Arizona State University
Computer Science and Engineering
CSE 340 - Fall 2018

Homework 2

Due Monday September 24 2018 by 11:59 PM

Please read the problems completely before asking questions.

1. **FIRST and FOLLOW** Calculate the FIRST and FOLLOW sets for the following grammar.

$$
\begin{array}{lll}
S & \rightarrow A\,B\,S & (1)\\
S & \rightarrow C\,S\,A & (2)\\
S & \rightarrow C\,B & (3)\\
A & \rightarrow A\,a & (4)\\
A & \rightarrow C\,S & (5)\\
C & \rightarrow c\,C & (6)\\
C & \rightarrow B & (7)\\
B & \rightarrow B\,b & (8)\\
B & \rightarrow \epsilon & (9)
\end{array}
$$

   - For FIRST sets
     (a) Do an initialization pass by applying FIRST sets rules I and II.
     (b) Do successive passes, on the grammar rules in the order they are listed and apply to each grammar rule FIRST set rules III, then FIRST set rule IV then FIRST set rule V until there is no change in any of the FIRST sets.

     In your answer, you should show the FIRST sets after each pass. You do not have to use the notation I used in class to indicate the order in which the elements are added to the sets.
   - You should calculate FOLLOW sets as follows:
     (a) Do an initialization pass by applying FOLLOW sets rule I
     (b) Do one pass on all grammar rules, in the order they are listed, and apply to each grammar rule FOLLOW set rules IV and V.
     (c) Do successive passes on all grammar rules in the order they are listed and apply to each grammar rule FOLLOW set rules II and III until there is no change in any of the FOLLOW sets.

   In your answer, you should show the FOLLOW sets after each pass. You do not have to use the notation I used in class to indicate the order in which the elements are added to the sets.

Solution.

- FIRST sets
  (a) Do an initialization pass by applying FIRST sets rules I and II. We get:
  $$\begin{aligned}
  \text{FIRST}(\epsilon) &= \{\,\epsilon\,\} \\
  \text{FIRST}(a) &= \{\,a\,\} \\
  \text{FIRST}(b) &= \{\,b\,\} \\
  \text{FIRST}(c) &= \{\,c\,\} \\
  \text{FIRST}(S) &= \{\ \} \\
  \text{FIRST}(A) &= \{\ \} \\
  \text{FIRST}(B) &= \{\ \} \\
  \text{FIRST}(C) &= \{\ \}
  \end{aligned}$$
  (b) Do successive passes, on the grammar rules in the order they are listed and apply to each grammar rule FIRST set rules III, then FIRST set rule IV then FIRST set rule V until there is no change in any of the FIRST sets.
    i. At the end of the first pass we get the following:
    $$\begin{aligned}
    \text{FIRST}(\epsilon) &= \{\,\epsilon\,\} \\
    \text{FIRST}(a) &= \{\,a\,\} \\
    \text{FIRST}(b) &= \{\,b\,\} \\
    \text{FIRST}(c) &= \{\,c\,\} \\
    \text{FIRST}(S) &= \{\ \} \\
    \text{FIRST}(A) &= \{\ \} \\
    \text{FIRST}(B) &= \{\,\epsilon\,\} \\
    \text{FIRST}(C) &= \{\,c\,\}
    \end{aligned}$$
    ii. At the end of the second pass we get the following:
    $$\begin{aligned}
    \text{FIRST}(\epsilon) &= \{\,\epsilon\,\} \\
    \text{FIRST}(a) &= \{\,a\,\} \\
    \text{FIRST}(b) &= \{\,b\,\} \\
    \text{FIRST}(c) &= \{\,c\,\} \\
    \text{FIRST}(S) &= \{\,c\,\} \\
    \text{FIRST}(A) &= \{\,c\,\} \\
    \text{FIRST}(B) &= \{\,\epsilon\,,\,b\,\} \\
    \text{FIRST}(C) &= \{\,c\,,\,\epsilon\,\}
    \end{aligned}$$
    iii. At the end of the third pass we get the following:
    $$\begin{aligned}
    \text{FIRST}(\epsilon) &= \{\,\epsilon\,\} \\
    \text{FIRST}(a) &= \{\,a\,\} \\
    \text{FIRST}(b) &= \{\,b\,\} \\
    \text{FIRST}(c) &= \{\,c\,\} \\
    \text{FIRST}(S) &= \{\,c\,,\,\epsilon\,\} \\
    \text{FIRST}(A) &= \{\,c\,,\,\epsilon\,\} \\
    \text{FIRST}(B) &= \{\,\epsilon\,,\,b\,\} \\
    \text{FIRST}(C) &= \{\,c\,,\,\epsilon\,,\,b\,\}
    \end{aligned}$$

iv. At the end of the fourth pass we get the following:

$$\begin{aligned}
\text{FIRST}(\epsilon) &= \{\, \epsilon \,\} \\
\text{FIRST}(a) &= \{\, a \,\} \\
\text{FIRST}(b) &= \{\, b \,\} \\
\text{FIRST}(c) &= \{\, c \,\} \\
\text{FIRST}(S) &= \{\, c \,,\, \epsilon,\, b \,\} \\
\text{FIRST}(A) &= \{\, c \,,\, \epsilon,\, a \,,\, b\} \\
\text{FIRST}(B) &= \{\, \epsilon \,,\, b \,\} \\
\text{FIRST}(C) &= \{\, c \,,\, \epsilon \,,\, b \,\}
\end{aligned}$$

v. At the end of the fifth pass we get the following:

$$\begin{aligned}
\text{FIRST}(\epsilon) &= \{\, \epsilon \,\} \\
\text{FIRST}(a) &= \{\, a \,\} \\
\text{FIRST}(b) &= \{\, b \,\} \\
\text{FIRST}(c) &= \{\, c \,\} \\
\text{FIRST}(S) &= \{\, c \,,\, \epsilon,\, b \,,\, a\} \\
\text{FIRST}(A) &= \{\, c \,,\, \epsilon,\, a \,,\, b\} \\
\text{FIRST}(B) &= \{\, \epsilon \,,\, b \,\} \\
\text{FIRST}(C) &= \{\, c \,,\, \epsilon \,,\, b \,\}
\end{aligned}$$

vi. At the end of the sixth pass, there is no change and we stop.

- FOLLOW sets:

(a) Do an initialization pass by applying FOLLOW sets rule I

$$\begin{aligned}
\text{FOLLOW}(S) &= \{\, \$ \,\} \\
\text{FOLLOW}(A) &= \{\ \} \\
\text{FOLLOW}(B) &= \{\ \} \\
\text{FOLLOW}(C) &= \{\ \}
\end{aligned}$$

(b) Do one pass on all grammar rules, in the order they are listed, and apply to each grammar rule FOLLOW set rules IV and V. At the end of this pass we get the following:

$$\begin{aligned}
\text{FOLLOW}(S) &= \{\, \$\, c \,,\, a \,,\, b\} \\
\text{FOLLOW}(A) &= \{\, b \,,\, c \,,\, a \,\} \\
\text{FOLLOW}(B) &= \{\, c \,,\, b \,,\, a \,\} \\
\text{FOLLOW}(C) &= \{\, c \,,\, b \,,\, a \,\}
\end{aligned}$$

(c) Do successive passes on all grammar rules in the order they are listed and apply to each grammar rule FOLLOW set rules II and III until there is no change in any of the FOLLOW sets.

i. At the end of the first pass we get

$$\begin{aligned}
\text{FOLLOW}(S) &= \{\, \$\, c \,,\, a \,,\, b\} \\
\text{FOLLOW}(A) &= \{\, b \,,\, c \,,\, a \,,\, \$ \,\} \\
\text{FOLLOW}(B) &= \{\, c \,,\, b \,,\, a \,,\, \$\} \\
\text{FOLLOW}(C) &= \{\, c \,,\, b \,,\, a \,,\, \$ \,\}
\end{aligned}$$

(d) At the end of the second pass there is no change and we stop

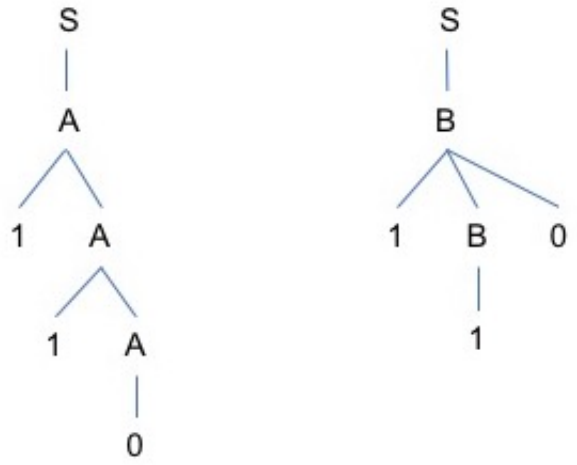
□

2. **Ambiguity**.

   Show that the following grammar is ambiguous

$$
\begin{array}{lll}
S & \to A & (1) \\
S & \to B & (2) \\
A & \to 0\,A\,1 & (3) \\
A & \to 1\,A & (4) \\
A & \to 0 & (5) \\
B & \to 1\,B\,0 & (6) \\
B & \to 0\,B & (7) \\
B & \to 1 & (8)
\end{array}
$$

   (a) Show that the following grammar is ambiguous by giving two parse trees for some input. The following are two different parse tress for the string 01



   (b) Show that the following grammar is ambiguous by giving two leftmost derivations for some input (you can use the same input as in part 1). The following are two leftmost derivations for the input 01

      i. $S \overset{(2)}{\Rightarrow} B \overset{(6)}{\Rightarrow} 1\,B\,0 \overset{(8)}{\Rightarrow} 1\,1\,B$

      ii. $S \overset{(1)}{\Rightarrow} A \overset{(4)}{\Rightarrow} 1\,A \overset{(4)}{\Rightarrow} 1\,1\,A \overset{(5)}{\Rightarrow} 1\,1\,0$

3. **Parsing**. Consider the grammar

$$
\begin{array}{lll}
S & \rightarrow A\,B\,C & \\
A & \rightarrow a\,A\,B\,d & \mid \quad e\,B\,d \\
B & \rightarrow c\,B\,d & \mid \quad \epsilon \\
C & \rightarrow e\,C & \mid \quad \epsilon
\end{array}
$$

   (a) Show that the grammar has a predictive recursive descent parser

   (b) Write the parser for the grammar. **Your parser should detect syntax error as soon as possible by checking for FOLLOW set when reducing to** ϵ

   (c) Give a full execution trace for your parser from part (a) above on input $e\,d\,c\,d$.

4. **Useless Symbols**. Consider the following grammar G

$$
\begin{array}{lllllll}
S & \rightarrow & A\,C & \mid & A\,E & \mid & A\,D \\
A & \rightarrow & a\,A & \mid & B & \mid & a \\
B & \rightarrow & b\,D & & & & \\
C & \rightarrow & c\,B & \mid & A & & \\
D & \rightarrow & d\,E & \mid & d\,B & & \\
D & \rightarrow & a\,D & \mid & c\,B & &
\end{array}
$$

   (a) What are the generating symbols of this grammar?

   (b) What are the reachable symbols of this grammar?

   (c) What are the useless symbols of the grammar (note that this part has to be done by first eliminating non-generating symbols from G to obtain a new grammar G' in which all symbols are generating, then eliminating non-reachable symbols from G' to obtain the answer)?

   Explain!