

ECONOMICS NOTES

Advanced Macroeconomics

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Contents

1	Solow Growth Model	2
1.1	Setup	2
1.2	Stable Solution	2
1.3	Consumption	4
1.4	The Impact of a Change in Saving Rate	5
1.4.1	The Impact on Output	5
1.5	Typical Example	6
2	The Ramsey-Cass-Koopmans Model	8
2.1	Setup	8
3	Content Section	8
3.1	Subsection 1	8
3.1.1	Subsubsection 1	8
3.1.2	Subsubsection 2	9
3.1.3	Subsubsection 3	9
4	Conclusion	10

1 Solow Growth Model

1.1 Setup

Given the assumption of constant returns to scale, the production function is $Y(t) = F(K(t), A(t)L(t))$ or alternatively in intensive form $y(t) = f(k(t))$ in which $y = Y/(AL)$ and $k = K/(AL)$. $f(k)$ is assumed to satisfy $f(0) = 0$, $f'(k) > 0$, $f''(0) < 0$ and the Inada conditions: $\lim_{k \rightarrow 0} f'(k) = \infty$, $\lim_{k \rightarrow \infty} f'(k) = 0$. The evolution of the inputs into production are determined by

$$\begin{aligned}\dot{L}(t) &= nL(t), \\ \dot{A}(t) &= gA(t), \\ \dot{K}(t) &= sY(t) - \delta K(t).\end{aligned}$$

These equations yield solution as follows

$$\begin{aligned}L(t) &= L(0)e^{nt} \\ A(t) &= A(0)e^{gt}.\end{aligned}$$

Labor and knowledge grow at constant rates n and g respectively. Since the production function $F(K, AL)$ is not specified, we cannot give an explicit solution of $K(t)$.

1.2 Stable Solution

For the sake of qualitative analysis, the system of differential equations can be simplified to a single differential equation with respect to $k(t)$:

$$\dot{k}(t) = sf(k) - (n + g + \delta)k(t).$$

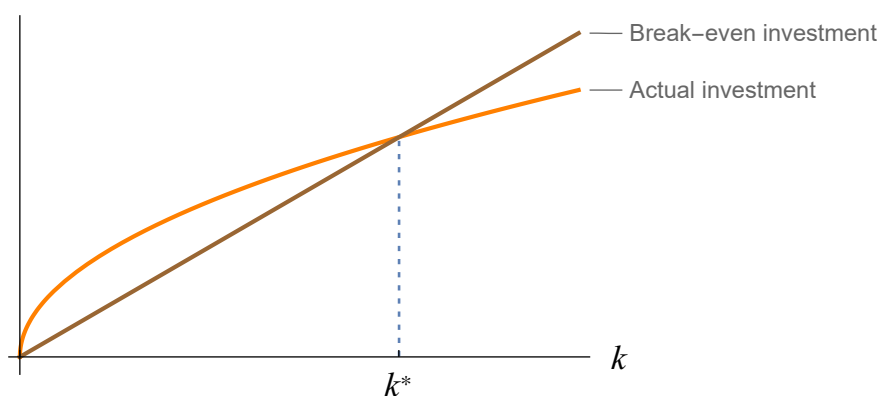


Figure 1: Actual and break-even investment

As the figure illustrates, the equation $sf(k) - (n + g + \delta)k(t) = 0$ has unique solution $k^* = k^*(s, n, g, \delta)$. Then we can readily employ the diagrammatic analysis to find the stable solution. It is clear to see that regardless of where k starts, it converge to k^* and remains there.

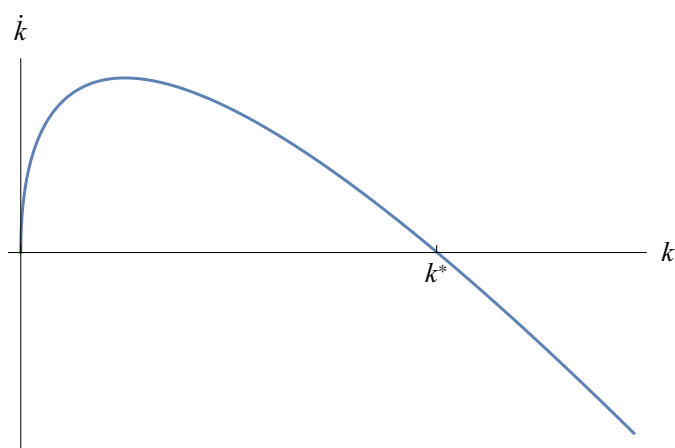


Figure 2: Phase diagram for k

When $t \rightarrow \infty$, the economy reaches its balanced growth path and thus we see

$$\begin{aligned}k(t) &\rightarrow k^* \\y(t) &\rightarrow f(k^*) \\L(t) &= L(0)e^{nt} \\A(t) &= A(0)e^{gt} \\K(t) &\sim K(0)e^{(n+g)t} \\Y(t) &\sim Y(0)e^{(n+g)t}.\end{aligned}$$

1.3 Consumption

While s of the production $Y(t)$ are invested for more consumption in the future, the current consumption $C(t)$ accounts for $1 - s$ of the production $Y(t)$. Let $c(t)$ denote the consumption per unit of effective labor, that is

$$c(t) = (1 - s)f(k).$$

On the balanced growth path it follows that

$$c^* = (1 - s)f(k^*) = f(k^*) - (n + g + \delta)k^*.$$

1.4 The Impact of a Change in Saving Rate

1.4.1 The Impact on Output

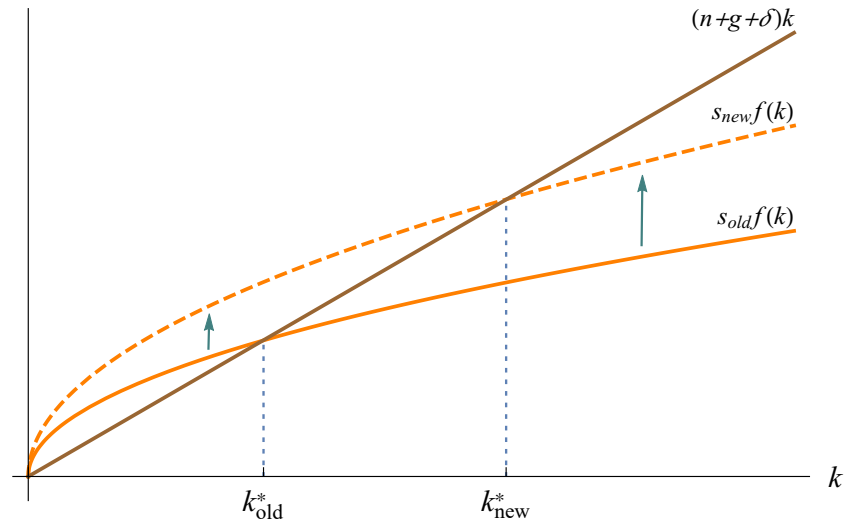


Figure 3: The effects of an increase in saving rate on investment

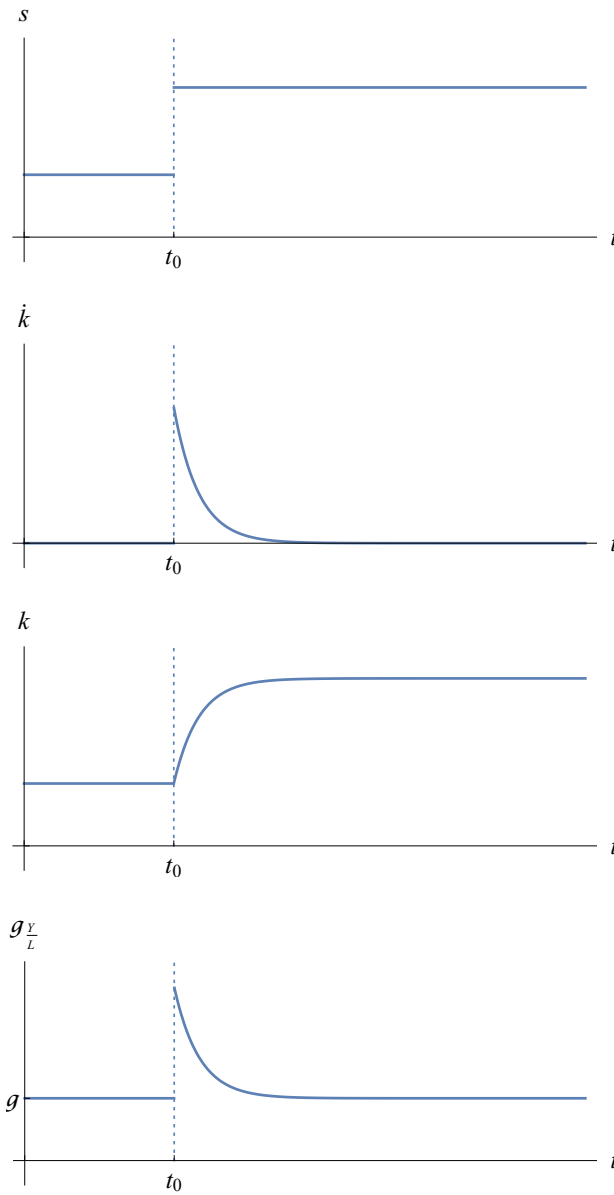


Figure 4: The effects of an increase in saving rate

1.5 Typical Example

Setting $Y = K^\alpha(AL)^{1-\alpha}$ ($0 < \alpha < 1$) and accordingly $y(t) = k(t)^\alpha$, we get the differential equation with respect to $k(t)$:

$$\dot{k} = sk^\alpha - (n + g + \delta)k.$$

The capital per unit of effective labor

$$k(t) = \left[\frac{\tilde{C}e^{-(1-\alpha)(n+g+\delta)t} + s}{n + g + \delta} \right]^{\frac{1}{1-\alpha}}.$$

solves the equation, where \tilde{C} is a constant to be specified by the initial condition $k(0) = k_0$. On the balanced growth path,

$$\lim_{t \rightarrow +\infty} k(t) = k^* = \left(\frac{s}{n + g + \delta} \right)^{1/(1-\alpha)}$$

2 The Ramsey-Cass-Koopmans Model

2.1 Setup

The production function is $Y(t) = F(K(t), A(t)L(t))$ or alternatively in intensive form $y(t) = f(k(t))$ in which $y = Y/(AL)$ and $k = K/(AL)$.

$$\begin{aligned}\dot{L}(t) &= nL(t) \\ \dot{A}(t) &= gA(t) \\ \dot{K}(t) &= sY(t) - \delta K(t).\end{aligned}$$

3 Content Section

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4 Conclusion

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References

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