Learning Curves and Automatic Convergence in Gradient Descent

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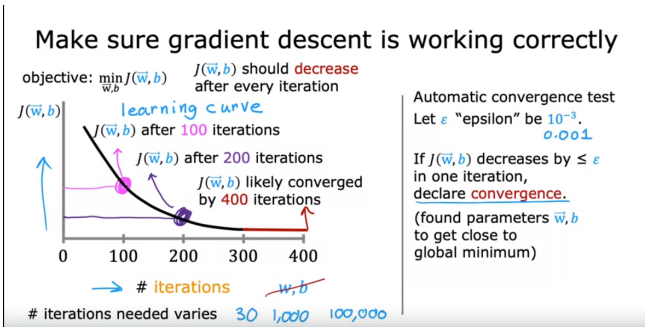
August 29, 2024

1. Learning Curve

The **learning curve** is a graphical representation of the performance of a model as a function of the number of iterations or the number of training examples. It is often used to assess how many iterations of updating the parameters (e.g., w and b) it takes to approach the minimum of the cost function. The cost function, denoted as $J(\mathbf{w}, b)$, typically decreases as the number of iterations increases, until it converges to a local or global minimum.

Example of a Learning Curve

A typical learning curve plots the cost function $J(\mathbf{w}, b)$ against the number of iterations:



This curve helps in visualizing whether the algorithm is converging and how many iterations are required to approach the minimum. The learning curve can also indicate if the algorithm is overfitting or underfitting, depending on its behavior.

2. Automatic Convergence Test

To automate the process of stopping gradient descent when it has nearly reached the minimum of the cost function, an **automatic convergence test** can be employed. The idea is to use a threshold value, known as **epsilon** (ϵ), which defines how small the decrease in the cost function should be for the algorithm to stop iterating.

Convergence Condition

During gradient descent, we monitor the difference between the cost function value at the current iteration and the previous iteration. If the change is smaller than a predefined value ϵ, the algorithm is assumed to have converged:

$$|J(\theta^{(t+1)}) - J(\theta^{(t)})| < \epsilon$$

Where:

- $J(\theta^{(t)})$ is the cost function value at iteration t,
- ϵ is a small threshold value chosen to indicate when the cost function is close enough to its minimum.

When this condition is met, the gradient descent process stops, assuming that the cost function is at or very near its minimum.

3. Choosing the Value of Epsilon (ϵ)

Choosing an appropriate value for ϵ can vary depending on the problem. There is no fixed rule, but here are some general guidelines:

- **Scale of the Cost Function:** If the cost function is large (e.g., thousands), then ϵ should be larger (e.g., 0.01 or 0.001). For smaller cost functions, ϵ can be smaller (e.g., 0.00001).
- **Desired Precision:** If a high level of precision is needed, ϵ should be set to a smaller value (e.g., 0.00001). If less precision is acceptable, a larger ϵ (e.g., 0.01) can be used.
- **Computational Resources:** Smaller values of ϵ require more iterations, consuming more computational power. If resources are limited, a larger ϵ might be appropriate.
- **Empirical Testing:** Often, it is best to start with a reasonable guess (e.g., $\epsilon = 10^{-4}$) and adjust based on the algorithm's performance.

Typical Range for Epsilon

In practice, values for ϵ typically range between 10^{-1} and 10^{-6}. A common starting point for many machine learning problems is:

$$\epsilon = 10^{-4}$$

Adjustments can be made depending on whether the algorithm converges too early or takes too long to converge.

4. Conclusion

The learning curve and automatic convergence tests are powerful tools for evaluating and improving the performance of gradient descent in machine learning. The learning curve helps to visualize how the model is learning over time, while the automatic convergence test allows for efficient stopping of the gradient descent iterations once the minimum of the cost function is near. Properly choosing the value of ϵ ensures a balance between computational efficiency and the precision of the solution.