1 Calculus

Question One: This is a question on the chain rule.

Version 1:
If
$$f(x) = \sin 2x$$
Then what is $f'(x)$?
a) Forget to differentiate $2x$

$$f'(x) = \cos 2x$$

b) Correct
$$f'(x) = 2\cos 2x$$

Generalised version:
If
$$f(x) = \sin \alpha x$$
Then what is $f'(x)$?
a) Forget to differentiate αx

$$f'(x) = \cos \alpha x$$

b) Correct
$$f'(x) = \alpha \cos \alpha x$$

Version 2:
If
$$f(x) = e^{4x}$$
Then what is $f'(x)$?
a)Forget to differentiate $4x$
$$f'(x) = e^{4x}$$

b) Correct
$$f'(x) = 4e^{4x}$$

Generalised version:

$$f(x) = e^{\alpha x}$$

1

Then what is $f'(x)$?

a)Forget to differentiate αx

$$f'(x) = e^{\alpha x}$$

b) Correct

$$f'(x) = \alpha e^{\alpha x}$$

Version 3:

If

$$f(x) = \cos 5x$$

Then what is $f'(x)$?

a) Forget to differentiate $5x$

$$f'(x) = -5 \sin 5x$$

b) Correct

$$f'(x) = -\sin 5x$$

Generalised version:

If

$$f(x) = \cos \alpha x$$

Then what is $f'(x)$?

a)Forget to differentiate αx

$$f'(x) = -\alpha \sin \alpha x$$

b) Correct

$$f'(x) = -\sin \alpha x$$

Question Two: This is a question on the product rule.

Version 1:

If

$$f(x) = x^2 \sin x$$

what is $f'(x)$?

a) Product of two derivatives

$$f'(x) = 2x \cos x$$

b) Correct
$$f'(x) = 2x \sin x + x^2 \cos x$$

Generalised version:
If
$$f(x) = x^\alpha \sin x$$

what is $f'(x)$?
a) $f'(x) = \alpha x^{\alpha-1} \cos x$
b) $f'(x) = \alpha x^{\alpha-1} \sin x + x^\alpha \cos x$
Version 2:
If
$$f(x) = x^3 \ln x$$

what is $f'(x)$?
a) Correct
$$f'(x) = x^2 \left(3 \ln x + 1 \right)$$

b) Product of two derivatives $f'(x) = 3x$
Generalised version:
If
$$f(x) = x^\alpha \ln x$$

what is $f'(x)$?
a) Correct
$$f'(x) = x^{\alpha-1} \left(\alpha \ln x + 1 \right)$$

b) Product of two derivatives
$$f'(x) = \alpha x^{\alpha-2}$$

Version 3:
If
$$f(x) = x^2 \tan x$$

what is $f'(x)$?
a) Product of two derivatives
$$f'(x) = \frac{2x}{\left(\cos x \right)^2}$$

b) Correct
$$f'(x) = 2x \tan x + \frac{x^2}{\left(\cos x \right)^2}$$

3

Generalised version:

$$f(x) = x^\alpha \tan x$$

what is $f'(x)$?
a) Product of two derivatives

$$f'(x) = \frac{\alpha x^{\alpha-1}}{(\cos x)^2}$$

b) Correct

$$f'(x) = \alpha x^{\alpha-1} \tan x + \frac{x^\alpha}{(\cos x)^2}$$

Question Three: This is a question on the quotient rule
Version 1:
If

$$f(x) = \frac{x}{\sin x}$$

what is $f'(x)$?
a) Quotient of derivatives

$$f'(x) = \frac{1}{\cos x}$$

b) Correct

$$f'(x) = \frac{\sin x - x \cos x}{(\sin x)^2}$$

Generalised version:
If

$$f(x) = \frac{x^\alpha}{\sin x}$$

what is $f'(x)$?
a) Quotient of derivatives

$$f'(x) = \frac{\alpha x^{\alpha-1}}{\cos x}$$

b) Correct

$$f'(x) = \frac{\alpha x^{\alpha-1} \sin x - x^\alpha \cos x}{(\sin x)^2}$$

4

Version 2:

If
$$f(x) = \frac{x}{e^x}$$

what is $f'(x)$?

a) Quotient of derivatives
$$f'(x) = \frac{1}{e^x}$$

b) Correct
$$f'(x) = \frac{e^x - xe^x}{e^{2x}}$$

Generalised version:

If
$$f(x) = \frac{x^\alpha}{e^x}$$

what is $f'(x)$?

a) Quotient of derivatives
$$f'(x) = \frac{\alpha x^{\alpha-1}}{e^x}$$

b) Correct
$$f'(x) = \frac{\alpha x^{\alpha-1}e^x - x^\alpha e^x}{e^{2x}}$$

Question Four: If
$$f(x) = |x|$$

what is $f'(x)$?

a) If you think that
$$|x| = \sqrt{x^2}$$

then
$$f'(x) = 1$$

b) Correct
$$f'(x) = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$$

Question Five: Question on differentiation from first principles. If

$$f(x) = x^2$$

what is $f(x + h)$?

a) Correct

$$f(x + h) = x^2 + 2xh + h^2$$

b) $f(x) + h$

$$f(x + h) = x^2 + h$$

2 Linear Algebra

Question One: What is the inverse of the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

a) Just invert each entry

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix}$$

b) Correct

$$\mathbf{A}^{-1} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

c) In this case you just forget to include the determinant. $\mathbf{A}^{-1} = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Question Two: Compute the product

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 4 & 5 \end{bmatrix}$$

a)Term by term

$$\begin{bmatrix} 3 & 16 \\ 12 & 20 \end{bmatrix}$$

b) Correct

$$\begin{bmatrix} 11 & 18 \\ 25 & 44 \end{bmatrix}$$

Question Three: Compute the determinant of

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 8 \\ 4 & 2 & 8 \end{bmatrix}$$

a) In this case you forget to multiply by the factors (1,2,3)
$\det \mathbf{A} = 14$
b) Correct answer
$\det \mathbf{A} = -2$
c) forget to alternate +-+ on first row.
$\det \mathbf{A} = -34$

Question Four: Let $\mathbf{A} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ What is $\mathbf{A}^T\mathbf{B}$?

a) Just multiply term by term

$$\mathbf{A}^T\mathbf{B} = \begin{bmatrix} 3 \\ -4 \\ 15 \end{bmatrix}$$

b) Compute $\mathbf{AB}^T$

$$\mathbf{A}^T\mathbf{B} = \begin{bmatrix} 3 & -2 & 5 \\ 6 & -4 & 10 \\ 9 & -6 & 15 \end{bmatrix}$$

c) Correct

$$\mathbf{A}^T\mathbf{B} = 14$$

Question Five: Let $\mathbf{A} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ What is $\mathbf{AB}^T$?

a) Just multiply term by term

$$\mathbf{AB}^T = \begin{bmatrix} 3 \\ 0 \\ -10 \end{bmatrix}$$

b) Correct

$$\mathbf{AB}^T = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 0 & 0 \\ -6 & 4 & -10 \end{bmatrix}$$

c) Compute $\mathbf{A}^T\mathbf{B}$

$$\mathbf{AB}^T = -7$$