

# TỔNG HỢP CÁC BÀI TOÁN TÍCH PHÂN TRÊN BOXMATH

21 Tìm nguyên hàm

$$I = \int \frac{dx}{\sin^3 x + \cos^3 x}$$

Ta có 
$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{\text{Lời giải}}{(\sin x + \cos x)} = \frac{(\sin x + \cos x)}{(\sin x + \cos x)^2(1 - \sin x \cos x)} = \frac{(\sin x + \cos x)}{(1 + \sin 2x)(1 - \sin x \cos x)}$$

Đặt  $t = \sin x - \cos x, \quad \sin x \cos x = \frac{1 - t^2}{2}, dt = (\cos x + \sin x) dx$

$$I = \int \frac{dt}{(2 - t^2) \left(1 - \frac{1 - t^2}{2}\right)} = 2 \int \frac{dt}{(2 - t^2)(1 + t^2)} = \frac{2}{3} \int \left( \frac{1}{2 - t^2} + \frac{1}{1 + t^2} \right) dt$$

$$I = \frac{2}{3} \int \frac{dt}{2 - t^2} + \frac{2}{3} \int \frac{dt}{1 + t^2}$$

21 Tính Tích Phân

$$I = \int_{-\frac{\pi}{4}}^0 \frac{\sin 4x}{(1 + \sin x)(1 + \cos x)} dx$$

$$\text{Lời giải}$$

$$2(1 + \sin x)(1 + \cos x) = (\sin x + \cos x + 1)^2 = \frac{4 \sin 2x (\cos x + \sin x)(\cos x - \sin x)}{(\sin x + \cos x + 1)^2}$$

Đặt  $t = \cos x + \sin x, \quad \sin 2x = t^2 - 1, \quad dt = (\cos x - \sin x) dx, \quad x = -\frac{\pi}{4}, t = 0, \quad x = 0, t = 1$

$$I = \int_0^1 \frac{4(t^2 - 1)t}{(t + 1)^2} dt = 4 \int_0^1 \frac{t^2 - t}{t + 1} dt = 4 \int_0^1 \left( t - 2 + \frac{2}{t + 1} \right) dt$$

$$I = (2t^2 - 8t + 8 \ln(t + 1)) \Big|_0^1 = 2(4 \ln 2 - 3)$$

22 Tính Tích Phân

$$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{1 + x^2 + x^{98} + x^{100}}$$

$$\text{Lời giải}$$

$$I = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{(1 + x^2)(1 + x^{98})} \stackrel{x = \frac{1}{x}}{=} \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{x^2 \left(1 + \frac{1}{x^2}\right) \left(1 + \frac{1}{x^{98}}\right)} = \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{x^{98} dx}{(x^2 + 1)(x^{98} + 1)}$$

$$\Rightarrow I = \frac{1}{2} \int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{1 + x^2}$$