CSE340 FALL 2020 HOMEWORK 5
Due Monday 30 November 2020 by 11:59 PM

1. The homework has 3 problems
2. Your answers must be typed.
3. On Gradescope, you should submit the answers to separate question separately.

Problem 1 (Static and Dynamic Scoping)

Consider the following program written in a C-like syntax (well not quite C-like but should not be hard to understand). Assume parameters are passed by value.

```
a, b, x : int;        // global variables

int g(a : int; d: int)
{
        print a;
        print b;
        print x;
        print d;
        return a + b+ x + d;
}

int f(x : int)
{
        b : int;
        b = 3;
        b = g(a,b);
        return b;
}

void main()
{
        int a;
        int b;
        a = 4;
        b = 5;
        a = f(b);
        g(b,a);
}
```

Solution

1. What is the output of this program if static scoping is used

I am going to assume that global variables are initialized to 0. If you did not assume that in your solution and simply assumed that the initial value is unknown, that is fine.

The execution is equivalent to the following execution

```
a_global = 0
b_global = 0
x_global = 0

a_main = 4
b_main = 5
a_main = f(b_main) = f(5)
                    f(5):     x_f = 5
                              b_f = 3
                              b_f = g(a_global,b_f) = g(0,3)
                                        g(0,3):  a_g = 0
                                                 d_g = 3
                                                 print a_g                                  0 printed
                                                 print b_global                             0 printed
                                                 print x_global                             0 printed
                                                 print d_g                                  3 printed
                                                 return a_g+b_global+x_global+d_g =     0+0+0+3  = 3
                              b_f = 3
                              return b_f = 3
a_main = 3
g(b_main,a_main) = g(5,3)
          g(5,3):  a_g = 5
                   d_g = 3
                   print a_g                                  5 printed
                   print b_global                             0 printed
                   print x_global                             0 printed
                   print d_g                                  3 printed
                   return a_g+b_global+x_global+d_g =     5+0+0+3  = 8
```

2. What is the output of this program if dynamic scoping is used

```
a_global = 0
b_global = 0
x_global = 0

        a_main = 4 => 11
        b_main = 5
        a_main = f(b_main) = f(5)
                        f(5):     x_f = 5
                                  b_f = 3 => 15
                                  b_f = g(a_main,b_f) = g(4,3)
                                            g(4,3):   a_g = 4
                                                      d_g = 3
                                                      print a_g                                  4 printed
                                                      print b_f                                  3 printed
                                                      print x_f                                  5 printed
                                                      print d_g                                  3 printed
                                                      return a_g+b_global+x_global+d_g =     4+3+5+3  = 15
                                  b_f = 15
                                  return b_f = 15
        a_main = 15
        g(b_main,a_main) = g(5,15)
                  g(5,11): a_g = 5
                           d_g = 15
                           print a_g                                  5 printed
                           print b_main                               5 printed
                           print x_global                             0 printed
                           print d_g                                  15 printed
                           return a_g+b_global+x_global+d_g =     5+5+0+15  = 25
```

Problem 2 (Structural, Name and Internal Name Equivalence)

Consider the following type declarations

```
TYPE
   T0  =    int;
   T1  =    real;
   T2  =    pointer to int;
   T3  =    pointer to real;
   T4  =    pointer to T0;
   T5  =    pointer to T1;
   T6  =    struct {
                  a: int;
            };
   T7  =    struct {
                  b: int;
                  a: pointer to T8;
            };
   T8  =    struct {
                  a: T0;
                  b: pointer to T7;
            };
   T9  =    struct {
                  x: T7 * T8 -> T7; // function of T7 and T8 that returns T7
                  y: T11 * T12 -> T8;
            };
   T10 =    struct {
                  x: T8 * T7 -> T8; // function of T7 and T8 that returns T8
                  y: T12 * T11 -> T7;
            };
   T11 =    array [4][5] of T9;
   T12 =    array [4][5] of T10
```

For each of the following types, list the types that are equivalent, assuming structural equivalence:

1. **T0:** not equivalent to any other type. The only other "simple" type is T1 which is real and is not equivalent to T0
2. **T2:** can only be equivalent to another pointer. It is equivalent to T4 (pointer to T0) because T0 is structurally equivalent to int.
3. **T4:** equivalent to T2
4. **T6:** is a structure with only one field. There is no other structure with one field, so T6 is not equivalent to any other type in the list.
5. **T8:** T8 can only be equivalent to T7 because it is the only other type whose first field is equivalent to int. Doing the iterative process and assuming T7 and T8 are equivalent, when we compare them, we find int equivalent to T0 and pointer to T7 is equivalent to pointer to T8, so T7 and T8 are equivalent.
6. **T10:** T10 is equivalent to T9. This result and the result for T12 and T11 are obtained through the iterative process.
7. **T12:** is equivalent to T11

Consider the following variable declarations (this is a continuation of the previous declarations

VARS

```
x    : pointer to int;
y    : pointer to int;
z    : T3;
p, q : T9 -> pointer to int;
r    : T9 -> T3;
s    : T9;
t    : T10;
```

For each of the following assignment statements write if the statement is valid under structural equivalence, name equivalence, or internal name equivalence. Write all that apply.

1. x = y is valid under structural equivalence only
2. y = z not valid because pointer to int and pointer to real (T3) are not structurally equivalent
3. p = q since p and q are part of the same declaration, this is valid under internal name equivalence and structural equivalence
4. q = r this is not valid because the type of q is a function that returns pointer to int and the type of r is a function that returns T3 which is pointer to real. Since the return types are not equivalent under structural equivalence, this assignment is not valid
5. z = p(s) there are two parts to this statement: (1) function call and (2) assigning return value to z. (1) is valid under name equivalence because s has type T9 and p() expects an argument of type T9. (2) is not valid because z has type T3 and p() returns pointer to int which is not equivalent to T3.
   We conclude that the statement is not valid
6. x = p(t) there are two parts to this statement: (1) function call and (2) assigning return value to x. (1) is valid under structural equivalence because t has type T9 and p() expects an argument of type T10 which is structurally equivalent to T9. (2) the assignment is valid under structural equivalence because the return type is pointer to int and x has type pointer to int.
   We conclude that the assignment is valid under structural equivalence.
7. z = r(t) there are two parts to this statement: (1) function call and (2) assigning return value to z. (1) is valid under structural equivalence only because t has type T10 and r() expects an argument of type T9 which is structurally equivalent to T10. (2) is valid under name equivalence because z has type T3 and r() returns a value of type T3.
   We conclude that the assignment is valid under structural equivalence.

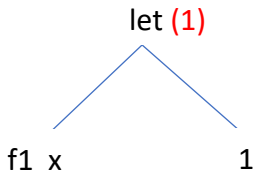
8. z = r(s) There are two parts to this statement. (1) function call and (2) assigning return value to z. (1) valid under name equivalence because s has type T9 and r() expects an argument of type T9. (2) is also valid under name equivalence because r() returns a value of type T3 and z has type T3.
We conclude that the whole statement is valid under name equivalence (and therefore under internal name and structural equivalence)

Problem 3 (Hindley Milner Type checking)

For this problem, you should give the answers and you do not need to show your work if there is no type checking error. You can use an online OCaml editor to check your answers, but you should not solely rely on that. If you do, you will not do well on the final.

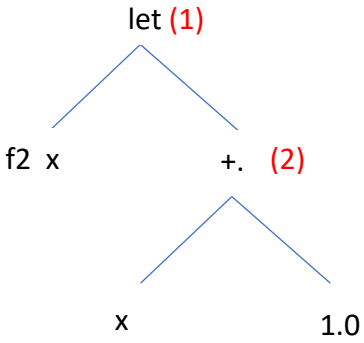
For each of the following determine the type of the function.

1. let f1 x = 1 ;;



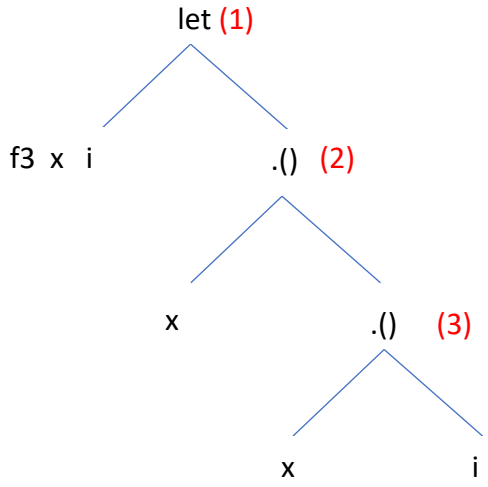
visiting node (1) : $T_{f1} = T_x \rightarrow int$ where T_x is unconstrained

2. let f2 x = x +. 1.0 ;;



visiting node (1) : $T_{f2} = T_x \rightarrow T_{(2)}$
visiting node (2) : $T_{(2)} = T_x = T_{1.0}$ = float
So : T_{f2} = float → float

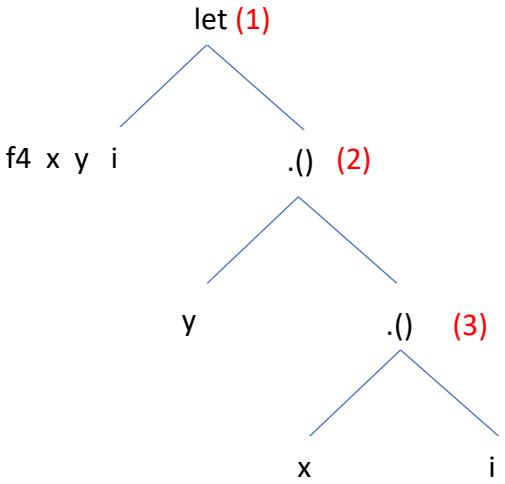
3. let f3 x i = x.(x.(i)) ;;



visiting node (1) : $T_{f3} = T_x \rightarrow T_i \rightarrow T_{(2)}$
visiting node (2) : $T_x = T_{(2)}$ array
$T_{(3)}$ = int
visiting node (3) : $T_x = T_{(3)}$ array = int array because $T_{(3)}$ = int
T_i = int
also, we conclude that $T_{(2)}$ = int because $T_x = T_{(2)}$ array and T_x = int array

So, T_{f3} = int array → int → int

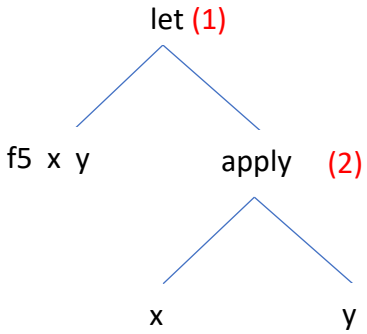
4. let f4 x y i = y.(x.(i)) ;;



visiting node (1) : $T_{f4} = T_x \rightarrow T_y \rightarrow T_i \rightarrow T_{(2)}$
visiting node (2) : $T_y = T_{(2)}$ array
$T_{(3)}$ = int
visiting node (3) : $T_x = T_{(3)}$ array = int array because $T_{(3)}$ = int
T_i = int

So, T_{f4} = int array → $T_{(2)}$ array → int → $T_{(2)}$ where $T_{(2)}$ is unconstrained

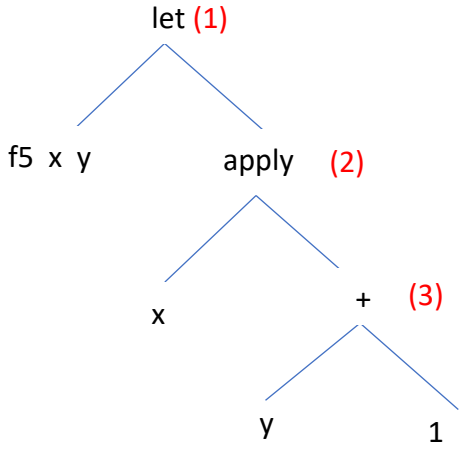
5. let f5 x y = x y ;;



visiting node (1) : $T_{f5} = T_x \rightarrow T_y \rightarrow T_{(2)}$
visiting node (2) : $T_x = T_y \rightarrow T_{(2)}$

So, $T_{f5} = (T_y \rightarrow T_{(2)}) \rightarrow T_y \rightarrow T_{(2)}$ for some unconstrained type $T_{(2)}$

6. let f6 x y = x (y+1) ;;



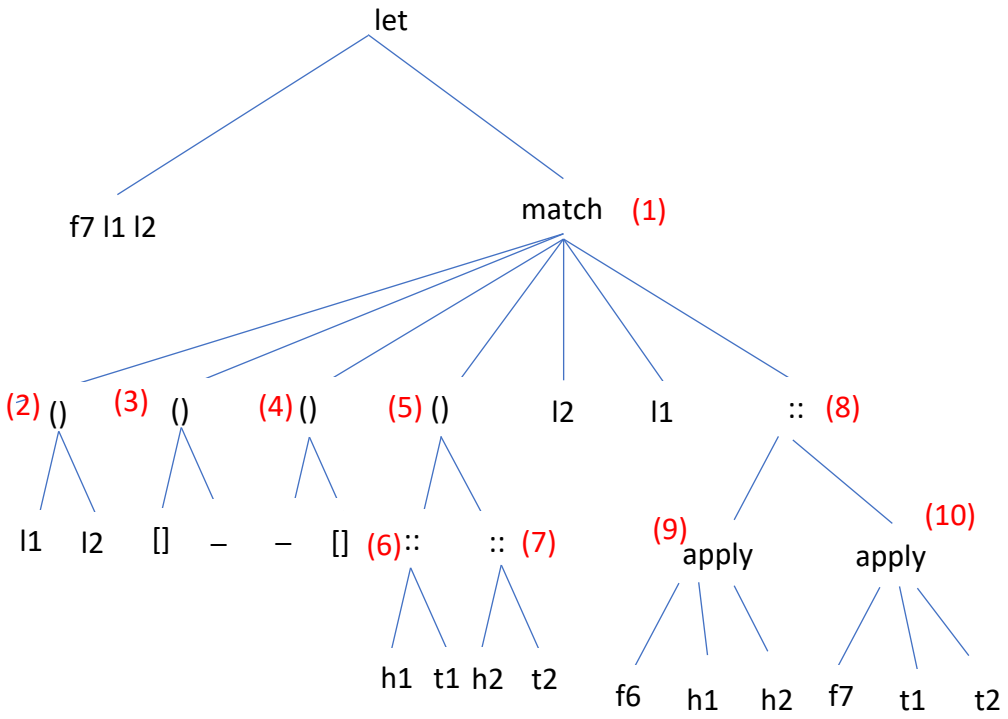
visiting node (1) : $T_{f6} = T_x \rightarrow T_y \rightarrow T_{(2)}$
visiting node (3) : $T_{(3} = T_y = \text{int}$
visiting node (2) : $T_x = T_{(3)} \rightarrow T_{(2)} = \text{int} \rightarrow T_{(2)}$

So, $T_{f6} = (\text{int} \rightarrow T_{(2)}) \rightarrow \text{int} \rightarrow T_{(2)}$

So, we can write $T_{f6} = (\text{int} \rightarrow T) \rightarrow \text{int} \rightarrow T$ for some unconstrainted type T

7. The following declaration results in a type checking error. Explain why

```
let rec f7 l1 l2 = match (l1,l2) with
        ([],_) -> l2
      | (_,[]) -> l1
      | (h1::t1,h2::t2) -> f6 h1 h2 :: f7 t1 t2 ;;
```



Visiting node (1) : $T_{(2)} = T_{(2)} = T_{(4)} = T_{(5)}$
$T_{(1)} = T_{l2} = T_{l1} = T_{(8)} = T$
Visiting node (2) : $T_{(2)} = T_{l1} * T_{l2} = T * T$
Visiting node (5) : $T_{(5)} = T_{(6)} * T_{(7)} = T_{(2)} = T * T$, so $T_{(6)} = T_{(7)} = T$
Visiting node (6) : $T_{(6)} = T_{t1} = T_{h1}$ list $= T$
Visiting node (7) : $T_{(7)} = T_{t2} = T_{h2}$ list $= T$

so T_{h1} list $= T_{h1}$ list and $T_{h1} = T_{h2} = T_h$ and $T_{l1} = T_{l2} = T_h$ list
$T_{t1} = T_{t2} = T_h$ list

Visiting nodes (3) and (4) makes no difference

Visiting node (9) : $T_{f6} = T_{h1} \rightarrow T_{h2} \rightarrow T_{(9)} = T_h \rightarrow T_h \rightarrow T_{(9)}$
but we know that $T_{f6} = (\text{int} \rightarrow T') \rightarrow \text{int} \rightarrow T'$ for some unconstrained type T'
(I use T' because T is used above = T_h list)

$T_h \rightarrow T_h \rightarrow T_{(9)} \equiv (\text{int} \rightarrow T') \rightarrow \text{int}$ gives us $(\text{int} \rightarrow T') \equiv \text{int}$ which is not possible

8. let rec f8 l1 l2 = match (l1,l2) with
   ([],_) -> l2
   | (_,[]) -> l1
   | (h1::t1,h2::t2) -> h1 + h2 :: f8 t1 t2 ;;

   T_{f8} = int list → int list → int list

9. what does f8 calculate? You can try different examples using the OCaml command line

   Function f8 takes two lists l1 and l2 as input. f8 is equivalent to the following. The shorter of the two lists is extended by appending 0's to obtain a list that has the same length as the longer list. Then, f8 creates a new list where the i'th element of the result is obtained by adding the i'th element of the extended list to the i'th element of the other list.