CSE340 Spring 2019
Homework 3 Solution

Problem 1 (Operator Precedence Parsing).
Consider the following operator grammar

E → E & E
E → ~ E
E → (E) | id

& is a left associative binary operator
~ is a unary operator and has higher precedence than &

The "yields precedence to" and "takes precedence over" relationship between (, id, and operators is given on page 7 of the operator precedence parsing notes. There relationships are highlighted in pink below.

& is a binary operator that is left associative, so we have & $\cdot\!>$ & (highlighted in blue below)

~ is a unary operator, on page 77 of the operator precedence parsing notes, we have op $<\!\cdot$ uop relationship between every operator op and every unary operator uop. So, we have the relationships highlighted in green below.

Finally, ~ is a unary operator that has higher precedence level than &, so we have the entry highlighted in yellow below.

1. Draw the precedence table for this grammar

	&	**~**	**(**	**)**	**Id**	**$**
&	$\cdot\!>$	$<\!\cdot$	$<\!\cdot$	$\cdot\!>$	$<\!\cdot$	$\cdot\!>$
~	$\cdot\!>$	$<\!\cdot$	$<\!\cdot$	$\cdot\!>$	$<\!\cdot$	$\cdot\!>$
(	$<\!\cdot$	$<\!\cdot$	$<\!\cdot$	$\doteq$	$<\!\cdot$	
)	$\cdot\!>$	$\cdot\!>$		$\cdot\!>$		$\cdot\!>$
Id	$\cdot\!>$	$\cdot\!>$		$\cdot\!>$		$\cdot\!>$
$	$<\!\cdot$	$<\!\cdot$	$<\!\cdot$		$<\!\cdot$	

2. Show step by step how ~a&b is parsed.

Stack	**Input**	**Action**	**Justification**
$	~a&b	Shift	$ ⋖ ~
$~	a&b	Shift	~ ⋖ id (a)
$~a	&b	Reduce E ➝ id	id (a) ⋗ &
$~E	&b	Reduce E ➝ ~E	~ ⋗ &
$E	&b	Shift	$ ⋖ &
$E&	b	Shift	& ⋖ id (b)
$E&b	$	Reduce E ➝ id	id (b) ⋗ $
$E&E	$	Reduce E ➝ E & E	& ⋗ $
$E	$	(return)	$ on stack and next token is $

Problem 2 (binding).

a. x (λx. x x) x

1 I 2 3 4

Answer:

I → 2, 3

b. x λx. x λx. x λx. x

1 I 2 II 3 III 4

Answer:

I → 2

II → 3

III → 4

c. λx. x (λx. x) (λy. x) y

I 1 II 2 III 3 4

Answer:

I → 1, 3

II → 2

d. λx. (λy. x) λx. x x

I II 1 III 2 3

Answer:

I → 1

II → No variables are bound to this

III → 2, 3

e. λx. (λx. (λx. x x (λx. x) x) (λx. x x) (λx. x)) x

I II III 1 2 IV 3 4 V 5 6 VI 7 8

Answer:

I → 8

II → No variables are bound to this.

III → 1 , 2, 4

IV → 3

V → 5, 6

VI → 7

Problem 2 (reducible expressions). In this problem you are asked to determine the reducible expressions in each of the following lambda expressions. The format of the answer is shown in the examples below

a. $x\ \lambda x.\ (\ \lambda x.\ \lambda x.\ x)\ x\ x$

Answer:

$$x\ (\lambda x.\ (\ (\underbrace{\lambda x.}_{(\ \lambda x.}\ \underbrace{(\lambda x.\ \ x)}_{t}\ \underset{)}{)}\ \ \underbrace{x}_{t'})\ \ x)$$

b. $(\lambda x.\ x)\ \lambda x.\ x\ x\ (\lambda x.\ x)$

Answer:

$$(\ \underbrace{\lambda x.}_{(\ \lambda x.}\ \ \underbrace{x}_{t\ )}\ )\ \underbrace{(\ \lambda x.\ (\ (\ x\ \ x\ )\ (\lambda x.\ \ x\ )\ )\ x\ )}_{t'}$$

c. $\lambda x.\ (\lambda y.\ x)\ \lambda x.\ \ (\lambda x.\ x)\ x$

Answer:

$$\lambda x.\ (\ (\ \lambda y.\ \ x\ )\ (\lambda x.\ (\ (\ \underbrace{\lambda x.}_{(\lambda x.}\ \ \underbrace{x}_{t\ )}\ )\ \underbrace{x}_{t'}\ )\ )\ )$$

$$\lambda x.\ (\ (\ \underbrace{\lambda y.}_{(\lambda x.}\ \ \underbrace{x}_{t)})\ \ \underbrace{(\ \lambda x.\ (\ \lambda x.\ \ x\ \ x\ )\ )}_{t'}\ )$$

d. $\lambda x.\ \lambda x.\ x\ \lambda x.\ x\ x$

Answer:

No reducible expression. Remember: an expression with no parentheses cannot have a redex

e. $(\lambda x.\ x\ )x\ \ (\lambda x.\ x\ )\ x$

Answer:

$$(\ (\ (\ \underbrace{\lambda x.}_{(\lambda x.}\ \ \underbrace{x}_{t\ )}\ )\ \underbrace{x}_{t'}\ )\ \ (\lambda x.\ x\ )\ )\ x$$

f. $(\lambda x.\ x\ x\ )\ (\lambda x.\ x\ )\ (\lambda x.\ (\lambda x.\ (\lambda x.\ x\ )\ x\ (\lambda x.\ x\ )\ x\ )\ x\ )$

Answer:

$$(\ (\underbrace{\lambda x.}_{(\lambda x.}\ \underbrace{x\ x}_{t\ \ )}\ )\ \underbrace{(\lambda x.\ x\ )}_{t'}\ )\ (\ \lambda x.\ (\lambda x.\ (\ (\underbrace{\lambda x.}_{(\lambda x.}\ \underbrace{x}_{t\ )}\ )\ \underbrace{x}_{t'}\ )\ (\lambda x.\ x\ )\ x\ )\ x\ )$$

$$(\ (\lambda x.\ x\ x\ )\ (\lambda x.\ x\ )\ )\ (\lambda x.\ (\ (\underbrace{\lambda x.}_{(\lambda x.}\ \underbrace{(\lambda x.\ x\ )\ x\ (\lambda x.\ x\ )\ x}_{t}\ \underset{)}{)}\ \underbrace{x}_{t'}\ )\ )$$

Problem 3 (Beta Reductions). For each of the following, give the resulting expression after executing a beta reduction (with renaming only if needed) of the highlighted regex

a. $(\underbrace{\lambda x.}_{(\lambda x.}\ \underbrace{x\ y\ x}_{t)})\ \underbrace{\lambda y.\ y\ y}_{t'} \xrightarrow{\beta\text{-reduction}} (\lambda y.\ y\ y)\ y\ (\lambda y.\ y\ y)$

Note: We should add parentheses when we do the beta reduction

b. $(\underbrace{\lambda x.}_{(\lambda x.}\ \underbrace{x\ (\lambda y.\ x\ \lambda y.\ y\ )\ x}_{t}\ \underset{)}{)}\ \underbrace{\lambda y.\ y\ x}_{t'} \xrightarrow{\beta\text{-reduction}} (\lambda y.\ y\ x\ )\ (\lambda y\ (\lambda y.\ y\ x\ )\ \lambda y.\ y\ )\ (\lambda y.\ y\ x)$

Note: Renaming is not needed because y does not change binding and x remains free.
Also, we should add parentheses when we do the beta reduction.

c. $(\underbrace{\lambda x.}_{(\lambda x.}\ \underbrace{y\ (\lambda y.\ \lambda z.\ x)\ x}_{t)})\ \underbrace{\lambda x.\ y\ x}_{t'} \xrightarrow{\alpha\text{-renaming}} (\lambda x.\ y\ (\lambda w.\ \lambda z.\ x)\ x\ )\ \lambda x.\ y\ x$

$\xrightarrow{\beta\text{-reduction}} (y\ (\lambda w.\ \lambda z.\ (\lambda x.\ y\ x))\ (\lambda x.\ y\ x\ )$

Note: Renaming is needed because y, which is free, will become bound if we do not do renaming before the beta reduction. Also, note that we should add parentheses when we do the beta reduction.

d. $(\underbrace{\lambda y.}_{(\lambda y.}\ \underbrace{x\ y\ \lambda z.\ y\ \lambda y.\ y}_{t}\ \underset{)}{)}\ \underbrace{\lambda y.z\ \lambda z.\ x\ y\ x\ z}_{t'} \xrightarrow{\alpha\text{-renaming}} (\lambda y.\ x\ y\ \lambda w.\ y\ \lambda y.\ y\ )\ \lambda y.z\ \lambda z.\ x\ y\ x\ z$

$\xrightarrow{\beta\text{-reduction}} (x\ (\lambda y.z\ \lambda z.\ x\ y\ x\ z)\ \lambda w.\ (\lambda y.z\ \lambda z.\ x\ y\ x\ z)\ (\lambda y.\ y))$

Note: Renaming is needed because the first **z** in $\lambda y.\mathbf{z}\ \lambda z.\ x\ y\ x\ z$, which is free, will become bound if we do not do renaming before the beta reduction. Also, note that we should add parentheses when we do the beta reduction.