HW4

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حوری دهش ۹۸۲۱۴۱۳

س۱ ←

$$T = 5 \longrightarrow \omega_0 = \frac{2\pi}{5}$$

$$x[n] = \sum_{k=0}^{n-1} a_k e^{jk\omega_0 n} = a_{-2} e^{-2j\left(\frac{2\pi}{5}\right)n} + a_0 + a_2 e^{2j\left(\frac{2\pi}{5}\right)n} + a_{-4} e^{-4j\left(\frac{2\pi}{5}\right)n} + a_4 e^{4j\left(\frac{2\pi}{5}\right)n} =$$

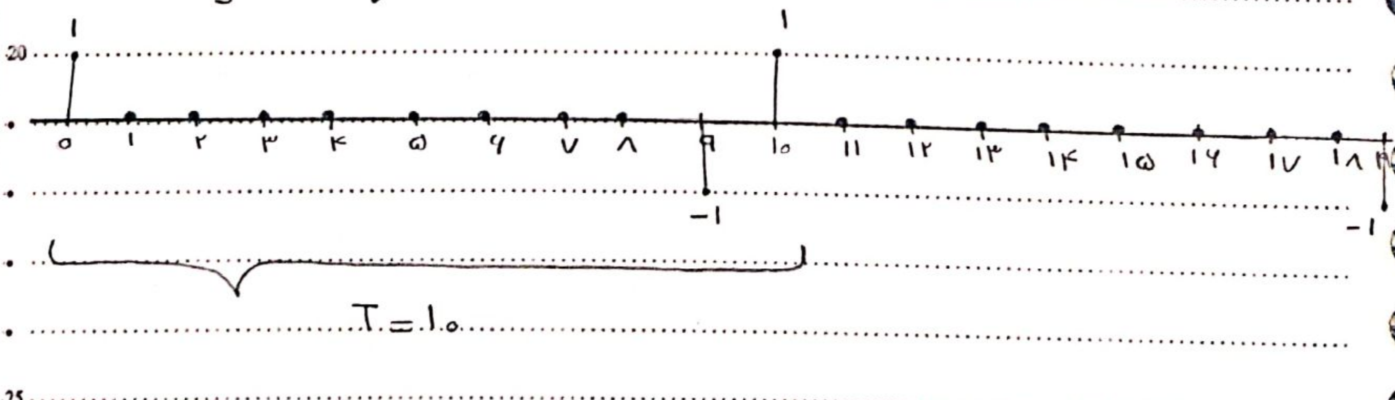
$$1 + e^{j\left(\frac{\pi}{4}\right)} e^{j\left(\frac{4\pi}{5}\right)n} + e^{j\left(\frac{\pi}{4}\right)} e^{-j\left(\frac{4\pi}{5}\right)n} + 2e^{j\left(\frac{\pi}{3}\right)} e^{j\left(\frac{8\pi}{5}\right)n} + 2e^{j\left(\frac{\pi}{3}\right)} e^{-j\left(\frac{8\pi}{5}\right)n} =$$

$$1 + 2\cos\left(\frac{4\pi n}{5} + \frac{\pi}{4}\right) + 4\cos\left(\frac{8\pi n}{5} + \frac{\pi}{3}\right) =$$

$$1 + 2\sin\left(\frac{4\pi n}{5} + \frac{3\pi}{4}\right) + 4\sin\left(\frac{8\pi n}{5} + \frac{5\pi}{6}\right)$$

س۲ ←

الف) $g[n] \rightarrow$



$T = 10$

ب) $b_k = \frac{1}{N}\sum_{n=0}^{9} g(n)\, e^{-jk\omega_0 n} =$

$\frac{1}{10}\left(1 + 0\times 8 - 1\times e^{-jk\frac{2\pi}{10}\times 9}\right) = \frac{1}{10}\left(1 - e^{-jk\,9\frac{\pi}{5}}\right)$

ج) $b_k = a_k - e^{-j\left(\frac{2\pi}{10}\right)k} a_k \longrightarrow a_k = \frac{b_k}{1 - e^{-j\left(\frac{2\pi}{10}\right)k}} =$

$$\frac{\frac{1}{10}\left(1 - e^{-j\left(\frac{\pi}{5}\right)9k}\right)}{1 - e^{-j\left(\frac{\pi}{5}\right)k}}$$

س ۳ $\leftarrow$

الف) $x[n] = \sin\left(\frac{2\pi n}{3}\right)\cos\left(\frac{\pi n}{2}\right)$

$z[n] = \cos\left(\frac{\pi n}{2}\right) = \frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$

$y[n] = \sin\left(\frac{2\pi n}{3}\right) = \frac{e^{j\frac{2\pi}{3}n} - e^{-j\frac{2\pi}{3}n}}{2j}$

$\rightarrow x[n] = y[n]\,z[n] = \left(\frac{1}{2j} e^{j\frac{2\pi}{3}n} - \frac{1}{2j} e^{-j\frac{2\pi}{3}n}\right)$

$\left(\frac{1}{2} e^{j\frac{\pi}{2}n} + \frac{1}{2} e^{-j\frac{\pi}{2}n}\right) = \frac{1}{4j}\left(e^{j\frac{7\pi}{6}n} + e^{j\frac{\pi}{6}n}\right) -$

$\frac{1}{4j}\left(e^{-j\frac{\pi}{6}n} + e^{-j\frac{7\pi}{6}n}\right) \Longrightarrow \begin{cases} a_1 = a_7 = \frac{1}{4j} \\ a_{-1} = a_{-7} = \frac{-1}{4j} \end{cases} \Longrightarrow$

$$\begin{cases} a_{۱} = a_{۷} = \dfrac{۱}{۴j} \\ a_{۱۱} = a_{۵} = \dfrac{-۱}{۴j} \\ a_{k} = ۰ \end{cases}$$

k از صفر تا ۱۱

$N = ۱۲ \;,\; \omega_0 = \frac{۲\pi}{۱۲} = \frac{\pi}{۶}$

ب)

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ج) $x[n] = \begin{cases} 3 & -2 \leq n < 0 \\ -3 & 1 \leq n \leq 3 \end{cases}$ $\quad N = 4 \longrightarrow \omega_0 = \frac{\pi}{3}$

$$a_k = \frac{1}{4} \sum_{n=-2}^{3} x(n) e^{-jk\omega_0 n} =$$

$$\frac{1}{4} \times 3 \left(e^{2jk\frac{\pi}{3}} + e^{jk\frac{\pi}{3}} + 1 + e^{-jk\frac{\pi}{3}} + e^{-2jk\frac{\pi}{3}} + e^{-3jk\frac{\pi}{3}} \right) =$$

$$\frac{1}{2} \left(e^{jk\frac{2\pi}{3}} + e^{jk\frac{\pi}{3}} + \cancel{1} + e^{-jk\frac{\pi}{3}} + e^{-jk\frac{2\pi}{3}} \cancel{1} \right) =$$

$$\cos\left(\frac{\pi}{3} k\right) + \cos\left(\frac{2\pi}{3} k\right)$$

د) $x[n] = \begin{cases} -2 & n = 0 \\ 2 & n = \pm 1 \\ 1 & n = \pm 2 \end{cases}$ $\quad N = 5 \longrightarrow \omega_0 = \frac{2\pi}{5}$

$$a_k = \frac{1}{5} \sum_{n=-2}^{2} x(n) e^{-jk\omega_0 n} =$$

$$\frac{1}{5} \left(e^{2jk\frac{2\pi}{5}} + 2e^{jk\frac{2\pi}{5}} - 2 + 2e^{-jk\frac{2\pi}{5}} + e^{-2jk\frac{2\pi}{5}} \right) =$$

$$\frac{1}{5} \left(-2 + \frac{1}{2} \cos\left(\frac{4\pi k}{5}\right) + \frac{1}{2} \cos \frac{2\pi k}{5} \right) = \frac{-2}{5} +$$

$$\frac{1}{10} \left(\cos\left(\frac{4\pi k}{5}\right) + \cos\left(\frac{2\pi k}{5}\right) \right)$$

س ۴ ←

الف) $x[n] \xleftrightarrow{FS} a_k \quad , \quad T_x = 6$

$$x[n] = a_0 + a_1 e^{j\frac{\pi}{3}n} + a_{-1} e^{-j\frac{\pi}{3}n} = a_0 + 2a_1 \cos(\frac{2\pi}{6}n) = \sin(\frac{2\pi n}{6} + \frac{\pi}{4})$$

$$\longrightarrow a_0 = 0 \;,\; a_1 = a_{-1} = \frac{e^{-j(\frac{\pi}{4})}}{2}$$

$y[n] \xleftrightarrow{FS} b_k \qquad T_y = 6$

$$y[n] = b_0 + b_1 e^{j\frac{\pi}{3}n} + b_{-1} e^{-j\frac{\pi}{3}n} = b_0 + \underbrace{2b_1 \cos(\frac{2\pi n}{6})}$$

$$\longrightarrow b_0 = 1 \;,\; b_1 = b_{-1} = \frac{1}{2} \qquad \longleftarrow 2b_1 = 1$$

ب) $z[n] = x[n]\,y[n] \xleftrightarrow{FS} c_k = \sum_{t=-2}^{2} a_t\, b_{k-t}$

$$c_0 = \cancelto{0}{a_{-2}b_2} + a_{-1}b_{-1} + \cancelto{0}{a_0 b_0} + a_1 b_1 + \cancelto{0}{a_2 b_2} = 2 \times \frac{e^{-j\frac{\pi}{4}}}{4} = \frac{1}{2}\cos(\frac{\pi}{4})$$

$$c_1 = c_{-1} = \cancelto{0}{a_{-2}b_1} + \cancelto{0}{a_{-1}b_0} + a_0 b_{-1} + \cancelto{0}{a_1 b_{-2}} + \cancelto{0}{a_2 b_{-3}} =$$

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$$\frac{1}{2} e^{-j\left(\frac{\pi}{4}\right)}$$

$$C_2 = C_{-2} = \cancel{a_{-2} b_0} + a_{-1} b_{-1} + \cancel{a_0 b_{-2}} + \cancel{a_1 b_{-3}} + \cancel{a_2 b_{-4}} =$$

$$\frac{1}{4} e^{-j\left(\frac{\pi}{4}\right)}$$

ج) $z[n] = \sin\left(\frac{2\pi n}{9} + \frac{\pi}{4}\right) + \sin\left(\frac{2\pi n}{9} + \frac{\pi}{4}\right) \cos\left(\frac{2\pi n}{9}\right) =$

$$\sin\left(\frac{2\pi n}{9} + \frac{\pi}{4}\right) + \frac{1}{2}\left(\sin\left(\frac{4\pi n}{9} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)\right) =$$

$$\frac{1}{2}\sin\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi n}{9} + \frac{\pi}{4}\right) + \frac{1}{2}\sin\left(\frac{4\pi n}{9} + \frac{\pi}{4}\right)$$

$$C_0 = \frac{1}{2}\sin\left(\frac{\pi}{4}\right) = \frac{1}{2}\cos\left(\frac{\pi}{4}\right)$$

$$C_1 = C_{-1} = \frac{1}{2} e^{-j\frac{\pi}{4}}$$

$$C_2 = C_{-2} = \frac{1}{4} e^{-j\frac{\pi}{4}}$$

س۵ ←

الف) $x[n] = x[-n] \longrightarrow a_n = a_{-n}$

ب) $N = 10$

ج) $a_{11} = 5 = a_1$

۵) مقایسه با ج سوال $\longrightarrow \frac{1}{10} \sum_{n=0}^{9} |x(n)|^2 = 50 \longrightarrow$

$$\sum_{k=\langle N \rangle} |a_k|^2 = 50 = \sum_{k=-4}^{5} |a_k|^2 = 2|a_4|^2 + 2|a_3|^2 +$$

$$2|a_2|^2 + 2|a_1|^2 + |a_0|^2 + |a_5|^2 \Longrightarrow a_1 = a_{-1} = 5 ,$$

$$a_4 = a_{-4} = a_3 = a_{-3} = a_{-2} = a_2 = a_0 = a_5 = 0$$

$$x(n) = \sum_{k=-4}^{5} a_k e^{jk\omega_0 n} = a_{-1} e^{-j\omega_0 n} + a_1 e^{j\omega_0 n} =$$

$$5 e^{-j\frac{\pi}{5} n} + 5 e^{j\frac{\pi}{5} n} = 10 \cos\left(\frac{\pi}{5} n\right) \longrightarrow A = 10 , \; B = \frac{\pi}{5} , \; C = 0$$

← س ۴

$$H(j\omega) = \begin{cases} 1 & |\omega| < 2000\pi \longrightarrow -2000\pi < \omega < 2000\pi \\ 0 & 2000\pi < |\omega| \longrightarrow \omega > 2000\pi \text{ and } \omega < -2000\pi \end{cases}$$

$$a_k = \begin{cases} 2 & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|} & \text{other} \end{cases} \longleftarrow ??$$

$$y(t) \xleftrightarrow{FS} b_k$$

$$T = 1.5\,ms \longrightarrow \omega_0 = \frac{2\pi}{15 \times 10^{-4}} = \frac{4000\pi}{3}$$

$$b_K = a_K \, H(K\omega_0) = a_K \, H\left(\frac{۴۰۰ K\pi}{۳}\right) \longrightarrow$$

$$b_0 = a_0 \, H(0) = a_0 = b_0 = ۲$$

$$b_1 = a_1 \, H\left(\frac{۴۰۰\pi}{۳}\right) \longrightarrow b_1 = a_1 = \frac{1}{۲} j$$

$$b_{-1} = a_{-1} \, H\left(\frac{-۴۰۰\pi}{۳}\right) \longrightarrow b_{-1} = a_{-1} = \frac{1}{۲} j$$

$$\text{if: } |K| \geq ۲ \longrightarrow \left|\frac{۴۰۰\pi}{۳}\right| > ۲۰۰\pi \longrightarrow H(j\omega) = 0 \longrightarrow$$

$$b_K = a_K \times 0 = 0$$

$$b_K = \begin{cases} ۲ & K = 0 \\ \frac{1}{۲} j & K = \pm 1 \\ 0 & \text{other} \end{cases}$$

$$y(t) = \sum_{K=-\infty}^{+\infty} b_K e^{j\omega_0 t} \longrightarrow y(t) = \frac{j}{۲} e^{-j\left(\frac{۴۰۰\pi}{۳}\right)t} + ۲ + \frac{j}{۲} e^{j\left(\frac{۴۰۰\pi}{۳}\right)t} = ۲ + j \cos\left(\frac{۴۰۰\pi t}{۳}\right)$$

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$h[n] = \left(\frac{1}{2}\right)^{|n|}$ ← س ۷

$x[n] = \sum_{k=-\infty}^{+\infty} \delta[n-4k]$

$H(e^{jk\omega_0}) = \sum_{n=-\infty}^{+\infty} h[n] e^{-jk\omega_0 n} = \sum_{n=-\infty}^{+\infty} \left(\frac{1}{2}\right)^{|n|} e^{-jk\omega_0 n}$

$= \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} e^{-jk\omega_0 n} + \sum_{n=1}^{+\infty} \left(\frac{1}{2}\right)^{n} e^{-jk\omega_0 n} =$

$\frac{1}{1-\frac{1}{2}e^{-jk\omega_0}} - \frac{1}{1-2e^{-jk\omega_0}}$

$x[n] \xleftrightarrow{FS} a_k = \frac{1}{4}$

طبق این در حالت کلی ضرایب سری فوریه $\sum_{k=-\infty}^{+\infty} \delta[n-Tk]$

برابر با $\frac{1}{T}$ می شود

$b_k = a_k H(jk\omega_0) = \frac{1}{4}\left(\frac{1}{1-\frac{1}{2}e^{-jk\omega_0}} - \frac{1}{1-2e^{-jk\omega_0}}\right) =$

$\frac{1}{4}\left(\frac{1}{1-\frac{1}{2}e^{-jk\frac{\pi}{2}}} - \frac{1}{1-2e^{-jk\frac{\pi}{2}}}\right)$ $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

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