THÁI BÌNH----------BÀI TẬP ĐẦY ĐỦ - TÍCH PHÂN

I. TÍNH TÍCH PHÂN BẰNG CÁCH SỬ DỤNG TÍNH CHẤT VÀ NGUYÊN HÀM CƠ BẢN:

1. $\int_0^1 (x^3 + x + 1)dx$

2. $\int_1^e (x + \frac{1}{x} + \frac{1}{x^2} + x^2)dx$

2. $\int_1^3 |x-2|dx$

3. $\int_1^2 \sqrt{x+1}dx$

4. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (2\sin x + 3cosx + x)dx$

5. $\int_0^1 (e^x + x)dx$

6. $\int_0^1 (x^3 + x\sqrt{x})dx$

7. $\int_1^2 (\sqrt{x}+1)(x-\sqrt{x}+1)dx$

8. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (3\sin x + 2cosx + \frac{1}{x})dx$

9. $\int_0^1 (e^x + x^2 + 1)dx$

10. $\int_1^2 (x^2 + x\sqrt{x} + \sqrt[3]{x})dx$

11. $\int_1^2 (\sqrt{x}-1)(x+\sqrt{x}+1)dx$

12. $\int_{-1}^{3} (x^3 + 1).dx$

13. $\int_{-1}^{2} \frac{x.dx}{x^2+2}$

14. $\int_1^{e^2} \frac{7x - 2\sqrt{x} - 5}{x} dx$

15. $\int_2^5 \frac{dx}{\sqrt{x+2} + \sqrt{x-2}}$

16. $\int_1^2 \frac{(x+1).dx}{x^2 + x\ln x}$

17. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos^3 x.dx}{\sqrt[3]{\sin x}}$

18. $\int_0^{\frac{\pi}{4}} \frac{\sqrt{tgx}.dx}{\cos^2 x}$

19. $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

20. $\int_0^1 \frac{\sqrt{e^x}.dx}{\sqrt{e^x + e^{-x}}}$

21. $\int_1^2 \frac{dx}{\sqrt{4x^2 + 8x}}$

22. $\int_0^{\ln\sqrt{3}} \frac{.dx}{e^x + e^{-x}}$

22. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$

II. PHƯƠNG PHÁP ĐẶT ẨN PHỤ:

1. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3 x cos^2 x dx$

2. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2 x cos^3 x dx$

3. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+3\cos x}\,dx$

3. $\int_0^{\frac{\pi}{4}} tgx\,dx$

4. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot g x\,dx$

5. $\int_0^{\frac{\pi}{6}} \sqrt{1+4\sin x}\cos x\,dx$

6. $\int_0^1 x\sqrt{x^2+1}\,dx$

7. $\int_0^1 x\sqrt{1-x^2}\,dx$

8. $\int_0^1 x^3\sqrt{x^2+1}\,dx$

9. $\int_0^1 \frac{x^2}{\sqrt{x^3+1}}\,dx$

10. $\int_0^1 x^3\sqrt{1-x^2}\,dx$

11. $\int_1^2 \frac{1}{x\sqrt{x^3+1}}\,dx$

12. $\int_0^1 \frac{1}{1+x^2}dx$

13. $\int_{-1}^1 \frac{1}{x^2+2x+2}dx$

14. $\int_0^1 \frac{1}{\sqrt{x^2+1}}dx$

15. $\int_0^1 \frac{1}{(1+3x^2)^2}dx$

16. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\sin x}\cos x\,dx$

17. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\cos x}\sin x\,dx$

18. $\int_0^1 e^{x^2+2}x\,dx$

19. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3 x\cos^2 x\,dx$

20. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\sin x}\cos x\,dx$

21. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\cos x}\sin x\,dx$

22. $\int_0^1 e^{x^2+2}x\,dx$

23. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^3 x\cos^2 x\,dx$

24. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2 x\cos^3 x\,dx$

25. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+3\cos x}\,dx$

26. $\int_0^{\frac{\pi}{4}} tgx\,dx$

27. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot g x\,dx$

28. $\int_0^{\frac{\pi}{6}} \sqrt{1+4\sin x}\cos x\,dx$

29. $\int_0^1 x\sqrt{x^2+1}\,dx$

30. $\int_0^1 x\sqrt{1-x^2}\,dx$

31. $\int_0^1 x^3\sqrt{x^2+1}dx$

32. $\int_0^1 \frac{x^2}{\sqrt{x^3+1}}dx$

33. $\int_0^1 x^3\sqrt{1-x^2}\,dx$

34. $\int_1^2 \frac{1}{x\sqrt{x^3+1}}dx$

35. $\int_1^e \frac{\sqrt{1+\ln x}}{x}dx$

36. $\int_1^e \frac{\sin(\ln x)}{x}dx$

37. $\int_1^e \frac{\sqrt{1+3\ln x}\ln x}{x}dx$

38. $\int_1^e \frac{e^{2\ln x+1}}{x}dx$

39. $\int_e^{e^2} \frac{1+\ln^2 x}{x\ln x}dx$

40. $\int_e^{e^2} \frac{1}{cos^2(1+\ln x)}dx$

41. $\int_1^2 \frac{x}{1+\sqrt{x-1}}dx$

42. $\int_0^1 \frac{x}{\sqrt{2x+1}}dx$

43. $\int_0^1 x\sqrt{x+1}dx$

44. $\int_0^1 \frac{1}{\sqrt{x+1}+\sqrt{x}}dx$

45. $\int_0^1 \frac{1}{\sqrt{x+1}-\sqrt{x}}dx$

46. $\int_1^3 \frac{\sqrt{x+1}}{x}dx$

46. $\int_1^e \frac{\sqrt{1+\ln x}}{x}dx$

47. $\int_1^e \frac{\sin(\ln x)}{x}dx$

48. $\int_1^e \frac{\sqrt{1+3\ln x}\ln x}{x}dx$

49. $\int_1^e \frac{e^{2\ln x+1}}{x}dx$

50. $\int_e^{e^2} \frac{1+\ln^2 x}{x\ln x}dx$

51. $\int_e^{e^2} \frac{1}{cos^2(1+\ln x)}dx$

52. $\int_0^1 x^2\sqrt{x^3+5}dx$

53. $\int_0^{\frac{\pi}{2}} \left(\sin^4 x+1\right)\cos x dx$

54. $\int_0^4 \sqrt{4-x^2}\,dx$

55. $\int_0^4 \sqrt{4-x^2}\,dx$

56. $\int_0^1 \frac{dx}{1+x^2}$

II. PHƯƠNG PHÁP TÍCH PHÂN TỪNG PHẦN:

Công thức tích phân từng phần : $\int_a^b \mathrm{u}(x)\mathrm{v}'(\mathrm{x})dx = u(x)v(x)\Big|_a^b - \int_a^b v(x)u'(x)dx$

Tích phân các hàm số dễ phát hiện u và dv

@ **Dạng 1** $\int_{\alpha}^{\beta} f(x)\begin{bmatrix}\sin ax\\ cosax\\ e^{ax}\end{bmatrix}dx$

$$\begin{cases}u=f(x)\\ dv=\begin{bmatrix}\sin ax\\ \cos ax\\ e^{ax}\end{bmatrix}dx\end{cases}\Rightarrow\begin{cases}du=f'(x)dx\\ v=\int\begin{bmatrix}\sin ax\\ cosax\\ e^{ax}\end{bmatrix}dx\end{cases}$$

@ **Dạng 2:** $\int_{\alpha}^{\beta} f(x)\ln(ax)dx$

Đặt $\begin{cases}u=\ln(ax)\\ dv=f(x)dx\end{cases}\Rightarrow\begin{cases}du=\dfrac{dx}{x}\\ v=\int f(x)dx\end{cases}$

@ **Dạng 3:** $\int_{\alpha}^{\beta} e^{ax}.\begin{bmatrix}\sin ax\\ cosax\end{bmatrix}dx$

Ví dụ ***1***: tính các tích phân sau

a/ $\int_0^1 \dfrac{x^2e^x}{(x+1)^2}dx$ đặt $\begin{cases}u=x^2e^x\\ dv=\dfrac{dx}{(x+1)^2}\end{cases}$

b/ $\int_2^3 \dfrac{x^8dx}{(x^4-1)^3}$ đặt $\begin{cases}u=x^5\\ dv=\dfrac{x^3dx}{(x^4-1)^3}\end{cases}$

c/ $\int_0^1 \dfrac{dx}{(1+x^2)^2}=\int_0^1 \dfrac{1+x^2-x^2}{(1+x^2)^2}dx=\int_0^1 \dfrac{dx}{1+x^2}-\int_0^1 \dfrac{x^2dx}{(1+x^2)^2}=I_1-I_2$

Tính I_1 $=\int_0^1 \dfrac{dx}{1+x^2}$ bằng phương pháp đổi biến số

Tính I_2 = $\int_0^1 \dfrac{x^2dx}{(1+x^2)^2}$ bằng phương pháp từng phần : đặt $\begin{cases}u=x\\ dv=\dfrac{x}{(1+x^2)^2}dx\end{cases}$

Bài tập

1. $\int_1^e \dfrac{\ln^3 x}{x^3}dx$
2. $\int_1^e x\ln xdx$
3. $\int_0^1 x\ln(x^2+1)dx$
4. $\int_1^e x^2\ln xdx$
5. $\int_1^e \dfrac{\ln^3 x}{x^3}dx$
6. $\int_1^e x\ln xdx$

7. $\int_0^1 x\ln(x^2+1)dx$

8. $\int_1^e x^2\ln x dx$

9. $\int_0^{\frac{\pi}{2}} (x+c\text{osx})\,\text{s inx}dx$

10. $\int_1^e (x+\frac{1}{x})\ln x dx$

11. $\int_1^2 \ln(x^2+x)dx$

12. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} x\tan^2 x dx$

13. $\int_1^2 \frac{\ln x}{x^5}dx$

14. $\int_0^{\frac{\pi}{2}} x\cos x dx$

15. $\int_0^1 xe^x dx$

16. $\int_0^{\frac{\pi}{2}} e^x\cos x dx$

III. TÍCH PHÂN HÀM HỮU TỶ:

1. $\int_3^5 \frac{2x-1}{x^2-3x+2}dx$

2. $\int_a^b \frac{1}{(x+a)(x+b)}dx$

3. $\int_0^1 \frac{x^3+x+1}{x+1}dx$

4. $\int_0^1 \frac{x^3+x+1}{x^2+1}dx$

5. $\int_0^1 \frac{x^2}{(3x+1)^3}dx$

6. $\int_0^1 \frac{1}{(x+2)^2(x+3)^2}dx$

7. $\int_1^2 \frac{1-x^{2008}}{x(1+x^{2008})}dx$

8. $\int_{-1}^0 \frac{2x^3-6x^2+9x+9}{x^2-3x+2}dx$

9. $\int_2^3 \frac{x^4}{(x^2-1)^2}dx$

10. $\int_0^1 \frac{x^{2n-3}}{(1+x^2)^n}dx$

11. $\int_1^2 \frac{x^2-3}{x(x^4+3x^2+2)}dx$

12. $\int_1^2 \frac{1}{x(1+x^4)}dx$

13. $\int_0^2 \frac{1}{4+x^2}dx$

14. $\int_0^1 \frac{x}{1+x^4}dx$

15. $\int_0^2 \frac{1}{x^2-2x+2}dx$

16. $\int_0^1 \frac{x}{(1+x^2)^3}dx$

17. $\int_2^4 \frac{1}{x^3-2x^2+x}dx$

18. $\int_2^3 \frac{3x^2+3x+3}{x^3-3x+2}dx$

19. $\int_1^2 \frac{1-x^2}{1+x^4}dx$

20. $\int_0^1 \frac{1}{1+x^3}dx$

21. $\int_0^1 \frac{x^6+x^5+x^4+2}{x^6+1}dx$

22. $\int_0^1 \frac{2-x^4}{1+x^2}dx$

23. $\int_0^1 \frac{1+x^4}{1+x^6}dx$

24. $\int_0^1 \frac{4x+11}{x^2+5x+6}dx$

25. $\int_0^1 \frac{dx}{x^2+x+1}$

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IV. TÍCH PHÂN HÀM LƯỢNG GIÁC:

1. $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^4 x dx$

2. $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^3 x dx$

3. $\int_0^{\frac{\pi}{2}} \sin^4 x\cos^5 x dx$

4. $\int_0^{\frac{\pi}{2}} (\sin^3 x+\cos^3)dx$

5. $\int_0^{\frac{\pi}{2}} \cos 2x(\sin^4 x+\cos^4 x)dx$

6. $\int_0^{\frac{\pi}{2}} (2\sin^2 x-\sin x\cos x-\cos^2 x)dx$

7. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin x}dx$

8. $\int_0^{\frac{\pi}{2}} (\sin^{10} x+\cos^{10} x-\cos^4 x\sin^4 x)dx$

9. $\int_0^{\frac{\pi}{2}} \frac{dx}{2-\cos x}$

10. $\int_0^{\frac{\pi}{2}} \frac{1}{2+\sin x}dx$

11. $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{1+\cos^2 x}dx$

12. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin^4 x.\cos x}$

13. $\int_0^{\frac{\pi}{4}} \frac{dx}{\sin^2 x+2\sin x\cos x-\cos^2 x}$

14. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\cos x}dx$

15. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{2-\cos x}dx$

16. $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2+\sin x}dx$

17. $\int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{1+\cos x}dx$

18. $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x+\cos x+1}dx$

19. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos x dx}{(1-\cos x)^2}$

20. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x-\cos x+1}{\sin x+2\cos x+3}dx$

21. $\int_0^{\frac{\pi}{4}} tg^3 x dx$

22. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot g^3 x dx$

23. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} tg^4 x dx$

24. $\int_0^{\frac{\pi}{4}} \frac{1}{1+tgx}dx$

25. $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos x\cos(x+\frac{\pi}{4})}$

26. $\int_0^{\frac{\pi}{2}} \frac{\sin x+7\cos x+6}{4\sin x+5\cos x+5}dx$

27. $\int_0^{2\pi} \sqrt{1+\sin x}dx$

28. $\int_0^{\frac{\pi}{4}} \frac{dx}{2\sin x+3\cos x+\sqrt{13}}$

29. $\int_0^{\frac{\pi}{4}} \frac{4\sin^3 x}{1+\cos^4 x}dx$

30. $\int_0^{\frac{\pi}{2}} \frac{1+\cos 2x+\sin 2x}{\sin x+\cos x}dx$

31. $\int_0^{\frac{\pi}{2}} \frac{\sin 3x}{1+\cos x}dx$

32. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{dx}{\sin 2x-\sin x}$

33. $\int_0^{\frac{\pi}{4}} \frac{\sin^3 x}{\cos^2 x}dx$

34. $\int_0^{\frac{\pi}{2}} \sin 2x(1+\sin^2 x)^3 dx$

35. $\int_0^{\pi} |\cos x|\sqrt{\sin x}dx$

36. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sqrt[3]{\sin^3 x-\sin x}}{\sin^3 x tgx}dx$

37. $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x+\cos x}$

38. $\int_0^{\frac{\pi}{2}} \frac{dx}{2\sin x+1}$

39. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^3 x \sin^5 x dx$

40. $\int_{0}^{\frac{\pi}{4}} \frac{\sin 4x dx}{1+\cos^2 x}$

41. $\int_{0}^{\frac{\pi}{2}} \frac{dx}{5\sin x + 3}$

2. $\int_{\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{dx}{\sin^4 x \cos x}$

43. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin x \sin(x+\frac{\pi}{6})}$

4. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{dx}{\sin x \cos(x+\frac{\pi}{4})}$

45. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin^2 x dx}{\cos^6 x}$

46. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tgxtg(x+\frac{\pi}{6})dx$

47. $\int_{0}^{\frac{\pi}{3}} \frac{4\sin x dx}{(\sin x + \cos x)^3}$

48. $\int_{-\frac{\pi}{2}}^{0} \frac{\sin 2x}{(2+\sin x)^2}$

49. $\int_{0}^{\frac{\pi}{2}} \sin \sqrt[3]{x} dx$

50. $\int_{0}^{\frac{\pi}{2}} x^2 \cos x dx$

51. $\int_{0}^{\frac{\pi}{2}} \sin 2x.e^{2x+1} dx$

52. $\int_{0}^{\frac{\pi}{2}} \frac{1+\sin x}{1+\cos x} e^x dx$

53. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin 3x \sin 4x}{tgx + \cot g 2x} dx$

54. $\int_{0}^{\frac{\pi}{2}} \frac{\sin 2x dx}{\sin^2 x - 5\sin x + 6}$

55. $\int_{1}^{2} \cos(\ln x) dx$

56. $\int_{\pi/6}^{\pi/3} \frac{\ln(\sin x)}{\cos^2 x} dx$

57. $\int_{0}^{\frac{\pi}{2}} (2x-1)\cos^2 x dx$

58. $\int_{0}^{\pi} x \sin x \cos^2 x dx$

59. $\int_{0}^{\frac{\pi}{4}} xtg^2 x dx$

60. $\int_{0}^{\pi} e^{2x} \sin^2 x dx$

61. $\int_{0}^{\frac{\pi}{2}} e^{\sin^2 x} \sin x \cos^3 x dx$

62. $\int_{0}^{\frac{\pi}{4}} \ln(1+tgx) dx$

63. $\int_{0}^{\frac{\pi}{4}} \frac{dx}{(\sin x + 2\cos x)^2}$

64. $\int_{0}^{\frac{\pi}{2}} \frac{(1-\sin x)\cos x}{(1+\sin x)(2-\cos^2 x)} dx$

65. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x \sin 7x dx$

66. $\int_{0}^{\frac{\pi}{2}} \cos x(\sin^4 x + \cos^4 x)dx$

67. $\int_{0}^{\frac{\pi}{2}} \frac{4\sin^3 x}{1+\cos x} dx$

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V. TÍCH PHÂN HÀM VÔ TỶ:

$\int_a^b R(x, f(x))dx$

Trong đó R(x, f(x)) có các dạng:

+) R(x, $\sqrt{\frac{a-x}{a+x}}$) Đặt x = a cos2t, t $\in [0;\frac{\pi}{2}]$

+) R(x, $\sqrt{a^2-x^2}$) Đặt x = $|a|\sin t$ hoặc x = $|a|\cos t$

+) R(x, $\sqrt[n]{\frac{ax+b}{cx+d}}$) Đặt t = $\sqrt[n]{\frac{ax+b}{cx+d}}$

+) R(x, f(x)) = $\frac{1}{(ax+b)\sqrt{\alpha x^2+\beta x+\gamma}}$ Với $(\alpha x^2+\beta x+\gamma)' = k(ax+b)$

Khi đó đặt t = $\sqrt{\alpha x^2+\beta x+\gamma}$, hoặc đặt t = $\frac{1}{ax+b}$

+) R(x, $\sqrt{a^2+x^2}$) Đặt x = $|a|tgt$, t $\in [-\frac{\pi}{2};\frac{\pi}{2}]$

+) R(x, $\sqrt{x^2-a^2}$) Đặt x = $\frac{|a|}{\cos x}$, t $\in [0;\pi]\setminus\{\frac{\pi}{2}\}$

+) R$\left(\sqrt[n_1]{x};\sqrt[n_2]{x};...;\sqrt[n_i]{x}\right)$ Gọi k = BCNH(n_1; n_2; ...; n_i)

Đặt $x = t^k$

1. $\int_{\sqrt{5}}^{2\sqrt{3}} \frac{dx}{x\sqrt{x^2+4}}$

2. $\int_{\frac{2}{\sqrt{3}}}^{\sqrt{2}} \frac{dx}{x\sqrt{x^2-1}}$

3. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{dx}{(2x+3)\sqrt{4x^2+12x+5}}$

4. $\int_{1}^{2} \frac{dx}{x\sqrt{x^3+1}}$

5. $\int_1^2 \sqrt{x^2+2008}dx$

6. $\int_1^2 \frac{dx}{\sqrt{x^2+2008}}$

7. $\int_0^1 x^2\sqrt{1+x^2}\,dx$

8. $\int_0^1 \sqrt{(1-x^2)^3}\,dx$

9. $\int_1^{\sqrt{3}} \frac{x^2+1}{x^2\sqrt{x^2+1}}dx$

10. $\int_0^{\frac{\sqrt{2}}{2}} \sqrt{\frac{1+x}{1-x}}dx$

11. $\int_0^1 \frac{dx}{\sqrt{(1+x^2)^3}}$

12. $\int_0^{\frac{\sqrt{2}}{2}} \frac{dx}{\sqrt{(1-x^2)^3}}$

13. $\int_0^1 \sqrt{1+x^2}\,dx$

14. $\int_0^{\frac{\sqrt{2}}{2}} \frac{x^2dx}{\sqrt{1-x^2}}$

15. $\int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\sqrt{7+\cos 2x}}$

16. $\int_0^{\frac{\pi}{2}} \sin x\sqrt{\cos x-\cos^2 x}dx$

17. $\int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\sqrt{2+\cos^2 x}}$

18. $\int_0^{\frac{\pi}{2}} \frac{\sin 2x+\sin x}{\sqrt{1+3\cos x}}dx$

19. $\int_0^{\sqrt{7}} \frac{x^3dx}{\sqrt[3]{1+x^2}}$

20. $\int_0^3 x^3\sqrt{10-x^2}\,dx$

21. $\int_0^1 \frac{xdx}{\sqrt{2x+1}}$

22. $\int_0^1 \frac{x^3dx}{x+\sqrt{x^2+1}}$

23. $\int_2^7 \frac{dx}{\sqrt{2x+1}+1}$

24. $\int_0^1 x^{15}\sqrt{1+3x^8}\,dx$

25. $\int_0^{\frac{\pi}{2}} \sqrt[6]{1-\cos^3 x}\sin x\cos^5 x dx$

26. $\int_0^{\ln 3} \frac{dx}{\sqrt{e^x+1}}$

27. $\int_{-1}^1 \frac{dx}{1+x+\sqrt{x^2+1}}$

28. $\int_0^{\ln 2} \frac{e^{2x}dx}{\sqrt{e^x+1}}$

29. $\int_{\frac{5}{4}}^1 \sqrt{12x-4x^2-8}dx$

30. $\int_1^e \frac{\sqrt{1+3\ln x}\ln x}{x}dx$

31. $\int_0^{\sqrt{3}} \frac{x^5+x^3}{\sqrt{1+x^2}}dx$

32. $\int_0^4 \sqrt{x^3-2x^2+x}dx$

33. $\int_{-1}^0 x(e^{2x}+\sqrt[3]{x+1})dx$

34. $\int_{\ln 2}^{\ln 3} \frac{\ln^2 x}{x\sqrt{\ln x+1}}dx$

35. $\int_0^{\frac{\pi}{3}} \frac{\sqrt{\frac{\cos 2x}{\cos^2 x} + 2\sqrt{3}tgx}}{\cos^2 x} dx$

36. $\int_0^{\ln 2} \frac{e^x dx}{\sqrt{(e^x+1)^3}}$

37. $\int_0^{\frac{\pi}{3}} \frac{\cos x dx}{\sqrt{2+\cos 2x}}$

38. $\int_0^{\frac{\pi}{2}} \frac{\cos x dx}{\sqrt{1+\cos^2 x}}$

39. $\int_0^{7} \frac{x+2}{\sqrt[3]{x+3}} dx$

40. $\int_0^{2a} \sqrt{x^2+a^2}\, dx$

VI. MỘT SỐ TÍCH PHÂN ĐẶC BIỆT:

Bài toán mở đầu: Hàm số f(x) liên tục trên [-a; a], khi đó: $\int_{-a}^{a} f(x)dx = \int_0^a [f(x)+f(-x)]dx$

Ví dụ: +) Cho f(x) liên tục trên [$-\frac{3\pi}{2}; \frac{3\pi}{2}$] thỏa mãn f(x) + f(-x) = $\sqrt{2-2\cos 2x}$,

Tính: $\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} f(x)dx$

+) Tính $\int_{-1}^{1} \frac{x^4+\sin x}{1+x^2} dx$

Bài toán 1: Hàm số y = f(x) liên tục và lẻ trên [-a, a], khi đó: $\int_{-a}^{a} f(x)dx = 0$.

Ví dụ: Tính: $\int_{-1}^{1} \ln(x+\sqrt{1+x^2})dx$ $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x \ln(x+\sqrt{1+x^2})dx$

Bài toán 2: Hàm số y = f(x) liên tục và chẵn trên [-a, a], khi đó: $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

Ví dụ: Tính $\int_{-1}^{1} \frac{|x|dx}{x^4-x^2+1}$ $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x+\cos x}{4-\sin^2 x} dx$

Bài toán 3: Cho hàm số y = f(x) liên tục, chẵn trên [-a, a], khi đó: $\int_{-a}^{a} \frac{f(x)}{1+b^x} dx = \int_0^a f(x)dx$ ($1 \neq b>0$, $\forall$ a)

Ví dụ: Tính: $\int_{-3}^{3} \frac{x^2+1}{1+2^x} dx$ $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin x \sin 3x \cos 5x}{1+e^x} dx$

Bài toán 4: Nếu y = f(x) liên tục trên [0; $\frac{\pi}{2}$], thì $\int_0^{\frac{\pi}{2}} f(\sin x) = \int_0^{\frac{\pi}{2}} f(\cos x)dx$

Ví dụ: Tính $\int_0^{\frac{\pi}{2}} \frac{\sin^{2009} x}{\sin^{2009} x + \cos^{2009} x} dx$ $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

Bài toán 5: Cho f(x) xác định trên [-1; 1], khi đó: $\int_0^{\pi} xf(\sin x)dx = \frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$

Ví dụ: Tính $\int_0^{\pi} \frac{x}{1+\sin x} dx$ $\int_0^{\pi} \frac{x\sin x}{2+\cos x} dx$

Bài toán 6: $\int_a^b f(a+b-x)dx = \int_a^b f(x)dx \quad \Rightarrow \quad \int_0^b f(b-x)dx = \int_0^b f(x)dx$

Ví dụ: Tính $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$ $\int_0^{\frac{\pi}{4}} \sin 4x \ln(1+tgx)dx$

Bài toán 7: Nếu f(x) liên tục trên R và tuần hoàn với chu kì T thì:

$$\int_a^{a+T} f(x)dx = \int_0^T f(x)dx \quad \Rightarrow \quad \int_0^{nT} f(x)dx = n\int_0^T f(x)dx$$

Ví dụ: Tính $\int_0^{2008\pi} \sqrt{1-\cos 2x}dx$

Các bài tập áp dụng:

1. $\int_{-1}^{1} \frac{\sqrt{1-x^2}}{1+2^x} dx$
2. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^7 - x^5 + x^3 - x + 1}{\cos^4 x} dx$
3. $\int_{-1}^{1} \frac{dx}{(1+e^x)(1+x^2)}$
4. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x + \cos x}{4 - \sin^2 x} dx$
5. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos 2x \ln(\frac{1-x}{1+x})dx$
6. $\int_0^{2\pi} \sin(\sin x + nx)dx$
7. $\int_{-\pi/2}^{\pi/2} \frac{\sin^5 x}{\sqrt{1+\cos x}} dx$
8. $\int_{\frac{1}{e}}^{tga} \frac{xdx}{1+x^2} + \int_{\frac{1}{e}}^{\cot ga} \frac{dx}{x(1+x^2)} = 1$ (tga>0)

VII. TÍCH PHÂN HÀM GIÁ TRỊ TUYỆT ĐỐI:

1. $\int_{-3}^{3} |x^2 - 1|dx$
2. $\int_0^2 |x^2 - 4x + 3|dx$

3. $\int_0^2 |x^2 - x|dx \int_0^1 x|x - m|dx$

4. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|dx$

5. $\int_{-\pi}^{\pi} \sqrt{1-\sin x}dx$

6. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{tg^2 x + \cot g^2 x - 2}dx$

7. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} |\sin 2x|dx$

8. $\int_0^{2\pi} \sqrt{1+\cos x}dx$

9. $\int_{-2}^{5} (|x+2| - |x-2|)dx$

10. $\int_0^3 |2^x - 4|dx$

11. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{3}} \cos x\sqrt{\cos x - \cos^3 x}dx$

12.