

BTVN BÀI BIẾN ĐỔI VÀ ĐỔI BIẾN NÂNG CAO TÍCH PHÂN HÀM

PHÂN THỨC HỮU TỈ

A. TÁCH CÁC MẪU SỐ CHỨA CÁC NHÂN TỬ KHÔNG ĐỒNG BẬC:

$$B_1 = \int \frac{dx}{x^3 + 5x} = \frac{1}{5} \int \frac{(x^2 + 5) - x^2}{(x^2 + 5)x} dx = \frac{1}{5} \int \frac{dx}{x} - \frac{1}{10} \int \frac{d(x^2 + 5)}{x^2 + 5} = \frac{1}{10} \ln \frac{x^2}{x^2 + 5} + C$$

$$B_3 = \int \frac{dx}{x^{11} - 8x^5} = \frac{1}{8} \int \frac{x^6 - (x^6 - 8)}{x^5(x^6 - 8)} dx = \frac{1}{8} \left(\int \frac{xdx}{x^6 - 8} - \int \frac{dx}{x^5} \right) = \frac{1}{64} \int \frac{d\left(\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)^3 - 1} - \frac{1}{8} \int \frac{dx}{x^5}$$

$$= \frac{1}{64} \int \frac{du}{u^3 - 1} - \frac{1}{8} \int \frac{dx}{x^5} = \frac{1}{64} \left[\frac{1}{6} \ln \left| \frac{u^2 - 2u + 1}{u^2 + u + 1} \right| - \frac{1}{2\sqrt{3}} \arctan \frac{2u + 1}{\sqrt{3}} \right] + \frac{1}{32x^4} + C$$

$$= \frac{1}{384} \ln \left| \frac{x^4 - 4x^2 + 4}{x^4 + 2x^2 + 4} \right| - \frac{1}{128\sqrt{3}} \arctan \frac{x^2 + 1}{\sqrt{3}} + \frac{1}{32x^4} + C$$

$$B_4 = \int \frac{dx}{x^6 + 9x} = \frac{1}{9} \int \frac{(x^5 + 9) - x^5}{x(x^5 + 9)} dx = \frac{1}{9} \int \frac{dx}{x} - \frac{1}{45} \int \frac{d(x^5 + 9)}{x^5 + 9} = \frac{1}{45} \ln \left| \frac{x^5}{x^5 + 9} \right| + C$$

$$B_5 = \int \frac{dx}{x^7 + 13x} = \frac{1}{13} \int \frac{(x^6 + 13) - x^6}{x(x^6 + 13)} dx = \frac{1}{13} \int \frac{dx}{x} - \frac{1}{78} \int \frac{d(x^6 + 13)}{x^6 + 13} = \frac{1}{78} \ln \left| \frac{x^6}{x^6 + 13} \right| + C$$

$$B_6 = \int \frac{dx}{x^3 + 6x^2 + 19x + 22} = \int \frac{d(x + 2)}{(x + 2)(x^2 + 4x + 11)} = \int \frac{du}{u(u^2 + 7)}$$

$$= \frac{1}{14} \ln \frac{u^2}{u^2 + 7} + C = \frac{1}{14} \ln \frac{(x + 2)^2}{x^2 + 4x + 11} + C$$

$$B_7 = \int \frac{dx}{x^3 - 3x^2 + 14x - 12} = \int \frac{d(x - 1)}{(x - 1)(x^2 - 2x + 12)} = \int \frac{du}{u(u^2 + 11)}$$

$$= \frac{1}{22} \ln \frac{u^2}{u^2 + 11} + C = \frac{1}{22} \ln \frac{(x - 1)^2}{x^2 - 2x + 12} + C$$

$$B_8 = \int \frac{dx}{x^4 + 4x^3 + 6x^2 + 7x + 4} = \int \frac{d(x+1)}{(x+1)(x^3 + 3x^2 + 3x + 4)} = \int \frac{du}{u(u^3 + 3)}$$

$$= \frac{1}{9} \ln \left| \frac{u^3}{u^3 + 3} \right| + C = \frac{1}{9} \ln \left| \frac{(x+1)^3}{x^3 + 3x^2 + 3x + 4} \right| + C$$

B. KỸ THUẬT NHẢY TẦNG LẦN KHI MẪU LÀ HÀM ĐA THỨC BẬC CAO

$$G_1 = \int \frac{x dx}{x^8 - 1} = \frac{1}{2} \int \frac{d(x^2)}{x^8 - 1} = \frac{1}{2} \int \frac{du}{u^4 - 1} = \frac{1}{2} C_1 = \frac{1}{2} \left(\frac{1}{4} \ln \left| \frac{u-1}{u+1} \right| - \frac{1}{2} \arctan u \right) + C$$

$$= \frac{1}{8} \ln \left| \frac{x^2 - 1}{x^2 + 1} \right| - \frac{1}{4} \arctan x^2 + C$$

$$G_2 = \int \frac{x^5 - x}{x^8 + 1} dx = \int \frac{x(x^4 - 1)}{x^8 + 1} dx = \frac{1}{2} \int \frac{(u^2 - 1) du}{u^4 + 1} = \frac{1}{2} C_8$$

$$= \frac{1}{2} \cdot \frac{1}{2\sqrt{2}} \ln \left| \frac{\left(u + \frac{1}{u}\right) - \sqrt{2}}{\left(u + \frac{1}{u}\right) + \sqrt{2}} \right| + C = \frac{1}{4\sqrt{2}} \ln \left| \frac{x^4 - \sqrt{2}x^2 + 1}{x^4 + \sqrt{2}x^2 + 1} \right| + C$$

$$G_3 = \int \frac{dx}{x^8 - 1} = \frac{1}{2} \int \frac{(x^4 + 1) - (x^4 - 1)}{(x^4 + 1)(x^4 - 1)} dx = \frac{1}{2} \left(\int \frac{dx}{x^4 - 1} - \int \frac{dx}{x^4 + 1} \right) = \frac{1}{2} (C_1 - C_{10})$$

$$= \frac{1}{8} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{4} \arctg x - \frac{1}{4} \left(\frac{1}{\sqrt{2}} \arctg \frac{x^2 - 1}{x\sqrt{2}} - \frac{1}{2\sqrt{2}} \ln \left| \frac{x^2 - x\sqrt{2} + 1}{x^2 + x\sqrt{2} + 1} \right| \right) + C$$

$$G_4 = \int \frac{x dx}{x^8 + 1} = \frac{1}{2} \int \frac{du}{u^4 + 1} dx = \frac{1}{2} C_{10} = \frac{1}{4} \left(\frac{1}{\sqrt{2}} \arctg \frac{x^2 - 1}{x\sqrt{2}} - \frac{1}{2\sqrt{2}} \ln \left| \frac{x^2 - x\sqrt{2} + 1}{x^2 + x\sqrt{2} + 1} \right| \right) + C$$

$$G_5^{****} = \int \frac{dx}{x^8 + 1} = \int \frac{\left[(x^5 + x) - (x^5 - x) + 2x \right] dx}{x^8 + 1} = \int \frac{x + \frac{1}{x^3}}{x^4 + \frac{1}{x^4}} dx - \int \frac{x - \frac{1}{x^3}}{x^4 + \frac{1}{x^4}} dx + \int \frac{d(x^2)}{x^8 + 1}$$

$$\begin{aligned}
 &= \frac{1}{2} \int \frac{d\left(x^2 - \frac{1}{x^2}\right)}{\left(x^2 - \frac{1}{x^2}\right)^2 + 2} - \frac{1}{2} \int \frac{d\left(x^2 + \frac{1}{x^2}\right)}{\left(x^2 + \frac{1}{x^2}\right)^2 - 2} + \int \frac{d\left(x^2\right)}{x^8 + 1} \\
 &= \frac{1}{2\sqrt{2}} \arctan\left(x^2 - \frac{1}{x^2}\right) - \frac{1}{4\sqrt{2}} \ln \frac{x^2 + \frac{1}{x^2} - \sqrt{2}}{x^2 + \frac{1}{x^2} + \sqrt{2}} + \frac{1}{2} \left(\frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x^4 - 1}{x^2 \sqrt{2}} - \frac{1}{2\sqrt{2}} \ln \left| \frac{x^4 - x^2 \sqrt{2} + 1}{x^4 + x^2 \sqrt{2} + 1} \right| \right) + C
 \end{aligned}$$

C. ****KĨ THUẬT CHỒNG NHI THỨC**

$$\begin{aligned}
 H_1 &= \int \frac{dx}{(3x-2)^7 (3x+4)^3} = \int \frac{dx}{\left(\frac{3x-2}{3x+4}\right)^7 (3x+4)^{10}} = \int \frac{1}{\left(\frac{3x-2}{3x+4}\right)^7} \cdot \frac{1}{(3x+4)^8} \cdot \frac{dx}{(3x+4)^2} \\
 &= \frac{1}{3 \cdot 6^9} \int \frac{u^6 - 6u^5 + 15u^4 - 20u^3 + 15u^2 - 6u + 1}{u^7} du \\
 &= \frac{1}{3 \cdot 6^9} \int \left(\frac{1}{u} - \frac{6}{u^2} + \frac{15}{u^3} - \frac{20}{u^4} + \frac{15}{u^5} - \frac{6}{u^6} + \frac{1}{u^7} \right) du \\
 &= \frac{1}{3 \cdot 6^9} \left(\ln|u| + \frac{6}{u} - \frac{15}{2u^2} + \frac{20}{3u^3} - \frac{15}{4u^4} + \frac{6}{5u^5} - \frac{1}{6u^6} \right) + c \\
 &= \frac{1}{3 \cdot 6^9} \left(\ln \left| \frac{3x-2}{3x+4} \right| + 6 \left(\frac{3x+4}{3x-2} \right) - \frac{15}{2} \left(\frac{3x+4}{3x-2} \right)^2 \right) \\
 &\quad + \frac{1}{3 \cdot 6^9} \left[\frac{20}{3} \left(\frac{3x+4}{3x-2} \right)^3 - \frac{15}{4} \left(\frac{3x+4}{3x-2} \right)^4 + \frac{6}{5} \left(\frac{3x+4}{3x-2} \right)^5 - \frac{1}{6} \left(\frac{3x+4}{3x-2} \right)^6 \right] + C
 \end{aligned}$$

$$\begin{aligned}
 H_2 &= \int \frac{dx}{(2x-1)^3 (3x-1)^4} = \int \frac{dx}{\left(\frac{2x-1}{3x-1}\right)^3 (3x-1)^7} = \int \frac{1}{\left(\frac{2x-1}{3x-1}\right)^3} \cdot \frac{1}{(3x-1)^5} \cdot \frac{dx}{(3x-1)^2} \\
 &= 2^5 \int \frac{1}{\left(\frac{2x-1}{3x-1}\right)^7} \cdot \left[\frac{(3x-1) - \frac{3}{2}(2x-1)}{3x-1} \right]^5 d\left(\frac{2x-1}{3x-1}\right) = \int \frac{1}{u^7} \cdot \left(1 - \frac{3}{2}u\right)^5 du = \dots
 \end{aligned}$$

$$H_3 = \int \frac{dx}{(3x+2)^5 (4x-1)^4} = \int \frac{dx}{\left(\frac{3x+2}{4x-1}\right)^5 (4x-1)^9} = \int \frac{1}{\left(\frac{3x+2}{4x-1}\right)^5} \cdot \frac{1}{(4x-1)^8} \cdot \frac{dx}{(4x-1)^2}$$

$$= \left(\frac{4}{11}\right)^8 \cdot \frac{-1}{11} \int \frac{1}{\left(\frac{3x+2}{4x-1}\right)^5} \cdot \left[\frac{(3x+2) - \frac{3}{4}(4x-1)}{4x-1}\right]^8 d\left(\frac{3x+2}{4x-1}\right) = \frac{-4^8}{11^9} \int \frac{1}{u^5} \cdot \left(u - \frac{3}{4}\right)^8 du = \dots$$

.....**Hết**.....

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