Dynamics Convergence to Pure Nash Equilibrium in Content Curation Game

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1 Experiment Methodology

We run simulations of the content distribution games on randomly generated networks with different properties and parameters. Then by partially or entirely isolating the parameters of the network and the game we investigate the effects of different properties of the network on the dynamics of convergence to Pure Nash equilibrium of this particular game. The networks are generated given a particular number for the number of nodes. These node then are randomly assigned to be a particular type of node (content, publisher or reader) according to a multinomial distribution. For convenience let us denote the probability by which the nodes have been assigned to be contents and publishers, α and β; obviously, the probabilty of a node being labeled reader in that case is $1-\alpha-\beta$. The approach followed to draw the connections between readers and publishers (the scenario of fixed followers) remains unaltered during the course of this experiment. The degree of every reader node is sampled from a uniform distribution over the possible degree values i.e. the number of publishers. Followed by this, a set of publishers are picked according to a uniform distribution as well. The intrinsic values of contens to users too, are assigned using a uniform distribution.

Follwing the construction of every singe or set of network, readers, take turn and choose a strategy according to a sorted ordered set of strategies (the purpose is to guarantee the termination of the algorithm) which yields the optimum objective for that particular individual with respect to current strategy of the other users. In upcoming sections, we report statistical data on the number of steps required to reach a Pure Nash equilibrium and correlate it to various properties af a network.

2 Size of the network

In this section we specifically investigate how the size of the network affect the rate of convergence to PNE in content curations game. The setting of the experiments follows the same framework that we described in the previous section. We randomly generate networks with particular sizes. However, the ratio of the classes (i.e. publishers, contents, and readers) remain the same as well as the ratio of present subscriptions to all potential subscriptions which is $R \times P$ if R and P respectively denote the cardinality of the sets of readers and publishers. Also, the limit of the number of contents that a publisher can share is fixed too. Note that the subscriptions remain unmodified through out a single game since the users are not strategic. Although the ratio remains the same, the edges change even for a graph of some size. This ratio if fixed grows quadratically in the size of the network since all other factors are fixed. In other words, if the ratio of edges is η we will have $\eta\beta N \times (1 - \beta - \alpha)N$ which is of order $O(N^2)$. Recall that β and α are the ratio of publisher and content users in the network. The result of the experiment shows sign of growth of $\Omega(N)$ which can be related to the quadratic growth of subscriptions.

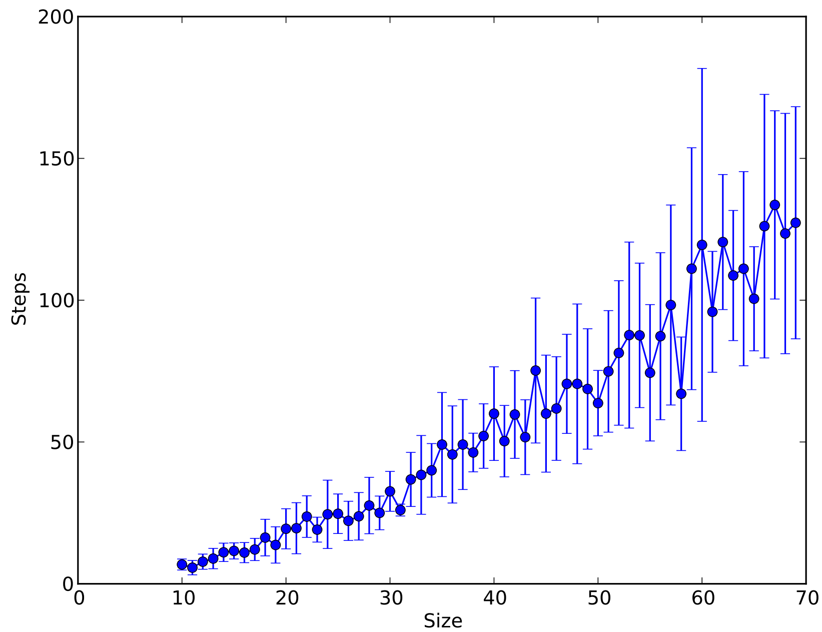

Figure 1:

Rate of convergence to the Pure Nash Equillibrium for different networks having the identical properties (parameters) except for their size.

3 The ratio of available contents and publishers to the size of the network

In this section we look at the effects of tempering with the ration of classes across networks on the rate of convergence to the Pure Nash Equillibrium. Again following the main framework, we fix the size of the network, and iterate through descrete values for the ratio of different classes. As you can see, this mesh of values has been chosen to have 0.5 over the two parameters α and β. Obviously, by fixing these to the other ratio is determined. We observed the following statistics for the rate of convergence for different values of these ratios. This data demonstrates the intuition that the size of the publishers is a major factor in deciding the rate in which the game converges to the pure Nash Equillibrium.

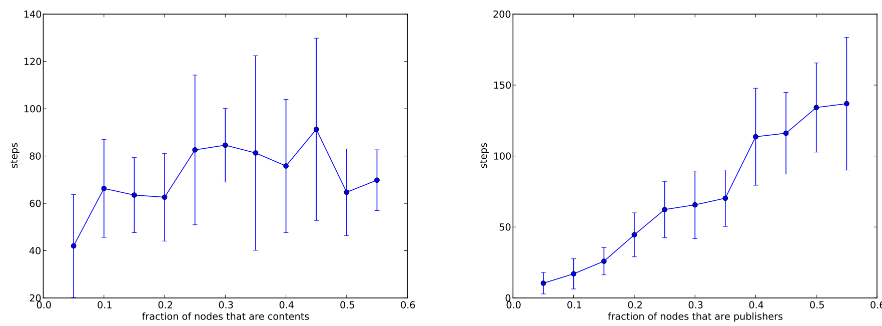

Figure 2:

Marginal convergence rate of the network curation games with respect to content node and publisher node ration.

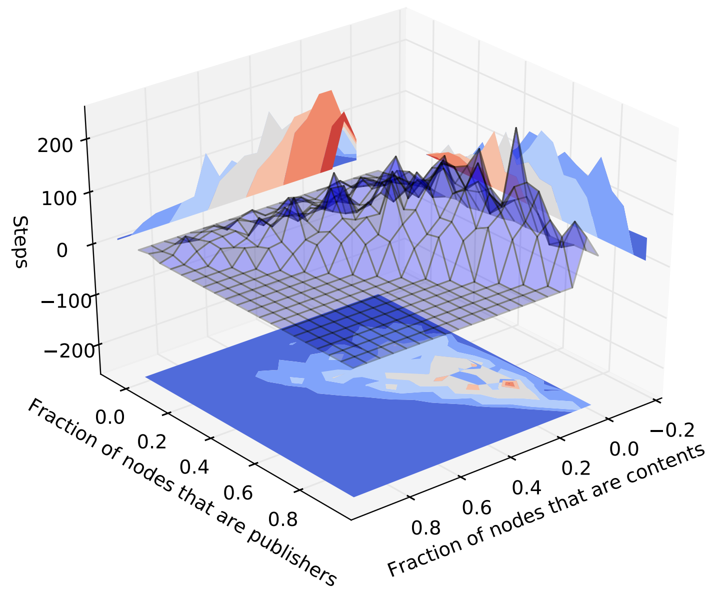

Figure 3:
Joint convergence rate of the network curation games with respect to content node and publisher node ration

4 Attention budget of the readers

In this set of expermients we want to demonstrate the correlation of attention budget (i.e. the limit of the number of contents that each individual publisher shares in terms of ratio to the entire contents) on the dynamics of convergence. The rough concavity of the rate of convergence with respect to the variable and reaching a peak in the middle corresponds to the fact that as the number of options which can be shown by $\binom{C}{\ell}$ by every individual publisher attains its maximum when $\ell = 2$. Simply the more strategies there are for publishers, the larger the rate of convergence will be.

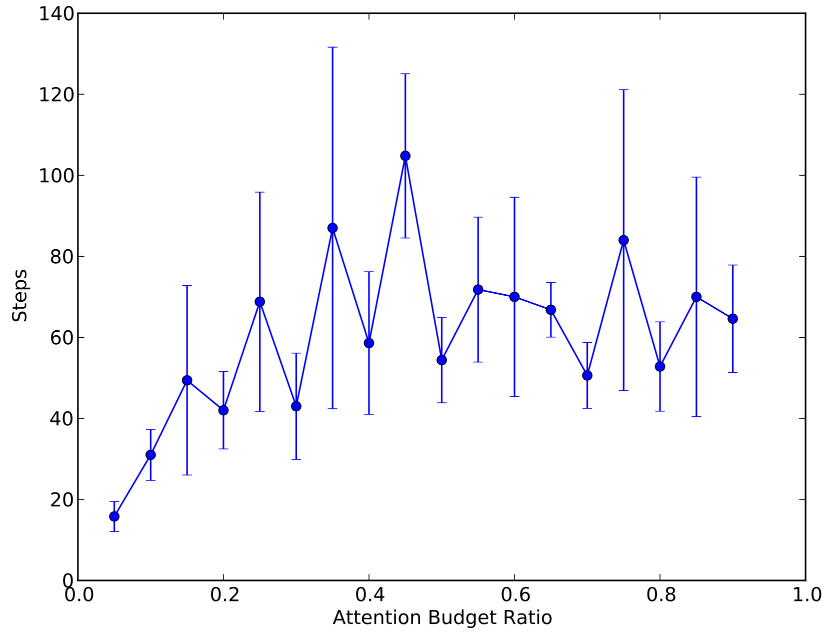

Figure 4:
Rate of convergence reported for networks with fixed parameters except for attention budget ratio.

5 Distribution of Intrinsic Values

Finally, in the last section we see some interesting results on the influence of the distribution of intrinsic values of the contents to the useres to the rate of convergence. The following figure shows the difference of rate of convergence on the same networks for two different form of distribution. Uniform distribution of the values as opposed to power law distribution of the values of contents to the readers, respectively shown by blue and red data points. We see that, the average rate of convergence is strictly less for the power law which is a more reallistic representation of real networks. Showing that in real networks the game converges in faster pace rather that random generated graphs.

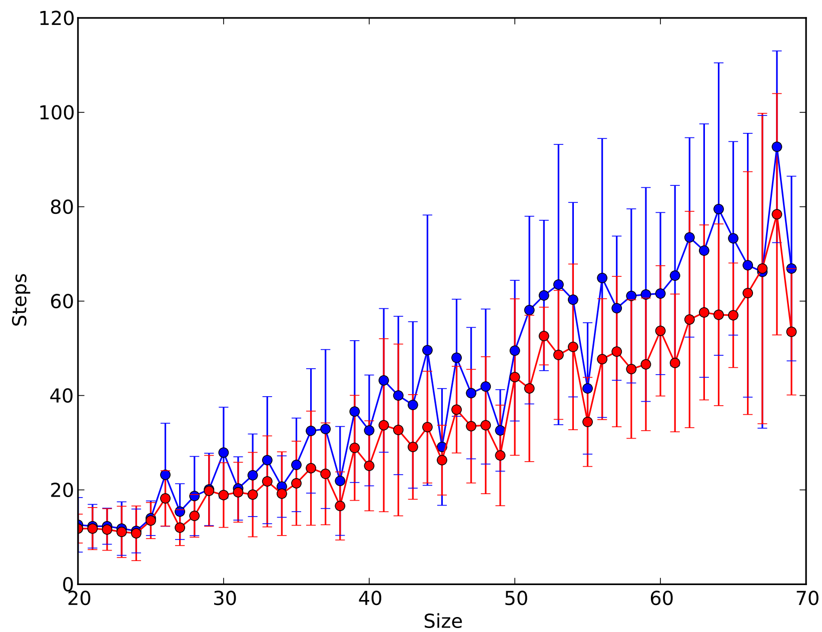

Figure 5:
Two different convergence rates on the same randomly generated networks. For every network, once we see the result of the game on a uniformly distributed intrinsic values, and once on a set of value parameters generated according to a pareto distribution.