

# Commutative Algebra for Algebraic Geometry

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## Part I

# Algebraic Geometry, Hartshorne

In this part, I will be going through the work of Hartshorne as he laid out in [Har13]. The book is sectioned into 5 distinct chapters - 'Varieties', 'Schemes', 'Cohomology', 'Curves' and 'Surfaces'. We study them in order as they flow nicely.

## 1 Varieties

**Definition 1 Affine  $n$ -Space over  $k$ :** Let  $k$  be a field and  $n \in \mathbb{N}$ . Then the affine  $n$ -space over  $k$  is defined

$$\mathbb{A}_k^n = \{(k_1, \dots, k_n) \in k^n\}$$

It seems that this is a silly definition, but later on we will see that it is useful to have a distinction between the variety  $\mathbb{A}_k^n$  and the set of points in  $k^n$ . We later view  $\mathbb{A}_k^n$  as an affine variety - an object in some arbitrary space rather than the set of  $k$ -tuples.

Let  $A = k[x_1, \dots, x_n]$  be the polynomial ring over  $k$  in  $n$  variables. Then  $f \in A$  is a map  $f : k^n \rightarrow k$ . We define the vanishing locus of this function in the following way:

**Definition 2 Vanishing Locus of a Polynomial:** Let  $f \in A = k[x_1, \dots, x_n]$ , then the vanishing locus is

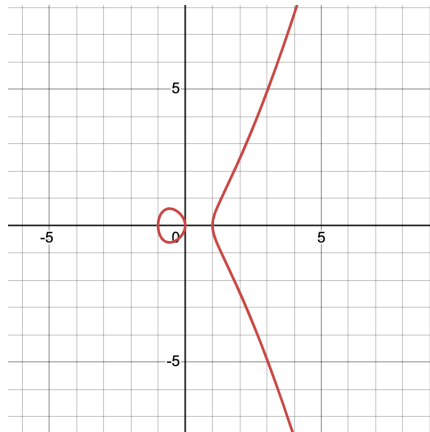
$$\mathbb{V}(f) := \{p \in \mathbb{A}_k^n : f(p) = 0\}$$

We can develop a more advanced analogue of this:

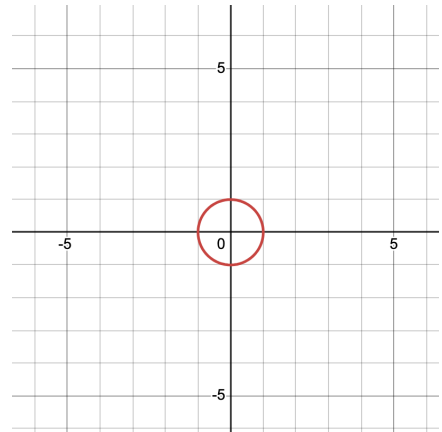
**Definition 3 Vanishing Locus of a Set of Polynomials:** Let  $T = \{f_i\}_{i \in I} \subset A = k[x_1, \dots, x_n]$ , then the vanishing locus of  $T$  is

$$\mathbb{V}(T) := \{p \in \mathbb{A}_k^n : f(p) = 0, \quad \forall f \in T\}$$

Some examples:



(a)  $\mathbb{V}(y^2 - x(x+1)(x-1))$



(b)  $\mathbb{V}(x^2 + y^2 - 1)$

Figure 1: The vanishing locus of two separate polynomials plotted in  $\mathbb{R}^2$



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