Japan Today's Calculation Of Integral 2010

522 Find $\lim_{a\to\infty}\frac{1}{a^2}\int_0^a \ln(1+e^x)dx.$

523 Prove the following inequality.

$$\ln\frac{\sqrt{2009}+\sqrt{2010}}{\sqrt{2008}+\sqrt{2009}}<\int_{\sqrt{2008}}^{\sqrt{2009}}\frac{\sqrt{1-e^{-x^2}}}{x}\,dx<\sqrt{2009}-\sqrt{2008}$$

524 Evaluate the following definite integral.

$$2^{2009}\frac{\int_0^1 x^{1004}(1-x)^{1004}\,dx}{\int_0^1 x^{1004}(1-x^{2010})^{1004}\,dx}$$

525 Let $a,\ b$ be real numbers satisfying $\int_0^1 (ax+b)^2dx=1.$

Determine the values of $a,\ b$ for which $\int_0^1 3x(ax+b)\,dx$ is maximized.

526 For a function satisfying $f'(x)>0$ for $a\leq x\leq b$, let $F(x)=\int_a^b |f(t)-f(x)|\,dt$. For what value of x is $F(x)$ is minimized?

527 Let $n,\ m$ be positive integers and $\alpha,\ \beta$ be real numbers.
Prove the following equations.

(1) $\int_\alpha^\beta (x-\alpha)(x-\beta)\,dx=-\frac{1}{6}(\beta-\alpha)^3$

(2) $\int_\alpha^\beta (x-\alpha)^n(x-\beta)\,dx=-\frac{n!}{(n+2)!}(\beta-\alpha)^{n+2}$

(3) $\int_\alpha^\beta (x-\alpha)^n(x-\beta)^m\,dx=(-1)^m\frac{n!m!}{(n+m+1)!}(\beta-\alpha)^{n+m+1}$

528 Consider a function $f(x)=xe^{-x^3}$ defined on any real numbers.

(1) Examine the variation and convexity of $f(x)$ to draw the garph of $f(x)$.

(2) For a positive number C, let D_1 be the region bounded by $y=f(x)$, the x-axis and $x=C$. Denote $V_1(C)$ the volume obtained by rotation of D_1 about the x-axis. Find $\lim_{C\to\infty}V_1(C)$.

(3) Let M be the maximum value of $y=f(x)$ for $x\geq 0$. Denote D_2 the region bounded by $y=f(x)$, the y-axis and $y=M$. Find the volume V_2 obtained by rotation of D_2 about the y-axis.

529 Prove that the following inequality holds for each natural number n.

$$\int_0^{\frac{\pi}{2}}\sum_{k=1}^n\left(\frac{\sin kx}{k}\right)^2dx<\frac{61}{144}\pi$$

530 Answer the following questions.

(1) By setting $x+\sqrt{x^2-1}=t$, find the indefinite integral $\int\sqrt{x^2-1}\,dx$.

(2) Given two points $P(p,\ q)\ (p>1,\ q>0)$ and $A(1,\ 0)$ on the curve $x^2-y^2=1$. Find the area S of the figure bounded by two lines $OA,\ OP$ and the curve in terms of p.

(3) Let $S=\frac{\theta}{2}$. Express $p,\ q$ in terms of θ.

531 (1) Let $f(x)$ be a continuous function defined on $[a,\ b]$, it is known that there exists some c such that

$$\int_a^b f(x)\,dx=(b-a)f(c)\ (a<c<b)$$

Explain the fact by using graph. Note that you don't need to prove the statement.

(2) Let $f(x)=a_0+a_1x+a_2x^2+\cdots\cdots+a_nx^n$,

Prove that there exists θ such that

$$f(\sin\theta)=a_0+\frac{a_1}{2}+\frac{a_3}{3}+\cdots\cdots+\frac{a_n}{n+1},\ 0<\theta<\frac{\pi}{2}.$$

532 For a curve $C: y=x\sqrt{9-x^2}\ (x\geq 0)$,

(1) Find the maximum value of the function.

(2) Find the area of the figure bounded by the curve C and the x-axis.

(3) Find the volume of the solid by revolution of the figure in (2) around the y-axis.

Please find the volume without using cylindrical shells for my students.

Last Edited.

533 Let C be the circle with radius 1 centered on the origin. Fix the endpoint of the string with length 2π on the point $A(1,\ 0)$ and put the other end point P on the point $P_0(1,\ 2\pi)$. From this situation, when we twist the string around C by moving the point P in anti clockwise with the string streched tightly, find the length of the curve that the point P draws from sarting point P_0 to reaching point A.

534 Find the indefinite integral $\int \frac{x^3}{(x-1)^3(x-2)}\ dx$.

535 Let C be the parameterized curve for a given positive number r and $0\leq t\leq \pi$,

$$C: \begin{cases} x=2r(t-\sin t\cos t) \\ y=2r\sin^2 t \end{cases}$$

When the point P moves on the curve C,

(1) Find the magnitude of acceleralation of the point P at time t.

(2) Find the length of the locus by which the point P sweeps for $0\leq t\leq \pi$.

(3) Find the volume of the solid by rotation of the region bounded by the curve C and the x-axis about the x-axis.

Edited.

536 Evaluate $\int_0^{\frac{\pi}{4}} \frac{x+\sin x}{1+\cos x}\ dx$.

537 Evaluate $\int_0^{\frac{\pi}{6}} \frac{\sqrt{1+\sin x}}{\cos x}\ dx$.

538 Evaluate $\int_1^{\sqrt{2}} \frac{x^2+1}{x\sqrt{x^4+1}}\ dx$.

539 Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^3 x}\ dx$.

540 Evaluate $\int_1^{e} \frac{\sqrt[3]{x}}{x(\sqrt{x}+\sqrt[3]{x})}\ dx$.

541 Find the functions $f(x),\ g(x)$ satisfying the following equations.

(1) $f'(x)=2f(x)+10,\ f(0)=0$

(2) $\int_0^x u^3 g(u)du=x^4+g(x)$

542 Find continuous functions $f(x),\ g(x)$ which takes positive value for any real number x, satisfying $g(x)=\int_0^x f(t)\ dt$ and $\{f(x)\}^2-\{g(x)\}^2=1$.

543 Let y be the function of x satisfying the differential equation $y''-y=2\sin x$.

(1) Let $y=e^x u-\sin x$, find the differential equation with which the function u with respect to x satisfies.

(2) If $y(0)=3,\ y'(0)=0$, then determine y.

544 (1) Evaluate $\int_{-\sqrt{3}}^{\sqrt{3}}(x^2-1)dx,\ \int_{-\sqrt{3}}^{\sqrt{3}}(x-1)^2dx,\ \int_{-\sqrt{3}}^{\sqrt{3}}(x+1)^2dx$.

(2) If a linear function $f(x)$ satifies $\int_{-\sqrt{3}}^{\sqrt{3}}(x-1)f(x)dx=5\sqrt{3},\ \int_{-\sqrt{3}}^{\sqrt{3}}(x+1)f(x)dx=3\sqrt{3}$, then we have $f(x)=\boxed{A}(x-1)+\boxed{B}(x+1)$, thus we have $f(x)=\boxed{C}$.

545 (1) Evaluate $\int_0^1 xe^{x^2}dx$.

(2) Let $I_n=\int_0^1 x^{2n-1}e^{x^2}dx$. Express I_{n+1} in terms of I_n.

546 Find the minimum value of $\int_0^{\pi}\left(x-\pi a-\frac{b}{\pi}\cos x\right)^2 dx$.

547 Find the minimum value of $\int_0^1 |e^{-x}-a|dx\ (-\infty<a<\infty)$.

548 For $f(x)=e^{\frac{x}{2}}\cos\frac{x}{2}$, evaluate $\sum_{n=0}^{\infty}\int_{-\pi}^{\pi}f(x)f(x-2n\pi)dx\ (n=0,\ 1,\ 2,\ \cdots)$.

549 Let $f(x)$ be a function defined on $[0,\ 1]$. For $n=1,\ 2,\ 3,\ \cdots$, a polynomial $P_n(x)$ is defined by $P_n(x)=\sum_{k=0}^{n}{}_nC_k f\left(\frac{k}{n}\right)x^k(1-x)^{n-k}$. Prove that $\lim_{n\to\infty}\int_0^1 P_n(x)dx=\int_0^1 f(x)dx$.

550 Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{(1+\cos x)^2}$.

551 In the coordinate plane, find the area of the region bounded by the curve $C:y=\frac{x+1}{x^2+1}$ and the line $L:y=1$.

552 Find the positive value of a such that the curve $C_1:x=\sqrt{2y^2+\frac{25}{2}}$ tangent to the parabola $C_2:y=ax^2$, then find the equation of the tangent line of C_1 at the point of tangency.

553 Find the continuous function such that $f(x)=\frac{e^{2x}}{2(e-1)}\int_0^1 e^{-y}f(y)dy+\int_0^{\frac12}f(y)dy+\int_0^{\frac12}\sin^2(\pi y)dy$.

554 Use $\frac{d}{dx}\ln(2x+\sqrt{4x^2+1}),\ \frac{d}{dx}(x\sqrt{4x^2+1})$ to evaluate $\int_0^1\sqrt{4x^2+1}dx$.

555 For $\frac{1}{e}<t<1$, find the minimum value of $\int_0^1 |xe^{-x}-tx|dx$.

556 Prove the following inequality.

$$\sqrt[3]{\int_0^{\frac{\pi}{4}}\frac{x}{\cos^2 x\cos^2(\tan x)\cos^2(\tan(\tan x))\cos^2(\tan(\tan(\tan x)))}dx}<\frac{4}{\pi}$$

Last Edited.

Sorry, I have changed the problem.

kunny

557 Find the folllowing limit.

$$\lim_{n\to\infty} \frac{(2n+1)\int_0^1 x^{n-1}\sin\left(\frac{\pi}{2}x\right)dx}{(n+1)^2\int_0^1 x^{n-1}\cos\left(\frac{\pi}{2}x\right)dx}\ \ (n=1,\ 2,\ \cdots).$$

558 For a positive constant t, let $\alpha,\ \beta$ be the roots of the quadratic equation $x^2+t^2x-2t=0$.

Find the minimum value of $\int_{-1}^2 \left\{\left(x+\frac{1}{\alpha^2}\right)\left(x+\frac{1}{\beta^2}\right)+\frac{1}{\alpha\beta}\right\}dx.$

559 In xyz space, consider two points $P(1,\ 0,\ 1),\ Q(-1,\ 1,\ 0)$. Let S be the surface generated by rotation the line segment PQ about x axis. Answer the following questions.

(1) Find the volume of the solid bounded by the surface S and two planes $x=1$ and $x=-1$.

(2) Find the cross-section of the solid in (1) by the plane $y=0$ to sketch the figure on the palne $y=0$.

(3) Evaluate the definite integral $\int_0^1 \sqrt{t^2+1}\ dt$ by substitution $t=\frac{e^s-e^{-s}}{2}$.

Then use this to find the area of (2).

560 Let K be the figure bounded by the graph of function $y=\frac{x}{\sqrt{1-x^2}}$, x axis and the line $x=\frac{1}{2}$.

(1) Find the volume V_1 of the solid generated by rotation of K around x axis.

(2) Find the volume V_2 of the solid generated by rotation of K around y axis.

Please solve question (2) without using the shell method for Japanese High School Students those who don't learn it.

561 Evaluate

$$\int_{-1}^1 \frac{1+2x^2+3x^4+4x^6+5x^8+6x^{10}+7x^{12}}{\sqrt{(1+x^2)(1+x^4)(1+x^6)}}dx.$$

562 (1) Show the following inequality for every natural number k.

$$\frac{1}{2(k+1)}<\int_0^1 \frac{1-x}{k+x}dx<\frac{1}{2k}$$

(2) Show the following inequality for every natural number $m,\ n$ such that $m>n$.

$$\frac{m-n}{2(m+1)(n+1)}<\log \frac{m}{n}-\sum_{k=n+1}^m \frac{1}{k}<\frac{m-n}{2mn}$$

563 Determine the pair of constant numbers $a,\ b,\ c$ such that for a quadratic function $f(x)=x^2+ax+b$, the following equation is identity with respect to x.

$$f(x+1)=c\int_0^1 (3x^2+4xt)f'(t)dt$$

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564 In the coordinate plane with $O(0,\ 0)$, consider the function $C:\ y=\frac{1}{2}x+\sqrt{\frac{1}{4}x^2+2}$ and two distinct points $P_1(x_1,\ y_1),\ P_2(x_2,\ y_2)$ on C.

(1) Let $H_i\ (i=1,\ 2)$ be the intersection points of the line passing through $P_i\ (i=1,\ 2)$, parallel to x axis and the line $y=x$.
Show that the area of $\triangle OP_1H_1$ and $\triangle OP_2H_2$ are equal.

(2) Let $x_1 < x_2$. Express the area of the figure bounded by the part of $x_1 \leq x \leq x_2$ for C and line segments $P_1O,\ P_2O$ in terms of $y_1,\ y_2$.

565 Prove that $f(x) = \int_0^1 e^{-|t-x|}t(1-t)dt$ has maximal value at $x = \frac{1}{2}$.

566 In the coordinate space, consider the cubic with vertices $O(0,\ 0,\ 0),\ A(1,\ 0,\ 0),\ B(1,\ 1,\ 0),\ C(0,\ 1,\ 0),\ D(0,\ 0,\ 1),\ E(1,\ 0,\ 1),\ F(1,\ 1,\ 1),\ G(0,\ 1,\ 1)$. Find the volume of the solid generated by revolution of the cubic around the diagonal OF as the axis of rotation.

567 Let a be a positive real numbers. In the coordinate plane denote by S the area of the figure bounded by the curve $y = \sin x\ (0 \leq x \leq \pi)$ and the x-axis and denote T by the area of the figure bounded by the curves

$y = \sin x\ \left(0 \leq x \leq \frac{\pi}{2}\right), y = a\cos x\ \left(0 \leq x \leq \frac{\pi}{2}\right)$ and the x-axis. Find the value of a such that $S : T = 3 : 1$.

568 Throw n balls in to $2n$ boxes.□Suppose each ball comes into each box with equal probability of entering in any boxes. Let p_n be the probability such that any box has ball less than or equal to one. Find the limit $\lim_{n\to\infty} \frac{\ln p_n}{n}$

569 In the coordinate plane, denote by $S(a)$ the area of the region bounded by the line passing through the point $(1,\ 2)$ with the slope a and the parabola $y = x^2$. When a varies in the range of $0 \leq a \leq 6$, find the value of a such that $S(a)$ is minimized.

570 Let $f(x) = 1 - \cos x - x\sin x$.

(1) Show that $f(x) = 0$ has a unique solution in $0 < x < \pi$.

(2) Let $J = \int_0^{\pi} |f(x)|dx$. Denote by α the solution in (1), express J in terms of $\sin\alpha$.

(3) Compare the size of J defined in (2) with $\sqrt{2}$.

571 Evaluate $\int_0^{\pi} \frac{x\sin^3 x}{\sin^2 x + 8}dx$.

572 For integer $n,\ a_n$ is difined by $a_n = \int_0^{\frac{\pi}{4}} (\cos x)^n dx$.

(1) Find $a_{-2},\ a_{-1}$.

(2) Find the relation of a_n and a_{n-2}.

(3) Prove that $a_{2n} = b_n + \pi c_n$ for some rational number $b_n,\ c_n$, then find c_n for $n < 0$.

573 Find the area of the figure bounded by three curves

$C_1 : y = \sin x\ \left(0 \leq x < \frac{\pi}{2}\right)$

$C_2 : y = \cos x\ \left(0 \leq x < \frac{\pi}{2}\right)$

$C_3 : y = \tan x\ \left(0 \leq x < \frac{\pi}{2}\right)$.

574 Let n be a positive integer. Prove that $x^n e^{1-x} \leq n!$ for $x \geq 0$,

575

For a function $f(x) = \int_x^{\frac{\pi}{4}-x} \log_4(1 + \tan t)dt\ \left(0 \leq x \leq \frac{\pi}{8}\right)$, answer the following questions.

(1) Find $f'(x)$.

(2) Find the n th term of the sequence a_n such that $a_1 = f(0),\ a_{n+1} = f(a_n)\ (n = 1,\ 2,\ 3,\ \cdots)$.

576 For a function $f(x) = (\ln x)^2 + 2\ln x$, let C be the curve $y = f(x)$. Denote $A(a,\ f(a)),\ B(b,\ f(b))\ (a < b)$ the points of tangency of two tangents drawn from the origin O to C and the curve C. Answer the following questions.

(1) Examine the increase and decrease, extremal value and inflection point , then draw the approximate garph of the curve C .

(2) Find the values of $a,\ b$.

(2) Find the values of $a,\ b$.

(3) Find the volume by a rotation of the figure bounded by the part from the point A to the point B and line segments $OA,\ OB$ around the y-axis.

577 Prove the following inequality for any integer $N\geq 4$.

$$\sum_{p=4}^N \frac{p^2+2}{(p-2)^4}<5$$

578 Find the range of k for which the following inequality holds for $0\leq x\leq 1$.

$$\int_0^x \frac{dt}{\sqrt{(3+t^2)^3}}\geq k\int_0^x \frac{dt}{\sqrt{3+t^2}}$$

If necessary, you may use $\ln 3=1.10$.

579 Let a be a positive real number. Find $\lim_{n\to\infty} \frac{(n+1)^a+(n+2)^a+\cdots+(n+n)^a}{1^a+2^a+\cdots+n^a}$

580 Let k be a positive constant number. Denote $\alpha,\ \beta\ (0<\beta<\alpha)$ the x coordinates of the curve $C: y=kx^2\ (x\geq 0)$ and two lines $l: y=kx+\frac{1}{k},\ m: y=-kx+\frac{1}{k}$.□Find the minimum area of the part bounded by the curve C and two lines $l,\ m$.

581 For real numer c for which $cx^2\geq \ln (1+x^2)$ for all real numbers x, find the value of c such that the area of the figure bounded by two curves $y=cx^2$ and $y=\ln (1+x^2)$ and two lines $x=1,\ x=-1$ is 4.

582 Prove the following inequality.

$$\frac{\pi}{4}\sqrt{\frac{3}{2}+\sqrt{2}}<\int_0^{\frac{\pi}{2}} \sqrt{1-\frac{1}{2}\sin ^ 2 x}\ dx<\frac{\sqrt{3}}{4}\pi$$

583 Find the values of k such that the areas of the three parts bounded by the graph of $y=-x^4+2x^2$ and the line $y=k$ are all equal.

584 Find $\lim_{x\to\infty} \left(\int_0^x \sqrt{1+e^{2t}}\ dt-e^x\right)$.

585 Evaluate $\int_0^{\ln 2} (x-\ln 2)e^{-2\ln (1+e^x)+x+\ln 2}dx$.

586 Evaluate $\int_0^1 \frac{x^{14}}{x^2+1}\ dx$.

587 Evaluate $\int_0^1 \frac{(x^2+3x)e^x-(x^2-3x)e^{-x}+2}{\sqrt{1+x(e^x+e^{-x})}}\ dx$.

588 Evaluate $\int_0^{\frac{\pi}{2}} e^{xe^x}\{(x+1)e^x(\cos x+\sin x)+\cos x-\sin x\}dx$.

589 Evaluate $\int_0^1 \frac{x}{\{(2x-1)\sqrt{x^2+x+1}+(2x+1)\sqrt{x^2-x+1}\}\sqrt{x^4+x^2+1}}\ dx$.

590 Evaluate $\int_0^{\frac{\pi}{8}} \frac{(\cos \theta+\sin \theta)^{\frac{3}{2}}-(\cos \theta-\sin \theta)^{\frac{3}{2}}}{\sqrt{\cos 2\theta}}\ d\theta$.

591 Let $a,\ b,\ c$ be real numbers such that $a\geq b\geq c\geq 1$.

Prove the following inequality:

$$\int_0^1 \{(1-ax)^3+(1-bx)^3+(1-cx)^3-3x\}\ dx \geq ab+bc+ca-\frac{3}{2}(a+b+c)-\frac{3}{4}abc.$$

592 Prove the following inequality.

$$\frac{\sqrt{2}}{4}-\frac{1}{2}-\frac{1}{4}\ln 2<\int_0^{\frac{\pi}{4}}\ln \cos x\ dx<\frac{3}{8}\pi+\frac{1}{2}-\ln{(3+2\sqrt{2})}$$

593 For a positive integer m, prove the following ineqaulity.

$$0\leq \int_0^1 \left(x+1-\sqrt{x^2+2x\cos \frac{2\pi}{2m+1}+1}\right)dx\leq 1.$$

1996 Osaka University entrance exam

594 In the x-y plane, two variable points $P,\ Q$ stay in $P(2t,\ -2t^2+2t),\ Q(t+2,-3t+2)$ at the time t.
Let denote t_0 as the time such that $\overline{PQ}=0$. When t varies in the range of $0\leq t\leq t_0$, find the area of the region swept by the line segment PQ in the x-y plane.

595 Evaluate $\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}} \left|\frac{4\sin x}{\sqrt{3}\cos x-\sin x}\right|dx.$

2009 Kumamoto University entrance exam/Medicine

596 Find the minimum value of $\int_0^{\frac{\pi}{2}} |a\sin 2x-\cos ^2 x|dx\ (a>0).$

2009 Shimane University entrance exam/Medicine

597 In space given a board shaped the equilateral triangle PQR with vertices $P\left(1,\ \frac{1}{2},\ 0\right)$, $Q\left(1,-\frac{1}{2},\ 0\right)$, $R\left(\frac{1}{4},\ 0,\ \frac{\sqrt{3}}{4}\right)$. When S is revolved about the z-axis, find the volume of the solid generated by the whole points through which S passes.

1984 Tokyo University entrance exam/Science

598 For a constant a, denote $C(a)$ the part $x\geq 1$ of the curve $y=\sqrt{x^2-1}+\frac{a}{x}$.

(1) Find the maximum value a_0 of a such that $C(a)$ is contained to lower part of $y=x$, or $y<x$.

(2) For $0<\theta <\frac{\pi}{2}$, find the volume $V(\theta)$ of the solid V obtained by revolving the figure bounded by $C(a_0)$ and three lines $y=x,\ x=1,\ x=\frac{1}{\cos \theta}$ about the x-axis.

(3) Find $\lim_{\theta\rightarrow \frac{\pi}{2}-0} V(\theta)$.

1992 Tokyo University entrance exam/Science, 2nd exam

599 Evaluate $\int_0^{\frac{\pi}{6}} \frac{e^x(\sin x+\cos x+\cos 3x)}{\cos ^ 2 2x}\ dx.$

created by kunny

600 Evaluate $\int_{-a}^a \left(x+\frac{1}{\sin x+\frac{1}{e^x-e^{-x}}}\right)\ dx\ (a>0).$

created by kunny

601 Evaluate $\int_0^{\frac{\pi}{4}} (\tan x)^{\frac{3}{2}}dx.$

created by kunny

Prove the following inequality.

602 Prove the following inequality.

$$\frac{e-1}{n+1} \leqq \int_1^e (\log x)^n dx \leqq \frac{(n+1)e+1}{(n+1)(n+2)}\ (n=1,2,\cdots)$$

1994 Kyoto University entrance exam/Science

603 Find the minimum value of $\int_0^1 \{\sqrt{x}-(a+bx)\}^2 dx.$

Please solve the problem without using partial differentiation for those who don't learn it.

1961 Waseda University entrance exam/Science and Technology

604 Let r be a positive integer. Determine the value of a for which the limit value $\lim_{n\to\infty} \frac{\sum_{k=1}^n k^r}{n^a}$ has a non zero finite value, then find the limit value.

1956 Tokyo Institute of Technology entrance exam

605 Let $f(x)$ be a differentiable function. Find the following limit value:

$$\lim_{n\to\infty} \binom{n}{k} \left\{f\left(\frac{x}{n}\right)-f(0)\right\}^k.$$

Especially, for $f(x)=(x-\alpha)(x-\beta)$ find the limit value above.

1956 Tokyo Institute of Technology entrance exam

606 Find the area of the part bounded by two curves $y=\sqrt{x},\ \sqrt{x}+\sqrt{y}=1$ and the x-axis.

1956 Tokyo Institute of Technology entrance exam

607 On the coordinate plane, Let C be the graph of $y=(\ln x)^2\ (x>0)$ and for $\alpha>0$, denote $L(\alpha)$ be the tangent line of C at the point $(\alpha,\ (\ln \alpha)^2)$.

(1) Draw the graph.

(2) Let $n(\alpha)$ be the number of the intersection points of C and $L(\alpha)$. Find $n(\alpha)$.

(3) For $0<\alpha<1$, let $S(\alpha)$ be the area of the region bounded by $C,\ L(\alpha)$ and the x-axis. Find $S(\alpha)$.

2010 Tokyo Institute of Technology entrance exam, Second Exam.

608 For $a>0$, find the minimum value of $\int_0^1 \frac{ax^2+(a^2+2a)x+2a^2-2a+4}{(x+a)(x+2)}dx.$

2010 Gakusyuin University entrance exam/Science

609 Prove that for positive number t, the function $F(t)=\int_0^t \frac{\sin x}{1+x^2}dx$ always takes positive number.

1972 Tokyo University of Education entrance exam

610 Evaluate $\int_2^a \frac{x^a-1-xa^x\ln a}{(x^a-1)^2}dx.$

proposed by kunny

611 Let $g(t)$ be the minimum value of $f(x)=x2^{-x}$ in $t\leq x\leq t+1$.

Evaluate $\int_0^2 g(t)dt.$

2010 Kumamoto University entrance exam/Science

612 For $f(x)=\frac{1}{x}\ (x>0)$, prove the following inequality.

$$f\left(t+\frac{1}{2}\right) \leq \int_t^{t+1} f(x)\ dx \leq \frac{1}{6}\left\{f(t)+4f\left(t+\frac{1}{2}\right)+f(t+1)\right\}$$

613 Find the area of the part, in the x-y plane, enclosed by the curve $|ye^{2x}-6e^x-8|=-(e^x-2)(e^x-4).$

2010 Tokyo University of Agriculture and Technology entrance exam

614 Evaluate $\int_0^1 \{x(1-x)\}^{\frac{3}{2}}dx.$

2010 Hirosaki University School of Medicine entrance exam

615 For $0\leq a\leq 2$, find the minimum value of $\int_0^2 \left|\frac{1}{1+e^x}-\frac{1}{1+e^a}\right|\ dx.$

2010 Kyoto Institute of Technology entrance exam/Textile e.t.c.

616 Evaluate $\int_1^3 \frac{\ln(x+1)}{x^2}dx.$

2010 Hirosaki University entrance exam

617 Let $y=f(x)$ be a function of the graph of broken line connected by points $(-1,\ 0),\ (0,\ 1),\ (1,\ 4)$ in the x -y plane.

Find the minimum value of $\int_{-1}^1 \{f(x)-(a|x|+b)\}^2dx.$

2010 Tohoku University entrance exam/Economics, 2nd exam

618 Find the minimu value of $\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{x\cos t+(1-x)\sin t\}^2dt.$

2010 Ibaraki University entrance exam/Science

619 Consider a function $f(x)=\frac{\sin x}{9+16\sin^2 x}\ \left(0\leq x\leq \frac{\pi}{2}\right)$. Let a be the value of x for which $f(x)$ is maximized.

Evaluate $\int_a^{\frac{\pi}{2}} f(x)\ dx.$

2010 Saitama University entrance exam/Mathematics

Last Edited

620 Let $a,\ b$ be real numbers. Suppose that a function $f(x)$ satisfies $f(x)=a\sin x+b\cos x+\int_{-\pi}^{\pi} f(t)\cos t\ dt$ and has the maximum value 2π for $-\pi \leq x\leq \pi$.

Find the minimum value of $\int_{-\pi}^{\pi} \{f(x)\}^2dx.$

2010 Chiba University entrance exam

621 Find the limit $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^n k\ln\left(\frac{n^2+(k-1)^2}{n^2+k^2}\right)$.

2010 Yokohama National University entrance exam/Engineering, 2nd exam

622 For $0<k<2$, consider two curves $C_1: y=\sin 2x\ (0\leq x\leq \pi),\ C_2: y=k\cos x\ (0\leqq x\leqq \pi).$
Denote by $S(k)$ the sum of the areas of four parts enclosed by $C_1,\ C_2$ and two lines $x=0,\ x=\pi.$
Find the minimum value of $S(k).$

2010 Nagoya Institute of Technology entrance exam

623 Find the continuous function satisfying the following equation.

$$\int_0^x f(t)dt+\int_0^x tf(x-t)dt=e^{-x}-1.$$

1978 Shibaura Institute of Technology entrance exam

624 Find the continuous function $f(x)$ such that the following equation holds for any real number x.

$$\int_0^x \sin t\cdot f(x-t)dt=f(x)-\sin x.$$

1977 Keio University entrance exam/Medicine

625 Find $\lim_{t\rightarrow 0}\frac{1}{t^3}\int_0^{t^2} e^{-x}\sin \frac{x}{t}\ dx\ (t\neq 0).$

2010 Kumamoto University entrance exam/Medicine

626 Find $\lim_{a\rightarrow +0}\int_a^1 \frac{x\ln x}{(1+x)^3}dx.$

2010 Nara Medical University entrance exam

627 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(2\sin \theta+1)\cos ^ 3 \theta}{(\sin ^ 2 \theta+1)^2}d\theta.$

Proposed by kunny

628 (1) Evaluate the following definite integrals.

(a) $\int_0^{\frac{\pi}{2}} \cos ^ 2 x\sin x\ dx$

(b) $\int_0^{\frac{\pi}{2}} (\pi -2x)\cos x\ dx$

(c) $\int_0^{\frac{\pi}{2}} x\cos ^ 3 x\ dx$

(2) Let a be a positive constant. Find the area of the cross section cut by the plane $z=\sin \theta \ \left(0\leq \theta \leq \frac{\pi}{2}\right)$ of the solid such that

$$x^2+y^2+z^2\leq a^2,\ \ x^2+y^2\leq ax,\ \ z\geq 0$$

, then find the volume of the solid.

1984 Yamanashi Medical University entrance exam

Please slove the problem without multi integral or arcsine function for Japanese high school students aged 17-18 those who don't study them.

Thanks in advance.

kunny

629 Evaluate $\int_0^{\infty} \frac{1}{e^x(1+e^{4x})}dx.$

630 Evaluate $\int_0^{\infty} \frac{\ln (1+e^{4x})}{e^x}dx.$

631 Evaluate $\int_{\sqrt{2}}^{\sqrt{3}} (x^2+\sqrt{x^4-1})\left(\frac{1}{\sqrt{x^2+1}}+\frac{1}{\sqrt{x^2-1}}\right)dx.$

Proposed by kunny

632 Find $\lim_{n\to\infty}\int_0^{\pi} |\sin nx|^a dx\ (n=1,\ 2,\ \cdots).$

2010 Kyoto Institute of Technology entrance exam/Textile, 2nd exam

633 Let $f(x)$ be a differentiable function. Find the value of x for which

$$\{f(x)\}^2+(e+1)f(x)+1+e^2-2\int_0^x f(t)dt-2f(x)\int_0^x f(t)dt+2\left\{\int_0^x f(t)dt\right\}^2$$

is minimized.

1978 Tokyo Medical College entrance exam

634 Prove that :

$$\int_1^{\sqrt{e}}(\ln x)^n dx=(-1)^{n-1}n!+\sqrt{e}\sum_{m=0}^n(-1)^{n-m}\frac{n!}{m!}\left(\frac{1}{2}\right)^m\ (n=1,\ 2,\ \cdots)$$

2010 Miyazaki University entrance exam/Medicine

635 Suppose that a function $f(x)$ defined in $-1<x<1$ satisfies the following properties (i) , (ii), (iii).

(i) $f'(x)$ is continuous.

(ii) When $-1<x<0,\ f'(x)<0,\ f'(0)=0$, when $0<x<1,\ f'(x)>0.$

(iii) $f(0)=-1$

Let $F(x)=\int_0^x \sqrt{1+\{f'(t)\}^2}dt\ (-1<x<1)$. If $F(\sin\theta)=c\theta\ (c:\text{constant})$ holds for $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$, then find $f(x)$.

1975 Waseda University entrance exam/Science and Technology

636 Let $a>1$ be a constant. In the xy-plane, let $A(a,\ 0),\ B(a,\ \ln a)$ and C be the intersection point of the curve $y=\ln x$ and the x-axis. Denote by S_1 the area of the part bounded by the x-axis, the segment BA and the curve $y=\ln x$

(1) For $1\leq b\leq a$, let $D(b,\ \ln b)$. Find the value of b such that the area of quadrilateral $ABDC$ is the closest to S_1 and find the area S_2.

(2) Find $\lim_{a\to\infty}\frac{S_2}{S_1}$.

1992 Tokyo University entrance exam/Science

637 For a non negative integer n, set t $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\ dx$ to answer the following questions:

(1) Calculate $I_{n+2}+I_n$.

(2) Evaluate the values of $I_1,\ I_2$ and I_3.

1978 Niigata university entrance exam

638 Let $(a,\ b)$ be a point on the curve $y=\frac{x}{1+x}\ (x\geq 0)$. Denote U the volume of the figure enclosed by the curve , the x axis and the line $x=a$, revolved around the the x axis and denote V the volume of the figure enclosed by the curve , the y axis and th line $y=b$, revolved around the y axis. What's the relation of U and V?

1978 Chuo university entrance exam/Science and Technology

639 Evaluate $\int_0^1 (x+3)\sqrt{xe^x}\ dx.$

640 Evaluate $\int_0^{\frac{\pi}{4}}\frac{1}{1-\sin x}\sqrt{\frac{\cos x}{1+\cos x+\sin x}}dx.$

Own

641 Evaluate

$$\int_{e^e}^{e^{e^e}} \left\{\ln(\ln(\ln x)) + \frac{1}{(\ln x)\ln(\ln x)}\right\} dx.$$

Own

642 Evaluate

$$\int_0^{\frac{\pi}{6}} \frac{(\tan^2 2x)\sqrt{\cos 2x}+2}{(\cos^2 x)\sqrt{\cos 2x}} dx.$$

Own

643 Evaluate

$$\int_0^{\pi} \frac{x}{\sqrt{1+\sin^3 x}}\{(3\pi\cos x+4\sin x)\sin^2 x+4\}dx.$$

Own

644 For a constant p such that $\int_1^p e^x dx = 1$, prove that

$$\left(\int_1^p e^x\cos x\ dx\right)^2+\left(\int_1^p e^x\sin x\ dx\right)^2>\frac{1}{2}.$$

Own

645 Prove the following inequality.

$$\int_{-1}^1 \frac{e^x+e^{-x}}{e^{e^{e^x}}}dx < e-\frac{1}{e}$$

Own

646 Evaluate

$$\int_0^{\pi} a^x\cos bx\ dx,\ \int_0^{\pi} a^x\sin bx\ dx\ (a>0,\ a\neq 1,\ b\in\mathbb{N}^+)$$

Own

647 Evaluate

$$\int_0^{\pi} xp^x\cos qx\ dx,\ \int_0^{\pi} xp^x\sin qx\ dx\ (p>0,\ p\neq 1,\ q\in\mathbb{N}^+)$$

Own

648 Consider a function real-valued function with C^{∞}-class on $\mathbb{R}$ such that:

(a) $f(0)=\frac{df}{dx}(0)=0,\ \frac{d^2f}{dx^2}(0)\neq 0.$

(b) For $x\neq 0,\ f(x)>0.$

Judge whether the following integrals $(i),\ (ii)$ converge or diverge, justify your answer.

(i)

$$\int\int_{|x_1|^2+|x_2|^2\leq 1}\frac{dx_1dx_2}{f(x_1)+f(x_2)}.$$

(ii)

$$\int\int_{|x_1|^2+|x_2|^2+|x_3|^2\leq 1}\frac{dx_1dx_2dx_3}{f(x_1)+f(x_2)+f(x_3)}.$$

2010 Kyoto University, Master Course in Mathematics

649 Let $f_n(x,\ y)=\frac{n}{r\cos \pi r+n^2r^3}\ (r=\sqrt{x^2+y^2})$,

$I_n=\int\int_{r\leq 1}f_n(x,\ y)\ dxdy\ (n\geq 2)$.

Find $\lim_{n\to\infty} I_n$.

2009 Tokyo Institute of Technology, Master Course in Mathematics

650 Find the values of $p,\ q,\ r\ (-1<p<q<r<1)$ such that for any polynomials with degree≤ 2, the following equation holds:

$$\int_{-1}^p f(x)\ dx-\int_p^q f(x)\ dx+\int_q^r f(x)\ dx-\int_r^1 f(x)\ dx=0.$$

1995 Hitotsubashi University entrance exam/Law, Economics etc.

651 Find

$$\lim_{n\to\infty}\int_0^{2n}e^{-2x}\left|x-2\lfloor\frac{x+1}{2}\rfloor\right|\ dx.$$

1985 Tohoku University entrance exam/Mathematics, Physics, Chemistry, Biology

652 Let $a,\ b,\ c$ be positive real numbers such that $b^2>ac.$
Evaluate

$$\int_0^{\infty}\frac{dx}{ax^4+2bx^2+c}.$$

1981 Tokyo University, Master Course

653 Sign me in. If is not to late :)

654 A function $f(x)$ defined in $x\geq 0$ satisfies $\lim_{x\to\infty}\frac{f(x)}{x}=1.$

Find $\int_0^{\infty}\{f(x)-f'(x)\}e^{-x}dx.$

1997 Hokkaido University entrance exam/Science

655 Find the area of the region of the points such that the total of three tangent lines can be drawn to two parabolas $y=x-x^2,\ y=a(x-x^2)\ (a\geq 2)$ in such a way that there existed the points of tangency in the first quadrant.

656 Find $\lim_{n\to\infty} n\int_0^{\frac{\pi}{2}}\frac{1}{(1+\cos x)^n}dx\ (n=1,\ 2,\ \cdots).$

657 A sequence a_n is defined by $\int_{a_n}^{a_{n+1}} (1+|\sin x|)dx = (n+1)^2\ (n=1,\ 2,\ \cdots),\ a_1=0.$

Find $\lim_{n\to\infty} \frac{a_n}{n^3}$.

658 Consider a parameterized curve $C: x=e^{-t}\cos t,\ y=e^{-t}\sin t\left(0\leq t\leq \frac{\pi}{2}\right).$

(1) Find the length L of C.

(2) Find the area S of the region enclosed by the $x,\ y$ axis and C.

Please solve the problem without using the formula of area for polar coordinate for Japanese High School Students who don't study it in High School.

1997 Kyoto University entrance exam/Science

659 Evaluate $\int_0^1 \frac{\ln(x+2)}{x+1}dx.$

660 Let $a,\ b$ be given positive constants.

Evaluate

$$\int_0^1 \frac{\ln\ (x+a)^{x+a}(x+b)^{x+b}}{(x+a)(x+b)}dx.$$

Own

661 Consider a sequence $1^{0.01},\ 2^{0.02},\ 2^{0.02},\ 3^{0.03},\ 3^{0.03},\ 3^{0.03},\ 4^{0.04},\ 4^{0.04},\ 4^{0.04},\ 4^{0.04},\ \cdots.$

(1) Find the 36th term.

(2) Find $\int x^2\ln x\ dx.$

(3) Let A be the product of from the first term to the 36th term. How many digits does A have integer part?

If necessary, you may use the fact $2.0<\ln 8<2.1,\ 2.1<\ln 9<2.2,\ 2.30<\ln 10<2.31.$

2010 National Defense Medical College Entrance Exam, Problem 4

662 In xyz space, let A be the solid generated by a rotation of the figure, enclosed by the curve $y=2-2x^2$ and the x-axis about the y-axis.

(1) When the solid is cut by the plane $x=a\ (|a|\leq 1)$, find the inequality which expresses the figure of the cross-section.

(2) Denote by L the distance between the point $(a,\ 0,\ 0)$ and the point on the perimeter of the cross-section found in (1), find the maximum value of L.

(3) Find the volume of the solid by a rotation of the solid A about the x-axis.

1987 Sophia University entrance exam/Science and Technology

663 Given are the curve $y=x^2+x-2$ and a curve which is obtained by tranfering the curve symmetric with respect to the point $(p,\ 2p)$. Let p change in such a way that these two curves intersects, find the maximum area of the part bounded by these curves.
1978 Nagasaki University entrance exam/Economics

664 For a positive integer n, let $I_n=\int_{-\pi}^{\pi} \left(\frac{\pi}{2}-|x|\right)\cos nx\ dx.$

Find $I_1+I_2+I_3+I_4$.

1992 University of Fukui entrance exam/Medicine

665 Find $\lim_{n\to\infty} \int_0^{\pi} x|\sin 2nx|dx\ (n=1,\ 2,\ \cdots).$

1992 Japan Women's University entrance exam/Physics, Mathematics