TỔNG HỢP CÁC BÀI TOÁN TÍCH PHÂN TRÊN BOXMATH

1 Tìm nguyên hàm
$$I=\int \frac{6x^3+8x+1}{(3x^2+4)\sqrt{x^2+1}}\,\mathrm{d}x$$

Lời giải

Ta có $\dfrac{6x^3+8x+1}{3x^2+4}=2x+\dfrac{1}{3x^2+4}$
$$\Rightarrow I=\int\left(2x+\frac{1}{3x^2+4}\right)\frac{1}{\sqrt{x^2+1}}\,\mathrm{d}x=\int\frac{2x}{\sqrt{x^2+1}}\,\mathrm{d}x+\int\frac{1}{(3x^2+4)\sqrt{x^2+1}}\,\mathrm{d}x$$
Tính $I_1=\displaystyle\int\frac{2x}{\sqrt{x^2+1}}\,\mathrm{d}x$

Đặt $\sqrt{x^2+1}=t,\ x^2+1=t^2,\ 2\,\mathrm{d}t=2x\,\mathrm{d}x\Rightarrow I_1=2\displaystyle\int\frac{\mathrm{d}t}{t}=2\ln|t|=2\ln\sqrt{x^2+1}$

Tính $I_2=\displaystyle\int\frac{1}{(3x^2+4)\sqrt{x^2+1}}.\,\mathrm{d}x$

Đặt $t=\dfrac{\sqrt{x^2+1}}{x},\ xt=\sqrt{x^2+1},\ x^2t^2=x^2+1,\ x^2=\dfrac{1}{t^2-1},\ 3x^2+4=\dfrac{4t^2-1}{t^2-1}$
$$x\,\mathrm{d}x=-\frac{t\,\mathrm{d}t}{(t^2-1)^2},\ \frac{\mathrm{d}x}{xt}=-\frac{t\,\mathrm{d}t}{(t^2-1)^2x^2t},\ \frac{\mathrm{d}x}{\sqrt{x^2+1}}=\frac{\mathrm{d}t}{1-t^2}$$
$$I_2=\int\frac{t^2-1}{4t^2-1}\frac{\mathrm{d}t}{1-t^2}=\int\frac{\mathrm{d}t}{1-4t^2}=\frac{1}{2}\int\left(\frac{1}{2t+1}-\frac{1}{2t-1}\right)\mathrm{d}t=\frac{1}{4}\ln\frac{2t+1}{2t-1}=\frac{1}{4}\ln\frac{2\sqrt{x^2+1}+x}{2\sqrt{x^2+1}-x}$$
Vậy
$$I=2\ln\sqrt{x^2+1}+\frac{1}{4}\ln\frac{2\sqrt{x^2+1}+x}{2\sqrt{x^2+1}-x}+C$$

2 Tìm nguyên hàm
$$I=\int\frac{\cos^2x}{\sin x+\sqrt{3}\cos x}\,\mathrm{d}x$$

Lời giải

Dùng pp hệ số bất định $\cos^2x=(a\sin x+b\cos x)(\sin x+\sqrt{3}\cos x)+c(\sin^2x+\cos^2x)$
$$\cos^2x=\left(\frac{-1}{4}\sin x+\frac{\sqrt{3}}{4}\cos x\right)(\sin x+\sqrt{3}\cos x)+\frac{1}{4}=\frac{-1}{4}(\sin x-\sqrt{3}\cos x)(\sin x+\sqrt{3}\cos x)+\frac{1}{4}$$
$$I=\int\frac{\frac{-1}{4}(\sin x-\sqrt{3}\cos x)(\sin x+\sqrt{3}\cos x)+\frac{1}{4}}{\sin x+\sqrt{3}\cos x}\,\mathrm{d}x$$
$$I=\frac{-1}{4}\int(\sin x-\sqrt{3}\cos x)\,\mathrm{d}x+\frac{1}{4}\int\frac{1}{\sin x+\sqrt{3}\cos x}\,\mathrm{d}x$$
$$I=\frac{1}{4}(\cos x+\sqrt{3}\sin x)+\frac{1}{4}\int\frac{1}{\sin x+\sqrt{3}\cos x}\,\mathrm{d}x$$
Ta tính $J=\dfrac{1}{4}\displaystyle\int\frac{\mathrm{d}x}{\sin x+\sqrt{3}\cos x}=\frac{1}{8}\int\frac{\mathrm{d}x}{\cos(x-\frac{\pi}{6})}=\frac{1}{8}\int\frac{\cos(x-\frac{\pi}{6})}{1-\sin^2(x-\frac{\pi}{6})}\,\mathrm{d}x$

Đặt $t=\sin(x-\frac{\pi}{6})\Rightarrow\ \mathrm{d}t=\cos(x-\frac{\pi}{6})\,\mathrm{d}x$
$$\Rightarrow J=\frac{1}{8}\int\frac{\mathrm{d}t}{1-t^2}=\frac{1}{16}\int\left(\frac{1}{t+1}-\frac{1}{t-1}\right)\,\mathrm{d}t=\frac{1}{16}\ln\frac{t+1}{t-1}=\frac{1}{16}\ln\frac{\sin(x-\frac{\pi}{6})+1}{\sin(x-\frac{\pi}{6})-1}$$
Vậy
$$I=\frac{1}{4}(\cos x+\sqrt{3}\sin x)+\frac{1}{16}\ln\frac{\sin(x-\frac{\pi}{6})+1}{\sin(x-\frac{\pi}{6})-1}+C$$

3 Tìm nguyên hàm
$$I=\int\frac{x^3+x^2}{\sqrt[4]{4x+5}}\,\mathrm{d}x$$

Lời giải

$$I=\int\frac{x^3+x^2}{\sqrt[4]{4x+5}}\,\mathrm{d}x=\int\frac{x^4+x^3}{\sqrt[4]{4x^5+5x^4}}\,\mathrm{d}x=\frac{1}{20}\int\left(4x^5+5x^4\right)^{-\frac{1}{4}}\mathrm{d}(4x^5+5x^4)=\frac{1}{15}\sqrt[4]{(4x^5+5x^4)^3}+C$$

$\boxed{4}$ Tìm nguyên hàm

$$I=\int\left(\cos 2x+\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)\right)e^{\sin x+\cos x+1}\,\mathrm{d}x$$

Lời giải

Ta có
$$\cos 2x+\sqrt{2}\cos\left(x+\tfrac{\pi}{4}\right)=(\cos x-\sin x)(\sin x+\cos x+1)$$

$$I=\int(\cos x-\sin x)(\sin x+\cos x+1)e^{\sin x+\cos x+1}\,\mathrm{d}x$$

$$I=\int(\sin x+\cos x+1)e^{\sin x+\cos x+1}\,\mathrm{d}\left(\sin x+\cos x+1\right)$$

$$I=\int(\sin x+\cos x+1)\,\mathrm{d}\left(e^{\sin x+\cos x+1}\right)$$

$$I=(\sin x+\cos x+1)e^{\sin x+\cos x+1}-\int e^{\sin x+\cos x+1}\,\mathrm{d}\left(\sin x+\cos x+1\right)$$

$$I=(\sin x+\cos x+1)e^{\sin x+\cos x+1}-e^{\sin x+\cos x+1}+C$$

$$I=(\sin x+\cos x)e^{\sin x+\cos x+1}+C$$

$\boxed{5}$ Tìm nguyên hàm

$$I=\int\sqrt[3]{3x-x^3}\,\mathrm{d}x$$

Lời giải

Đặt $t=\dfrac{\sqrt[3]{3x-x^3}}{x}\Rightarrow x^2=\dfrac{3}{t^3+1}\Rightarrow 2x\,\mathrm{d}x=\dfrac{-9t^2\,\mathrm{d}t}{(t^3+1)^2}$

$$I=\frac{1}{2}\int\frac{\sqrt[3]{3x-x^3}}{x}2x\,\mathrm{d}x=\frac{-9}{2}\int\frac{t^3\,\mathrm{d}t}{(t^3+1)^2}=\frac{3}{2}\int t\,\mathrm{d}\left(\frac{1}{t^3+1}\right)=\frac{3t}{2(t^3+1)}-\frac{3}{2}\int\frac{\mathrm{d}t}{t^3+1}$$

Tính $J=\displaystyle\int\frac{\mathrm{d}t}{t^3+1}=\int\frac{\mathrm{d}(t+1)}{(t+1)[(t+1)^2-3(t+1)+3]}=\frac{1}{2}\left(\ln 3(1-t)-2\ln 3t+\ln(1+t)\right)$

Vậy $I=\dfrac{1}{2}x\sqrt[3]{3x-x^3}-\dfrac{3}{4}\left(\ln 3\left(1-\dfrac{\sqrt[3]{3x-x^3}}{x}\right)-2\ln 3\dfrac{\sqrt[3]{3x-x^3}}{x}+\ln\left(1+\dfrac{\sqrt[3]{3x-x^3}}{x}\right)\right)+C$

$\boxed{6}$ Tìm nguyên hàm

$$I=\int\frac{1}{x^4+4x^3+6x^2+7x+4}\,\mathrm{d}x$$

Lời giải

Tổng các hệ số bậc chẵn bằng tổng các hệ số bậc lẻ nên đa thức ở mẫu nhận $x=-1$ làm nghiệm

$$I=\int\frac{\mathrm{d}x}{(x+1)[(x+1)^3+3]}$$

$$I=\frac{1}{3}\int\frac{(x+1)^3+3-(x+1)^3}{(x+1)[(x+1)^3+3]}\,\mathrm{d}x$$

$$I=\frac{1}{3}\left[\int\frac{\mathrm{d}x}{x+1}-\int\frac{(x+1)^2}{(x+1)^3+3}\,\mathrm{d}x\right]$$

$$I=\frac{1}{3}\left[\ln|x+1|-\frac{1}{3}\int\frac{\mathrm{d}((x+1)^3)}{(x+1)^3+3}\right]$$

$$I=\frac{1}{3}\ln|x+1|-\frac{1}{9}\ln|(x+1)^3+3|+C$$

$\boxed{7}$ Tính tích phân

$$I=\int_0^1\frac{x\ln\left(x+\sqrt{1+x^2}\right)}{x+\sqrt{1+x^2}}\,\mathrm{d}x$$

Lời giải

Đặt $u = \ln(x + \sqrt{x^2+1})$, $\quad$ $dv = \dfrac{x\ dx}{x + \sqrt{x^2+1}} = x(\sqrt{x^2+1} - x)\ dx$

Suy ra $du = \dfrac{1 + \dfrac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}}\ dx = \dfrac{dx}{\sqrt{x^2+1}}$, $\quad v = \dfrac{1}{2}\displaystyle\int (1+x^2)^{\frac{1}{2}} d(1+x^2) - \int x^2 dx = \dfrac{1}{3}[(1+x^2)^{\frac{3}{2}} - x^3]$

$$I = \frac{1}{3}[(1+x^2)^{\frac{3}{2}} - x^3]\ln(x+\sqrt{1+x^2})\Big|_0^1 - \frac{1}{3}\int_0^1 [(1+x^2)^{\frac{3}{2}} - x^3]\frac{dx}{\sqrt{1+x^2}}$$

Mà $\quad J = \displaystyle\int [(1+x^2)^{\frac{3}{2}} - x^3]\frac{dx}{\sqrt{1+x^2}} = \int \frac{dx}{1+x^2} - \int \frac{x^3 dx}{\sqrt{1+x^2}} = \arctan x - \frac{1}{3}(x^2-2)\sqrt{x^2+1}$

Nên $\quad I = \dfrac{1}{3}[(1+x^2)^{\frac{3}{2}} - x^3]\ln(x+\sqrt{1+x^2})\Big|_0^1 - \dfrac{1}{3}\arctan x\Big|_0^1 + \dfrac{1}{9}(x^2-2)\sqrt{x^2+1}\Big|_0^1$

Vậy $\quad I = \dfrac{1}{3}(\sqrt{8}-1)\ln(1+\sqrt{2}) - \dfrac{\pi}{12} + \dfrac{1}{9}(2+\sqrt{2})$

$\boxed{8}$ Tính tích phân

$$I = \int_0^{\frac{1}{2}} x\ln\frac{1+x}{1-x}\ dx$$

Lời giải

Với $u = \ln\dfrac{1+x}{1-x}$, $\quad dv = x\ dx \quad$ nên $\ du = \dfrac{2}{1-x^2}\ dx$, $\quad v = \dfrac{1}{2}x^2$

$$I = \frac{1}{2}x^2\ln\frac{1+x}{1-x}\Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} \frac{x^2}{1-x^2}\ dx$$

$$I = \frac{1}{8}\ln 3 + \int_0^{\frac{1}{2}} \frac{1-x^2-1}{1-x^2}\ dx$$

$$I = \frac{1}{8}\ln 3 + \frac{1}{2} - \frac{1}{2}\int_0^{\frac{1}{2}} \left(\frac{1}{1+x} + \frac{1}{1-x}\right)\ dx$$

$$I = \frac{1}{8}\ln 3 + \frac{1}{2} - \frac{1}{2}\ln\frac{1+x}{1-x}\Big|_0^{\frac{1}{2}}$$

$$I = \frac{1}{2} - \frac{3}{8}\ln 3$$

$\boxed{9}$ Tính tích phân

$$I = \int_0^{\pi} e^{-x}\cos 2x\ dx$$

Lời giải

$$I = \int_0^{\pi} e^{-x}\cos 2x\ dx$$

$$I = -\int_0^{\pi} \cos 2x\ d(e^{-x})$$

$$I = -e^{-x}\cos 2x\Big|_0^{\pi} - 2\int_0^{\pi} e^{-x}\sin 2x\ dx$$

$$I = e^{-\pi} + 1 + 2\int_0^{\pi} \sin 2x\ d(e^{-x})$$

$$I = e^{-\pi} + 1 + 2e^{-x}\sin 2x\Big|_0^{\pi} - 4\int_0^{\pi} e^{-x}\cos 2x\ dx$$

$$I = \frac{1}{5}(e^{-\pi} + 1)$$

10 Tính tích phân

$$I=\int_0^{\sqrt{3}}\frac{x^5+2x^3}{\sqrt{x^2+1}}\,\mathrm{d}x$$

Lời giải

$$I=\int_0^{\sqrt{3}}\frac{x(x^4+2x^2)}{\sqrt{x^2+1}}\,\mathrm{d}x=\int_0^{\sqrt{3}}(x^4+2x^2)\,\mathrm{d}(\sqrt{x^2+1})$$

$$I=(x^4+2x^2)\sqrt{x^2+1}\Big|_0^{\sqrt{3}}-\int_0^{\sqrt{3}}\sqrt{x^2+1}\,\mathrm{d}(x^4+2x^2)$$

Tính
$$J=\int\sqrt{x^2+1}\,\mathrm{d}(x^4+2x^2)=\int 4x(x^2+1)\sqrt{x^2+1}\,\mathrm{d}x=4\int\frac{x(x^2+1)^2}{\sqrt{x^2+1}}\,\mathrm{d}x$$
$$=4\int(\sqrt{x^2+1})^4\,\mathrm{d}(\sqrt{x^2+1})=\frac{4}{5}(x^2+1)^2\sqrt{x^2+1}$$

Nên
$$I=(x^4+2x^2)\sqrt{x^2+1}\Big|_0^{\sqrt{3}}-\frac{4}{5}(x^2+1)^2\sqrt{x^2+1}\Big|_0^{\sqrt{3}}$$

11 Tính tích phân

$$I=\int_1^e\frac{1+x^2\ln x}{x+x^2\ln x}\,\mathrm{d}x$$

Lời giải

$$\begin{aligned}I&=\int_1^e\frac{1+x^2\ln x}{x+x^2\ln x}\,\mathrm{d}x\\
&=\int_1^e\frac{\frac{1}{x^2}+\ln x}{\frac{1}{x}+\ln x}\,\mathrm{d}x\\
&=\int_1^e\frac{\frac{1}{x}+\ln x}{\frac{1}{x}+\ln x}\,\mathrm{d}x+\int_1^e\frac{\frac{1}{x^2}-\frac{1}{x}}{\frac{1}{x}+\ln x}\,\mathrm{d}x\\
&=\int_1^e\mathrm{d}x-\int_1^e\frac{\mathrm{d}\left(\frac{1}{x}+\ln x\right)}{\frac{1}{x}+\ln x}\\
&=x\Big|_1^e-\ln\left(\frac{1}{x}+\ln x\right)\Big|_1^e\\
&=e-1-\ln\left(\frac{1}{e}+1\right)\end{aligned}$$

12 Tính nguyên hàm

$$I=\int\frac{2(1+\ln x)+x\ln x(1+\ln x)}{1+x\ln x}\,\mathrm{d}x$$

Lời giải

Đặt
$$u=1+x\ln x\Rightarrow\ \mathrm{d}u=(1+\ln x)\ \mathrm{d}x$$
$$I=\int\frac{(2+x\ln x)(1+\ln x)}{1+x\ln x}\,\mathrm{d}x=\int\frac{u+1}{u}\,\mathrm{d}u=u+\ln|u|+C=1+x\ln x+\ln|1+x\ln x|+C$$

13 Tính tích phân

$$I=\int_0^{\frac{\pi}{4}}\frac{x^2(x^2\sin 2x+1)-(x-1)\sin 2x}{\cos x(x^2\sin x+\cos x)}\,\mathrm{d}x$$

Lời giải

$$
\begin{aligned}
I &= \int \frac{x^4 \sin 2x + x^2 - (x-1)\sin 2x}{x^2 \sin x \cos x + \cos^2 x}\ \mathrm{d}x \\
&= \int_0^{\frac{\pi}{4}} \frac{2x^4 \sin 2x + 2x^2 - 2x \sin x + 2\sin 2x}{x^2 \sin 2x + \cos 2x + 1}\ \mathrm{d}x \\
&= \int_0^{\frac{\pi}{4}} \frac{2x^2(x^2 \sin 2x + \cos 2x + 1) - (x^2 \sin 2x + \cos 2x + 1)'}{x^2 \sin 2x + \cos 2x + 1}\ \mathrm{d}x \\
&= \int_0^{\frac{\pi}{4}} 2x^2\ \mathrm{d}x - \int_0^{\frac{\pi}{4}} \frac{\mathrm{d}(x^2 \sin 2x + \cos 2x + 1)}{x^2 \sin 2x + \cos 2x + 1} \\
&= \frac{2}{3}x^3\Big|_0^{\frac{\pi}{4}} - \ln|x^2 \sin 2x + \cos 2x + 1|\Big|_0^{\frac{\pi}{4}} \\
&= \frac{\pi^3}{96} + \ln 2 - \ln\left(\frac{\pi^2}{16} + 1\right)
\end{aligned}
$$

$\boxed{14}$ Tính nguyên hàm

$$I = \int \frac{(x^2+1) + (x^3 + x\ln x + 2)\ln x}{1 + x\ln x}\ \mathrm{d}x$$

Lời giải

$$I = \int \frac{(x^2 + \ln x) + x\ln x(x^2 + \ln x) + (1 + \ln x)}{1 + x\ln x}\ \mathrm{d}x$$

$$I = \int \frac{(x^2 + \ln x)(1 + x\ln x) + (1 + \ln x)}{1 + x\ln x}\ \mathrm{d}x$$

$$I = \int (x^2 + \ln x)\ \mathrm{d}x + \int \frac{\mathrm{d}(1 + x\ln x)}{1 + x\ln x}$$

$$I = \frac{1}{3}.x^3 + x\ln x - x + \ln|1 + x\ln x| + C$$

$\boxed{15}$ Tính nguyên hàm

$$I = \int \frac{x^2(x^2\sin^2 x + \sin 2x + \cos x) + \sin x(2x - 1 - \sin x) + 1}{x^2 \sin x + \cos x}\ \mathrm{d}x$$

Lời giải

Vì $x^2(x^2\sin^2 x + \sin 2x + \cos x) + \sin x(2x - 1 - \sin x) + 1 = (x^2\sin x + \cos x)^2 + (x^2 \sin x + \cos x)'$

$I = \displaystyle\int (x^2 \sin x + \cos x)\ \mathrm{d}x + \int \frac{\mathrm{d}(x^2 \sin x + \cos x)}{x^2 \sin x + \cos x} = \int x^2 \sin x\ \mathrm{d}x + \sin x + \ln|x^2 \sin x + \cos x|$

Tính $J = \displaystyle\int x^2 \sin x\ \mathrm{d}x = -\int x^2\ \mathrm{d}(\cos x) = -x^2\cos x + 2\int x\cos x\ \mathrm{d}x = -x^2\cos x + 2\int x\ \mathrm{d}(\sin x)$

$$J = -x^2\cos x + 2x\sin x - 2\int \sin x\ \mathrm{d}x = -x^2\cos x + 2x\sin x + 2\cos x$$

Vậy
$$I = -x^2\cos x + 2x\sin x + 2\cos x + \sin x + \ln|x^2\sin x + \cos x| + C$$

$\boxed{16}$ Tìm nguyên hàm

$$I = \int \Big(x(x+2)(3\sin x - 4\sin^3 x) + 2\cos x(\cos x - 2\sin x) + 3x^2\cos 3x - 1\Big)e^x\ \mathrm{d}x$$

Lời giải

$$
\begin{aligned}
&\Big(x(x+2)(3\sin x - 4\sin^3 x) + 2\cos x(\cos x - 2\sin x) + 3x^2\cos 3x - 1\Big)e^x \\
&= \Big(x^2\sin 3x + (x^2\sin 3x)' + \cos 2x + (\cos 2x)'\Big)e^x \\
&\Rightarrow I = (x^2\sin 3x + \cos 2x)e^x
\end{aligned}
$$

$\boxed{17}$ Tìm nguyên hàm

$$I = \int \frac{2x^4\ln^2 x + x\ln x(x^3+1) + x - \frac{1}{x^2}}{1 + x^3\ln x}\ \mathrm{d}x$$

Lời giải

$$\frac{2x^6\ln^2 x + x^6\ln x + x^3\ln x + x^3 - 1}{x^2 + x^5\ln x}$$
$$= \frac{2[(x^3\ln x)^2 - 1] + x^3(x^3\ln x + 1) + (x^3\ln x + 1)}{x^2(1 + x^3\ln x)}$$
$$= \frac{(x^3\ln x + 1)(2x^3\ln x + x^3 - 1)}{x^2(1 + x^3\ln x)} = 2x\ln x + x - \frac{1}{x^2}$$

Nên $$I = \int\left(2x\ln x + x - \frac{1}{x^2}\right)\,\mathrm{d}x = \frac{1}{2}x^2 + \frac{1}{x} + \int 2x\ln x\,\mathrm{d}x = \frac{1}{2}x^2 + \frac{1}{x} + \int \ln x\,\mathrm{d}(x^2)$$
$$I = \frac{1}{2}x^2 + \frac{1}{x} + x^2\ln x - \int x\,\mathrm{d}x = \frac{1}{x} + x^2\ln x + C$$

$\boxed{18}$ Tìm nguyên hàm
$$I = \int x^2\sin(\ln x)\,\mathrm{d}x$$

Lời giải

Đặt $x = e^t, \quad \ln x = t, \quad \mathrm{d}x = e^t\mathrm{d}t$
$$\Rightarrow I = \int e^{3t}\sin t\,\mathrm{d}t = -e^{3t}\cos t + \int 3e^{3t}\cos t\,\mathrm{d}t = -e^{3t}\cos t + 3e^{3t}\sin t - \int 9e^{3t}\sin t\,\mathrm{d}t$$
$$\Rightarrow 10I = 3e^{3t}\sin t - e^{3t}\cos t \Rightarrow I = \frac{1}{10}\Big(3.e^{3\ln x}\sin(\ln x) - e^{3\ln x}\cos(\ln x)\Big) + C$$

$\boxed{19}$ Tìm nguyên hàm
$$I = \int \frac{e^x(x-1) + 2x^3 + x^3(e^x + x(x^2+1))}{e^x.x + x^2(x^2+1)}\,\mathrm{d}x$$

Lời giải

$$\frac{e^x(x-1) + 2x^3 + x^3(e^x + x(x^2+1))}{e^x.x + x^2(x^2+1)} = \frac{x^3-1}{x} + \frac{3x^2 + e^x + 1}{x^3 + x + e^x} = x^2 - \frac{1}{x} + \frac{(x^3 + x + e^x)'}{x^3 + x + e^x}$$

Do đó
$$I = \frac{x^3}{3} - \ln|x| + \ln|x^3 + x + e^x| + C$$

$\boxed{20}$ Tính tích phân
$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \ln(\tan x)\,\mathrm{d}x$$

Lời giải

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \ln(\tan x)\,\mathrm{d}x \overset{\text{đổi biến } (x = \frac{\pi}{2} - x)}{=} \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \ln(\cot x)\,\mathrm{d}x \Rightarrow 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \ln(\tan x.\cot x)\,\mathrm{d}x = 0 \Rightarrow I = 0$$