Due: Monday October 26, 2020 by 11:59 PM on Canvas

All submissions should be typed, no exceptions.

1. **(Lambda calculus: binding)**. For each of the following expressions. determine for each variable x the λx. it is bound to. I have numbered the variables and the abstractions. The variables are numbered using Arabic numerals and the abstractions are numbered using roman numerals and you should specify which variable numbers are bound to which abstraction number. For example, if variables 4 and 7 are bound to abstraction I, your answer should be of the form I → 4 7 to indicate that abstraction I has variables 4, and 7 bound by it. If variable 3 is free, then your answer should have the form free → 3.

Example	(λy. x y λx. y) x
	I 1 2 II 3 4

Answer	free → 1 4
	I → 2 3
	II →

Your answers need not be colored

1.1. λy. x z
        I   1 2

1.2. λy. x y (λy. y) (λz. y)
        I   1 2 II   3   III 4

1.3. (λy. x z λx. y (λy. x y λx. y)   y) x
        I   1 2 II   3   III   4 5 IV   6   7 8

1.4. ((λx. x λx. x) x (λx. x x λx. x) x) x
        I   1 II   2   3 III   4 5 IV   6   7 8

1.5. λx. (λx. λx. x λx. x (λx. x x) λx. x) x x
        I   II III 1 IV   2   V   3 4 VI   5   6 7

2. **(Reducible Expressions)** For each of the following, identify all the reducible expressions by highlighting the λx. , the t and the t' of the reducible expressions. If there is more than one redex in the given expression, you should identify all the redexes, each one on a separate line. If there is no reducible expression, you should say so in your answer.

Example 1	(λx. λx. x x)
Answer	no redex

Example 2	((λx. x x)(λx. x x))((λx. x x)(λx. x x))
Answer	((λx. x x)(λx. x x))((λx. x x)(λx. x x))
	((λx. x x)(λx. x x))((λx. x x)(λx. x x))

2.1. (λx. x (λx. x) x) (x x) (λx. x x) x
2.2.   x (λx. (λx. x)   x) x
2.3. (λx. (λx. (λx. x) y) z) λx. w (λx. x) x
2.4. λz. (λx. (λy. z y) x) y
2.5. λx. (λy. (λz. x y) x) λx. (λx. w) (y z)
2.6. (λx. (λy. (λz. x y) x)) y z

3. (**Alpha Renaming**) For each of the following redexes, do alpha renaming if it is needed. If renaming is not needed, you should say so. <u>You should highlight the renamed variable(s) in your answer</u>. You should not do beta reduction.

    Example 1   (λx. x x) x
    Answer   no renaming needed

    Example 2   (λx. λy. λz. x y λw. z) (x y z w)
    Answer   (λx. λu. λv. x u λw. v) (x y z w)

    Example 3   (λx. λy. λz. x y λw. z) (x z) (z w)
    Answer   (λx. λy. λv. x y λw. v) (x z) (z w)

  3.1. (λx. λy. λz. y z) (y z)
  3.2. (λx. λy. x λz. y z) (y z)
  3.3. (λx. λy. x λx. λz. x y z) (y z)
  3.4. (λx. λy. x λx. λz. x y z) (λy. y z)
  3.5. (λy. x λz. x y z) (λy. y z)
  3.6. (λx. λy. x λx. λz. x y z) (y z)

4. (**Beta reductions**) For each of the following expressions, identify the redexes and, for each redex, do <u>only one</u> beta reduction step to reduce the redex. If renaming is needed you should do renaming first before doing the beta reduction.

  // The examples below clearly show that you should answer by highlighting the redexes and also highlight the result of the
  // reduction (and the renaming if applicable). You should follow the same format as the examples below

    Example 1   (λx. x x) x
    Answer   (λx. x x) x →^{β} x x

    Example 2   (λx. λy. λz. x y λw. z) (x y z w)
    Answer   (λx. λy. λz. x y λw. z) (x y z w) →^{α} (λx. λu. λv. x u λw. v) (x y z w)
               (λx. λu. λv. x u λw. v) (x y z w) →^{β} λu. λv. (x y z w) u λw. v

    Example 3   (λx. (λy. x) y) z
    Answer   // This expression has two redexes you should show one beta reduction for each redex separately
               (λx. (λy. x) y) z →^{β} (λy. z) y   // beta reduction for first redex
               (λx. (λy. x) y) z →^{β} (λx. x) z   // beta reduction for second redex

  4.1. (λx. (λy. λz. y) z) (y z)
  4.2. (λx. (λy. x) (λz. y z)) (y z)
  4.3. (λx. λy. x λx. λz. x y z) (y z)
  4.4. (λx. λx. x λx. λz. x y z) ((λy. y z) z)
  4.5. (λy. y λz. x y z) (λy. (λy. y) z)
  4.6. (λx. λy. x λx. λz. x y z) (y z)

    // 4.3 and 4.6 are the same so you better have the same answers for them. If you have different answers for 4.3 and
    // 4.6, you will not get credit for them

5. (**Call by value**) repeat problem 5 but do beta reductions according to call by value.

  // This is asking you to repeat problem 4. The answer to this question should have the same format as the answer to problem 4.

6. (**Normal Order**) repeat problem 5 but do beta reductions according to normal order evaluation

  // This is asking you to repeat problem 4. The answer to this question should have the same format as the answer to problem 4.

7. (**Evaluating Expressions**) Evaluate each of the following expressions

  7.1. tru fls tru tru fls
  7.2. 2 (plus 2) 3
  7.3. 2 fst (pair (pair 4 5) (pair (1 3)))   // this had a missing parenthesis that I added
  7.4. 2 fst (pair (pair (plus 2) 5) (pair 1 3)) 5   // this had an extra parenthesis that I removed and a missing "pair" that I added
  7.5. snd (pair (plus 2) (plus 3)) (tru 4 5)

8. (**Operator precedence parsing**) No question on this homework assignment. I will ask about it in the next homework assignment, closer to the second exam!