Economics Notes

Advanced Macroeconomics

by David Romer

Author:
Chen

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Contents

1 Solow Growth Model

1.1 Setup

Given the assumption of constant returns to scale, the production function is $Y(t) = F(K(t), A(t)L(t))$ or alternatively in intensive form $y(t) = f(k(t))$ in which $y = Y/(AL)$ and $k = K/(AL)$. $f(k)$ is assumed to satisfy $f(0) = 0, f'(k) > 0, f"(0) < 0$ and the Inada conditions: $\lim_{k\to 0} f'(k) = \infty, \lim_{k\to\infty} f'(k) = 0$. The evolution of the inputs into production are determined by

$$\begin{aligned}
\dot{L}(t) &= nL(t),\\
\dot{A}(t) &= gA(t),\\
\dot{K}(t) &= sY(t) - \delta K(t).
\end{aligned}$$

These equations yield solution as follows

$$\begin{aligned}
L(t) &= L(0)\mathrm{e}^{nt}\\
A(t) &= A(0)\mathrm{e}^{gt}.
\end{aligned}$$

Labor and knowledge grow at constant rates n and g respectively. Since the production function $F(K, AL)$ is not specified, we cannot give an explicit solution of $K(t)$.

1.2 Stable Solution

For the sake of qualitative analysis, the system of differential equations can be simplified to a single differential equation with respect to $k(t)$:

$$\dot{k}(t) = sf(k) - (n + g + \delta)k(t).$$

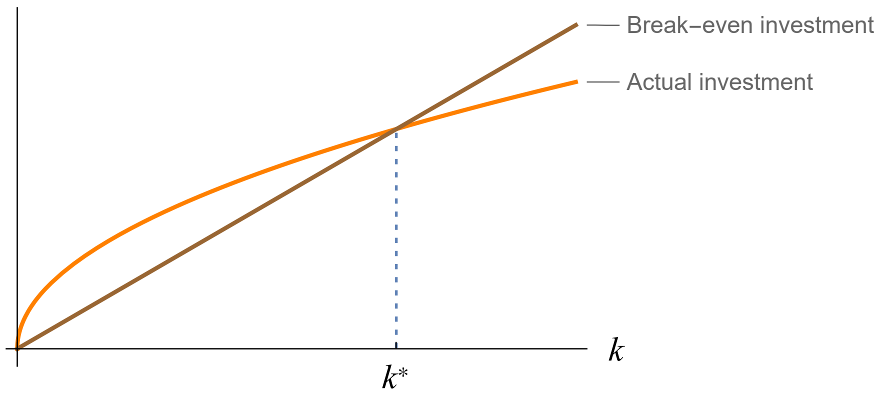

Figure 1: Actual and break-even investment

As the figure illustrates, the equation $sf(k) - (n + g + \delta)k(t) = 0$ has unique solution $k^* = k^*(s, n, g, \delta)$. Then we can readily employ the diagrammatic analysis to find the stable solution. It is clear to see that regardless of where k starts, it converge to k^* and remains there.

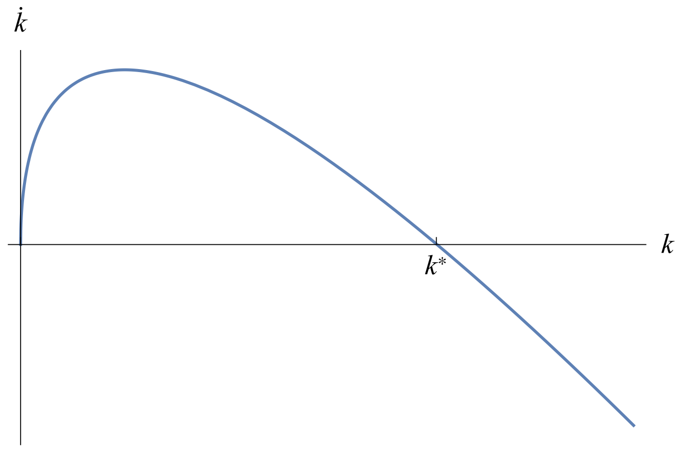

Figure 2: Phase diagram for k

When $t \to \infty$, the economy reaches its balanced growth path and thus we see

$$\begin{aligned}
k(t) &\to k^* \\
y(t) &\to f(k^*) \\
L(t) &= L(0)\mathrm{e}^{nt} \\
A(t) &= A(0)\mathrm{e}^{gt} \\
K(t) &\sim K(0)\mathrm{e}^{(n+g)t} \\
Y(t) &\sim Y(0)\mathrm{e}^{(n+g)t}.
\end{aligned}$$

1.3 Consumption

While s of the production $Y(t)$ are invested for more consumption in the future, the current consumption $C(t)$ accounts for $1 - s$ of the production $Y(t)$. Let $c(t)$ denote the consumption per unit of effective labor, that is

$$c(t) = (1 - s)f(k).$$

On the balanced growth path it follows that

$$c^* = (1 - s)f(k^*) = f(k^*) - (n + g + \delta)k^*.$$

1.4 The Impact of a Change in Saving Rate

1.4.1 The Impact on Output

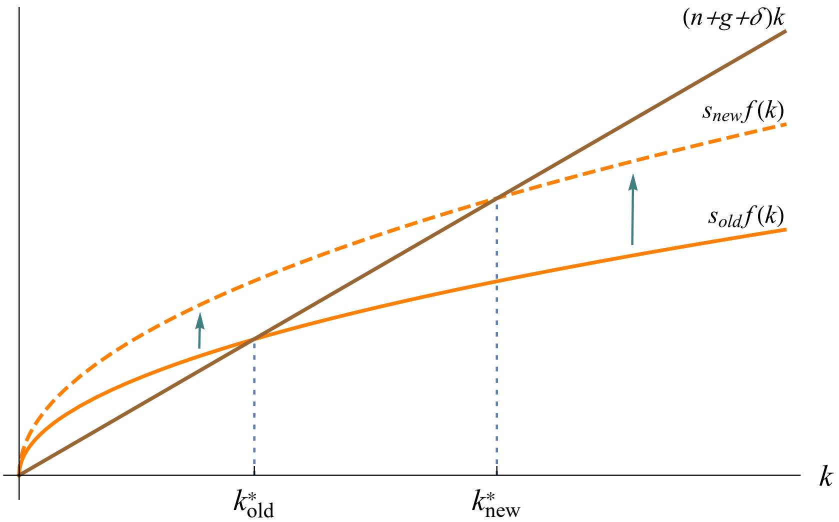

Figure 3: The effects of an increase in saving rate on investment

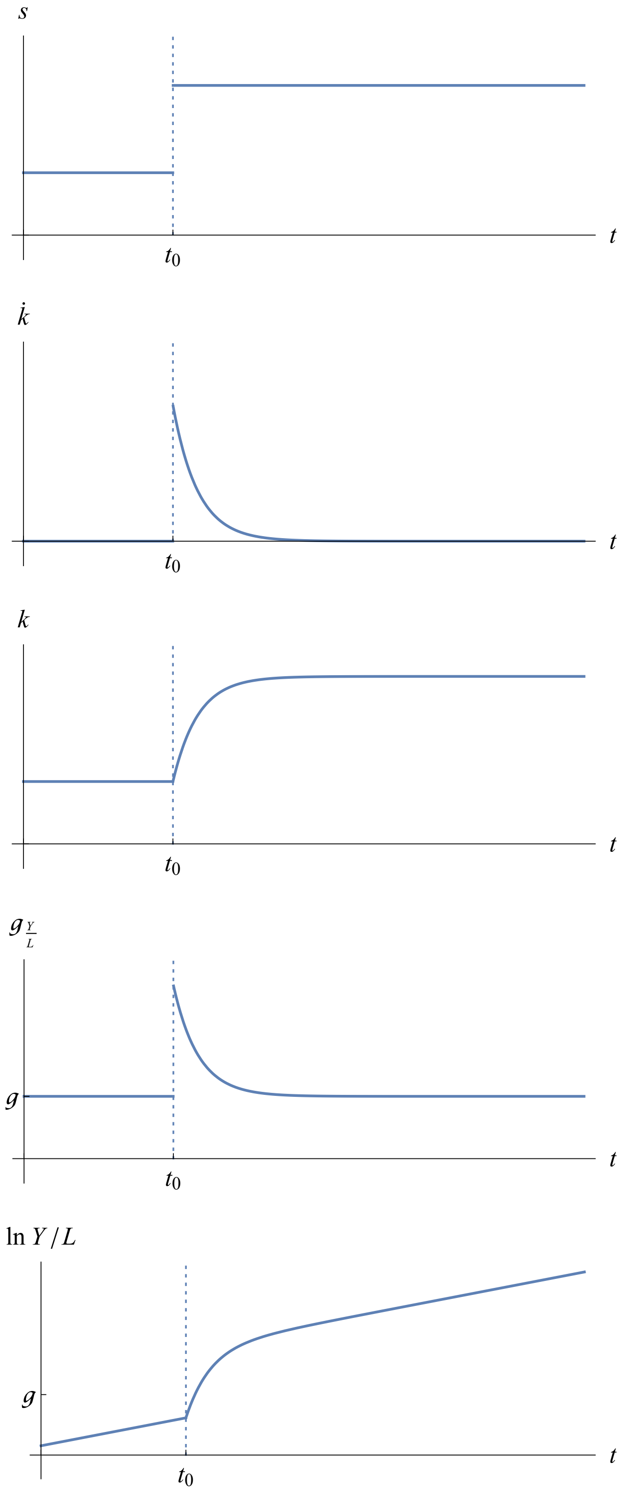

Figure 4: The effects of an increase in saving rate

1.4.2 The Impact on Consumption

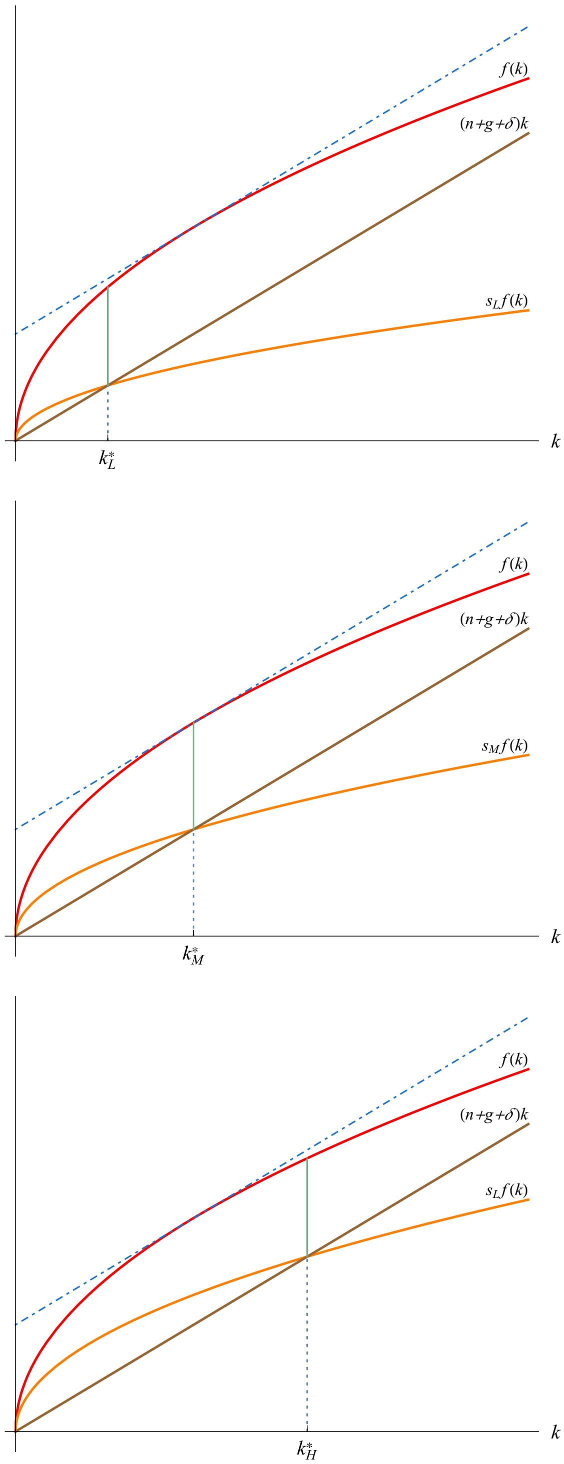

Figure 5: The effects of an increase in saving rate on consumption

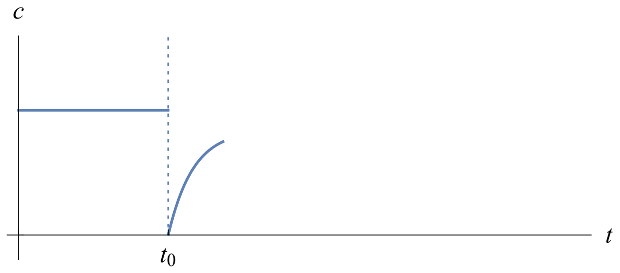

Figure 6: The effects of an increase in saving rate on consumption

1.5 Typical Example

Setting $Y = K^\alpha (AL)^{1-\alpha}$ $(0 < \alpha < 1)$ and accordingly $y(t) = k(t)^\alpha$, we get the differential equation with respect to $k(t)$:

$$\dot{k} = sk^\alpha - (n+g+\delta)k.$$

The capital per unit of effective labor

$$k(t) = \left[\frac{\widetilde{C}\mathrm{e}^{-(1-\alpha)(n+g+\delta)t} + s}{n+g+\delta}\right]^{\frac{1}{1-\alpha}}.$$

solves the equation, where $\widetilde{C}$ is a constant to be specified by the initial condition $k(0) = k_0$. On the balanced growth path,

$$\lim_{t\to+\infty} k(t) = k^* = \left(\frac{s}{n+g+\delta}\right)^{1/(1-\alpha)}$$

1.6 Quantitative Implications

Since $y^*(s,n,g,\delta) = f(k^*(s,n,g,\delta))$,

$$\frac{\partial y^*}{\partial s} = \frac{\partial k^*}{\partial s} f'(k^*)$$

$$\frac{\partial k^*}{\partial s} = \frac{f(k^*)}{(n+g+\delta) - sf'(k^*)}$$

$$\begin{aligned}\frac{s}{y^*}\frac{\partial y^*}{\partial s} &= \frac{s}{y^*}\frac{f'(k^*)f(k^*)}{(n+g+\delta) - sf'(k^*)} \\ &= \frac{\alpha_K(k^*)}{1-\alpha_K(k^*)}\end{aligned}$$

2 The Ramsey-Cass-Koopmans Model

2.1 Setup

Households' maximization problem is

$$\max\ B\int_0^\infty e^{-\beta t}\frac{c(t)^{1-\theta}}{1-\theta}dt$$

$$\text{s.t. } k'(t) = f(k(t)) + c(t) - (n+g)k(t)$$

where $B = A(0)^{1-\theta}L(0)/H$, $\beta = \rho - n - (1-\theta)g$. Hamilton function is

$$H = e^{-\beta t}\frac{c(t)^{1-\theta}}{1-\theta} + \lambda(t)[f(k(t)) + c(t) - (n+g)k(t)],$$

which leads to Hamilton equations

$$\frac{\partial H}{\partial c} = e^{-\beta t}c^{-\theta} + \lambda = 0,$$
$$\frac{\partial H}{\partial k} = \lambda[f'(k) - (n+g)] = -\lambda'.$$

Substituting β into it yields the Euler equation

$$\frac{c'}{c} = \frac{f'(k) - \rho - \theta g}{\theta}$$

2.2 Stable Solution

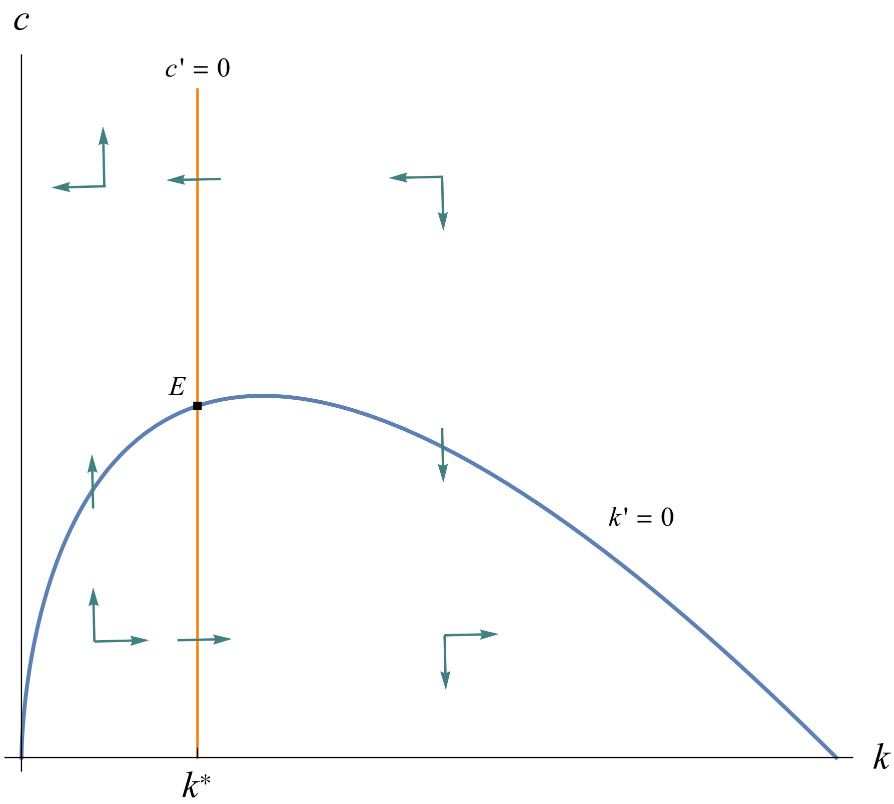

Figure 7: The dynamics of k and c

3 The Diamond Model

3.1 Setup

Let $C_{1,t}$ and $C_{2,t+1}$ denote the consumption in period t of young and old individuals. Households' maximization problem is

$$\max \frac{C_{1,t}^{1-\theta}}{1-\theta} + \frac{1}{1+\rho}\frac{C_{2,t+1}^{1-\theta}}{1-\theta}$$

$$\text{s.t. } C_{1,t} + \frac{1}{1+r_{t+1}}C_{2,t+1} = A_t w_t$$

The Euler equation is

$$\frac{C_{2,t+1}}{C_{1,t}} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\frac{1}{\theta}}$$

4 Content Section

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5 Conclusion

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References

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