CSE340 Spring 2020 - Homework 1

Due: Friday February 7 2020 by 11:59 PM on Gradescope

All submissions should be typed. Exception can only be made for drawing parse trees, which can be hand drawn and scanned in the submitted document.

When you submit your solution on Gradescope, you should indicate for each problem the page on which the solution is.

Problem 1. Consider the grammar

$$S \rightarrow Y X Y$$

$$X \rightarrow a Y \mid Y$$

$$Y \rightarrow b b Y \mid X \mid \epsilon$$

where a and b are tokens. Remember that ϵ represent the empty string. $Y \to \epsilon$ means that Y does not have to match any tokens.

1. Give a leftmost derivation for the string (sequence of tokens): bbabbabb

Answer

| | <u>S</u> | $(S \rightarrow YXY)$ |
|----|---|-------------------------------|
| => | YXY | $(Y \rightarrow b b Y)$ |
| => | b b <u>Y</u> X Y | $(Y \rightarrow X)$ |
| => | b b <u>X</u> X Y | $(X \rightarrow a Y)$ |
| => | b b <mark>a <u>Y</u> X Y</mark> | $(Y \rightarrow b b Y)$ |
| => | b b a b b <u>Y</u> X Y | $(Y \rightarrow \varepsilon)$ |
| => | b b a b b <u>X</u> Y | $(X \rightarrow a Y)$ |
| => | b b a b b <mark>a <u>Y</u> Y</mark> | $(Y \rightarrow b b Y)$ |
| => | b b a b b a <mark>b b <u>Y</u> Y</mark> | $(Y \rightarrow \varepsilon)$ |
| => | b b a b b a b b Y | $(Y \rightarrow \varepsilon)$ |
| => | b b a b b a b b | |

2. Give a rightmost derivation for the string (sequence of tokens): bbabbabb

Answer

| <u>S</u> | $(S \rightarrow YXY)$ |
|-----------------------------------|-------------------------------|
| => Y X <u>Y</u> | $(Y \rightarrow b b Y)$ |
| => Y X b b <u>Y</u> | $(Y \rightarrow \varepsilon)$ |
| \Rightarrow Y X b b | $(X \rightarrow a Y)$ |
| => Y a <u>Y</u> b b | $(Y \rightarrow \varepsilon)$ |
| $\Rightarrow \underline{Y} a b b$ | $(Y \rightarrow b b Y)$ |
| => b b <u>Y</u> a b b | $(Y \rightarrow X)$ |
| => b b X a b b | $(X \rightarrow a Y)$ |
| => b b a <u>Y</u> a b b | $(Y \rightarrow b b Y)$ |
| => b b a b b <u>Y</u> a b b | $(Y \rightarrow \varepsilon)$ |
| => b b a b b a b b | |

Problem 2. Consider the grammar

$$S \rightarrow X X X$$

$$X \rightarrow a Y \mid Y$$

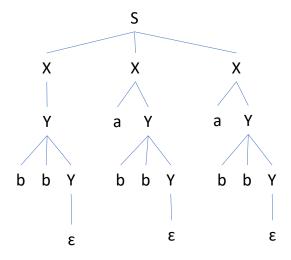
$$Y \rightarrow b b Y \mid X \mid \epsilon$$

Draw a parse tree for the sequence of tokens

bbabbabb

The parse tree should have height less than or equal to 5.

Answer



Problem 3. Consider the grammar

$$S \rightarrow a S b S c S$$

$$S \rightarrow A$$

$$A \rightarrow a S b S$$

$$A \rightarrow d$$

1. What are the non-terminals?

Answer. By convention, and unless otherwise noted, the non-terminals are the LHS of the grammar rules. Hence, for **S** and **A** are the non-terminals for the given grammar.

2. What is the start symbol?

Answer. By convention, and unless otherwise noted, the start symbol is the LHS of the first rule. Hence, **S** is the start symbol for the given grammar.

3. What are the terminals?

Answer. By convention, and unless otherwise noted, the terminals are the symbols that appear only on the RHS of the rules. For the given grammar, the terminals are \mathbf{a} , \mathbf{b} , \mathbf{c} and \mathbf{d} .

Explain your answers!

Problem 4. Show that the following grammar is ambiguous by giving a string that has two different leftmost derivations

$$S \rightarrow AD \mid BC$$

 $A \rightarrow aA \mid bC$
 $B \rightarrow ACD$
 $C \rightarrow cC \mid D \mid \epsilon$
 $D \rightarrow CD \mid DD \mid a$

You should give the two derivations for the string you propose.

Answer. A string that has two different leftmost derivations is the string abca

| <u>S</u> | $(S \rightarrow AD)$ |
|--|---|
| => <u>A</u> D | $(A \rightarrow a A)$ |
| => a <u>A</u> D | $(A \rightarrow b C)$ |
| => a b <u>C</u> D | $(C \rightarrow c C)$ |
| => a b c <u>C</u> D | $(C \rightarrow \epsilon)$ |
| => a b c <u>D</u> | $(D \rightarrow a)$ |
| => a b c a | |
| | |
| | |
| <u>S</u> | $(S \rightarrow BC)$ |
| <u>S</u> => <u>B</u> C | $(S \to BC)$ $(B \to A C D)$ |
| _ | , |
| => <u>B</u> C | $(B \rightarrow A C D)$ |
| $=> \frac{\mathbf{B}}{\mathbf{C}}\mathbf{C}$ $=> \mathbf{A}\mathbf{C}\mathbf{D}\mathbf{C}$ | $(B \to A C D)$ $(A \to a A)$ |
| => <u>B</u> C => ACDC => aACDC | $(B \rightarrow A C D)$ $(A \rightarrow a A)$ $(A \rightarrow b C)$ |
| => <u>B</u> C => ACDC => aACDC => a b CCDC | $(B \rightarrow A C D)$ $(A \rightarrow a A)$ $(A \rightarrow b C)$ $(C \rightarrow c C)$ |
| => <u>B</u> C => ACDC => aACDC => a b CCDC => a b c <u>C</u> CDC | $(B \rightarrow A C D)$ $(A \rightarrow a A)$ $(A \rightarrow b C)$ $(C \rightarrow c C)$ $(C \rightarrow \epsilon)$ |
| => <u>B</u> C => ACDC => aACDC => abCCDC => abc <u>C</u> CDC => abc <u>C</u> DC | $(B \rightarrow A C D)$ $(A \rightarrow a A)$ $(A \rightarrow b C)$ $(C \rightarrow c C)$ $(C \rightarrow \epsilon)$ $(C \rightarrow \epsilon)$ |

This string that has two different leftmost derivations. This proves that the given grammar is ambiguous.