Wednesday, September 29, 2021 12:37 PM

Lambda Calculus.

- Syntax : how to parse λ expressions
- Semantics: how to evaluate λ expressions

Syntax. Context-free grammer

$t \longrightarrow x$ // x is a variable name // from a set of names

(t ↑ term)

$t \longrightarrow \lambda x.\ t$ // You can think of this // as function of x // whose code is t // t is the body of the abstraction

($\lambda x.\ t$: abstraction)

$t \longrightarrow t\ t$ // apply t to t

($t\ t$: application)

$t \longrightarrow (\ t\ )$ // parentheses for // grouping

this grammer is ambiguous.

1. Identifying the body of abstraction $t \rightarrow \lambda x.\ t$

Example $\lambda x.\ x\ x$

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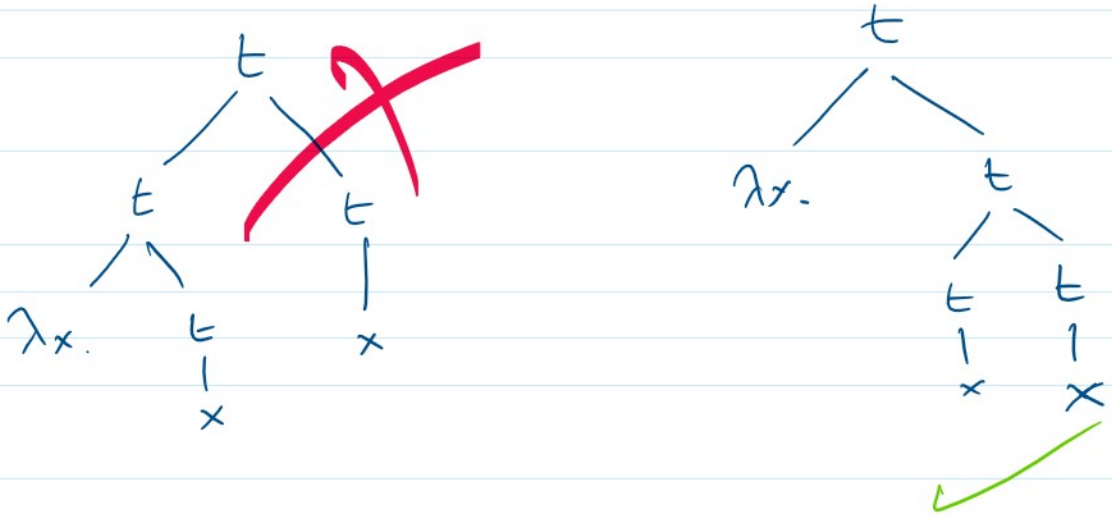
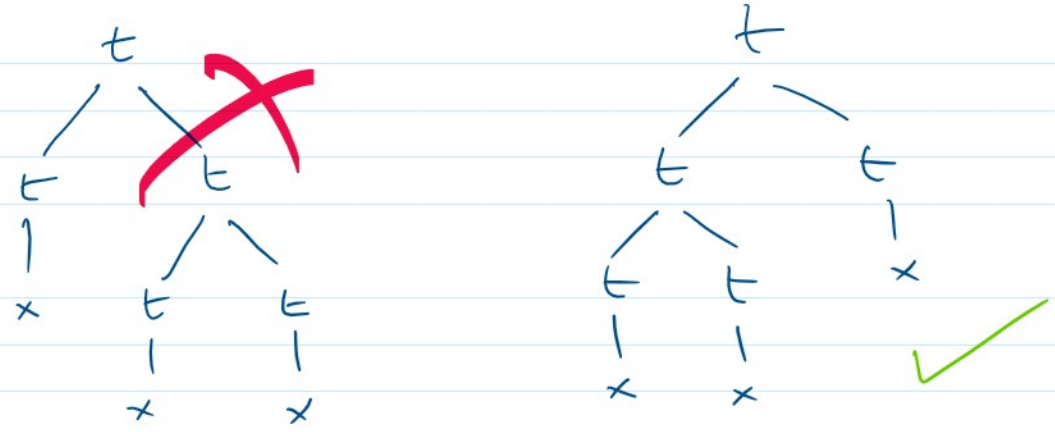


2. Grouping terms for the rule $t \to t\ t$

Example. $x\ x\ x$



danger

left associative

Side discussion: $(((1 - 2) - 3) - 4)$

-1

-4

$$\underline{1 - 2} - \underline{3 - 4}$$
$$-1 \qquad -1 = 0$$

Syntax: Disambiguation rules

1. Abstraction extends as far to the right as possible without crossing enclosing parentheses. . .
2. Application is left associative.

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1. Abstraction extends as far to the right as possible without crossing a right parentheses that is part of a pair of matching parentheses enclosing the $\lambda x.$ of the abstraction.

The goal is to determine the body of the abstraction

Example.

1. $x \; x \; x \; \lambda x. \; x \; ((\lambda x. \; x \; (\lambda x. \; x \; \lambda x. \; x) \; x) \; x) \; x$

2. $\lambda x.\ x\ (\lambda x.\ x\ (\lambda x.\ x\ \lambda x.\ x)\ x)\ x$

3. $x\ (x\ x\ \lambda x.\ x\ (\lambda x.\ x\ (\lambda x.\ x\ \lambda x.\ x)\ x)\ x)\ x$

.

.

6. $\lambda x.\ x\ (x\ \lambda x.\ x\ x)\ x$

Grouping applications.

2. Application is left associative

· if a term has the form

$t_1\ t_2\ t_3$

where t_1, t_2 and t_3 have already

[illegible] Then we

where t_1, t_2 ... have been determined to be terms, then we group them as follows

$$((t_1 \; t_2) \; t_3)$$

Example. $(\lambda x.\, x)(\lambda x\, x)(\lambda x.\, x)$ ✗

$$\lambda x.(x \quad (\lambda x.(x \quad (\lambda x.\, x))))$$

Parsing General Expressions

1. Identify the bodies of all abstractions
2. If an abstraction does not have parentheses around it, add parentheses around it.
3.
4.
5.