Japan Today's Calculation Of Integral 2009

396 Evaluate $\int_0^{2008} \left(3x^2 - 8028x + 2007^2 + \frac{1}{2008}\right)\ dx.$

397 In xy plane, find the minimum volume of the solid by rotating the region boubded by the parabola $y = x^2 + ax + b$ passing through the point $(1,\ -1)$ and the x axis about the x axis

398 In xyz space, find the volume of the solid expressed by the sytem of inequality:

$0 \leqq x \leqq 1,\ 0 \leqq y \leqq 1,\ 0 \leqq z \leqq 1$

$x^2 + y^2 + z^2 - 2xy - 1 \geqq 0$

399 Evaluate $\int_0^{\sqrt{2}-1} \frac{1+x^2}{1-x^2} \ln\left(\frac{1+x}{1-x}\right)\ dx.$

400 (1) A function is defined $f(x) = \ln(x + \sqrt{1+x^2})$ for $x \geq 0$. Find $f'(x)$.

(2) Find the arc length of the part $0 \leq \theta \leq \pi$ for the curve defined by the polar equation: $r = \theta\ (\theta \geq 0)$.

Remark:

You may not directly use the integral formula of $\frac{1}{\sqrt{1+x^2}},\ \sqrt{1+x^2}$ here.

401 For real number a with $|a| > 1$, evaluate $\int_0^{2\pi} \frac{d\theta}{(a+\cos\theta)^2}$.

402 Consider a right circular cylinder with radius r of the base, hight h. Find the volume of the solid by revolving the cylinder about a diameter of the base.

403 Evaluate $\int_0^1 \frac{2e^{2x} + xe^x + 3e^x + 1}{(e^x+1)^2(e^x+x+1)^2}\ dx.$

404 Evaluate $\int_{-\pi}^{\pi} \frac{\sin nx}{(1+2009^x)\sin x}\ dx\ (n = 0,\ 1,\ 2,\ \cdots).$

405 Calculate $\left|\frac{\int_0^{\frac{\pi}{2}} (x\cos x + 1)e^{\sin x}\ dx}{\int_0^{\frac{\pi}{2}} (x\sin x - 1)e^{\cos x}\ dx}\right|$.

406 Find $\lim_{n\to\infty} \int_0^{\frac{\pi}{2}} x|\cos(2n+1)x|\ dx.$

407 Evaluate $\int_0^1 (x+3)\sqrt{xe^x}\ dx.$

408 Evaluate $\int_1^e \{(1+x)e^x + (1-x)e^{-x}\}\ln x\ dx.$

409 Evaluate $\int_0^1 \sqrt{\frac{x+\sqrt{x^2+1}}{x^2+1}}\ dx.$

410 Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\cos\theta}\sqrt{\frac{1+\sin\theta}{\cos\theta}}\ d\theta.$

411 Find the area bounded by $y = x^2 - |x^2 - 1| + |2|x| - 2| + 2|x| - 7$ and the x axis.

412 Let the definite integral $I_n = \int_0^{\frac{\pi}{4}} \frac{dx}{(\cos x)^n}\ (n = 0,\ \pm 1,\ \pm 2,\ \cdots).$

(1) Find $I_0,\ I_{-1},\ I_2$.

(2) Find I[illegible]

(2) Find I_1.

(3) Express I_{n+2} in terms of I_n.

(4) Find $I_{-3},\ I_{-2},\ I_3$.

(5) Evaluate the definite integrals $\int_0^1 \sqrt{x^2+1}\ dx,\ \int_0^1 \frac{dx}{(x^2+1)^2}\ dx$ in using the avobe results.

You are not allowed to use the formula of integral for $\sqrt{x^2+1}$ directively here.

413 Find the maximum and minimum value of $F(x)=\frac{1}{2}x+\int_0^x (t-x)\sin t\ dt$ for $0\leq x\leq \pi$.

414 Evaluate $\int_0^{2(2+\sqrt{3})} \frac{16}{(x^2+4)^2}\ dx$.

415 For a function $f(x)=6x(1-x)$, suppose that positive constant c and a linear function $g(x)=ax+b\ (a,\ b:\mathrm{constants}\ \ a>0)$ satisfy the following 3 conditions: $c^2\int_0^1 f(x)\ dx=1,\ \int_0^1 f(x)\{g(x)\}^2\ dx=1,\ \int_0^1 f(x)g(x)\ dx=0$. Answer the following questions.

(1) Find the constants $a,\ b,\ c$.

(2) For natural number n, let $I_n=\int_0^1 x^n e^x\ dx$. Express I_{n+1} in terms of I_n. Then evaluate $I_1,\ I_2,\ I_3$.

(3) Evaluate the definite integrals $\int_0^1 e^x f(x)\ dx$ and $\int_0^1 e^x f(x)g(x)\ dx$.

(4) For real numbers $s,\ t$, define $J=\int_0^1 \{e^x-cs-tg(x)\}^2\ dx$. Find the constants $A,\ B,\ C,\ D,\ E$ by setting $J=As^2+Bst+Ct^2+Ds+Et+F$.
(You don't need to find the constant F).

(5) Find the values of $s,\ t$ for which J is minimal.

416 Answer the following questions.

(1) $0<x\leq 2\pi$, prove that $|\sin x|<x$.

(2) Let $f_1(x)=\sin x$, a be the constant such that $0<a\leq 2\pi$.

Define $f_{n+1}(x)=\frac{1}{2a}\int_{x-a}^{x+a} f_n(t)\ dt\ (n=1,\ 2,\ 3,\ \cdots)$. Find $f_2(x)$.

(3) Find $f_n(x)$ for all n.

(4) For a given x, find $\sum_{n=1}^{\infty} f_n(x)$.

417 The functions $f(x),\ g(x)$ satify that $f(x)=\frac{x^3}{2}+1-x\int_0^x g(t)\ dt,\ g(x)=x-\int_0^1 f(t)\ dt$.
Let $l_1,\ l_2$ be the tangent lines of the curve $y=f(x)$, which pass through the point $(a,\ g(a))$ on the curve $y=g(x)$.
Find the minimum area of the figure bounded by the tangent tlines $l_1,\ l_2$ and the curve $y=f(x)$.

418 (1) 2009 Kansai University entrance exam

Calculate $\int \frac{e^{-2x}}{1+e^{-x}}\ dx$.

(2) 2009 Rikkyo University entrance exam/Science

Evaluate $\int_0^1 \frac{2x^3}{1+x^2}\ dx$.

419 In the xy plane, the line l touches to 2 parabolas $y=x^2+ax,\ y=x^2-2ax$, where a is positive constant.

(1) Find the equation of l.

(2) Find the area S bounded by the parabolas and the tangent line l.

420 Let K be the figure bounded by the curve $y=e^x$ and 3 lines $x=0,\ x=1,\ y=0$ in the xy plane.

420 Let K be the figure bounded by the curve $y=e^x$ and 3 lines $x=0,\ x=1,\ y=0$ in the xy plane.

(1) Find the volume of the solid formed by revolving K about the x axis.

(2) Find the volume of the solid formed by revolving K about the y axis.

421 Let $f(x)=e^{(p+1)x}-e^x$ for real number $p>0$. Answer the following questions.

(1) Find the value of $x=s_p$ for which $f(x)$ is minimal and draw the graph of $y=f(x)$.

(2) Let $g(t)=\int_t^{t+1} f(x)e^{t-x}\ dx$. Find the value of $t=t_p$ for which $g(t)$ is minimal.

(3) Use the fact $1+\frac{p}{2}\leq \frac{e^p-1}{p}\leq 1+\frac{p}{2}+p^2\ (0<p\leq 1)$ to find the limit $\lim_{p\rightarrow +0}(t_p-s_p)$.

422 There are 10 cards, labeled from 1 to 10. Three cards denoted by $a,\ b,\ c\ (a>b>c)$ are drawn from the cards at the same time.
Find the probability such that $\int_0^a (x^2-2bx+3c)\ dx=0$.

423 Let $f(x)=x^2+3$ and $y=g(x)$ be the equation of the line with the slope a, which pass through the point $(0,\ f(0))$.
Find the maximum and minimum values of $I(a)=3\int_{-1}^1 |f(x)-g(x)|\ dx$.

424 Let n be positive integer. For $n=1,\ 2,\ 3,\ \cdots n$, let denote S_k be the area of $\triangle AOB_k$ such that
$\angle AOB_k=\frac{k}{2n}\pi,\ OA=1,\ OB_k=k$. Find the limit $\lim_{n\to\infty}\frac{1}{n^2}\sum_{k=1}^n S_k$.

425 The coordinate of P at time t, moving on a plane, is expressed by $x=f(t)=\cos 2t+t\sin 2t,\ y=g(t)=\sin 2t-t\cos 2t$.

(1) Find the acceleration vector $\overrightarrow{\alpha}$ of P at time t .

(2) Let L denote the line passing through the point P for the time t, which is parallel to the acceleration vector $\overrightarrow{\alpha}$ at the time. Prove that L always touches to the unit circle with center the origin, then find the point of tangency Q.

(3) Prove that $f(t)$ decreases in the interval $0\leq t\leq \frac{\pi}{2}$.

(4) When t varies in the range $\frac{\pi}{4}\leq t\leq \frac{\pi}{2}$, find the area S of the figure formed by moving the line segment PQ.

426 Consider the polynomial $f(x)=ax^2+bx+c$, with degree less than or equal to 2.

When f varies with subject to the constrain $f(0)=0,\ f(2)=2$, find the minimum value of $S=\int_0^2 |f'(x)|\ dx$.

427 Let a be a positive real number, in Euclidean space, consider the two disks:
$D_1=\{(x,\ y,\ z)|x^2+y^2\leq 1,\ z=a\}$,
$D_2=\{(x,\ y,\ z)|x^2+y^2\leq 1,\ z=-a\}$.
Let D_1 overlap to D_2 by rotating D_1 about the y axis by 180°. Note that the rotational direction is supposed to be the direction such that we would lean the postive part of the z axis to into the direction of the postive part of x axis. Let denote E the part in which D_1 passes while the rotation, let denote $V(a)$ the volume of E and let $W(a)$ be the volume of common part of E and $\{(x,\ y,\ z)|x\geq 0\}$.

(1) Find $W(a)$.

(2) Find $\lim_{a\rightarrow\infty} V(a)$.

428 Let $f(x)$ be a polynomial and C be a real number.
Find the $f(x)$ and C such that $\int_0^x f(y)dy+\int_0^1 (x+y)^2f(y)dy=x^2+C$.

429 Find the length of the curve expressed by the polar equation: $r=1+\cos\theta\ (0\leq \theta \leq \pi)$.

430 For a natural number n, let $a_n=\int_0^{\frac{\pi}{4}}(\tan x)^{2n}dx$.
Answer the following questions.

(1) Find a_1.

(2) Express a_{n+1} in terms of a_n.

(3) Find $\lim a_n$.

(4) Find $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(-1)^{k+1}}{2k-1}$.

431 Consider the function $f(\theta)=\int_0^1|\sqrt{1-x^2}-\sin\theta|dx$ in the interval of $0\leq\theta\leq\frac{\pi}{2}$.

(1) Find the maximum and minimum values of $f(\theta)$.

(2) Evaluate $\int_0^{\frac{\pi}{2}}f(\theta)\ d\theta$.

432 Define the function $f(t)=\int_0^1(|e^x-t|+|e^{2x}-t|)dx$. Find the minimum value of $f(t)$ for $1\leq t\leq e$.

433 Evaluate $\int_0^{\frac{\pi}{2}}\frac{(\sin x)^{\cos x}}{(\cos x)^{\sin x}+(\sin x)^{\cos x}}dx$.

434 Evaluate $\int_0^1\frac{x-e^{2x}}{x^2-e^{2x}}dx$.

435 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1}{(\sin x+\cos x+2\sqrt{\sin x\cos x})\sqrt{\sin x\cos x}}dx$.

436 Find the minimum area bounded by the graphs of $y=x^2$ and $y=kx(x^2-k)\ (k>0)$.

437 Evaluate $\int_0^1\frac{1}{\sqrt{x}\sqrt{1+\sqrt{x}}\sqrt{1+\sqrt{1+\sqrt{x}}}}\ dx$.

438 Evaluate $\int_{\sqrt{2}-1}^{\sqrt{2}+1}\frac{x^4+x^2+2}{(x^2+1)^2}\ dx$.

439 Find the volume of the solid defined by the inequality $x^2+y^2+\ln(1+z^2)\leq\ln 2$.
Note that you may not directively use double integral here for Japanese high school students who don't study it.

440 For $a>1$, find $\lim_{n\to\infty}\int_0^a\frac{e^x}{1+x^n}dx$.

441 Evaluate $\int_1^e\frac{(x^2\ln x-1)e^x}{x}\ dx$.

442 Evaluate $\int_0^{\frac{\pi}{2}}\frac{\cos\theta-\sin\theta}{(1+\cos\theta)(1+\sin\theta)}\ d\theta$

443 Evaluate $\int_1^{e^2}\frac{(e^{\sqrt{x}}-e^{-\sqrt{x}})\cos\left(e^{\sqrt{x}}+e^{-\sqrt{x}}+\frac{\pi}{4}\right)+(e^{\sqrt{x}}+e^{-\sqrt{x}})\cos\left(e^{\sqrt{x}}-e^{-\sqrt{x}}+\frac{\pi}{4}\right)}{\sqrt{x}}\ dx$.

444 Evaluate $\int_0^{\frac{\pi}{6}}\frac{\sin x+\cos x}{1-\sin 2x}\ln\left(2+\sin 2x\right)\ dx$.

445 Evaluate $\int_0^1\frac{(1-2x)e^x+(1+2x)e^{-x}}{(e^x+e^{-x})^3}\ dx$.

446 Evaluate $\int_0^1\frac{(1-2x)e^x+(1+2x)e^{-x}}{(e^x+e^{-x})^3}\ dx$.

447 Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{x^2}{(1+x\tan x)(x-\tan x)\cos^2 x}\ dx$.

448 Evaluate $\int^{\ln 2}\frac{2e^x+1}{\cdots}\ dx$.

$\int_0 \quad e^{3x}+2e^{2x}+e^{x}-e^{-x}$

449 Evaluate $\sum_{k=1}^{n}\int_0^{\pi}(\sin x-\cos kx)^2\,dx.$

450 Let $a,\ b$ be postive real numbers. Find $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{(k+an)(k+bn)}.$

451 Find $\lim_{n\to\infty}\sum_{k=1}^{n}\ln\left(1+\frac{k^a}{n^{a+1}}\right).$

452 Let $a,\ b$ are postive constant numbers.

(1) Differentiate $\ln(x+\sqrt{x^2+a})\ (x>0).$

(2) For $a=\frac{4b^2}{(e-e^{-1})^2}$, evaluate $\int_0^b\frac{1}{\sqrt{x^2+a}}\,dx.$

453 Find the minimum value of $\int_0^{\frac{\pi}{2}}|x\sin t-\cos t|\,dt\ (x>0).$

454 Let a be positive constant number. Evaluate $\int_{-a}^{a}\frac{x^2\cos x+e^x}{e^x+1}\,dx.$

455 (1) Evaluate $\int_1^{3\sqrt{3}}\left(\frac{1}{\sqrt[3]{x^2}}-\frac{1}{1+\sqrt[3]{x^2}}\right)\,dx.$

(2) Find the positive real numbers $a,\ b$ such that for $t>1,\ \lim_{t\to\infty}\left(\int_1^t\frac{1}{1+\sqrt[3]{x^2}}\,dx-at^b\right)$ converges.

456 Find $\lim_{n\to\infty}\frac{\pi}{n}\left\{\frac{1}{\sin\frac{\pi(n+1)}{4n}}+\frac{1}{\sin\frac{\pi(n+2)}{4n}}+\cdots+\frac{1}{\sin\frac{\pi(n+n)}{4n}}\right\}$

457 Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{1}{1+\sin\theta-\cos\theta}\,d\theta$

458 Let $S(t)$ be the area of the traingle OAB with $O(0,\ 0,\ 0),\ A(2,\ 2,\ 1),\ B(t,\ 1,\ 1+t).$
Evaluate $\int_1^e S(t)^2\ln t\,dt.$

459 Find $\lim_{x\to\infty}\int_{e^{-x}}^{1}\left(\ln\frac{1}{t}\right)^n\,dt\ (x\geq 0,\ n=1,\ 2,\ \cdots).$

460 $\int_{-\frac{\pi}{3}}^{\frac{\pi}{6}}\left|\frac{4\sin x}{\sqrt{3}\cos x-\sin x}\right|\,dx.$

461 Let $I_n=\int_0^{\sqrt{3}}\frac{1}{1+x^n}\,dx\ (n=1,\ 2,\ \cdots).$

(1) Find $I_1,\ I_2.$

(2) Find $\lim_{n\to\infty}I_n.$

462 Evaluate $\int_0^1\frac{(1-x+x^2)\cos\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}\sin\ln(x+\sqrt{1+x^2})}{(1+x^2)^{\frac{3}{2}}}\,dx.$

463 Evaluate $\int_0^{\frac{\pi}{4}} \frac{e^{\frac{1}{\cos^2 x}}\sin x}{\cos^3 x}\ dx.$

464 Evaluate $\int_1^e \frac{(1+2x^2)\ln x}{\sqrt{1+x^2}}\ dx.$

465 Compute $\int_0^1 x^{2n+1}e^{-x^2}dx\ (n=1,\ 2,\ \cdots)$, then use this result, prove that $\sum_{n=0}^{\infty}\frac{1}{n!}=e.$

466 For $n=1,\ 2,\ 3,\ \cdots$, let $(p_n,\ q_n)\ (p_n>0,\ q_n>0)$ be the point of intersection of $y=\ln(nx)$ and $\left(x-\frac{1}{n}\right)^2+y^2=1.$

(1) Show that $1-q_n^2\leq \frac{(e-1)^2}{n^2}$ to find $\lim_{n\to\infty} q_n.$

(2) Find $\lim_{n\to\infty} n\int_{\frac{1}{n}}^{p_n}\ln(nx)\ dx.$

467 Let the curve $C: y=x\sqrt{9-x^2}\ (x\geq 0).$

(1) Find the maximum value of $y.$

(2) Find the area of the figure bounded by the curve C and the x axis.

(3) Find the volume of the solid generated by rotation of the figure about the y axis.

468 Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x}{\{(2x+1)\sqrt{x^2-x+1}+(2x-1)\sqrt{x^2+x+1}\}\sqrt{x^4+x^2+1}}\ dx.$

469 Evaluate $\int_0^1 \frac{t}{(1+t^2)(1+2t-t^2)}\ dt.$

470 Determin integers $m,\ n\ (m>n>0)$ for which the area of the region bounded by the curve $y=x^2-x$ and the lines $y=mx,\ y=nx$ is $\frac{37}{6}.$

471 Evaluate $\int_1^e \frac{1-x(e^x-1)}{x(1+xe^x\ln x)}\ dx.$

472 Given a line segment PQ moving on the parabola $y=x^2$ with end points on the parabola. The area of the figure surrounded by PQ and the parabola is always equal to $\frac{4}{3}.$ Find the equation of the locus of the mid point M of $PQ.$

473 For nonzero real numbers $r,\ l$ and the positive constant number c, consider the curve on the xy plane :

$$y=\begin{cases} x^2 & (0\leq x\leq r) \\ r^2 & (r\leq x\leq l+r) \\ (x-l-2r)^2 & (l+r\leq x\leq l+2r)\end{cases}$$

Denote V the volume of the solid by revolvering the curve about the x axis. Let $r,\ l$ vary in such a way that $r^2+l=c.$ Find the values of $r,\ l$ which gives the maxmimum volume.

474 Calculate the following indefinite integrals.

(1) $\int \frac{3x+4}{x^2+3x+2}dx$

(2) $\int \sin 2x\cos 2x\cos 4x\ dx$

(3) $\int xe^x dx$

(4) $\int 5^x dx$

475 For a positive constant number t, let denote D the region surrounded by the curve $y=e^x$, the line $x=t$, the x axis and the y axis. Let $V_x,\ V_y$ be the volumes of the solid obtained by rotating D about the x axis and the y axis respectively. Compare the size of $V_x,\ V_y.$

476 Suppose a parabola with the axis as the y axis, concave up and touches the graph $y = 1 - |x|$. Find the equation of the parabola such that the area of the region surrounded by the parabola and the x axis is maximal.

477 Suppose that $P_1(x) = \frac{d}{dx}(x^2-1),\ P_2(x) = \frac{d^2}{dx^2}(x^2-1)^2,\ P_3(x) = \frac{d^3}{dx^3}(x^2-1)^3$.

Find all possible values for which $\int_{-1}^1 P_k(x)P_l(x)\ dx\ (k = 1,\ 2,\ 3,\ l = 1,\ 2,\ 3)$ can be valued.

478 Evaluate $\int_0^{\frac{\pi}{4}} \{(x\sqrt{\sin x} + 2\sqrt{\cos x})\sqrt{\tan x} + (x\sqrt{\cos x} - 2\sqrt{\sin x})\sqrt{\cot x}\}\ dx$.

479 Let $a,\ b$ be real constants. Find the minimum value of the definite integral:

$I(a,\ b) = \int_0^{\pi} (1 - a\sin x - b\sin 2x)^2 dx.$

480 Let $a,\ b$ be positive real numbers.

Prove that

$$\int_{a-2b}^{2a-b} \left|\sqrt{3b(2a-b) + 2(a-2b)x - x^2} - \sqrt{3a(2b-a) + 2(2a-b)x - x^2}\right| dx$$
$$\leq \frac{\pi}{3}(a^2+b^2).$$

Edited by moderator.

481 For real numbers $a,\ b$ such that $|a| \neq |b|$, let $I_n = \int \frac{1}{(a+b\cos\theta)^n}\ (n \geq 2)$.

Prove that : $\boxed{\boxed{I_n = \frac{a}{a^2-b^2}\cdot\frac{2n-3}{n-1}I_{n-1} - \frac{1}{a^2-b^2}\cdot\frac{n-2}{n-1}I_{n-2} - \frac{b}{a^2-b^2}\cdot\frac{1}{n-1}\cdot\frac{\sin\theta}{(a+b\cos\theta)^{n-1}}}}$

482 Let n be natural number. Find the limit value of $\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{5}} + \cdots\cdots + \frac{n}{\sqrt{n^2+1}}\right)$.

483 Let $n \geq 2$ be natural number. Answer the following questions.

(1) Evaluate the definite integral $\int_1^n x\ln x\ dx$.

(2) Prove the following inequality.

$$\frac{1}{2}n^2\ln n - \frac{1}{4}(n^2-1) < \sum_{k=1}^n k\ln k < \frac{1}{2}n^2\ln n - \frac{1}{4}(n^2-1) + n\ln n.$$

(3) Find $\lim_{n\to\infty}(1^1\cdot 2^2\cdot 3^3\cdots\cdots n^n)^{\frac{1}{n^2\ln n}}$.

484 Let $C : y = \ln x$. For natural number n, denote A_n the area of the region bounded by the line passing through two points $(n,\ \ln n),\ (n+1,\ \ln(n+1))$ on C,
and let B_n be the area of the region bounded by the tangent line at $(n,\ \ln n)$ on C and the line $x = n+1$. Set $g(x) = \ln(x+1) - \ln x$.

(1) Express $A_n,\ B_n$ in terms of $n,\ g(n)$.

(2) Use $A_n > 0,\ B_n > 0$ to find the limit $\lim_{n\to\infty} n\{1 - ng(n)\}$.

485 In the x-y plane, for the origin O, given an isosceles triangle OAB with $AO = AB$ such that A is on the first quadrant and B is on the x axis.
Denote the area by s. Find the area of the common part of the traingle and the region expressed by the inequality $xy \leq 1$ to give the area as the function of s.

486 Let H be the piont of midpoint of the cord PQ that is on the circle centered the origin O with radius 1.
Suppose the length of the cord PQ is $2\sin\frac{t}{2}$ for the angle t $(0 \leq t \leq \pi)$ that is formed by half-ray OH and the positive direction of the x axis. Answer the following questions.

(1) Express the coordiante of H in terms of t.

(2) When t moves in the range of $0 \leq t \leq \pi$, find the minimum value of x coordinate of H.

(3) When t moves in the range of $0\leq t\leq \frac{\pi}{2}$, find the area S of the region bounded by the curve drawn by the point H and the x axis and the y axis.

487 Suppose two functions $f(x)=x^4-x,\ g(x)=ax^3+bx^2+cx+d$ satisfy $f(1)=g(1),\ f(-1)=g(-1).$

Find the values of $a,\ b,\ c,\ d$ such that $\int_{-1}^1 (f(x)-g(x))^2dx$ is minimal.

488 For $0\leq x<\frac{\pi}{2}$, prove the following inequality.

$$x+\ln(\cos x)+\int_0^1 \frac{t}{1+t^2}\,dt\leq \frac{\pi}{4}$$

489 Find the following limit.

$$\lim_{n\to\infty}\int_{-1}^1 |x|\left(1+x+\frac{x^2}{2}+\frac{x^3}{3}+\cdots+\frac{x^{2n}}{2n}\right)\,dx.$$

490 For a positive real number $a>1$, prove the following inequality.

$$\frac{1}{a-1}\left(1-\frac{\ln a}{a-1}\right)<\int_0^1 \frac{x}{a^x}\,dx<\frac{1}{\ln a}\left\{1-\frac{\ln(\ln a+1)}{\ln a}\right\}$$

491 Let $f(x)=\sin 3x+\cos x,\ g(x)=\cos 3x+\sin x.$

(1) Evaluate $\int_0^{2\pi}\{f(x)^2+g(x)^2\}\,dx.$

(2) Find the area of the region bounded by two curves $y=f(x)$ and $y=g(x)$ $(0\leq x\leq \pi).$

492 Find the volume formed by the revolution of the region satisfying $0\leq y\leq (x-p)(q-x)$ $(0<p<q)$ in the coordinate plane about the y -axis.

You are not allowed to use the formula: $V=\boxed{\int_a^b 2\pi x|f(x)|\,dx\ (a<b)}$ here.

493 In the $x-y$ plane, let l be the tangent line at the point $A\left(\frac{a}{2},\ \frac{\sqrt{3}}{2}b\right)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (0<b<1<a)$. Let denote S be the area of the figure bounded by $l,$ the x axis and the ellipse.

(1) Find the equation of $l.$

(2) Express S in terms of $a,\ b.$

(3) Find the maximum value of S with the constraint $a^2+3b^2=4.$

494 Suppose the curve $C:y=ax^3+4x\ (a\neq 0)$ has a common tangent line at the point P with the hyperbola $xy=1$ in the first quadrant.

(1) Find the value of a and the coordinate of the point $P.$

(2) Find the volume formed by the revolution of the solid of the figure bounded by the line segment OP and the curve C about the line $OP.$

[Edited.]

495 Evaluate the following definite integrals.

(1) $\int_0^{\frac{1}{2}}\frac{x^2}{\sqrt{1-x^2}}\,dx$

(2) $\int_0^1 \frac{1-x}{(1+x^2)^2}\,dx$

(3) $\int_{-1}^7 \frac{dx}{1+\sqrt[3]{1+x}}$

496 Evaluate $\int^{a^2} \frac{1}{x^2+a^2}\,dx\ (a>0).$

You may not use $\tan^{-1} x$ or Complex Integral here.

497 Consider a parameterized curve $C: x = e^{-t}\cos t,\ y = e^{-t}\sin t\ \left(0 \leq t \leq \frac{\pi}{2}\right)$.

(1) Find the length L of C.

(2) Find the area S of the region bounded by C, the x axis and y axis.

You may not use the formula $\boxed{\int_a^b \frac{1}{2} r(\theta)^2 d\theta}$ here.

498 Let $f(x)$ be a continuous function defined in the interval $0 \leq x \leq 1$.

Prove that $\int_0^1 xf(x)f(1-x)\ dx \leq \frac{1}{4}\int_0^1 \{f(x)^2 + f(1-x)^2\}\ dx.$

499 Evaluate

$$\int_0^{\pi} \left(\sqrt[2009]{\cos x} + \sqrt[2009]{\sin x} + \sqrt[2009]{\tan x}\right) dx.$$

500 Let $a,\ b,\ c$ be positive real numbers. Prove the following inequality.

$$\int_1^e \frac{x^{a+b+c-1}[2(a+b+c)+(c+2a)x^{a-b}+(a+2b)x^{b-c}+(b+2c)x^{c-a}+(2a+b)x^{a-c}+(2b+c)x^{b-a}+(2c+a)x^{c-b}]}{(x^a+x^b)(x^b+x^c)(x^c+x^a)} \geq a+b+c.$$

I have just posted 500 th post.

Thank you for your cooperations, mathLinkers and AOPS users.

I will keep posting afterwards.

Japanese Communities Modeartor

kunny

501 Find the volume of the uion $A\cup B\cup C$ of the three subsets $A,\ B,\ C$ in xyz space such that:

$$A = \{(x,\ y,\ z)\ |\ |x| \leq 1,\ y^2+z^2 \leq 1\}$$

$$B = \{(x,\ y,\ z)\ |\ |y| \leq 1,\ z^2+x^2 \leq 1\}$$

$$C = \{(x,\ y,\ z)\ |\ |z| \leq 1,\ x^2+y^2 \leq 1\}$$

502 (1) For $0 < x < 1$, prove that $(\sqrt{2}-1)x+1 < \sqrt{x+1} < \sqrt{2}.$

(2) Find $\lim_{a\rightarrow 1-0} \frac{\int_a^1 x\sqrt{1-x^2}\ dx}{(1-a)^{\frac{3}{2}}}$.

503 Prove the following inequality.

$$\frac{2}{2+e^{\frac{1}{2}}} < \int_0^1 \frac{dx}{1+xe^x} < \frac{2+e}{2(1+e)}$$

504 Let $a,\ b$ are positive constants. Determin the value of a positive number m such that the areas of four parts of the region bounded by two parabolas $y = ax^2 - b,\ y = -ax^2 + b$ and the line $y = mx$ have equal area.

505 In the xyz space with the origin O, given a cuboid $K: |x| \leq \sqrt{3},\ |y| \leq \sqrt{3},\ 0 \leq z \leq 2$ and the plane $\alpha: z = 2$. Draw the perpendicular PH from P to the plane. Find the volume of the solid formed by all points of P which are included in K such that $OP \leq PH$

506 Let $a,\ b$ be the real numbers such that $0\leq a\leq b\leq 1.$ Find the minimum value of $\int_0^1 |(x-a)(x-b)|\ dx.$

507 Evaluate

$$\int_e^{e^{2009}} \frac{1}{x}\left\{1+\frac{1-\ln x}{\ln x\cdot \ln \frac{x}{\ln(\ln x)}}\right\}\ dx$$

508 Compare the size of the definite integrals?

$$\int_0^{\frac{\pi}{4}} x^{2008}\tan^{2008} x\ dx,\ \int_0^{\frac{\pi}{4}} x^{2009}\tan^{2009} x\ dx,\ \int_0^{\frac{\pi}{4}} x^{2010}\tan^{2010} x\ dx$$

509 Evaluate $\int_0^{\frac{\pi}{4}} \frac{\tan x}{1+\sin x}\ dx.$

510 (1) Evaluate $\int_0^{\frac{\pi}{2}} (x\cos x+\sin^2 x)\sin x\ dx.$

(2) For $f(x)=\int_0^x e^t\sin (x-t)\ dt$, find $f''(x)+f(x).$

511 Suppose that $f(x),\ g(x)$ are differential fuctions and their derivatives are continuous.
Find $f(x),\ g(x)$ such that $f(x)=\frac{1}{2}-\int_0^x \{f'(t)+g(t)\}\ dt\ \ g(x)=\sin x-\int_0^{\pi} \{f(t)-g'(t)\}\ dt.$

512 Evaluate $\int_0^{n\pi} \sqrt{1-\sin t}\ dt\ (n=1,\ 2,\ \cdots).$

513 Find the constants $a,\ b,\ c$ such that a function $f(x)=a\sin x+b\cos x+c$ satisfies the following equation for any real numbers $x.$

$$5\sin x+3\cos x+1+\int_0^{\frac{\pi}{2}} (\sin x+\cos t)f(t)\ dt=f(x).$$

514 Prove the following inequalities:

(1) $x-\sin x\leq \tan x-x\ \ \left(0\leq x<\frac{\pi}{2}\right)$

(2) $\int_0^x \cos (\tan t-t)\ dt\leq \sin (\sin x)+\frac{1}{2}\left(x-\frac{\sin 2x}{2}\right)\ \ \left(0\leq x\leq \frac{\pi}{3}\right)$

515 Find the maximum and minimum values of $\int_0^{\pi} (a\sin x+b\cos x)^3dx$ for $|a|\leq 1,\ |b|\leq 1.$

Note that you are not allowed to solve in using partial differentiation here.

516 Let $f(x)=\frac{1}{\sin x\sqrt{1-\cos x}}\ (0<x<\pi).$

(1) Find the local minimum value of $f(x).$

(2) Evaluate $\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} f(x)\ dx.$

517 Consider points P which are inside the square with side length a such that the distance from P to the center of the square equals to the least distance from P to each side of the square.Find the area of the figure formed by the whole points $P.$

518 Evaluate $\int_0^{\frac{\pi}{8}} \frac{\cos x}{\cos\left(x-\frac{\pi}{8}\right)}\ dx.$