

## CSE340 Fall 2018 Homework 3

Due Tuesday October 2 2018 by 11:59 PM on Blackboard

**Note: your answers can be handwritten, but you need to have clear handwriting. If we cannot read it we cannot grade it!**

**Problem 1 (binding).** For each of the following determine for each variable  $x$  the  $\lambda x$ . it is bound to. I have numbered the variables and the abstractions. The variables are numbered using Arabic numerals and the abstractions are numbered using roman numerals. If variables 4 and 7 are bound to abstraction I, your answer should be of the form  $I \rightarrow 4, 7$  to indicate that abstraction I has variables 4, and 7 bound by it.

**Example**       $(\lambda x. x \ x \ \lambda x. x) \ x$   
                  I   1 2   II   3   4  
  
**Answer**        I  $\rightarrow$  1, 2  
                  II  $\rightarrow$  3

a.  $x \ \lambda x. (\lambda x. \lambda x. x \ x) \ x$   
      1   I   II   III   2   3   4

b.  $\lambda x. x \ \lambda x. x \ x$   
      I   1   II   2   3

c.  $\lambda x. (\lambda y. x) \ \lambda x. x \ x$   
      I   II   1   III   2   3

f.  $(\lambda x. (\lambda x. (\lambda x. x) \ x \ (\lambda x. x) \ x) \ (\lambda x. x \ x) \ (\lambda x. x) \ ) \ x$   
      I   II   III   1   2   IV   3   4   V   5   6   VI   7   8

**Problem 2 (reducible expressions).** In this problem you are asked to determine the reducible expressions in each of the following lambda expressions. The format of the answer is shown in the examples below

a.  $x \lambda x. (\lambda x. \lambda x. x \ x) x$

b.  $\lambda x. x \lambda x. x \ x$

c.  $\lambda x. (\lambda y. x) \lambda x. x \ x$

d.  $\lambda x. \lambda x. x \ x \ x$

e.  $(\lambda x. x \ x) (\lambda x. x) x$

f.  $(\lambda x. (\lambda x. (\lambda x. x) x (\lambda x. x) x) (\lambda x. x \ x) (\lambda x. x) ) x$

### Examples to show the format of the answer

**Example 1:** for the expressions

$$\lambda x. x \ x (\lambda x. x) x \lambda x. x \ x$$

Your answer should be that there are no reducible expressions.

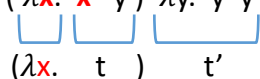
**Example 2:** for the expression

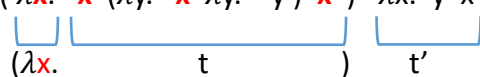
$$(\lambda x. (\lambda x. x) x) \lambda x. x \ x$$

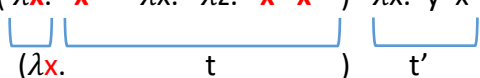
Your answer should be

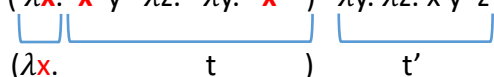
$$\begin{array}{l} \boxed{\lambda x. (\lambda x. x) x} \boxed{\lambda x. x \ x} \\ (\lambda x. \quad t \quad) \quad t' \\ (\lambda x. \boxed{\lambda x. x} \boxed{x}) \lambda x. x \ x \\ (\lambda x. t) t' \end{array}$$

**Problem 3 (Beta Reductions).** For each of the following expressions, give the resulting expression after executing a beta reduction of the highlighted redex (with renaming only if needed). You should not do renaming if it is not needed). Examples are shown below.

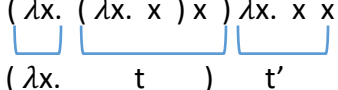
a.  $(\lambda x. x \ y) \ \lambda y. y \ y$   


b.  $(\lambda x. x \ (\lambda y. x \ \lambda y. y) \ x) \ \lambda x. y \ x$   


c.  $(\lambda x. x \ \lambda x. \lambda z. x \ x) \ \lambda x. y \ x$   


d.  $(\lambda x. x \ y \ \lambda z. \lambda y. x) \ \lambda y. \lambda z. x \ y \ z$   


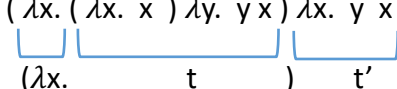
**Example 1:** for the expression

$$(\lambda x. (\lambda x. x) x) \lambda x. x \ x$$


your answer should be

$$(\lambda x. x) (\lambda x. x \ x) \quad // \text{ beta reduction}$$

**Example 2:** for the expression

$$(\lambda x. (\lambda x. x) \lambda y. y \ x) \lambda x. y \ x$$


your answer should be

$$(\lambda x. (\lambda x. x) \lambda w. w \ x) \lambda x. y \ x \quad // \text{ renaming}$$

$$(\lambda x. x) \lambda w. w (\lambda x. y \ x) \quad // \text{ beta reduction}$$

**Problem 4 (Call by Value).** For each of the following, identify the redex that should be reduced first **according to call by value** and give the resulting expression after one beta reduction. If there is more than one possibility, you should show all possibilities.

- a.  $(\lambda x. (\lambda z. \lambda y. x) x y z)$
- b.  $x (\lambda x. x \lambda x. \lambda y. x x) ((\lambda x. y) x) ((\lambda x. y) x)$
- c.  $((\lambda x. x) ((\lambda x. y) x)) ((\lambda x. y) x)$
- d.  $(\lambda x. (\lambda x. x) x (\lambda x. x) x) (\lambda x. y) x$

**Problem 5 (Normal Order of Evaluation).** For each of the following, identify the redex that should be reduced first **according to normal order evaluation** and give the resulting expression after one beta reduction.

- a.  $(\lambda x. x (\lambda z. \lambda y. x) x y)$
- b.  $(\lambda x. x \lambda x. \lambda y. x x) ((\lambda x. y) x) ((\lambda x. y) x)$
- c.  $\lambda x. ((\lambda x. y) x) ((\lambda x. y) x) (\lambda x. y z)$
- d.  $(\lambda x. (\lambda x. x) x (\lambda x. x) x) (\lambda x. y) x$

**Problem 6 (Calculation with Lambda Expressions).** For each of the following, write a lambda expression to calculate the function. You can use any of the functions I covered in class or in the slides, but you should not use recursion for any of the functions. For all questions the functions are for numbers represented as Church numerals.

a.  $\text{ONETRUE} = (\lambda a. \lambda b. \lambda c. \dots)$  should calculate  $\text{ONETRUE } a \ b \ c$  defined as follows

1.  $\text{ONETRUE } \text{fls } \text{fls } \text{fls} = \text{fls}$
2.  $\text{ONETRUE } \text{fls } \text{fls } \text{tru} = \text{tru}$
3.  $\text{ONETRUE } \text{fls } \text{tru } \text{fls} = \text{tru}$
4.  $\text{ONETRUE } \text{fls } \text{tru } \text{tru} = \text{fls}$
5.  $\text{ONETRUE } \text{tru } \text{fls } \text{fls} = \text{tru}$
6.  $\text{ONETRUE } \text{tru } \text{fls } \text{tru} = \text{fls}$
7.  $\text{ONETRUE } \text{tru } \text{tru } \text{fls} = \text{fls}$
8.  $\text{ONETRUE } \text{tru } \text{tru } \text{tru} = \text{fls}$

b.  $\text{sum\_products} = (\lambda n. \dots)$  should calculate  $1*2 + 3*4 + 5*6 + n*(n+1)$

c.  $\text{DIV}_3 =$  integer division by 3.

**Examples**

$\text{DIV}_3$	2	= 0
$\text{DIV}_3$	5	= 1
$\text{MOD}_3$	6	= 0
$\text{MOD}_3$	7	= 1

e. (Bonus) Can you generalize this for integer division by n?