TỔNG HỢP CÁC BÀI TOÁN TÍCH PHÂN TRÊN BOXMATH

16 Tìm nguyên hàm

$$I=\int \Big(x(x+2)(3\sin x-4\sin^3 x)+2\cos x(\cos x-2\sin x)+3x^2\cos 3x-1\Big)e^x\ \mathrm{d}x$$

Lời giải

$$\Big(x(x+2)(3\sin x-4\sin^3 x)+2\cos x(\cos x-2\sin x)+3x^2\cos 3x-1\Big)e^x$$
$$=\Big(x^2\sin 3x+(x^2\sin 3x)'+\cos 2x+(\cos 2x)'\Big)e^x$$
$$\Rightarrow I=(x^2\sin 3x+\cos 2x)e^x$$

17 Tìm nguyên hàm

$$I=\int \frac{2x^4\ln^2 x+x\ln x(x^3+1)+x-\frac{1}{x^2}}{1+x^3\ln x}\,\mathrm{d}x$$

Lời giải

$$\frac{2x^6\ln^2 x+x^6\ln x+x^3\ln x+x^3-1}{x^2+x^5\ln x}$$
$$=\frac{2[(x^3\ln x)^2-1]+x^3(x^3\ln x+1)+(x^3\ln x+1)}{x^2(1+x^3\ln x)}$$
$$=\frac{(x^3\ln x+1)(2x^3\ln x+x^3-1)}{x^2(1+x^3\ln x)}=2x\ln x+x-\frac{1}{x^2}$$

Nên
$$I=\int\left(2x\ln x+x-\frac{1}{x^2}\right)\mathrm{d}x=\frac{1}{2}x^2+\frac{1}{x}+\int 2x\ln x\ \mathrm{d}x=\frac{1}{2}x^2+\frac{1}{x}+\int \ln x\ \mathrm{d}(x^2)$$
$$I=\frac{1}{2}x^2+\frac{1}{x}+x^2\ln x-\int x\ \mathrm{d}x=\frac{1}{x}+x^2\ln x+C$$

18 Tìm nguyên hàm

$$I=\int x^2\sin(\ln x)\ \mathrm{d}x$$

Lời giải

Đặt $x=e^t,\quad \ln x=t,\quad \mathrm{d}x=e^t\mathrm{d}t$

$$\Rightarrow I=\int e^{3t}\sin t\ \mathrm{d}t=-e^{3t}\cos t+\int 3e^{3t}\cos t\ \mathrm{d}t=-e^{3t}\cos t+3e^{3t}\sin t-\int 9e^{3t}\sin t\ \mathrm{d}t$$
$$\Rightarrow 10I=3e^{3t}\sin t-e^{3t}\cos t\Rightarrow I=\frac{1}{10}\Big(3.e^{3\ln x}\sin(\ln x)-e^{3\ln x}\cos(\ln x)\Big)+C$$

19 Tìm nguyên hàm

$$I=\int \frac{e^x(x-1)+2x^3+x^3(e^x+x(x^2+1))}{e^x.x+x^2(x^2+1)}\,\mathrm{d}x$$

Lời giải

$$\frac{e^x(x-1)+2x^3+x^3(e^x+x(x^2+1))}{e^x.x+x^2(x^2+1)}=\frac{x^3-1}{x}+\frac{3x^2+e^x+1}{x^3+x+e^x}=x^2-\frac{1}{x}+\frac{(x^3+x+e^x)'}{x^3+x+e^x}$$

Do đó

$$I=\frac{x^3}{3}-\ln|x|+\ln|x^3+x+e^x|+C$$

20 Tính tích phân

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\ln(\tan x)\ \mathrm{d}x$$

Lời giải

$$I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\ln(\tan x)\ \mathrm{d}x\overset{\text{đổi biến }(x=\frac{\pi}{2}-x)}{=}\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\ln(\cot x)\ \mathrm{d}x\Rightarrow 2I=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\ln(\tan x.\cot x)\ \mathrm{d}x=0\Rightarrow I=0$$