TỔNG HỢP CÁC BÀI TOÁN TÍCH PHÂN TRÊN BOXMATH

10 Tính tích phân

$$I=\int_0^{\sqrt{3}}\frac{x^5+2x^3}{\sqrt{x^2+1}}\,\mathrm{d}x$$

Lời giải

$$I=\int_0^{\sqrt{3}}\frac{x(x^4+2x^2)}{\sqrt{x^2+1}}\,\mathrm{d}x=\int_0^{\sqrt{3}}(x^4+2x^2)\,\mathrm{d}(\sqrt{x^2+1})$$

$$I=(x^4+2x^2)\sqrt{x^2+1}\Big|_0^{\sqrt{3}}-\int_0^{\sqrt{3}}\sqrt{x^2+1}\,\mathrm{d}(x^4+2x^2)$$

Tính $$J=\int\sqrt{x^2+1}\,\mathrm{d}(x^4+2x^2)=\int 4x(x^2+1)\sqrt{x^2+1}\,\mathrm{d}x=4\int\frac{x(x^2+1)^2}{\sqrt{x^2+1}}\,\mathrm{d}x$$

$$=4\int(\sqrt{x^2+1})^4\,\mathrm{d}(\sqrt{x^2+1})=\frac{4}{5}(x^2+1)^2\sqrt{x^2+1}$$

Nên $$I=(x^4+2x^2)\sqrt{x^2+1}\Big|_0^{\sqrt{3}}-\frac{4}{5}(x^2+1)^2\sqrt{x^2+1}\Big|_0^{\sqrt{3}}$$

11 Tính tích phân

$$I=\int_1^e\frac{1+x^2\ln x}{x+x^2\ln x}\,\mathrm{d}x$$

Lời giải

$$I=\int_1^e\frac{1+x^2\ln x}{x+x^2\ln x}\,\mathrm{d}x$$

$$=\int_1^e\frac{\dfrac{1}{x^2}+\ln x}{\dfrac{1}{x}+\ln x}\,\mathrm{d}x$$

$$=\int_1^e\frac{\dfrac{1}{x}+\ln x}{\dfrac{1}{x}+\ln x}\,\mathrm{d}x+\int_1^e\frac{\dfrac{1}{x^2}-\dfrac{1}{x}}{\dfrac{1}{x}+\ln x}\,\mathrm{d}x$$

$$=\int_1^e\mathrm{d}x-\int_1^e\frac{\mathrm{d}\left(\dfrac{1}{x}+\ln x\right)}{\dfrac{1}{x}+\ln x}$$

$$=x\Big|_1^e-\ln\left(\frac{1}{x}+\ln x\right)\Big|_1^e$$

$$=e-1-\ln\left(\frac{1}{e}+1\right)$$

12 Tính nguyên hàm

$$I=\int\frac{2(1+\ln x)+x\ln x(1+\ln x)}{1+x\ln x}\,\mathrm{d}x$$

Lời giải

Đặt $$u=1+x\ln x\Rightarrow\ \mathrm{d}u=(1+\ln x)\ \mathrm{d}x$$

$$I=\int\frac{(2+x\ln x)(1+\ln x)}{1+x\ln x}\,\mathrm{d}x=\int\frac{u+1}{u}\,\mathrm{d}u=u+\ln|u|+C=1+x\ln x+\ln|1+x\ln x|+C$$

13 Tính tích phân

$$I=\int_0^{\frac{\pi}{4}}\frac{x^2(x^2\sin 2x+1)-(x-1)\sin 2x}{\cos x(x^2\sin x+\cos x)}\,\mathrm{d}x$$

Lời giải

$$I = \int \frac{x^4 \sin 2x + x^2 - (x-1)\sin 2x}{x^2 \sin x \cos x + \cos^2 x}\,\mathrm{d}x$$
$$= \int_0^{\frac{\pi}{4}} \frac{2x^4 \sin 2x + 2x^2 - 2x \sin x + 2\sin 2x}{x^2 \sin 2x + \cos 2x + 1}\,\mathrm{d}x$$
$$= \int_0^{\frac{\pi}{4}} \frac{2x^2(x^2 \sin 2x + \cos 2x + 1) - (x^2 \sin 2x + \cos 2x + 1)'}{x^2 \sin 2x + \cos 2x + 1}\,\mathrm{d}x$$
$$= \int_0^{\frac{\pi}{4}} 2x^2\,\mathrm{d}x - \int_0^{\frac{\pi}{4}} \frac{\mathrm{d}(x^2 \sin 2x + \cos 2x + 1)}{x^2 \sin 2x + \cos 2x + 1}$$
$$= \frac{2}{3}x^3\Big|_0^{\frac{\pi}{4}} - \ln|x^2 \sin 2x + \cos 2x + 1|\Big|_0^{\frac{\pi}{4}}$$
$$= \frac{\pi^3}{96} + \ln 2 - \ln\left(\frac{\pi^2}{16} + 1\right)$$

14 Tính nguyên hàm

$$I = \int \frac{(x^2+1) + (x^3 + x\ln x + 2)\ln x}{1 + x\ln x}\,\mathrm{d}x$$

Lời giải

$$I = \int \frac{(x^2 + \ln x) + x\ln x(x^2 + \ln x) + (1 + \ln x)}{1 + x\ln x}\,\mathrm{d}x$$
$$I = \int \frac{(x^2 + \ln x)(1 + x\ln x) + (1 + \ln x)}{1 + x\ln x}\,\mathrm{d}x$$
$$I = \int (x^2 + \ln x)\,\mathrm{d}x + \int \frac{\mathrm{d}(1 + x\ln x)}{1 + x\ln x}$$
$$I = \frac{1}{3}.x^3 + x\ln x - x + \ln|1 + x\ln x| + C$$

15 Tính nguyên hàm

$$I = \int \frac{x^2(x^2\sin^2 x + \sin 2x + \cos x) + \sin x(2x - 1 - \sin x) + 1}{x^2 \sin x + \cos x}\,\mathrm{d}x$$

Lời giải

Vì $x^2(x^2\sin^2 x + \sin 2x + \cos x) + \sin x(2x - 1 - \sin x) + 1 = (x^2 \sin x + \cos x)^2 + (x^2 \sin x + \cos x)'$

$$I = \int (x^2 \sin x + \cos x)\,\mathrm{d}x + \int \frac{\mathrm{d}(x^2 \sin x + \cos x)}{x^2 \sin x + \cos x} = \int x^2 \sin x\,\mathrm{d}x + \sin x + \ln|x^2 \sin x + \cos x|$$

Tính $J = \int x^2 \sin x\,\mathrm{d}x = -\int x^2\,\mathrm{d}(\cos x) = -x^2\cos x + 2\int x\cos x\,\mathrm{d}x = -x^2\cos x + 2\int x\,\mathrm{d}(\sin x)$

$$J = -x^2\cos x + 2x\sin x - 2\int \sin x\,\mathrm{d}x = -x^2\cos x + 2x\sin x + 2\cos x$$

Vậy $$I = -x^2\cos x + 2x\sin x + 2\cos x + \sin x + \ln|x^2 \sin x + \cos x| + C$$