Japan Today's Calculation Of Integral 2007

168 Prove that $\sum_{n=1}^{\infty}\int_{\frac{1}{n+1}}^{\frac{1}{n}}\left|\frac{1}{x}\sin\frac{\pi}{x}\right|dx$ diverge for $x>0.$

169 (1) Let $f(x)$ be the differentiable and increasing function such that $f(0)=0$.Prove that
$\int_0^1 f(x)f'(x)dx \geq \frac{1}{2}\left(\int_0^1 f(x)dx\right)^2.$

(2) $g_n(x)=x^{2n+1}+a_nx+b_n\ (n=1,\ 2,\ 3,\ \cdots)$ satisfies $\int_{-1}^1 (px+q)g_n(x)dx=0$ for all linear equations $px+q.$
Find $a_n,\ b_n.$

170 Let $a,\ b$ be constant numbers such that $a^2\geq b.$
Find the following definite integrals.

(1) $I=\int\frac{dx}{x^2+2ax+b}$

(2) $J=\int\frac{dx}{(x^2+2ax+b)^2}$

171 Evaluate $\int_0^1 x^{2007}(1-x^2)^{1003}dx.$

172 Evaluate $\int_{-1}^0\sqrt{\frac{1+x}{1-x}}dx.$

173 Find the function $f(x)$ such that $f(x)=\cos(2mx)+\int_0^{\pi} f(t)|\cos t|\ dt$ for positive inetger $m.$

174 Let a be a positive number. Assume that the parameterized curve $C:\ x=t+e^{at},\ y=-t+e^{at}\ (-\infty<t<\infty)$ is touched to x axis.

(1) Find the value of $a.$
(2) Find the area of the part which is surrounded by two straight lines $y=0, y=x$ and the curve $C.$

175 Evaluate $\sum_{n=0}^{\infty}\frac{1}{(2n+1)2^{2n+1}}.$

176 Let $f_n(x)=\sum_{k=1}^n\frac{\sin kx}{\sqrt{k(k+1)}}.$

Find $\lim_{n\to\infty}\int_0^{2\pi}\{f_n(x)\}^2dx.$

177 On xyplane the parabola $K:\ y=\frac{1}{d}x^2\ (d:\ positive\ constant\ number)$ intersects with the line $y=x$ at the point P that is different from the origin.
Assumed that the circle C is touched to K at P and y axis at the point $Q.$
Let S_1 be the area of the region surrounded by the line passing through two points $P,\ Q$ and $K,$ or S_2 be the area of the region surrounded by the line which is passing through P and parallel to x axis and $K.$ Find the value of $\frac{S_1}{S_2}.$

178 Let $f(x)$ be a differentiable function such that $f'(x)+f(x)=4xe^{-x}\sin 2x,\quad f(0)=0.$

Find $\lim_{n\to\infty}\sum_{k=1}^n f(k\pi).$

179 Evaluate the following integrals.

(1) Meiji University

$\int^e\frac{(\log x)^2}{}dx.$

$\int_{\frac{1}{e}} \quad x$

(2) Tokyo University of Science

$$\int_0^1 \frac{7x^3+23x^2+21x+15}{(x^2+1)(x+1)^2}dx.$$

180 Let a_n be the area surrounded by the curves $y=e^{-x}$ and the part of $y=e^{-x}|\cos x|,\ (n-1)\pi \leq x \leq n\pi\ (n=1,\ 2,\ 3,\ \cdots).$
Evaluate $\lim_{n\to\infty}(a_1+a_2+\cdots+a_n).$

181 For real number $a,$ find the minimum value of $\int_0^{\frac{\pi}{2}}\left|\frac{\sin 2x}{1+\sin^2 x}-a\cos x\right|dx.$

182 Find the area of the domain of the system of inequality

$$y(y-|x^2-5|+4)\leq 0,\quad y+x^2-2x-3\leq 0.$$

183 Let $n\geq 2$ be integer. On a plane there are $n+2$ points $O,\ P_0,\ P_1,\ \cdots P_n$ which satisfy the following conditions as follows.

[1] $\angle P_{k-1}OP_k=\frac{\pi}{n}\ (1\leq k\leq n),\ \angle OP_{k-1}P_k=\angle OP_0P_1\ (2\leq k\leq n).$

[2] $\overline{OP_0}=1,\ \overline{OP_1}=1+\frac{1}{n}.$

Find $\lim_{n\to\infty}\sum_{k=1}^n \overline{P_{k-1}P_k}.$

184 (1) For real numbers $x,\ a$ such that $0<x<a,$ prove the following inequality.

$$\frac{2x}{a}<\int_{a-x}^{a+x}\frac{1}{t}\,dt<x\left(\frac{1}{a+x}+\frac{1}{a-x}\right).$$

(2) Use the result of (1) to prove that $0.68<\ln 2<0.71.$

185 Evaluate the following integrals.

(1) $\int_0^{\frac{\pi}{4}}\frac{dx}{1+\sin x}.$

(2) $\int_{\frac{4}{3}}^2\frac{dx}{x^2\sqrt{x-1}}.$

186 For $a>0,$ find $\lim_{a\to\infty}a^{-(\frac{3}{2}+n)}\int_0^a x^n\sqrt{1+x}\,dx\ (n=1,\ 2,\ \cdots).$

187 For a constant $a,$ let $f(x)=ax\sin x+x+\frac{\pi}{2}.$ Find the range of a such that $\int_0^{\pi}\{f'(x)\}^2\ dx\geq f\left(\frac{\pi}{2}\right).$

188 Find the volume of the solid obtained by revolving the region bounded by the graphs of $y=xe^{1-x}$ and $y=x$ around the x axis.

189 Let n be positive integers. Denote the graph of $y=\sqrt{x}$ by $C,$ and the line passing through two points $(n,\ \sqrt{n})$ and $(n+1,\ \sqrt{n+1})$ by $l.$ Let V be the volume of the solid obtained by revolving the region bounded by C and l around the x axis.Find the positive numbers $a,\ b$ such that $\lim_{n\to\infty}n^aV=b.$

190 In xyz space, let l be the segment joining two points $(1,\ 0,\ 1)$ and $(1,\ 0,\ 2),$ and A be the figure obtained by revolving l around the z axis. Find the volume of the solid obtained by revolving A around the x axis.

Note you may not use double integral.

191 (1) For integer $n=0,\ 1,\ 2,\ \cdots$ and positive number a_n, let $f_n(x)=a_n(x-n)(n+1-x).$ Find a_n such that the curve $y=f_n(x)$ touches to the curve $y=e^{-x}.$

(2) For $f_n(x)$ defined in (1), denote the area of the figure bounded by $y=f_0(x), y=e^{-x}$ and the y-axis by $S_0,$ for $n\geq 1,$ the area of the figure bounded by $y=f_{n-1}(x),\ y=f_n(x)$ and $y=e^{-x}$ by $S_n.$ Find $\lim_{n\to\infty}(S_0+S_1+\cdots+S_n).$

192 Let t be positive number. Draw two tangent lines to the palabola $y=x^2$ from the point $(t,-1)$. Denote the area of the region bounded by these tangent lines and the parabola by $S(t)$. Find the minimum value of $\frac{S(t)}{\sqrt{t}}$.

193 For $a>0$, let l be the line created by rotating the tangent line to parabola $y=x^2$, which is tangent at point $A(a,a^2)$, around A by $-\frac{\pi}{6}$.

Let B be the other intersection of l and $y=x^2$. Also, let C be $(a,0)$ and let O be the origin.

(1) Find the equation of l.

(2) Let $S(a)$ be the area of the region bounded by OC, CA and $y=x^2$. Let $T(a)$ be the area of the region bounded by AB and $y=x^2$. Find $\lim_{a\to\infty}\frac{T(a)}{S(a)}$.

194 Evaluate

$$\sum_{n=0}^{2006}\int_0^1 \frac{dx}{2(x+n+1)\sqrt{(x+n)(x+n+1)}}$$

195 Find continuous functions $x(t),\ y(t)$ such that

$$x(t)=1+\int_0^t e^{-2(t-s)}x(s)ds$$

$$y(t)=\int_0^t e^{-2(t-s)}\{2x(s)+3y(s)\}ds$$

196 Calculate

$$\frac{\int_0^{\pi} e^{-x}\sin^n x\,dx}{\int_0^{\pi} e^{x}\sin^n x\,dx}\ (n=1,\ 2,\ \cdots).$$

197 Let $|a|<\frac{\pi}{2}$. Evaluate the following definite integral.

$$\int_0^{\frac{\pi}{2}}\frac{dx}{\{\sin(a+x)+\cos x\}^2}$$

198 Compare the values of the following definite integrals.

$$\int_0^{\infty}\ln\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2},\ \int_0^{\frac{\pi}{2}}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta$$

199 Let $m,\ n$ be non negative integers.
Calculate

$$\sum_{k=0}^{n}(-1)^k\frac{n+m+1}{k+m+1}\ nC_k.$$

where ${}_iC_j$ is a binomial coefficient which means $\frac{i\cdot(i-1)\cdots(i-j+1)}{j\cdot(j-1)\cdots 2\cdot 1}$.

200 Evaluate the following definite integral.

$$\int_0^{\pi}\frac{\cos nx}{2-\cos x}dx\ (n=0,\ 1,\ 2,\ \cdots)$$

201 Evaluate the following definite integral.

$$\int_{-1}^{1} \frac{e^{2x}+1-(x+1)(e^x+e^{-x})}{x(e^x-1)}dx$$

202 Let $a,\ b$ are real numbers such that $a+b=1$.

Find the minimum value of the following integral.

$$\int_0^{\pi} (a\sin x + b\sin 2x)^2\ dx$$

203 Let $\alpha,\ \beta$ be the distinct positive roots of the equation of $2x=\tan x$.

Evaluate the following definite integral.

$$\int_0^1 \sin\alpha x \sin\beta x\ dx$$

204 Evaluate

$$\int_0^1 \frac{x\ dx}{(x^2+x+1)^{\frac{3}{2}}}$$

205 Evaluate the following definite integral.

$$\int_{e^2}^{e^3} \frac{\ln x\cdot\ln(x\ln x)\cdot\ln\{x\ln(x\ln x)\}+\ln x+1}{\ln x\cdot\ln(x\ln x)}\ dx$$

206 Calculate $\displaystyle\int \frac{x^3}{(x-1)^3(x-2)}\ dx$

207 Evaluate the following definite integral.

$$\int_{e^e}^{e^{e+1}} \left\{\frac{1}{\ln x\cdot\ln(\ln x)}+\ln(\ln(\ln x))\right\}dx$$

208 Find the values of real numbers $a,\ b$ for which the function $f(x)=a|\cos x|+b|\sin x|$ has local minimum at $x=-\frac{\pi}{3}$ and satisfies $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\{f(x)\}^2dx=2.$

209 Let $m,\ n$ be the given distinct positive integers. Answer the following questions.

(1) Find the real number $\alpha\ (|\alpha|<1)$ such that $\displaystyle\int_{-\pi}^{\pi}\sin(m+\alpha)x\ \sin(n+\alpha)x\ dx=0.$

(2) Find the real number β satifying the sytem of equation $\displaystyle\int_{-\pi}^{\pi}\sin^2(m+\beta)x\ dx=\pi+\frac{2}{4m-1},$ $\displaystyle\int_{-\pi}^{\pi}\sin^2(n+\beta)x\ dx=\pi+\frac{2}{4n-1}.$

210 Evaluate $\displaystyle\int_1^{\pi}\left(x^3\ln x-\frac{6}{x}\right)\sin x\ dx.$

211 When the parabola which has the axis parallel to y -axis and passes through the origin touch to the rectangular hyperbola $xy=1$ in the first quadrant moves,
prove that the area of the figure sorrounded by the parabola and the x-axis is constant.

212 For integers $k\ (0\leq k\leq 5)$, positive numbers $m,\ n$ and real numbers $a,\ b$, let
$f(k)=\displaystyle\int_{-\pi}^{\pi}(\sin kx-a\sin mx-b\sin nx)^2\ dx,$

$p(k)=\frac{5!}{k!(5-k)!}\left(\frac{1}{2}\right)^5,\ E=\sum_{k=0}^5 p(k)f(k)$. Find the values of $m,\ n,\ a,\ b$ for which E is minimized.

213 Find the minimum value of $f(a)=\int_0^1 x|x-a|\ dx$.

214 Find the area of the region surrounded by the two curves $y=\sqrt{x},\ \sqrt{x}+\sqrt{y}=1$ and the x axis.

215 For $a\in\mathbb{R}$, let $M(a)$ be the maximum value of the function $f(x)=\int_0^{\pi}\sin(x-t)\sin(2t-a)\ dt$.

Evaluate $\int_0^{\frac{\pi}{2}} M(a)\sin(2a)\ da$.

216 Let a_n is a positive number such that $\int_0^{a_n}\frac{e^x-1}{1+e^x}\ dx=\ln n$.

Find $\lim_{n\to\infty}(a_n-\ln n)$.

217 Evaluate $\int_0^1 e^{\sqrt{e^x}}\ dx+2\int_e^{e^{\sqrt{e}}}\ln(\ln x)\ dx$.

218 For any quadratic functions $f(x)$ such that $f'(2)=1$, evaluate $\int_{2-\pi}^{2+\pi} f(x)\sin\left(\frac{x}{2}-1\right)dx$.

219 Let $f(x)=\left(1+\frac{1}{x}\right)^x\ \ (x>0)$.

Find $\lim_{n\to\infty}\left\{f\left(\frac{1}{n}\right)f\left(\frac{2}{n}\right)f\left(\frac{3}{n}\right)\cdots\cdots f\left(\frac{n}{n}\right)\right\}^{\frac{1}{n}}$.

220 Prove that $\frac{\pi}{2}-1<\int_0^1 e^{-2x^2}\ dx$.

221 Evaluate $\int_2^6 \ln\frac{-1+\sqrt{1+4x}}{2}\ dx$.

222 Find $\lim_{a\to\infty}\int_a^{a+1}\frac{x}{x+\ln x}\ dx$.

223 Evaluate $\int_0^{\pi}\sqrt{(\cos x+\cos 2x+\cos 3x)^2+(\sin x+\sin 2x+\sin 3x)^2}\ dx$.

224 Let $f(x)=x^2+|x|$. Prove that $\int_0^{\pi} f(\cos x)\ dx=2\int_0^{\frac{\pi}{2}} f(\sin x)\ dx$.

225 2 Points $P\left(a,\ \frac{1}{a}\right),\ Q\left(2a,\ \frac{1}{2a}\right)\ \ (a>0)$ are on the curve $C: y=\frac{1}{x}$. Let $l,\ m$ be the tangent lines at $P,\ Q$ respectively. Find the area of the figure surrounded by $l,\ m$ and C.

226 Evaluate $\int_0^{\frac{\pi}{2}}\frac{x^2}{(\cos x+x\sin x)^2}\ dx$

Virgil Nicula have already posted the integral :oops:

227 Evaluate $\frac{1}{\int_0^{\frac{\pi}{2}}\cos^{2006}x\cdot\sin 2008x\ dx}$

228 Let $x_n=\int_0^{\frac{\pi}{2}}\sin^n\theta\ d\theta\ (n=0,\ 1,\ 2,\ \ldots)$

228 Let $x_n = \int_0 \sin^n x \, dx \ (n = 0,\ 1,\ 2,\ \cdots)$.

(1) Show that $x_n = \frac{n-1}{n} x_{n-2}$.

(2) Find the value of $n x_n x_{n-1}$.

(3) Show that a sequence $\{x_n\}$ is monotone decreasing.

(4) Find $\lim_{n\to\infty} n x_n^2$.

229 Find $\lim_{a\to+\infty} \frac{\int_0^a \sin^4 x \, dx}{a}$.

230 Prove that $\frac{(-1)^n}{n!}\int_1^2 (\ln x)^n \ dx = 2\sum_{k=1}^n \frac{(-\ln 2)^k}{k!} + 1$.

231 Evaluate $\int_0^{\frac{\pi}{3}} \frac{1}{\cos^7 x} \ dx$.

232 For $f(x) = 1 - \sin x$, let $g(x) = \int_0^x (x-t)f(t) \ dt$.

Show that $g(x+y) + g(x-y) \geq 2g(x)$ for any real numbers $x,\ y$.

233 Find the minimum value of the following definite integral.

$\int_0^\pi (a\sin x + b\sin 3x - 1)^2 \ dx$.

234 For $x \geq 0$, define a function $f(x) = \sin\left(\frac{n\pi}{4}\right)\sin x \ (n\pi \leq x < (n+1)\pi) \ (n = 0,\ 1,\ 2,\ \cdots)$.

Evaluate $\int_0^{100\pi} f(x) \ dx$.

Show that a function $f(x) = \int_{-1}^1 (1 - |\ t\ |)\cos(xt) \ dt$ is continuous at $x = 0$.

235

236 Let a be a positive constant. Evaluate the following definite integrals $A,\ B$.

$$A = \int_0^\pi e^{-ax}\sin^2 x \ dx,\ \ B = \int_0^\pi e^{-ax}\cos^2 x \ dx$$

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1998 Shinsyu University entrance exam/Textile Science

237 Calculate $\int \frac{dx}{x^{2008}(1-x)}$

238 Find $\lim_{a\to\infty} \frac{1}{a^2}\int_0^a \log(1+e^x) \ dx$.

239 Evaluate $\int_0^\pi \sin(\pi\cos x) \ dx$.

240 2 curves $y = x^3 - x$ and $y = x^2 - a$ pass through the point P and have a common tangent line at P. Find the area of the region bounded by these curves.

241 1.Let $x = \alpha,\ \beta\ (\alpha < \beta)$ are x coordinates of the intersection points of a parabola $y = ax^2 + bx + c\ (a \neq 0)$ and the line $y = ux + v$.

Prove that the area of the region bounded by these graphs is $\boxed{\frac{|a|}{6}(\beta-\alpha)^3}$.

2. Let $x=\alpha,\ \beta\ (\alpha<\beta)$ are x coordinates of the intersection points of parabolas $y=ax^2+bx+c$ and $y=px^2+qx+r\ (ap\neq 0)$.

Prove that the area of the region bounded by these graphs is $\boxed{\frac{|a-p|}{6}(\beta-\alpha)^3}$.

242 A cubic function $y=ax^3+bx^2+cx+d\ (a\neq 0)$ touches a line $y=px+q$ at $x=\alpha$ and intersects $x=\beta\ (\alpha\neq\beta)$. Find the area of the region bounded by these graphs in terms of $a,\ \alpha,\ \beta$.

243 A cubic funtion $y=ax^3+bx^2+cx+d\ (a\neq 0)$ intersects with the line $y=px+q$ at $x=\alpha,\ \beta,\ \gamma\ (\alpha<\beta<\gamma)$. Find the area of the region bounded by these graphs in terms of $a,\ \alpha,\ \beta,\ \gamma$.

244 A quartic funtion $y=ax^4+bx^3+cx^2+dx+e\ (a\neq 0)$ touches the line $y=px+q$ at $x=\alpha,\ \beta\ (\alpha<\beta)$. Find the area of the region bounded by these graphs in terms of $a,\ \alpha,\ \beta$.

245 A sextic funtion $y=ax^6+bx^5+cx^4+dx^3+ex^2+fx+g\ (a\neq 0)$ touches the line $y=px+q$ at $x=\alpha,\ \beta,\ \gamma\ (\alpha<\beta<\gamma)$.
Find the area of the region bounded by these graphs in terms of $a,\ \alpha,\ \beta,\gamma$.

created by kunny

246 An eighth degree polynomial funtion $y=ax^8+bx^7+cx^6+dx^5+ex^4+fx^3+gx^2+hx+i\ (a\neq 0)$ touches the line $y=px+q$ at $x=\alpha,\ \beta,\ \gamma,\ \delta\ (\alpha<\beta<\gamma<\delta)$.
Find the area of the region bounded by these graphs in terms of $a,\ \alpha,\ \beta,\gamma,\ \delta$.

247 Evaluate $\displaystyle\int_{\frac{\pi}{8}}^{\frac{3}{8}\pi}\frac{11+4\cos 2x+\cos 4x}{1-\cos 4x}\,dx.$

248 Evaluate $\displaystyle\int_{\frac{\pi}{4}}^{\frac{3}{4}\pi}\cos\frac{1}{\sin\left(\frac{1}{\sin x}\right)}\cdot\cos\left(\frac{1}{\sin x}\right)\cdot\frac{\cos x}{\sin^2 x\cdot\sin^2\left(\frac{1}{\sin x}\right)}\,dx$

Last Edited, Sorry

kunny

249 Determine the sign of $\displaystyle\int_{\frac{1}{2}}^{2}\frac{\ln t}{1+t^n}\,dt\ (n=1,2,\cdots)$.

250 For a positive constant number p, find $\displaystyle\lim_{n\to\infty}\frac{1}{n^{p+1}}\sum_{k=0}^{n-1}\int_{2k\pi}^{(2k+1)\pi}x^p\sin^3 x\cos^2 x\,dx.$

251 Evaluate $\displaystyle\int_0^{n\pi}e^x\sin^4 x\,dx\ (n=1,\ 2,\ \cdots)$.

252 Compare $f(\theta)=\displaystyle\int_0^1(x+\sin\theta)^2\,dx$ and $g(\theta)=\displaystyle\int_0^1(x+\cos\theta)^2\,dx$ for $0\leqq\theta\leqq 2\pi$.

253 Evaluate $\displaystyle\int_0^1(1+x+x^2+\cdots+x^{n-1})\{1+3x+5x^2+\cdots+(2n-3)x^{n-2}+(2n-1)x^{n-1}\}\,dx.$

254 Evaluate $\displaystyle\int_e^{e^2}\frac{(\ln x)^7-7!}{(\ln x)^8}\,dx.$

Sorry, I have deleted my first post because that was wrong.

kunny

255 Find the value of a for which the area of the figure surrounded by $y=e^{-x}$ and $y=ax+3\ (a<0)$ is minimized.

256 Find the value of a for which $\displaystyle\int_0^{\pi}\{ax(\pi^2-x^2)-\sin x\}^2\,dx$ is minimized.