

# (Quasi) Periodic Signals in Regularly Sampled Data



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# Outline

- Evenly-spaced time series
- Recap: Fourier transforms, power spectra
- Wavelets, Hilbert-Huang transform
- Dynamical power spectrum
- Signals from:
  - Stellar-mass black holes
  - Neutron stars
  - Regular stars
  - Active galactic nuclei (super-massive black holes)
- Cross-spectral analysis techniques (“spectral-timing”)
- Stingray: open-source analysis software

# Evenly-spaced time series

- Signal period  $\ll$  observation length
- In X-rays and gamma-rays, we count photons. It's very possible to have zero counts -- “sparse” light curves are common
  - Instead of saving light curves with lots of zeroes, we use event lists
  - Analysis  $dt$  might be milliseconds, but detector  $dt$  can be microseconds or nanoseconds!
- In optical etc., bright-enough sources mean you detect flux above background in every time bin (e.g., every 30 seconds)



# Fourier transforms

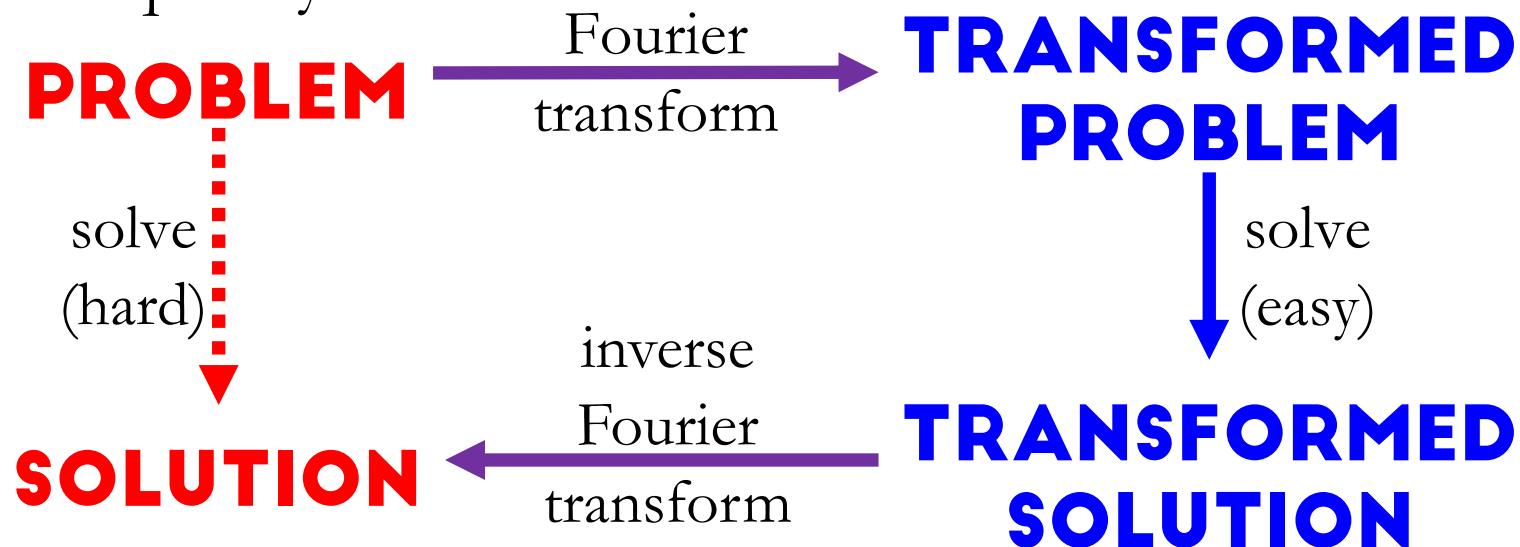
- Observed light curves can vary on timescales from microseconds to years
- Shorter (< 1 minute) variability: Fourier analysis!
  - Study **time domain  $f$**  in the frequency domain  $\hat{f}$
  - Break down **light curve** into **sine waves**, take amplitude of sines at each frequency



# Fourier transforms

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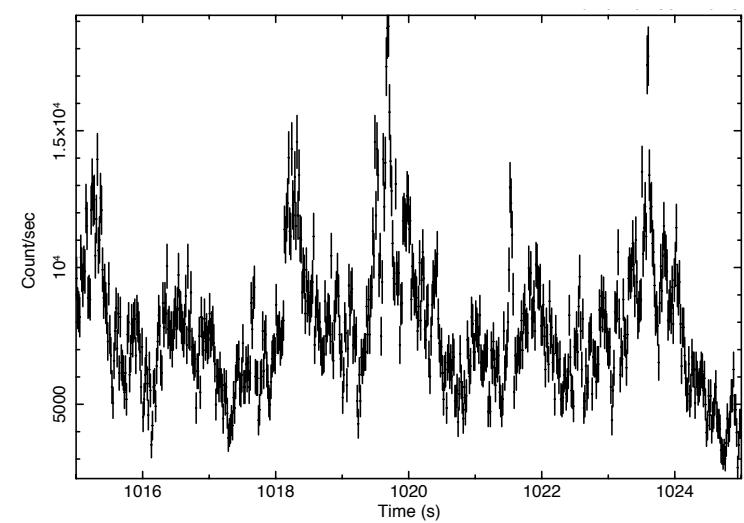
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# Applying Fourier transforms to data

Time domain

Light curve



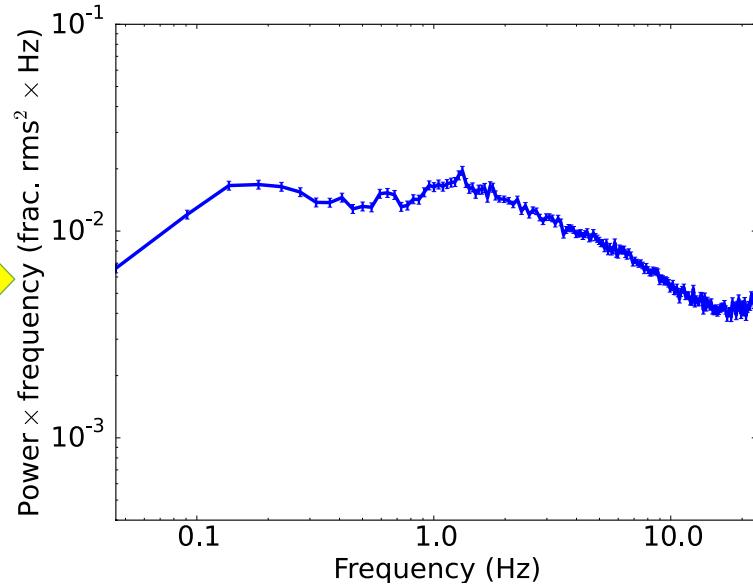
$$x(t) \rightarrow X(\nu)$$

$$\begin{aligned} P(\nu) &= X(\nu)X^*(\nu) \\ &= |X(\nu)|^2 \end{aligned}$$

FOURIER  
TRANSFORM<sup>2</sup>

Frequency/Fourier domain

Power density spectrum



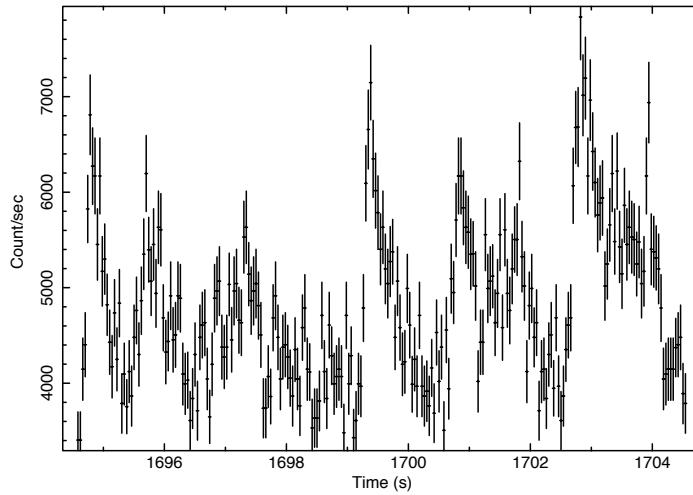
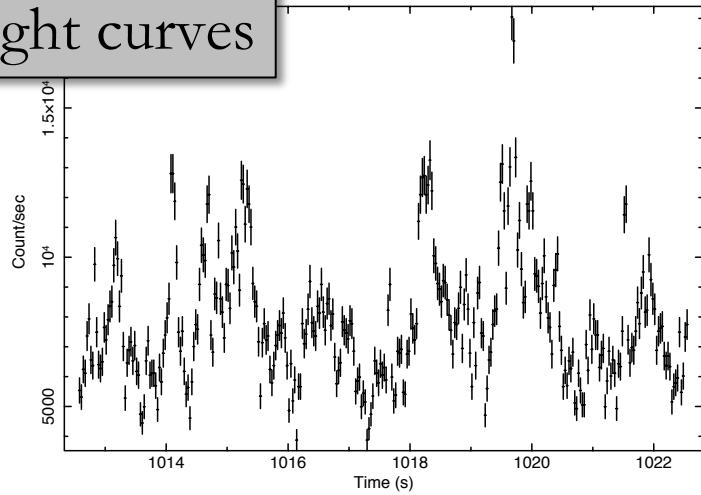
Light curve broken into equal-length chunks,  
take power spectrum of each chunk,  
average those together

# X-ray variability: Hard to see by eye

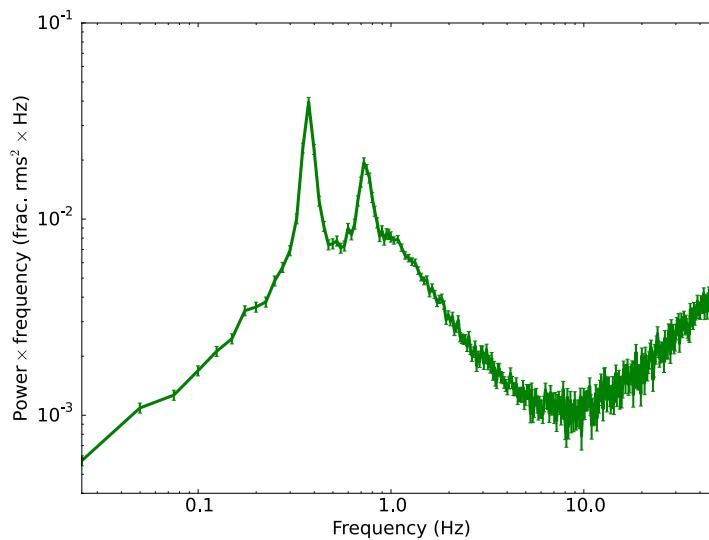
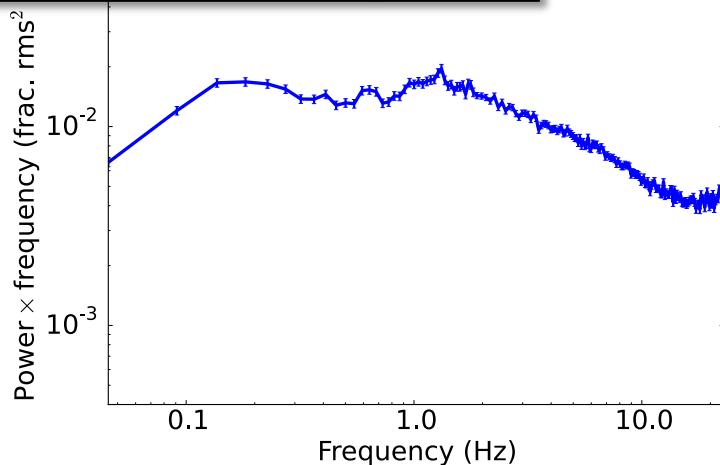
Noise: Cygnus X-1

Signal: GRS 1915+105

Light curves

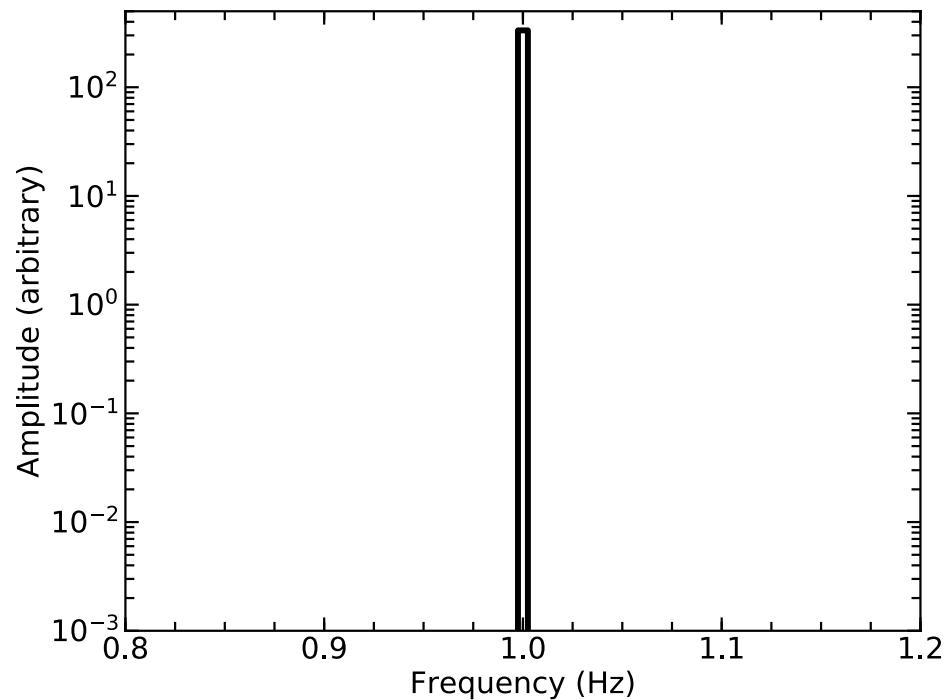
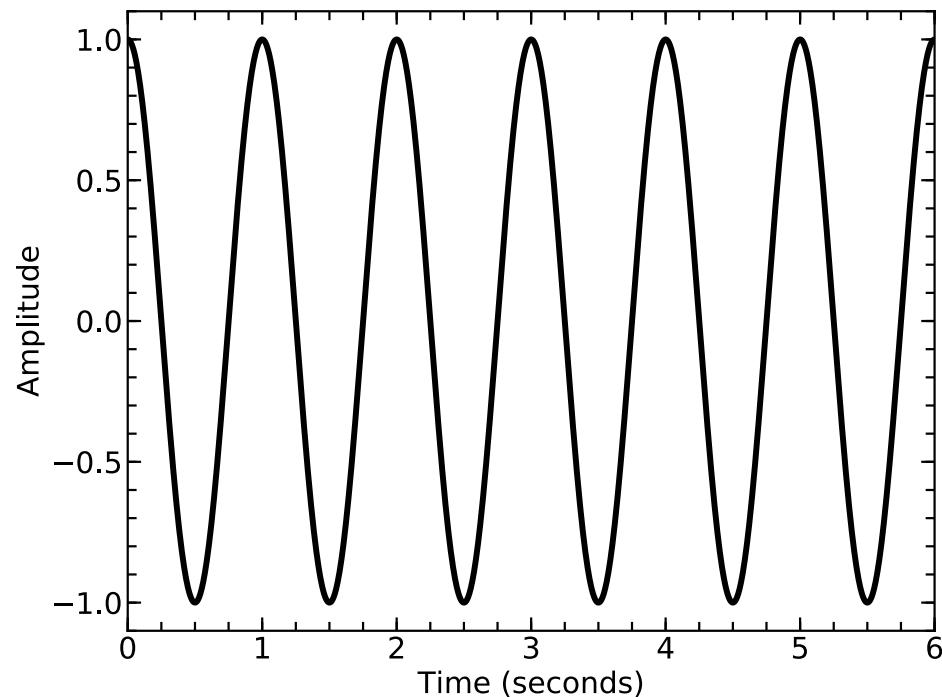


Power density spectra



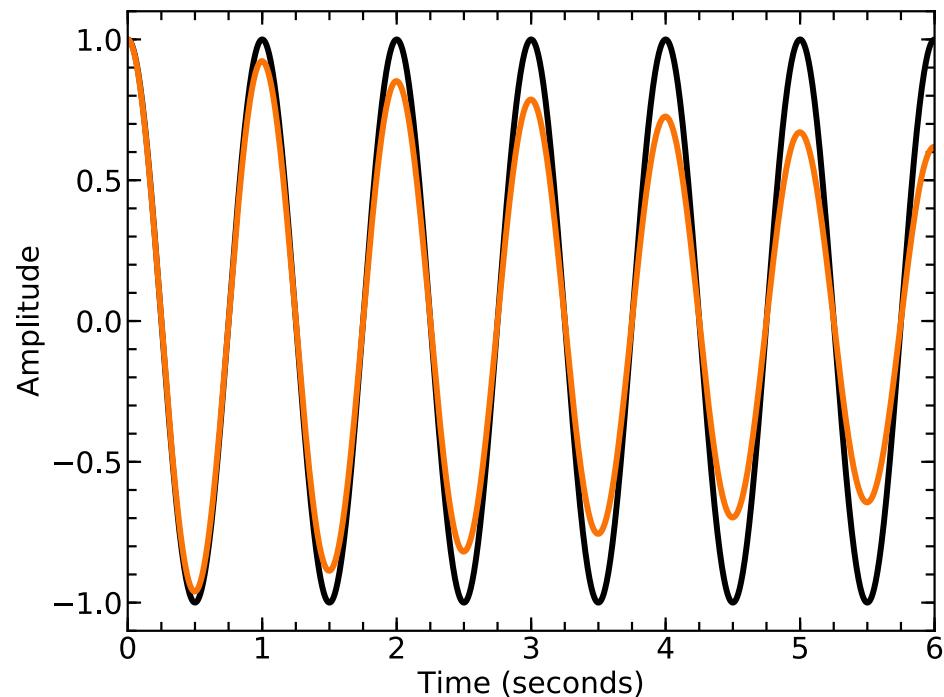
# QPOs → Damped harmonic oscillators

$$y = \cos(\omega t)$$



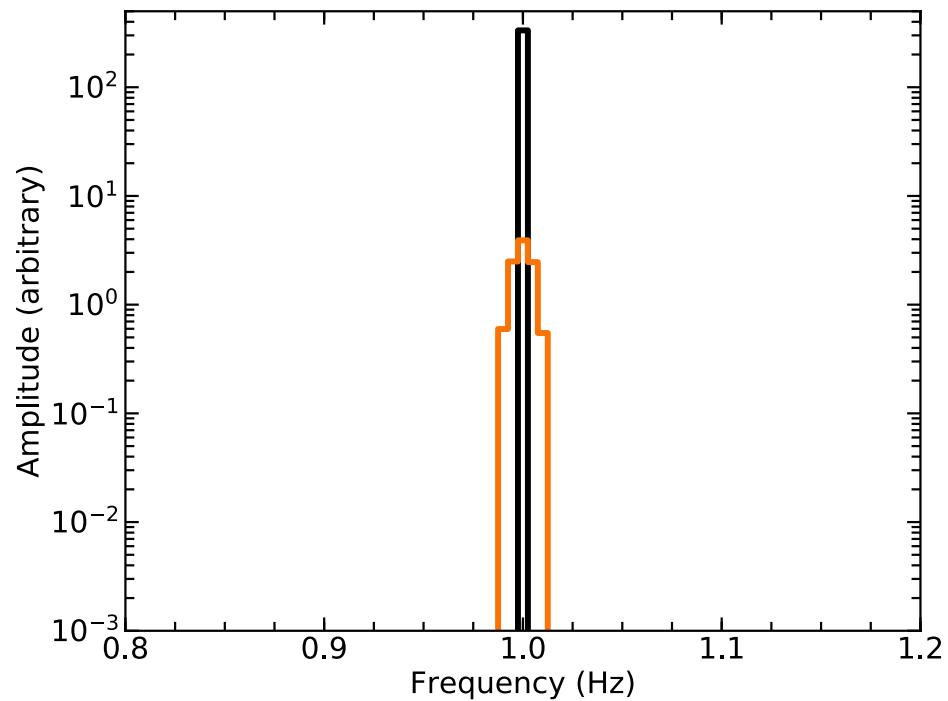
# QPOs → Damped harmonic oscillators

$$y = \cos(\omega t) \times e^{-bt}$$



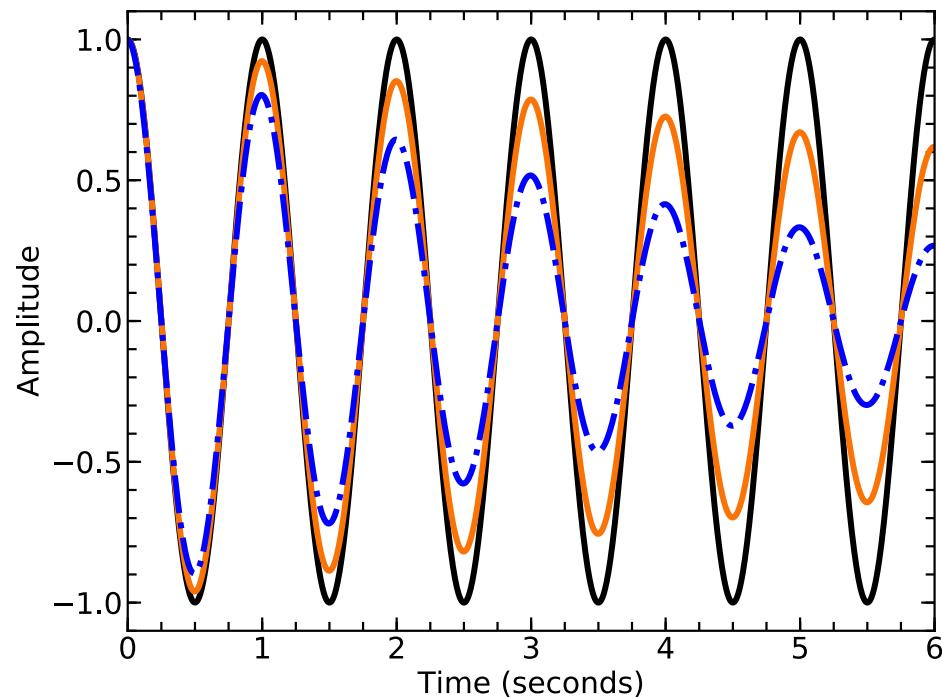
$b=0$

$b=0.08$



# QPOs → Damped harmonic oscillators

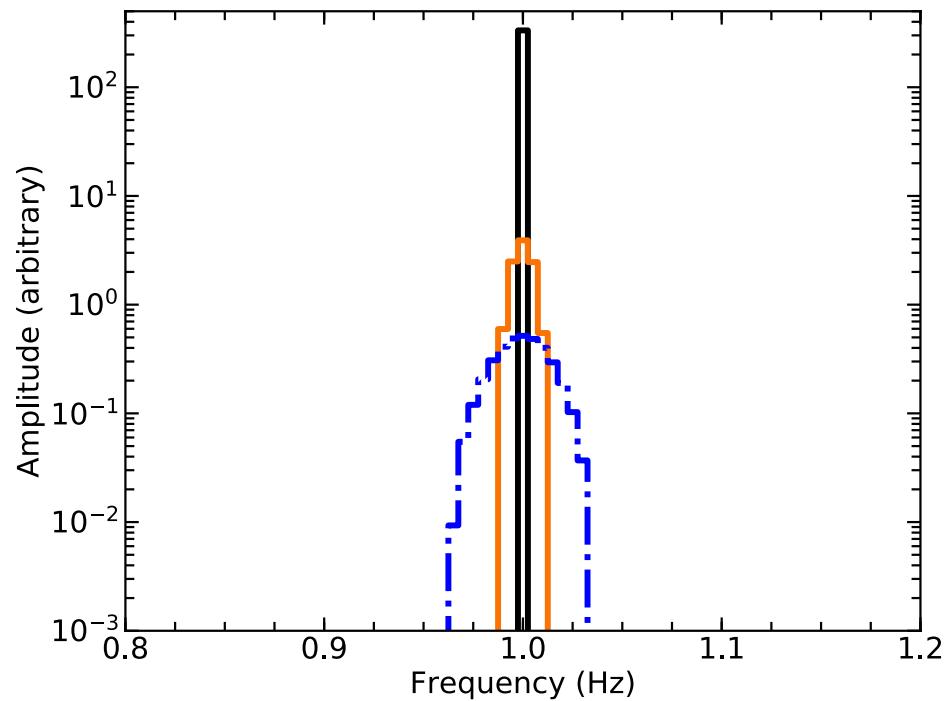
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$b=0$

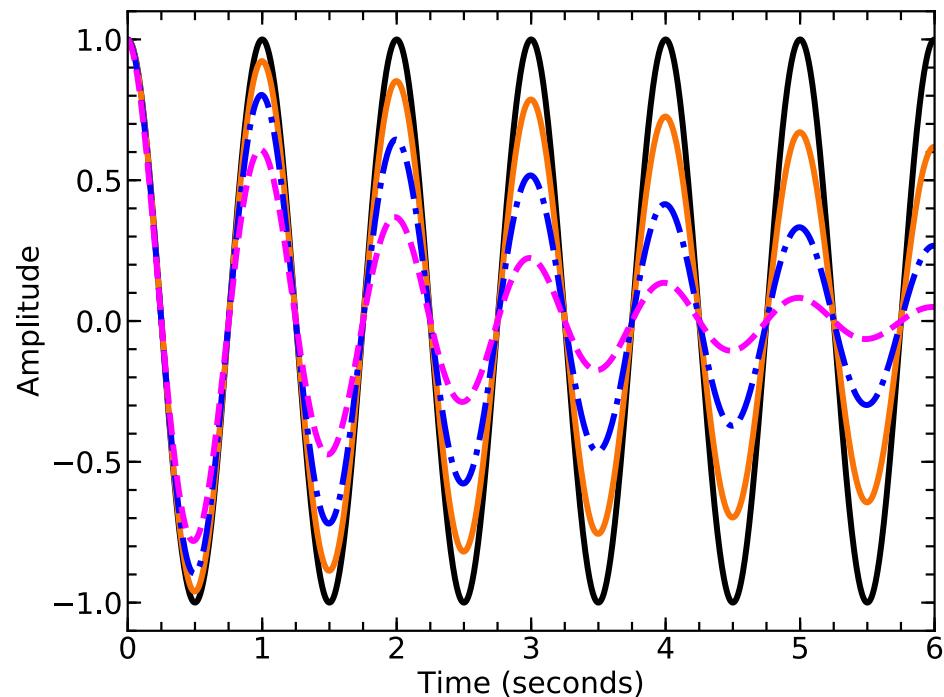
$b=0.08$

$b=0.22$



# QPOs → Damped harmonic oscillators

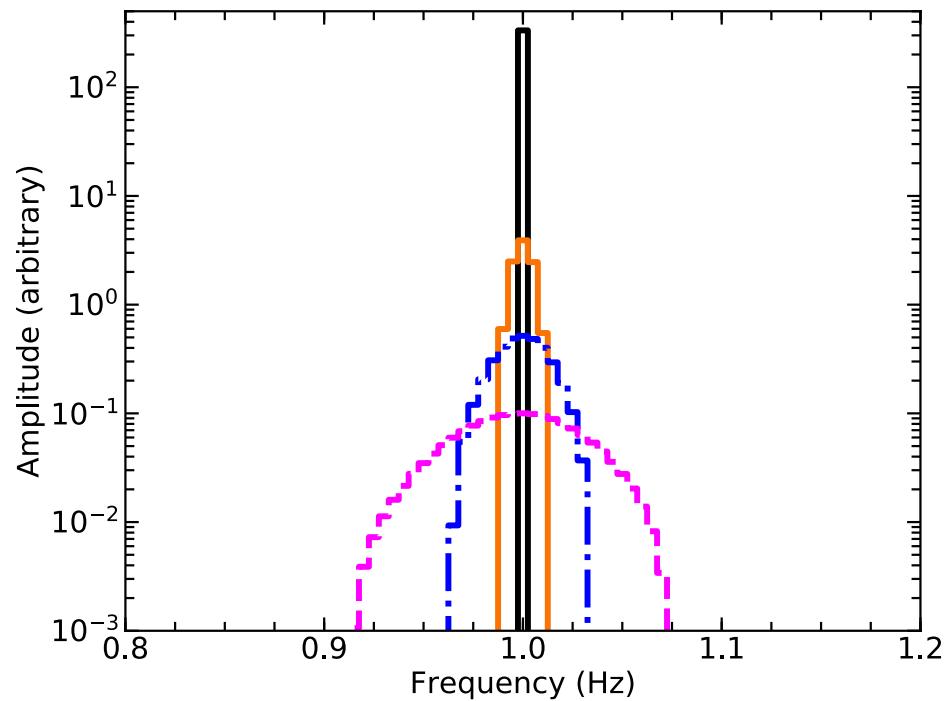
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$b=0$

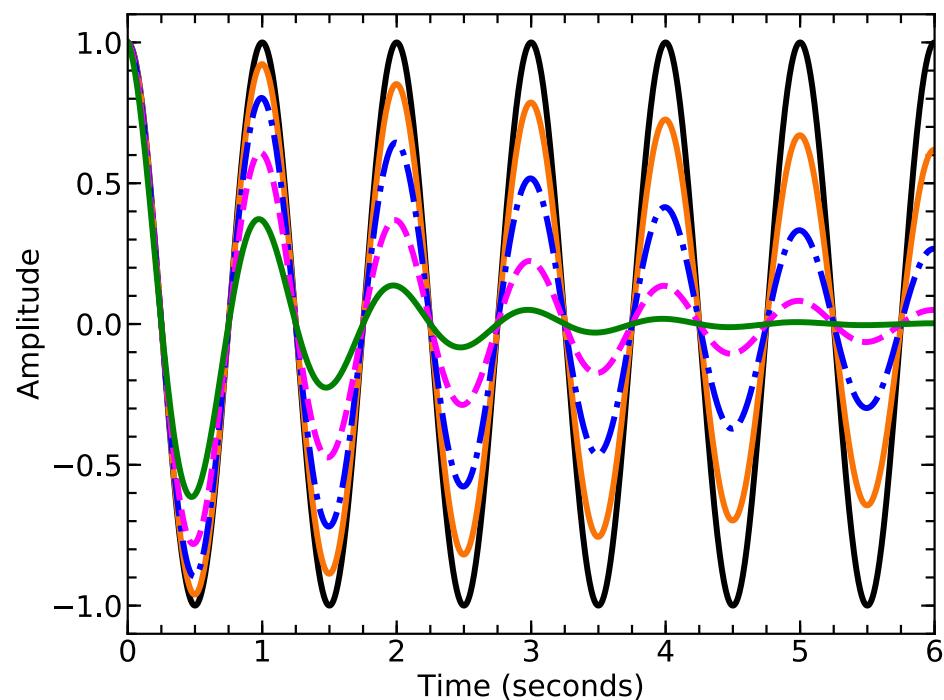
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# QPOs → Damped harmonic oscillators

$$y = \cos(\omega t) \times e^{-bt}$$

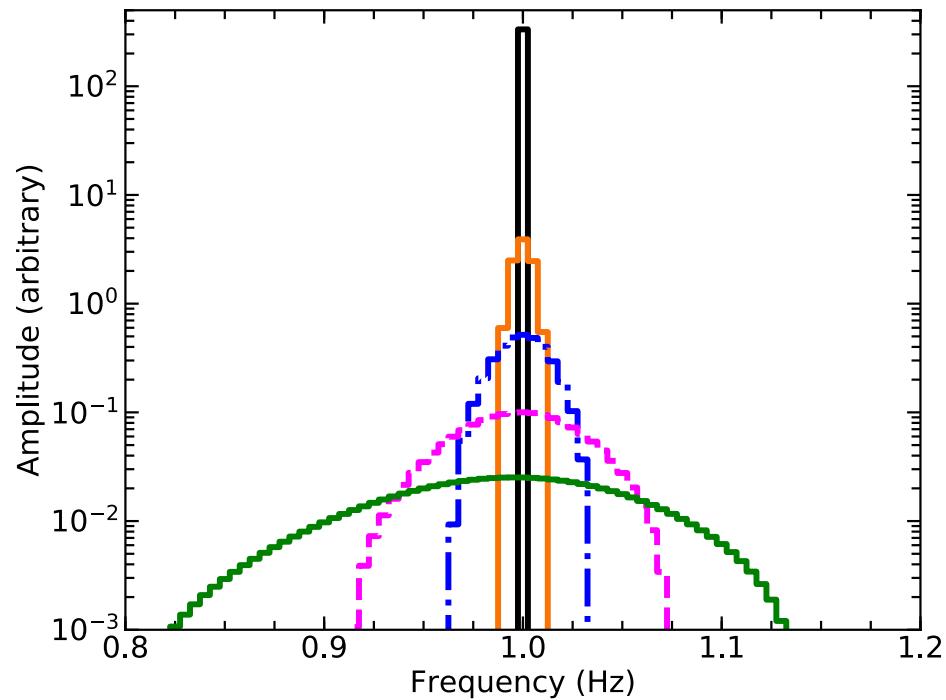


$b=0$

$b=0.08$   $b=1.0$

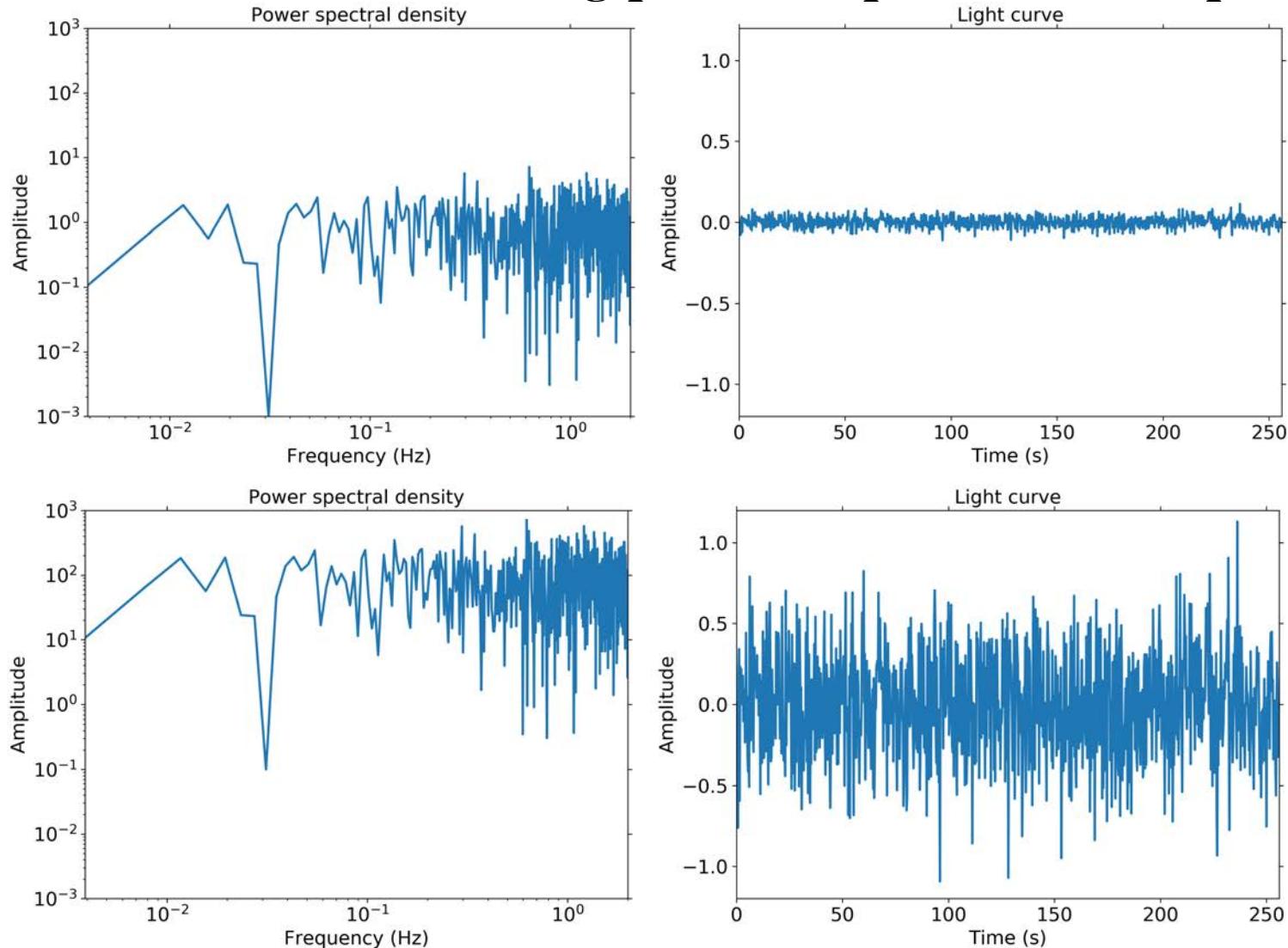
$b=0.22$

The stronger the damping,  
the wider the peak

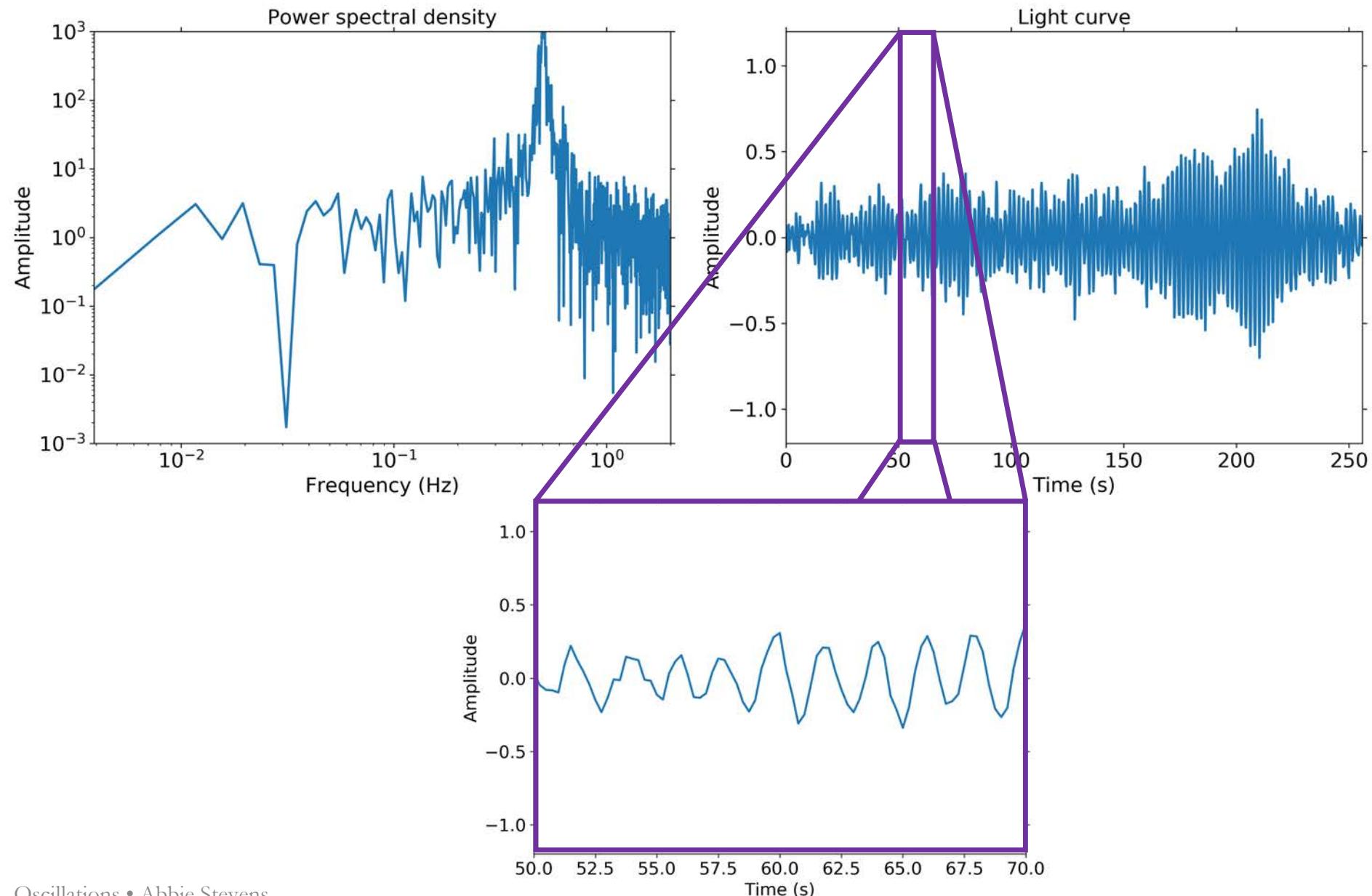


# Poisson noise (“white noise”)

Poisson noise from counting photons; power-law slope=0

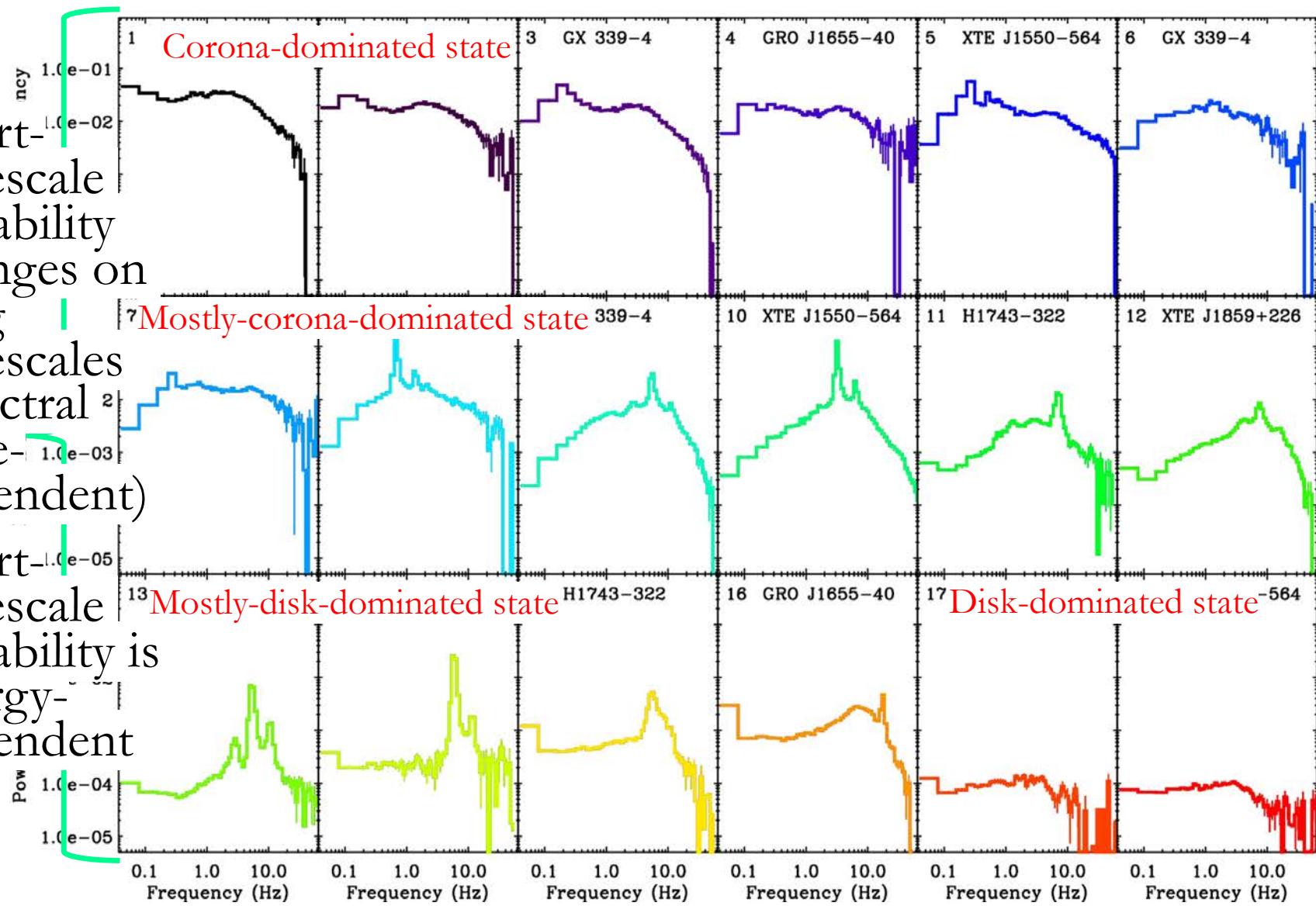


# Poisson noise + Lorentzian QPO



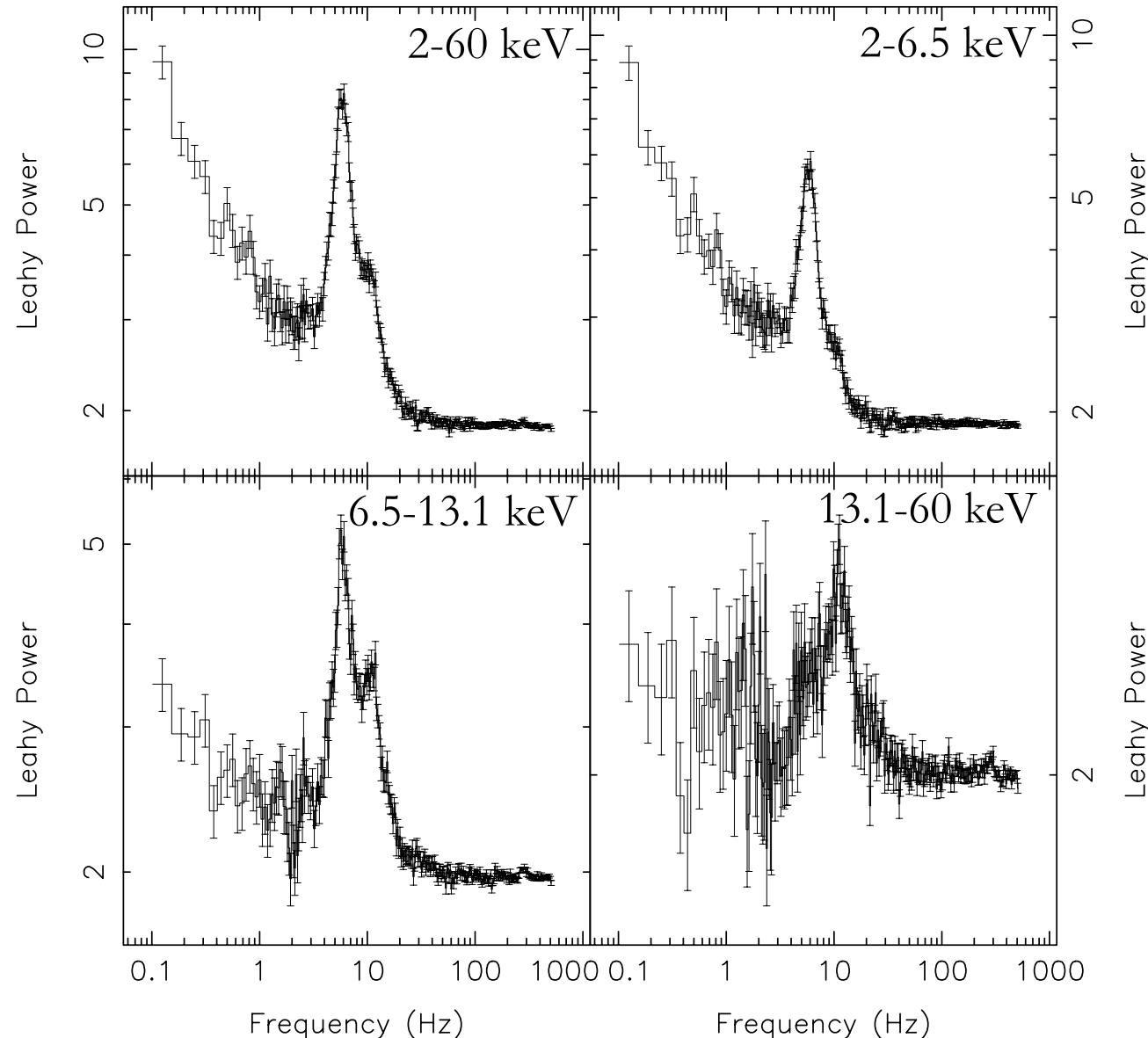
# Power spectral density (periodograms)

- Short-timescale variability changes on long timescales for (spectral state-dependent)
- Short-timescale variability is energy-dependent



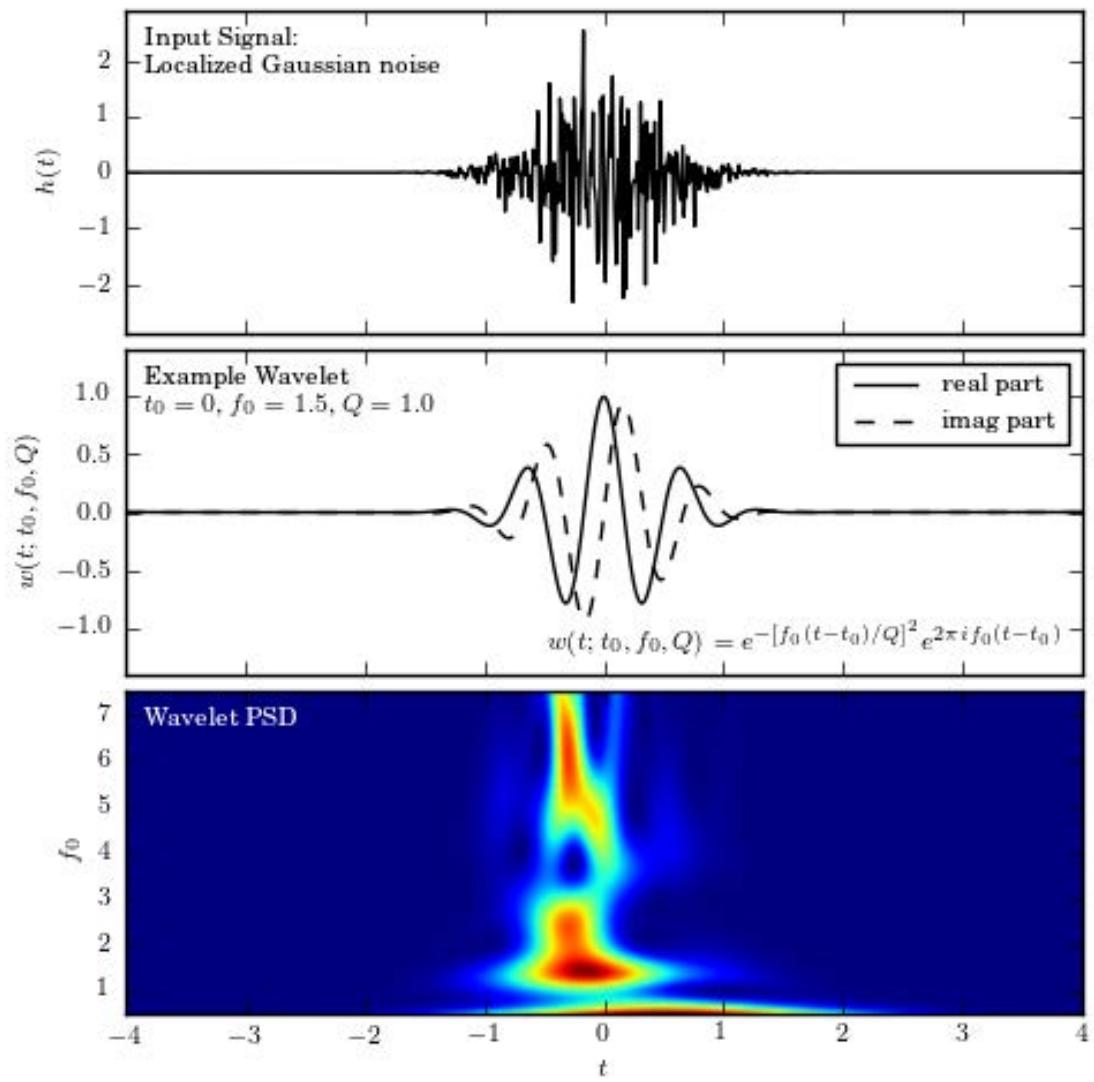
# Power spectral density (periodograms)

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# Wavelets are a thing

- Fourier products (like power spectra) don't have an intrinsic way to tell time resolution (i.e., when in the light curve the signal is present)
- Wavelets represent a signal in the time domain as well as the frequency domain



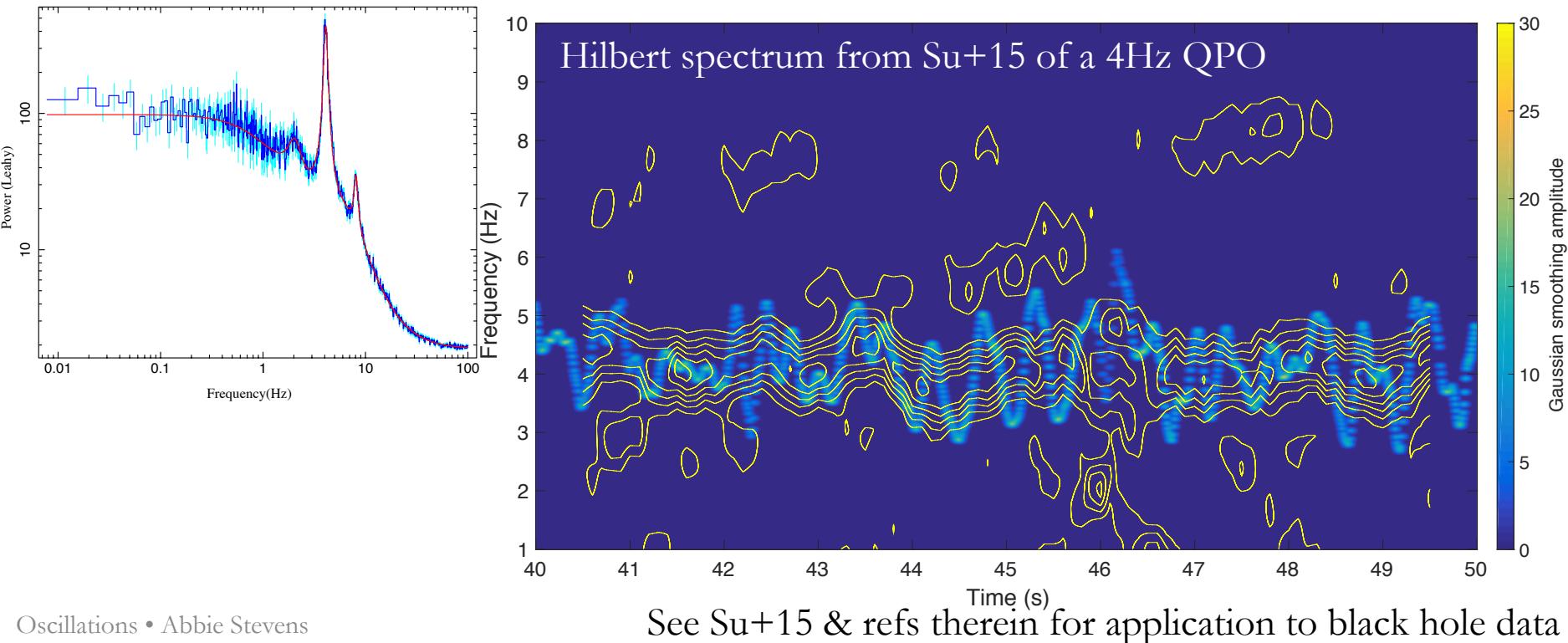
Resource: "A really friendly guide to wavelets", C. Valens, 1999

# Wavelets aren't great though

- Averaged power density spectra ( $\sim 50+$  segments) follow a chi-squared distribution with 2 degrees of freedom, about the underlying true power spectrum
  - Errors are statistically well-defined and well-understood (and easy to compute!)
- Wavelets do not follow such a well-defined and well-known distribution
- No clear way to assess statistical significance of a signal (which is one of the things we often want to do)

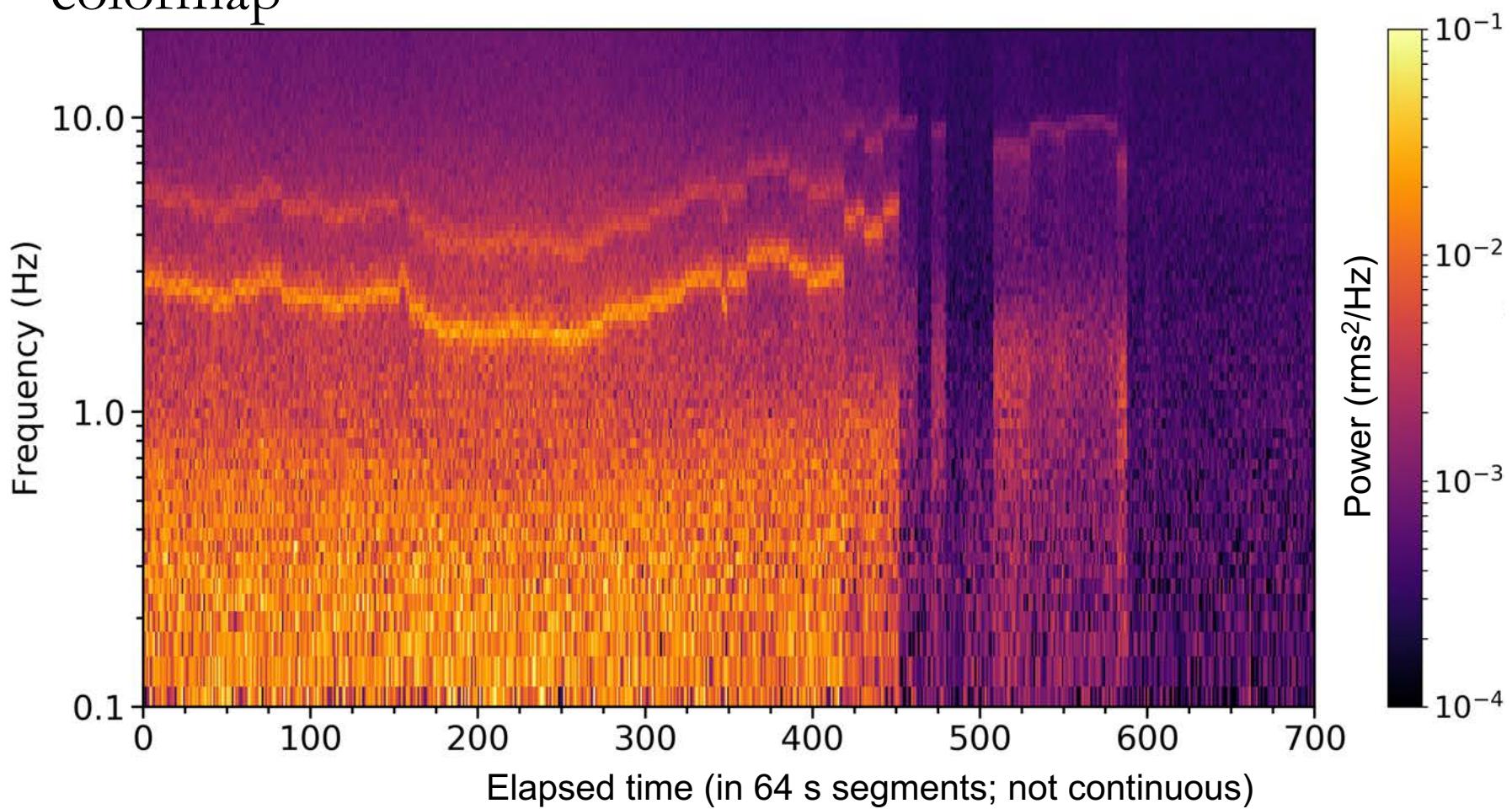
# Hilbert-Huang transform

- Frequency-domain product designed for data that are non-stationary and non-linear
- Like an instantaneous Fourier transform → gives (some) time localization!
- Error from standard deviation of (1000+) simulations



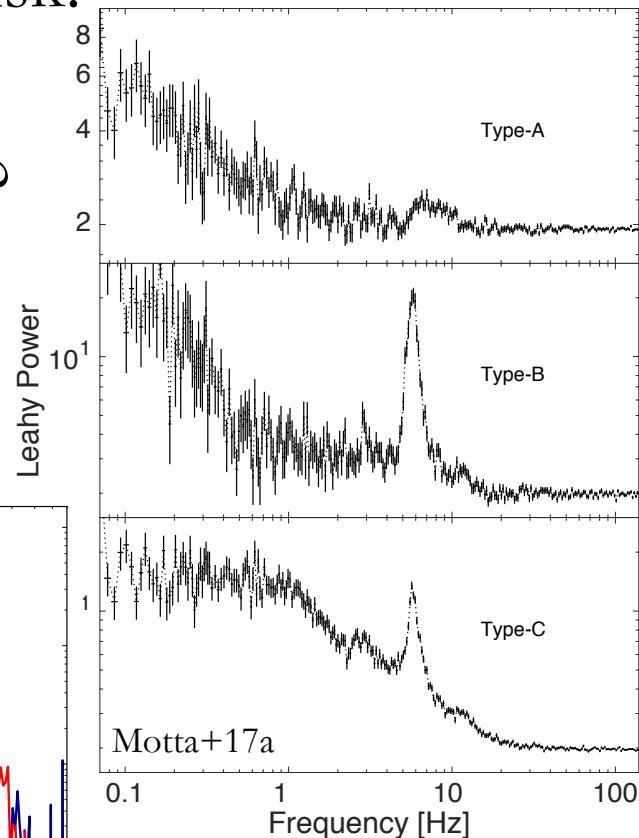
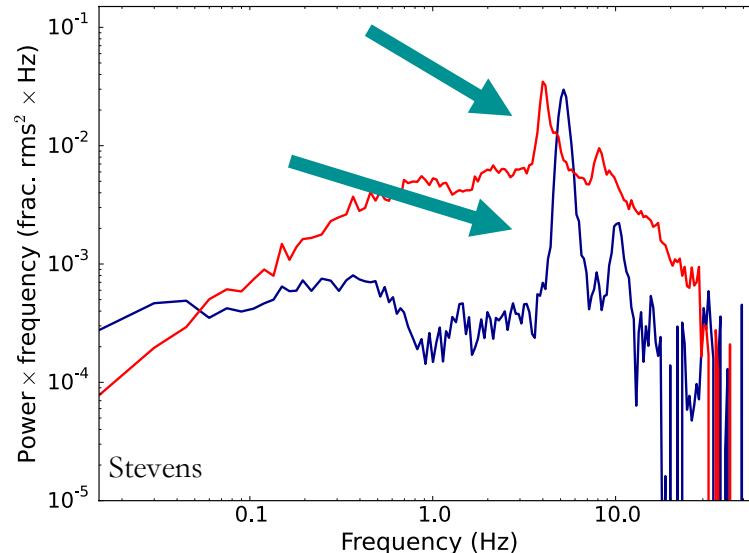
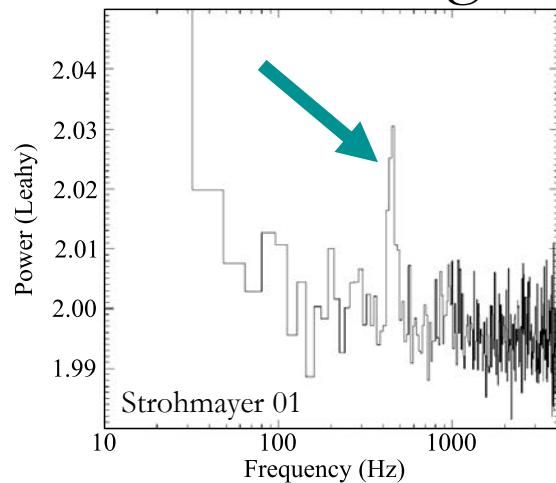
# Dynamical power spectrum

- Evolution of power spectrum in time
- Instead of averaging the segments together, plot them in a colormap

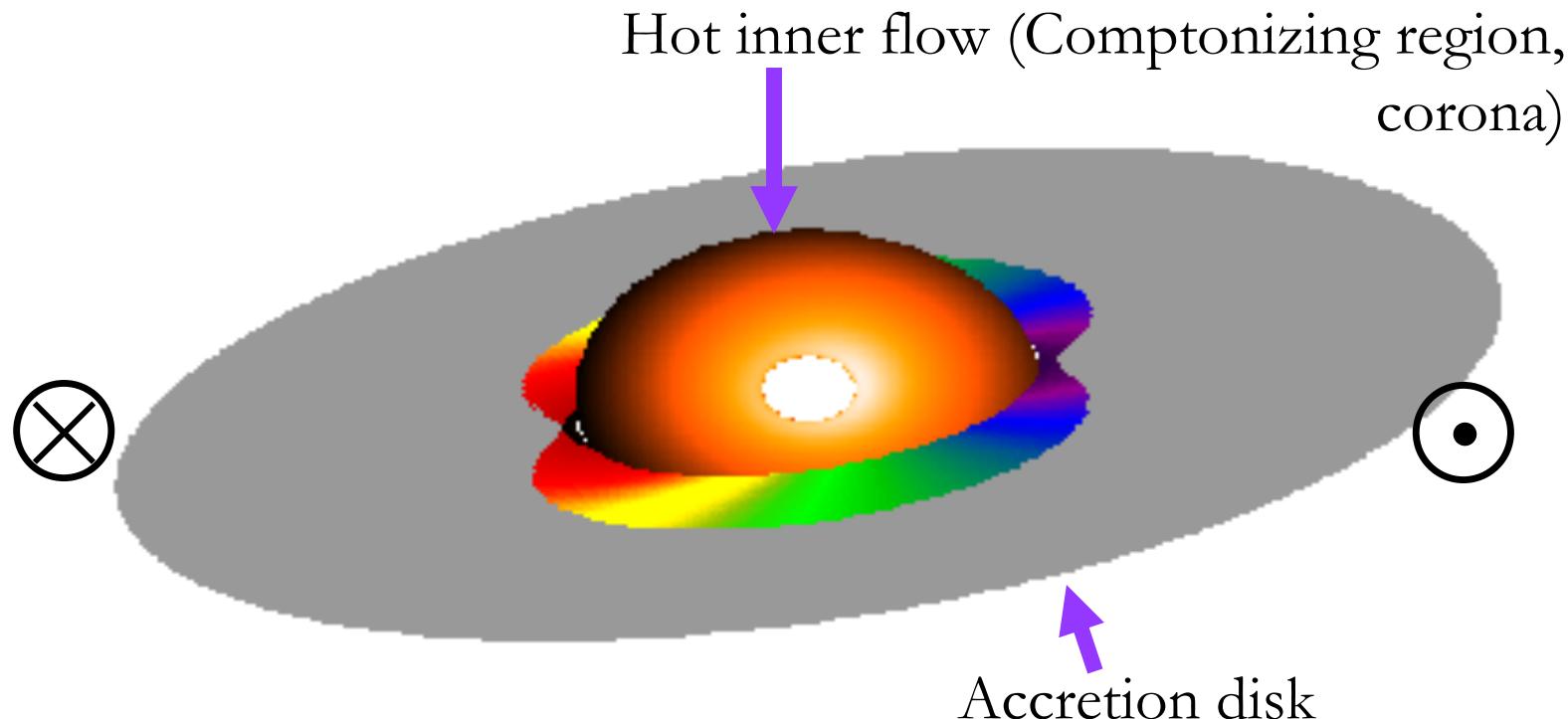


# QPOs in black holes and neutron stars

- High-frequency: 100's Hz
  - Hot Keplerian blobs in inner disk?
- Low-frequency:  $\sim$ 0.01-10's Hz
  - Precession of corona/hot flow?
  - Magnetic warps in disk?



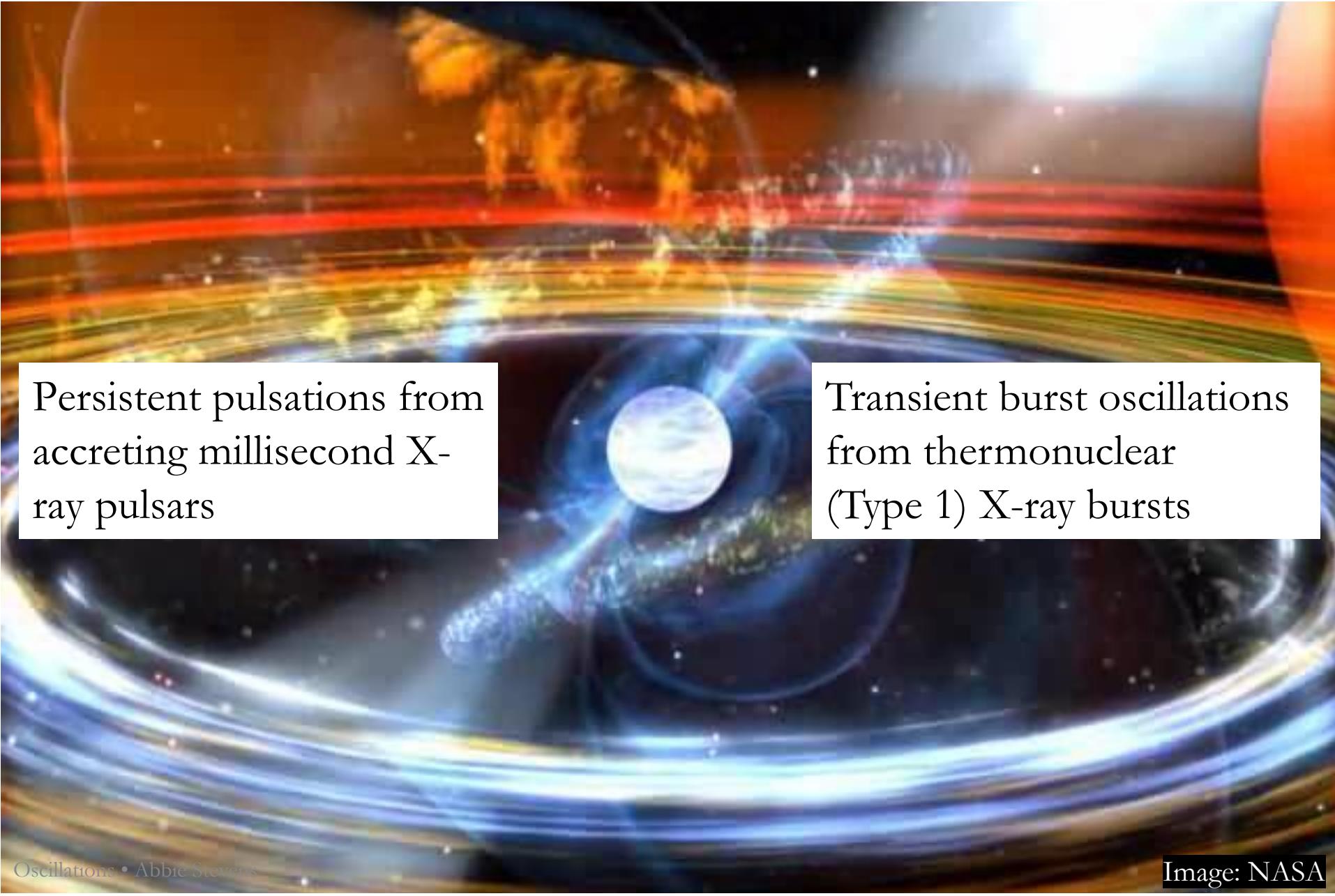
# Low-freq QPOs: Lense-Thirring precession?



Disk color pattern: Doppler shifting and boosting of emission

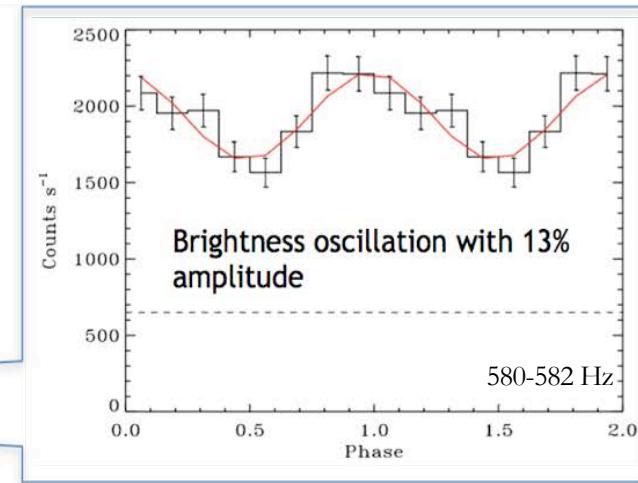
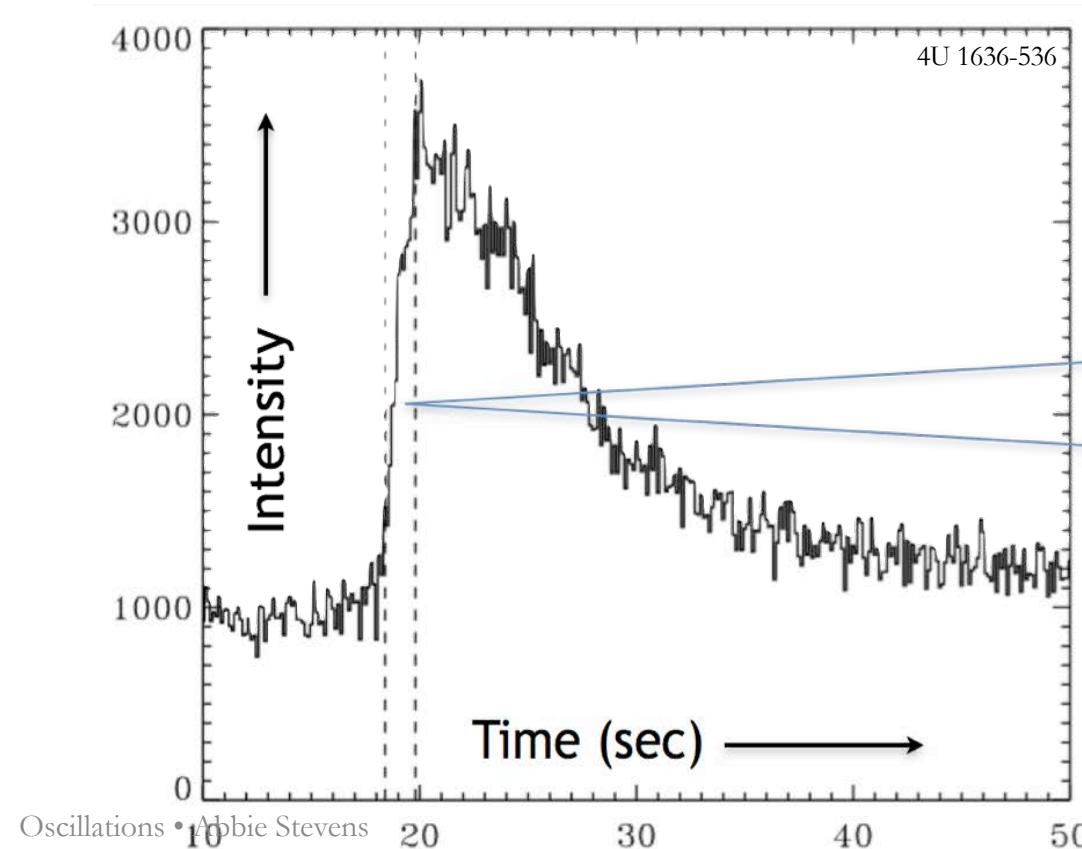
Stella+Vietri '98; Fragile+Anninos '05; Schnittman, Homan+Miller '06;  
Ingram+09; Ingram+van der Klis '15; Fragile+16; Ingram+16a,b; Liska+18

# Accreting neutron stars



# Thermonuclear X-ray bursts

- Accreting material builds up on surface → nuclear burning
- Spectral evidence of coronal reprocessing of persistent emission or disk reprocessing of burst emission (Keek+18a)
- X-ray emission is coming from NS surface



Discovered by Strohmayer+96 with RXTE

Reviews: Galloway+08, Watts12, Bilous+Watts (arxiv:1812.10684)

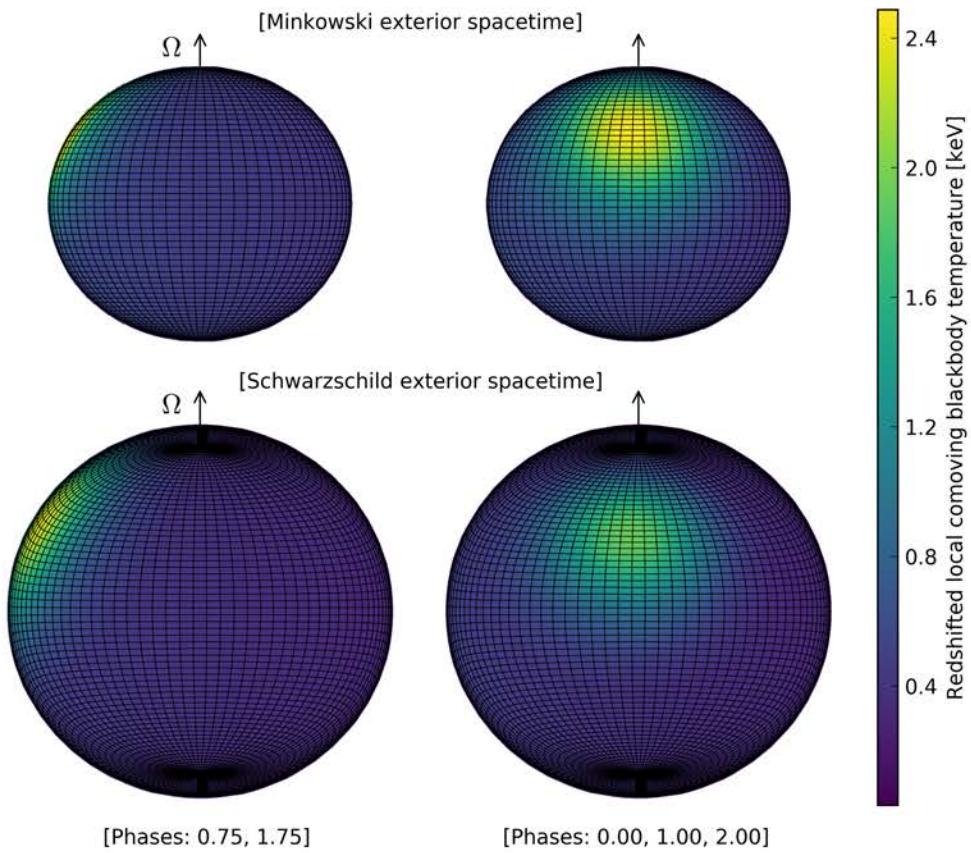
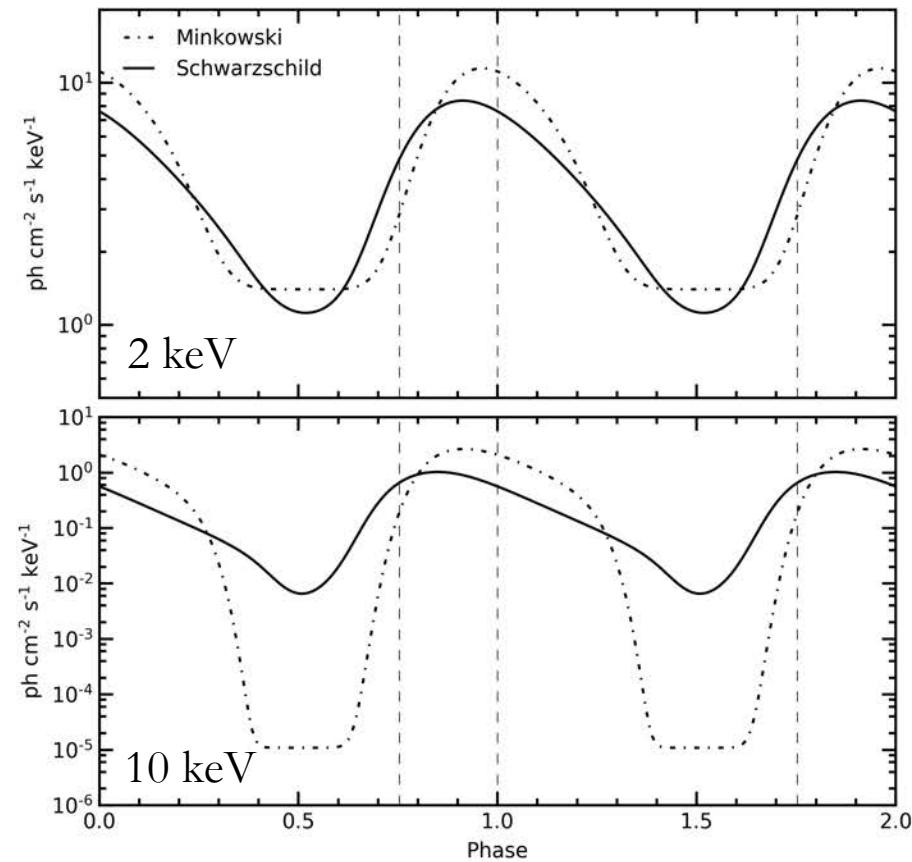
See MINBAR catalog by Galloway

# Neutron stars and relativity

Slide thanks to  
Anna Watts

- Lightbending: able to see  $\sim 3/4$  of NS surface
- Typically, hotspot is visible for entire rotation

Watts et al. 2019,  
arXiv:1812.04021

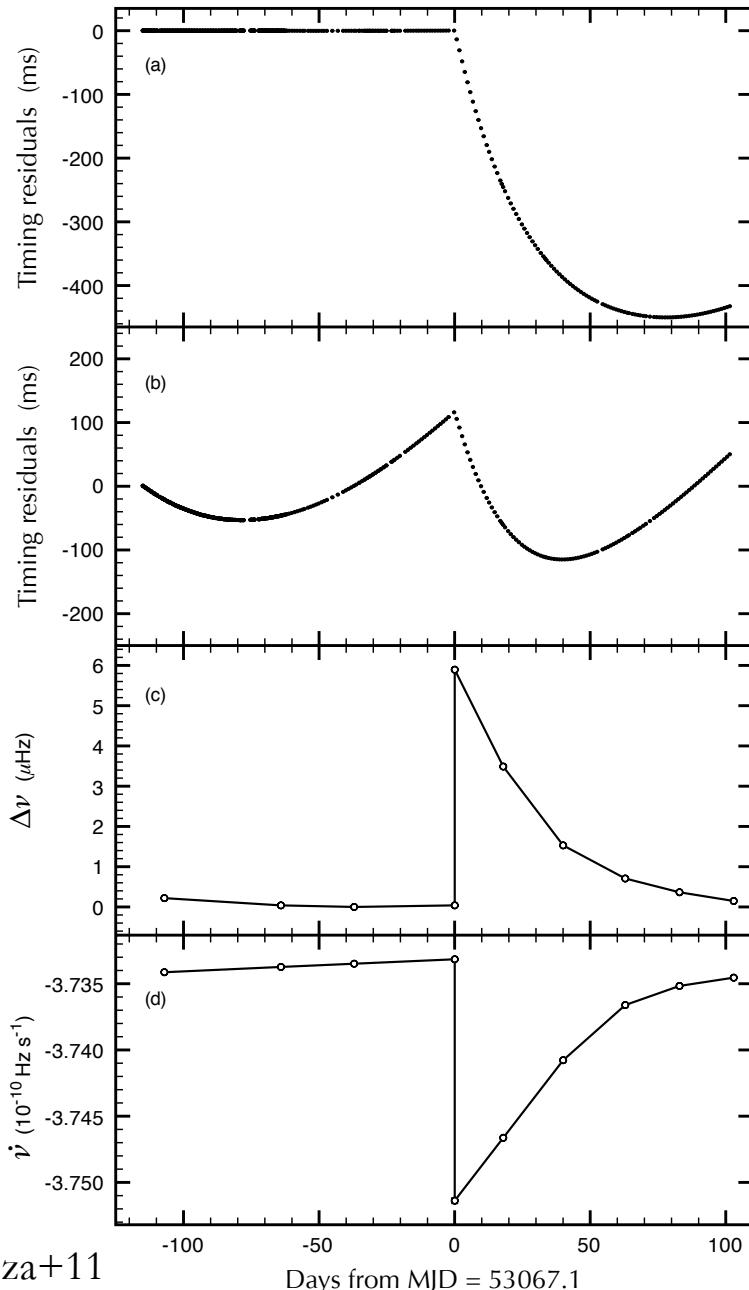


Mass  $1.8 M_\odot$ , Equatorial radius 14 km, Spin 600 Hz

For papers on relativistic ray-tracing around rapidly rotating neutron stars, see references in Watts+16

# Pulsations in neutron stars

- Spin-down: decreasing spin frequency (e.g., losing rotational energy to the environment)
- Spin-up: increasing spin frequency (e.g., accreting material and thus increasing angular momentum)
- Glitch: sudden change in spin frequency (due to superfluid NS core?)
- Seen in residuals of frequency or pulse timing

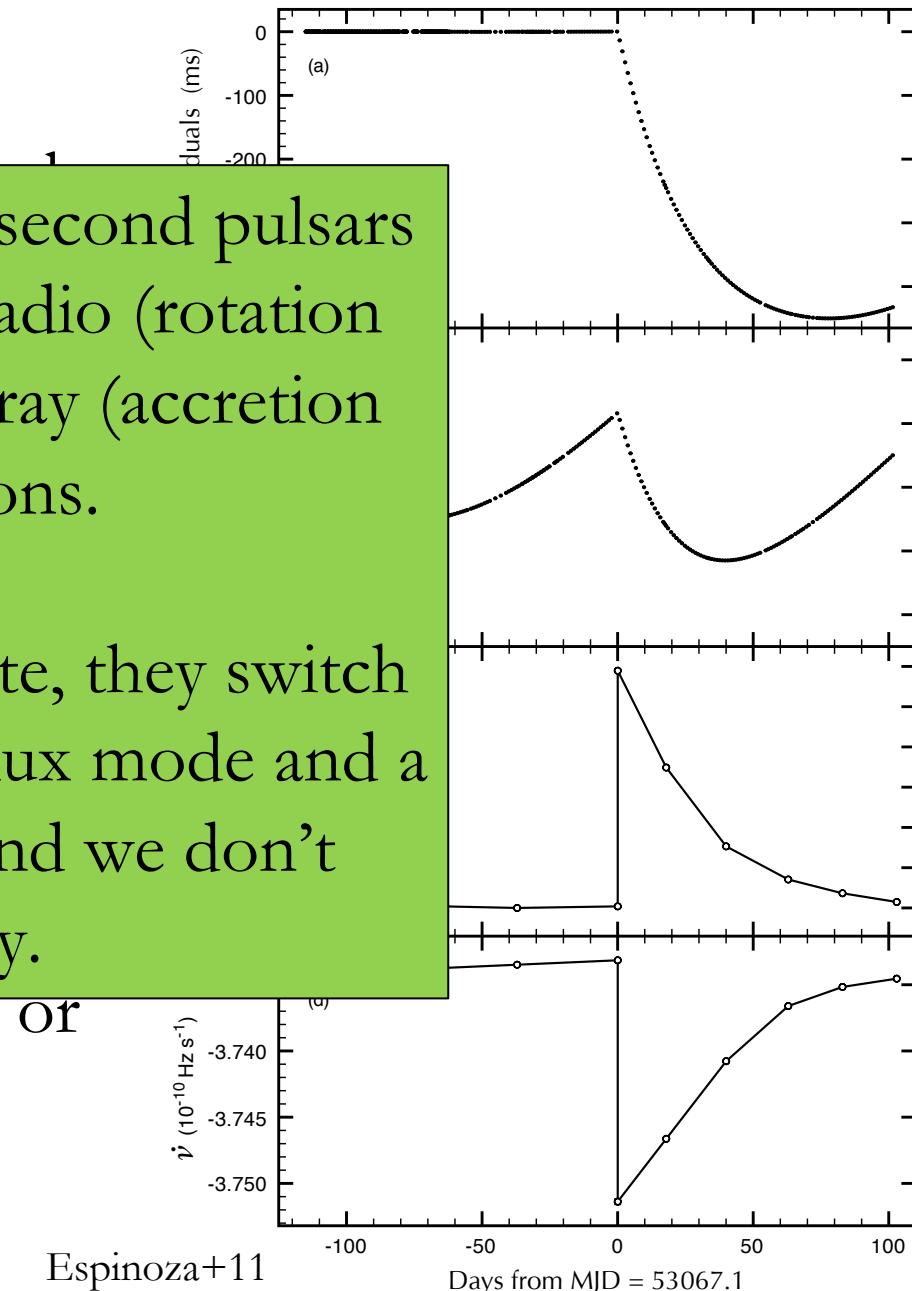


# Pulsations in neutron stars

- Spin-down: decreasing spin frequency (e.g., due to loss of energy to the environment)
- Spin-up: increasing spin frequency (e.g., accretion-powered) or increasing angular momentum (core?)
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Transitional millisecond pulsars switch between radio (rotation powered) and X-ray (accretion powered) pulsations.

In their X-ray state, they switch between a high-flux mode and a low-flux mode, and we don't know how or why.



# Stellar pulsations

- Cepheid variables, RR Lyrae stars, Delta Scuti variables, Blahzko effect (long-period modulation of the periodicity)
- Period-luminosity relation makes them standard candles used as “cosmic distance ladder”
- Slow enough (periods > hours) that time-domain photometry is often used

M15 - Globular Cluster with RR-Lyrae Stars

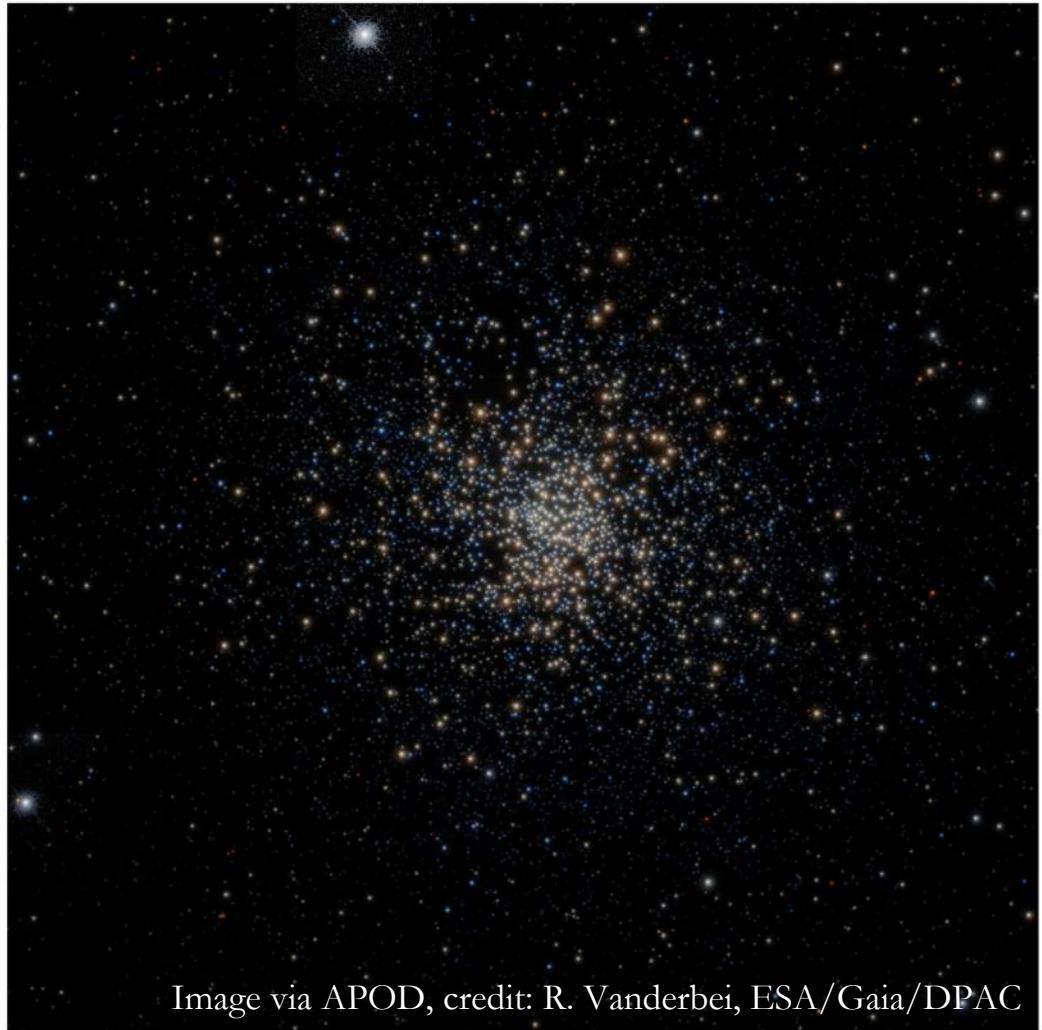
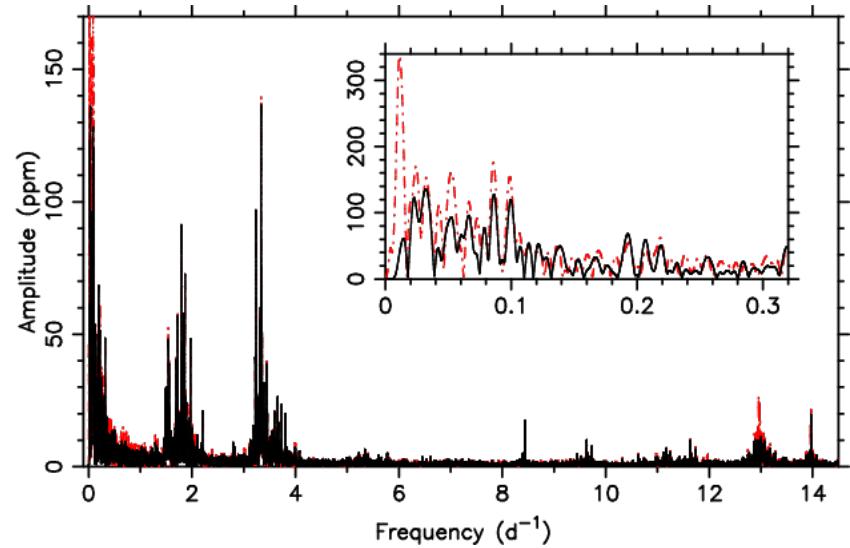
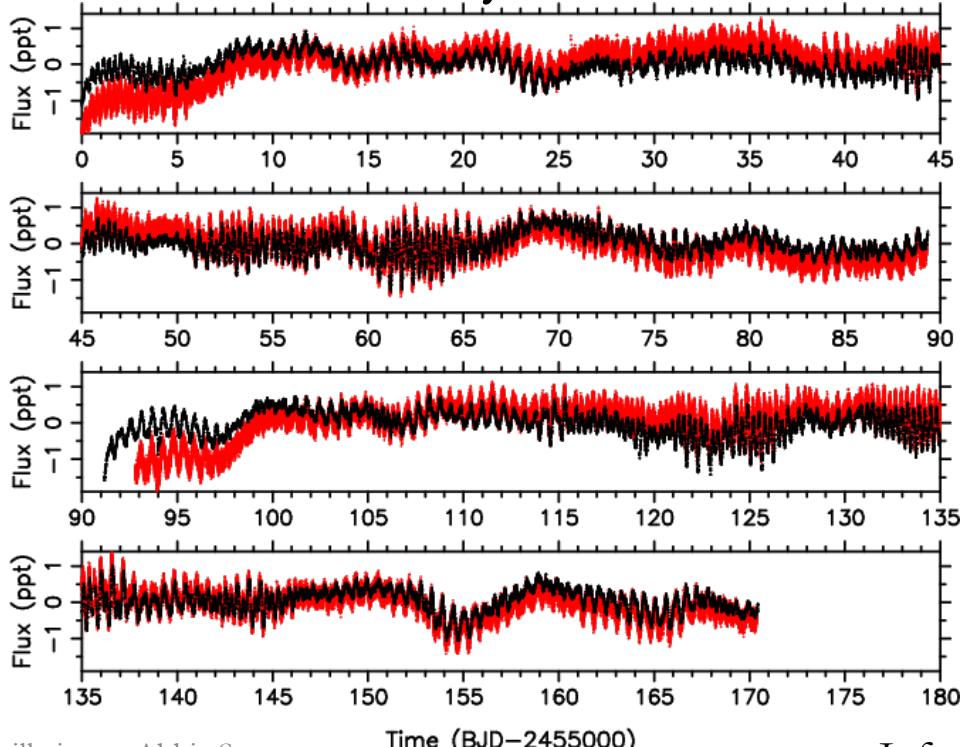


Image via APOD, credit: R. Vanderbei, ESA/Gaia/DPAC

# Asteroseismology (“starquakes”)

- Understanding the internal structure of stars using their brightness oscillations
- Convective zone excites oscillations
- Fourier analysis of light curves:  
often see many harmonics

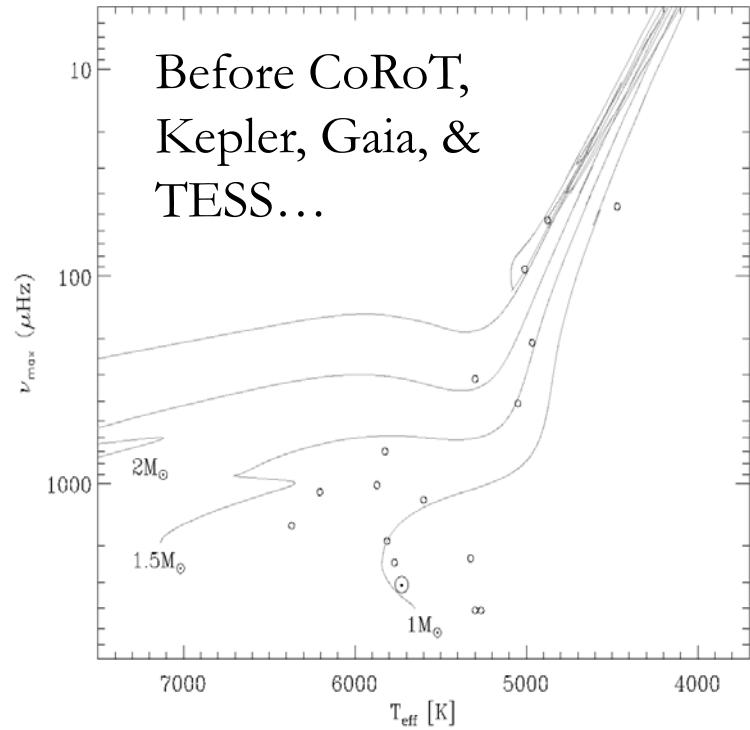


Aerts+19

Info thanks to online slides by T. Bedding and refs therein

# Asteroseismology (“starquakes”)

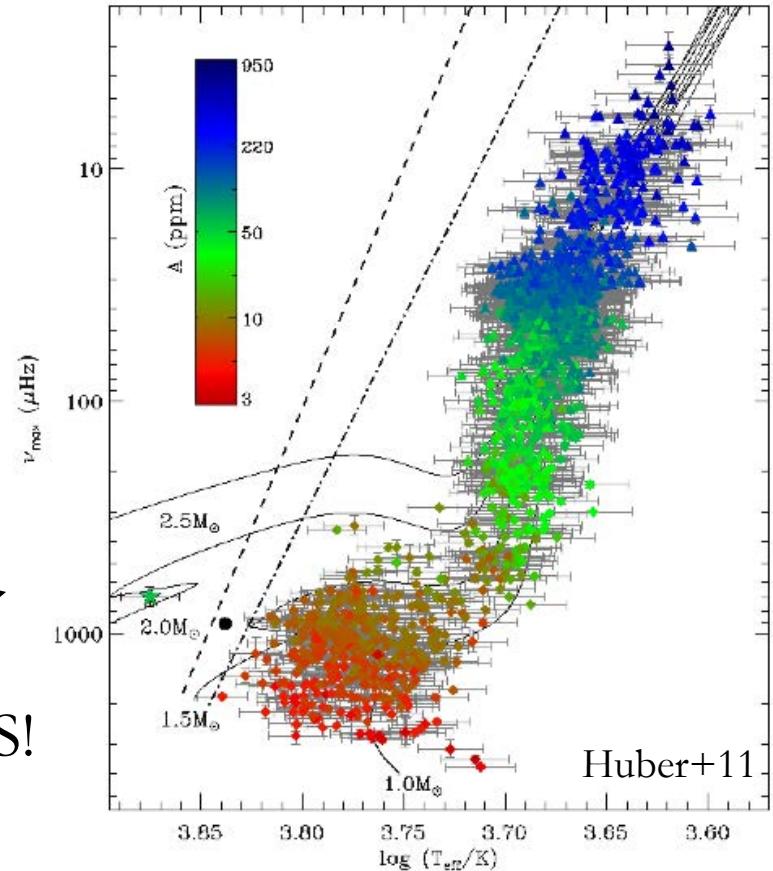
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- With frequencies and spectral temperature, can derive mass, radius, size of core and thus stellar age, etc.



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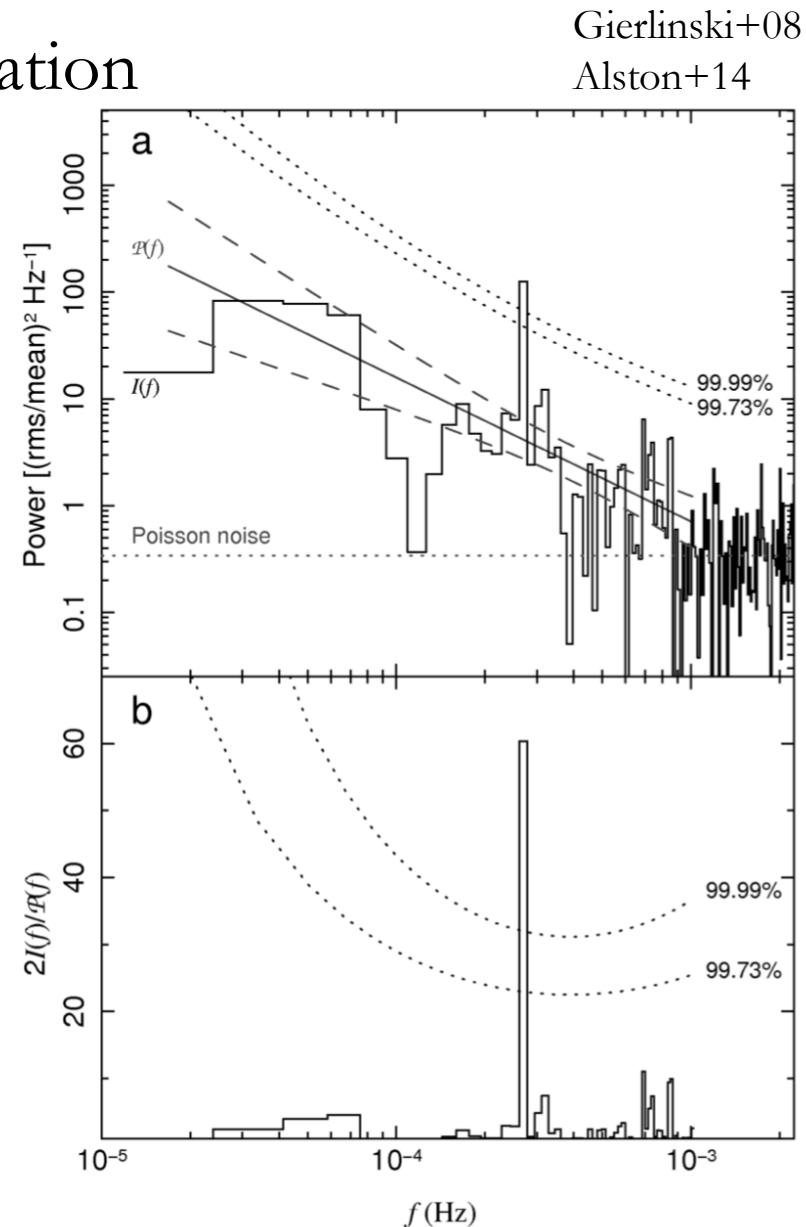
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This doesn't even have Gaia or TESS!



# QPOs in active galactic nuclei (AGN)

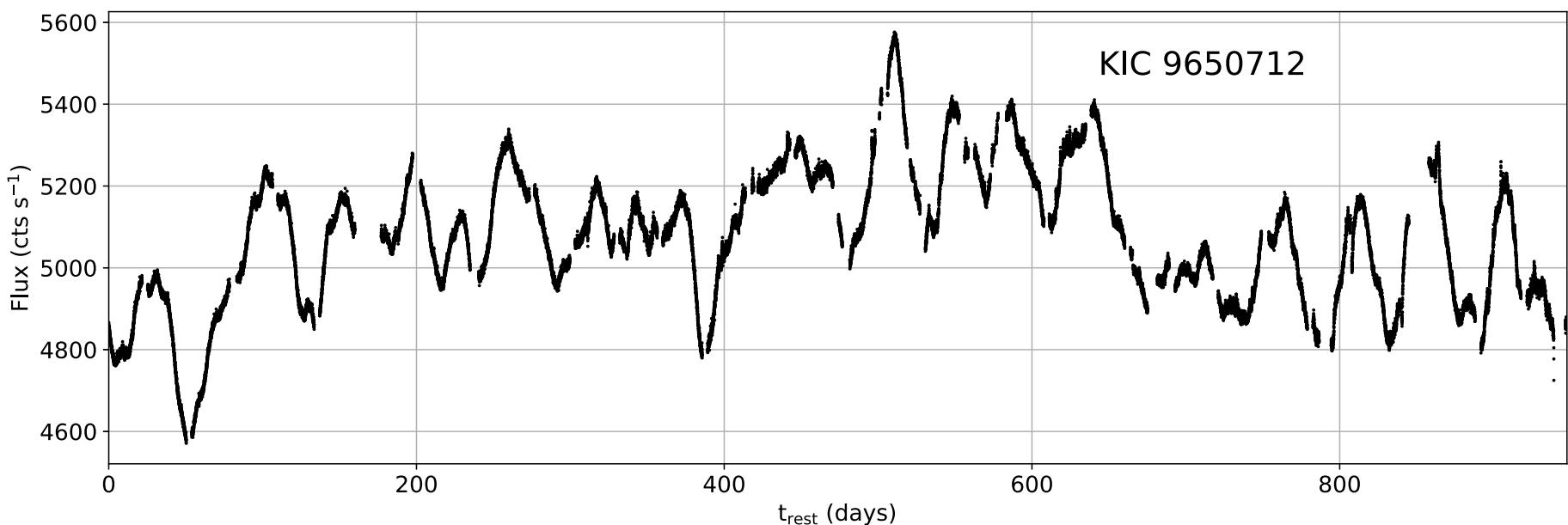
- $\sim 1$  hr “periodicity”, 91ks observation
- RE J1034+396 is a narrow-line Seyfert 1 AGN
- Saw 16 ‘cycles’ (periods) in one uninterrupted observation!
- Evenly-sampled time bins
- Signal attributed to high-freq. QPO
  - If at innermost stable circular orbit,  
 $M_{\text{BH}} \sim 7 \times 10^6 - 1 \times 10^7 M_{\odot}$



# QPOs in active galactic nuclei (AGN)

- 44 day low-freq. QPO in KIC 9650712
- NLS1 in original Kepler field
- 30 minute cadence over 3.5 years:  $\sim 30$  cycles
- Tested periodicity via simulations (Uttley+02) and Lomb-Scargle periodogram

Smith+18b

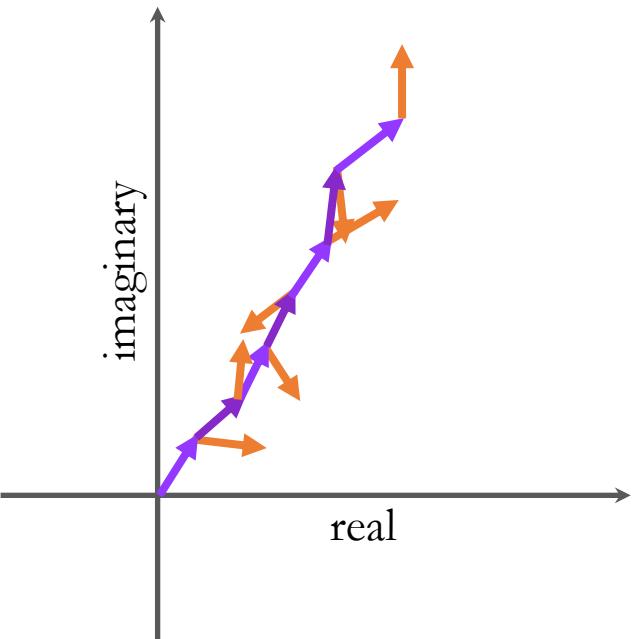


# Cross spectra

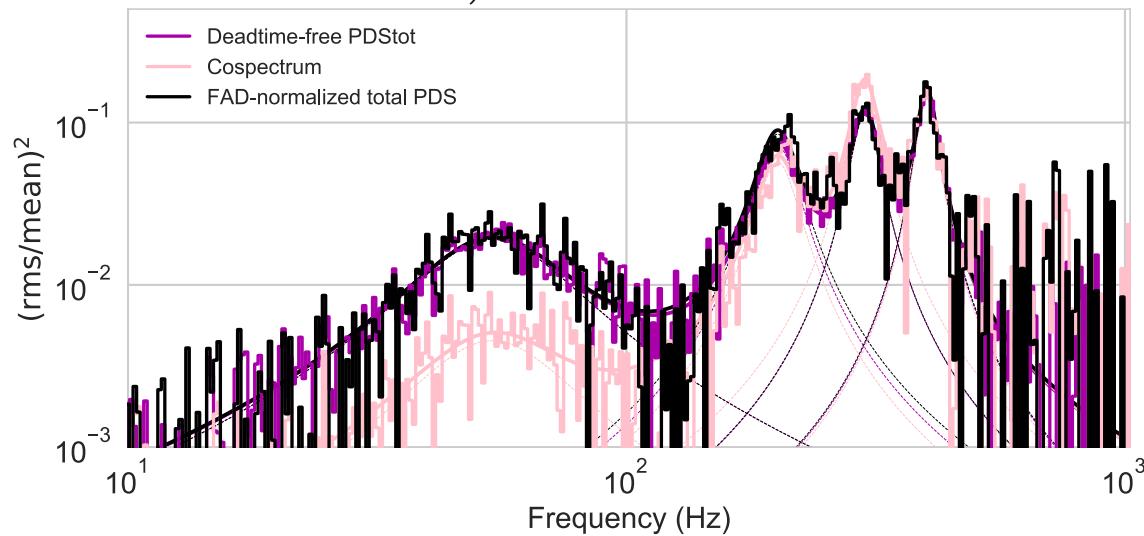
$x(t) \rightarrow X(\nu)$  for many narrow energy bands

$y(t) \rightarrow Y(\nu)$  for a broad-energy reference band

$C_{XY}(\nu) = X(\nu)Y^*(\nu)$  Average segments together: **signal adds, noise cancels**



Cospectrum: real part of the cross spectrum (see Bachetti+15, Bachetti+Huppenkothen 18 and Huppenkothen+Bachetti 18 for statistical details)



Also used: amplitude of the cross spectrum

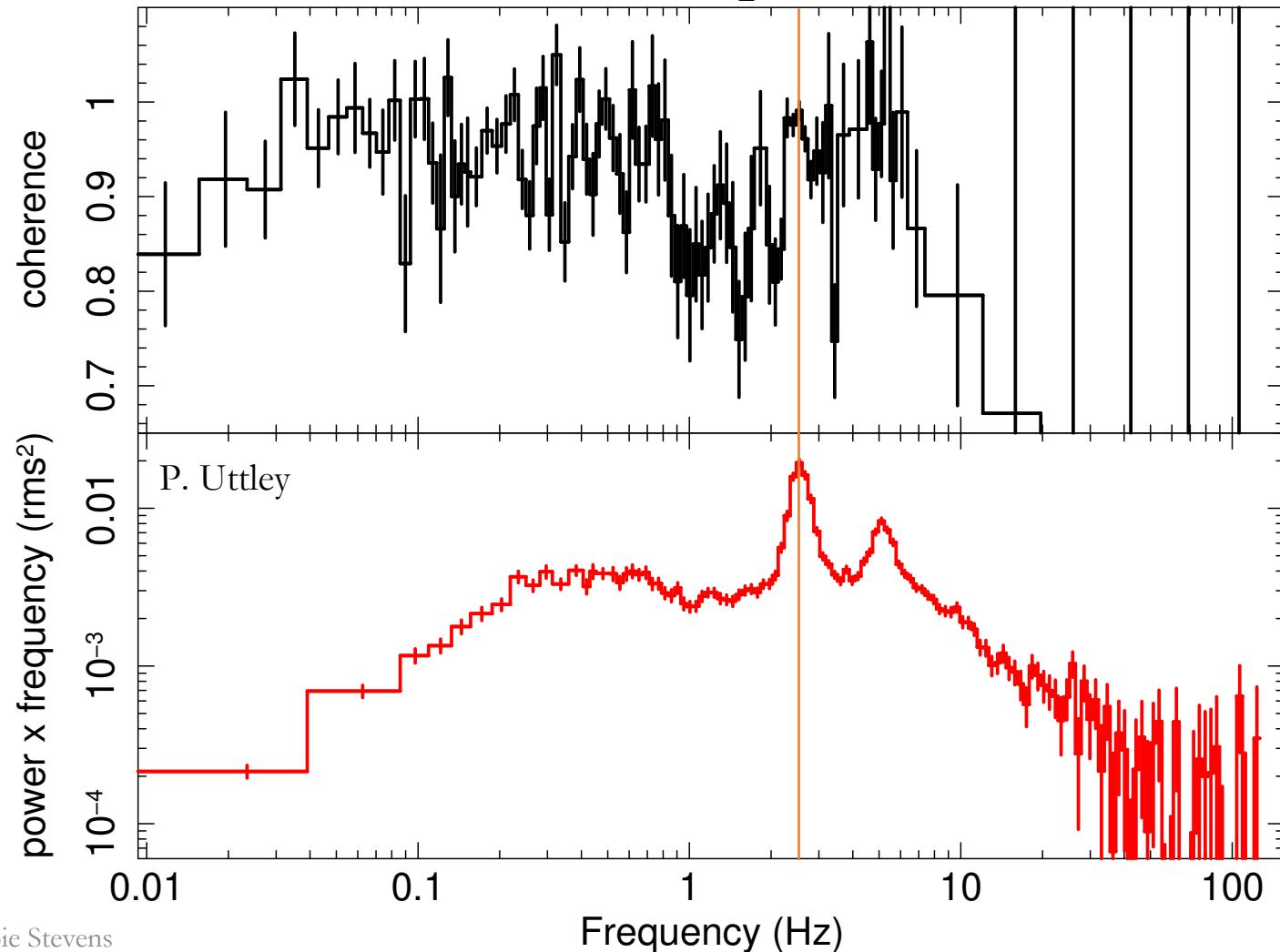
Note: for  $X(\nu)=Y(\nu)$ ,

cospectrum=cross amplitude= power spectrum

# Coherence

Fraction of the variance in light curve that can be predicted by a linear transformation (like amplitude modulation)

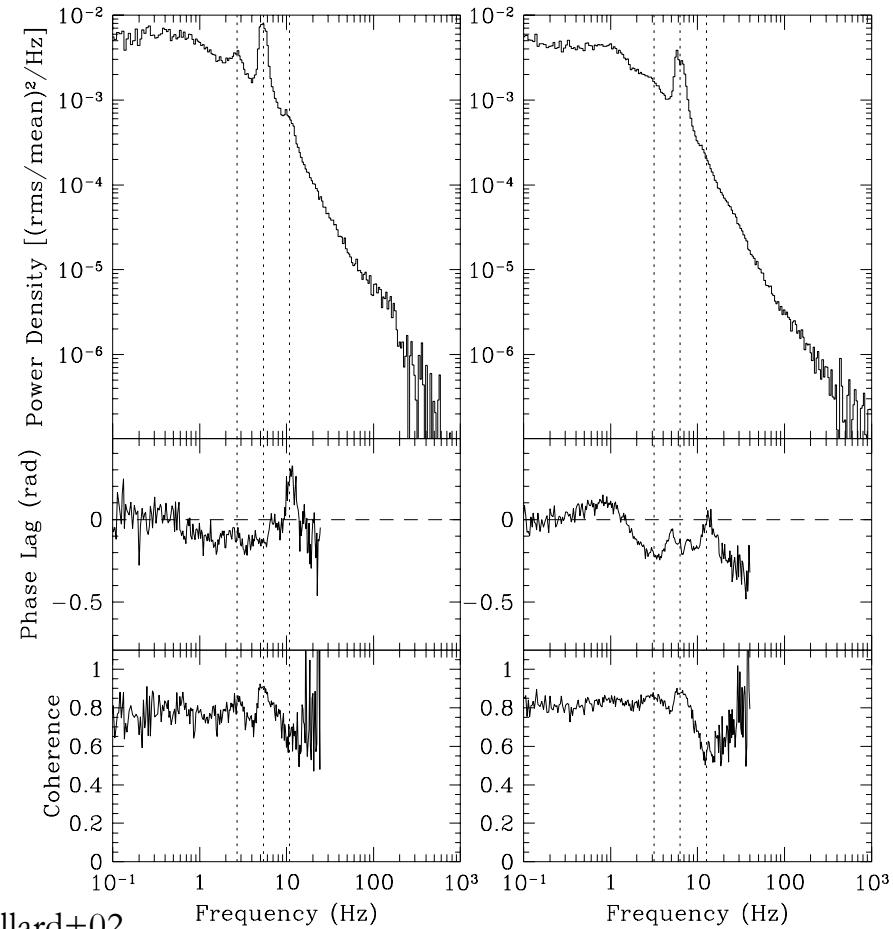
Vaughan+  
Nowak97



# Lag-frequency spectra

Cross-spectral phase lags between two light curves in different energy bands at Fourier frequencies

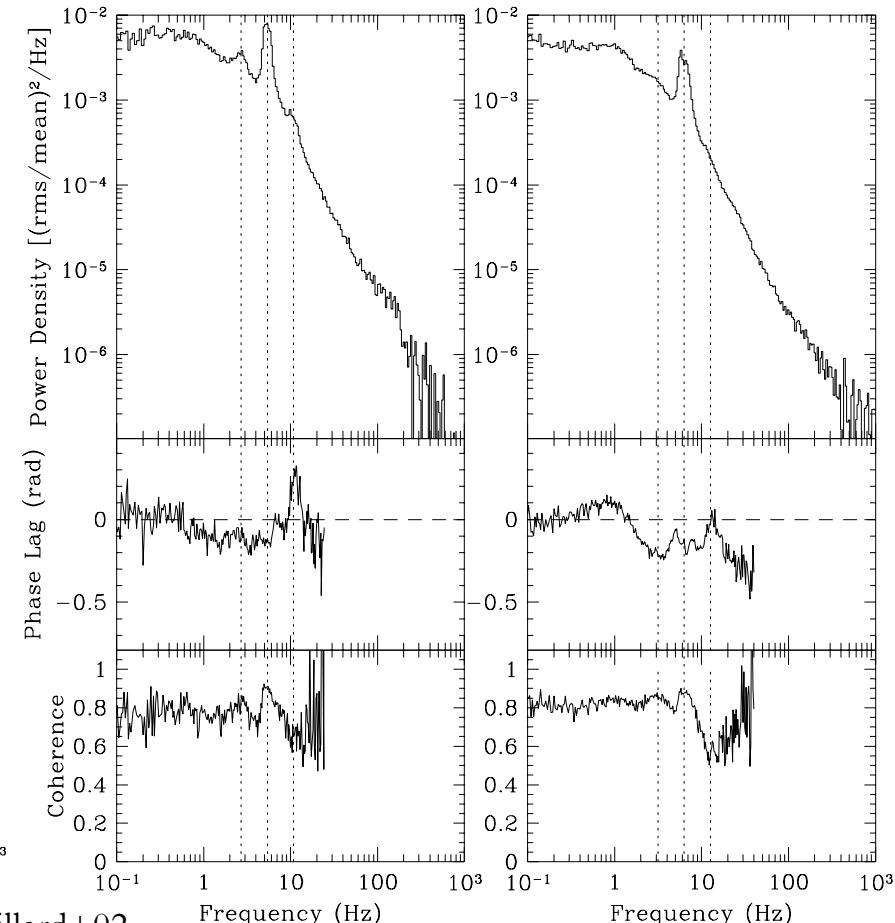
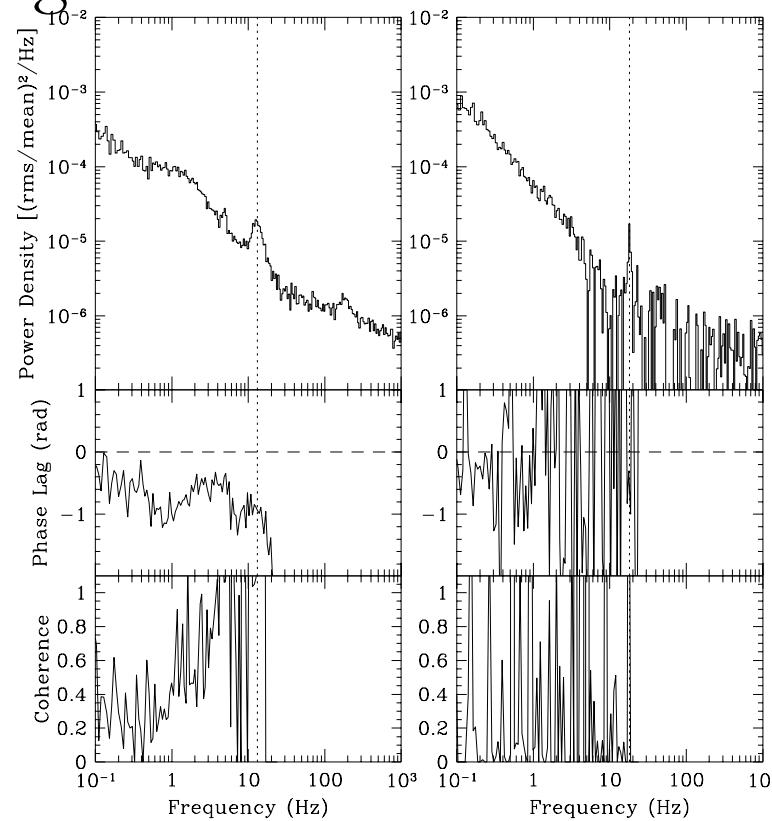
- Negative → low-energy light curve lags high-energy light curve



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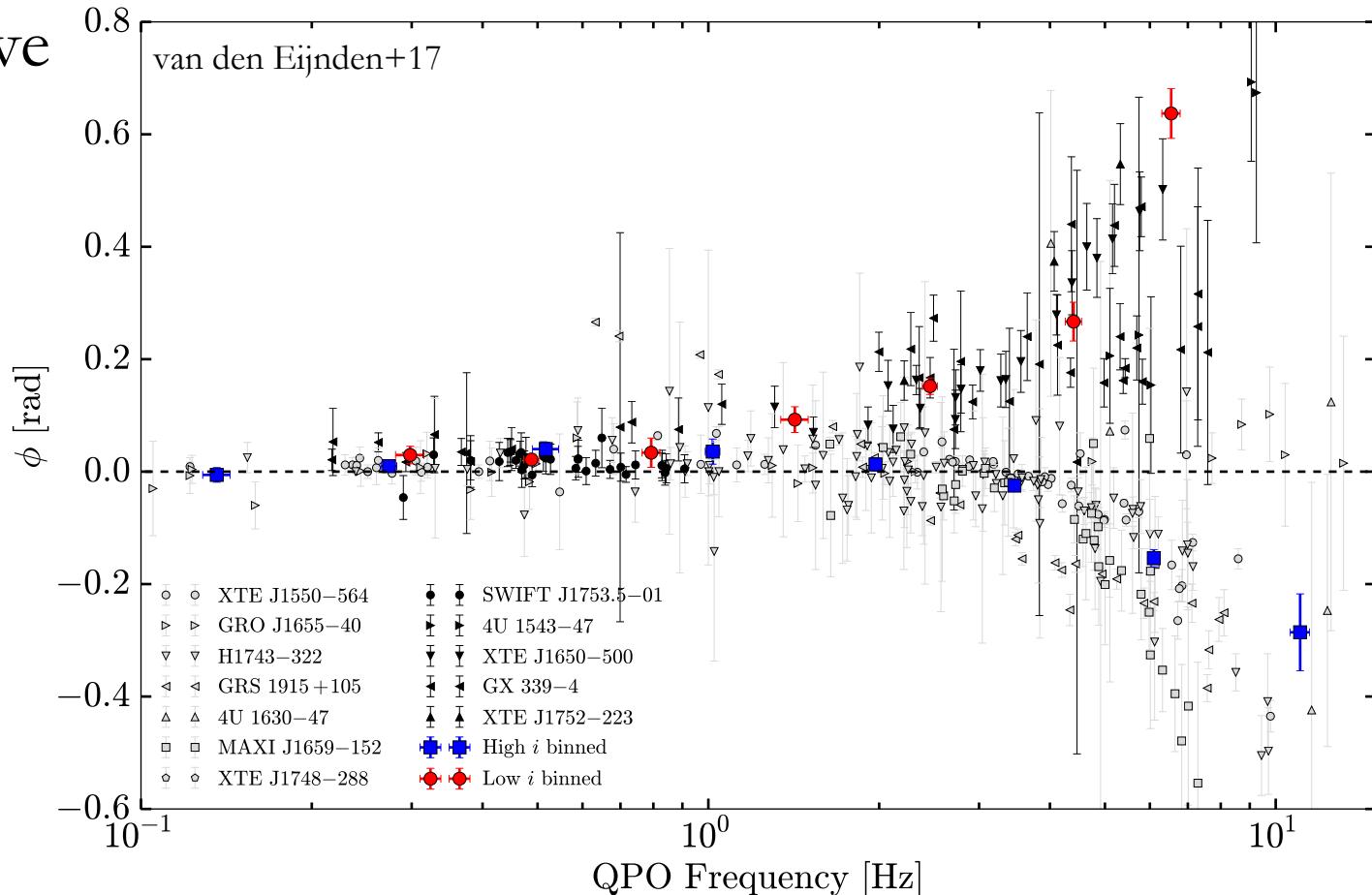
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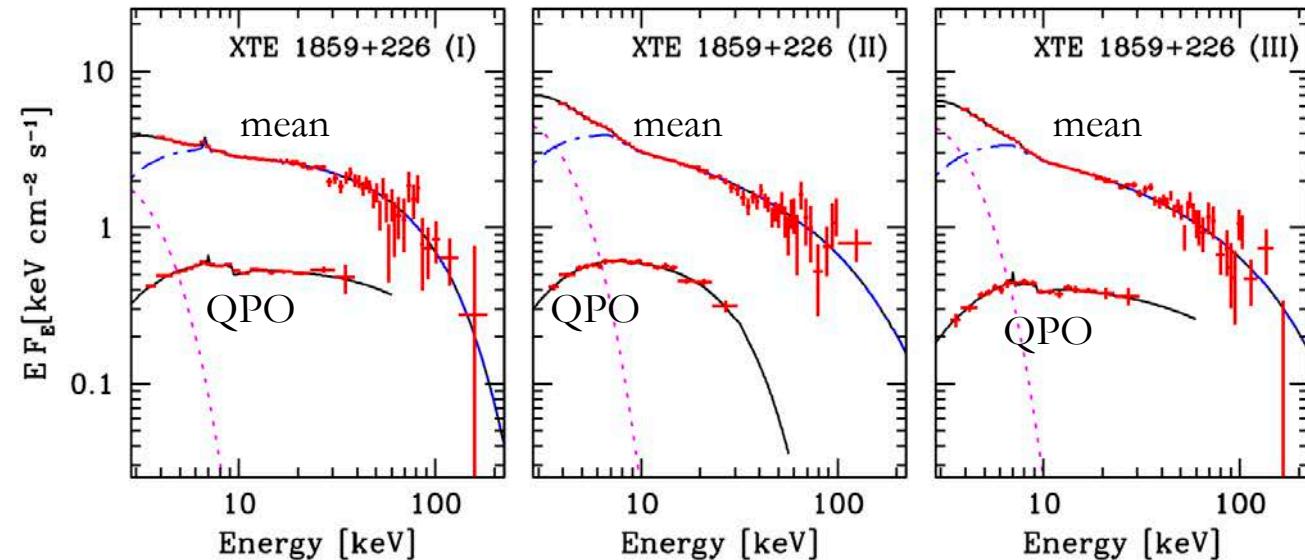
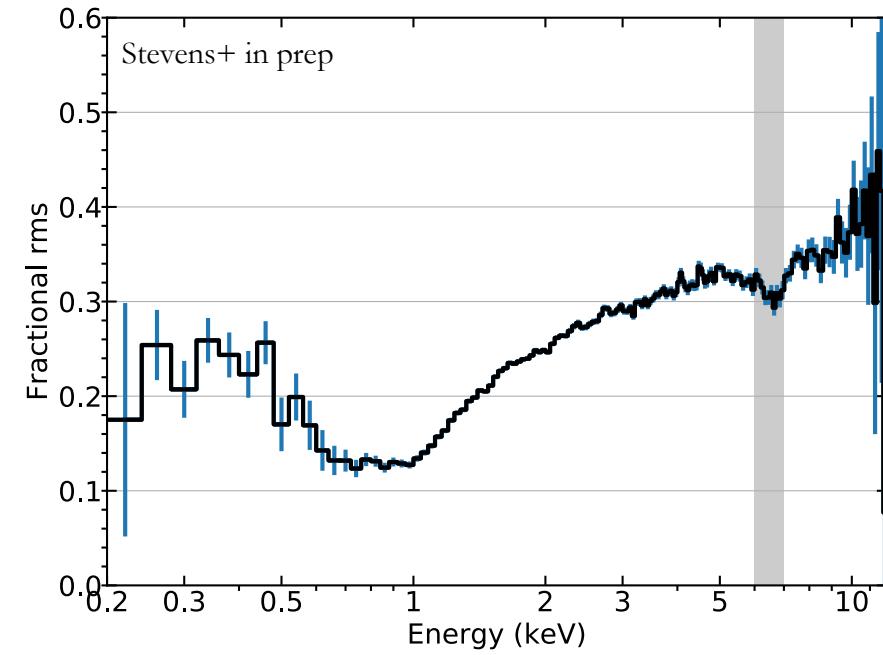
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# Fourier-resolved spectroscopy

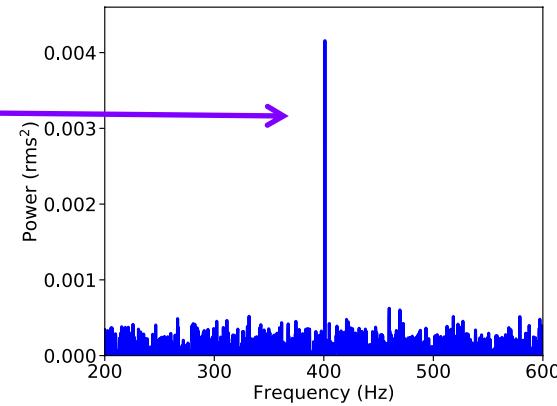
- Rms spectra: fractional variance of a light curve in narrow energy bands (Revnivtsev+99)
- Covariance spectra: fractional variance of a light curve with respect to a reference band (Wilkinson+Uttley 09)



# Phase-resolved spectroscopy

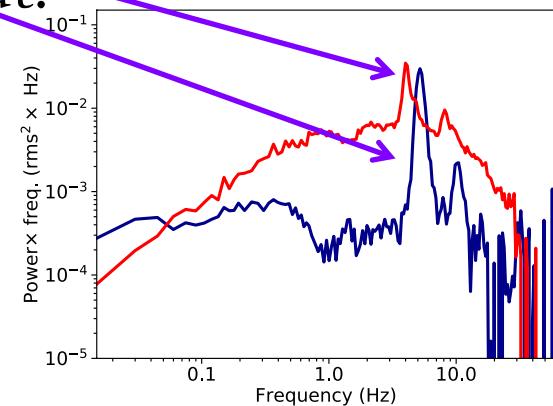
## Periodic signals:

- fold light curve at pulse period, stack signal in time domain
- need to know ephemeris of source

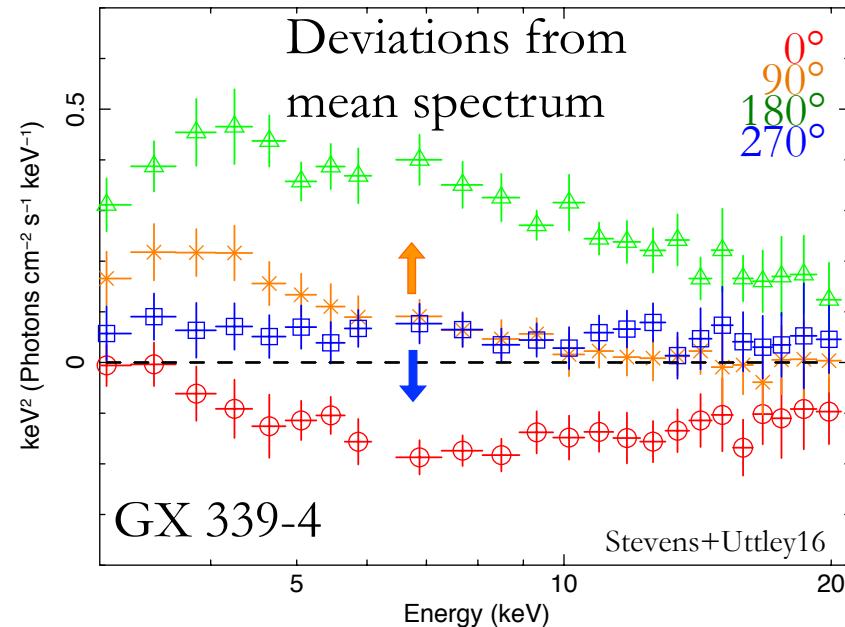
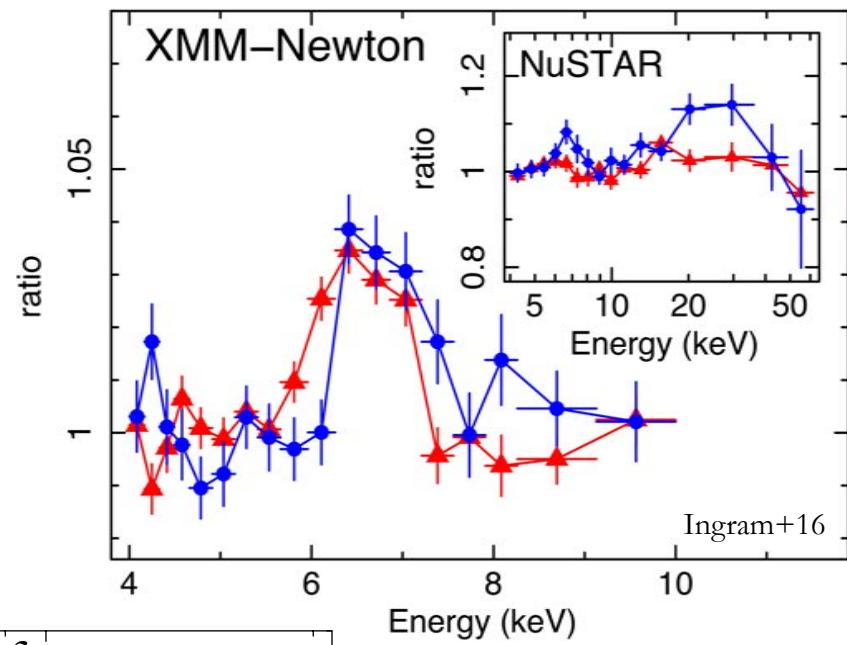
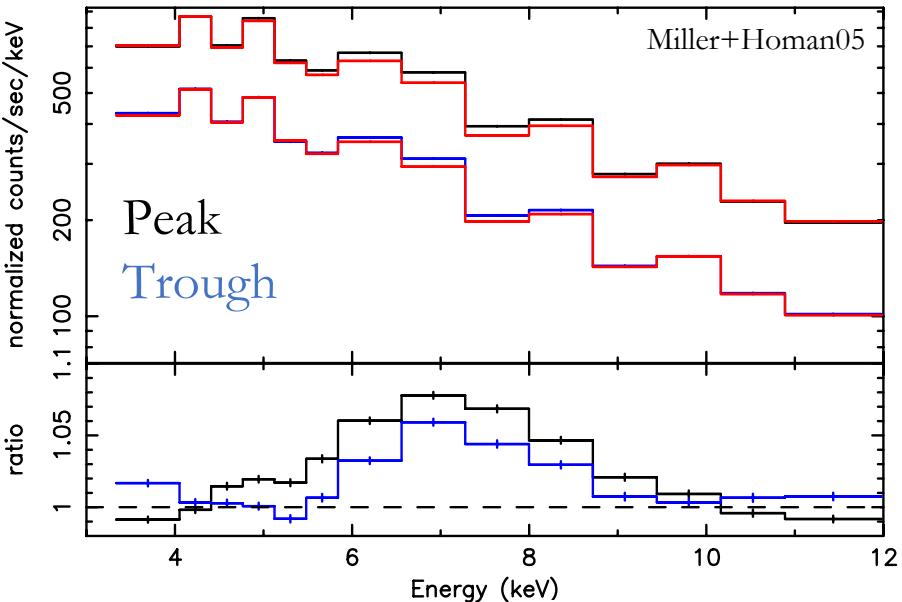


## Quasi-periodic signals:

- not coherent enough to fold light curve
- in time domain, signal would smear out!
- ➔ average together signals in frequency domain
- ephemeris not needed

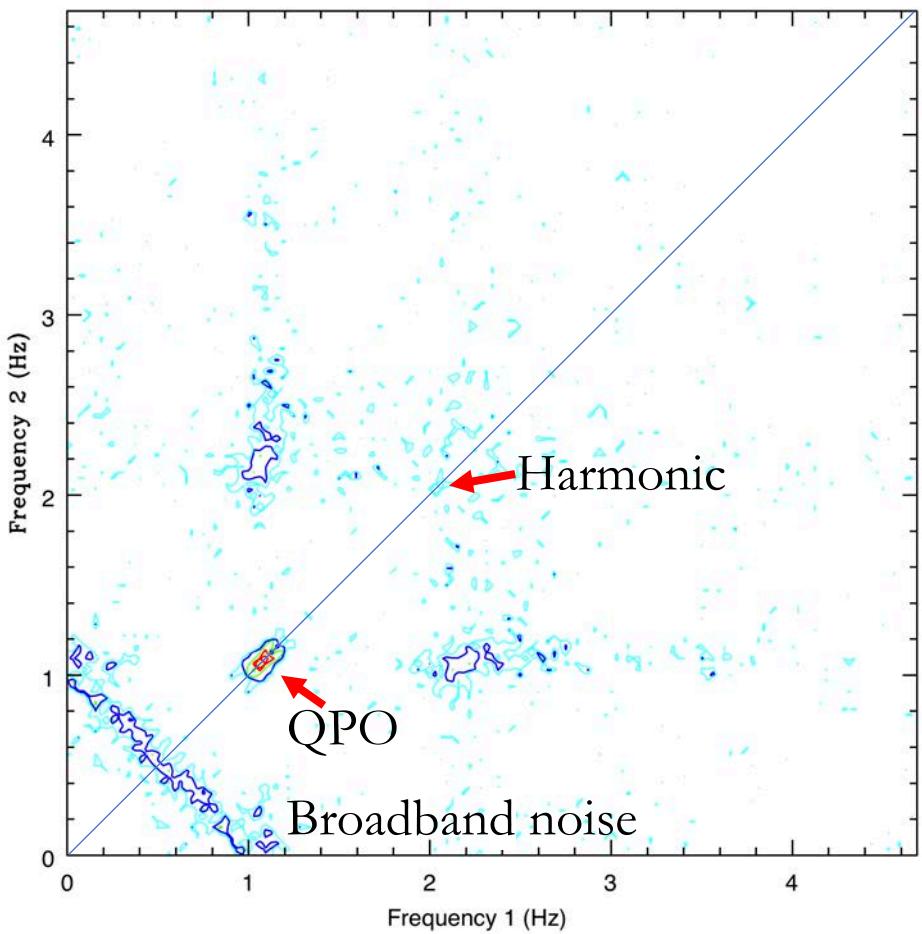


# Phase-resolved QPO spectroscopy



# Bispectrum (& biphase, bicoherence)

- Nonlinear (e.g., multiplicative) interactions between variability components (e.g., broadband noise and QPO)  
(see e.g. papers by T. Maccarone et al., S. Scaringi et al.)

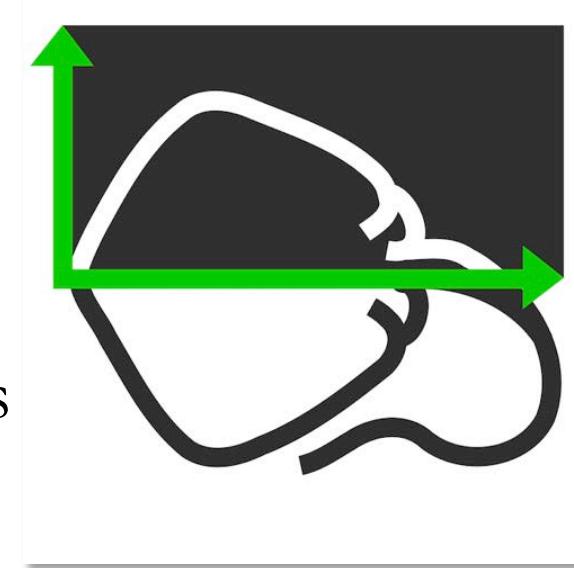


The bispectrum is defined for a frequency pair  $\nu_i$  and  $\nu_j$  as:

$$B(\nu_i, \nu_j) = F(\nu_i) F(\nu_j) F^*(\nu_{i+j})$$

# Stingray: spectral-timing software

- Open-source, community-driven and -developed, python, Astropy-affiliated package
- Stingray: Python library of analysis tools
- HENDRICS: shell scripting interface
- DAVE: graphical user interface
- Tutorials in Jupyter(/iPython) notebooks
- [github.com/StingraySoftware](https://github.com/StingraySoftware)

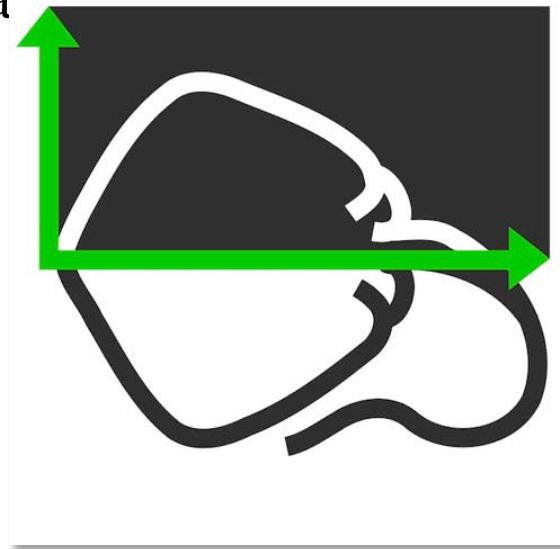


- Leads: D. Huppenkothen, M. Bachetti, A.L. Stevens, S. Migliari, P. Balm
  - Google Summer of Code students: S. Sharma ('18); O. Hammad and H. Rashid ('17); U. Khan, H. Mishra, and D. Sodhi ('16)
  - Other major contributors: E. Martinez Ribeiro, R. Valles
- Now published: Huppenkothen et al. 2019, ApJ, in press!



# Stingray: spectral-timing software

- Library of time series analysis methods
  - Power spectra, cross spectra, bispectra
  - Lag-frequency & lag-energy spectra
  - Rms & covariance spectra
  - Coherence, cross-correlation
  - Handles GTIs, pulsar & QPO searches
  - Phase-resolved spectroscopy of QPOs
- Simulator, modeling
- Well-tested on X-ray timing data (RXTE, NuSTAR, XMM, some NICER); also used by a few people for radio timing



# Summary and resources

- There are many types of periodic and quasi-periodic signals in evenly-spaced time-domain astronomy data
  - Compact object quasi-periodic oscillations (QPOs)
  - Pulsars and burst oscillations
  - Stellar pulsations and asteroseismology
- In-depth Fourier technique review: Uttley+2014
  - Power spectra, cross spectra
  - Coherence
  - Phase lags
  - Frequency- and phase-resolved spectroscopy
  - Bispectra
- [StingraySoftware.github.io](https://StingraySoftware.github.io)



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