Notes on 'Sheaf Theory' by Bredon

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Contents

Part I
Sheaf Theory, Bredon

1 Sheaves and Presheaves

1.1 Definitions

Definition 1 ***Presheaf:*** *A presheaf of abelian groups on a topological space X is a functor*

$$A : \mathfrak{Opn}_X^{op} \to \mathfrak{Ab}$$

i.e. an assignment to each open set $U \in Ob(\mathfrak{Opn}_X^{op})$ an abelian group $A(U) \in Ob(\mathfrak{Ab})$, and on inclusion functions $V \overset{\iota_U^V}{\hookrightarrow} U$ restriction functions $r_V^U : A(U) \to A(V)$ such that:

- $r_U^U = id_U$,
- $U \subset V \subset W \implies r_V^W r_U^V = r_U^W$

One can switch out the target category with other algebraic categories to obtain the definitions of presheaves over R-modules, algebras, etc.

Definition 2 ***Germ:*** *Let $x \in X$ be a point and A a presheaf over X. The set $\mathfrak{M}$ of elements $s \in A(U)$, $x \in U \in Ob\mathfrak{Opn}_X^{op}$. The germs of the presheaf A at x is thus the quotient*

$$\mathfrak{G}_{A,x} := \mathfrak{M}/$$

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