Wednesday, October 6, 2021 11:56 AM

last time.

- Grammar for λ-calculus
- Disambiguation rules
  .. not quite finished

Today

- Disambiguation rules (syntax)
- Bound and free variable (semantics)
- Reducible expressions (Syntax)
- β-reductions

Reminder.

$$t \rightarrow x$$
$$t \rightarrow \lambda x.\, t \quad // \text{ abstraction}$$
$$t \rightarrow t \ t \quad // \text{ application}$$
$$t \rightarrow (t)$$

The grammar is ambiguous.

Two disambiguation rules.

1. Abstractions extend as far to the right as possible without crossing a right parentheses that is part of a pair of matching parentheses enclosing the $\lambda x.$ of the abstraction.

of the abstraction.

2. Application is left associative

Example $x \; (x \; \lambda x. \; x \; (\lambda x. \; x \; x) \; x) \; x$

$(((a \;\; b) \;\; c) \;\; d)$

Parsing General Expressions

1. Identify the bodies of all abstractions
2. If an abstraction does not have parentheses around it, add parentheses
3. Within the body of each abstraction group terms using left associative grouping. Treat any terms within parentheses as one term.
4. Within any pair of parentheses, group terms using left associative grouping
5. Outside all abstractions and parentheses

5. Outside all abstractions and parentheses group terms using left associative grouping.

Example

$$(\lambda x. (x (\lambda x. ((x (\lambda x. x)) (\lambda x. ((x\ x) (\lambda x. x)))))))$$

$$(((a\ c)\ b)\ d)$$

$$(((\lambda x. x) (\lambda x. x)) (\lambda x. x))$$

$$(\lambda x. x (\lambda x. x (\lambda x. x)))$$

$$((x (\lambda x. x))\ x)$$

$$((x\ x) (\lambda x. x))$$

$$(((x ((x\ x) (\lambda x. ((x (\lambda x. x))\ x))))\ x)\ x)$$

Bound and free variables (semantics)

Bound an free variables (semantics)

Aside

Syntax vs semantics

```
int x;          int x;
int y;    vs.   int y;

x = z;          x = y;
```

syntax correct, but not semantics

End Aside

AFTER WE GROUP THE TERM, we determine bound and free variables

x is bound to λx if:

same name

- x is in the body of the abstraction on λx.

and

- $\lambda x.$ is the closest $\lambda x.$ to the left of x in whose body x appears

If x is not bound to any λx, it is free

Example 1.

$$\underset{\mathrm{I}}{\lambda x}.\ \underset{1}{x}\ (\underset{\mathrm{II}}{\lambda x}.\ \underset{2}{x}\ (\underset{\mathrm{III}}{\lambda x}.\ \underset{3}{x}\ \underset{4}{x}))\ \underset{5}{x}\ (\underset{\mathrm{IV}}{\lambda x}.\ \underset{6}{x})\ \underset{7}{x}$$

I 1 II 2 III 3 4 5 IV 6 7

four = 1 + 1 + 1 + 1

not seven

```
{ int i; // 1
  i = 2;
  { int i; // 2
    i = 3;
  }
}
```

Example 2

$x \ (\lambda x.\ x \ x) \ x$

free

free

Example 3.

$\lambda x.\ y \ \lambda y.\ x$

free

```
{ int i;
  j = 5;
  { int j;
    i = 2;
  }
}
```

Example 4.

free

$$\lambda x.\ x\ (\lambda y.\ x\ y)\ y\ x$$

Example 5.

$$(\underset{\mathrm{I}}{\lambda x}.\ y\ (\underset{\mathrm{II}}{\lambda x}.\ y\ \underset{\mathrm{III}}{\lambda y}.\ x\ y)\ x\ \underset{\mathrm{IV}}{\lambda z}.\ x)\ x$$

free

free

<u>Recap</u> So far

- Grammer (Syntax)
- Disambiguation rules (Syntax)
- bound and free variables (semantics)

<u>Next.</u>
- Reducible Expressions (Syntax)
- β - reductions (Semantics)

<u>DEFINITION.</u> A Reducible Expression, also called redex is a term of the

also called redex, is a term of the form $(\lambda x. t)\ t'$

where t and t' are terms.

Examples.

1. $(\lambda x.(x\ \ x))$ NO redex

2. $((x\ \ (\lambda x.\, x))\, x)$ No redex

3. $(x\ \ (\lambda x.(x\ \ (\lambda x.\, x))))$ NO REDEX

$$((\lambda x.\ t)\ t')$$

4. $((\lambda x.\, y)\ \ z)$ One redex

5. $$(((\underbrace{(\lambda x.\, x)\ \ (\lambda x.\, x)}_{\text{Redex}})\ (\lambda y.\, y))(\lambda y.\, y))$$

$$(((a \quad b) \quad c) \quad d)$$

$$(((\lambda x.\ x \quad x) \quad (x \quad y)) \quad x)$$

$$(\lambda x.\ (x \quad x)) \quad (\lambda x.\ x \quad y \quad (\lambda x \quad x))$$

$$((\lambda x.\ (t)) \quad (t'))$$

$$(((\lambda x.\ x) \quad x) \quad x)$$

$$(((\lambda x.\ (((\lambda x.\ x) \quad x) \quad x)) \quad x) \quad (\lambda x.\ x)) \quad x)$$

two redexes

$$((\lambda x.\ (x) \quad (\lambda x.\ x)) \quad (\lambda y.\ y))$$