I_0; in other words, $i(0) = I_0$. We therefore have

$$Ri + v_L = Ri + L\frac{di}{dt} = 0$$

or

$$\frac{di}{dt} + \frac{R}{L}i = 0 \qquad [1]$$

Our goal is an expression for $i(t)$ which satisfies this equation *and* also has the value I_0 at $t = 0$. The solution may be obtained by several different methods.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL : روش مستقیم حل معادله دیفرانسیل همگن

$$\frac{di}{i} = -\frac{R}{L}\,dt \qquad [2]$$

Since the current is I_0 at $t = 0$ and $i(t)$ at time t, we may equate the two definite integrals which are obtained by integrating each side between the corresponding limits:

$$\int_{I_0}^{i(t)} \frac{di'}{i'} = \int_0^t -\frac{R}{L}\,dt'$$

Performing the indicated integration,

$$\ln i'\Big|_{I_0}^{i} = -\frac{R}{L}t'\Big|_0^t$$

which results in

$$\ln i - \ln I_0 = -\frac{R}{L}(t - 0)$$

After a little manipulation, we find that the current $i(t)$ is given by

$$i(t) = I_0 e^{-Rt/L} \qquad [3]$$

We check our solution by first showing that substitution of Eq. [3] in Eq. [1] yields the identity $0 = 0$, and then showing that substitution of $t = 0$ in Eq. [3] produces $i(0) = I_0$. Both steps are necessary; the solution must satisfy the differential equation which characterizes the circuit, and it must also satisfy the initial condition.

مدارهای مرتبه اول
مدار RL : مثال و تمرین

If the inductor of Fig. 8.2 has a current $i_L = 2$ A at $t = 0$, find an expression for $i_L(t)$ valid for $t > 0$, and its value at $t = 200\ \mu$s.

This is the identical type of circuit just considered, so we expect an inductor current of the form

$$i_L = I_0 e^{-Rt/L}$$

where $R = 200\ \Omega$, $L = 50$ mH and I_0 is the initial current flowing through the inductor at $t = 0$. Thus,

$$i_L(t) = 2e^{-4000t}$$

Substituting $t = 200 \times 10^{-6}$ s, we find that $i_L(t) = 898.7$ mA, less than half the initial value.

PRACTICE

8.1 Determine the current i_R through the resistor of Fig. 8.3 at $t = 1$ ns if $i_R(0) = 6$ A.

Ans: 812 mA.

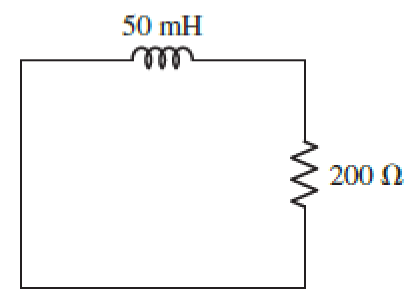


FIGURE 8.2 A simple *RL* circuit in which energy is stored in the inductor at $t = 0$.

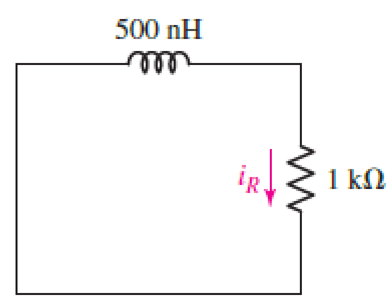


FIGURE 8.3 Circuit for Practice Problem 8.1.

مدارهای مرتبه اول

مدار RL : روش جایگزین (حل معادله دیفرانسیل به روش تشکیل معادله مشخصه)

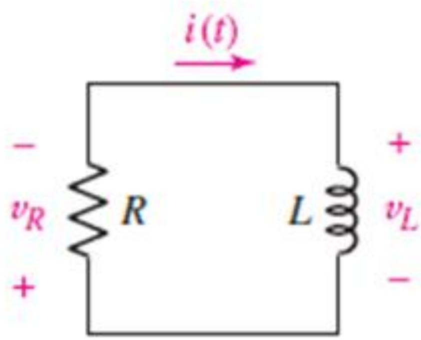


FIGURE 8.1 A series *RL* circuit for which *i*(*t*) is to be determined, subject to the initial condition that $i(0) = I_0$.

$$Ri + L\frac{di}{dt} = 0$$

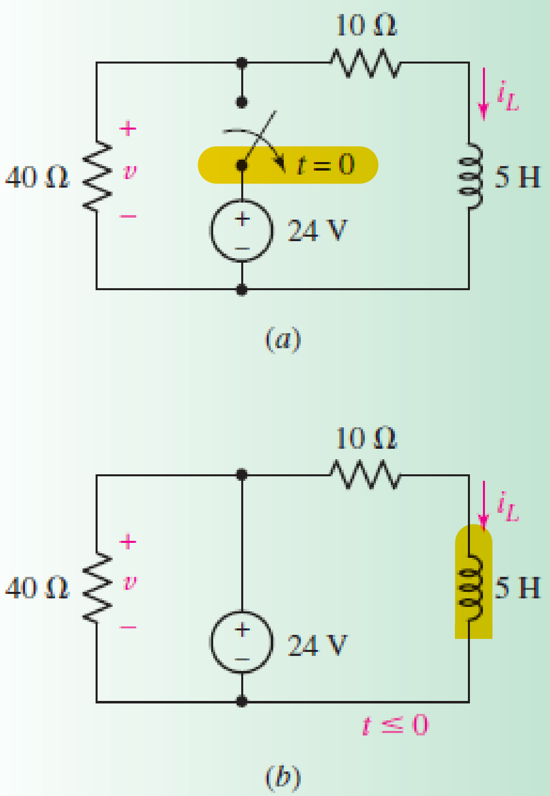
$$R + Lr = 0 \rightarrow r = -\frac{R}{L} \rightarrow i(t) = i_h(t) = Ke^{rt} = Ke^{-\frac{R}{L}t}$$

$$\xrightarrow{i_L(0)=I_0} I_0 = K \rightarrow i(t) = I_0 e^{-\frac{R}{L}t}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL : مثال (پاسخ ورودی صفر)



For the circuit of Fig. 8.5*a*, find the voltage labeled v at $t = 200$ ms.

Identify the goal of the problem.
The schematic of Fig. 8.5*a* actually represents *two different* circuits: one with the switch closed (Fig. 8.5*b*) and one with the switch open (Fig. 8.5*c*). We are asked to find $v(0.2)$ for the circuit shown in Fig. 8.5*c*.

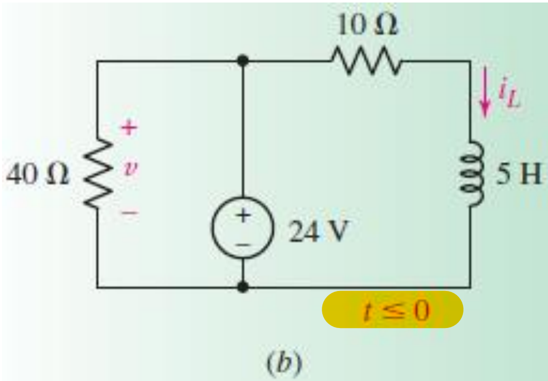
Collect the known information.
Both new circuits are drawn and labeled correctly. We next make the assumption that the circuit in Fig. 8.5*b* has been connected for a long time, so that any transients have dissipated. We may make such an assumption as a general rule unless instructed otherwise. This circuit determines $i_L(0)$.

Devise a plan.
The circuit of Fig. 8.5*c* may be analyzed by writing a KVL equation. Ultimately we want a differential equation with only v and t as variables; we will then solve the differential equation for $v(t)$.

Construct an appropriate set of equations.
Referring to Fig. 8.5*c*, we write

مدارهای مرتبه اول

مدار RL: (پاسخ ورودی صفر) ادامه مثال



$$t < 0$$

$$KVL: 24 = 10 i_L \rightarrow i_L(0^-) = 2.4A, v(0^-) = 24v$$

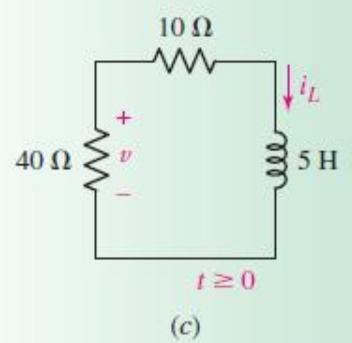


FIGURE 8.5 (*a*) A simple *RL* circuit with a switch thrown at time $t = 0$. (*b*) The circuit as it exists prior to $t = 0$. (*c*) The circuit after the switch is thrown, and the 24 V source is removed.

$$t > 0$$

$$KVL: 40 i_L + 10 i_L + 5\frac{di_L}{dt} = 0 \rightarrow 5\frac{di_L}{dt} + 50 i_L = 0$$

$$5r + 50 = 0 \rightarrow r = -10 \rightarrow i_L = Ke^{-10t} \xrightarrow{i_L(0^-) = i_L(0^+) = 2.4A} K = 2.4$$

$$i_L(t) = 2.4e^{-10t} \rightarrow v(t) = -40 i_L(t) = -96e^{-10t}$$

$$i_L(t) = \begin{cases} 2.4 & t \le 0 \\ 2.4e^{-10t} & t > 0 \end{cases}, v(t) = \begin{cases} 24 & t \le 0 \\ -96e^{-10t} & t > 0 \end{cases}$$

$$v(0.2s) = -12.99v$$

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مدارهای مرتبه اول

مدار RL : (پاسخ ورودی صفر) انرژی **اولیه** ذخیره شده در سلف

Before we turn our attention to the interpretation of the response, let us return to the circuit of Fig. 8.1, and check the power and energy relationships. The power being dissipated in the resistor is

$$p_R = i^2 R = I_0^2 R e^{-2Rt/L}$$

and the total energy turned into heat in the resistor is found by integrating the instantaneous power from zero time to infinite time:

$$w_R = \int_0^\infty p_R \, dt = I_0^2 R \int_0^\infty e^{-2Rt/L} \, dt$$

$$= I_0^2 R \left(\frac{-L}{2R} \right) e^{-2Rt/L} \Big|_0^\infty = \frac{1}{2} L I_0^2$$

This is the result we expect, because the total energy stored initially in the inductor is $\frac{1}{2} L I_0^2$, and there is no longer any energy stored in the inductor at infinite time since its current eventually drops to zero. All the initial energy therefore is accounted for by dissipation in the resistor.

مدارهای مرتبه اول
مدار RL : (پاسخ ورودی صفر) خواص پاسخ نمایی مدار RL

$$i(t) = I_0 e^{-Rt/L}$$

At $t = 0$, the current has value I_0, but as time increases, the current decreases and approaches zero. The shape of this decaying exponential is seen by the plot of $i(t)/I_0$ versus t shown in Fig. 8.7. Since the function we are plotting is $e^{-Rt/L}$, the curve will not change if R/L remains unchanged. Thus, the same curve must be obtained for every series RL circuit having the same L/R ratio. Let us see how this ratio affects the shape of the curve.

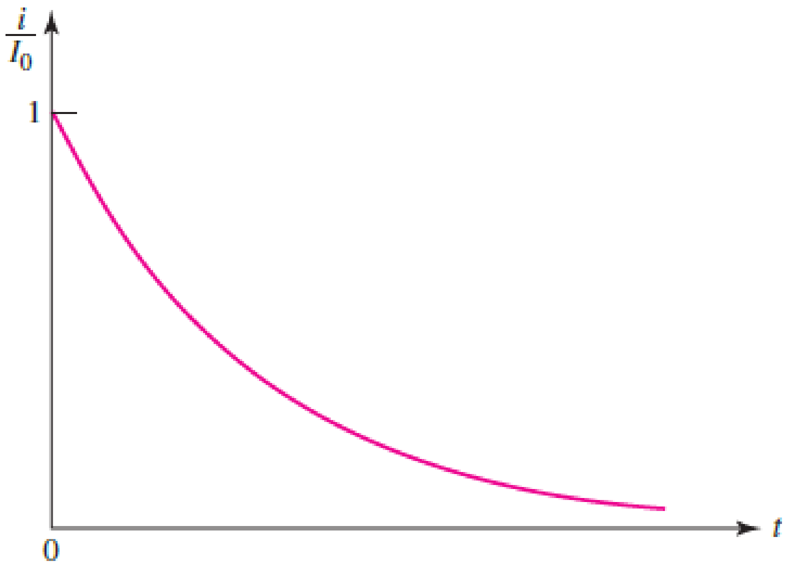

FIGURE 8.7 A plot of $e^{-Rt/L}$ versus t.

$$\boxed{\tau = \frac{L}{R}}$$

مدارهای مرتبه اول

مدار RL : (پاسخ ورودی صفر) خواص پاسخ نمایی مدار RL

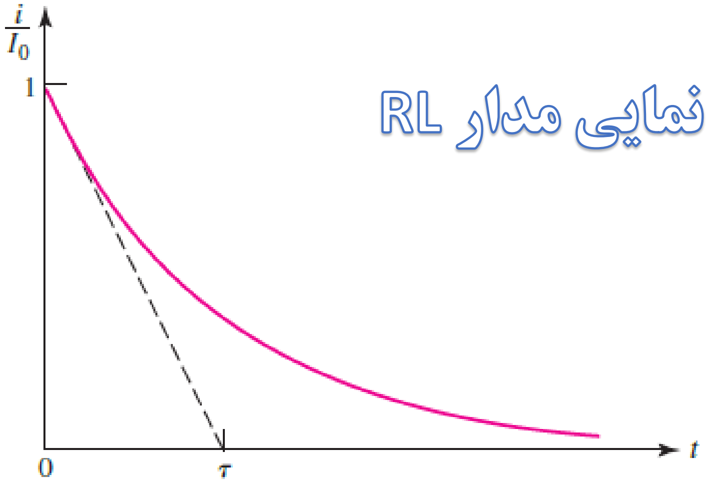

FIGURE 8.8 The time constant τ is L/R for a series *RL* circuit. It is the time required for the response curve to drop to zero if it decays at a constant rate equal to its initial rate of decay.

An equally important interpretation of the time constant τ is obtained by determining the value of $i(t)/I_0$ at $t = \tau$. We have

$$\frac{i(\tau)}{I_0} = e^{-1} = 0.3679 \qquad \text{or} \qquad i(\tau) = 0.3679 I_0$$

Thus, in one time constant the response has dropped to 36.8 percent of its initial value; the value of τ may also be determined graphically from this fact, as indicated by Fig. 8.9. It is convenient to measure the decay of the current at intervals of one time constant, and recourse to a hand calculator shows that $i(t)/I_0$ is 0.3679 at $t = \tau$, 0.1353 at $t = 2\tau$, 0.04979 at $t = 3\tau$, 0.01832 at $t = 4\tau$, and 0.006738 at $t = 5\tau$. At some point three to five time constants after zero time, most of us would agree that the current is a negligible fraction of its former self. Thus, if we are asked, "*How long does it take for the current to decay to zero?*" our answer might be, "*About five time constants.*" At that point, the current is less than 1 percent of its original value!

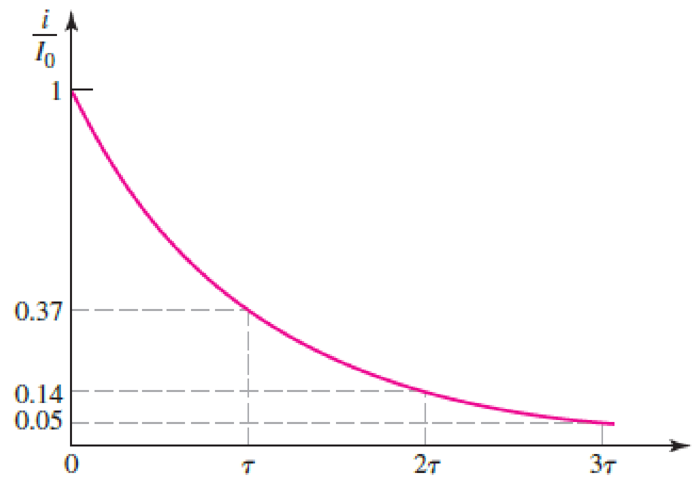

FIGURE 8.9 The current in a series *RL* circuit is reduced to 37 percent of its initial value at $t = \tau$, 14 percent at $t = 2\tau$, and 5 percent at $t = 3\tau$.

مدارهای مرتبه اول

مدار RC (پاسخ ورودی صفر)

Let us see how closely the analysis of the parallel (or is it series?) *RC* circuit shown in Fig. 8.15 corresponds to that of the *RL* circuit. We will assume an initial stored energy in the capacitor by selecting

$$v(0) = V_0$$

$$KVL: Ri_R(t) - v = 0 \rightarrow -Ri_C(t) - v = 0 \rightarrow -RC\frac{dv}{dt} - v = 0$$

$$KCL\ A: i_C(t) + i_R(t) = 0 \rightarrow C\frac{dv}{dt} + \frac{v}{R} = 0$$

$$Cr + \frac{1}{R} = 0 \rightarrow r = -\frac{1}{RC} \Rightarrow v(t) = Ke^{-\frac{t}{RC}}$$

$$\xrightarrow{v(0)=V_0} V_0 = K$$

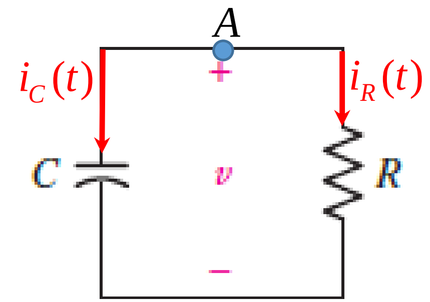

FIGURE 8.15 A parallel *RC* circuit for which $v(t)$ is to be determined, subject to the initial condition that $v(0) = V_0$.

enables us to immediately write

$$v(t) = v(0)e^{-t/RC} = V_0 e^{-t/RC} \qquad [14]$$

ثابت زمانی مدار RC

$$\boxed{\tau = RC}$$

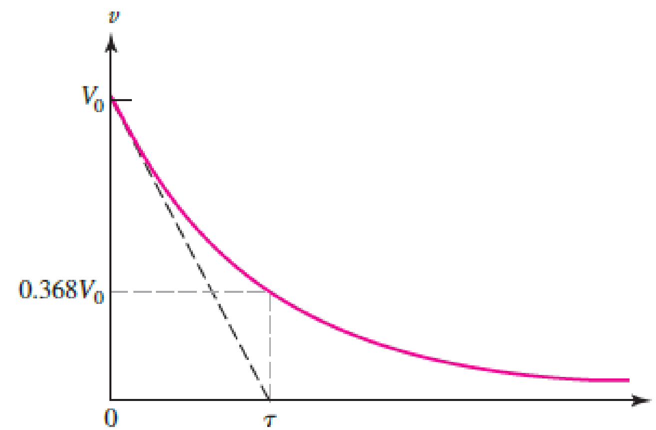

FIGURE 8.16 The capacitor voltage $v(t)$ in the parallel *RC* circuit is plotted as a function of time. The initial value of $v(t)$ is V_0.

Let us discuss the physical nature of the voltage response of the *RC* circuit as expressed by Eq. [14]. At $t = 0$ we obtain the correct initial condition, and as t becomes infinite, the voltage approaches zero. This latter result agrees with our thinking that if there were any voltage remaining across the capacitor, then energy would continue to flow into the resistor and be dissipated as heat. *Thus, a final voltage of zero is necessary*. The time constant of the *RC*

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مدارهای مرتبه اول

روش ثابت زمانی

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \tag{7.53}$$

where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value. Thus, to find the step response of an RC circuit requires three things:

1. The initial capacitor voltage $v(0)$.
2. The final capacitor voltage $v(\infty)$.
3. The time constant τ.

We obtain item 1 from the given circuit for $t < 0$ and items 2 and 3 from the circuit for $t > 0$. Once these items are determined, we obtain the

response using Eq. (7.53). This technique equally applies to RL circuits, as we shall see in the next section.

Note that if the switch changes position at time $t = t_0$ instead of at $t = 0$, there is a time delay in the response so that Eq. (7.53) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau} \tag{7.54}$$

where $v(t_0)$ is the initial value at $t = t_0^+$. Keep in mind that Eq. (7.53) or (7.54) applies only to step responses, that is, when the input e xcitation is constant.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

روش ثابت زمانی در حالت پاسخ ورودی صفر

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}} \qquad (7.53)$$

$$v(\infty) = 0v \rightarrow v(t) = 0 + \left[v(0) - 0\right]e^{-\frac{t}{\tau}}$$

$$\boxed{v(t) = v(0)e^{\frac{-t}{\tau}}}$$

مدارهای مرتبه اول

مدار RC : مثال (پاسخ ورودی صفر)

For the circuit of Fig. 8.17*a*, find the voltage labeled v at $t = 200\ \mu$s.

To find the requested voltage, we will need to draw and analyze two separate circuits: one corresponding to before the switch is thrown (Fig. 8.17*b*), and one corresponding to after the switch is thrown (Fig. 8.17*c*).

The sole purpose of analyzing the circuit of Fig. 8.17*b* is to obtain an initial capacitor voltage; we assume any transients in that circuit died out long ago, leaving a purely dc circuit. With no current through either the capacitor or the 4 Ω resistor, then,

$$v(0) = 9\ \text{V} \qquad [17]$$

روش اول: تشکیل معادله دیفرانسیل

$$KVL: v + 2i_C(t) + 4i_C(t) = 0 \rightarrow v + 6C\frac{dv}{dt} = 0$$

$$1 + 6Cr = 0 \rightarrow r = -\frac{1}{6C} \Rightarrow v(t) = Ke^{\frac{-t}{6C}}$$

$$\xrightarrow{v(0)=9} K = 9 \rightarrow v(t) = 9e^{\frac{-t}{60\times10^{-6}}}$$

روش دوم: ثابت زمانی کل مدار

We next turn our attention to the circuit of Fig. 8.17*c*, recognizing that

$$\tau = RC = (2+4)(10\times10^{-6}) = 60\times10^{-6}\ \text{s}$$

Thus, from Eq. [14],

$$v(t) = v(0)e^{-t/RC} = v(0)e^{-t/60\times10^{-6}} \qquad [18]$$

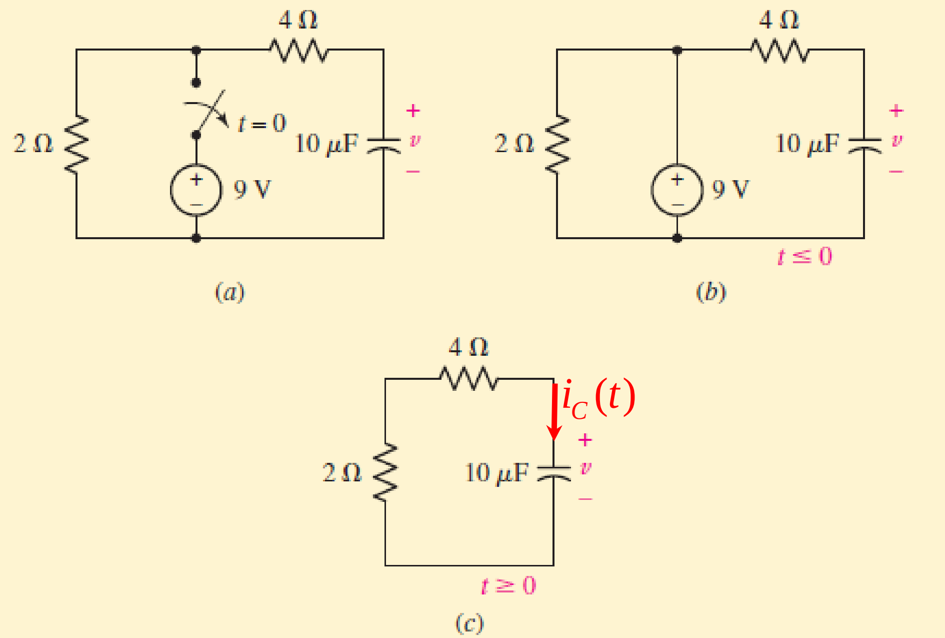


FIGURE 8.17 (*a*) A simple *RC* circuit with a switch thrown at time $t = 0$. (*b*) The circuit as it exists prior to $t = 0$. (*c*) The circuit after the switch is thrown, and the 9 V source is removed.

The capacitor voltage must be the same in both circuits at $t = 0$; no such restriction is placed on any other voltage or current. Substituting Eq. [17] into Eq. [18],

$$v(t) = 9e^{-t/60\times10^{-6}}\ \text{V}$$

so that $v(200\times10^{-6}) = 321.1$ mV (less than 4 percent of its maximum value).

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL : حالت کلی تر

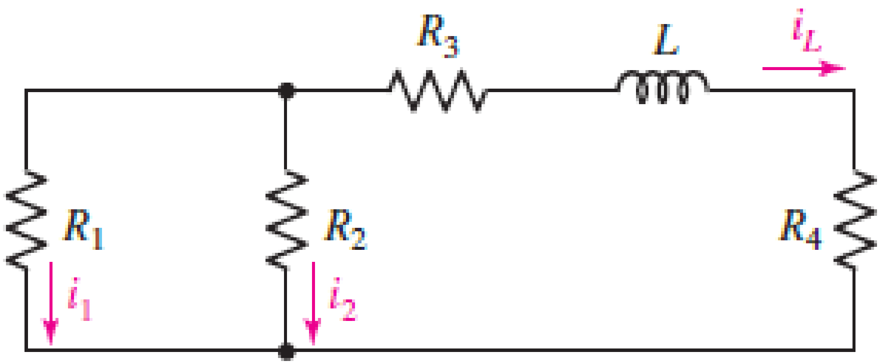


FIGURE 8.19 A source-free circuit containing one inductor and several resistors is analyzed by determining the time constant $\tau = L/R_{eq}$.

$$R_{eq} = R_3 + R_4 + \frac{R_1 R_2}{R_1 + R_2}$$

and the time constant is therefore

$$\tau = \frac{L}{R_{eq}} \qquad [19]$$

If several inductors are present in a circuit and can be combined using series and/or parallel combination, then Eq. [19] can be further generalized to

$$\tau = \frac{L_{eq}}{R_{eq}} \qquad [20]$$

where L_{eq} represents the equivalent inductance.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول: مدار RL : (پاسخ ورودی صفر) پیوستگی شرایط اولیه

Slicing Thinly: The Distinction Between 0^+ and 0^-

Let's return to the circuit of Fig. 8.19, and assume that some finite amount of energy is stored in the inductor at $t = 0$, so that $i_L(0) \neq 0$.

The inductor current i_L is

$$i_L = i_L(0)e^{-t/\tau}$$

and this represents what we might call the basic solution to the problem. It is quite possible that some current or voltage other than i_L is needed, such as the current i_2 in R_2. We can always apply Kirchhoff's laws and Ohm's law to the resistive portion of the circuit without any difficulty, but current division provides the quickest answer in this circuit:

$$i_2 = -\frac{R_1}{R_1 + R_2}[i_L(0)e^{-t/\tau}]$$

It may also happen that we know the initial value of some current *other* than the inductor current. Since *the current in a resistor may change instantaneously,* we will indicate the instant after any change that might have occurred at $t = 0$ by the use of the symbol 0^+; in more mathematical language, $i_1(0^+)$ is the limit from the right of $i_1(t)$ as t approaches zero.[1] Thus, if we are given the initial value of i_1 as $i_1(0^+)$, then the initial value of i_2 is

$$i_2(0^+) = i_1(0^+)\frac{R_1}{R_2}$$

From these values, we obtain the necessary initial value of i_L:

$$i_L(0^+) = -[i_1(0^+) + i_2(0^+)] = -\frac{R_1 + R_2}{R_2}i_1(0^+)$$

and the expression for i_2 becomes

$$i_2 = i_1(0^+)\frac{R_1}{R_2}e^{-t/\tau}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL : مثال (پاسخ ورودی صفر)

Determine both i_1 and i_L in the circuit shown in Fig. 8.20*a* for $t > 0$.

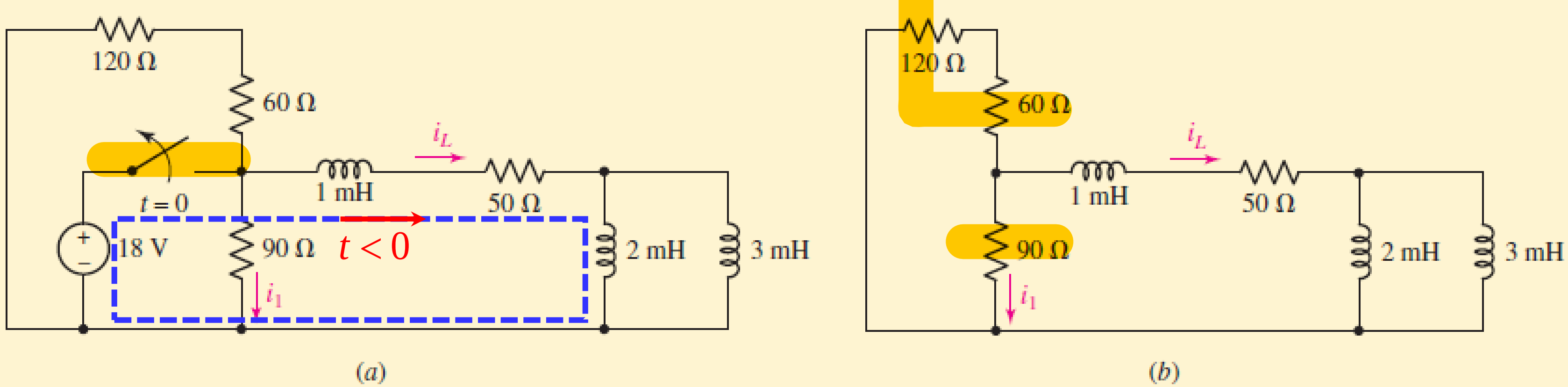


FIGURE 8.20 (*a*) A circuit with multiple resistors and inductors. (*b*) After $t = 0$, the circuit simplifies to an equivalent resistance of 110 Ω in series with $L_{eq} = 2.2$ mH.

After $t = 0$, when the voltage source is disconnected as shown in Fig. 8.20*b*, we easily calculate an equivalent inductance,

$$L_{eq} = \frac{2 \times 3}{2 + 3} + 1 = 2.2 \text{ mH}$$

an equivalent resistance, in series with the equivalent inductance,

$$R_{eq} = \frac{90(60 + 120)}{90 + 180} + 50 = 110\ \Omega$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL : ادامه مثال (پاسخ ورودی صفر)

and the time constant,

$$\tau = \frac{L_{\text{eq}}}{R_{\text{eq}}} = \frac{2.2 \times 10^{-3}}{110} = 20\ \mu\text{s}$$

Thus, the form of the natural response is $Ke^{-50,000t}$, where K is an unknown constant. Considering the circuit just prior to the switch opening ($t = 0^-$), $i_L = 18/50$ A. Since $i_L(0^+) = i_L(0^-)$, we know that $i_L = 18/50$ A or 360 mA at $t = 0^+$ and so

$$i_L = \begin{cases} 360 \text{ mA} & t < 0 \\ 360e^{-50,000t} \text{ mA} & t \geq 0 \end{cases}$$

There is no restriction on i_1 changing instantaneously at $t = 0$, so its value at $t = 0^-$ (18/90 A or 200 mA) is not relevant to finding i_1 for $t > 0$. Instead, we must find $i_1(0^+)$ through our knowledge of $i_L(0^+)$. Using current division,

$$i_1(0^+) = -i_L(0^+)\frac{120 + 60}{120 + 60 + 90} = -240 \text{ mA}$$

Hence,

$$i_1 = \begin{cases} 200 \text{ mA} & t < 0 \\ -240e^{-50,000t} \text{ mA} & t \geq 0 \end{cases}$$

مدارهای مرتبه اول

پاسخ ورودی صفر مدار RL حالت کلی‌تر: تمرین (به عهده دانشجو)

Example 7.5

In the circuit shown in Fig. 7.19, find i_o, v_o, and i for all time, assuming that the switch was open for a long time.

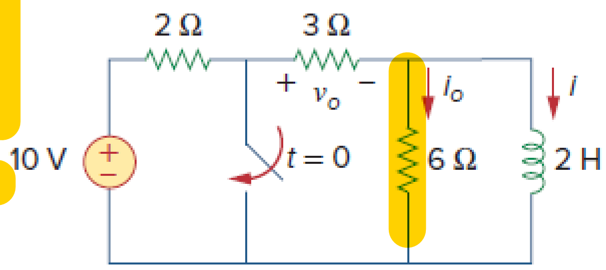


Figure 7.19
For Example 7.5.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ ورودی صفر مدار RL حالت کلی‌تر: ادامه **تمرین** **(به عهده دانشجو)**

دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ ورودی صفر مدار RL حالت کلی‌تر: ادامه **تمرین** **(به عهده دانشجو)**

مدارهای مرتبه اول

مدار RC: (پاسخ ورودی صفر) حالت کلی تر

Many of the *RC* circuits for which we would like to find the natural response contain more than a single resistor and capacitor. Just as we did for the *RL* circuits, we first consider those cases in which the given circuit may be reduced to an equivalent circuit consisting of only one resistor and one capacitor.

Let us suppose first that we are faced with a circuit containing a single capacitor, but any number of resistors. It is possible to replace the two-terminal resistive network which is across the capacitor terminals with an equivalent resistor, and we may then write down the expression for the capacitor voltage immediately. In such instances, the circuit has an effective time constant given by

$$\tau = R_{\text{eq}}C$$

where R_{eq} is the equivalent resistance of the network. An alternative perspective is that R_{eq} is in fact the Thévenin equivalent resistance "seen" by the capacitor.

If the circuit has more than one capacitor, but they may be replaced somehow using series and/or parallel combinations with an equivalent capacitance C_{eq}, then the circuit has an effective time constant given by

$$\tau = RC_{\text{eq}}$$

with the general case expressed as

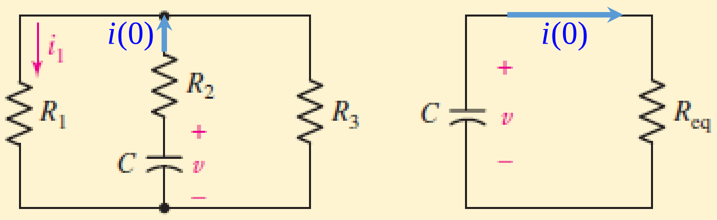
$$\tau = R_{\text{eq}}C_{\text{eq}}$$

It is worth noting, however, that parallel capacitors replaced by an equivalent capacitance would have to have identical initial conditions.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول
مدار RC: حالت کلی تر: مثال
(پاسخ ورودی صفر): روش ثابت زمانی کل مدار

Find $v(0^+)$ and $i_1(0^+)$ for the circuit shown in Fig. 8.22*a* if $v(0^-) = V_0$.



(*a*) (*b*)

FIGURE 8.22 (*a*) A given circuit containing one capacitor and several resistors. (*b*) The resistors have been replaced by a single equivalent resistor; the time constant is simply $\tau = R_{eq}C$.

We first simplify the circuit of Fig. 8.22*a* to that of Fig. 8.22*b*, enabling us to write

$$v = V_0 e^{-t/R_{eq}C}$$

where

$$v(0^+) = v(0^-) = V_0 \qquad \text{and} \qquad R_{eq} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

Every current and voltage in the resistive portion of the network must have the form $Ae^{-t/R_{eq}C}$, where A is the initial value of that current or voltage. Thus, the current in R_1, for example, may be expressed as

$$i_1 = i_1(0^+)e^{-t/\tau}$$

where

$$\tau = \left(R_2 + \frac{R_1 R_3}{R_1 + R_3}\right) C$$

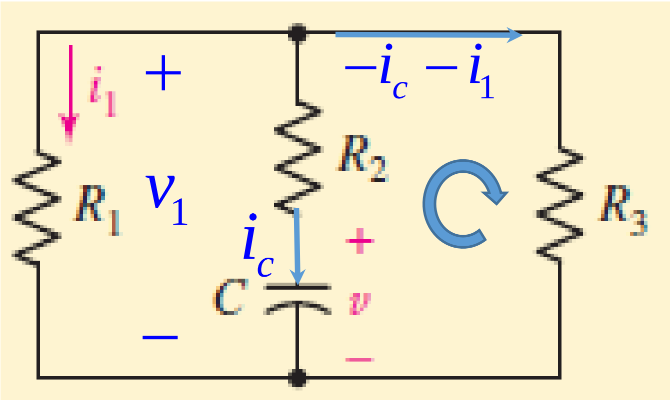
$$-V_0 + R_{eq} i(0) = 0 \rightarrow i(0) = \frac{V_0}{R_{eq}}$$

$$i_1(0) = i(0) \times \frac{R_3}{R_1 + R_3} = \frac{V_0 R_3}{R_{eq}\left(R_1 + R_3\right)}$$

$$i_1(t) = \frac{V_0 R_3}{R_{eq}\left(R_1 + R_3\right)} e^{\frac{-t}{R_{eq}C}}$$

$$i_1(t) = \frac{R_3 V_0}{R_2\left(R_1 + R_3\right) + R_1 R_3} e^{\frac{-t}{R_{eq}C}}$$

$$i_1(0) = \frac{R_3 V_0}{R_2\left(R_1 + R_3\right) + R_1 R_3}$$



مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه مثال

(پاسخ ورودی صفر): روش تشکیل معادله دیفرانسیل

$$KVL: -v - R_2 i_c + R_3 \times -\left(i_1 + i_c\right) = 0 \rightarrow -v - R_2 C\frac{dv}{dt} - R_3\left(\frac{v + R_2 C\frac{dv}{dt}}{R_1} + C\frac{dv}{dt}\right) = 0$$

$$-\left(R_2 + R_3 + \frac{R_3 R_2}{R_1}\right)C\frac{dv}{dt} - \left(1 + \frac{R_3}{R_1}\right)v = 0 \rightarrow v(t) = V_0 e^{-\frac{R_1+R_3}{\left(R_1\left(R_2+R_3\right)+R_3R_2\right)C}t} = V_0 e^{-\frac{1}{R_{eq}C}t}$$

$$i_C(t) = C\frac{dv}{dt} = C \times \frac{-V_0}{R_{eq}C} e^{\frac{-t}{R_{eq}C}} = \frac{-V_0}{R_{eq}} e^{\frac{-t}{R_{eq}C}} \rightarrow i_1(t) = -i_C(t)\frac{R_3}{R_1 + R_3} = \frac{V_0}{R_{eq}}\frac{R_3}{R_1 + R_3} e^{\frac{-t}{R_{eq}C}}$$

$$i_1(t) = \frac{R_3 V_0}{R_2\left(R_1 + R_3\right) + R_1 R_3} e^{\frac{-t}{R_{eq}C}} \rightarrow i_1(0) = \frac{R_3 V_0}{R_2\left(R_1 + R_3\right) + R_1 R_3}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: تمرین (به عهده دانشجو)

PRACTICE

8.6 Find values of v_C and v_o in the circuit of Fig. 8.23 at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) 1.3 ms.

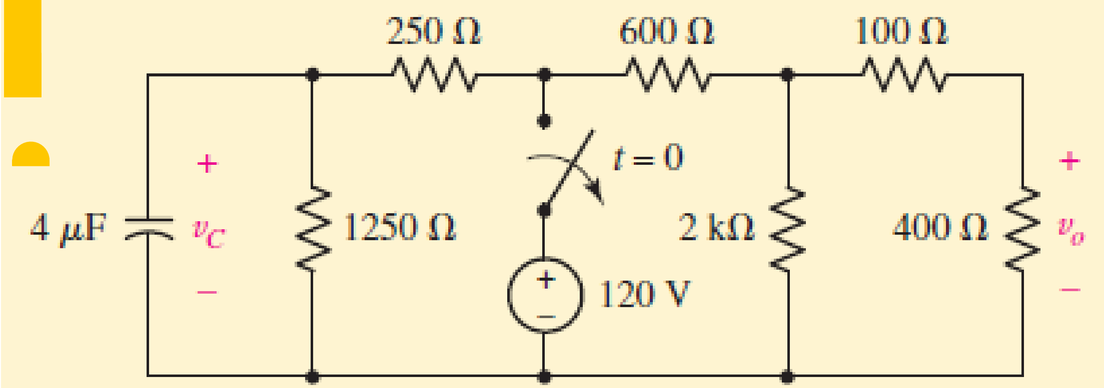


FIGURE 8.23

Ans: 100 V, 38.4 V; 100 V, 25.6 V; 59.5 V, 15.22 V.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه تمرین (به عهده دانشجو)

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر:(پاسخ ورودی صفر) ادامه **تمرین (به عهده دانشجو)**

روش اول:

محاسبه ثابت زمانی کل مدار

مدارهای مرتبه اول

مدار RC: حالت کلی تر: (پاسخ ورودی صفر) ادامه **تمرین (به عهده دانشجو)**

روش دوم:

تشکیل معادله دیفرانسیل

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: مثال **منبع وابسته**: راه حل اول **(پاسخ ورودی صفر)**

For the circuit of Fig. 8.24*a*, find the voltage labeled v_C for $t > 0$ if $v_C(0^-) = 2$ V.

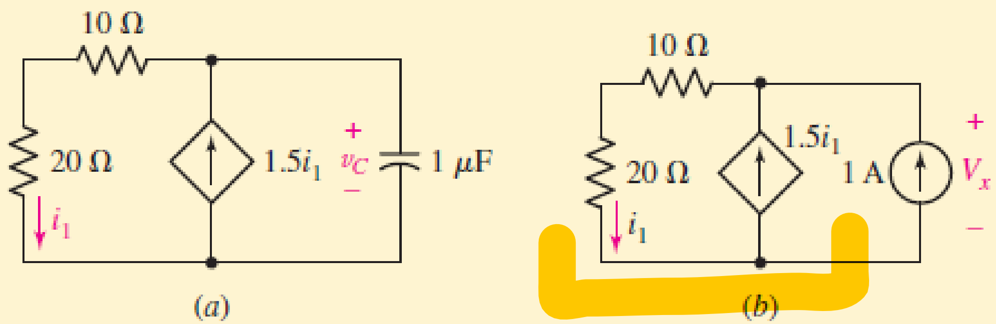


FIGURE 8.24 (*a*) A simple *RC* circuit containing a dependent source not controlled by a capacitor voltage or current. (*b*) Circuit for finding the Thévenin equivalent of the network connected to the capacitor.

The dependent source is not controlled by a capacitor voltage or current, so we can start by finding the Thévenin equivalent of the network to the left of the capacitor. Connecting a 1 A test source as in Fig. 8.24*b*,

$$V_x = (1 + 1.5i_1)(30)$$

where

$$i_1 = \left(\frac{1}{20}\right)\frac{20}{10+20}V_x = \frac{V_x}{30}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه مثال منبع وابسته (راه حل اول: ثابت زمانی کل مدار)

Performing a little algebra, we find that $V_x = -60$ V, so the network has a Thévenin equivalent resistance of $-60\ \Omega$ (unusual, but not impossible when dealing with a dependent source). Our circuit therefore has a *negative* time constant

$$\tau = -60(1 \times 10^{-6}) = -60\ \mu s$$

The capacitor voltage is therefore

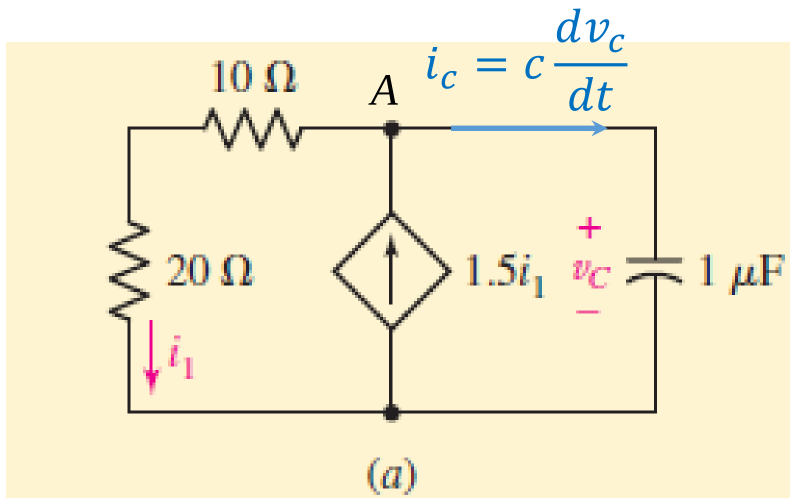
$$v_C(t) = Ae^{t/60\times 10^{-6}} \qquad \text{V}$$

where $A = v_C(0^+) = v_C(0^-) = 2$ V. Thus,

$$v_C(t) = 2e^{t/60\times 10^{-6}} \qquad \text{V} \qquad [21]$$

which, interestingly enough is unstable: it grows exponentially with time. This cannot continue indefinitely; one or more elements in the circuit will eventually fail.

مدار RC: حالت کلی تر: ادامه مثال منبع وابسته: راه حل دوم (تشکیل معادله دیفرانسیل)



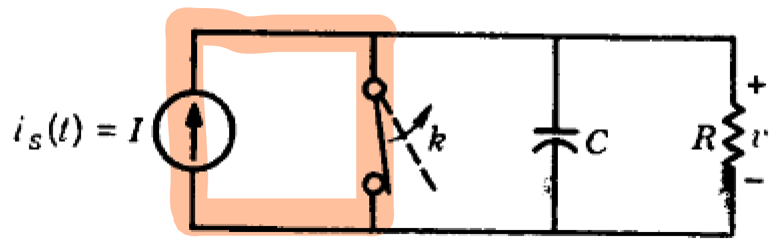
$$KCL\ A: 1.5i_1 = i_1 + i_c \rightarrow 0.5i_1 = C\frac{dv_C}{dt}$$

$$i_1 = \frac{v_C}{30} \rightarrow C\frac{dv_C}{dt} - \frac{v_C}{60} = 0, Cr - \frac{1}{60} = 0 \rightarrow r = \frac{1}{60C}$$

$$v_C(t) = Ke^{\frac{t}{60C}} \xrightarrow{v_C(0)=2v} K = 2, v_C(t) = 2e^{\frac{t}{60C}} = 2e^{\frac{t}{60\times 10^{-6}}}$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر: مثال: (پاسخ حالت صفر)



شکل ۱-۲- مدار RC با ورودی منبع جریان . درلحظه $t=0$ کلید باز میشود .

میشود . از KVL می‌بینیم که ولتاژ دوسر هرسه عنصر یکی است . این ولتاژ را با v نشان داده وفرض‌میکنیم v پاسخ موردنظر باشد. بانوشتن KCL برحسب v معادله زیر:

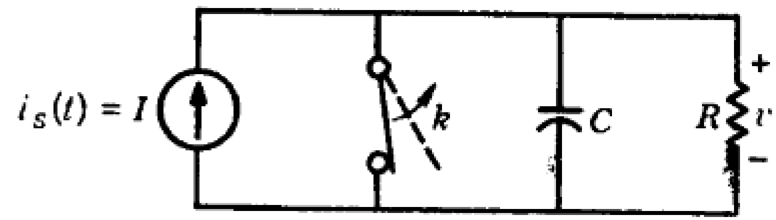
$$C\frac{dv}{dt}+\frac{v}{R}=i_s(t)=I \qquad t\geqslant 0 \tag{۲—۱}$$

که درآن I یک ثابت است برای شبکه بدست می‌آید . فرض میکنیم خازن بدون باراولیه باشد پس شرط اولیه چنین خواهد بود :

$$v(0)=0 \tag{۲—۲}$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه مثال: (پاسخ حالت صفر)



شکل ۱-۲- مدار RC با ورودی منبع جریان . درلحظه $t=0$
کلید باز میشود .

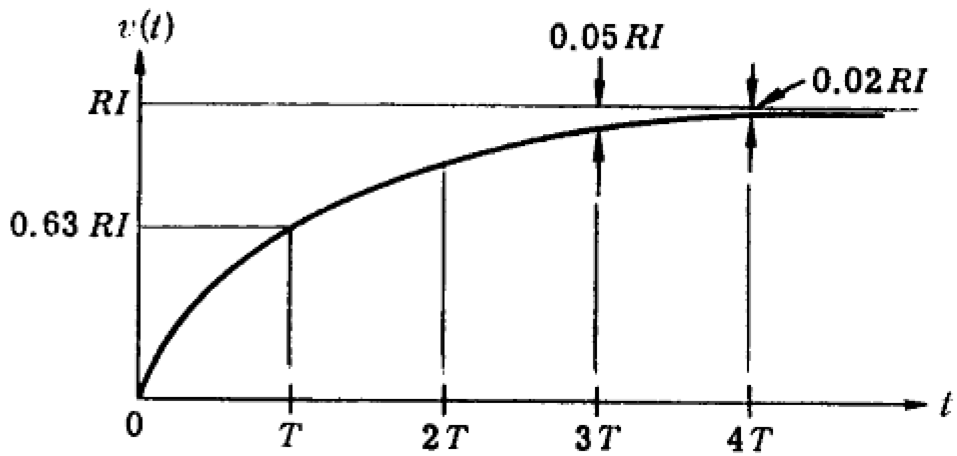
$$C\frac{dv}{dt}+\frac{v}{R}=I \to \begin{cases} v_h : Cr+\frac{1}{R}=0 \to r=-\frac{1}{RC} \to v_h = Ke^{-\frac{t}{RC}} \\ v_p : v_p = A \to C\times 0+\frac{A}{R}=I \to A=RI \to v_p = RI \end{cases}$$

$$v(t)=RI+Ke^{-\frac{t}{RC}} \xrightarrow{v(0^-)=v(0^+)=0} K=-RI \to v(t)=RI-RIe^{-\frac{t}{RC}}=RI\left(1-e^{-\frac{t}{RC}}\right)$$

مدارهای مرتبه اول

مدار RC: حالت کلی تر: ادامه مثال: (پاسخ حالت صفر)

$$\boxed{v(t) = RI\left(1 - e^{-\left(\frac{1}{RC}\right)t}\right)} \qquad t \geq 0 \qquad (۲-۱۰)$$



شکل ۳-۲- پاسخ ولتاژ مدار RC ناشی ازمنبع ثابت I چنانکه درشکل (۱-۲) با $v(0)=0$ نشان داده شده است .

دانشکده برق و کامپیوتر

مدارهای مرتبه اول
تعریف **پاسخ حالت صفر**

در دوحالتی که دراین بخش دیدیم ولتاژ v را پاسخ و منبع جریان i_s را ورودی درنظر گرفتیم . شرط اولیه درمدار صفر بوده یعنی پیش از وارد آوردن ورودی، ولتاژ دوسر خازن برابر با صفر بود . درحالت کلی اگر همه شرطهای اولیه درمدار صفر باشند گوئیم مدار در حالت‌صفر (۱) است+. پاسخ مداری که ازحالت‌صفر شروع میکند منحصراً معلول ورودی آنست. بموجب تعریف ، **پاسخ حالت صفر** یک مدار پاسخ آن به یک ورودی است که در زمان دلخواه t_0 بمدار وارد شود بشرط آنکه مدار درست پیش از وارد آوردن این ورودی (یعنی در زمان t_{0-}) درحالت صفر باشد . درمحاسبهٔ پاسخ حالت صفر هدف اصلی، رفتار پاسخ برای $t \geqslant t_0$ است. بدین‌منظورچنین «قرارمیگذاریم» : برای $t < t_0$ ورودی و پاسخ حالت صفر را متحد باصفر میگیریم .

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

روش ثابت زمانی در پاسخ حالت صفر

$$\boxed{v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}} \qquad (7.53)$$

سوئیچ زنی در زمان صفر

$$v(0) = 0v \rightarrow v(t) = v(\infty) + \left[0 - v(\infty)\right]e^{\frac{-t}{\tau}}$$

$$v(t) = v(\infty)\left[1 - e^{\frac{-t}{\tau}}\right]$$

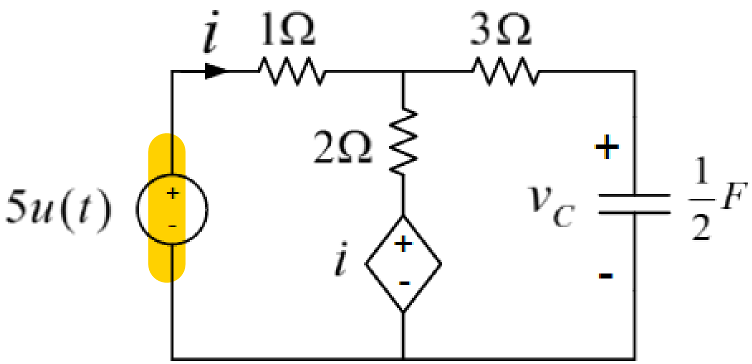
سوئیچ زنی در زمان $t = t_0$

$$v(0) = 0v \rightarrow v(t) = v(\infty) + \left[0 - v(\infty)\right]e^{\frac{-(t-t_0)}{\tau}}$$

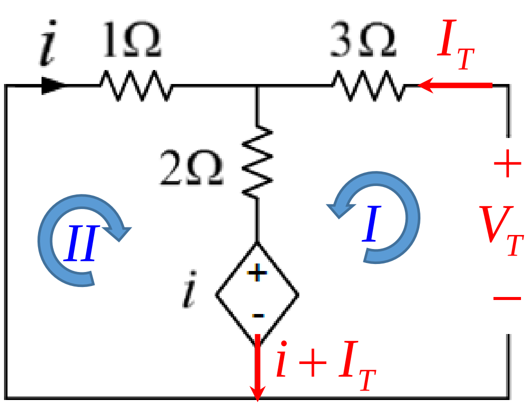
$$v(t) = v(\infty)\left[1 - e^{\frac{-(t-t_0)}{\tau}}\right]$$

مدارهای مرتبه اول

مدار RC: (پاسخ حالت صفر) محاسبه ثابت زمانی: مثال:



ثابت زمانی مدار شکل روبرو را بدست آورید؟



$$KVL\ I: V_T = 3I_T + 2(i+I_T) + i \rightarrow \boxed{V_T = 5I_T + 3i}$$

$$KVL\ II: i + 2(i+I_T) + i = 0 \rightarrow 4i + 2I_T = 0 \rightarrow \boxed{i = -0.5I_T}$$

$$V_T = 5I_T + 3\left(-0.5I_T\right) = 3.5I_T \rightarrow \boxed{R_{th} = \frac{V_T}{I_T} = 3.5\Omega}$$

$$\boxed{\tau = R_{th}C = 0.5\times 3.5 = 1.75s}$$

مدارهای مرتبه اول

مدار RL: مثال: (پاسخ حالت صفر) – روش اول: حل معادله دیفرانسیل

PRACTICE

8.11 The circuit shown in Fig. 8.41 has been in the form shown for a very long time. The switch opens at $t = 0$. Find i_R at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) ∞; (*d*) 1.5 ms.

Ans: 0; 10 mA; 4 mA; 5.34 mA.

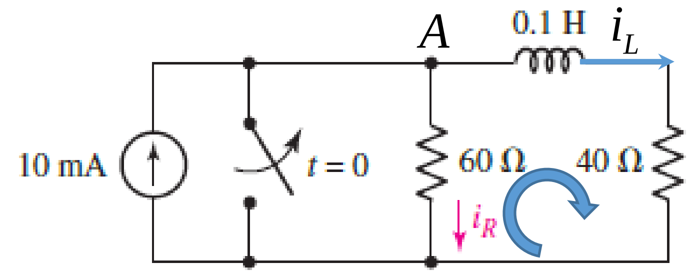


FIGURE 8.41

$$t>0: KCL\ A: 0.01 = i_R + i_L \rightarrow i_R = 0.01 - i_L$$

$$KVL: -60 i_R + L\frac{di_L}{dt} + 40 i_L = 0 \rightarrow 0.1\frac{di_L}{dt} + 40 i_L - 60\left(0.01 - i_L\right) = 0$$

$$0.1\frac{di_L}{dt} + 100 i_L = 0.6 \rightarrow \begin{cases} i_{Lh}: 0.1r + 100 = 0 \rightarrow r = -1000 \rightarrow i_{Lh} = Ke^{-1000t} \\ i_{Lp}: i_{Lp} = 0.006 \end{cases}$$

$$i_L(t) = i_{Lh} + i_{Lp} = Ke^{-1000t} + 0.006 \xrightarrow{i_L(0^-) = i_L(0^+) = 0} i_L(t) = 0.006\left(1 - e^{-1000t}\right)$$

$$i_R(t) = 0.01 - i_L = 0.01 - 0.006\left(1 - e^{-1000t}\right) = 0.004 + 0.006e^{-1000t}$$

$$i_R(t) = \begin{cases} 0 & t < 0 \\ 0.004 + 0.006e^{-1000t} & t > 0 \end{cases}$$

$$\rightarrow i_R(0^-) = 0A, i_R(0^+) = 0.01A, i_R(\infty) = 0.01\frac{40}{60+40} = 0.004A, i_R(0.0015) = 0.00533A$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه مثال: (پاسخ حالت صفر) – روش دوم: ثابت زمانی

PRACTICE

8.11 The circuit shown in Fig. 8.41 has been in the form shown for a very long time. The switch opens at $t = 0$. Find i_R at t equal to (*a*) 0^-; (*b*) 0^+; (*c*) ∞; (*d*) 1.5 ms.

Ans: 0; 10 mA; 4 mA; 5.34 mA.

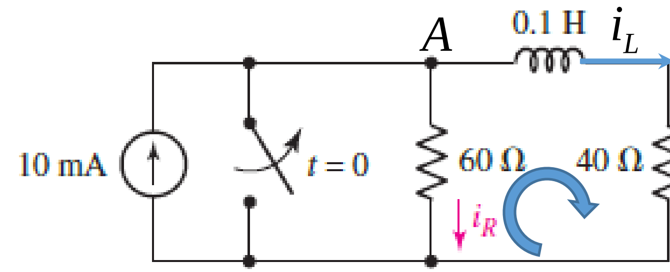


FIGURE 8.41

$$i_L(0^-) = 0A, i_L(\infty) = 0.01\times\frac{60}{100} = 0.006A$$

$$R_{th} = 100\Omega \rightarrow \tau = \frac{L}{R_{th}} = \frac{0.1}{100} = 0.001s$$

$$i_L(t) = i_L(\infty) + \left[i_L(0^+) - i_L(\infty)\right]e^{\frac{-t}{\tau}} = 0.006 - 0.006e^{-1000t} = 0.006\left(1 - e^{-1000t}\right)$$

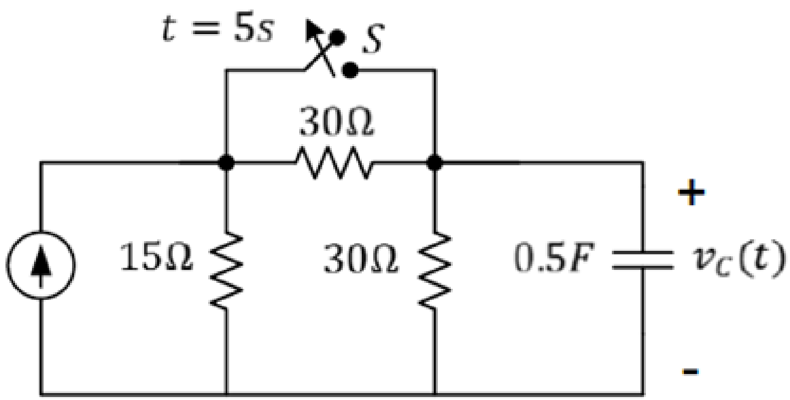
$$i_R(t) = 0.01 - i_L = 0.01 - 0.006\left(1 - e^{-1000t}\right) = 0.004 + 0.006e^{-1000t}$$

$$i_R(t) = \begin{cases} 0 & t < 0 \\ 0.004 + 0.006e^{-1000t} & t > 0 \end{cases}$$

$$\rightarrow i_R(0^-) = 0A, i_R(0^+) = 0.01A, i_R(\infty) = 0.01\frac{40}{60+40} = 0.004A, i_R(0.0015) = 0.00533A$$

مدارهای مرتبه اول

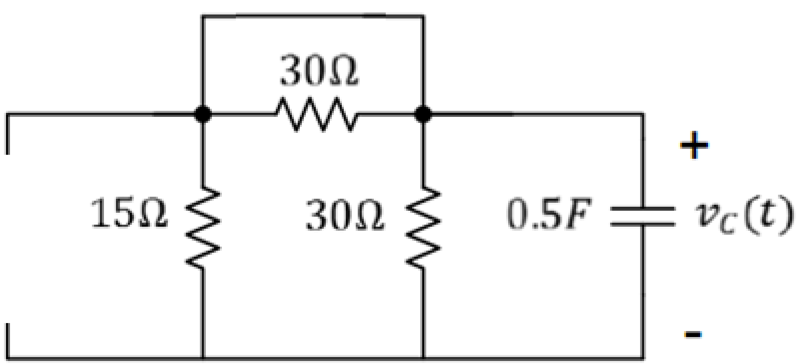
مدار RC: مثال: (پاسخ حالت صفر در حالت سوئیچ زنی در زمان غیرصفر)



$$\begin{cases} 0A & t<0 \\ 10A & t>0 \end{cases}$$

در مدار شکل مقابل، کلید S بسته بوده و در زمان ۵ ثانیه باز می‌شود. رابطه‌ی تغییرات ولتاژ خازن را برای همه‌ی زمان های قبل و بعد از باز شدن کلید، به دست آورده و آن را رسم کنید.

$t<0$



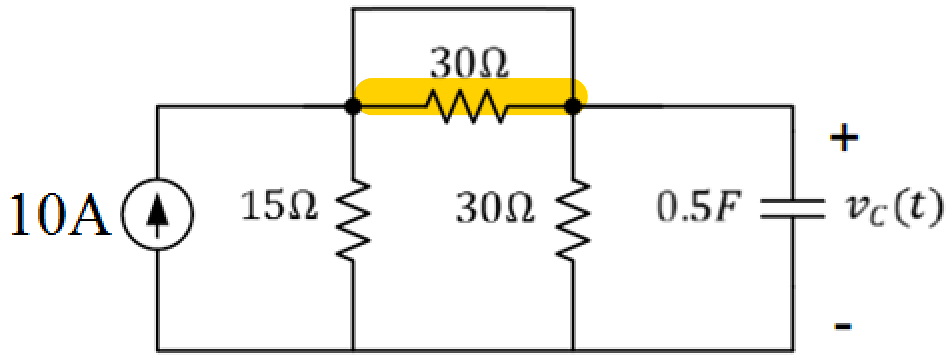
$$v_C(0^-)=0$$

مدارهای مرتبه اول

مدار RC: ادامه مثال: (پاسخ حالت صفر در حالت سوئیچ زنی در زمان غیرصفر)

روش اول: روش ثابت زمانی

$0 < t < 5s$



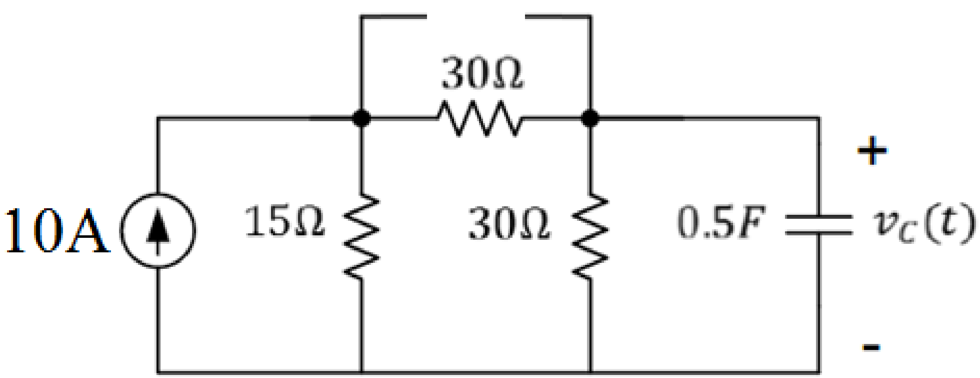
$$R_{th} = \frac{15\times 30}{45} = 10\Omega \rightarrow \tau = 10\times 0.5 = 5s$$

$$v_C(\infty) = 30\left(10\times\frac{15}{45}\right) = 100v$$

$$v_C(t) = 100 + (0-100)e^{\frac{-t}{5}} = 100\left(1-e^{-0.2t}\right)$$

$$v_C(5) = 100\left(1-e^{-1}\right) = 63.21v$$

$t > 5s$



$$R_{th} = \frac{45\times 30}{75} = 18\Omega \rightarrow \tau = 18\times 0.5 = 9s$$

$$v_C(\infty) = 30\left(10\times\frac{15}{75}\right) = 60v$$

$$v_C(t) = 60 + (63.21-60)e^{\frac{-(t-5)}{9}} = 60 + 3.21e^{\frac{-(t-5)}{9}}$$

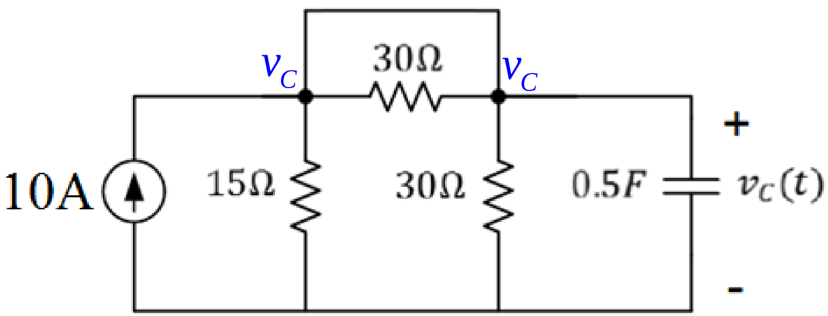
$$v_C(t) = 60 + 5.59e^{\frac{-t}{9}}$$

مدارهای مرتبه اول

مدار RC: ادامه مثال: (پاسخ حالت صفر در حالت سوئیچ زنی در زمان غیرصفر)

روش دوم: روش تشکیل و حل معادله دیفرانسیل

$0 < t < 5s$



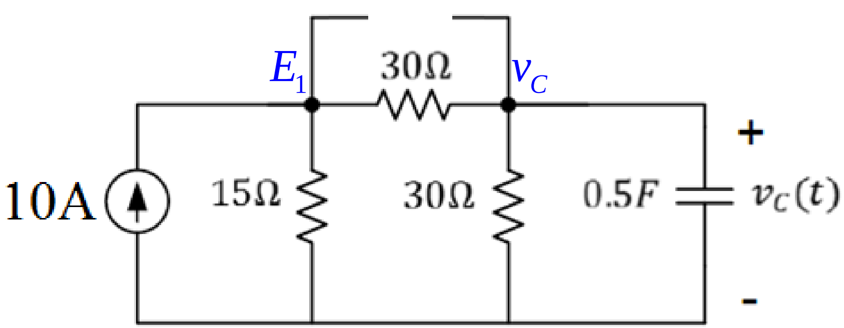
$$KCL\ v_C: 10 = \frac{v_C(t)}{15} + \frac{v_C(t)}{30} + C\frac{dv_C}{dt}$$

$$10 = 0.5\frac{dv_C}{dt} + 0.1v_C$$

$$\begin{cases} v_{Ch}(t): 0.5r + 0.1 = 0 \rightarrow r = -0.2 \rightarrow v_{Ch}(t) = Ke^{-0.2t} \\ v_{Cp} = 100v \end{cases}$$

$$v_C(t) = 100 + Ke^{-0.2t} \xrightarrow{v_C(0^+)=0v} v_C(t) = 100 - 100e^{-0.2t}$$

$t > 5s$



$$\boxed{\begin{cases} KCL\ E_1: 10 = \frac{E_1}{15} + \frac{E_1 - v_C}{30} \\ KCL\ v_C: \frac{E_1 - v_C}{30} = \frac{v_C}{30} + C\frac{dv_C}{dt} \end{cases}}$$

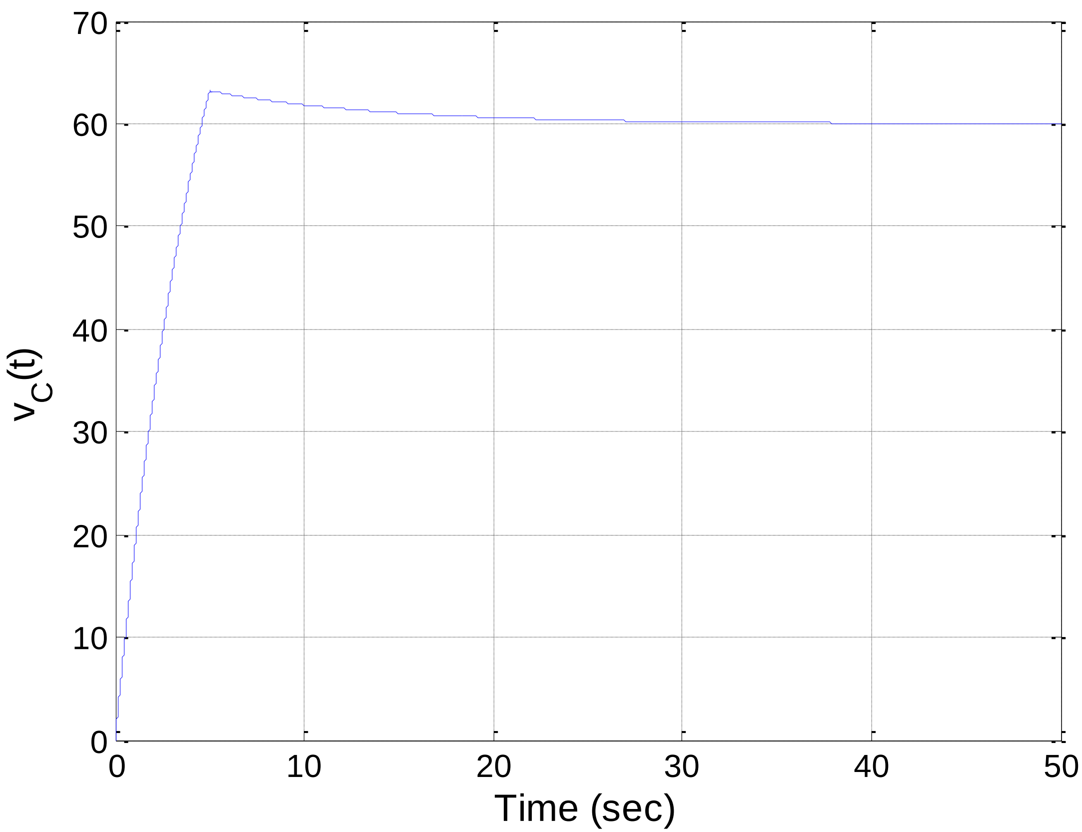
$$45\frac{dv_C}{dt} + 5v_C = 300$$

$$v_C(t) = 60 + Ke^{\frac{-t}{9}} \xrightarrow{v_C(5)=63.21v} v_C(t) = 60 + 5.59e^{\frac{-t}{9}}$$

مدارهای مرتبه اول

مدار RC: ادامه مثال: (پاسخ حالت صفر در حالت سوئیچ زنی در زمان غیرصفر)

$$v_C(t) = \begin{cases} 0 & t < 0 \\ 100 - 100e^{-0.2t} & 0 < t < 5 \\ 60 + 5.59e^{\frac{-t}{9}} & t > 5 \end{cases}$$

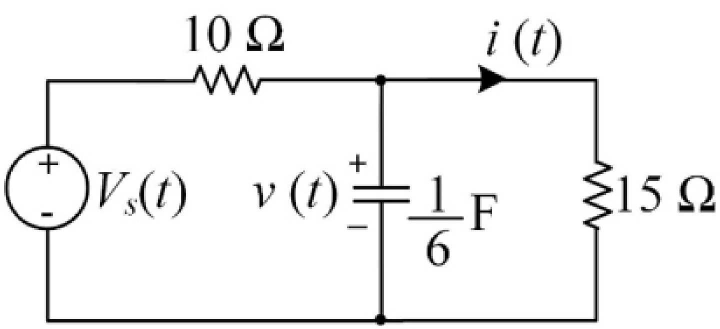


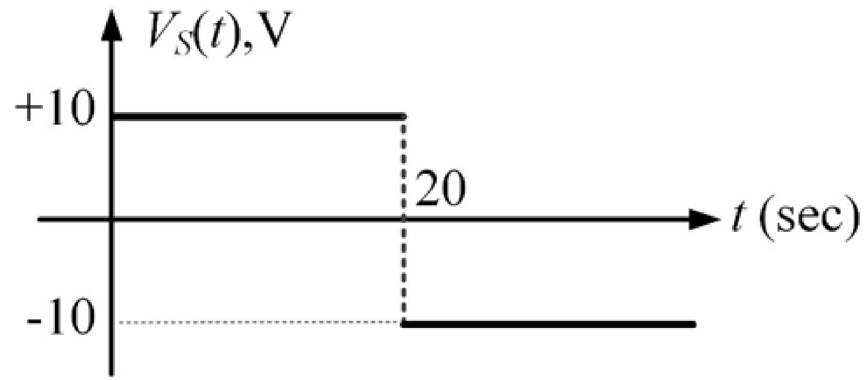
مدارهای مرتبه اول

مدار RC: تمرین (به عهده دانشجو) : استفاده از روش حل ثابت زمانی

1- در مدار شکل زیر فرض کنید $v_c(0^-) = 0$ است.

با فرض ولتاژ $v_s(t)$ نشان داده شده در شکل زیر، جریان $i(t)$ را برای $t \geq 0$ به دست آورده و رسم نمائید.





مدارهای مرتبه اول

مدار RC: ادامه **تمرین (به عهده دانشجو)** : استفاده از روش حل **ثابت زمانی**

دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

مدارهای مرتبه اول
تعریف پاسخ کامل

۳- پاسخ کامل : حالت گذرا و حالت دائمی

۳-۱- پاسخ کامل

پاسخ یک مدار به تحریک ورودی و شرطهای اولیه رویهم ، پاسخ کامل[۱] نام دارد . بنابراین پاسخ ورودی صفر و پاسخ حالت صفر حالتهای خاص پاسخ کامل هستند . دراین بخش نشان خواهیم داد که :

« برای مدار ساده خطی تغییرناپذیر با زمان RC پاسخ کامل برابر است با مجموع پاسخ ورودی صفر و پاسخ حالت صفر آن مدار». »

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مدارهای مرتبه اول

پاسخ کامل : مثال: روش اول: قضیه جمع آثار

مدار شکل (۱-۳) ، که درآن خازن دارای بار اولیه میباشد یعنی :

$$v(o)=V_0\neq o$$

را درنظرگرفته یک ورودی جریان درلحظهٔ $t=o$ بمدار وصل میکنیم . بموجب تعریف، پاسخ کامل شکل موج $v(\cdot)$ است که معلول تحریک ورودی $i_s(\cdot)$ و حالت اولیه V_0 رویهم میباشد . از لحاظ ریاضی این پاسخ جواب معادله زیر است :

$$C\frac{dv}{dt}+Gv=i_s(t) \qquad t\geqslant o \tag{۳-۱}$$

با شرط

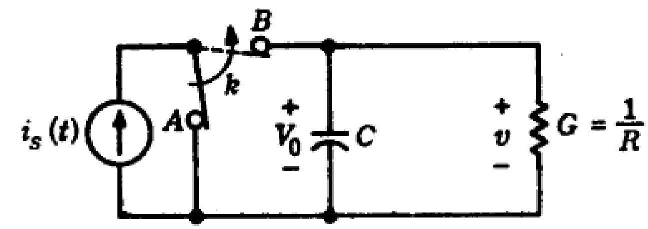
$$v(o)=V_0 \tag{۳-۲}$$

که درآن V_0 ولتاژ اولیه دوسر خازن است . گیریم v_i پاسخ ورودی صفر باشد ، بنا به تعریف v_i جواب معادله زیر است :

$$C\frac{dv_i}{dt}+Gv_i=o \qquad t\geqslant o$$

با شرط

$$v_i(o)=V_0$$



شکل ۱-۳- مدار RC با $v(o)=V_0$ با یک منبع جریان $i_s(t)$ تحریک میشود. درلحظه $t=o$ کلید k ازنقطه A به نقطه B چرخانیده میشود .

$$v_i(t)=V_0e^{-\left(\frac{1}{RC}\right)t} \qquad t\geqslant o$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ کامل : ادامه مثال: روش اول: قضیه جمع آثار

گیریم v_0 پاسخ حالت صفر باشد . بنا بتعریف ، این پاسخ جواب معادله زیر است :

$$C\frac{dv_0}{dt}+Gv_0=i_s(t) \qquad t\geqslant 0$$

با شرط

$$v_0(0)=0$$

$$v_0(t)=RI(1-e^{-\left(\frac{1}{RC}\right)t}) \qquad t\geqslant 0$$

درنتیجه پاسخ کامل چنین است :

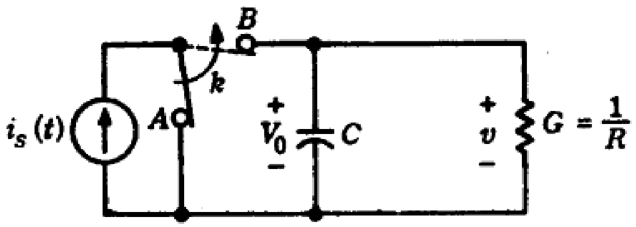
$$(۳-۳) \qquad \underbrace{v(t)}_{\text{پاسخ کامل}}=\underbrace{V_0e^{-\left(\frac{1}{RC}\right)t}}_{\text{پاسخ ورودی صفر } v_i}+\underbrace{RI(1-e^{-\left(\frac{1}{RC}\right)t})}_{\text{پاسخ حالت صفر } v_0} \quad t\geqslant 0$$

دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ کامل : ادامه مثال

روش دوم: روش مستقیم: تشکیل معادله دیفرانسیل



شکل ۱-۳- مدار RC با $v(o)=V_0$ با یک منبع جریان $i_s(t)$ تحریک میشود. درلحظه $t=o$ کلید k ازنقطه A به نقطه B چرخانیده میشود.

$$KCL\ B: i_s(t)=I=C\frac{dv}{dt}+\frac{v}{R},\ v(0)=V_0 \rightarrow \begin{cases} v_h : cr+\frac{1}{R}=0 \rightarrow r=-\frac{1}{RC} \rightarrow v_h(t)=Ke^{-\frac{t}{RC}} \\ v_p : v_p=RI \end{cases}$$

$$v(t)=Ke^{-\frac{t}{RC}}+RI \xrightarrow{v(0^-)=v(0^+)=V_0} V_0=K+RI \rightarrow K=V_0-RI$$

$$v(t)=\left(V_0-RI\right)e^{-\frac{t}{RC}}+RI=V_0e^{-\frac{t}{RC}}+RI\left(1-e^{-\frac{t}{RC}}\right)$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

روش ثابت زمانی در پاسخ کامل مدارهای مرتبه اول

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \tag{7.53}$$

where $v(0)$ is the initial voltage at $t = 0^+$ and $v(\infty)$ is the final or steady-state value. Thus, to find the step response of an *RC* circuit requires three things:

Note that if the switch changes position at time $t = t_0$ instead of at $t = 0$, there is a time delay in the response so that Eq. (7.53) becomes

$$v(t) = v(\infty) + [v(t_0) - v(\infty)]e^{-(t-t_0)/\tau} \tag{7.54}$$

where $v(t_0)$ is the initial value at $t = t_0^+$. Keep in mind that Eq. (7.53) or (7.54) applies only to step responses, that is, when the input e xcitation is constant.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ کامل: مثال:

Example 7.10

The switch in Fig. 7.43 has been in position A for a long time. At $t = 0$, the switch moves to B. Determine $v(t)$ for $t > 0$ and calculate its value at $t = 1$ and 4 s.

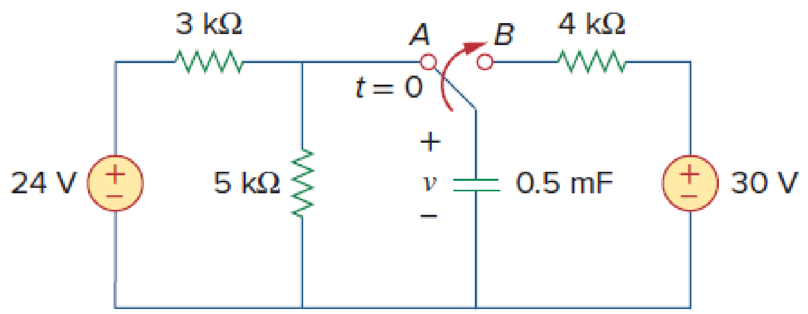


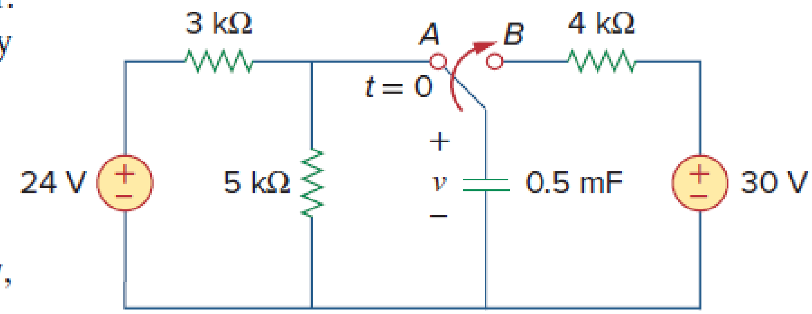
Figure 7.43
For Example 7.10.

مدارهای مرتبه اول

پاسخ کامل: ادامه مثال: (روش ثابت زمانی)

Solution:

For $t < 0$, the switch is at position A. The capacitor acts like an open circuit to dc, but v is the same as the voltage across the 5-kΩ resistor. Hence, the voltage across the capacitor just before $t = 0$ is obtained by voltage division as



$$v(0^-) = \frac{5}{5+3}(24) = 15 \text{ V}$$

Using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15 \text{ V}$$

For $t > 0$, the switch is in position B. The Thevenin resistance connected to the capacitor is $R_{Th} = 4$ kΩ, and the time constant is

$$\tau = R_{Th}C = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2 \text{ s}$$

Since the capacitor acts like an open circuit to dc at steady state, $v(\infty) = 30$ V. Thus,

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t}) \text{ V}$$

At $t = 1$,

$$v(1) = 30 - 15e^{-0.5} = 20.9 \text{ V}$$

At $t = 4$,

$$v(4) = 30 - 15e^{-2} = 27.97 \text{ V}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

پاسخ کامل: ادامه مثال: (روش تشکیل معادله دیفرانسیل)

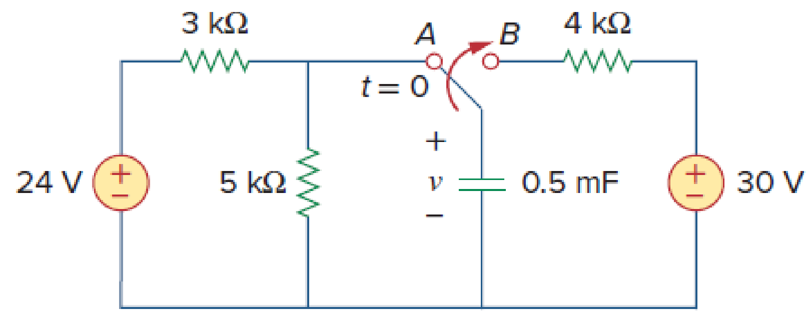
$$t<0: v(0^-)=15v$$

$$t>0: KVL: -v-4K\times 0.5m\frac{dv}{dt}+30=0$$

$$v+2\frac{dv}{dt}=30 \rightarrow \begin{cases} v_h: 1+2r=0 \rightarrow r=-0.5 \\ v_p=30 \end{cases}$$

$$v(t)=Ke^{-0.5t}+30 \xrightarrow{v(0^+)=v(0^-)=15v} K=-15$$

$$v(t)=\begin{cases} 15 & t\le 0 \\ 30-15e^{-0.5t} & t>0 \end{cases}$$



At $t = 1$,

$$v(1) = 30 - 15e^{-0.5} = 20.9\text{ V}$$

At $t = 4$,

$$v(4) = 30 - 15e^{-2} = 27.97\text{ V}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

معرفی تابع پله واحد برای جایگزین سوئیچ در مدار

We define the unit-step forcing function as a function of time which is zero for all values of its argument less than zero and which is unity for all positive values of its argument. If we let $(t - t_0)$ be the argument and represent the unit-step function by u, then $u(t - t_0)$ must be zero for all values of t less than t_0, and it must be unity for all values of t greater than t_0. At $t = t_0$, $u(t - t_0)$ changes *abruptly* from 0 to 1. Its value at $t = t_0$ is not defined, but its value is known for all instants of time that are arbitrarily close to $t = t_0$. We often indicate this by writing $u(t_0^-) = 0$ and $u(t_0^+) = 1$. The concise mathematical definition of the unit-step forcing function is

$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

and the function is shown graphically in Fig. 8.26. Note that a vertical line of unit length is shown at $t = t_0$. Although this "riser" is not strictly a part of the definition of the unit step, it is usually shown in each drawing.

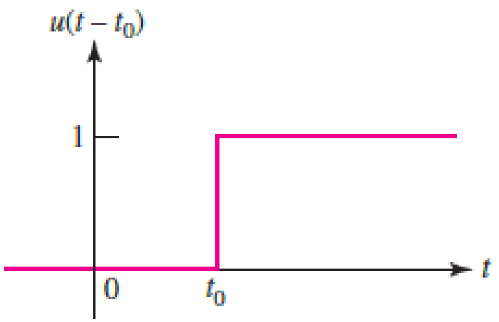


FIGURE 8.26 The unit-step forcing function, $u(t - t_0)$.

دانشکده برق و کامپیوتر

مدارهای مرتبه اول
معرفی تابع پله واحد: ادامه

We also note that the unit step need not be a time function. For example, $u(x - x_0)$ could be used to denote a unit-step function where x might be a distance in meters, for example, or a frequency.

Very often in circuit analysis a discontinuity or a switching action takes place at an instant that is defined as $t = 0$. In that case $t_0 = 0$, and we then represent the corresponding unit-step forcing function by $u(t - 0)$, or more simply $u(t)$. This is shown in Fig. 8.27. Thus

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

The unit-step forcing function is in itself dimensionless. If we wish it to represent a voltage, it is necessary to multiply $u(t - t_0)$ by some constant voltage, such as 5 V. Thus, $v(t) = 5u(t - 0.2)$ V is an ideal voltage source which is zero before $t = 0.2$ s and a constant 5 V after $t = 0.2$ s. This forcing function is shown connected to a general network in Fig. 8.28*a*.

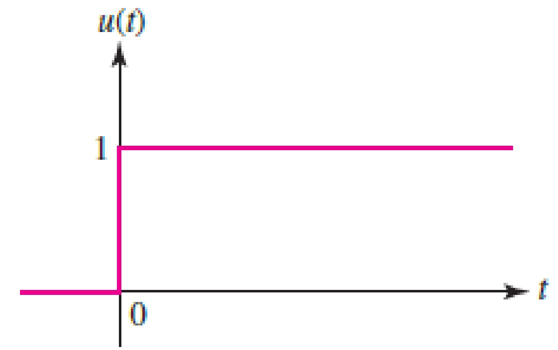


FIGURE 8.27 The unit-step forcing function $u(t)$ is shown as a function of t.

مدارهای مرتبه اول
معرفی تابع پالس واحد

$$v(t) = \begin{cases} 0 & t < t_0 \\ V_0 & t_0 < t < t_1 \\ 0 & t > t_1 \end{cases}$$

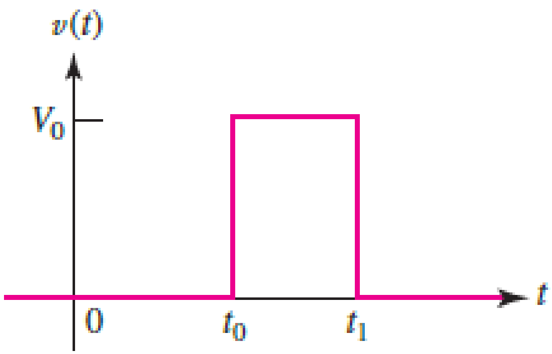


FIGURE 8.30 A useful forcing function, the rectangular voltage pulse.

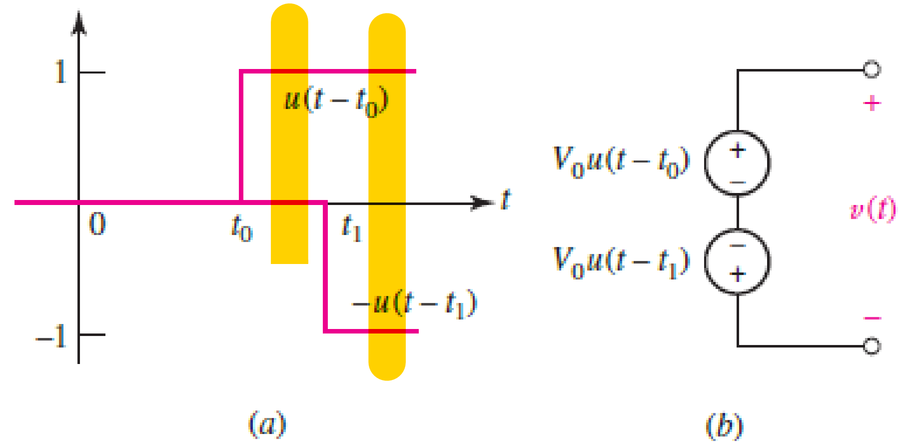


FIGURE 8.31 (*a*) The unit steps $u(t-t_0)$ and $-u(t-t_1)$. (*b*) A source which yields the rectangular voltage pulse of Fig. 8.30.

مدارهای مرتبه اول

راه انداز مدار RL با استفاده از تابع پله واحد

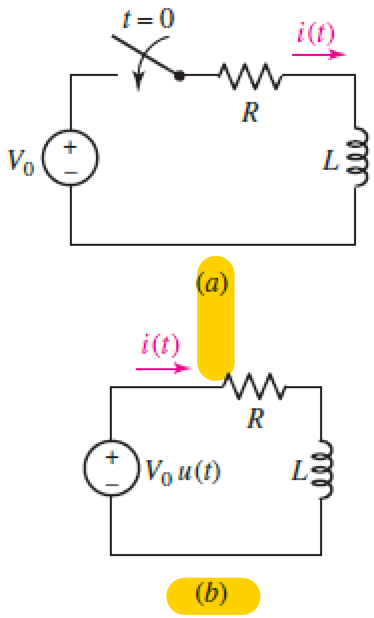


FIGURE 8.33 (*a*) The given circuit. (*b*) An equivalent circuit, possessing the same response *i*(*t*) for all time.

$$Ri + L\frac{di}{dt} = V_0 u(t)$$

Since the unit-step forcing function is discontinuous at $t = 0$, we will first consider the solution for $t < 0$ and then for $t > 0$. The application of zero voltage since $t = -\infty$ forces a zero response, so that

$$i(t) = 0 \qquad t < 0$$

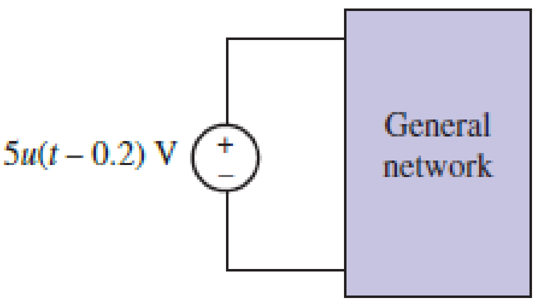
For positive time, however, $u(t)$ is unity and we must solve the equation

$$Ri + L\frac{di}{dt} = V_0 \qquad t > 0$$

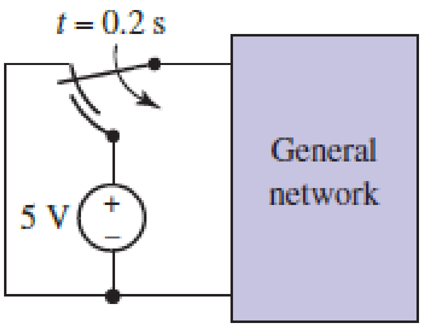
$$\begin{cases} i_h(t) : R + Lr = 0 \rightarrow r = -R/L \rightarrow i_h(t) = Ke^{-R/Lt} \\ i_P(t) = V_0/R \end{cases} \rightarrow i(t) = Ke^{-R/Lt} + V_0/R$$

$$\xrightarrow{i(0^-)=i(0^+)=0} i(t) = -V_0/Re^{-R/Lt} + V_0/R = V_0/R\left(1 - e^{-R/Lt}\right)u(t)$$

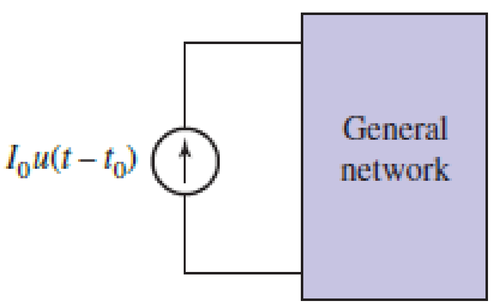
مدارهای مرتبه اول
معرفی تابع پله واحد: ادامه



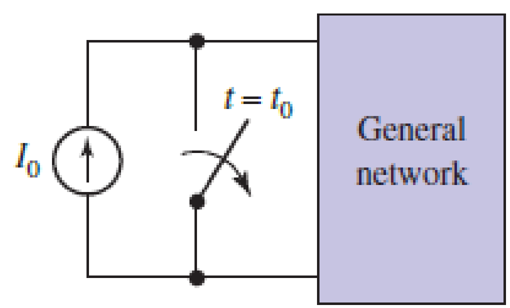
(a)



(c)



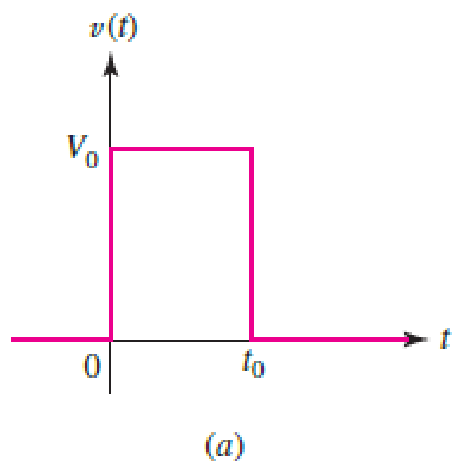
(a)



(b)

مدارهای مرتبه اول
مدار RL: استفاده از تابع پالس واحد

Find the current response in a simple series *RL* circuit when the forcing function is a rectangular voltage pulse of amplitude V_0 and duration t_0.



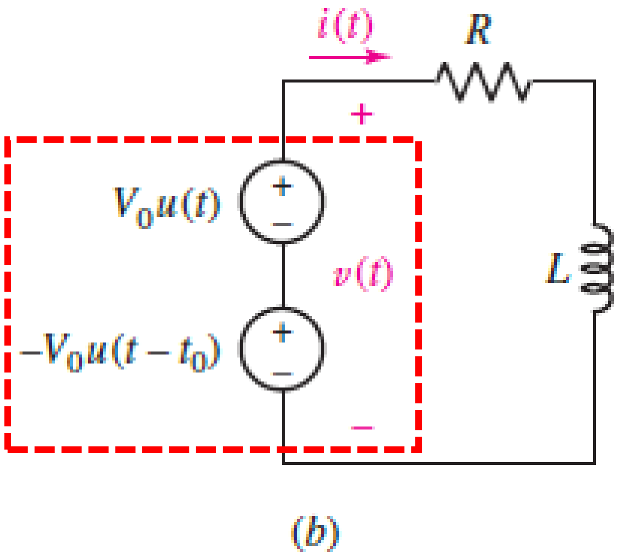


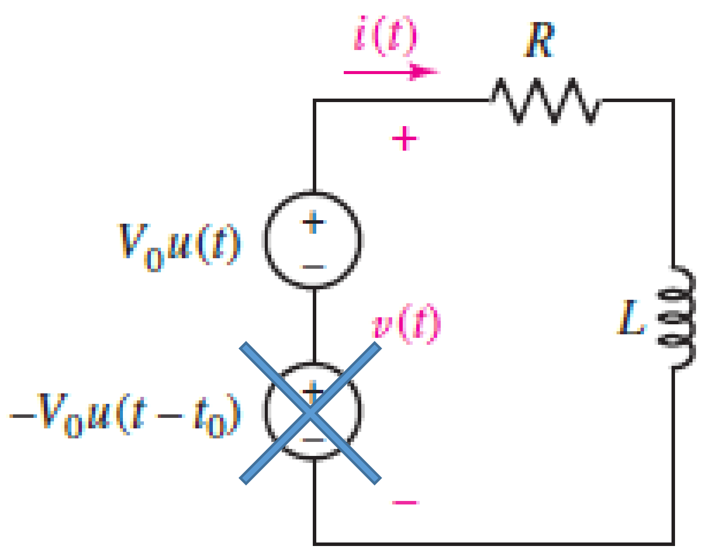
FIGURE 8.39 (*a*) A rectangular voltage pulse which is to be used as the forcing function in a simple series *RL* circuit. (*b*) The series *RL* circuit, showing the representation of the forcing function by the series combination of two independent voltage-step sources. The current *i*(*t*) is desired.

مدارهای مرتبه اول: مدار RL: ادامه مثال: (روش اول) تاثیر منبع اول

$$KVL: Ri + L\frac{di}{dt} = V_0 u(t) \qquad t>0 \rightarrow Ri + L\frac{di}{dt} = V_0$$

$$\begin{cases} i_h(t): R + Lr = 0 \rightarrow r = -\dfrac{R}{L} \rightarrow i_h(t) = Ke^{-R/Lt} \\ i_P(t) = \dfrac{V_0}{R} \end{cases} \rightarrow i(t) = Ke^{-R/Lt} + \frac{V_0}{R}$$

$$\xrightarrow{i(0^-)=i(0^+)=0} i(t) = -\frac{V_0}{R}e^{\frac{-R}{L}t} + \frac{V_0}{R} = \frac{V_0}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$



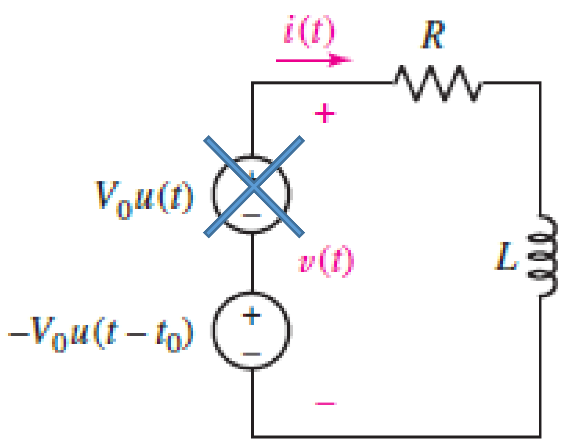
مدارهای مرتبه اول: مدار RL: ادامه مثال: (روش اول) تاثیر منبع دوم

$$KVL: Ri + Ldi/dt = -V_0 u(t-t_0) \qquad t > t_0 \to Ri + Ldi/dt = -V_0$$

$$\begin{cases} i_h(t): R + Lr = 0 \to r = -R/L \to i_h(t) = Ke^{\frac{-R}{L}t} \\ i_P(t) = -\dfrac{V_0}{R} \end{cases} \to i(t) = Ke^{\frac{-R}{L}t} - \frac{V_0}{R}$$

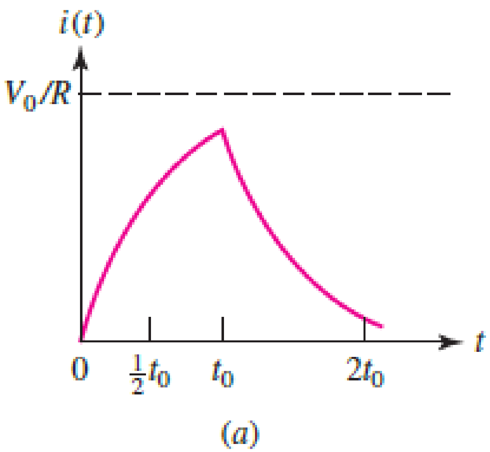
$$\xrightarrow{i(t_0^-)=i(t_0^+)=0} 0 = Ke^{\frac{-R}{L}t_0} - \frac{V_0}{R} \to K = \frac{\frac{V_0}{R}}{e^{\frac{-R}{L}t_0}} = \frac{V_0}{R}e^{\frac{R}{L}t_0}$$

$$i(t) = \frac{V_0}{R}e^{\frac{R}{L}t_0}e^{\frac{-R}{L}t} - \frac{V_0}{R} = \frac{V_0}{R}\left[e^{\frac{R}{L}t_0}e^{\frac{-R}{L}t} - 1\right] = \frac{V_0}{R}\left[e^{\frac{-R}{L}(t-t_0)} - 1\right]$$



مدارهای مرتبه اول

مدار RL: ادامه مثال (روش اول): قضیه جمع آثار



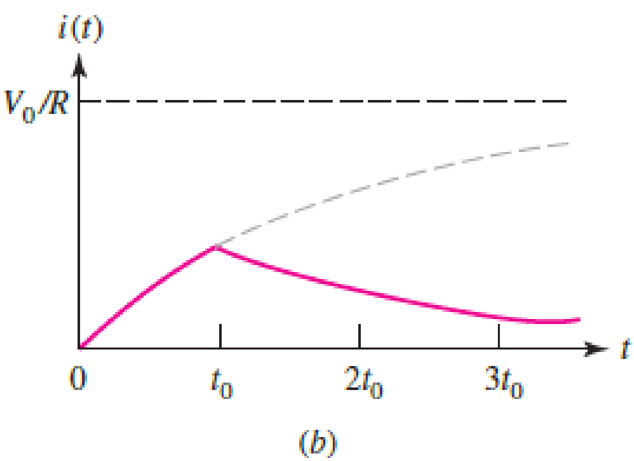


FIGURE 8.40 Two possible response curves are shown for the circuit of Fig. 8.39*b*. (*a*) τ is selected as $t_0/2$. (*b*) τ is selected as $2t_0$.

$$i_1(t) = V_0/R\left(1-e^{-R/Lt}\right)u(t)$$

$$i_2(t) = V_0/R\left(e^{-R/L(t-t_0)}-1\right)u(t-t_0)$$

$$i(t) = i_1(t) + i_2(t)$$

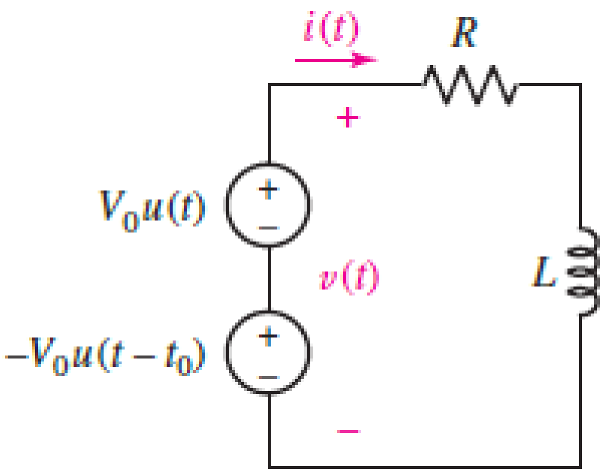
$$i(t) = \begin{cases} 0 & t \le 0 \\ V_0/R\left(1-e^{-R/Lt}\right) & 0 < t \le t_0 \\ V_0/R\left(1-e^{-R/Lt}\right) + V_0/R\left(e^{-R/L(t-t_0)}-1\right) & t > t_0 \end{cases}$$

$$i(t) = \begin{cases} 0 & t \le 0 \\ V_0/R\left(1-e^{-R/Lt}\right) & 0 < t \le t_0 \\ V_0/R\left(e^{R/Lt_0}-1\right)e^{-R/Lt} & t > t_0 \end{cases}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه مثال (روش دوم)



$0<t<t_0$

$$KVL: Ri+Ldi/dt=V_0 \rightarrow \begin{cases} i_h(t): R+Lr=0 \rightarrow r=-\dfrac{R}{L} \rightarrow i_h(t)=Ke^{-R/Lt} \\ i_P(t)=\dfrac{V_0}{R} \end{cases}$$

$$\rightarrow i(t)=Ke^{-R/Lt}+\frac{V_0}{R} \xrightarrow{i(0^-)=i(0^+)=0} i(t)=-\frac{V_0}{R}e^{-R/Lt}+\frac{V_0}{R}=\frac{V_0}{R}\left(1-e^{-R/Lt}\right)$$

$t>t_0$

$$KVL: Ri+Ldi/dt=0 \rightarrow i_h(t): R+Lr=0 \rightarrow r=-\frac{R}{L} \rightarrow i_h(t)=Ke^{\frac{-R}{L}t}$$

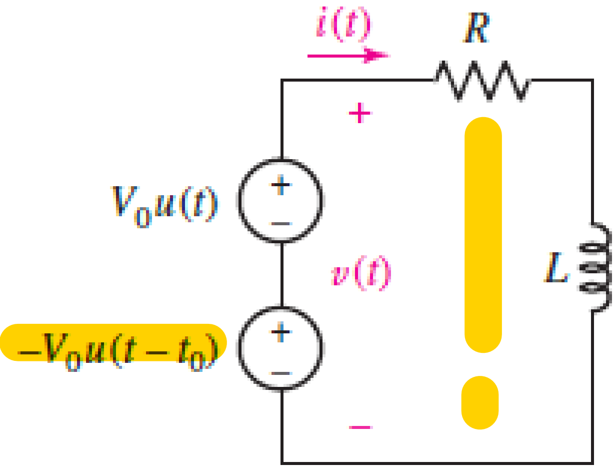
$$\xrightarrow{i(t_0^-)=i(t_0^+)=\frac{V_0}{R}\left(1-e^{\frac{-R}{L}t_0}\right)} \frac{V_0}{R}\left(1-e^{\frac{-R}{L}t_0}\right)=Ke^{\frac{-R}{L}t_0} \rightarrow K=\frac{\frac{V_0}{R}\left(1-e^{\frac{-R}{L}t_0}\right)}{e^{\frac{-R}{L}t_0}}=\frac{V_0}{R}\left(e^{\frac{R}{L}t_0}-1\right) \rightarrow i(t)=\frac{V_0}{R}\left(e^{\frac{R}{L}t_0}-1\right)e^{-}$$

$$i(t)=\begin{cases} 0 & t\le 0 \\ \dfrac{V_0}{R}\left(1-e^{-R/Lt}\right) & 0<t\le t_0 \\ \dfrac{V_0}{R}\left(e^{\frac{R}{L}t_0}-1\right)e^{\frac{-R}{L}t} & t>t_0 \end{cases}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه مثال (روش سوم)



$$i_L(t)=i_L(\infty)+\left[i_L(0^+)-i_L(\infty)\right]e^{\frac{-t}{\tau}}$$

$$i_L(t)=i_L(\infty)+\left[i_L(t_0)-i_L(\infty)\right]e^{\frac{-(t-t_0)}{\tau}}$$

$$0<t\le t_0:\tau=\frac{L}{R},i_L(0^-)=i_L(0^+)=0,i_L(\infty)=\frac{V_0}{R}\rightarrow i_L(t)=\frac{V_0}{R}+\left[0-\frac{V_0}{R}\right]e^{\frac{-R}{L}t}=\frac{V_0}{R}\left(1-e^{\frac{-R}{L}t}\right)$$

$$t>t_0:\tau=\frac{L}{R},i_L(t_0)=\frac{V_0}{R}\left(1-e^{\frac{-R}{L}t_0}\right),i_L(\infty)=0\rightarrow i_L(t)=0+\left[\frac{V_0}{R}\left(1-e^{\frac{-R}{L}t_0}\right)-0\right]e^{\frac{-R}{L}(t-t_0)}$$

$$i_L(t)=\frac{V_0}{R}\left(1-e^{\frac{-R}{L}t_0}\right)e^{\frac{-R}{L}(t-t_0)}=\frac{V_0}{R}\left(e^{\frac{-R}{L}(t-t_0)}-e^{\frac{-R}{L}t}\right)=\frac{V_0}{R}\left(e^{\frac{R}{L}t_0}-1\right)e^{\frac{-R}{L}t}$$

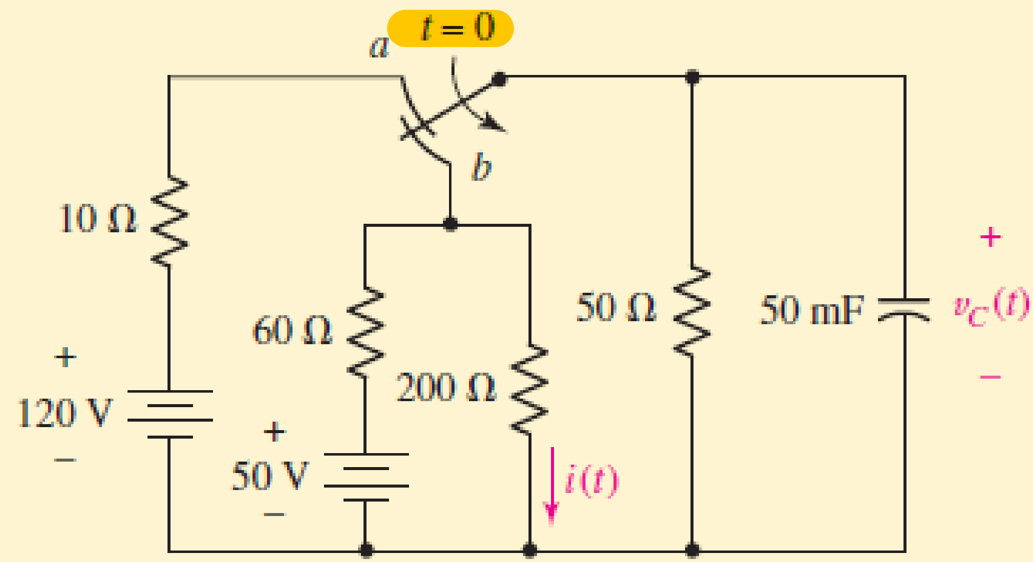
$$i(t)=\begin{cases}0 & t\le 0\\ \frac{V_0}{R}\left(1-e^{-R/Lt}\right) & 0<t\le t_0\\ \frac{V_0}{R}\left(e^{\frac{R}{L}t_0}-1\right)e^{\frac{-R}{L}t} & t>t_0\end{cases}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RC: مثال (پاسخ کامل)

Find the capacitor voltage $v_C(t)$ and the current $i(t)$ in the 200 Ω resistor of Fig. 8.42 for all time.

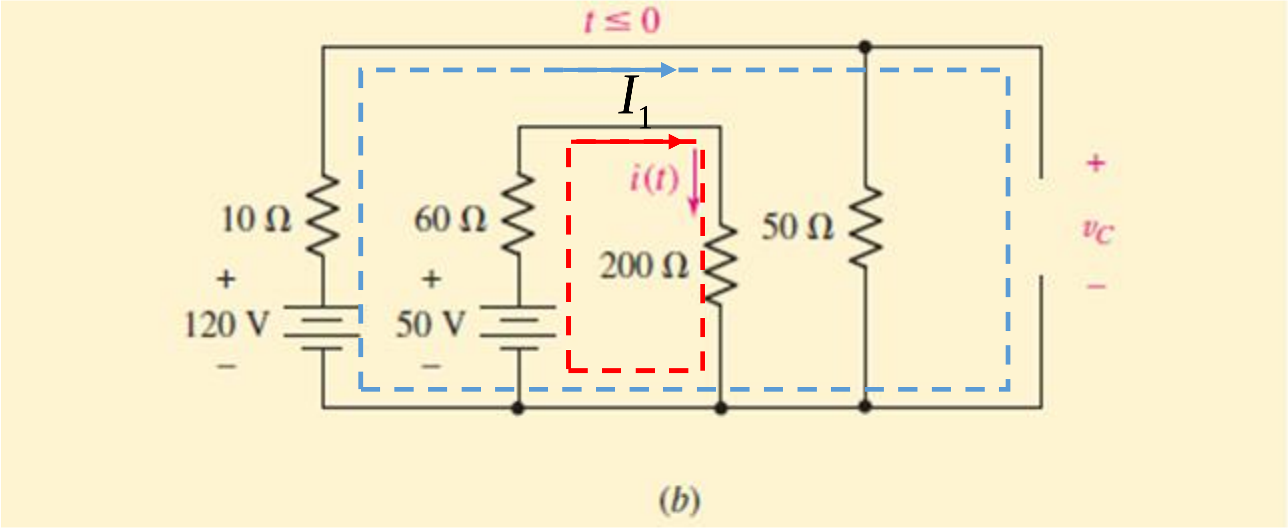


(a)

دانشکده برق و کامپیوتر

مدارهای مرتبه اول: مدار RC: ادامه مثال

$t < 0$



$$KVL: -120 + 10I_1 + 50I_1 = 0 \rightarrow I_1 = 2A$$

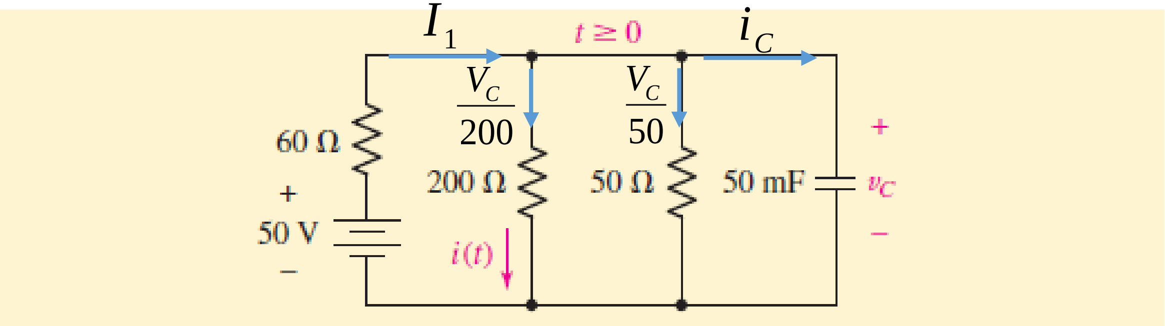
$$V_C(0^-) = 50 \times 2A = 100v = V_C(0^+)$$

$$KVL: -50 + 60i(t) + 200i(t) = 0 \rightarrow i(t) = 0.1923A$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول: مدار RC: ادامه مثال (روش اول)

$t > 0$



$$KCL: I_1 = \frac{V_C}{200} + \frac{V_C}{50} + C\frac{dV_C}{dt}$$

$$KVL: -50 + 60I_1 + V_C = 0 \rightarrow -50 + 60\left(\frac{V_C}{200} + \frac{V_C}{50} + C\frac{dV_C}{dt}\right) + V_C = 0$$

$$60C\frac{dV_C}{dt} + 2.5V_C = 50 \rightarrow V_C(t) = V_{Ch}(t) + V_{CP}(t) \rightarrow \begin{cases} 60Cr + 2.5 = 0 \rightarrow r = -0.833 \\ V_{CP}(t) = \dfrac{50}{2.5} \end{cases}$$

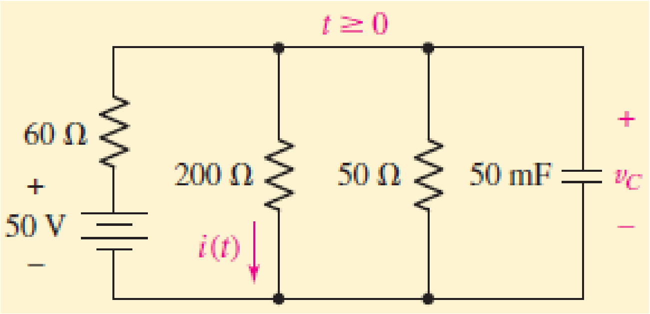
$$V_C(t) = Ke^{-0.833t} + \frac{50}{2.5} \xrightarrow{V_C(0^-)=100v=V_C(0^+)} 100 = K + \frac{50}{2.5} \rightarrow K = 80$$

$$V_C(t) = 80e^{-0.833t} + 20 \xrightarrow{i(t)=\frac{V_C(t)}{200}} i(t) = 0.4e^{-0.833t} + 0.1$$

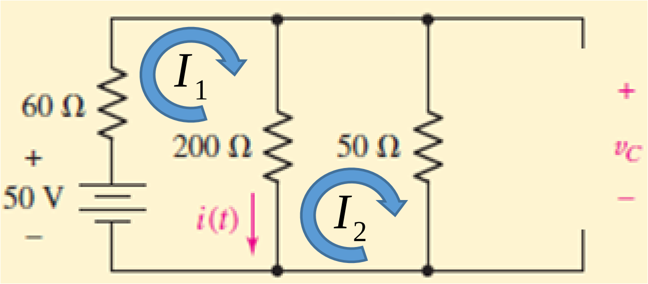
$$V_C(t) = \begin{cases} 100 & t < 0 \\ 80e^{-0.833t} + 20 & t \ge 0 \end{cases}, i(t) = \begin{cases} 0.1923 & t < 0 \\ 0.4e^{-0.833t} + 0.1 & t > 0 \end{cases}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول: مدار RC: ادامه مثال (روش دوم)



$t = \infty$



$$V_C(0^+) = 100v \rightarrow i(0^+) = \frac{100}{200} = 0.5A$$

$$KVL: \begin{cases} 50 = 60I_1 + 200\left(I_1 - I_2\right) \\ 0 = 50I_2 + 200\left(I_2 - I_1\right) \end{cases} \rightarrow I_1 = 0.5A; I_2 = 0.4A$$

$$v_C(\infty) = 50I_2(\infty) = 20v, i(\infty) = I_1(\infty) - I_2(\infty) = 0.1A$$

$$R_{eq} = 50 \| 200 \| 60 = 23.98\Omega \rightarrow \tau = R_{eq}C = 1.199s$$

$$v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right]e^{\frac{-t}{\tau}}$$

$$i(t) = i(\infty) + \left[i(0^+) - i(\infty)\right]e^{\frac{-t}{\tau}}$$

$$v_C(t) = 20 + 80e^{\frac{-t}{1.199}}$$

$$i(t) = 0.1 + 0.4e^{\frac{-t}{1.199}}$$

$$v_C(t) = \begin{cases} 100 & t < 0 \\ 80e^{-0.833t} + 20 & t \geq 0 \end{cases}$$

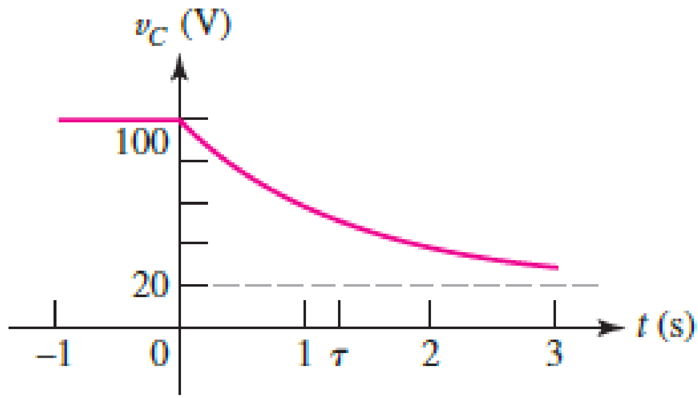
$$i(t) = \begin{cases} 0.1923 & t < 0 \\ 0.4e^{-0.833t} + 0.1 & t > 0 \end{cases}$$

دانشکده برق و کامپیوتر

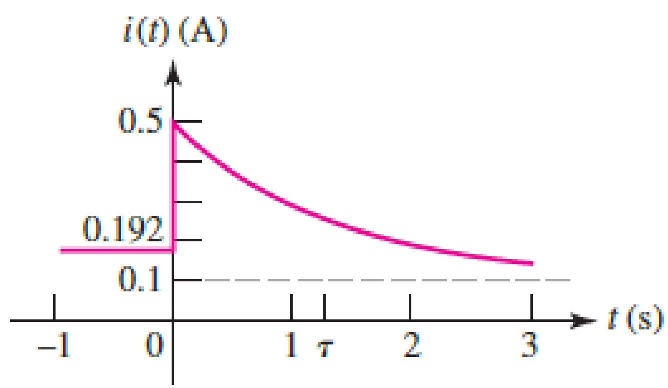
مدارهای مرتبه اول

مدار RC: ادامه مثال

$$\begin{cases} v_C(t \le 0) = 100v \\ v_C(t > 0) = 80e^{-0.833t} + 20 \end{cases}$$



$$\begin{cases} i(t < 0) = 0.1923A \\ i(t > 0) = 0.4e^{-0.833t} + 0.1 \end{cases}$$

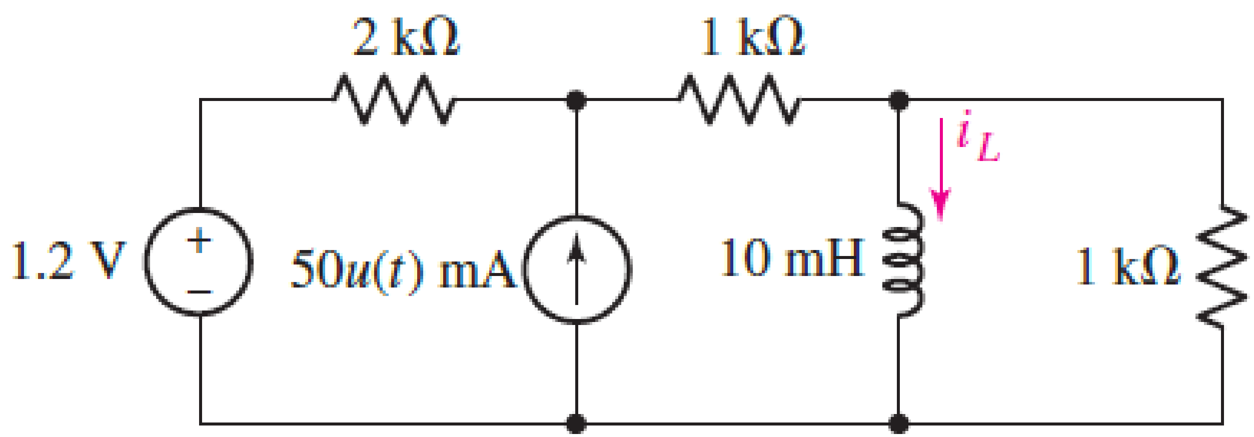


دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: تمرین (به عهده دانشجو)

The circuit depicted in Fig. 8.79 contains two independent sources, one of which is only active for $t > 0$. (*a*) Obtain an expression for $i_L(t)$ valid for all t; (*b*) calculate $i_L(t)$ at $t = 10\ \mu s$, $20\ \mu s$, and $50\ \mu s$.



■ FIGURE 8.79

مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو)

$t \geq 0$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو)

(روش اول): تشکیل و حل معادله دیفرانسیل

مدارهای مرتبه اول

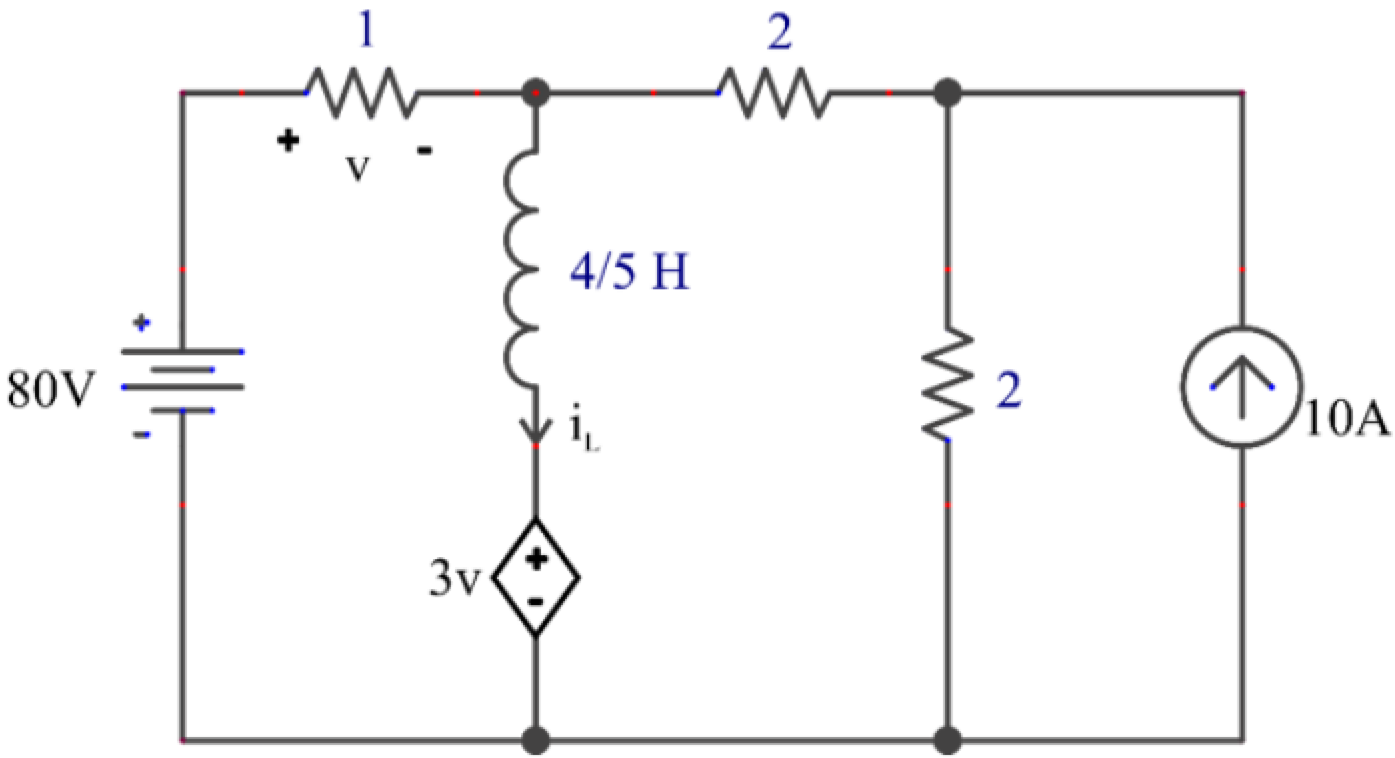
مدار RL: ادامه تمرین (به عهده دانشجو)

(روش دوم): روش ثابت زمانی

مدارهای مرتبه اول

مدار RL: تمرین (به عهده دانشجو)

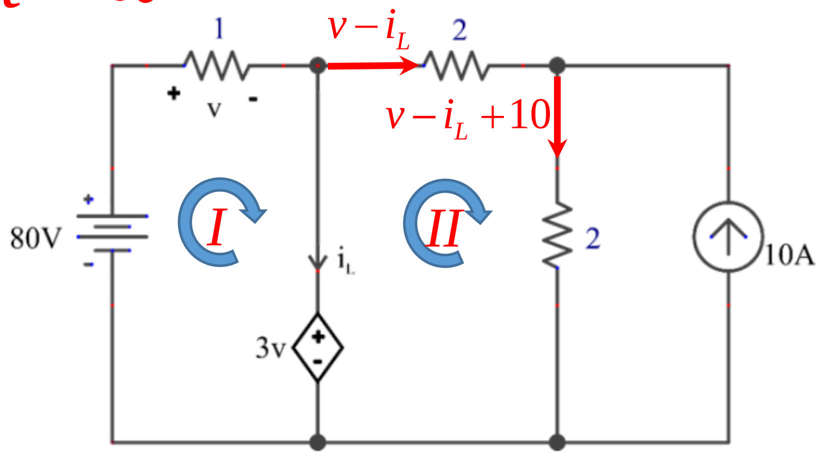
در مدار شکل زیر اگر بدانیم $i_L(0^-) = 2$ آمپر است، جریان $i_L(t)$ را برای $t > 0$ بیابید. (مقاومت ها بر حسب اهم هستند)



مدارهای مرتبه اول

مدار RL: ادامه تمرین (به عهده دانشجو) : استفاده از روش حل ثابت زمانی

$t=\infty$

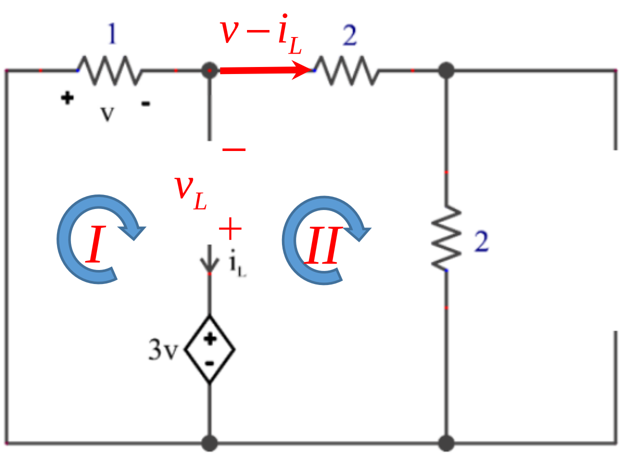


$$KVL\ I: 80 = v + 3v \rightarrow v = 20$$

$$KVL\ II: 2\left[v - i_L(\infty)\right] + 2\left[v - i_L(\infty) + 10\right] - 3v = 0$$

$$v + 20 = 4 i_L(\infty) \rightarrow \boxed{i_L(\infty) = \frac{40}{4} = 10A}$$

محاسبه مقاومت دیده شده از دو سر سلف



$$KVL\ I: 0 = v - v_L + 3v \rightarrow \boxed{4v = v_L}$$

$$KVL\ II: 4\left[v - i_L\right] - 3v + v_L = 0$$

$$v - 4i_L + v_L = 0 \rightarrow \frac{v_L}{4} - 4i_L + v_L = 0$$

$$\frac{5}{4} v_L = 4 i_L \rightarrow \boxed{R_{th} = \frac{v_L}{i_L} = \frac{8}{5}\,\Omega}$$

$$\boxed{\tau = \frac{L}{R_{th}} = \frac{4/5}{8/5} = 0.5s}$$

دانشکده برق و کامپیوتر

مدارهای مرتبه اول

مدار RL: ادامه **تمرین (به عهده دانشجو)** : استفاده از روش حل **ثابت زمانی**

$$\begin{cases} i_L(0) = 2A \\ i_L(\infty) = 10A \rightarrow i_L(t) = i_L(\infty) + \left[i_L(0) - i_L(\infty) \right] e^{\frac{-t}{\tau}} \rightarrow \boxed{i_L(t) = 10 - 8e^{-2t}} \\ R_{th} = \frac{8}{5} \Omega \end{cases}$$