

# CSE340 SPRING 2019 HOMEWORK 3

Due Monday 11 February 2019 by 11:59 PM on Blackboard

## PLEASE READ THE FOLLOWING CAREFULLY

1. Your answers can be handwritten, but you need to have clear handwriting. If we cannot read it we cannot grade it!
2. You should answer the questions in the order they are listed
3. You submit only one file containing all the answers not multiple files
4. Your submission should be in pdf format

### Problem 1 (Operator Precedence Parsing).

Consider the following operator grammar

$$\begin{aligned} E &\rightarrow E \& E \\ E &\rightarrow \sim E \\ E &\rightarrow ( E ) \mid \text{id} \end{aligned}$$

where  $\&$  is a left associative binary operator and  $\sim$  is a unary operator and has higher precedence than  $\&$ .

1. Draw the precedence table for this grammar
2. Show step by step how  $\sim a \& b$  is parsed.

**Problem 2 (Lambda calculus binding).** For each of the following determine for each variable  $x$  the  $\lambda x.$  it is bound to. I have numbered the variables and the abstractions. The variables are numbered using Arabic numerals and the abstractions are numbered using roman numerals. If variables 4 and 7 are bound to abstraction I, your answer should be of the form  $I \rightarrow 4, 7$  to indicate that abstraction I has variables 4, and 7 bound by it.

**Example**       $(\lambda x. x \ x \ \lambda x. x) \ x$   
                   I   1 2   II 3 4

**Answer**        I  $\rightarrow$  1, 2  
                       II  $\rightarrow$  3

a.  $x \ (\lambda x. x \ x) \ x$   
     1   I   2   3   4

b.  $x \ \lambda x. x \ \lambda x. x \ \lambda x. x$   
     1   I   2   II   3   III   4

c.  $\lambda x. x \ (\lambda x. x) \ (\lambda y. x) \ y$   
     I   1   II   2   III   3   4

d.  $\lambda x. (\lambda y. x) \ \lambda x. x \ x$   
     I   II   1   III   2   3

e.  $\lambda x. (\lambda x. (\lambda x. x \ x \ (\lambda x. x) \ x) \ (\lambda x. x \ x) \ (\lambda x. x)) \ x$   
     I   II   III   1   2   IV   3   4   V   5   6   VI   7   8

**Problem 2 (lambda calculus reducible expressions).** In this problem you are asked to determine the reducible expressions in each of the following lambda expressions. The format of the answer is shown in the examples below

a.  $x \lambda x. (\lambda x. \lambda x. x) x x$

b.  $(\lambda x. x) \lambda x. x x (\lambda x. x) x$

c.  $\lambda x. (\lambda y. x) \lambda x. (\lambda x. x) x$

d.  $\lambda x. \lambda x. x \lambda x. x x$

e.  $(\lambda x. x) x (\lambda x. x) x$

f.  $(\lambda x. x x) (\lambda x. x) (\lambda x. (\lambda x. (\lambda x. x) x (\lambda x. x) x) x)$

**Examples to show the format of the answer**

**Example 1:** for the expressions

$$\lambda x. x x (\lambda x. x) x \lambda x. x x$$

Your answer should be that there are no reducible expressions.

**Example 2:** for the expression

$$(\lambda x. (\lambda x. x) x) \lambda x. x x$$

Your answer should be

$$\begin{array}{c} (\lambda x. (\lambda x. x) x) \lambda x. x x \\ \underbrace{\hspace{1cm}} \underbrace{\hspace{1cm}} \underbrace{\hspace{1cm}} \\ (\lambda x. \quad t \quad) \quad t' \end{array}$$

$$\begin{array}{c} (\lambda x. (\lambda x. x) x) \lambda x. x x \\ \underbrace{\hspace{1cm}} \underbrace{\hspace{1cm}} \underbrace{\hspace{1cm}} \\ (\lambda x. t) t' \end{array}$$

**Problem 3 (lambda calculus Beta Reductions).** For each of the following expressions, give the resulting expression after executing a beta reduction of the highlighted redex. Examples are shown below

1. Do renaming **only if needed**. You should not do renaming if it is not needed.
2. Do not forget to add parentheses when  $t'$  is an non-parenthesized abstraction.
3. The bound variables of the redex are identified for you in parts a. and b. You should identify the bound variables in parts c. and d. as part of your answer.

a.  $(\lambda x. x \ y \ x) \ \lambda y. y \ y$   
 $(\lambda x. \quad t \quad) \quad t'$

b.  $(\lambda x. x \ (\lambda y. x \ \lambda y. y) \ x) \ \lambda y. y \ x$   
 $(\lambda x. \quad t \quad) \quad t'$

c.  $(\lambda x. y \ (\lambda y. \lambda z. x) \ x) \ \lambda x. y \ x$   
 $(\lambda x. \quad t \quad) \quad t'$

d.  $(\lambda y. x \ y \ \lambda z. y \ \lambda y. y) \ \lambda y. z \ \lambda z. x \ y \ x \ z$   
 $(\lambda y. \quad t \quad) \quad t'$

**Example 1:** for the expression

$$(\lambda x. (\lambda x. x) x) \lambda x. x \ x$$

$$(\lambda x. \quad t \quad) \quad t'$$

your answer should be

$$(\lambda x. x) (\lambda x. x \ x) \quad // \text{ beta reduction}$$

**Example 2:** for the expression

$$(\lambda x. (\lambda x. x) \lambda y. y \ x) \lambda x. y \ x$$

$$(\lambda x. \quad t \quad) \quad t'$$

your answer should be

$$(\lambda x. (\lambda x. x) \lambda w. w \ x) \lambda x. y \ x \quad // \text{ renaming}$$

$$(\lambda x. x) \lambda w. w \ (\lambda x. y \ x) \quad // \text{ beta reduction}$$