

Matrix Operations Learning Sheet

1. Element-wise Multiplication

Element-wise multiplication involves multiplying corresponding elements of two arrays of the same shape (or broadcastable).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

The element-wise product $A \circ B$ is:

$$A \circ B = \begin{bmatrix} 1 \times 7 & 2 \times 8 & 3 \times 9 \\ 4 \times 10 & 5 \times 11 & 6 \times 12 \end{bmatrix} = \begin{bmatrix} 7 & 16 & 27 \\ 40 & 55 & 72 \end{bmatrix}$$

2. Matrix Multiplication

Matrix multiplication involves taking the dot product of rows of the first matrix with columns of the second matrix. The inner dimensions must match.

Let:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

Matrix multiplication $A \times B$ results in:

$$A \times B = \begin{bmatrix} 1 \times 5 + 2 \times 7 & 1 \times 6 + 2 \times 8 \\ 3 \times 5 + 4 \times 7 & 3 \times 6 + 4 \times 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

3. Dot Product

The dot product is a special case of matrix multiplication for vectors and returns a scalar. It is the sum of the element-wise products of the two vectors.

Given vectors:

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

The dot product $\mathbf{a} \cdot \mathbf{b}$ is:

$$\mathbf{a} \cdot \mathbf{b} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 4 + 10 + 18 = 32$$

Summary of Key Distinctions

- **Element-wise Multiplication:** Operates on arrays of the same shape (or broadcastable) and produces an array of the same shape.
- **Matrix Multiplication:** Works on 2D matrices (or vectors as special cases) where the inner dimensions must match. The result is a new matrix.
- **Dot Product:** A special case of matrix multiplication applied to two vectors that returns a scalar.