Japan Today's Calculation Of Integral 2008

257 Evaluate $\sqrt{\pi\int_0^{2008} x|\sin \pi x|\ dx}$

258 Find the volume of the solid formed by the revolution of the curve $x=a(1+\cos\theta)\cos\theta,\ y=a(1+\cos\theta)\sin\theta\ (a>0,\ -\pi<\theta\leq\pi)$ about the x axis.

259 Evaluate $\int_0^{\frac{\pi}{12}} \frac{dx}{(\sin x+\cos x)^4}$

260 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{(\sin^3\theta-\cos^3\theta-\cos^2\theta)(\sin\theta+\cos\theta+\cos^2\theta)^{2007}}{\sin^{2009}\theta\cos^{2009}\theta}\ d\theta$

261 Find the continuous function $f(x)$ such that $f(x)=\int_0^x \{f(t)\cos t-\cos(t-x)\}\ dt.$

262 Answer the following questions for positive integer $n.$

(1) Find the maximum value of $f_n(x)=x^ne^{-x}$ for $x\geq 0.$

(2) Show that $\lim_{x\to\infty} f_n(x)=0.$

(3) Let $I_n(x)=\int_0^x f_n(t)\ dt,$ find $\lim_{x\to\infty} I_n(x).$

263 Let $F(t)=\frac{1}{t}\int_0^{\frac{\pi}{2}t} |\cos 2x|\ dx$ for $0<t\leq 1.$

(1) Find $\lim_{t\to 0} F(t).$

(2) Find the range of t such that $F(t)\geq 1.$

264 Find the area of the figure surrounded by the locus of the point P inside the square with side length a such that the distance to the center of the square is equal to the minimum distance to the side of the square.

265 Supposed that $f(x)$ has $f'(x)$ and for any real numbers $x,\ y,$ $\int_y^{x+y} f(t)\ dt=\int_0^x \{f(y)\cos t+f(t)\cos y\}\ dt$ holds.

(1) Express $f(x+y)$ in terms of $f(x),\ f(y).$
(2) Find $f'\left(x+\frac{\pi}{2}\right).$ Note that $f\left(\frac{\pi}{2}\right)=2.$
(3) Find $f(x).$

266 Find the area of the region expressed by the system of inequality $\frac{x^2}{4}+y^2\leq 1,\ \left(1+\frac{\sqrt{3}}{2}\right)x+y\leq 1.$

267 Let K be the curved surface obtained by rotating the parabola $y=\frac{3}{4}-x^2$ about the y-axis.Cut K by the plane H passing through the origin and forming angle 45° for the axis. Find the volume of the solid surrounded by K and $H.$

Note that you are not allowed to use Double Integral for the problem.

268 Find the constant numbers $u,\ v,\ s,\ t\ (s<t)$ such that $\int_{-1}^1 f(x)\ dx=uf(s)+vf(t)$ holds for any polynomials $f(x)=ax^3+bx^2+cx+d$ with the degree of 3 or less than.

269 For the curve $C: y=\frac{1}{1+x^2},$ Let $A(\alpha,\ f(\alpha)),\ B\left(-\frac{1}{\alpha},\ f\left(-\frac{1}{\alpha}\right)\right)\ (\alpha>0).$ Find the minimum area bounded by the line segments $OA,\ OB$ and $C,$ where O is the origin.

Note that you are not allowed to use the integral formula of $\frac{1}{1+x^2}$ for the problem.

270 Let $f(x)$ be the continuous function at $0\leq x\leq 1$ such that $\int_0^1 x^kf(x)\ dx=0$ for integers $k=0,\ 1,\ \cdots,\ n-1\ (n\geq 1).$

(1) For all real numbers t, find the minimum value of $g(t)=\int_0^1 |x-t|^n\ dx.$

(2) Show the following equation for all real real numbers t.

$$\int_0^1 (x-t)^nf(x)\ dx=\int_0^1 x^nf(x)\ dx$$

(3) Let M be the maximum value of the function $|f(x)|$ for $0\leq x\leq 1.$

Show that $\left|\int_0^1 x^nf(x)\ dx\right|\leq \frac{M}{2^n(n+1)}$

271 For a positive constant number a, the function $f(x)$ satisfies the following equation.

$$f(x)=xe^{-\frac{x}{a}}+\frac{1}{a+1}\int_0^a f(t)dt$$

(1) Express the value of $\int_0^a f(t)dt$ in terms of a.

(2) Find the maximum and minimum value of $f(x)$ for $-1\leq x\leq 1.$

272 Find $\lim_{n\to\infty}\int_0^{\pi} e^x|\sin nx|dx.$

273 Find the area bounded by $0\leq y\leq \frac{x|\sin x|}{1+|\cos x|},\ 0\leq x\leq n\pi\ (n=1,\ 2,\ \cdots).$

274 For real constant numbers $a,\ b,\ c,\ d,$ consider the function $f(x)=ax^3+bx^2+cx+d$ such that $f(-1)=0,\ f(1)=0,\ f(x)\geq 1-|x|$ for $|x|\leq 1.$

Find $f(x)$ for which $\int_{-1}^1 \{f'(x)-x\}^2\ dx$ is minimized.

275 The line $y=mx$ has three intersection points with the curve $y=|x(x-1)|$. Find the value of m such that the areas of two parts bounded by the line and the curve are equal.

276 Prove that $\int_{\frac{1}{n}}^{\pi-\frac{1}{n}}\frac{1}{\sqrt[3]{\sin x}}\ dx\leq \frac{3}{2}\pi\ (n=1,\ 2,\ 3,\ \cdots).$

277 Find the fuction such that $f(x)=x^2-x\left(2\int_0^1 |f(t)|\ dt\right)^{\frac{1}{2}}.$

278 Find the value of t such that $\frac{\int_0^{\frac{\pi}{2}}(\sin x+t\cos x)dx}{\sqrt{\int_0^{\frac{\pi}{2}}(\sin x+t\cos x)^2dx}}$ is maximized.

279 If the function $f(x)=3x^2+2ax+b$ satisfies $\int_{-1}^1 |f(x)|\ dx<2,$ then show that $f(x)=0$ has distinct real roots.

280 Let $p,\ q\ (p<q)$ be the x coordinates of curves $x^2+y^2=1,\ y\geq 0$ and $y=\frac{1}{4x}.$

(1) Find $\alpha,\ \beta$ such that $\cos\frac{\alpha}{2}=p,\ \cos\frac{\beta}{2}=q\ (0<\alpha<\pi,\ 0<\beta<\pi).$

(2) Find the area bounded by the curves.

281 Find the maximum and minimum values of $\int_0^{\pi}(a\sin x+b\cos x)^3dx$ for $|a|\leq 1,\ |b|\leq 1.$

282 $g(x)$ is a differentiable function for $0 \leq x \leq \pi$ and $g'(x)$ is a continuous function for $0 \leq x \leq \pi$.
Let $f(x) = g(x)\sin x$. Find $g(x)$ such that $\int_0^{\pi} \{f(x)\}^2 dx = \int_0^{\pi} \{f'(x)\}^2 dx$.

283 $f(x)$ is a continuous function with the piriodicity of 2π and c is a positive constant number.
Find $f(x)$ and c such that $\int_0^{2\pi} f(t-x)\sin t dt = cf(x)$ with $f(0) = 1$ for all real numbers x.

284 $f(x)$ is a continuous function which takes positive values for $x \geq 0$.
Find $f(x)$ such that $\int_0^x f(t)dt = x\sqrt{f(x)}$ with $f(1) = \frac{1}{2}$.

285 Find the minimum value of $\int_0^1 \frac{2}{|2t-x|+3}\ dt$ for $0 \leq x \leq 4$.

286 Evaluate $\int_{-2008}^{2008} \frac{f'(x)+f'(-x)}{2008^x+1}\ dx$

287 For constant numbers $a,\ b$, let $f(x) = e^{-x}(ax+b)$. Suppose that the curve $y = f(x)$ passes through two points $(t,\ 1),\ (t+1,\ 1)$.
Find the area of the part bounded by the curve $y = f(x)$ and the line $y = 1$.

288 Evaluate $\int_0^1 (1+2008x^{2008})e^{x^{2008}}\ dx$.

289 Let p be a positive constant number, find $\lim_{a\to\infty} \int_0^p |x\sin(ax^2)|\ dx$.

290 Let the curve $C : y = |x^2+2x-3|$ and the line l passing through the point $(-3,\ 0)$ with a slope m in $x-y$ plane. Suppose that C intersects to l at distinct two points other than the point $(-3,\ 0)$, find the value of m for which the area of the figure bounded by C and l is minimized.

291 Consider the parabola $C : y = x^2$ in the $x-y$ plane. Let l_1 be the tangent line of C with slope $\sqrt{3}$ and l_2 be the tangent line of C with negative slope which makes angle 45° with l_1. Find the area bounded by $C,\ l_1$ and l_2.

292 Let $x,\ y$ and t be real numbers such that

$$\begin{cases} t^3-(2y-1)t^2-t+2y-1=0 \\ xt-2t+2y=0 \end{cases}$$

In $x-y$ plane find the area of the figure bounded by the part of curves above with $x \geq 0$ and the line $y = 2$.

293 Consider the parabolas $C_a : y = \frac{1}{4}x^2 - ax + a^2 + a - 2$ and $C : y = -\frac{1}{4}x^2 + 2$ for real number a in the $x-y$ plane.

(1) Find the equation of the locus of the vertex for C_a.

(2) For $a = 3$, find the slope of the two common tangent lines of C and C_a, then the intersection points of the lines.

(3) Suppose that C_a intersects with C at two distinct points. Find the maximum area of the figure bounded by C and C_a.

294 Evaluate $\int_{\ln 2}^{\ln 3} \frac{\ln(e^x+1)}{e^{2x}}\ dx$.

295 Let $f_n(x) = x^n(1-x)^n,\ I_n = \int_0^1 f_n(x)\ dx\ (n = 1, 2, \cdots)$.

(1) Find a polynomial $g(x)$ such that $f'_{n+1}(x) = (n+1)f_n(x)g(x)$.

(2) Find constant numbers $A_n,\ B_n$ such that $f''_{n+2}(x) = A_n f_{n+1}(x) + B_n f_n(x)$.

(3) Find $A_n I_{n+1} + B_n I_n$.

(4) Let $J_n = (2n+1)!I_n$. Express J_{n+1} in terms of J_n.

(5) Find I_n.

296 Let $a_n = \int_0^{\frac{\pi}{2}} (1 - \sin t)^n \sin 2t\ dt.$

(1) Find $\sum_{n=1}^{\infty} a_n$.

(2) Find $\sum_{n=1}^{\infty} \frac{a_n}{n}$.

(3) Find $\sum_{n=1}^{\infty} (n+1)(a_n - a_{n+1})$.

297 Evaluate $\int_0^{\frac{\pi}{6}} \left(\sin \frac{3}{2}x\right)\left(\cos \frac{x}{2}\right)\ dx.$

298 Let A be the region : $\left\{(x,\ y)|x \geq 0,\ y \geq 0, \left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{1-a}\right)^{\frac{1}{2}} \leq 1\right\}$ for $0 < a < 1.$

(1) Find the area of A.

(2) Find the maximum volume of the solid obtained by revolving A around the x axis.

299 Let $I_n(x) = \int_1^x (\ln t)^n dt\ (x > 0)$ for $n = 1,\ 2,\ 3,\ \cdots$.

(1) Prove by mathematical induction that $I_n(x)$ is expressed by $I_n(x) = xf_n(\ln x) + C_n\ (n \geq 1)$ in terms of some polynomial $f_n(y)$ with degree n and some constant number C_n.

(2) Express the constant term of $f_n(y)$ interms of n.

300 In Euclidean space, take the point $N(0,\ 0,\ 1)$ on the sphere S with radius 1 centered in the origin. For moving points $P,\ Q$ on S such that $NP = NQ$ and $\angle PNQ = \theta\ \left(0 < \theta < \frac{\pi}{2}\right)$, consider the solid figure T in which the line segment PQ can be passed.

(1) Show that z coordinates of $P,\ Q$ are equal.

(2) When P is on the palne $z = h$, express the length of PQ in terms of θ and h.

(3) Draw the outline of the cross section by cutting T by the plane $z = h$, then express the area in terms of θ and h.

(4) Pay attention to the range for which h can be valued, express the volume V of T in terms of θ, then find the maximum V when let θ vary.

301 For the positive constant number a, let D be the part surrounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and the line $x + y = a$.

(1) Draw the outline of D and find the area.

(2) Find the volume of the solid by rotating D about the line $x + y = a$ as the rotation axis.

302 Evaluate $\int_0^{\infty} \frac{|x-1|}{(x+1)(x^2+1)}\ dx.$

303 In the $x - y$ plane, find the area of the region bounded by the parameterized curve as follows.

$$\begin{cases} x = \cos 2t \\ y = t\sin t \end{cases} \qquad (0 \leq t \leq 2\pi)$$

304 Let $\alpha,\ \beta$ be real numbers with $0 \leq \alpha \leq \beta$ and $f(x) = x^2 - (\alpha + \beta)x + \alpha\beta$ such that $\int_{-1}^{1} f(x)\ dx = 1.$ Find the maximum value of $\int_0^{\alpha} f(x)\ dx.$

305 Find $\int_0^{\infty} (1 - \sqrt{1 - e^{-2x}})\ dx.$

306 For positive real numbers $a,\ b$, two graphs of the function : x^a and $\ln bx$ have a tangency of point.

(1) Let $(s,\ t)$ be the tangency of point, express this in terms of a, then express b as the function of a.

(2) For h with $0 < h < s$, denote the area as $A(h)$ of the domain bounded by the line $x = h$ and two curves $y = x^a,\ y = \ln bx$.

Express $\lim_{h\rightarrow 0} A(h)$ in terms of a.

307 For real numbers $a,\ b,\ c$, let $f(x) = ax^2 + bx + c$.

Prove that : $\int_{-1}^1 (1 - x^2)\{f'(x)\}^2\,dx \leq 6\int_{-1}^1 \{f(x)\}^2\,dx$.

308 Let a be a positive constant number. For a positive integer n, define a function $I_n(t)$ by $I_n(t) = \int_0^t x^n e^{-ax}\,dx$. Answer the following questions.

Note that you may use $\lim_{t\rightarrow\infty} t^n e^{-at} = 0$ without proof.

(1) Evaluate $I_1(t)$.

(2) Find the relation of $I_{n+1}(t),\ I_n(t)$.

(3) Prove that there exists $\lim_{t\rightarrow\infty} I_n(t)$ for all natural number n by using mathematical induction.

(4) Find $\lim_{t\rightarrow\infty} I_n(t)$.

309 (1) Calculate the indefinite integral :

$$\int e^{-x}\sin 2x\ dx.$$

(2) Evaluate the definite integral :

$$\int_0^{\pi} e^{-x}|\sin 2x|\ dx.$$

310 Define the function $f(x)$ as : $f(x) = \begin{cases} \frac{\ln(1+x)}{1+x} & (x \geq 0) \\ \frac{\ln(1-x)}{1-x} & (-1 \leq x < 0) \end{cases}$

1. Examine the variation of $f(x)$ and find the maximum and minimum value of $f(x)$.

2. Find the value of a for which $\int_{-1}^{e-1} |f(x) - a|\ dx$ is minimized for $0 \leq a \leq \frac{1}{e}$.

311 Prove the following inequality.

$$\int_0^1 2008^{x^{2008}}\ dx \geq \left(1 - \frac{1}{2008^{2008}}\right)\frac{1}{\ln 2008}.$$

312 Let $a,\ b$ be postive real numbers. For a real number t, denote by $d(t)$ the distance between the origin and the line $(ae^t)x + (be^{-t})y = 1$.

Let $a,\ b$ vary with $ab = 1$, find the minimum value of $\int_0^1 \frac{1}{d(t)^2}\ dt$.

313 Find the area bounded by the graph of $y = \sqrt[3]{x + \sqrt{x^2+1}} + \sqrt[3]{x - \sqrt{x^2+1}}$, the line $x - y - 1 = 0$ and the x axis.

314 Evaluate $\int_{\frac{\pi}{2}}^{\pi} x^{\sin x}(1 + x\cos x\cdot\ln x + \sin x)\ dx$.

315 Evaluate $\int_0^1 \frac{(x^2 - 3x + 1)e^x - (x-1)e^{2x} - x}{(x + e^x)^3}\ dx$.

316 Evaluate $\int_0^1 \frac{e^{2x} - e^{-2x} + 4x}{(e^x + e^{-x})^2}\ dx$.

317 Evaluate $\int_0^1 \frac{x+\sqrt{x}-1}{(\sqrt{x}+1)^2}\ dx$.

318 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1+\cot x}{e^x \sin x}\ dx$.

319 Evaluate $\int_{-1}^1 \frac{xe^x+e^x+1}{x^2(e^x+1)^2}\ dx$.

320 Evaluate $\int_0^{\pi} e^{x\sin x}(x^2\cos x+x\sin x+1)\ dx$.

321 Evaluate $\int_{-1}^1 \frac{x^2e^x-x^2e^{2x}+2e^x+2}{(e^x+1)^2}\ dx$.

322 Evaluate $\int_1^e \frac{(x^4-4x^3+9x^2-6x+8)e^x}{x^3}\ dx$.

323 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sqrt{\frac{\sin x}{x}}+\sqrt{\frac{x}{\sin x}}\cos x\right)\ dx$.

324 Evaluate $\int_1^e \left(\frac{1}{\sqrt{x\ln x}}+\sqrt{\frac{\ln x}{x}}\right)\ dx$.

325 Prove that $\frac{1}{3}<\int_0^1 x^{(\sin x+\cos x)^2}\ dx<\frac{1}{2}$.

326 Evaluate $\int_1^e \frac{(x+1)(x+2-\ln x)}{(x+\ln x)^2}\ dx$.

327 Let $a,\ b$ be real numbers and C be the graph of the function $y=e^{a+bx^2}$.

(1) Find the values of $a,\ b$ such that C passes through the point $P(1,\ 1)$ and the slope of the tangent line of C at P is -2.

(2) For the values of $a,\ b$ found in (1), find the volume of the solid generated by the revolution of the part which lies in the right side for the y axis in the figure bounded by the parabola $y=x^2$ and the curve C about the y axis.

328 Let t be a negative real number and D be the part bounded by the curve $y=2^{2x+2t}$, the curve $y=2^{x+3t}$ and the y axis.

(1) Find the volume $V(t)$ generated by rotation of D about the x axis.

(2) When t moves in the range of negative real number, find the maximum value of $V(t)$.

329 Let $f(x),\ g(x)$ be continuous functions defined in the range of $0\leq x\leq 1$.

(1) Find $f(x)$ such that $f(x)=\int_0^1 e^{x+t}f(t)\ dt$.

(2) Find $g(x)$ such that $g(x)=\int_0^1 e^{x+t}g(t)\ dt+x$.

330 Find all real numbers x such that $\int_0^x t^2\sin(x-t)\ dt=x^2$.

331 For $a\in\mathbb{R}$, find the minimum value of $\int_0^{\frac{\pi}{2}}\left|\frac{\sin 2x}{1+\sin^2 x}-a\cos x\right|\ dx$.

332 Let $f(x)$ be a function such that $1-f(x)=f(1-x)$ for $0\leq x\leq 1$.

Evaluate $\int_0^1 f(x)\ dx$.

Evaluate $\int_0 \; f(x) \; dx.$

333 Find the functions $f(x),\ g(x)$ such that $f(x)+\int_0^x g(t)\ dt=\sin x(\cos x-\sin x),\ \ \{f'(x)\}^2+\{g(x)\}^2=1.$

334 Evaluate $\int_0^1 \frac{7x^3+23x^2+21x+15}{(x^2+1)(x+1)^2}\ dx.$

335 For $t>0$, prove that $\frac{1}{2t}\int_t^{3t} x\ln x\ dx\geq 2t\ln 2t.$

336 Let $P_n=\sqrt[n]{\frac{(3n)!}{(2n)!}}\ (n=1,\ 2,\ \cdots).$

Find the following limits.

(1) $\lim_{n\to\infty}\frac{P_n}{n}.$

(2) $\lim_{n\to\infty}\left(\frac{n+2}{n}\right)^{P_n}.$

337 Find $\lim_{n\to\infty}\int_0^{\frac{\pi}{2}}\sum_{k=1}^n \sin kx\sin x\ dx.$

338 Given a parameterized curve $C: x=e^t-e^{-t},\ y=e^{3t}+e^{-3t}.$

Find the area bounded by the curve C, the x axis and two lines $x=\pm 1.$

339 Find the minimum area of the part bounded by the parabola $y=a^3x^2-a^4x\ (a>0)$ and the line $y=x.$

340 Find the continuous function $f(x)$ such that $xf(x)-\int_0^x f(t)\ dt=x+\ln(\sqrt{x^2+1}-x)$ with $f(0)=\ln 2.$

341 Let c be a constant number. Find the real value of x such that $\int_0^c |t-x|\ dt=\int_0^x |t-c|\ dt.$

342 Prove the following inequality.

$$\left(\int_0^1 (x^2+px+q)\ dx\right)^2\leq \int_0^1 (x^2+px+q)^2\ dx-\frac{1}{180}$$

343 Find all continuous positive functions $f(x)$, for $0\leq x\leq 1$, such that

$$\int_0^1 f(x)\ dx=1$$

$$\int_0^1 xf(x)\ dx=\alpha$$

$$\int_0^1 x^2f(x)\ dx=\alpha^2$$

where α is given real number.

344 Find the value of $k\ (0<k<5)$ such that $\int_0^{\infty}\frac{x^k}{2+4x+3x^2+5x^3+3x^4+4x^5+2x^6}\ dx$ is minimal.

345 Given a continuous function $f(x)$ such that $\int_0^{2\pi} f(x)\ dx=0.$

Let $S(x)=A_0+A_1\cos x+B_1\sin x$, find constant numbers $A_0,\ A_1$ and B_1 for which $\int_0^{2\pi}\{f(x)-S(x)\}^2\ dx$ is minimized.

346 Suppose that two curves $C_1 : y = x^3 - x, C_2 : y = (x-a)^3 - (x-a)$ have two intersection points.
Find the maximum area of the figure bounded by $C_1,\ C_2$.

347 For real positive numbers a, find the minimum value of $\int_0^{\pi} \left(\frac{x}{a} + a\sin x\right)^2 dx$.

348 Find $\lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{\frac{1}{n^2}} \left(\frac{n+2}{n}\right)^{\frac{2}{n^2}} \left(\frac{n+3}{n}\right)^{\frac{3}{n^2}} \cdots \left(\frac{n+n}{n}\right)^{\frac{n}{n^2}}$ $(n = 1,\ 2,\ 3,\ \cdots)$.

349 For real numbers $a,\ b\ (ab \neq 0)$, evaluate $\int_0^{\pi} (a\sin x + b\cos x)^6\, dx$.

350 Evaluate $\int_0^1 (1-x^2)^{\frac{5}{2}}\, dx$.

351 Find the positive value of k for which $\int_0^{\frac{\pi}{2}} |\cos x - kx|\, dx$ is minimized.

352 Prove the following inequality.

$$1 \leq \lim_{n\to\infty} \int_{\frac{1}{n}}^{\frac{\pi}{2}} \frac{\pi \sin x}{x(\pi - x)}\, dx < 3$$

.

353 Consider a parabola $C : y = \frac{1}{4}x^2$ and the point $F(0,\ 1)$. For the origin O, take n points on the parabola C, $A_1(x_1,\ y_1),\ A_2(x_2,\ y_2),\ \cdots,A_n(x_n,\ y_n)$ such that $x_k > 0$ and $\angle OFA_k = \frac{k\pi}{2n}$ $(k = 1,\ 2,\ 3,\ \cdots,n)$. Find $\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} FA_k$.

354 Evaluate $\int_{-2}^{2} x^4\sqrt{4-x^2}\, dx$.

355 For a positive number n, find $\lim_{n\to\infty} n^2 \int_{e^{-\frac{1}{n}}}^{1} x^n \ln x\, dx$.

356 A continuous function $f(x)$ satisfies that $\frac{d}{dx}\left(\int_0^x f(x+t)\, dt\right) = 0$. Find the necessary and sufficient condition such that

$\sum_{n=1}^{\infty} f(2^n) = 1$.

357 For $0 < a < 1$, let $S(a)$ is the area of the figure bounded by three curves $y = e^x,\ y = e^{\frac{1+a}{1-a}x}$ and $y = e^{2-x}$.
Find $\lim_{a\to 0} \frac{S(a)}{a}$.

358 Let a be a postive constant number. Given a positive integer n, take an integer m such that $m \leq \frac{na}{\pi} < m+1$.
Find $\lim_{n\to\infty} \frac{1}{n}\int_0^{m\pi} \left(a - \frac{t}{n}\right)|\sin t|\, dt$.

359 Evaluate $\int_{-1}^{1} \frac{2x^4 - x^3 - 2x^2 + 1}{x^3 - x + \sqrt{1-x^2}}\, dx$.

360 Evaluate $\int_{-1}^{1} (1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}\, dx$.

361 Find the following indefinite integrals.

(1) $\int \frac{\sin x}{3+\sin^2 x}\ dx.$

(2) $\int e^{2x+e^x}\ dx.$

362 Evaluate the following definite integrals.

(1) $\int_0^{\frac{\pi}{2}} x\cos x\ dx.$

(2) $\int_1^2 \frac{1}{x(x+1)}\ dx.$

(3) $\int_0^1 \frac{1}{1+e^x}\ dx.$

(4) $\int_0^{\frac{\pi}{4}} \cos 2x\cos 3x\ dx.$

363 For $t\geq 0$, let $a_n=\int_0^t e^{nx}\ dx\ (n=0,\ 1,\ 2,\ 3).$

(1) Show that $a_3-3a_2+3a_1-a_0\geq 0.$

(2) Show that $e^ta_0+(e^t-1)a_1-a_2\geq 0.$

364 Evaluate $\int_0^1 x^2(x-1)^2e^{2x}\ dx.$

365 Let $a,\ b$ be postive constant numbers.

Evaluate $\int_0^{\frac{\pi}{2}} \frac{a\cos x-b\sin x}{b\sin x+a\cos x}\ dx.$

366 (1) Determine the constant numbers $a,\ b,\ c,\ p,\ q,\ r,\ s$ such that the following equation is equality.

$4x=\{a(x+1)^2+b(x+1)+c\}(x^2+1)^2+\{(px+q)(x^2+1)+(rx+s)\}(x+1)^3.$

(2) Evaluate the following definite integrals.

$(a)\int_0^1 \frac{dx}{x+1}$ $(b)\ \int_0^1 \frac{dx}{(x+1)^2}$ $(c)\ \int_0^1 \frac{dx}{(x+1)^3}$

$(d)\int_0^1 \frac{x}{x^2+1}\ dx$ $(e)\ \int_0^1 \frac{dx}{x^2+1}$ $(f)\ \int_0^1 \frac{x}{(x^2+1)^2}\ dx$

$(g)\int_0^1 \frac{dx}{(x^2+1)^2}$ $(h)\ \int_0^1 \frac{4x}{(x+1)^3(x^2+1)^2}\ dx$

367 Let $f(x)=x^2+\int_0^1 xf(t)\ dt+\int_{-1}^2 f(t)\ dt.$

Evaluate $\int_0^{\frac{\pi}{2}} f(x+\sin x)\ dx.$

368 For a real number a, evaluate $\int_0^a |x(x-a^2)|\ dx.$

369 Calculate

$$\int_{e^{e^e}}^{e^{e^{e^e}}} \frac{1}{xlnxln(lnx)ln(ln(lnx))}dx.$$

370 Calculate

$\int^{\pi/3} \frac{\cos^2 x+1}{\ldots}\ dx.$

$\int_0 \quad \cos x\sqrt{\cos x}$

371 Calculate

$$\int_{-1}^{1}\frac{1}{(1+e^x)(1+x^2)}\ dx.$$

372 Evaluate

$$\int_0^1 e^{x+e^{x+e^{x+e^x}}}\ dx.$$

373 Evaluate

$$\int_0^{\frac{\pi}{2}} e^x\left\{\cos\ (\sin x)\cos^2\frac{x}{2}+\sin\ (\sin x)\sin^2\frac{x}{2}\right\}\ dx.$$

374 Let $n\geq 2$ be positive integers.

(1) Prove that $n\ln n-n+1<\sum_{k=1}^n \ln k<(n+1)\ln n-n+1.$

(2) Find $\lim_{n\to\infty}(n!)^{\frac{1}{n\ln n}}.$

375 Prove the following inequality.

$$\frac{1}{n}<\int_0^{\frac{\pi}{2}}\frac{1}{(1+\cos x)^n}\ dx<\frac{n+5}{n(n+1)}\ (n=2,,\ \cdots).$$

376 Evaluate $\int_0^{\frac{\pi}{4}}\frac{1+2\sin x-\sin^2 x}{(1+x)\cos^4 x}\ dx.$

377 The line $y=mx$ has 3 intersection points with the curve $y=|x(x-1)|$. Find the value of m such that the area of the 2 regions bounded by the line and the curve are equal.

378 Evaluate $\int_0^{\pi} x|\sin nx|\ dx\ (n=1,\ 2,\ \cdots).$

379 Let $\alpha,\ r$ be real numbers such that $r>1,\ r\neq 3,\ r\neq 4.$

Find the values of $\alpha,\ r$ such that $\lim_{n\to\infty}\sum_{k=1}^n\frac{n^{\alpha}}{(n+k)^r}=\frac{r-3}{(r-1)(r-4)}.$

380 Find $\int_{-1}^1 {}_nC_k(1+x)^{n-k}(1-x)^k\ dx\ (k=0,\ 1,\ 2,\ \cdots n).$

381 A function $f(x)$ is defined as follows for $x\geq 0.$

$$f(x)=\sin\left(\frac{n\pi}{4}\right)\sin x\ (n\pi\leq x<(n+1)\pi)\ (n=0,\ 1,\ 2,\ \cdots).$$

Evaluate $\int_0^{100\pi} f(x)\ dx.$

382 (1) For $a>0,\ b\geq 0$, Compare

$$\int_b^{b+1}\frac{dx}{\sqrt{x+a}},\ \frac{1}{\sqrt{a+b}},\ \frac{1}{\sqrt{a+b+1}}.$$

(2) Find $\lim_{n\to\infty}\sum_{k=1}^n\frac{1}{\sqrt{n^2+k}}.$

$\frac{\pi}{}$

383 For a positive integer m, evaluate $\int_0^2 \cos^m x\ \sin 2mx\ dx$.

384 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x+\cos x)\sqrt{\frac{e^x}{\sin x}}\ dx$.

385 Evaluate $\int_0^{\frac{\pi^2}{4}} (2\sin\sqrt{x}+\sqrt{x}\cos\sqrt{x})\ dx$.

386 For $a=2007*2009$, evaluate

$$\int_0^1 \frac{6x+5a\sqrt{x}}{4\sqrt{x+a\sqrt{x}}}\ dx.$$

387 Let $l_1,\ l_2$ be the tangent and nomal line respectively at the point $(p,\ \ln(p+1))$ on the curve $C: y=\ln(x+1)$.
Denote by $T_i\ (i=1,\ 2)$ the areas bounded by $l_i\ (i=1,\ 2), C$ and the y axis respectively. Find the limit $\lim_{p\to 0}\frac{T_2}{T_1}$.

388 For $f(x)=\sin^3 x$, let $p(x)$ be quadratic polynomial.
Evaluate $\int_0^{2\pi} p(x)f''(x)\ dx$.

389 Find the values of $t\in[0,\ 2]$ for which $\int_{t-3}^{2t} 2^{x^2}\ dx$ is maximal and minimal.

390 Find the polynomials $f(x),\ g(x)$ such that:

$$\frac{1}{\pi}\int_0^{\frac{1}{2}} \frac{tf'(t)-6g(t)}{\sqrt{1-t^2}}\ dt=f(x)-g(x)+x$$

$$\frac{6}{\pi}\int_0^{\frac{1}{2}} \frac{8f(t)-5g'(t)}{\sqrt{1-t^2}}\ dt=2f(x)-3g(x)-x^2+2x.$$

391 Evaluate $\int_0^{\frac{\pi}{2}} \sin|2x-a|\ dx$ for a real number $a\in[0,\ \pi]$.

392 Evaluate $\int_{-1}^1 \frac{|x|}{1+e^x}\ dx$.

393 Let $V(a)$ be the volume of the solid obtained by revolvong the region bounded by the curve $y=x\sqrt{x\sin ax}\ (0\leq x\leq\frac{\pi}{a})$ and x axis around the x axis.

For $a>0$, find the minimum value of $V(a)+V\left(\frac{1}{a}\right)$.

394 Find $\lim_{x\to 0}\frac{1}{x}\int_{-x}^x (t\sin 2006t+2007t+1004)\ dt$.

395 3 points $O(0,\ 0),\ P(a,\ a^2), Q(-b,\ b^2)\ (a>0,\ b>0)$ are on the parabpla $y=x^2$.
Let S_1 be the area bounded by the line PQ and the parabola and let S_2 be the area of the triangle OPQ.

Find the minimum value of $\frac{S_1}{S_2}$.