دانشکده برق و کامپیوتر

به نام خدا

فصل سوم: تجزیه و تحلیل حالت دائمی سینوسی

ارائه کننده:

روحانی

آبان ۱۴۰۰

دانشکده برق و کامپیوتر

تعاریف اولیه: موج سینوسی

Consider a sinusoidally varying voltage

$$v(t) = V_m \sin \omega t$$

shown graphically in Figs. 10.1*a* and *b*. The *amplitude* of the sine wave is V_m, and the *argument* is ωt. The *radian frequency*, or *angular frequency*, is ω. In Fig. 10.1*a*, $V_m \sin \omega t$ is plotted as a function of the argument ωt, and the periodic nature of the sine wave is evident.

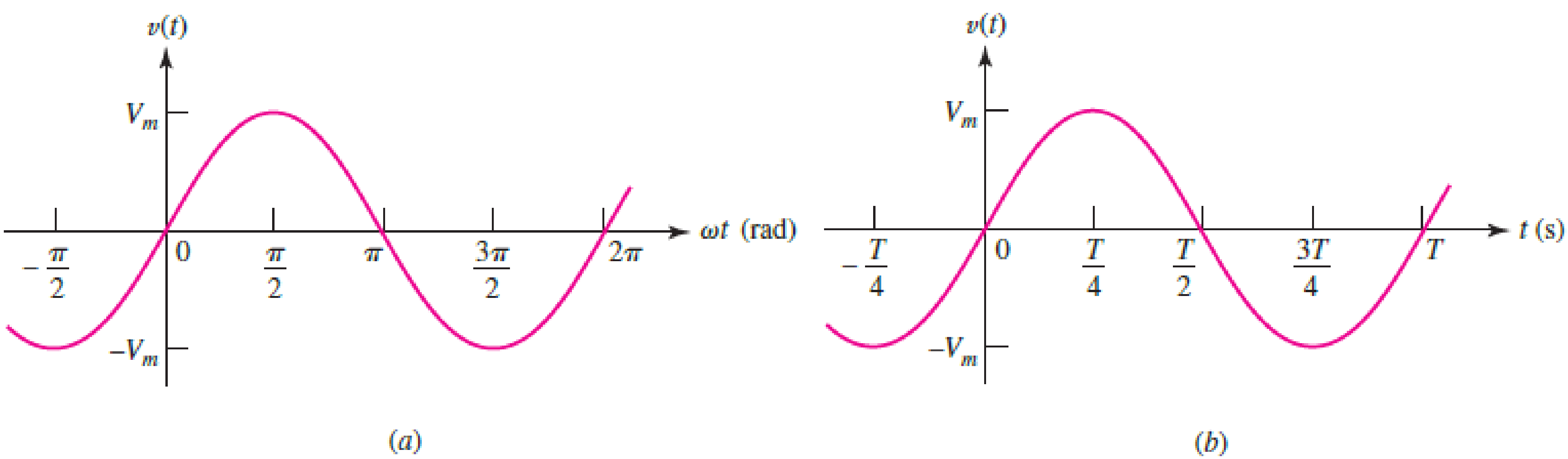


FIGURE 10.1 The sinusoidal function $v(t) = V_m \sin \omega t$ is plotted (*a*) versus ωt and (*b*) versus t.

تعاریف اولیه: موج سینوسی

The function repeats itself every 2π radians, and its ***period*** is therefore 2π radians. In Fig. 10.1*b*, $V_m \sin \omega t$ is plotted as a function of t and the *period* is now T. A sine wave having a period T must execute $1/T$ periods each second; its ***frequency*** f is $1/T$ hertz, abbreviated Hz. Thus,

$$f = \frac{1}{T}$$

and since

$$\omega T = 2\pi$$

we obtain the common relationship between frequency and radian frequency,

$$\boxed{\omega = 2\pi f}$$

دانشکده برق و کامپیوتر

تعاریف اولیه: موج سینوسی
تعریف تقدم (پیش فاز) و تاخر فاز (پس فاز)

A more general form of the sinusoid,

$$v(t) = V_m \sin(\omega t + \theta) \tag{1}$$

includes a *phase angle* θ in its argument. Equation [1] is plotted in Fig. 10.2 as a function of ωt, and the phase angle appears as the number of radians by which the original sine wave (shown in green color in the sketch) is shifted to the left, or earlier in time. Since corresponding points on the sinusoid $V_m \sin(\omega t + \theta)$ occur θ rad, or θ/ω seconds, earlier, we say that $V_m \sin(\omega t + \theta)$ *leads* $V_m \sin \omega t$ by θ rad. Therefore, it is correct to describe

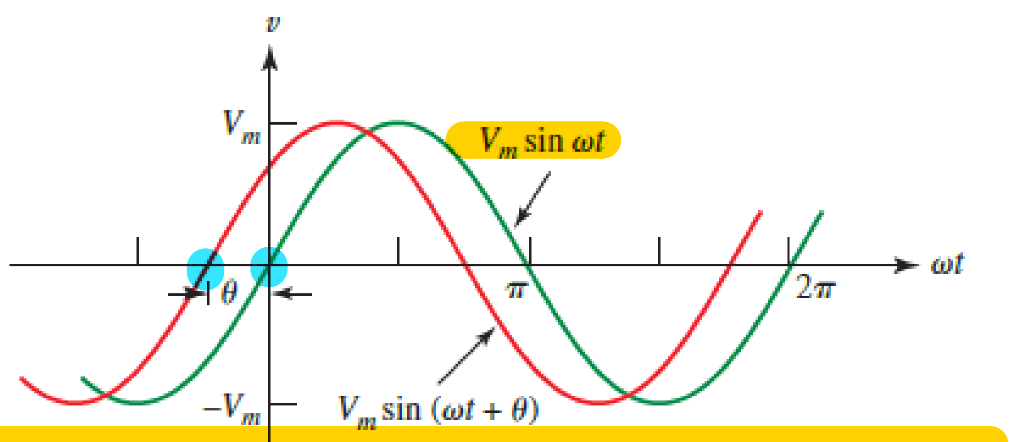


FIGURE 10.2 The sine wave $V_m \sin(\omega t + \theta)$ leads $V_m \sin \omega t$ by θ rad.

دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

تعاریف اولیه: موج سینوسی
تعریف تقدم (پیش فاز) و تاخر فاز (پس فاز)

sin ωt as ***lagging*** $\sin(\omega t + \theta)$ by θ rad, as ***leading*** $\sin(\omega t + \theta)$ by $-\theta$ rad, or as leading $\sin(\omega t - \theta)$ by θ rad.

In either case, leading or lagging, we say that the sinusoids are *out of phase*. If the phase angles are equal, the sinusoids are said to be *in phase*.

In electrical engineering, the phase angle is commonly given in degrees, rather than radians; to avoid confusion we should be sure to always use the degree symbol. Thus, instead of writing

$$v = 100 \sin\left(2\pi 1000t - \frac{\pi}{6}\right)$$

we customarily use

$$v = 100 \sin(2\pi 1000t - 30^\circ)$$

In evaluating this expression at a specific instant of time, e.g., $t = 10^{-4}$ s, $2\pi 1000t$ becomes 0.2π *radian*, and this should be expressed as 36° before 30° is subtracted from it. Don't confuse your apples with your oranges.

دانشکده برق و کامپیوتر

تعاریف اولیه: موج سینوسی
تبدیل موج کسینوسی به سینوسی

Converting Sines to Cosines

The sine and cosine are essentially the same function, but with a 90° phase difference. Thus, $\sin \omega t = \cos(\omega t - 90°)$. Multiples of 360° may be added to or subtracted from the argument of any sinusoidal function without changing the value of the function. Hence, we may say that

$$v_1 = V_{m_1} \cos(5t + 10°)$$
$$= V_{m_1} \sin(5t + 90° + 10°)$$
$$= V_{m_1} \sin(5t + 100°)$$

leads

$$v_2 = V_{m_2} \sin(5t - 30°)$$

by 130°. It is also correct to say that v_1 *lags* v_2 by 230°, since v_1 may be written as

$$v_1 = V_{m_1} \sin(5t - 260°)$$

We assume that V_{m_1} and V_{m_2} are both positive quantities. A graphical representation is provided in Fig. 10.3; note that the frequency of both sinusoids (5 rad/s in this case) must be the same, or the comparison is meaningless. Normally, the difference in phase between two sinusoids is expressed by that angle which is less than or equal to 180° in magnitude.

The concept of a leading or lagging relationship between two sinusoids will be used extensively, and the relationship is recognizable both mathematically and graphically.

Note that:

$$-\sin \omega t = \sin(\omega t \pm 180°)$$
$$-\cos \omega t = \cos(\omega t \pm 180°)$$
$$\mp\sin \omega t = \cos(\omega t \pm 90°)$$
$$\pm\cos \omega t = \sin(\omega t \pm 90°)$$

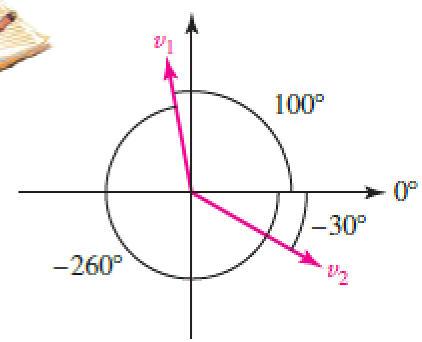


FIGURE 10.3 A graphical representation of the two sinusoids v_1 and v_2. The magnitude of each sine function is represented by the length of the corresponding arrow, and the phase angle by the orientation with respect to the positive *x* axis. In this diagram, v_1 leads v_2 by 100° + 30° = 130°, although it could also be argued that v_2 leads v_1 by 230°. It is customary, however, to express the phase difference by an angle less than or equal to 180° in magnitude.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول

Consider the series *RL* circuit shown in Fig. 10.4. The sinusoidal source voltage $v_s = V_m \cos \omega t$ has been switched into the circuit at some remote time in the past, and the natural response has died out completely. We seek the forced (or "steady-state") response, which must satisfy the differential equation

$$L\frac{di}{dt} + Ri = V_m \cos \omega t$$

obtained by applying KVL around the simple loop. At any instant where the derivative is equal to zero, we see that the current must have the form $i \propto \cos \omega t$. Similarly, at an instant where the current is equal to zero, the

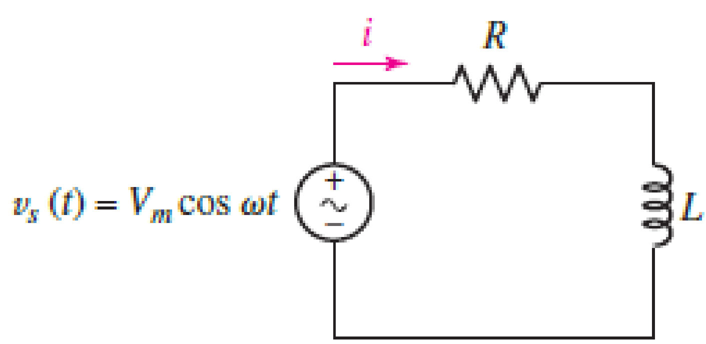


FIGURE 10.4 A series *RL* circuit for which the forced response is desired.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول: ادامه مدار RL

derivative must be proportional to $\cos \omega t$, implying a current of the form $\sin \omega t$. We might expect, therefore, that the forced response will have the general form

$$i(t) = I_1 \cos \omega t + I_2 \sin \omega t$$

where I_1 and I_2 are real constants whose values depend upon V_m, R, L, and ω. No constant or exponential function can be present. Substituting the assumed form for the solution in the differential equation yields

$$L(-I_1\omega \sin \omega t + I_2\omega \cos \omega t) + R(I_1 \cos \omega t + I_2 \sin \omega t) = V_m \cos \omega t$$

If we collect the cosine and sine terms, we obtain

$$(-LI_1\omega + RI_2) \sin \omega t + (LI_2\omega + RI_1 - V_m) \cos \omega t = 0$$

This equation must be true for all values of t, which can be achieved only if the factors multiplying $\cos \omega t$ and $\sin \omega t$ are each zero. Thus,

$$-\omega LI_1 + RI_2 = 0 \qquad \text{and} \qquad \omega LI_2 + RI_1 - V_m = 0$$

and simultaneous solution for I_1 and I_2 leads to

$$I_1 = \frac{RV_m}{R^2 + \omega^2 L^2} \qquad I_2 = \frac{\omega L V_m}{R^2 + \omega^2 L^2}$$

Thus, the forced response is obtained:

$$i(t) = \frac{RV_m}{R^2 + \omega^2 L^2} \cos \omega t + \frac{\omega L V_m}{R^2 + \omega^2 L^2} \sin \omega t \qquad [2]$$

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول: ادامه مدار RL

دانشکده برق و کامپیوتر

A More Compact and User-Friendly Form

Although accurate, this expression is slightly cumbersome; a clearer picture of the response can be obtained by expressing it as a single sinusoid or cosinusoid with a phase angle. We choose to express the response as a cosine function,

$$i(t) = A\cos(\omega t - \theta) \qquad [3]$$

At least two methods of obtaining the values of A and θ suggest themselves. We might substitute Eq. [3] directly in the original differential equation, or we could simply equate the two solutions, Eqs. [2] and [3]. Selecting the latter method, and expanding the function $\cos(\omega t - \theta)$:

$$A\cos\theta\cos\omega t + A\sin\theta\sin\omega t = \frac{RV_m}{R^2+\omega^2L^2}\cos\omega t + \frac{\omega LV_m}{R^2+\omega^2L^2}\sin\omega t$$

All that remains is to collect terms and perform a bit of algebra, an exercise left to the reader. The result is

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

and

$$A = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

and so the *alternative form of* the forced response therefore becomes

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right) \qquad [4]$$

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
مدار RL: مثال

Find the current i_L in the circuit shown in Fig. 10.5*a*, if the transients have already died out.

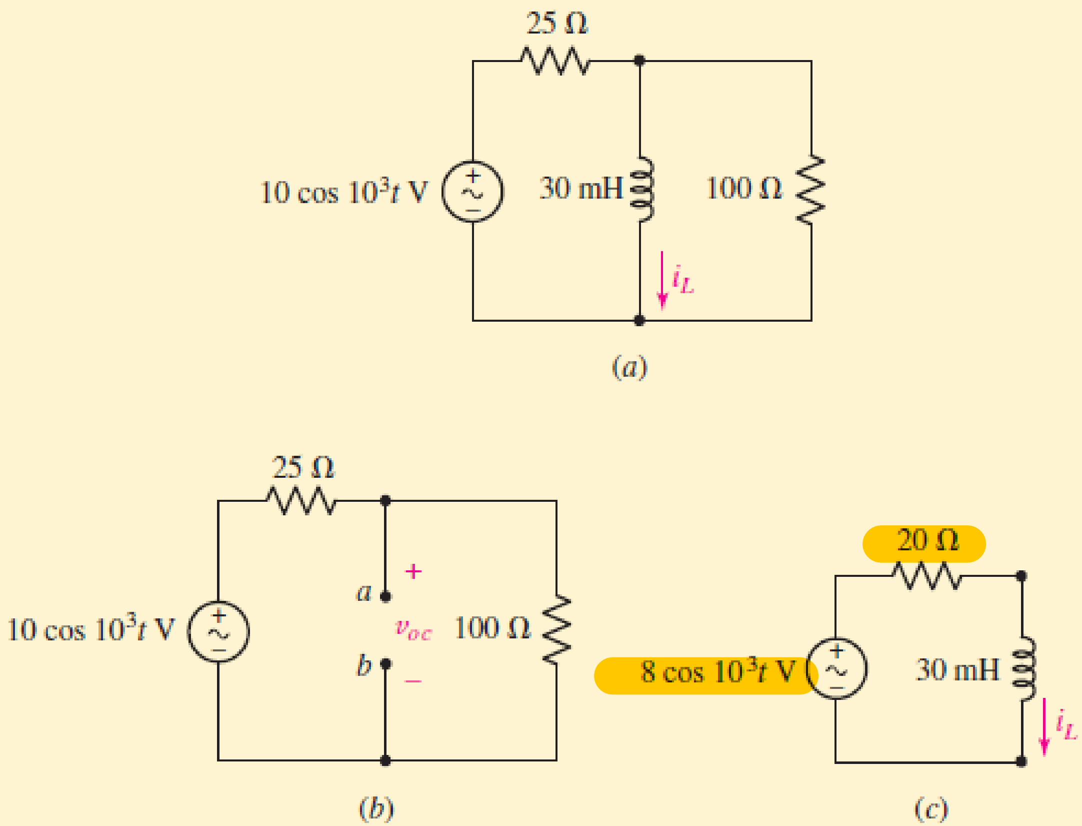


FIGURE 10.5 (*a*) The circuit for Example 10.1, in which the current i_L is desired. (*b*) The Thévenin equivalent is desired at terminals *a* and *b*. (*c*) The simplified circuit.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول: مدار RL: ادامه مثال

Although this circuit has a sinusoidal source and a single inductor, it contains two resistors and is not a single loop. In order to apply the results of the preceding analysis, we need to seek the Thévenin equivalent as viewed from terminals *a* and *b* in Fig. 10.5*b*.

The open-circuit voltage v_{oc} is

$$v_{oc} = (10\cos 10^3 t)\frac{100}{100+25} = 8\cos 10^3 t \qquad \text{V}$$

Since there are no dependent sources in sight, we find R_{th} by shorting out the independent source and calculating the resistance of the passive network, so $R_{th} = (25 \times 100)/(25+100) = 20\ \Omega$.

Now we do have a series *RL* circuit, with $L = 30$ mH, $R_{th} = 20\ \Omega$, and a source voltage of $8\cos 10^3 t$ V, as shown in Fig. 10.5*c*. Thus, applying Eq. [4], which was derived for a general *RL* series circuit,

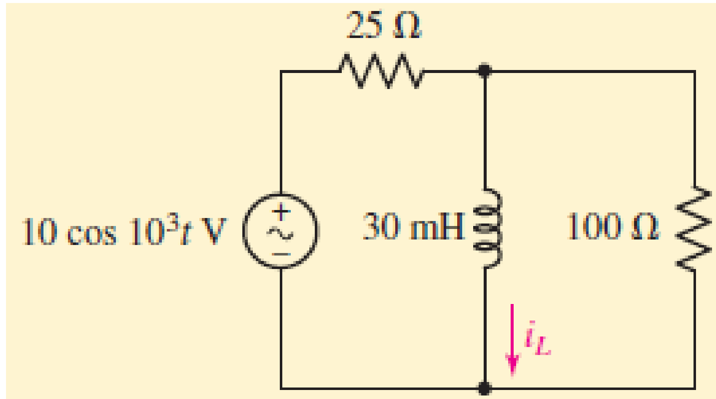
$$\begin{aligned} i_L &= \frac{8}{\sqrt{20^2 + (10^3 \times 30 \times 10^{-3})^2}} \cos\left(10^3 t - \tan^{-1}\frac{30}{20}\right) \\ &= 222\cos(10^3 t - 56.3^\circ) \qquad \text{mA} \end{aligned}$$

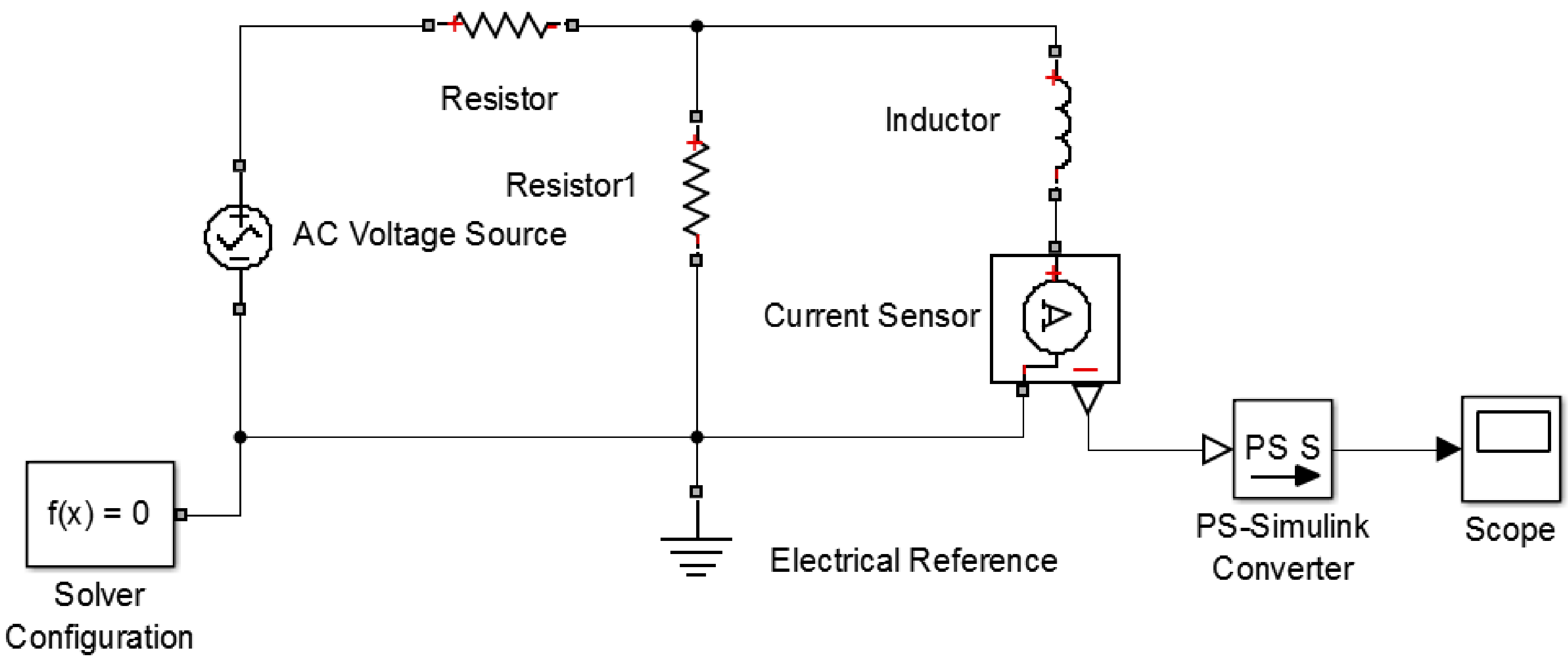
The voltage and current waveforms are plotted in Fig. 10.6.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:

مدار RL: ادامه مثال: **نتیجه شبیه سازی با نرم افزار MATLAB**

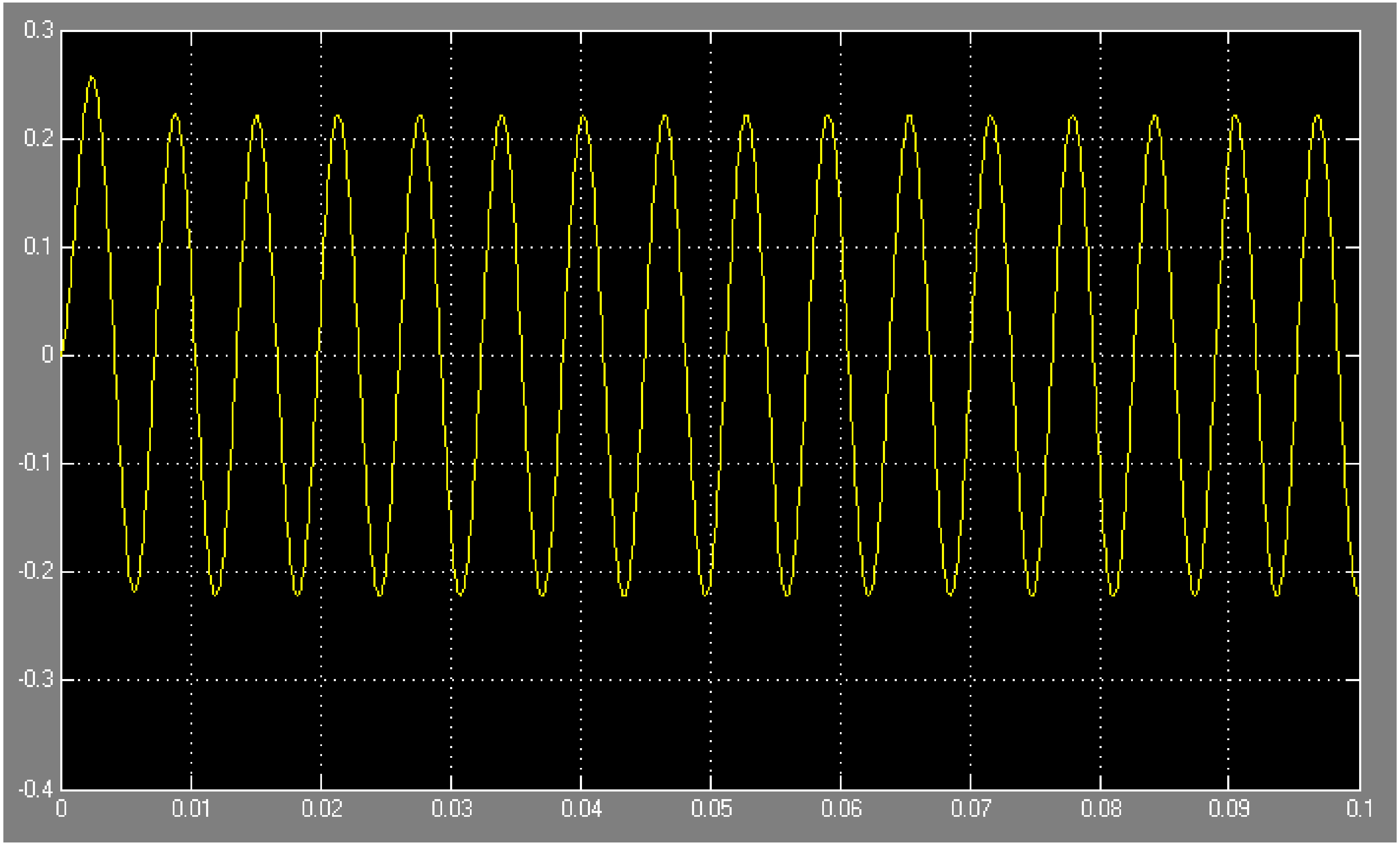




پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:

مدار RL: ادامه مثال: **نتیجه شبیه سازی با نرم افزار MATLAB**

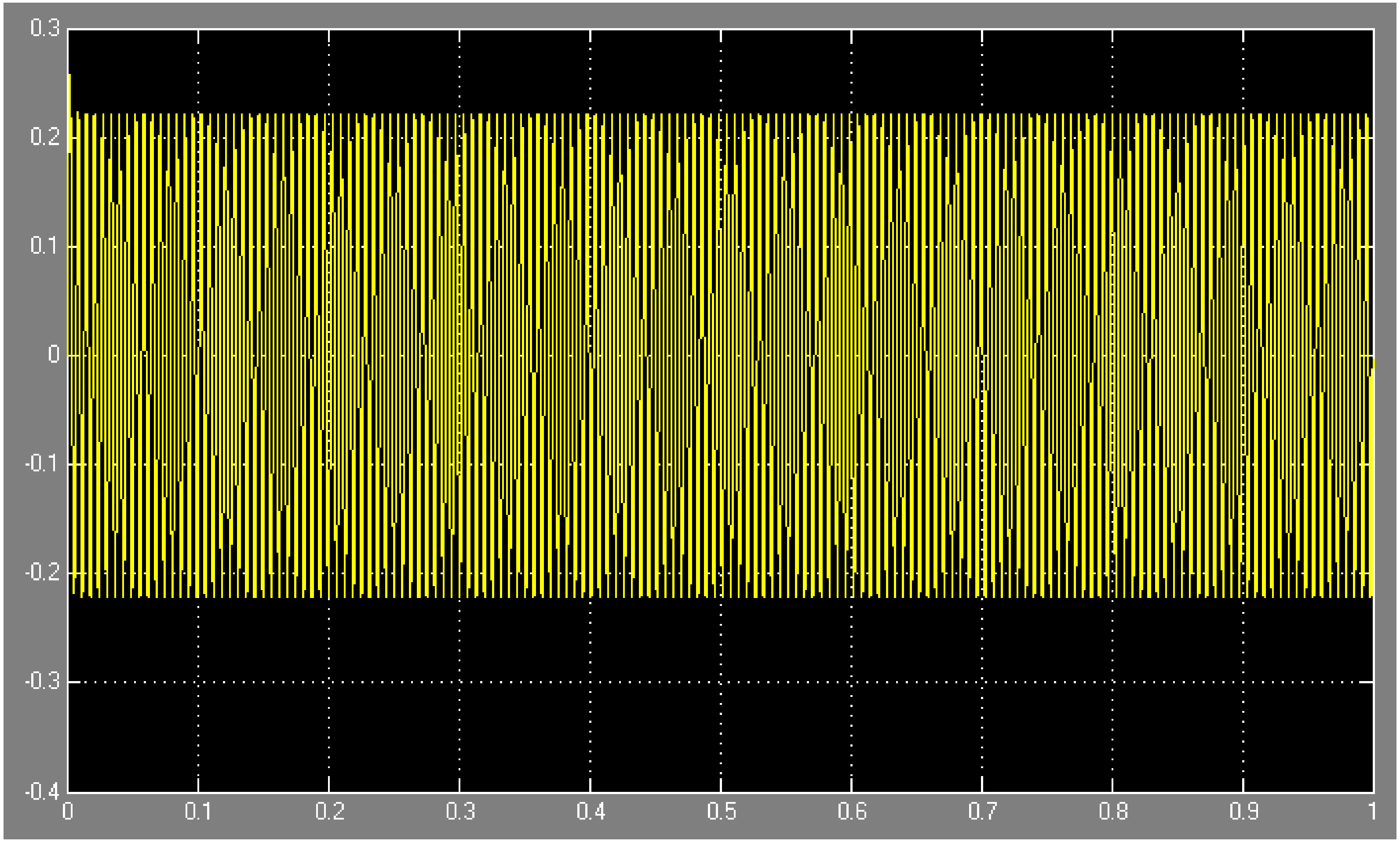
نتیجه شبیه سازی از زمان صفر تا ۰/۱ ثانیه



پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:

مدار RL: ادامه مثال: **نتیجه شبیه سازی با نرم افزار MATLAB**

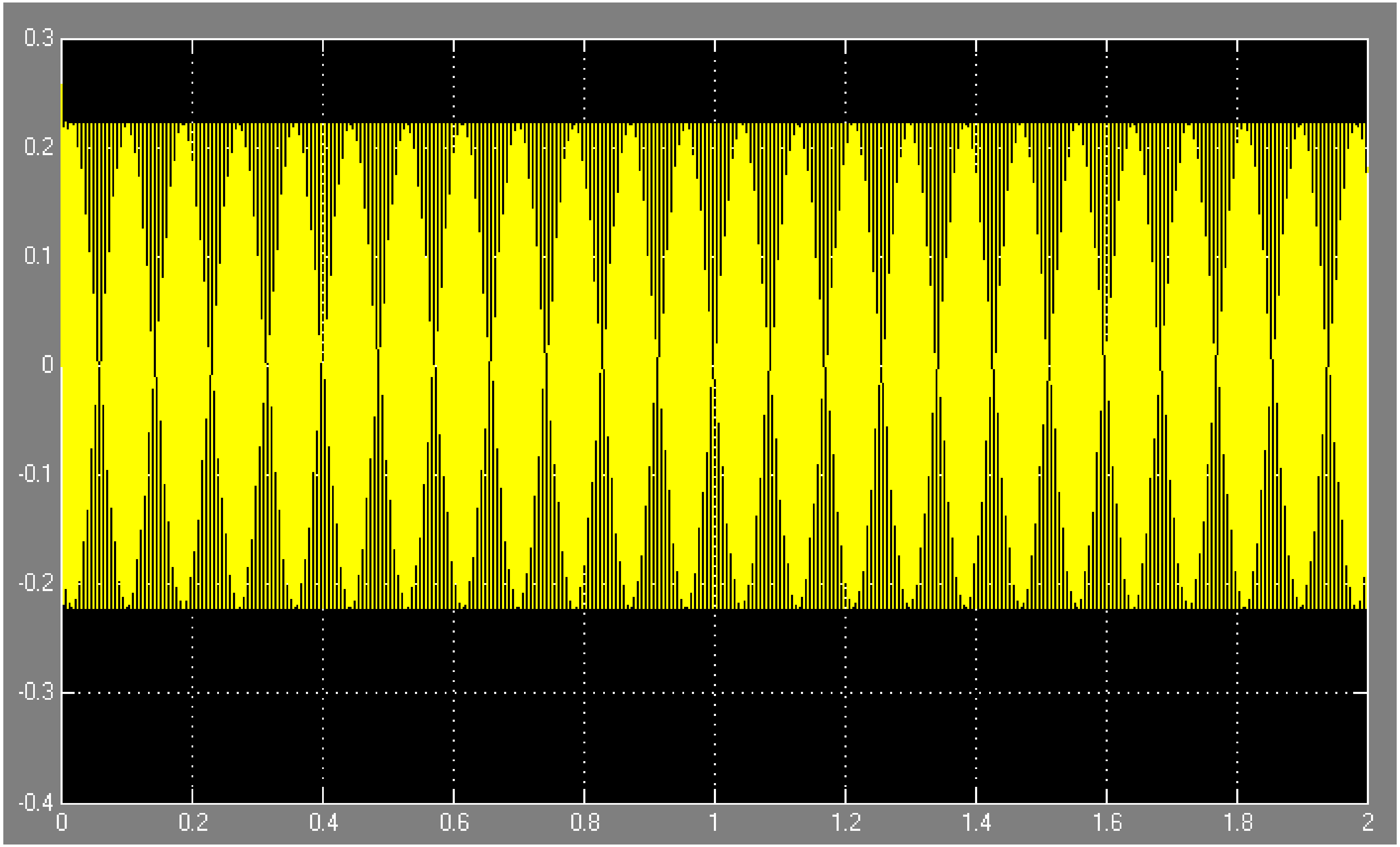
نتیجه شبیه سازی از زمان صفر تا ۱ ثانیه



پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:

مدار RL: ادامه مثال: **نتیجه شبیه سازی با نرم افزار MATLAB**

نتیجه شبیه سازی از زمان صفر تا ۲ ثانیه



پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
مدار RL: تمرین (به عهده دانشجو)

PRACTICE

10.3 Let $v_s = 40\cos 8000t$ V in the circuit of Fig. 10.7. Use Thévenin's theorem where it will do the most good, and find the value at $t = 0$ for (*a*) i_L; (*b*) v_L; (*c*) i_R; (*d*) i_s.

Ans: 18.71 mA; 15.97 V; 5.32 mA; 24.0 mA.

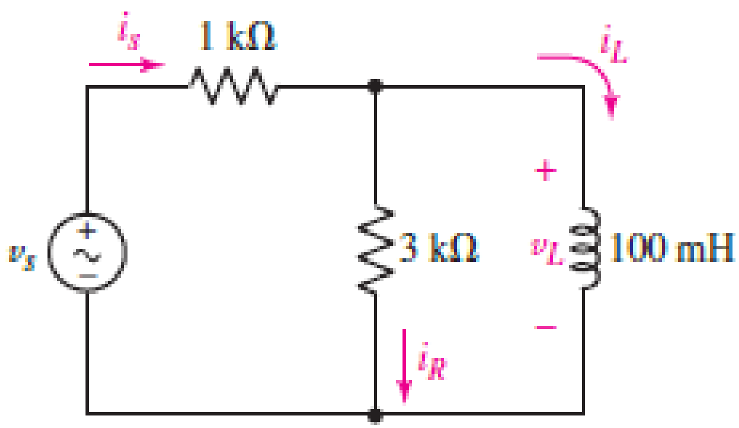


FIGURE 10.7

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
مقدمه ای در مورد اعداد مختلط و روش فازور

The basic idea is that sinusoids and exponentials are related through complex numbers. Euler's identity, for example, tells us that

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos(\omega t + \theta) + j\sin(\omega t + \theta) = e^{j(\omega t+\theta)}$$

$$V_m \cos\omega t = \mathrm{Re}\{V_m \cos\omega t + jV_m \sin\omega t\} = \mathrm{Re}\{V_m e^{j\omega t}\}$$

$3e^{j0^\circ}$ V (or even just 3 V)

$e^{j0} = \cos 0 + j\sin 0 = 1$

These complex quantities are usually written in polar form rather than exponential form in order to achieve a slight additional saving of time and effort. For example, a source voltage

$$v(t) = V_m \cos\omega t = V_m \cos(\omega t + 0^\circ)$$

we now represent in complex form as

$$V_m \underline{/0^\circ}$$

Remember that none of the steady-state circuits we are considering will respond at a frequency other than that of the excitation source, so that the value of ω is always known.

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
مقدمه ای در مورد اعداد مختلط و روش فازور

and its current response

$$i(t) = I_m \cos(\omega t + \phi)$$

becomes

$$I_m \underline{/\phi}$$

This abbreviated complex representation is called a ***phasor.***[1]

Let us review the steps by which a real sinusoidal voltage or current is transformed into a phasor, and then we will be able to define a phasor more meaningfully and to assign a symbol to represent it.

A real sinusoidal current

$$i(t) = I_m \cos(\omega t + \phi)$$

is expressed as the real part of a complex quantity by invoking Euler's identity

$$i(t) = \text{Re}\left\{I_m e^{j(\omega t + \phi)}\right\}$$

We then represent the current as a complex quantity by dropping the instruction Re{}, thus adding an imaginary component to the current without affecting the real component; further simplification is achieved by suppressing the factor $e^{j\omega t}$:

$$\mathbf{I} = I_m e^{j\phi}$$

and writing the result in polar form:

$$\mathbf{I} = I_m \underline{/\phi}$$

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
مقدمه ای در مورد اعداد مختلط و روش فازور

$$i(t) = I_m \cos(\omega t + \phi)$$

$$i(t) = \mathrm{Re}\{I_m e^{j(\omega t + \phi)}\}$$

$$\mathbf{I} = I_m e^{j\phi}$$

$$\mathbf{I} = I_m \underline{/\phi}$$

The process by which we change $i(t)$ into $\mathbf{I}$ is called a *phasor transformation* from the time domain to the frequency domain.

This abbreviated complex representation is the *phasor representation;* phasors are complex quantities and hence are printed in boldface type. Capital letters are used for the phasor representation of an electrical quantity because the phasor is not an instantaneous function of time; it contains only amplitude and phase information. We recognize this difference in viewpoint by referring to $i(t)$ as a *time-domain representation* and terming the phasor $\mathbf{I}$ a *frequency-domain representation*. It should be noted that the frequency-domain expression of a current or voltage does not explicitly include the frequency. The process of returning to the time domain from the frequency domain is exactly the reverse of the previous sequence. Thus, given the phasor voltage

$$\mathbf{V} = 115\underline{/-45^\circ} \text{ volts}$$

and the knowledge that $\omega = 500$ rad/s, we can write the time-domain equivalent directly:

$$v(t) = 115\cos(500t - 45^\circ) \qquad \text{volts}$$

If desired as a sine wave, $v(t)$ could also be written

$$v(t) = 115\sin(500t + 45^\circ) \qquad \text{volts}$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:

روش فازور: مقاومت

The Resistor

The resistor provides the simplest case. In the time domain, as indicated by Fig. 10.13*a*, the defining equation is

$$v(t) = Ri(t)$$

$$\mathbf{V} = R\mathbf{I} \qquad [18]$$

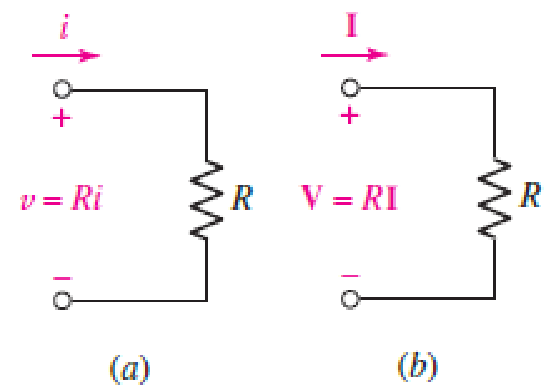


FIGURE 10.13 A resistor and its associated voltage and current in (*a*) the time domain, $v = Ri$; and (*b*) the frequency domain, $\mathbf{V} = R\mathbf{I}$.

روش فازور: سلف

The Inductor

Let us now turn to the inductor. The time-domain representation is shown in Fig. 10.14*a*, and the defining equation, a time-domain expression, is

$$v(t) = L\frac{di(t)}{dt} \qquad [19]$$

we obtain the desired phasor relationship

$$\boxed{\mathbf{V} = j\omega L\mathbf{I}} \qquad [20]$$

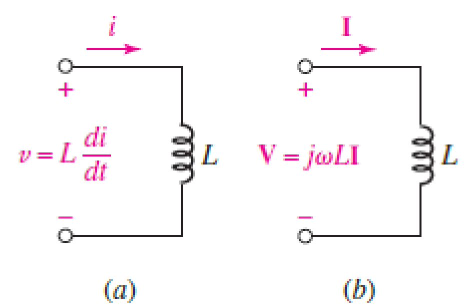


FIGURE 10.14 An inductor and its associated voltage and current in (*a*) the time domain, $v = L\,di/dt$; and (*b*) the frequency domain, $\mathbf{V} = j\omega L\mathbf{I}$.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: سلف: مثال

Apply the voltage $8\underline{/-50^\circ}$ V at a frequency $\omega = 100$ rad/s to a 4 H inductor, and determine the phasor current and the time-domain current.

We make use of the expression we just obtained for the inductor,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{8\underline{/-50^\circ}}{j100(4)} = -j0.02\underline{/-50^\circ} = (1\underline{/-90^\circ})(0.02\underline{/-50^\circ})$$

or

$$\mathbf{I} = 0.02\underline{/-140^\circ} \text{ A}$$

If we express this current in the time domain, it becomes

$$i(t) = 0.02\cos(100t - 140^\circ) \text{ A} = 20\cos(100t - 140^\circ) \text{ mA}$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول: روش فازور: خازن

The Capacitor

The final element to consider is the capacitor. The time-domain current-voltage relationship is

$$i(t) = C\,\frac{dv(t)}{dt}$$

The equivalent expression in the frequency domain is obtained once more by letting $v(t)$ and $i(t)$ be the complex quantities of Eqs. [16] and [17], taking the indicated derivative, suppressing $e^{j\omega t}$, and recognizing the phasors $\mathbf{V}$ and $\mathbf{I}$. Doing this, we find

$$\mathbf{I} = j\omega C\mathbf{V} \qquad [21]$$

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}$$

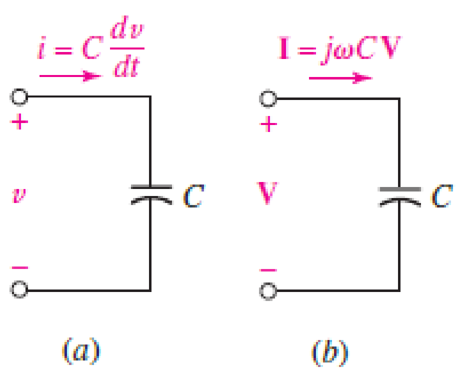
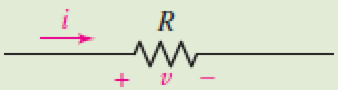
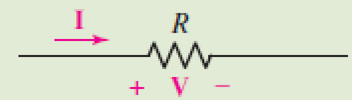
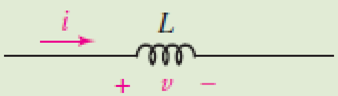
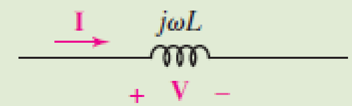
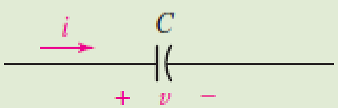
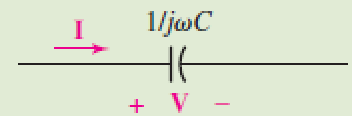


FIGURE 10.15 (*a*) The time-domain and (*b*) the frequency-domain relationships between capacitor current and voltage.

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: جدول مقاومت، سلف و خازن

TABLE **10.1** Comparison of Time-Domain and Frequency-Domain Voltage-Current Expressions

Time Domain		Frequency Domain	
	$v = Ri$	$\mathbf{V} = R\mathbf{I}$	
	$v = L\dfrac{di}{dt}$	$\mathbf{V} = j\omega L\mathbf{I}$	
	$v = \dfrac{1}{C}\displaystyle\int i\,dt$	$\mathbf{V} = \dfrac{1}{j\omega C}\mathbf{I}$	

Kirchhoff's Laws Using Phasors

Kirchhoff's voltage law in the time domain is

$$v_1(t) + v_2(t) + \cdots + v_N(t) = 0$$

We now use Euler's identity to replace each real voltage v_i by a complex voltage having the same real part, suppress $e^{j\omega t}$ throughout, and obtain

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_N = 0$$

Thus, we see that Kirchhoff's voltage law applies to phasor voltages just as it did in the time domain. Kirchhoff's current law can be shown to hold for phasor currents by a similar argument.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: مثال از یک مدار RL

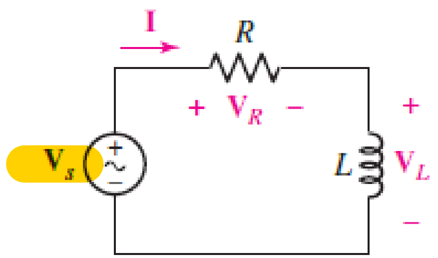


FIGURE 10.16 The series *RL* circuit with a phasor voltage applied.

Now let us look briefly at the series RL circuit that we have considered several times before. The circuit is shown in Fig. 10.16, and a phasor current and several phasor voltages are indicated. We may obtain the desired response, a time-domain current, by first finding the phasor current. From Kirchhoff's voltage law,

$$\mathbf{V}_R + \mathbf{V}_L = \mathbf{V}_s$$

and using the recently obtained **V**-**I** relationships for the elements, we have

$$R\mathbf{I} + j\omega L\mathbf{I} = \mathbf{V}_s$$

The phasor current is then found in terms of the source voltage $\mathbf{V}_s$:

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\omega L}$$

Let us select a source-voltage amplitude of V_m and phase angle of 0°. Thus,

$$\mathbf{I} = \frac{V_m\underline{/0^\circ}}{R + j\omega L}$$

The current may be transformed to the time domain by first writing it in polar form:

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: ادامه مثال از یک مدار RL

$$\mathbf{I}=\frac{V_m+0j}{R+j\omega L}=\frac{V_m+0j}{R+j\omega L}\times\frac{R-j\omega L}{R-j\omega L}\rightarrow \mathbf{I}=\frac{RV_m-j\left(V_m\omega L\right)}{R^2+\left(\omega L\right)^2}=\frac{RV_m}{R^2+\left(\omega L\right)^2}-j\frac{V_m\omega L}{R^2+\left(\omega L\right)^2}$$

$$\mathbf{I}=I_m e^{j\theta}=I_m\measuredangle\theta\Rightarrow i(t)=I_m\cos(\omega t+\theta)$$

$$I_m=\sqrt{\left[\frac{RV_m}{R^2+\left(\omega L\right)^2}\right]^2+\left[\frac{V_m\omega L}{R^2+\left(\omega L\right)^2}\right]^2}=\frac{1}{R^2+\left(\omega L\right)^2}\sqrt{\left(RV_m\right)^2+\left(V_m\omega L\right)^2}=\frac{V_m}{\sqrt{R^2+\omega^2L^2}}$$

$$\theta=\arctan\left(-\frac{\omega L}{R}\right)$$

$$\mathbf{I}=\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\,\angle\left[-\tan^{-1}(\omega L/R)\right]$$

and then following the familiar sequence of steps to obtain in a very simple manner the same result we obtained the “hard way” earlier in this chapter.

$$i(t)=\frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos\left[\omega t-\arctan\left(\frac{\omega L}{R}\right)\right]$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: مثال از یک مدار RLC

For the *RLC* circuit of Fig. 10.17, determine $\mathbf{I}_s$ and $i_s(t)$ if both sources operate at $\omega = 2$ rad/s, and $\mathbf{I}_C = 2\angle 28^\circ$ A.

The fact that we are given $\mathbf{I}_C$ and asked for $\mathbf{I}_s$ is all the prompting we need to consider applying KCL. If we label the capacitor voltage $\mathbf{V}_C$ consistent with the passive sign convention, then

$$\mathbf{V}_C = \frac{1}{j\omega C}\mathbf{I}_C = \frac{-j}{2}\mathbf{I}_C = \frac{-j}{2}(2\angle 28^\circ) = (0.5\angle -90^\circ)(2\angle 28^\circ) = 1\angle -62^\circ \text{ V}$$

This voltage also appears across the 2 Ω resistor, so that the current $\mathbf{I}_{R_2}$ flowing downward through that branch is

$$\mathbf{I}_{R_2} = \frac{1}{2}\mathbf{V}_C = \frac{1}{2}\angle -62^\circ \text{ A}$$

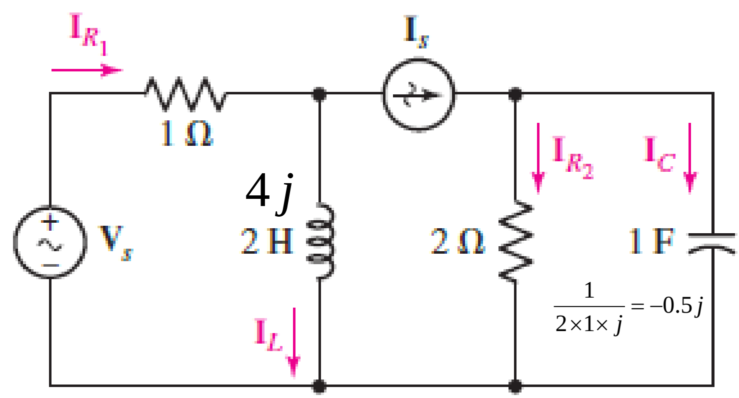


FIGURE 10.17 A three-mesh circuit. Each source operates at the same frequency ω.

$$\mathbf{I}_s = \mathbf{I}_{R_2} + \mathbf{I}_C = \frac{1}{2}\measuredangle -62^\circ + 2\measuredangle 28^\circ = 0.234 - 0.441j + 1.765 + 0.938j = 1.999 + j0.497$$

$$\mathbf{I}_s = 2.059\measuredangle 13.96^\circ \rightarrow i_s(t) = 2.059\cos(2t + 13.69^\circ)$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: تعریف امپدانس و ادمیتانس

$$\mathbf{V} = R\mathbf{I} \qquad \mathbf{V} = j\omega L\mathbf{I} \qquad \mathbf{V} = \frac{\mathbf{I}}{j\omega C}$$

If these equations are written as phasor voltage/phasor current ratios

$$\frac{\mathbf{V}}{\mathbf{I}} = R \qquad \frac{\mathbf{V}}{\mathbf{I}} = j\omega L \qquad \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}$$

اتصال سری مقاومت، سلف و خازن

$$\mathbf{Z}_L = j\omega L = j50\ \Omega$$

and the impedance of the capacitor is

$$\mathbf{Z}_C = \frac{1}{j\omega C} = \frac{-j}{\omega C} = -j1\ \Omega$$

The impedance of the series combination is therefore

$$\mathbf{Z}_{\text{eq}} = \mathbf{Z}_L + \mathbf{Z}_C = j50 - j1 = j49\ \Omega$$

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: تعریف راکتانس

Reactance

Of course, we may choose to express impedance in either *rectangular* ($\mathbf{Z} = R + jX$) or *polar* ($\mathbf{Z} = |\mathbf{Z}|\underline{/\theta}$) form. In rectangular form, we can see clearly the real part which arises only from real resistances, and an imaginary component, termed the ***reactance***, which arises from the energy storage elements. Both resistance and reactance have units of ohms, but reactance will always depend upon frequency. An ideal resistor has zero reactance; an ideal inductor or capacitor is purely reactive (i.e., characterized by zero resistance). Can a series or parallel combination include *both* a capacitor and an inductor, and yet have *zero reactance*? Sure! Consider the series connection of a 1 Ω resistor, a 1 F capacitor, and a 1 H inductor driven at $\omega = 1$ rad/s. $\mathbf{Z}_{eq} = 1 - j(1)(1) + j(1)(1) = 1\ \Omega$. At that particular frequency, the equivalent is a simple 1 Ω resistor. However, even small deviations from $\omega = 1$ rad/s lead to nonzero reactance.

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: مثال

Determine the equivalent impedance of the network shown in Fig. 10.18*a*, given an operating frequency of 5 rad/s.

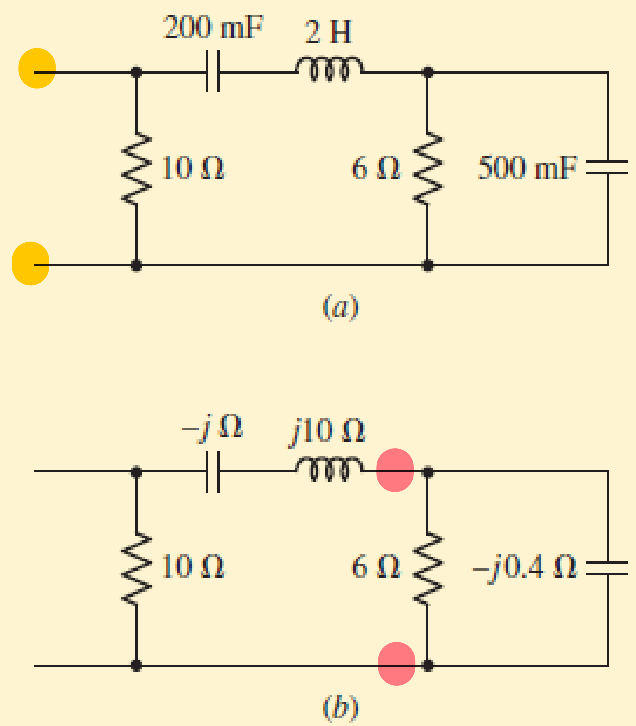


FIGURE 10.18 (*a*) A network that is to be replaced by a single equivalent impedance. (*b*) The elements are replaced by their impedances at $\omega = 5$ rad/s.

We begin by converting the resistors, capacitors, and inductor into the corresponding impedances as shown in Fig. 10.18*b*.

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: ادامه مثال

Upon examining the resulting network, we observe that the 6 Ω impedance is in parallel with $-j0.4$ Ω. This combination is equivalent to

$$\frac{(6)(-j0.4)}{6-j0.4} = 0.02655 - j0.3982\ \Omega$$

which is in series with both the $-j$ Ω and $j10$ Ω impedances, so that we have

$$0.0265 - j0.3982 - j + j10 = 0.02655 + j8.602\ \Omega$$

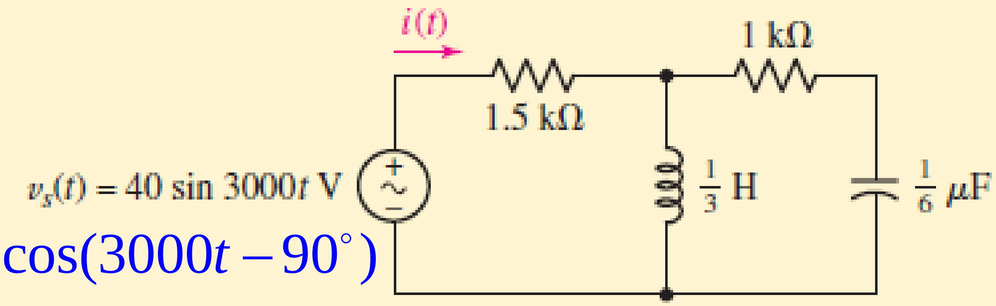
This new impedance is in parallel with 10 Ω, so that the equivalent impedance of the network is

$$10 \parallel (0.02655 + j8.602) = \frac{10(0.02655 + j8.602)}{10 + 0.02655 + j8.602}$$
$$= 4.255 + j4.929\ \Omega$$

Alternatively, we can express the impedance in polar form as $6.511\underline{/49.20^\circ}$ Ω.

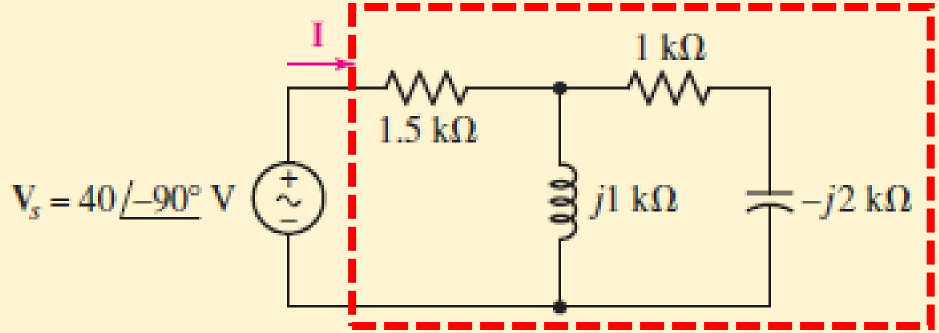
پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: مثال

Find the current $i(t)$ in the circuit shown in Fig. 10.20*a*.



$$40\sin(3000t) = 40\cos(3000t - 90^\circ)$$

(*a*)



(*b*)

FIGURE 10.20 (*a*) An *RLC* circuit for which the sinusoidal forced response $i(t)$ is desired. (*b*) The frequency-domain equivalent of the given circuit at $\omega = 3000$ rad/s.

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: ادامه مثال

$$\mathbf{Z}_{\mathrm{eq}} = 1.5 + \frac{(j)(1-2j)}{j+1-2j} = 1.5 + \frac{2+j}{1-j}$$

$$= 1.5 + \frac{2+j}{1-j}\,\frac{1+j}{1+j} = 1.5 + \frac{1+j3}{2}$$

$$= 2 + j1.5 = 2.5\underline{/36.87^\circ}\ \mathrm{k}\Omega$$

The phasor current is then simply

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}_{\mathrm{eq}}}$$

Determine if additional information is required.
Substituting known values, we find that

$$\mathbf{I} = \frac{40\underline{/-90^\circ}}{2.5\underline{/36.87^\circ}}\ \mathrm{mA}$$

which, along with the knowledge that $\omega = 3000$ rad/s, is sufficient to solve for $i(t)$.

Attempt a solution.
This complex expression is easily simplified to a single complex number in polar form:

$$\mathbf{I} = \frac{40}{2.5}\underline{/-90^\circ - 36.87^\circ}\ \mathrm{mA} = 16.00\underline{/-126.9^\circ\ \mathrm{mA}}$$

Upon transforming the current to the time domain, the desired response is obtained:

$$i(t) = 16\cos(3000t - 126.9^\circ) \qquad \mathrm{mA}$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:

روش فازور: تعریف کنداکتانس و سوسپتانس

Although the concept of impedance is very useful, and familiar in a way based on our experience with resistors, the reciprocal is often just as valuable. We define this quantity as the ***admittance*** $\mathbf{Y}$ of a circuit element or passive network, and it is simply the ratio of current to voltage:

The real part of the admittance is the ***conductance*** G, and the imaginary part is the ***susceptance*** B. All three quantities ($\mathbf{Y}$, G, and B) are measured in siemens.

The real part of the admittance is the ***conductance*** G, and the imaginary part of the admittance is the ***susceptance*** B. Thus,

$$\mathbf{Y} = G + jB = \frac{1}{\mathbf{Z}} = \frac{1}{R + jX} \qquad [22]$$

$$\mathbf{Y}_R = \frac{1}{R}$$

$$\mathbf{Y}_L = \frac{1}{j\omega L}$$

$$\mathbf{Y}_C = j\omega C$$

There is a general (unitless) term for both impedance and admittance—*immitance*—which is sometimes used, but not very often.

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: تجزیه و تحلیل مش و گره: مثال از تجزیه و تحلیل گره

دانشگاه صنعتی اصفهان
دانشکده برق و کامپیوتر

Find the time-domain node voltages $v_1(t)$ and $v_2(t)$ in the circuit shown in Fig. 10.22.

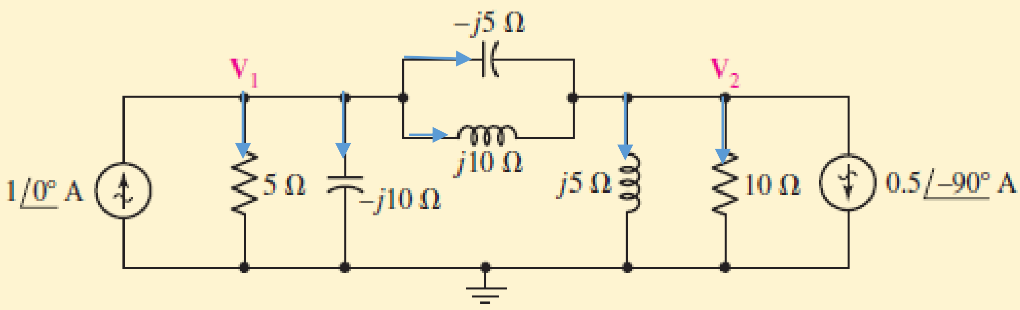


FIGURE 10.22 A frequency-domain circuit for which node voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ are identified.

Two current sources are given as phasors, and phasor node voltages $\mathbf{V}_1$ and $\mathbf{V}_2$ are indicated. At the left node we apply KCL, yielding:

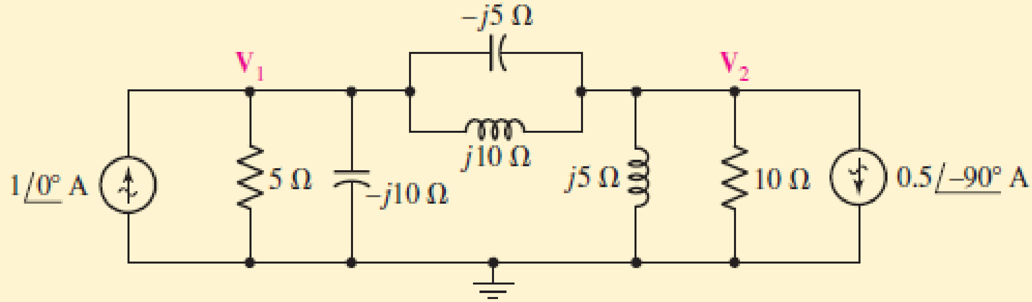
$$\frac{\mathbf{V}_1}{5} + \frac{\mathbf{V}_1}{-j10} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{-j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j10} = 1\underline{/0°} = 1 + j0$$

At the right node,

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{-j5} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{j10} + \frac{\mathbf{V}_2}{j5} + \frac{\mathbf{V}_2}{10} = -(0.5\underline{/-90°}) = j0.5$$

Combining terms, we have

$$(0.2 + j0.2)\mathbf{V}_1 - j0.1\mathbf{V}_2 = 1$$



and

$$-j0.1\mathbf{V}_1 + (0.1 - j0.1)\mathbf{V}_2 = j0.5$$

These equations are easily solved on most scientific calculators, resulting in $\mathbf{V}_1 = 1 - j2$ V and $\mathbf{V}_2 = -2 + j4$ V.

The time-domain solutions are obtained by expressing $\mathbf{V}_1$ and $\mathbf{V}_2$ in polar form:

$$\mathbf{V}_1 = 2.24\underline{/-63.4^\circ}$$
$$\mathbf{V}_2 = 4.47\underline{/116.6^\circ}$$

and passing to the time domain:

$$v_1(t) = 2.24\cos(\omega t - 63.4^\circ) \qquad \text{V}$$
$$v_2(t) = 4.47\cos(\omega t + 116.6^\circ) \qquad \text{V}$$

Note that the value of ω would have to be known in order to compute the impedance values given on the circuit diagram. Also, *both sources must be operating at the same frequency.*

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: تجزیه و تحلیل مش و گره: مثال از تجزیه و تحلیل مش

Obtain expressions for the time-domain currents i_1 and i_2 in the circuit given as Fig. 10.24*a*.

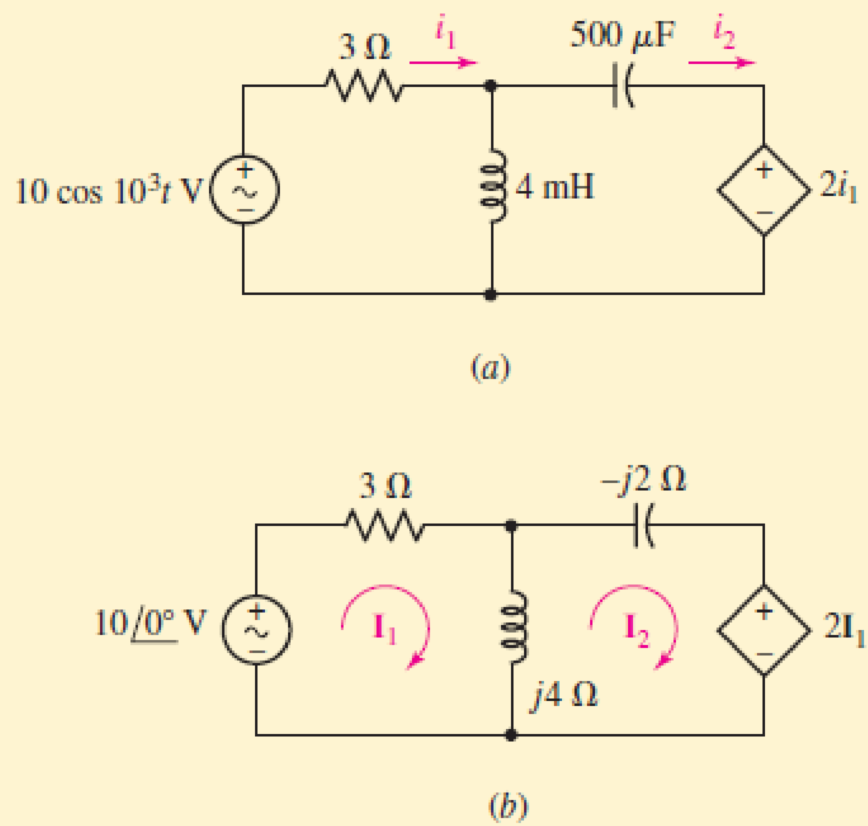
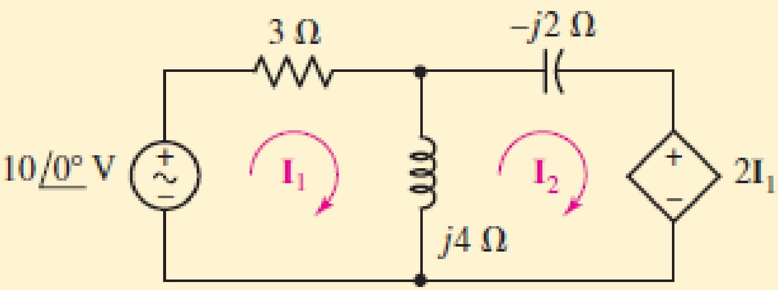


FIGURE 10.24 (*a*) A time-domain circuit containing a dependent source. (*b*) The corresponding frequency-domain circuit.

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: تجزیه و تحلیل مش و گره: ادامه مثال از تجزیه و تحلیل مش



Noting from the left source that $\omega = 10^3$ rad/s, we draw the frequency-domain circuit of Fig. 10.24*b* and assign mesh currents $\mathbf{I}_1$ and $\mathbf{I}_2$. Around mesh 1,

$$3\mathbf{I}_1 + j4(\mathbf{I}_1 - \mathbf{I}_2) = 10\underline{/0^\circ}$$

or

$$(3 + j4)\mathbf{I}_1 - j4\mathbf{I}_2 = 10$$

while mesh 2 leads to

$$j4(\mathbf{I}_2 - \mathbf{I}_1) - j2\mathbf{I}_2 + 2\mathbf{I}_1 = 0$$

or

$$(2 - j4)\mathbf{I}_1 + j2\mathbf{I}_2 = 0$$

Solving,

$$\mathbf{I}_1 = \frac{14 + j8}{13} = 1.24\underline{/29.7^\circ}\ \text{A}$$

$$\mathbf{I}_2 = \frac{20 + j30}{13} = 2.77\underline{/56.3^\circ}\ \text{A}$$

Hence,

$$i_1(t) = 1.24\cos(10^3 t + 29.7^\circ) \qquad \text{A}$$

$$i_2(t) = 2.77\cos(10^3 t + 56.3^\circ) \qquad \text{A}$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: مثال

Determine the power dissipated by the 10 Ω resistor in the circuit of Fig. 10.33*a*.

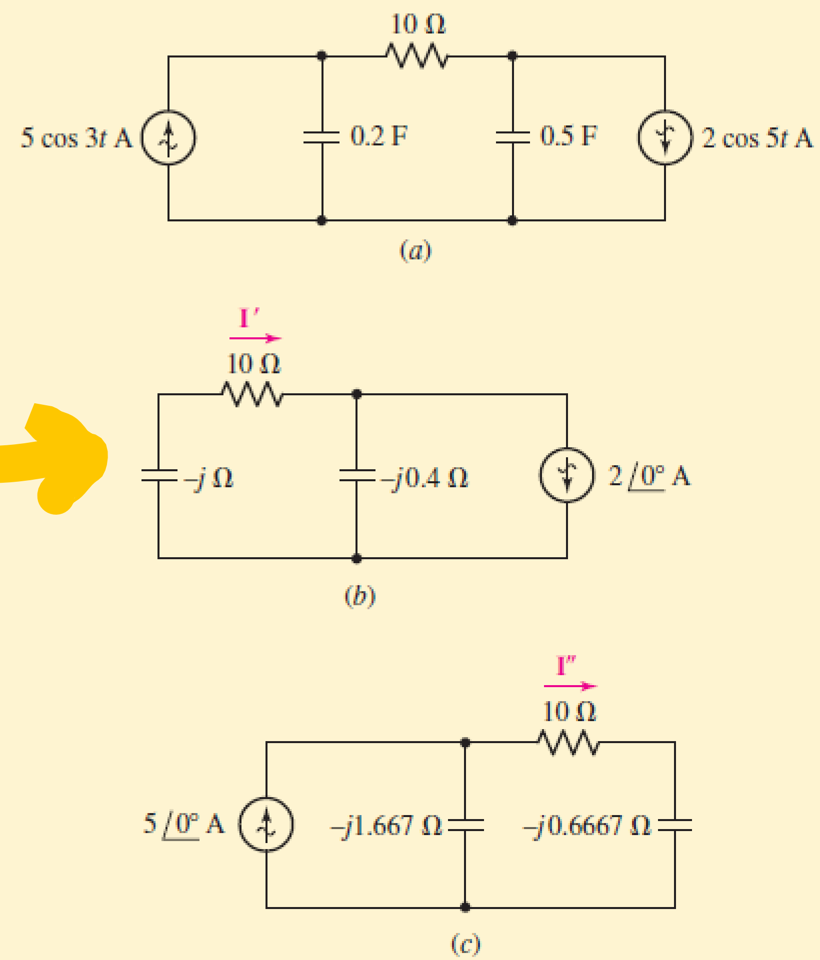
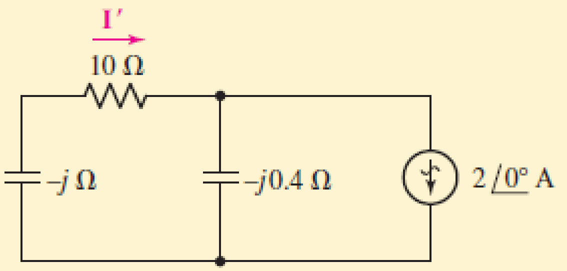


FIGURE 10.33 (*a*) A simple circuit having sources operating at different frequencies. (*b*) Circuit with the left source killed. (*c*) Circuit with the right source killed.

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: ادامه مثال



After glancing at the circuit, we might be tempted to write two quick nodal equations, or perhaps perform two sets of source transformations and launch immediately into finding the voltage across the 10 Ω resistor.

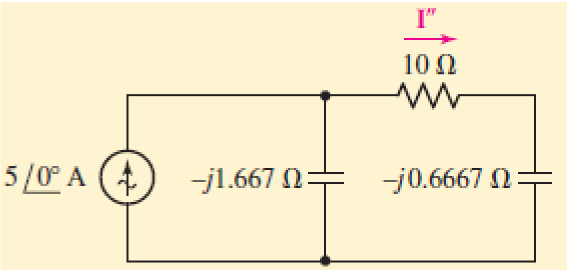
Unfortunately, this is impossible, since we have *two* sources operating at *different* frequencies. In such a situation, there is no way to compute the impedance of any capacitor or inductor in the circuit—which ω would we use?

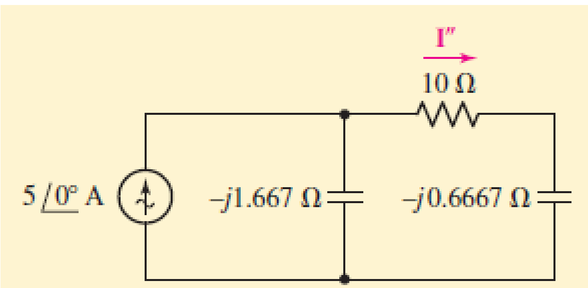
The only way out of this dilemma is to employ superposition, grouping all sources with the same frequency in the same subcircuit, as shown in Fig. 10.33*b* and *c*.

In the subcircuit of Fig. 10.33*b*, we quickly compute the current $\mathbf{I}'$ using current division:

$$\mathbf{I}' = 2\underline{/0^\circ}\left[\frac{-j0.4}{10 - j - j0.4}\right]$$
$$= 79.23\underline{/-82.03^\circ}\ \text{mA}$$

پاسخ حالت دائمی سینوسی مدارهای مرتبه اول:
روش فازور: ادامه مثال





so that

$$i' = 79.23\cos(5t - 82.03^\circ) \qquad \text{mA}$$

Likewise, we find that

$$\mathbf{I}'' = 5\underline{/0^\circ}\left[\frac{-j1.667}{10 - j0.6667 - j1.667}\right]$$
$$= 811.7\underline{/-76.86^\circ}\ \text{mA}$$

so that

$$i'' = 811.7\cos(3t - 76.86^\circ) \qquad \text{mA}$$

It should be noted at this point that no matter how tempted we might be to add the two phasor currents $\mathbf{I}'$ and $\mathbf{I}''$, in Fig. 10.33*b* and *c*, this *would be incorrect*. Our next step is to add the two time-domain currents, square the result, and multiply by 10 to obtain the power absorbed by the 10 Ω resistor in Fig. 10.33*a*:

$$p_{10} = (i' + i'')^2 \times 10$$
$$= 10[79.23\cos(5t - 82.03^\circ) + 811.7\cos(3t - 76.86^\circ)]^2 \qquad \mu\text{W}$$

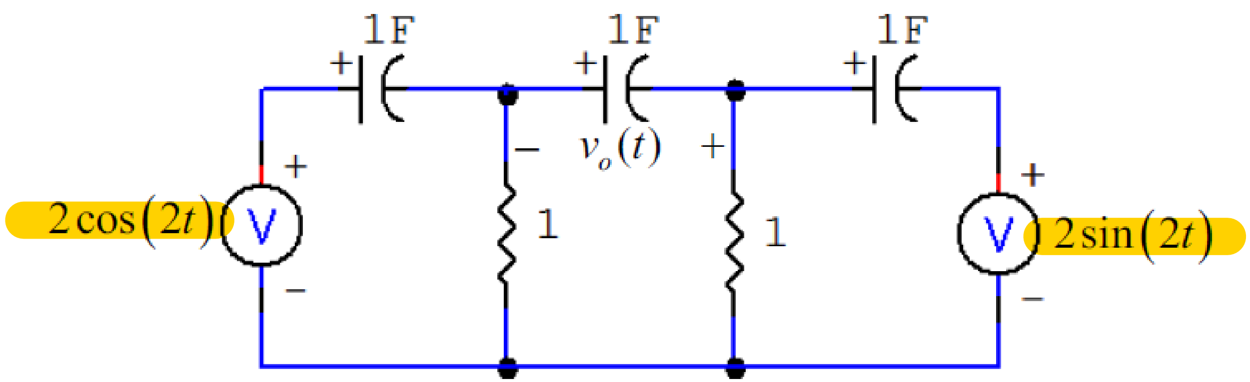
پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: قضایای جمع آثار، تبدیل منابع، تونن و نورتون

The circuits containing inductors and capacitors were still linear, and that the benefits of linearity were again available. Included among these were the superposition principle, Thévenin's and Norton's theorems, and source transformations. Thus, we know that these methods may be used on the circuits we are now considering. The fact that we are analyzing the circuits in terms of phasors is also immaterial; they are still linear circuits.

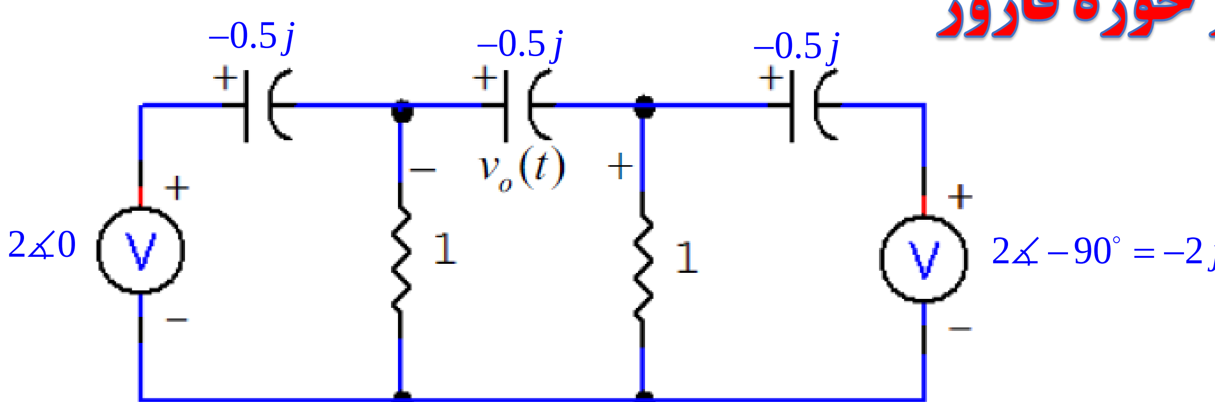
پاسخ حالت دائمی سینوسی مدارهای RLC: (نمونه سوال امتحانی ترمهای قبل)
روش فازور: مثال

دانشکده برق و کامپیوتر

مدار شکل زیر در حالت دائمی سینوسی قرار گرفته است. مقادیر تمام مقاومت ها بر حسب اهم است. در این صورت مقدار ولتاژ خروجی $v_o(t)$ را محاسبه کنید؟

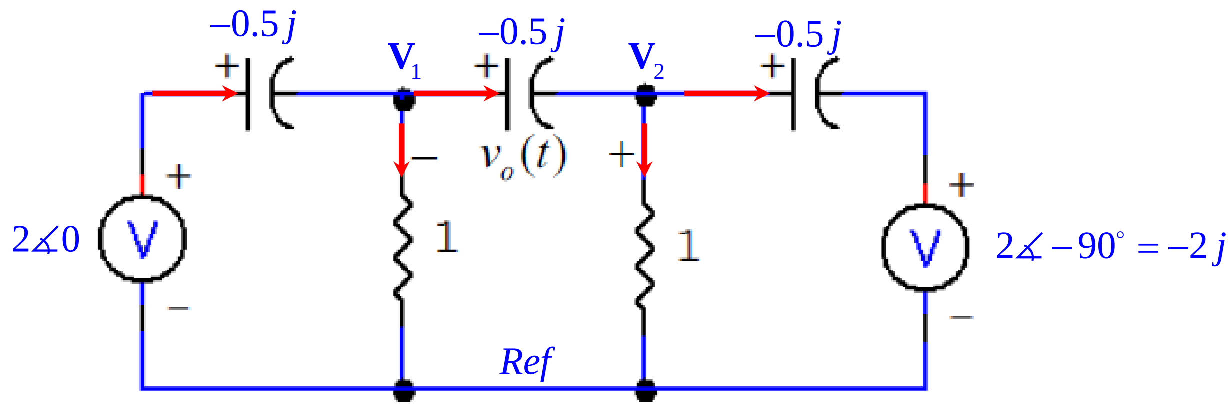


رسم مدار در حوزه فازور



دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: ادامه مثال



$$KCL\ \mathbf{V}_1: \frac{2-\mathbf{V}_1}{-0.5j} = \frac{\mathbf{V}_1}{1} + \frac{\mathbf{V}_1-\mathbf{V}_2}{-0.5j} \rightarrow 2-\mathbf{V}_1 = -0.5j\mathbf{V}_1 + \mathbf{V}_1 - \mathbf{V}_2$$

$$KCL\ \mathbf{V}_2: \frac{\mathbf{V}_1-\mathbf{V}_2}{-0.5j} = \frac{\mathbf{V}_2}{1} + \frac{\mathbf{V}_2-(-2j)}{-0.5j} \rightarrow \mathbf{V}_1 - \mathbf{V}_2 = -0.5j\mathbf{V}_2 + \mathbf{V}_2 + 2j$$

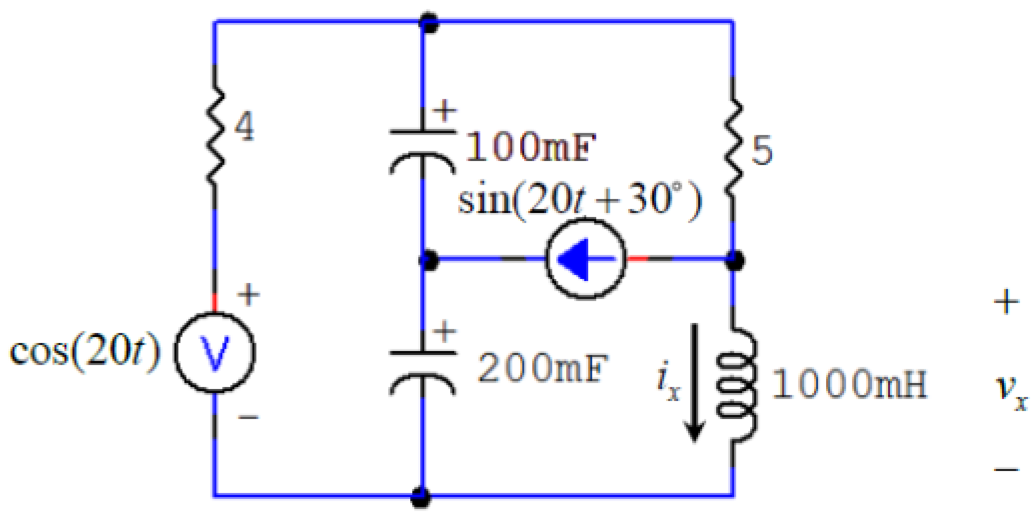
$$\begin{cases} (2-0.5j)\mathbf{V}_1 - \mathbf{V}_2 = 2 \\ \mathbf{V}_1 - (2-0.5j)\mathbf{V}_2 = 2j \end{cases} \xrightarrow{MATLAB} \mathbf{V}_1 = 1.4703 - 0.0216j, \mathbf{V}_2 = 0.9279 - 0.7784j$$

$$\mathbf{V}_O = \mathbf{V}_2 - \mathbf{V}_1 = -0.5406 - 0.7568j = 0.9301\measuredangle -125.5408^\circ$$

$$v_o(t) = 0.9301\cos(2t - 125.5408^\circ)$$

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: مثال

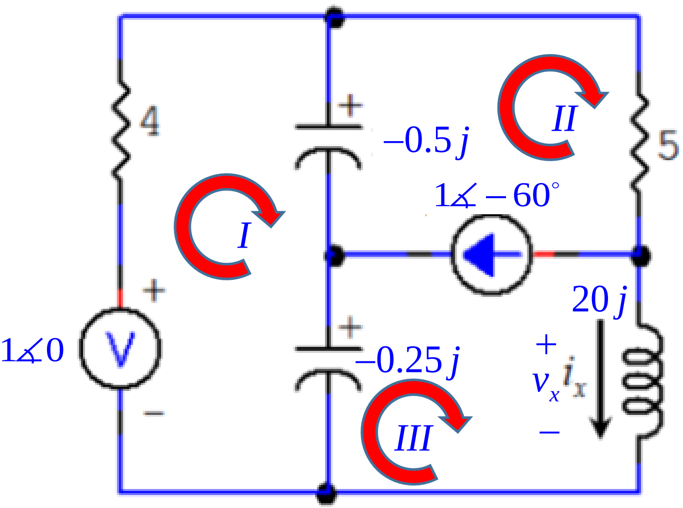
در شکل زیر مدار در حالت دائمی سینوسی قرار دارد. در این صورت مقدار ولتاژ v_x و جریان i_x را در طول زمان محاسبه کنید؟ مقادیر مقاومتها بر حسب اهم می باشند.



دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:

روش فازور: **ادامه مثال**



$$KVL\ I: 1 = 4\mathbf{I}_1 - 0.5j(\mathbf{I}_1 - \mathbf{I}_2) - 0.25j(\mathbf{I}_1 - \mathbf{I}_3)$$

$$KVL\ II, III: 5\mathbf{I}_2 + 20j\mathbf{I}_3 - 0.25j(\mathbf{I}_3 - \mathbf{I}_1) - 0.5j(\mathbf{I}_2 - \mathbf{I}_1) = 0$$

$$\mathbf{I}_2 - \mathbf{I}_3 = 1\measuredangle -60^\circ = 0.5 - 0.866j$$

$$\begin{cases} 1 = (4 - 0.75j)\mathbf{I}_1 + 0.5j\mathbf{I}_2 + 0.25j\mathbf{I}_3 \\ 0 = 0.75j\mathbf{I}_1 + (5 - 0.5j)\mathbf{I}_2 + 19.75j\mathbf{I}_3 \\ 0.5 - 0.866j = \mathbf{I}_2 - \mathbf{I}_3 \end{cases} \xrightarrow{MATLAB} \begin{cases} \mathbf{I}_1 = 0.1835 - 0.0636j \\ \mathbf{I}_2 = 0.6895 - 0.7069j \\ \mathbf{I}_3 = 0.1895 + 0.1591j \end{cases}$$

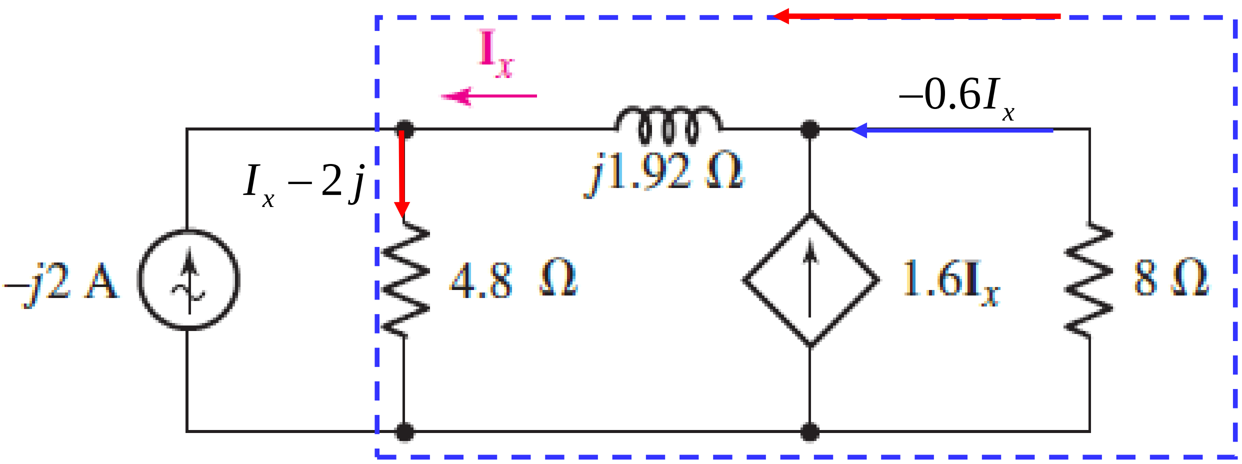
$$\mathbf{I}_3 = 0.1895 + 0.1591j = \mathbf{I}_x = 0.2474\measuredangle 40.01^\circ \rightarrow i_x(t) = 0.2474\cos(20t + 40.01^\circ)A$$

$$\mathbf{v}_x = 20j\mathbf{I}_3 = -3.182 + 3.790j = 4.9478\measuredangle 130.01^\circ \rightarrow v_x(t) = 4.9478\cos(20t + 130.01^\circ)v$$

پاسخ حالت دائمی سینوسی مدارهای RLC:

روش فازور: مثال

$$i_x(t) = ?$$



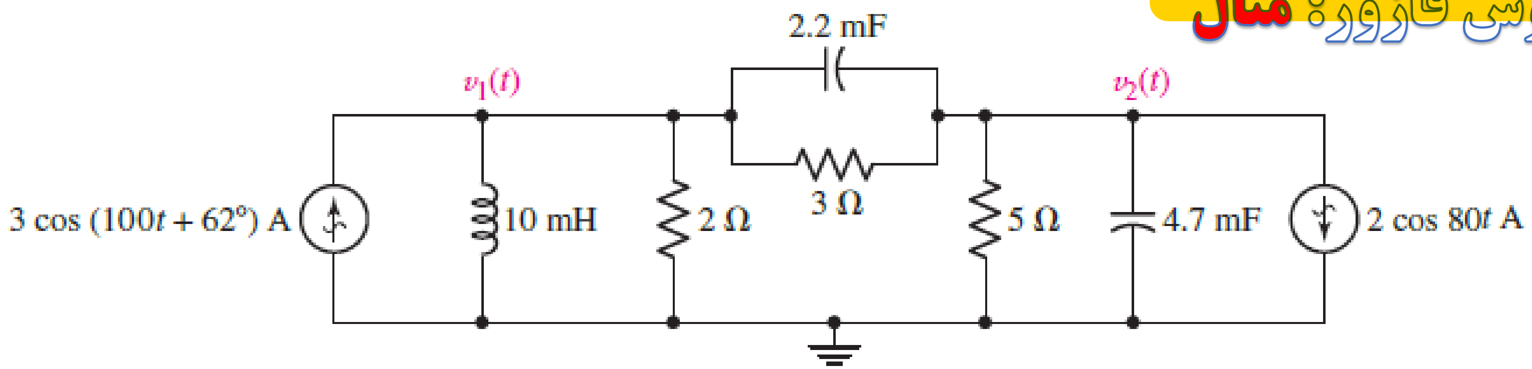
$$KVL: 8\times -0.6I_x + 1.92j \times I_x + 4.8\times(I_x - 2j) = 0$$

$$1.92jI_x = 9.6j \rightarrow I_x = \frac{9.6j}{1.92j} = 5 = 5\angle 0^\circ$$

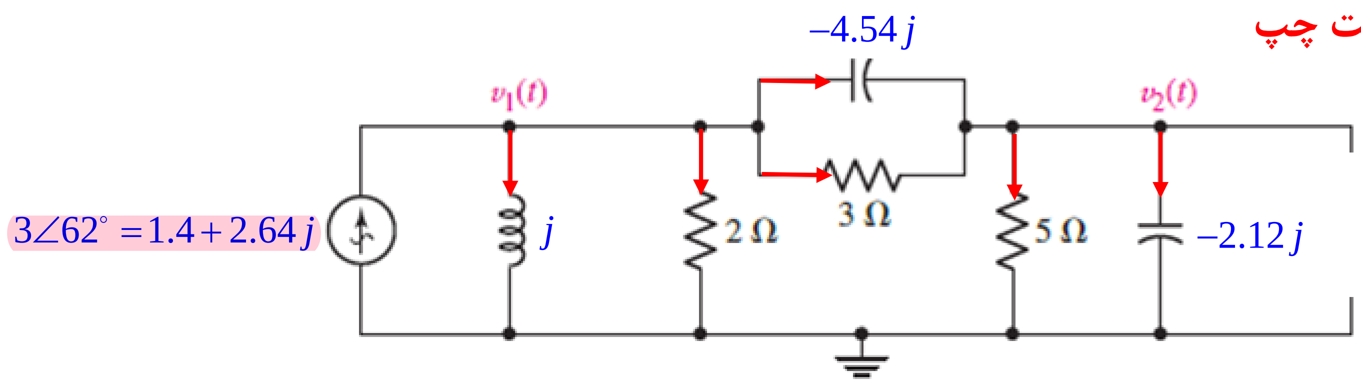
$$i_x = 5\cos(\omega t)$$

پاسخ حالت دائمی سینوسی مدارهای RLC:

روش فازور: مثال



در نظر گرفتن تنها منبع سمت چپ



$$KCL\ V_1: 1.4+2.64j=\frac{V_1}{j}+\frac{V_1}{2}+\frac{V_1-V_2}{-4.54j}+\frac{V_1-V_2}{3}\rightarrow 1.4+2.64j=(0.83-0.77j)V_1-(0.33+0.22j)V_2$$

$$KCL\ V_2: \frac{V_1-V_2}{-4.54j}+\frac{V_1-V_2}{3}=\frac{V_2}{-2.12j}+\frac{V_2}{5}\rightarrow 0=(0.33+0.22j)V_1-(0.53+0.69j)V_2$$

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: ادامه مثال

$$\begin{cases} 1.4+2.64j=(0.83-0.77j)V_1-(0.33+0.22j)V_2 \\ 0=(0.33+0.22j)V_1-(0.53+0.69j)V_2 \end{cases}$$

$$\rightarrow V_1=\frac{\begin{vmatrix} 1.4+2.64j & -0.33-0.22j \\ 0 & -0.53-0.69j \end{vmatrix}}{\begin{vmatrix} 0.83-0.77j & -0.33-0.22j \\ 0.33+0.22j & -0.53-0.69j \end{vmatrix}}, V_2=\frac{\begin{vmatrix} 0.83-0.77j & 1.4+2.64j \\ 0.33+0.22j & 0 \end{vmatrix}}{\begin{vmatrix} 0.83-0.77j & -0.33-0.22j \\ 0.33+0.22j & -0.53-0.69j \end{vmatrix}}$$

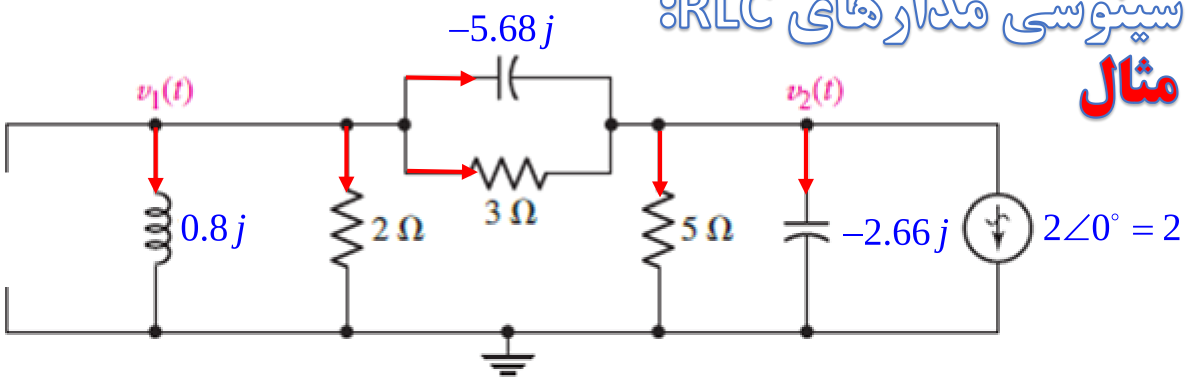
$$V_1=\frac{1.07-2.36j}{-0.91-0.02j}=-1.11+2.61j, V_2=\frac{0.11-1.17j}{-0.91-0.02j}=-0.09+1.28j$$

$$V_1=2.83\angle 113.04^\circ, V_2=1.28\angle 94.02^\circ$$

$$v_1(t)=2.83\cos(100t+113.04), v_2(t)=1.28\cos(100t+94.02)$$

پاسخ حالت دائمی سینوسی مدارهای RLC:

روش فازور: **ادامه مثال**



در نظر گرفتن تنها منبع سمت راست

$$KCL\ V_1: 0=\frac{V_1}{0.8j}+\frac{V_1}{2}+\frac{V_1-V_2}{-5.68j}+\frac{V_1-V_2}{3}\rightarrow 0=\left(0.83-1.07j\right)V_1-\left(0.33+0.17j\right)V_2$$

$$KCL\ V_2: \frac{V_1-V_2}{-5.68j}+\frac{V_1-V_2}{3}=\frac{V_2}{-2.66j}+\frac{V_2}{5}+2\rightarrow 2=\left(0.33+0.17j\right)V_1-\left(0.53+0.55j\right)V_2$$

$$\begin{cases}0=\left(0.83-1.07j\right)V_1-\left(0.33+0.17j\right)V_2\\ 2=\left(0.33+0.17j\right)V_1-\left(0.53+0.55j\right)V_2\end{cases}$$

$$\rightarrow V_1=\frac{\begin{vmatrix}0 & -0.33-0.17j\\ 2 & -0.53-0.55j\end{vmatrix}}{\begin{vmatrix}0.83-1.07j & -0.33-0.17j\\ 0.33+0.17j & -0.53-0.55j\end{vmatrix}},V_2=\frac{\begin{vmatrix}0.83-1.07j & 0\\ 0.33+0.17j & 2\end{vmatrix}}{\begin{vmatrix}0.83-1.07j & -0.33-0.17j\\ 0.33+0.17j & -0.53-0.55j\end{vmatrix}}$$

$$V_1=\frac{0.66+0.34j}{-0.94+0.22j}=-0.58-0.5j,V_2=\frac{1.66-2.14j}{-0.94+0.22j}=-2.18+1.76j$$

دانشکده برق و کامپیوتر

پاسخ حالت دائمی سینوسی مدارهای RLC:
روش فازور: **ادامه مثال**

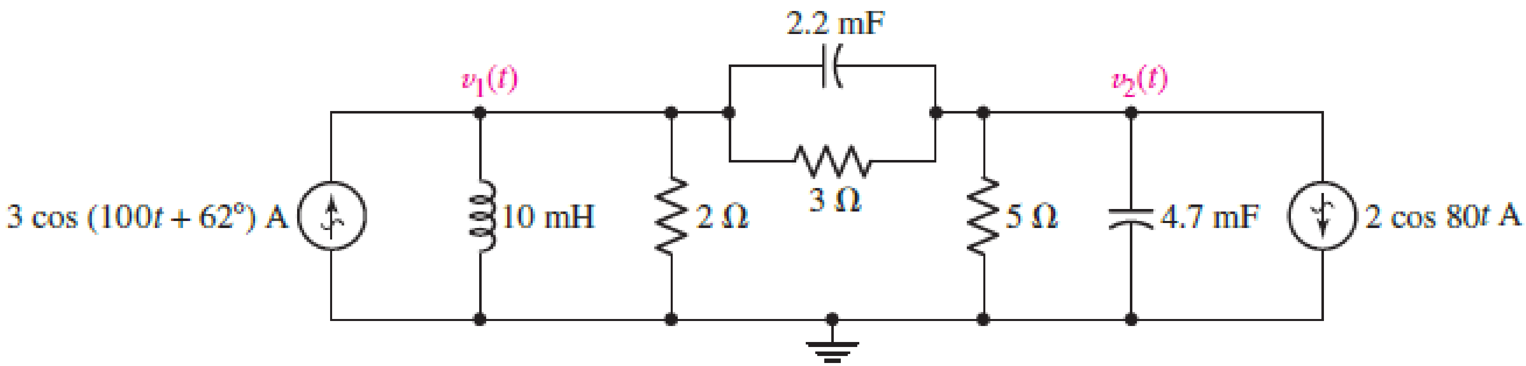
در نظر گرفتن تنها منبع سمت راست

$$V_1=\frac{0.66+0.34j}{-0.94+0.22j}=-0.58-0.5j, V_2=\frac{1.66-2.14j}{-0.94+0.22j}=-2.18+1.76j$$

$$V_1=0.76\angle-139.23^\circ, V_2=2.8\angle 141.08^\circ$$

$$v_1(t)=0.76\cos(80t-139.23), v_2(t)=2.8\cos(80t+141.08)$$

قضیه جمع آثار



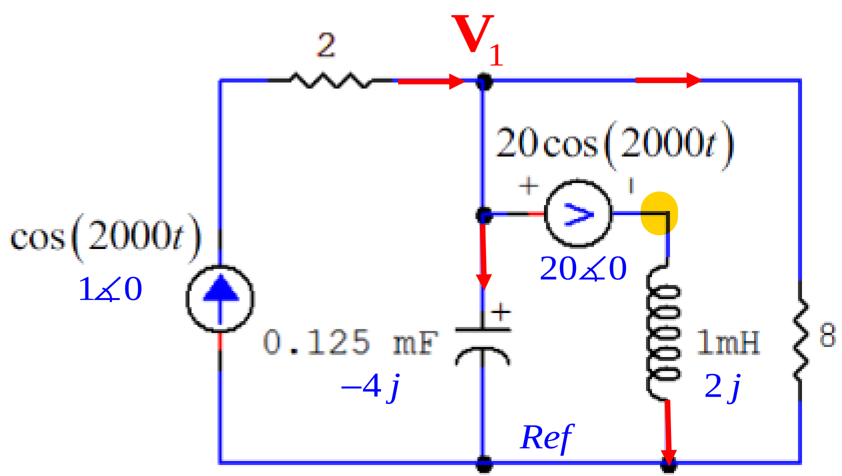
$$v_1(t)=2.83\cos(100t+113.04)+0.76\cos(80t-139.23)$$

$$v_2(t)=1.28\cos(100t+94.02)+2.8\cos(80t+141.08)$$

پاسخ حالت دائمی سینوسی مدارهای RLC:

روش فازور: مثال

مدار شکل زیر در حالت دائمی سینوسی قرار گرفته است. مقادیر تمام مقاومت ها بر حسب اهم است. در این صورت توان لحظه ای تحویل داده شده به مقاومت ۸ اهمی را محاسبه کنید؟



$$KCL\ \mathbf{V}_1 : 1 = \frac{\mathbf{V}_1}{-4j} + \frac{\mathbf{V}_1}{8} + \frac{\mathbf{V}_1 - 20}{2j} \rightarrow 1 - 10j = \mathbf{V}_1(-0.25j + 0.125)$$

$$\mathbf{V}_1 = \frac{1-10j}{-0.25j+0.125} = 33.6 - 12.8j = 35.95\measuredangle -20.85^\circ \rightarrow v_1(t) = 35.95\cos(2000t - 20.85^\circ)$$

$$P_{8\Omega} = \frac{v_1^2(t)}{8} = 161.55\cos^2(2000t - 20.85^\circ)$$